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Efficient Compression of Noisy Sparse Sources Based on Syndrome Encoding

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***Abstract*—Signal compression is essential for energy and bandwidth efficient communication and storage systems. In this paper, we provide two practical approaches for source compression of noisy sparse and non-strictly sparse (compressible) sources. The proposed schemes are based on channel coding theory to construct a source encoder that decreases the number of transmitted bits while preserving the fidelity of the reconstructed signal at the receiver by exploiting its sparsity. In addition, a model order selection scheme is proposed to detect the non-zero elements of sparse vectors embedded in noise, or to find a nonlinear sparse approximation of compressible signals. As illustrated by numerical results, our approach provides a lower distortion-rate function compared to previously known methods. For example, the proposed schemes achieve a lower distortion, about 2 orders of magnitude, compared to compressed sensing, for the same rate.**

I. Introduction

The classic problem of designing efficient lossy compression schemes for sources is gaining increasing interest, supported by the tremendous increase of data generated by the Internet of things (IoT). The data size can be significantly decreased by exploiting some of their structures. One of these structures is the sparsity/compressibility, i.e., the ability to describe/approximate signals with a fewer number of coefficients compared to their dimension in some domains, e.g., time, frequency, discrete cosine transform (DCT), and Wavelet. Most of the signals of interest such as image, audio, video, and IoT data are compressible [1], [2]. Hence, the growing challenge is to represent a compressible signal with a minimum number of bits while limiting the distortion due to quantization and sparse approximation.

Let us start by considering a signal, $\mathbf{x} \in \mathbb{R}^N$, emitted by a discrete-time continuous-valued source, the target is to encode it at the minimum rate which still guarantees that the distortion does not exceed a predefined limit, provided that at most k_0 elements of $\mathbf{x}$ are non-zero, i.e., $\mathbf{x}$ is a sparse vector with sparsity order k_0. The first intuitive approach, which we name address coding (AC), is to separately quantize the k_0 non-zero components using a uniform scalar quantizer with b bits/sample, then encode each of their locations with a fixed number of bits $\lceil \log_2 N \rceil$. The total number of required bits is calculated as

$$R_t = k_0 \lceil \log_2 N \rceil + k_0 \, b. \tag{1}$$

This approach is simple, but it requires the transmission of both the values and the locations separately, and also there exist more efficient approaches, in terms of compression gain.

Another approach is based on the well-known signal acquisition technique called compressed sensing (CS), where one can collect $M < N$ linear observations from $\mathbf{x}$, $\mathbf{y} = \mathbf{A}\,\mathbf{x}$, through an $M \times N$ measurement matrix $\mathbf{A}$. The necessary condition on the number of measurements needed to guarantee unique vector representation is $M \geq 2\,k_0$. Then, $\mathbf{x}$ can be reconstructed from $\mathbf{y}$ by solving an ℓ_0-minimization program. However, solving the ℓ_0 program is not feasible for the ranges of N usually used in practice. It is proved in [3], [4] that the solution provided by the ℓ_1-minimization is the same as that of ℓ_0, alleviating the computational burden of signal recovery, at the expense of taking more measurements. In particular, considering Gaussian measurement matrices, perfect recovery is guaranteed with high probability, for $m \geq C\,k_0 \log(N/k_0)$ [3]–[6]. The most important advantage of CS is that it does not require the knowledge about the basis at which the signal is sparse at the encoder, but only at the decoder. Moreover, it has been shown to be stable with respect to compressible (non-strictly sparse) signals. On the other hand, the number of measurements is still considerably higher than the signal sparsity and there is no practical scheme to accurately multiply the random measurement matrix with the signal in the analog domain, except for Rademacher and Bernoulli matrices [7].

In this paper, we provide two practical approaches for lossy source compression of noisy sparse and compressible sources. At first, we derive a blind estimator for the sparsity order based on a model order selection rule to detect the non-zero elements of sparse signals embedded in noise, or to obtain a nonlinear sparse approximation for compressible signals. Unlike, the conventional method of using a threshold to differentiate signal entries from noise, requiring perfect knowledge of noise statistics. Then, we propose two novel schemes based on exploiting the syndromes associated with channel block codes, i.e., Reed-Solomon (RS) and Bose, Chaudhuri, and Hocquenghem (BCH) codes, as lossless source encoders of quantized sparse signals.[1] As confirmed by numerical results, the proposed approaches achieve a better distortion-rate

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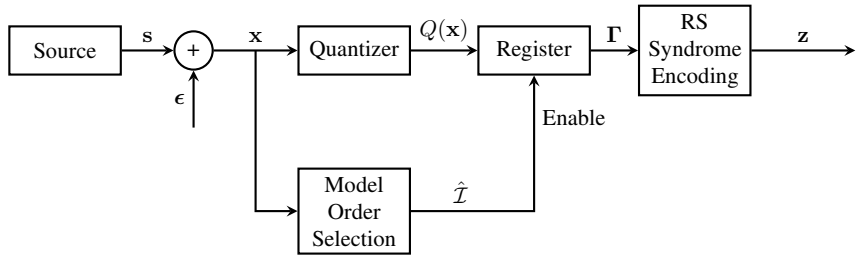
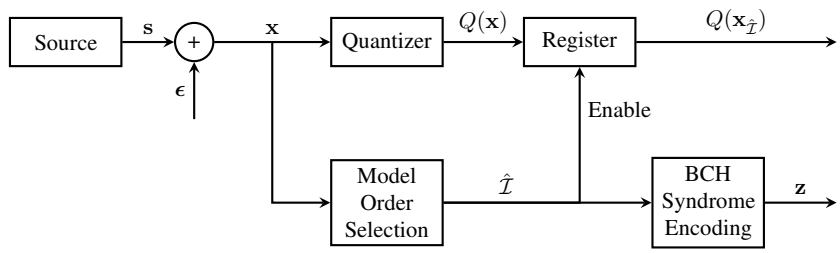
[1] Source encoders based on channel coding have been used for the different purpose of approaching the Slepian-Wolf/Wyner-Ziv bounds for the problems of lossless/lossy distributed source coding with side information only at the receiver [8], [9], and also for the lossy compression of binary symmetric sources [10].



(a) RS based source encoder.



(b) BCH based source encoder.

Fig. 1. The block diagram of the proposed compression schemes for noisy sparse sources.

performance compared to CS based source encoder.

Throughout this paper, we indicate with $\|\cdot\|_0$ the ℓ_0 quasi-norm of a vector indicating the number of its non-zero entries, with $\mathcal{N}(\boldsymbol{\mu}, \mathbf{C})$ the multivariate Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{C}$, with $\mathbf{I}_N$ the N-dimensional identity matrix, and with $\mathbb{F}_q$ the Galois field of order q.

II. Signal Model

Let us consider a sparse source model, which fits images, sounds, medical data, and sensor signals in appropriate transform domains, called sparse-land Gaussian model [11], [12]. In this model the source emits a sparse vector $\mathbf{s} \in \mathbb{R}^N$ with sparsity order $\|\mathbf{s}\|_0 = k_0 \ll N$. The signal support is chosen at random from all possible $\binom{N}{k_0}$ cardinalities. A variant to this model considers the sparsity order, K_0, as a random variable (r.v.) uniformly distributed within the set $\{0, 1, \cdots, k_{\max}\}$. Clearly, the entropy associated to the location of the non-zero elements of $\mathbf{s}$ is

$$\mathcal{H} = \log_2 \sum_{i=0}^{k_{\max}} \binom{N}{i} \tag{2}$$

which provides a lower bound on the number of bits needed to encode the signal support [13]. Let us further assume that the values of the non-zero elements, for both models, are drawn from a Gaussian distribution with zero mean and variance σ_{s}^2.

The acquisition device may induce noise to the input signal, thus the noisy sparse signal at the output of the sampler can be represented as

$$\mathbf{x} = \mathbf{s} + \boldsymbol{\epsilon} \tag{3}$$

where $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \sigma_{\mathrm{n}}^2 \mathbf{I}_N)$ represents the noise. This model fits also the case of compressible sources, where the signal is not exactly sparse, and the Gaussian vector $\boldsymbol{\epsilon}$ accounts for the insignificant components [14].

III. The Proposed Compression Schemes based on Syndrome Encoding

In this section, we describe the two novel schemes for efficient lossy compression shown in Fig. 1. At first, the signal is quantized and the samples are stored in a register of size N. At the same time, the signal support is estimated using model order selection, then the insignificant elements in the register are set to zero accordingly. Finally, the data is compressed by calculating the syndromes using the parity-check matrix of a RS or BCH code. In the following, we will separately illustrate each part of the proposed scheme.

A. Signal Support Estimation by Model Order Selection

We will derive a novel estimator for the number and the locations of the non-zero elements in the noisy sparse signal, based on model order selection theory [15].

At first, the vector $\mathbf{x}$ is sorted in descending order, according to the absolute values of its entries, $|x_i|$, such that

$$|x_{\pi_1}| \geq |x_{\pi_2}| \geq \cdots \geq |x_{\pi_N}|$$

where $\pi = (\pi_1, \pi_2, \cdots, \pi_N)$ is the permutation vector. We denote by $\tilde{\mathbf{x}} \triangleq (x_{\pi_1}, x_{\pi_2}, \cdots, x_{\pi_N})$ the ordered vector.[2]

When the signal-to-noise ratio (SNR) is high, the vector $\tilde{\mathbf{x}}$ can be represented as a concatenation of two vectors $(\tilde{\mathbf{x}}_{1:k_0}, \tilde{\boldsymbol{\epsilon}}_{k_0+1:N})$, where the first one, $\tilde{\mathbf{x}}_{1:k_0}$, contains signal-plus-noise elements and the remaining one, $\tilde{\boldsymbol{\epsilon}}_{k_0+1:N}$, contains noise-only terms. Hence, the estimated signal support is indicated by the indexes $\mathcal{I} = \{\pi_1, \pi_2, \cdots, \pi_{k_0}\}$. Therefore, detecting the location of the non-zero elements of $\mathbf{s}$ is equivalent to estimate k_0, i.e., the signal sparsity. To pursue this goal, we propose to reformulate the detection problem as a model order selection problem where the order of the model to be estimated

[2]We do not consider the effects of quantization in this step.

can be related with the unknown signal sparsity. A powerful solution to model order estimation is based on information-theoretic criteria, where the model order is estimated by minimizing a penalized likelihood [15], [16]. In this work we consider, in particular, the generalized information criterion (GIC) because of its versatility in controlling the estimation accuracy [17], [18].

Considering that the sparse signal is unknown and deterministic, from (3) its likelihood function can be expressed as

$$f(\tilde{\mathbf{x}}) = \frac{1}{(2\pi\,\sigma_{\mathrm{n}}^2)^{\frac{N}{2}}} \exp\left(-\frac{\sum_{i=1}^{N}(\tilde{x}_i - \tilde{s}_i)^2}{2\,\sigma_{\mathrm{n}}^2}\right) \tag{4}$$

where $\tilde{\mathbf{s}} \triangleq (s_{\pi_1}, s_{\pi_2}, \cdots, s_{\pi_N})$ is the permutation of the noiseless signal.

Let us define a family of models to fit the measured data with the kth model representing the case where the last signal-plus-noise sample is the kth one, i.e., $\tilde{\mathbf{s}}_{k+1:N} = \mathbf{0}$. The likelihood function in (4) depends on some parameters such as the noise variance, σ_{n}^2, and the sparse signal, $\tilde{\mathbf{s}}_{1:k}$, denoted by $\mathbf{\Theta}_{(k)} \triangleq (\sigma_{\mathrm{n}}^2, \tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_k)$. As a consequence, the number of degrees of freedom (the model order) in the kth model is $\phi(k) = k+1$.

As the sparsity estimator is blind, we consider the noise variance and the sparse signal unknown. Hence, for the kth hypothesis, the vector $\mathbf{\Theta}_{(k)}$ needs to be estimated. The conditional likelihood function based on the estimated parameters, $\widehat{\mathbf{\Theta}}_{(k)} \triangleq (\hat{\sigma}_{(k)}^2, \hat{\tilde{s}}_1, \hat{\tilde{s}}_2, \cdots, \hat{\tilde{s}}_k)$, is then

$$f\left(\tilde{\mathbf{x}}; \widehat{\mathbf{\Theta}}_{(k)}\right) = \left(2\pi\,\hat{\sigma}_{(k)}^2\right)^{-\frac{N}{2}} \exp\left(-\frac{\sum_{i=1}^{N}\left(\tilde{x}_i - \hat{\tilde{s}}_i\right)^2}{2\,\hat{\sigma}_{(k)}^2}\right). \tag{5}$$

In the kth hypothesis, the maximum likelihood (ML) estimate of the signal amplitudes are

$$\hat{\tilde{\mathbf{s}}}_{1:k} = \tilde{\mathbf{x}}_{1:k} \tag{6}$$

while the remaining components are set to zero, i.e.,

$$\hat{\tilde{\mathbf{s}}}_{k+1:N} = \mathbf{0}_{k+1:N}. \tag{7}$$

For the noise variance, the ML estimate results in

$$\hat{\sigma}_{(k)}^2 = \underset{\sigma_{\mathrm{n}}^2 > 0}{\arg\max} \log f(\tilde{\mathbf{x}}; \sigma_{\mathrm{n}}^2) = \frac{1}{N}\sum_{i=k+1}^{N} \tilde{x}_i^2\,. \tag{8}$$

Then, the log-likelihood function (LLF) of the kth model can be written by substituting (6), (7), and (8), in (5) as

$$\log f\left(\tilde{\mathbf{x}}; \widehat{\mathbf{\Theta}}_{(k)}\right) = -\frac{N}{2}\log\left(\frac{2\pi}{N}\sum_{i=k+1}^{N}\tilde{x}_i^2\right) + \frac{N}{2}$$

from which the GIC takes the form

$$\hat{k}_0 = \underset{k\in[0,\bar{k}]}{\arg\min}\left\{N\,\log\left(\frac{2\pi}{N}\sum_{i=k+1}^{N}\tilde{x}_i^2\right) + N + \nu\,(k+1)\right\}$$

where $\bar{k} \leq N-1$ is an upper bound on the sparsity order and ν is a penalty factor that will be investigated in Section IV.[3] The estimator can be further simplified by omitting all terms that do not depend on k, resulting in

$$\hat{k}_0 = \arg\min_{k\in[0,\bar{k}]}\left\{N\,\log\left(\sum_{i=k+1}^{N}\tilde{x}_i^2\right) + \nu\,k\right\}. \tag{9}$$

The estimated support can now be identified as $\hat{\mathcal{I}} = \{\pi_1, \pi_2, \cdots, \pi_{\hat{k}_0}\}$, and the corresponding filtered sparse signal becomes

$$x_i^* = \begin{cases} x_i & i \in \hat{\mathcal{I}}, \\ 0 & \text{otherwise.} \end{cases}$$

B. Scalar Uniform Quantizer

Due to the large number of zero elements in $\mathbf{x}^*$, we consider a scalar mid-tread uniform quantizer [19], whose zero-valued reconstruction level prevents the introduction of additional quantization noise out of the signal support. This quantizer maps each element x_i^* of $\mathbf{x}^*$ to a discrete quantization index

$$Q : \mathbb{R} \to \{0, 1, \cdots, q-2\}$$

where $q \triangleq 2^b$.[4] In this quantizer the range $[-A, A]$ is uniformly partitioned into $L = q-1$ levels, with a step size $\Delta = \frac{2A}{L}$. Note that for the Gaussian source, we choose $A = 4\sigma_s$ [19]. The index vector of the quantized signal is then

$$\mathbf{\Gamma} \triangleq Q(\mathbf{x}^*) = (Q(x_1^*), Q(x_2^*), \cdots, Q(x_N^*)) \in \mathbb{F}_q^N.$$

C. Syndrome Based Source Encoder

We propose a source encoder for quantized sparse vectors in $\mathbb{F}_q^N$ based on the syndrome vector of a RS code. Firstly, let us consider the *dual channel coding* problem, assuming that the transmitter sends a codeword, $\mathbf{c} \in \mathbb{F}_q^N$, from the RS code with minimum distance $2k_0+1$. If the channel changes at most k_0 symbols, then the received vector can be represented as $\mathbf{r} = \mathbf{c} + \mathbf{\Gamma}$, where $\mathbf{\Gamma} \in \mathbb{F}_q^N$ is the error vector with a maximum sparsity order k_0, and the summation is in $\mathbb{F}_q$. Hence, the receiver can estimate the error vector, which is the sparsest vector satisfying the computed syndrome. Regarding the *source coding* problem, $\mathbf{\Gamma}$ is compressed by calculating its syndrome vector through the parity check matrix of the k_0-error-correcting RS code at the encoder. Consequently, the receiver can perfectly reconstruct $\mathbf{\Gamma}$ from the syndromes, provided that its sparsity order is at most k_0.

More precisely, the syndrome $\mathbf{z} \in \mathbb{F}_q^{2\hat{k}_0}$ is computed by the source encoder as

$$\mathbf{z} = \mathbf{\Gamma}\,\mathbf{H}^T \tag{10}$$

where all the operations are performed in $\mathbb{F}_q$, $\hat{k}_0$ is the estimated sparsity from model order selection, (9), and

[3]The encoder may have prior information about the maximum sparsity order, e.g., $k_0 \leq \bar{k} = 0.5N$, in the absence of this information $\bar{k} = N-1$.

[4]Note that the number of quantization levels is odd in mid-tread uniform quantizers.

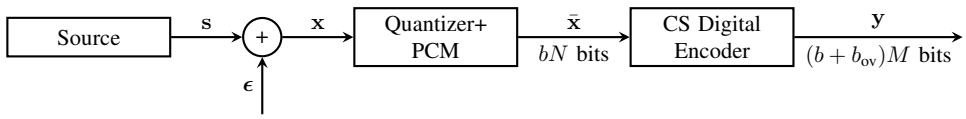


Fig. 2. The block diagram of the CS based source encoder [7], [20].

$$\mathbf{H} = \begin{bmatrix} 1 & \alpha & \alpha^2 & \cdots & \alpha^{N-1} \\ 1 & \alpha^2 & (\alpha^2)^2 & \cdots & (\alpha^2)^{N-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \alpha^{2\hat{k}_0} & (\alpha^{2\hat{k}_0})^2 & \cdots & (\alpha^{2\hat{k}_0})^{N-1} \end{bmatrix} \tag{11}$$

is the $2\,\hat{k}_0 \times N$ parity-check matrix for the $\hat{k}_0$-error-correcting RS code with $N = q-1$, while α is a primitive element in $\mathbb{F}_q$ [21]. The syndrome symbols can also be computed using efficient hardware [22].

The resulting total number of bits required to encode the sparse vector using RS syndrome coding is

$$R_t = 2\hat{k}_0\, b = 2\hat{k}_0\, \log_2(N+1). \tag{12}$$

A further compression gain can be achieved by separately sending the $\hat{k}_0$ quantized non-zero elements, then compressing the binary vector which determines their locations using the syndrome of a BCH code, see Fig.1(b).[5] In fact, since for BCH code the number of parity check bits $m \leq \hat{k}_0\, \log_2(N+1)$, the required number of bits is

$$R_t = m + \hat{k}_0\, b \leq \hat{k}_0\, \log_2(N+1) + \hat{k}_0\, b \tag{13}$$

where m is calculated from the design table of the BCH code, for a given sparsity order (i.e., error correcting capability) and dimension [22, Appendix C]. Clearly, form (12) and (13), the rate of this scheme is upper-bounded by the RS based approach, but the non-zero values and the syndrome vector should be transmitted separately.

In the following the proposed schemes will be referred as RS based source coding (RSSC) (Fig. 1(a)) and BCH based source coding (BCHSC) (Fig.1(b)).

D. Source Decoder

Regarding the RSSC approach, the locations and the values of the non-zero elements can be estimated at the receiver from the syndrome vector $\mathbf{z}$ using Berlekamp's iterative algorithm. Due to the minimum distance properties of the RS code and the maximum sparsity order of $\mathbf{\Gamma}$, the vector of the quantization indexes $\mathbf{\Gamma}$ is exactly recovered at the receiver.

Finally, the mapper

$$Q^{-1} : \{0, 1, \cdots, q-2\} \rightarrow \{i\,\Delta\}_{i=-\lfloor \frac{L-1}{2} \rfloor}^{\lfloor \frac{L-1}{2} \rfloor}$$

reconstructs the quantized signal from its indexes, and the reconstructed signal $\hat{\mathbf{x}}$ is then

$$\hat{\mathbf{x}} = Q^{-1}(\mathbf{\Gamma}) \triangleq \left(Q^{-1}(\Gamma_1), Q^{-1}(\Gamma_2), \cdots, Q^{-1}(\Gamma_N)\right). \tag{14}$$

[5]Note that the ones in the N-bit location vector indicates the locations of the non-zero elements.

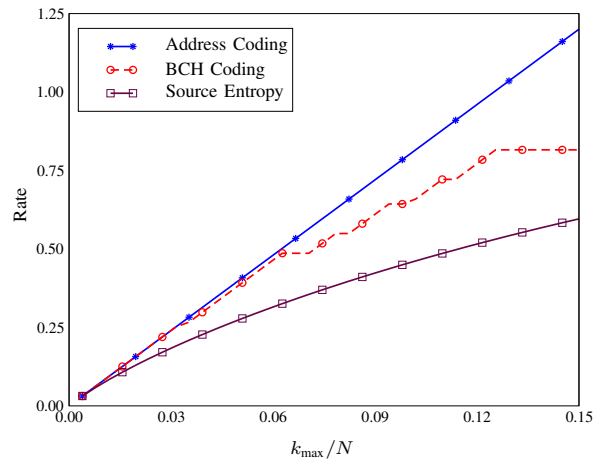


Fig. 3. The number of bits per dimension (rate) to encode the signal support of maximum size $k_{\max} \in \{1, 2, \cdots, 38\}$ with $N = 255$.

The proposed RSSC scheme requires only $2\,\hat{k}_0$ words ($2\,\hat{k}_0\, b$ bits). Hence, it can achieve a compression gain of $\frac{N}{2\,\hat{k}_0}$ over the non-compressed version. Moreover, the complexity for both the transmitter and the receiver are low, as there are efficient devices for coding and decoding the signal, and the location vector is implicitly embedded in the data.

Considering the BCHSC approach, the binary location vector can be recovered using the Berlekamp's algorithm, and the quantized non-zero entries are reconstructed from the quantization indexes as usual.

IV. Numerical Results

In this section, we compare the compression performance of the proposed schemes with source entropy, AC, and CS. We adopt the rate-distortion function usually considered as the performance metric for lossy source encoders, where the distortion between the noiseless and the reconstructed signals is defined as

$$D \triangleq \frac{1}{N}\, \mathbb{E}\left\{(\hat{\mathbf{x}} - \mathbf{s})\ (\hat{\mathbf{x}} - \mathbf{s})^T\right\}$$

while the rate is the expected number of transmitted bits per sample. The SNR is defined as $\mathrm{SNR} \triangleq \mathbb{E}\left\{\mathbf{s}\,\mathbf{s}^T\right\} / \mathbb{E}\left\{\boldsymbol{\epsilon}\,\boldsymbol{\epsilon}^T\right\}$.

As a benchmark for performance we choose the CS scheme employed in [20] (see Fig. 2), where the signal is first quantized then digitally multiplied by the measurement matrix generated from a Rademacher distribution with $\mathbb{P}\{A_{i,j} = 1\} = \mathbb{P}\{A_{i,j} = -1)\} = \frac{1}{2}$. The additional overload bits, b_{ov}, resulting from the binary multiplication and summation required to compute $\mathbf{y}$, should be considered for calculating the rate [20]. The number of measurements is

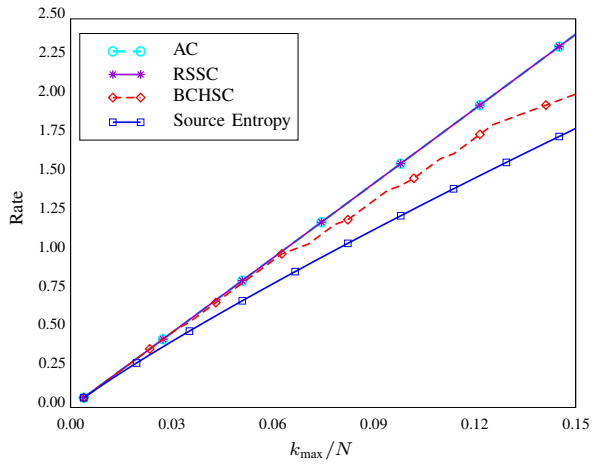


Fig. 4. The number of bits per sample to encode a non-binary sparse source with support of maximum size $k_{\max} \in \{1, 2, \cdots, 38\}$, and $b = 8$ bits/sample.

selected to minimize the rate-distortion function and the signal is recovered using the Basis Pursuit Denoising (BPDN) minimization program through `YALL1` Matlab package, assuming a perfect knowledge of noise statistics.

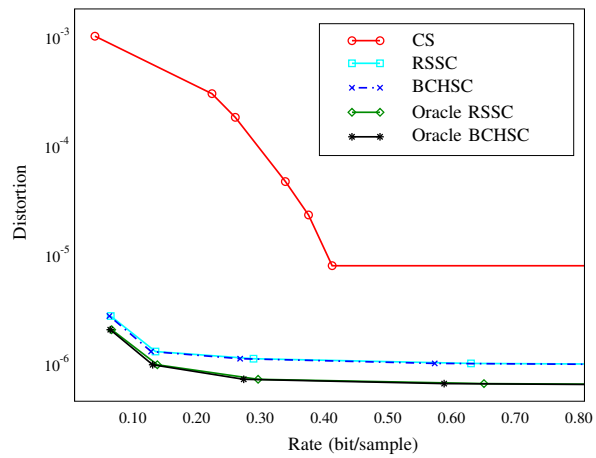
Firstly, we illustrate the lossless compression performance for the source, indicating the locations of the non-zero elements. In particular, the number of bits per dimension needed to encode the signal support for $N = 255$ and $k_{\max} \in \{1, 2, \cdots, 38\}$ using the syndromes of the BCH code, AC, and the lower bound indicated by the source entropy (2) is shown in Fig. 3. We can see that the BCH scheme approaches the source entropy, especially at lower sparsity ratios, and its rate is upper bounded by the AC, hence achieving a higher compression gain (more than 30% compared to AC for higher $k_{\max}/N$).

Secondly, the lossless compression for discrete-valued discrete-time sources is emphasized. Let us consider a quantized source S emitting a vector of length N with at most $k_{\max}$ non-zero elements chosen at random from the possible 2^b levels. The signal support is generated as in Section II. Since all source realizations are equiprobable, the source entropy is calculated as

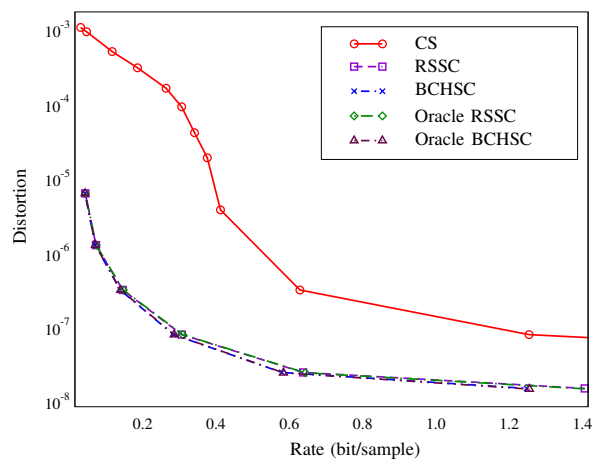
$$\mathcal{H} = \log_2 \sum_{i=0}^{k_{\max}} 2^{b\,i} \binom{N}{i}. \tag{15}$$

In Fig. 4, we report the number of bits per sample needed to encode S using AC (1), RSSC (12), BCHSC (13), and Shannon's lower bound (15) as a function of the maximum sparsity order $k_{\max} \in \{1, 2, \cdots, 38\}$, for $b = 8$ bits/sample and $N = 2^b - 1 = 255$. It is noticed that the rates indicated by both RSSC and AC are coincident, while the BCHSC method achieves a higher compression gain (up to 15% compared to RSSC).

To illustrate the performance of the lossy compression schemes for sparse noisy sources, Fig. 5 shows the rate-distortion function of the proposed RSSC and BCHSC schemes in Fig.1, along with the CS encoder in Fig. 2. The signal is generated according to the sparse-land Gaussian model



(a) $\mathsf{SNR} = 20$ dB

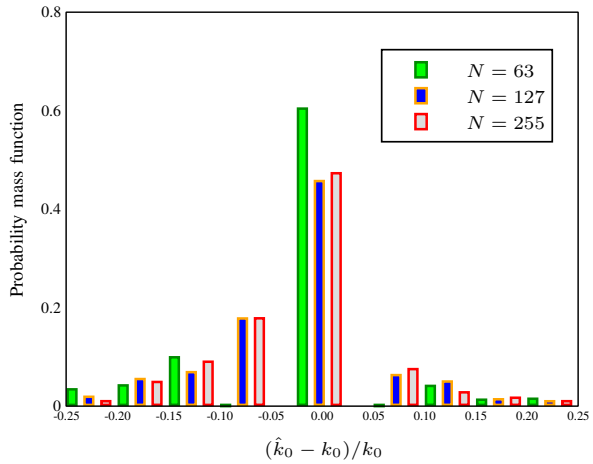


(b) $\mathsf{SNR} = 40$ dB

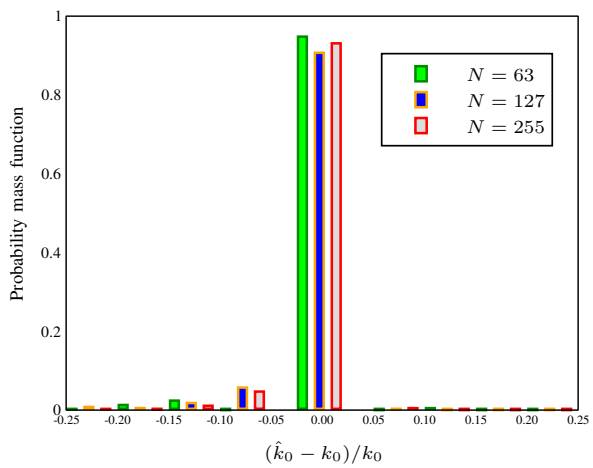
Fig. 5. The rate-distortion for lossy source compression of sparse noisy sources, with $k_{\max} = \lceil 0.14N \rceil$, $b \in \{5, 6, 7, 8, 9, 10\}$, and $N = 2^b - 1$.

presented in Section II, with $N = 2^b - 1$, and sparsity order K_0 uniformly distributed within $\{0, 1, \ldots, \lceil 0.14N \rceil\}$. Then, the signal is quantized with $b \in \{5, 6, 7, 8, 9, 10\}$ bits/sample. The signal sparsity order and support are estimated by the derived GIC (9) with penalty $\nu = 10$, and $\nu = 18$, for $\mathsf{SNR} = 20\,\text{dB}$, and $\mathsf{SNR} = 40\,\text{dB}$, respectively. Moreover, the rate-distortion function using syndrome encoding with perfect knowledge of the sparse signal support at the encoder is also presented as an *oracle* approach.

It can be noted that the syndrome encoding based approach achieves a reduction of around 2 orders of magnitude in distortion compared to CS, despite the latter requires knowledge of the noise statistics and optimization over all possible numbers of measurements. This gap is due to the denoising effect resulted from estimating the signal support using model order selection and discarding the insignificant elements attributed to the noise. Additionally, for the same distortion, the BCHSC suggests a reduced rate, about 10%, compared to the RSSC. Interestingly, model order selection based schemes are almost coincident with the oracle based approach.



(a) $\mathsf{SNR} = 20$ dB ($\nu = 10$)



(b) $\mathsf{SNR} = 40$ dB ($\nu = 18$)

Fig. 6. The probability mass function of the relative sparsity order estimation error using GIC for $k_{\max} = \lceil 0.14N \rceil$.

On this point, the performance of the GIC estimator for the sparsity order k_0 is examined, where the probability mass function of the sparsity estimation error, $\frac{\hat{k}_0 - k_0}{k_0}$, is presented in Fig. 6. As can be seen, for $\mathsf{SNR} = 20$ dB, the probability that the estimation error is within 20% of the correct order is more than 75%, while for $\mathsf{SNR} = 40$ dB, the proposed estimator correctly detects the sparsity order with high probability, more than 90%. We also note that unlike the conventional estimators with large number of realizations, here we are able to efficiently estimate the sparsity order from only one snapshot of $\mathbf{x}$. Hence, model order selection can be used to detect the non-zero elements, and obtain a non-linear sparse approximation for compressible signals, efficiently.[6]

V. Conclusion

This paper proposed two novel schemes for efficient encoding of noisy sparse sources. These approaches are based on exploiting the duality between the source coding of sparse sources and the channel coding. At first, the source sparsity and support are determined through the GIC, and the sparse signal is quantized using a uniform scalar quantizer. Then, the compression is achieved by sending the syndromes of the quantized/location vector computed from the parity check matrix of a RS/BCH code. The decoder perfectly recovers the quantized/location vector from only the received syndromes using conventional RS/BCH decoder. As illustrated by numerical results, the proposed approaches provide a lower distortion-rate compared to the previously known methods such as CS.

[6]Since the noise at the input of the transmitter is introduced by the acquisition device, the SNR is quite large [20].

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