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Frame Synchronization for M-ary Modulation with Phase Offsets

Ahmed Elzanaty*, Ksenia Koroleva+, Stanislav Gritsutenko+, and Marco Chiani*

* DEI/IEIIT University of Bologna, Bologna, Italy

+ IATIT, Omsk State Transport University, Omsk, Russia

***Abstract*—We study frame synchronization (FS) based on the transmission of known sequences (synchronization words) for M-PSK signals in the presence of additive white Gaussian noise and phase offset due to imperfect carrier phase estimation. In particular, we derive optimal and simple suboptimal metrics for noncoherent FS of M-PSK modulation with $M \geq 4$. We show that a simple ℓ_1-norm correction of the noncoherent correlation gives large improvements in terms of synchronization error probability. For example, more than 2 dB are gained with respect to usual correlation tests in terms of signal to noise ratio, assuming QPSK with a synchronization error probability 10^{-3}. Finally, we illustrate that the proposed technique is better than correlation based metric also for M-QAM systems, as well as in the presence of small frequency offsets.**

I. Introduction

Frame synchronization (FS) is a fundamental stage in most communication systems for achieving reliable radio links with low probability of error [1]–[18]. The FS mechanism considers finding the position of a known synchronization sequence, called here sync word (SW), which is inserted into a data stream composed of modulated symbols.

The optimal FS metric for binary transmission has been studied for binary symmetric channels (BSC) and additive white Gaussian noise (AWGN) channels in [1] and [2], respectively, where binary signalling with coherent demodulation and perfect carrier synchronization was assumed. The probability of correct frame synchronization for the metrics in [2] is analyzed in [12], where the concept of pairwise synchronization error probability (PSEP) is introduced. The derivation of the optimum metrics according to hypothesis testing theory and the acquisition time analysis is provided for variable (unknown) frame lengths in [9]–[11], where sequential FS is considered.

Nevertheless, in some systems FS is required to be performed prior to phase synchronization, referred throughout the paper as "noncoherent" FS. For this "noncoherent" FS setting, it has been shown that, assuming a phase offset and negligible frequency offset, the common noncoherent correlation detector is not the optimum one [13]. In the presence of large frequency offsets the situation is even more difficult. For example, in code division multiple access (CDMA) systems, suitable postdetection integration techniques, with combinations of successive partial noncoherent correlations, are employed to mitigate the effects of frequency offsets [19], [20]. Metrics for FS that are robust to frequency offsets have been provided for TDMA systems in [21], [22].

While the previous schemed are usually given for binary phase shift keying (BPSK), many modern systems use multilevel modulations. For example, in link adaptive schemes the number of levels in the modulation is chosen adaptively, depending on the channel characteristics and required throughput [23], [24]. Therefore, efficient FS is essential for such M-ary modulation based systems. Extensions of coherent FS of BPSK to multilevel modulation, frequency selective channels, and code-aided frame synchronization techniques are provided in several works [3]–[8].

On the other hand, less is known in the case of noncoherent FS for M-ary phase shift keying (M-PSK) modulated symbols, where the most common approach is to use a noncoherent correlator. The correlation is performed over a testing time equal to the duration of the SW if the phase offset is constant within that duration, while the performance significantly degrades when the phase varies considerably within the correlation time, due for example to the presence of frequency offset. Yet, optimal metrics for noncoherent FS of M-ary modulated symbols have not been fully addressed.

In this paper, we first derive an optimal metric for noncoherent FS of M-PSK modulations with $M \geq 4$, assuming a negligible frequency offset. We show that the optimal test requires numerical integration, which is not suitable for real-time implementations. Hence, suboptimal low-complexity metrics, i.e., accurate approximations of the optimal detector, are proposed for quadrature phase shift keying (QPSK) and 8-phase shift keying (8-PSK). Starting from the approach in [13], we also analyze for M-PSK a low-complexity metric consisting in the simple noncoherent correlator corrected by removing the ℓ_1-norm of the observed vector. The proposed metric shows a considerable performance improvement with respect to (w.r.t.) the (non-optimal) correlation based detector, also in the presence of small frequency offsets. Finally, we investigate the performance improvement of the new scheme when applied to M-ary quadrature amplitude modulation (M-QAM) systems.

II. Problem Statement

The frame structure is shown in Fig. 1 where a sync word composed of N_{SW} arbitrary symbols $(c_0, ..., c_{N_{\text{SW}}-1})$

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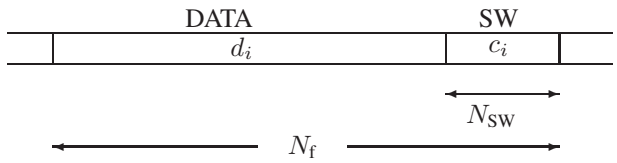

Fig. 1. Frame Structure.

is periodically inserted, with period N_f, in a random M-PSK data stream. The SW symbols c_i are completely arbitrary. The data stream is assumed to be composed of M-PSK symbols d_i which are independent, identically distributed (i.i.d.) and uniformly distributed over the set $\left\{e^{j0}, e^{j2\pi/M}, e^{j4\pi/M}, \ldots, e^{j(M-1)2\pi/M}\right\}$.

We assume matched filter (MF) and perfect clock synchronization with sampling period T seconds. The observation window is composed of the N_{f} samples $r_0, r_1, \ldots, r_{N_{\text{f}}-1}$. The SW is transmitted starting at position $m \in \{0, \cdots, N_{\text{f}}-1\}$. The received baseband complex samples, $r_i = r_i^{\text{I}} + j\, r_i^{\text{Q}}$, are

$$\begin{aligned} r_{i \bmod N_{\text{f}}} &= c_{i-m}\, e^{j\varphi} + n_{i \bmod N_{\text{f}}}, && i = m, \ldots, N_{\text{SW}}+m-1 \\ r_i &= d_i\, e^{j\varphi} + n_i, && \text{elsewhere.} \end{aligned}$$

where φ is a random variable (r.v.) uniformly distributed over $[-\pi, \pi)$ representing the carrier phase offset, assumed constant over the considered observation window, and n_i are the i.i.d. circularly symmetric complex Gaussian r.v.s, with zero mean and variance σ^2 per dimension, accounting for thermal noise. We normalize the constellation for the data so that the energy per symbol is unity, i.e., $E_s \triangleq \mathbb{E}\left\{|d_i|^2\right\} = 1$. Consequently, the signal-to-noise ratio (SNR) is $E_s/N_0 = 1/(2\sigma^2)$, where N_0 is the one-sided thermal noise power spectral density.

III. Testing Hypothesis for FS

In this section, we generalize the work in [13], which considered BPSK, to a broader class of multilevel modulation. More precisely, we derive optimal metrics for frame synchronization of M-PSK signals, $M \geq 4$. As described in [10], [12], [13], the metric for estimating if the SW position starts at position 0 (hypothesis $\mathcal{H}_1$) or not (hypothesis $\mathcal{H}_0$) is related to testing the following hypotheses:

$$\begin{aligned} \mathcal{H}_0 &: r_i = d_i\, e^{j\varphi} + n_i, && i = 0, \ldots, N_{\text{SW}}-1 \\ \mathcal{H}_1 &: r_i = c_i\, e^{j\varphi} + n_i, && i = 0, \ldots, N_{\text{SW}}-1\,. \end{aligned}$$

The common test used for synchronization is based on the so-called noncoherent correlation metric

$$\Lambda^{(\text{corr})}(\mathbf{r}) = \left| \sum_{i=0}^{N_{\text{SW}}-1} r_i^*\, c_i \right| \underset{\mathcal{D}_0}{\overset{\mathcal{D}_1}{\gtrless}} \lambda \tag{1}$$

where $\mathbf{r} = r_0, r_1 \ldots, r_{N_{\text{SW}}-1}$ and $\mathcal{D}_0, \mathcal{D}_1$, are respectively decisions for $\mathcal{H}_0, \mathcal{H}_1$. Since this test is widely adopted, we will use it as a benchmark for the comparison with the new proposed tests. However, we remark that, although commonly used, the test based on noncoherent correlation metric (1) is by no way the optimal one, as shown in [13] for binary modulation formats.

In order to derive the optimal test for M-PSK, we start from the general form of the likelihood ratio test (LRT)

$$\Lambda(\mathbf{r}) = \frac{f_{\mathbf{R}|\mathcal{H}_1}(\mathbf{r}|\mathcal{H}_1)}{f_{\mathbf{R}|\mathcal{H}_0}(\mathbf{r}|\mathcal{H}_0)} \underset{\mathcal{D}_0}{\overset{\mathcal{D}_1}{\gtrless}} \lambda \tag{2}$$

where $f_{\mathbf{R}|\mathcal{H}_l}(\mathbf{r}|\mathcal{H}_l)$ is the probability distribution function (p.d.f.) of the random vector $\mathbf{R} = (R_0, \ldots, R_{N_{\text{SW}}-1})$ in the hypothesis $\mathcal{H}_l$ and R_i is the r.v. of the received sample r_i [25]. We have now to specialize the general expression (2) to our noncoherent frame synchronization problem.

A. Case $\mathcal{H}_0$

Assuming the $\mathcal{H}_0$ hypothesis, when the data vector $\mathbf{d} = (d_0, \ldots, d_{N_{\text{SW}}-1})$ with phase offset φ is observed, we have the conditional p.d.f.

$$f_{\mathbf{R}|\mathcal{H}_0,\varphi,\mathbf{d}}(\mathbf{r}|\mathcal{H}_0, \varphi, \mathbf{d}) = \prod_{i=0}^{N_{\text{SW}}-1} \frac{1}{2\pi\sigma^2}\, e^{-\frac{\left|r_i - d_i\, e^{j\varphi}\right|^2}{2\sigma^2}}. \tag{3}$$

Taking the expectation with respect to the distribution of the M-PSK data symbols $\mathbf{d}$ and assuming M even, we have

$$\begin{aligned} &f_{\mathbf{R}|\mathcal{H}_0,\varphi}(\mathbf{r}|\mathcal{H}_0, \varphi) = \\ &\left(\frac{2}{M}\right)^{N_{\text{SW}}} K(\mathbf{r}) \prod_{i=0}^{N_{\text{SW}}-1} \sum_{p=0}^{M/2-1} \cosh \Re\left\{\tilde{r}_i e^{-jp2\pi/M} e^{-j\varphi}\right\} \end{aligned} \tag{4}$$

where $K(\mathbf{r}) = (2\pi\sigma^2)^{-N_{\text{SW}}} \prod_{i=0}^{N_{\text{SW}}-1} e^{-\frac{|r_i|^2+1}{2\sigma^2}}$.

To simplify the notation, we use the tilde to indicate normalization with respect to the noise variance, i.e., $\tilde{r}_i = r_i/\sigma^2$. Then, we evaluate $f_{\mathbf{R}|\mathcal{H}_0}(\mathbf{r}|\mathcal{H}_0)$ by averaging (4) with respect to φ, with different numerical approximation.

B. Case $\mathcal{H}_1$

For hypothesis $\mathcal{H}_1$, where we observe the known SW symbols c_i in additive Gaussian noise, we have the conditional p.d.f.

$$f_{\mathbf{R}|\mathcal{H}_1,\varphi}(\mathbf{r}|\mathcal{H}_1, \varphi) = \prod_{i=0}^{N_{\text{SW}}-1} \frac{1}{2\pi\sigma^2} e^{-\frac{\left|r_i - c_i\, e^{j\varphi}\right|^2}{2\sigma^2}}. \tag{5}$$

Averaging over φ, we obtain

$$f_{\mathbf{R}|\mathcal{H}_1}(\mathbf{r}|\mathcal{H}_1) = K(\mathbf{r})\, I_0\left(\left|\sum_{i=0}^{N_{\text{SW}}-1} \tilde{r}_i^*\, c_i\right|\right) \tag{6}$$

where $I_0(x) = (2\pi)^{-1} \int_{-\pi}^{\pi} \exp(x\cos\theta)\, d\theta$ is the zeroth-order modified Bessel function of first kind [26, (8.406)]. In order to use (2), we need (6) and the expected value of (4) with respect to φ. In the following sections we discuss different methods to integrate (4) over φ for QPSK and 8-PSK.

IV. Optimal and suboptimal tests for QPSK modulation

In this section we provide the optimal likelihood ratio test for the QPSK modulated signal. Putting $M = 4$ in (4) we get

$$f_{\mathbf{R}|\mathcal{H}_0,\varphi}(\mathbf{r}|\mathcal{H}_0,\varphi) = \left(\frac{1}{2}\right)^{N_{\text{sw}}} K(\mathbf{r}) \times \prod_{i=0}^{N_{\text{sw}}-1} \left(\cosh \Re\left\{\tilde{r}_i e^{-j\varphi}\right\} + \cosh \Re\left\{\tilde{r}_i e^{-j\pi/2} e^{-j\varphi}\right\}\right)$$
$$= \left(\frac{1}{2}\right)^{N_{\text{sw}}} K(\mathbf{r}) \prod_{i=0}^{N_{\text{sw}}-1} \left[\cosh\left(\tilde{r}_i^I \cos\varphi + \tilde{r}_i^Q \sin\varphi\right) + \cosh\left(\tilde{r}_i^I \sin\varphi - \tilde{r}_i^Q \cos\varphi\right)\right] \quad (7)$$

where $\tilde{r}_i = \tilde{r}_i^{\text{I}} + j\,\tilde{r}_i^{\text{Q}}$.

Averaging with respect to φ we get, after some manipulation, the metric for the (optimum) LRT in (8). Unfortunately, the LRT in the form (8) is not suitable for real time implementation as it requires numerical integration, thus we have to introduce some simplifications. At first, we approximate the denominator by applying both the quadrature rule with N_q points

$$\int_0^{\pi} f(\varphi) d\varphi \simeq \frac{\pi}{N_q} \sum_{l=0}^{N_q-1} f\left(l\frac{\pi}{N_q}\right) \quad (9)$$

and by considering that the hyperbolic cosine has exponential growth taking into account only the maximum contribution. Then, the corresponding approximate log-likelihood ratio (LLR) can be written as

$$\ln\Lambda^{(1)}(\mathbf{r}) \triangleq \ln I_0\left(\left|\sum_{i=0}^{N_{\text{sw}}-1} \tilde{r}_i^* c_i\right|\right) - \max_l \left[\sum_{i=0}^{N_{\text{sw}}-1} \ln\left(\cosh\frac{(\tilde{r}_i^I - \tilde{r}_i^Q)\cos\varphi_l + (\tilde{r}_i^I + \tilde{r}_i^Q)\sin\varphi_l}{2} \times \cosh\frac{(\tilde{r}_i^I + \tilde{r}_i^Q)\cos\varphi_l + (\tilde{r}_i^Q - \tilde{r}_i^I)\sin\varphi_l}{2}\right)\right] \quad (10)$$

where $\varphi_l = l\pi/N_q$.

Regarding the numerator of (8), $\ln\Lambda^{(1)}(\mathbf{r})$ can be approximated for high SNR as

$$\ln\Lambda^{(2)}(\mathbf{r}) \triangleq \left|\sum_{i=0}^{N_{\text{sw}}-1} \tilde{r}_i^* c_i\right| - \max_l \left[\sum_{i=0}^{N_{\text{sw}}-1} \left|\frac{(\tilde{r}_i^I - \tilde{r}_i^Q)\cos\varphi_l + (\tilde{r}_i^I + \tilde{r}_i^Q)\sin\varphi_l}{2}\right| + \sum_{i=0}^{N_{\text{sw}}-1} \left|\frac{(\tilde{r}_i^I + \tilde{r}_i^Q)\cos\varphi_l + (\tilde{r}_i^Q - \tilde{r}_i^I)\sin\varphi_l}{2}\right|\right] \quad (11)$$

where we used the approximations $\ln I_0(x) \simeq |x| - \ln\sqrt{2\pi|x|} \simeq |x|$ and $\ln\cosh(x) \simeq |x| - \ln 2$, valid for $|x| \gg 1$.

In fact, (11) can be interpreted as a modification of the noncoherent correlator with an additive correction term. The approximation accuracy of the numerical integration in (8) increases with N_q. For $N_q = 2$ the metrics given in (10) and (11) specialize to:

$$\ln\Lambda^{(1)}(\mathbf{r}) \triangleq \ln I_0\left(\left|\sum_{i=0}^{N_{SW}-1} \tilde{r}_i^* c_i\right|\right) - \max\left[\sum_{i=0}^{N_{\text{sw}}-1} \ln\left(\cosh\frac{\tilde{r}_i^I + \tilde{r}_i^Q}{2\sigma^2}\right), \sum_{i=0}^{N_{\text{sw}}-1} \ln\left(\cosh\frac{\tilde{r}_i^I - \tilde{r}_i^Q}{2\sigma^2}\right)\right] \quad (12)$$

$$\ln\Lambda^{(2)}(\mathbf{r}) \triangleq \left|\sum_{i=0}^{N_{\text{sw}}-1} r_i^* c_i\right| - \max\left[\sum_{i=0}^{N_{\text{sw}}-1} \left|\frac{\tilde{r}_i^I + \tilde{r}_i^Q}{2}\right|, \sum_{i=0}^{N_{\text{sw}}-1} \left|\frac{\tilde{r}_i^I - \tilde{r}_i^Q}{2}\right|\right]. \quad (13)$$

For $N_q = 4$ the metric for QPSK in (11) becomes

$$\ln\Lambda^{(2)}(\mathbf{r}) \triangleq -\max\left[\sum_{i=0}^{N_{\text{sw}}-1} \left|\frac{\tilde{r}_i^I + \tilde{r}_i^Q}{2}\right|, \sum_{i=0}^{N_{\text{sw}}-1} \left|\frac{\tilde{r}_i^I - \tilde{r}_i^Q}{2}\right|, \sum_{i=0}^{N_{\text{sw}}-1} \left|\frac{\sqrt{2}|\tilde{r}_i^I|}{2}\right|, \sum_{i=0}^{N_{\text{sw}}-1} \left|\frac{\sqrt{2}|\tilde{r}_i^Q|}{2}\right|\right] + \left|\sum_{i=0}^{N_{\text{sw}}-1} r_i^* c_i\right|. \quad (14)$$

In Section VII we numerically investigate the synchronization error probability as a function of the signal to noise ratio for the tests based on these metrics.

V. Optimal and Suboptimal tests for 8-PSK modulation

For 8-PSK signals the transmitted symbols are $d_i = \exp\{j\pi m/4\}$, $m \in \{0, 1, ..., 7\}$, and (4) becomes:

$$f_{\mathbf{R}|\mathcal{H}_0,\varphi}(\mathbf{r}|\mathcal{H}_0,\varphi) = \left(\frac{1}{2}\right)^{N_{\text{sw}}} K(\mathbf{r}) \prod_{i=0}^{N_{\text{sw}}-1} \Big(\cosh \Re\left\{\tilde{r}_i e^{-j\varphi}\right\} + \cosh \Re\left\{\tilde{r}_i e^{-j\pi/4} e^{-j\varphi}\right\} + \cosh \Re\left\{\tilde{r}_i e^{-j\pi/2} e^{-j\varphi}\right\} + \cosh \Re\left\{\tilde{r}_i e^{-j3\pi/4} e^{-j\varphi}\right\}\Big). \quad (15)$$

Taking into account that φ is uniformly distributed over the interval $[-\pi, \pi)$, we obtain from (15)

$$f_{\mathbf{R}|\mathcal{H}_0,\varphi}(\mathbf{r}|\mathcal{H}_0) = \frac{K(\mathbf{r})}{\pi} \int_0^{\pi} \prod_{i=0}^{N_{\text{sw}}-1} \Bigg[\cosh\left(\tilde{r}_i^I \cos\varphi + \tilde{r}_i^Q \sin\varphi\right) + \cosh\left(\frac{\sqrt{2}}{2}\left((\tilde{r}_i^I + \tilde{r}_i^Q)\cos\varphi + (\tilde{r}_i^Q - \tilde{r}_i^I)\sin\varphi\right)\right) + \cosh\left(\frac{\sqrt{2}}{2}\left((\tilde{r}_i^I + \tilde{r}_i^Q)\cos\varphi + (\tilde{r}_i^I - \tilde{r}_i^Q)\sin\varphi\right)\right) + \cosh\left(-\tilde{r}_i^I \sin\varphi + \tilde{r}_i^Q \cos\varphi\right)\Bigg] d\varphi. \quad (16)$$

$$\Lambda(\mathbf{r}) \triangleq \frac{I_0\left(\left|\sum_{i=0}^{N_{\rm SW}-1} \tilde{r}_i^* c_i\right|\right)}{\sum_{l=0}^{N_q-1}\prod_{i=0}^{N_{\rm SW}-1} \cosh\frac{(\tilde{r}_i^I-\tilde{r}_i^Q)\cos\varphi_l+(\tilde{r}_i^I+\tilde{r}_i^Q)\sin\varphi_l}{2}\cdot\cosh\frac{(\tilde{r}_i^I+\tilde{r}_i^Q)\cos\varphi_l+(\tilde{r}_i^Q-\tilde{r}_i^I)\sin\varphi_l}{2}} \tag{8}$$

The resulting exact metric (i.e., the LRT for 8-PSK modulation) is reported in (17), where we put

$$\tilde{r}_{i,\varphi}^S = \tilde{r}_i^I\cos\varphi + \tilde{r}_i^Q\sin\varphi \quad \tilde{r}_{i,\varphi}^D = \tilde{r}_i^Q\cos\varphi - \tilde{r}_i^I\sin\varphi$$
$$\tilde{r}_{i,\varphi}^{I+Q} = \left(\tilde{r}_i^I+\tilde{r}_i^Q\right)\cos\varphi \quad \tilde{r}_{i,\varphi}^{I-Q} = \left(\tilde{r}_i^I-\tilde{r}_i^Q\right)\cos\varphi\,.$$

To simplify (17), we again use the rectangular quadrature rule and the approximations. More precisely, the LLR can be approximated for large SNR as

$$\begin{aligned}\ln\Lambda^{(1)}(\mathbf{r}) \triangleq -\max_l \Bigg[&\sum_{i=0}^{N_{\rm SW}-1}\ln\left(\cosh\tilde{r}_i^I\cos\varphi_l+\tilde{r}_i^Q\sin\varphi_l\right)\\ &+\cosh\left(\frac{\sqrt{2}}{2}\left((\tilde{r}_i^I+\tilde{r}_i^Q)\cos\varphi_l+(\tilde{r}_i^Q-\tilde{r}_i^I)\sin\varphi_l\right)\right)\\ &+\cosh\left(\frac{\sqrt{2}}{2}\left((\tilde{r}_i^I+\tilde{r}_i^Q)\cos\varphi_l+(\tilde{r}_i^I-\tilde{r}_i^Q)\sin\varphi_l\right)\right)\\ &+\cosh\left(-\tilde{r}_i^I\sin\varphi_l+\tilde{r}_i^Q\cos\varphi_l\right)\Bigg]+\left|\sum_{i=0}^{N_{\rm SW}-1}\tilde{r}_i^* c_i\right|.\end{aligned} \tag{18}$$

For example, the formula based on the rectangular quadrature approximation with $N_q = 2$ is

$$\begin{aligned}\ln\Lambda^{(2)}(\mathbf{r}) \triangleq &\left|\sum_{i=0}^{N_{\rm SW}-1}\tilde{r}_i^* c_i\right| -\\ &\max\Bigg[\sum_{i=0}^{N_{\rm SW}-1}\ln\left(\cosh\tilde{r}_i^I+\cosh\tilde{r}_i^Q+2\cosh\frac{\tilde{r}_i^I+\tilde{r}_i^Q}{\sqrt{2}}\right),\\ &\sum_{i=0}^{N_{\rm SW}-1}\ln\left(\cosh\tilde{r}_i^I+\cosh\tilde{r}_i^Q+2\cosh\frac{\tilde{r}_i^I-\tilde{r}_i^Q}{\sqrt{2}}\right)\Bigg]\end{aligned} \tag{19}$$

and for $N_q = 4$ is

$$\begin{aligned}\ln\Lambda^{(2)}(\mathbf{r}) \triangleq &\left|\sum_{i=0}^{N_{\rm SW}-1}\tilde{r}_i^* c_i\right|\\ &-\max\Bigg[\sum_{i=0}^{N_{\rm SW}-1}\ln\left(\cosh\tilde{r}_i^I+\cosh\tilde{r}_i^Q+2\cosh\frac{\tilde{r}_i^I+\tilde{r}_i^Q}{\sqrt{2}}\right),\\ &\sum_{i=0}^{N_{\rm SW}-1}\ln\left(\cosh\frac{\tilde{r}_i^I-\tilde{r}_i^Q}{\sqrt{2}}+\cosh\frac{\tilde{r}_i^I+\tilde{r}_i^Q}{\sqrt{2}}+\cosh\tilde{r}_i^I+\cosh\tilde{r}_i^Q\right),\\ &\sum_{i=0}^{N_{\rm SW}-1}\ln\left(\cosh\tilde{r}_i^I+\cosh\tilde{r}_i^Q+2\cosh\frac{\tilde{r}_i^I-\tilde{r}_i^Q}{\sqrt{2}}\right)\Bigg].\end{aligned} \tag{20}$$

VI. A TEST BASED ON A UNIFORM PHASE APPROXIMATION FOR THE DATA SYMBOLS

The previous tests are based on the LRT and specialized to particular data formats. Alternatively, in this section we propose a slightly different approach, where we approximate the phase distribution of the data symbols with a uniform random variable. More precisely, since we are dealing with $M \geq 4$, we approximate the data symbols as $d_i = e^{j\theta_i}$ where we treat $\theta_0, \theta_1, ..., \theta_{N_{\rm SW}-1}$ as i.i.d. continuous r.v.s uniformly distributed over $[-\pi, \pi)$. In this way, (4) becomes

$$\begin{aligned}&f_{\mathbf{R}|\mathcal{H}_0,\varphi}(\mathbf{r}|\mathcal{H}_0,\varphi)\\ &\simeq K(\mathbf{r})\prod_{i=0}^{N_{\rm SW}-1}\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{-|\tilde{r}_i|\cos(\arg c_i-\varphi-\theta_i)}d\theta_i\\ &= K(\mathbf{r})\prod_{i=0}^{N_{\rm SW}-1}I_0(|\tilde{r}_i|).\end{aligned} \tag{21}$$

Now, substituting (21) and (6) into (2) we obtain the metric[1]

$$\ln\Lambda^{(3)}(\mathbf{r}) \triangleq \ln I_0\left(\left|\sum_{i=0}^{N_{\rm SW}-1}\tilde{r}_i^* c_i\right|\right) - \sum_{i=0}^{N_{\rm SW}-1}\ln I_0\left(|\tilde{r}_i^*|\right). \tag{22}$$

Thus, for large SNR the metric becomes simply

$$\ln\Lambda^{(4)}(\mathbf{r}) \triangleq \left|\sum_{i=0}^{N_{\rm SW}-1} r_i^*\, c_i\right| - \sum_{i=0}^{N_{\rm SW}-1}|r_i| \tag{23}$$

which coincides with [13, (25)]. This metric is thus the noncoherent correlation corrected by removing the ℓ_1-norm of the observed sampled vector, which accounts for the non-gaussianity of the data symbols.

For M-PSK with $M \geq 4$, we found that (23) gives performance very close to the optimal metric obtained in the previous sections. In the next section we also show that the same test is better than noncoherent correlation for M-QAM modulations.

VII. NUMERICAL RESULTS

In this section we investigate the performance of the proposed methods used for frame synchronization by adopting Monte Carlo simulations. All results shown in the following have been obtained by counting at least 100 synchronization errors.

Regarding FS by Peak Detection for QPSK signals, we assume that the received samples are due to randomly generated QPSK symbols ($M = 4$) with a constant phase offset over the frame, in the presence of additive complex Gaussian noise. We consider the algorithms described in Section IV: for each observation window of $N_{\rm f}$ samples, the detector analyzes all N_{SW}-length sequences, estimating the SW position as that

[1]Note that there is no more dependency on φ, so (21) is an approximation of $f_{\mathbf{R}|\mathcal{H}_0}(\mathbf{r}|\mathcal{H}_0)$.

$$\Lambda(\mathbf{r}) \triangleq \frac{I_0\left(\left|\sum_{i=0}^{N_{\mathrm{SW}}-1} \tilde{r}_i^* c_i\right|\right)}{\int_0^{\pi} \prod_{i=0}^{N_{\mathrm{SW}}-1} \left(\cosh\left(\tilde{r}_{i,\varphi}^S\right) + \cosh\left(\tilde{r}_{i,\varphi}^D\right) + \cosh\left(\frac{\sqrt{2}}{2}(\tilde{r}_{i,\varphi}^S + \tilde{r}_{i,\varphi}^D)\right) + \cosh\left(\frac{\sqrt{2}}{2}(\tilde{r}_{i,\varphi}^{I+Q} + \tilde{r}_{i,\varphi}^{I-Q})\right)\right) d\varphi} \tag{17}$$

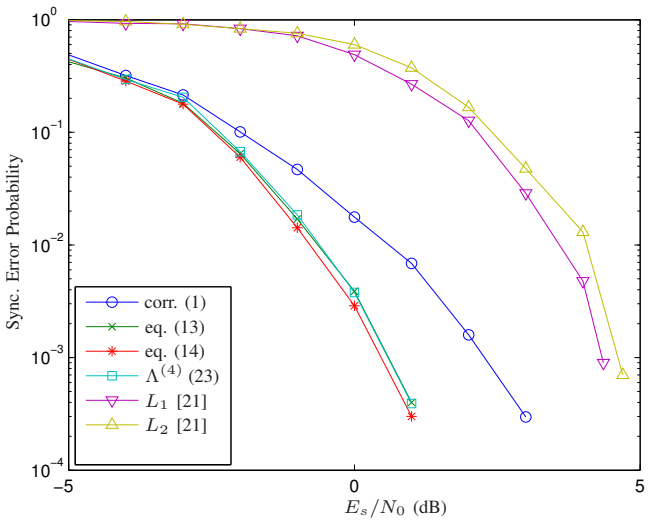

Fig. 2. Frame synchronization error probability for peak detectors, QPSK over AWGN channels with phase offset. Comparison between the noncoherent correlation metric, and the metrics from [21]. Frame composed of $N_{\mathrm{f}} = 240$ QPSK symbols, sync word of $N_{\mathrm{SW}} = 24$ QPSK symbols.

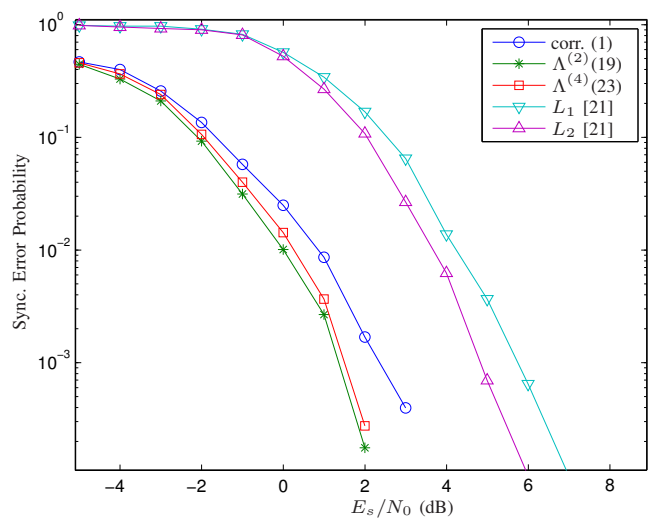

Fig. 3. Frame synchronization error probability for peak detectors, 8-PSK over AWGN channels with phase and frequency offsets. Comparison between the noncoherent correlation metric and the new metrics and the metrics from [21]. Frame composed of $N_{\mathrm{f}} = 1000$ 8-PSK symbols, sync word of $N_{\mathrm{SW}} = 32$ 8-PSK symbols, frequency offset uniformly distributed over the interval $[-\Delta f_{\max}T : \Delta f_{\max}T] = [-0.01; 0.01]$.

maximizing the metric. In particular, our metrics reported in (13) and (14) (high complexity), and (23) (low complexity) are compared with the common noncoherent correlation (1), and with the complex metrics L_1 and L_2 derived in [21].

Fig. 2 shows the values of the synchronization error probability for the derived metrics, as a function of the SNR, obtained with $N_f = 240$ and $N_{SW} = 24$. The metrics L_1 and L_2 have poor performance in this case, as they are designed to be robust w.r.t. large frequency offsets. On the contrary, the proposed metrics, which are designed for phase offsets but negligible frequency offsets, show large improvements with respect to the noncoherent correlation metric. For example, for a target synchronization error probability of 10^{-3}, our tests require 2 dB less than the noncoherent correlation test. As shown, the performance for the metrics (13) and (14) are very close, and slightly outperforming that obtained by using (23). The metric (23) seems therefore the most appealing due to its simplicity.

We now investigate the performance of the proposed synchronization metrics for 8-PSK modulation. Fig. 3 compares the synchronization error probability for the common noncoherent correlation (1), metrics L_1 and L_2 [21], and our proposed metrics, i.e., (19) as a high complexity test and (23) as the simplest one.[2] The performance is evaluated for various values of SNRs with $N_f = 1000$, $N_{SW} = 32$, and a maximum frequency offset $\Delta f_{\max}T = 0.01$. More precisely, we assume that the phase φ in Section II is replaced by $\varphi_0 + 2\pi\Delta fTi$, where the r.v.s φ_0 and ΔfT are uniformly distributed over $(-\pi,\pi]$ and $[-\Delta f_{\max}T, \Delta f_{\max}T]$, respectively. Again, the proposed metrics perform better than the others.

For example, for a target synchronization error probability of 10^{-3}, our tests require at least 2 dB less than the noncoherent correlation test, and at least 4 dB less than the tests designed for large frequency offsets [21]. We remark, however, that the metrics from [21] are expected to be more robust w.r.t. large frequency offsets.

Considering FS for M-QAM, the exact approach here used for M-PSK could be in theory extended to M-QAM data symbols. However, dealing with an alphabet of symbols with different phases and amplitudes would lead to complicated expressions of limited practical interest. Thus, it would be interesting to see if, e.g., the simple metric (23), which was derived assuming data symbols with constant amplitude and uniformly distributed phase, could give for M-QAM some

[2]Note that the implementation of the tests (19), L_1, and L_2 could be quite demanding.

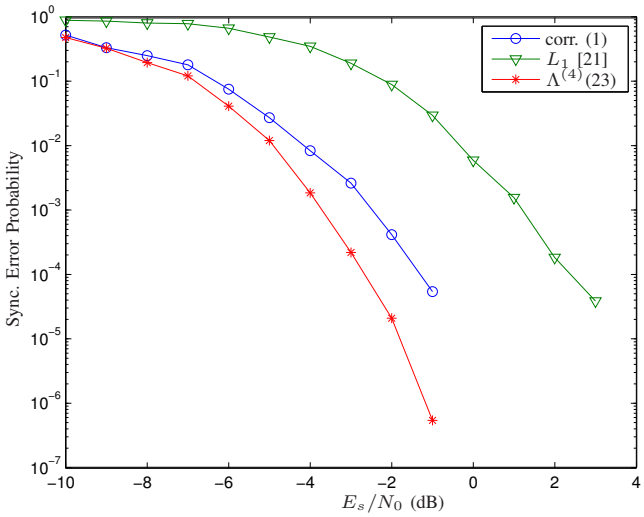

Fig. 4. Frame synchronization error probability for peak detectors, 16-QAM over AWGN channels with phase offset. Comparison between the noncoherent correlation metric, the new metric and the metrics from [21], labels as in previous figure. Frame composed of $N_{\rm f} = 240$ 16-QAM symbols, sync word of $N_{\rm SW} = 24$ 16-QAM symbols.

improvements with respect to the noncoherent correlator.

In this regard, fig. 4 shows some simulation results assuming a $16-$QAM constellation for the proposed test (23), the common noncoherent correlation rule (1), and L_2 from [21, eq. (11)], without frequency offsets. Again, we see that (23) outperforms the other approaches: for example, for a synchronization error probability of 10^{-4} we save more than 1.5 dB in terms of SNR.

VIII. Conclusion

We studied frame synchronization for M-PSK modulation in AWGN channels with phase offset, deriving the optimal synchronization rule and some suboptimal rules based on hypothesis testing theory. We verified numerically that our tests outperform the commonly used noncoherent correlation detector. Finally, we can conclude that the simple metric (23) (i.e., subtracting an ℓ_1-norm correction term from the noncoherent correlation) can be used in practice to replace the noncoherent correlation for a wide class of modulation formats, i.e., M-PSK, and M-QAM, in the presence of phase and small frequency offsets due to imperfect carrier recovery.

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