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Analogy as categorization: a support for model-based reasoning

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Abstract: Generally speaking, model-based reasoning refers to every reasoning that involves model of reality or physical world, and it is especially involved in scientific discovery. Analogy is a cognitive process involved in scientific discovery as well as in everyday thinking. I suggest to consider analogy as a type of model-based reasoning and in relation with models. Analogy requires models in order to connect a source situation and a target situation. A model in an analogy is required to establish salient properties and, mostly, relations that allow transfer of knowledge from the source domain to the target domain. In another sense, analogy is the model itself, or better, analogy provides the elements of model of reality that enable the processes of scientific discovery or knowledge increase. My suggestion is that some insight on how an analogy is a model and is connected to model-based reasoning is provided by recently proposed theories about analogy as a categorization phenomenon. Seeing analogy as a categorization phenomenon is a fruitful attempt to solve the problem of feature relevance in analogies, especially in the case of conceptual innovation and knowledge increase in scientific domain.

Keywords: analogy, model-based reasoning, concept, categorization, models, scientific discovery.

1 Introduction

Analogy is a kind of model-based reasoning, at least in some senses. Model-based reasoning involves models that are used in inferences, from a formal and logical point of view (for example, the traditional expert systems of artificial intelligence[2]). But, in a wider sense, model-based reasoning refers to every reasoning that involves models of reality or the physical world, and it is especially involved in scientific discovery (Magnani et al. 1999). Analogy is a cognitive process involved in scientific discovery as well as in everyday thinking. Analogy requires models in order to connect a source situation and a target situation. Roughly speaking, a model is required in an analogy to establish salient properties and relations that allow the transfer of knowledge from the source domain to the target domain. In another sense, analogy is the model itself, or better, analogy provides the elements of the model of reality that allow the processes of scientific discovery or increase in knowledge to take place. My suggestion is that some insight into how an analogy is a model and is connected to model-based reasoning is provided

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[2] See Russell and Norvig (2010).

by recently proposed theories on analogy as a categorization phenomenon, which are consistent with a more general cognitive thesis according to which analogy-making is categorization *and* categorization is analogy-making (Hofstadter and Sander 2013).

To support my claim, in the next sections I deal with the relationship between analogy and models in general (§ 2); I then comment on the connection between categorization and concept creation (§ 3) and I illustrate the relationship between analogy-making and categorization, explaining how we can speak about analogy-making as categorization (§ 4). Lastly, I combine analogy-making as categorization and models to clearly describe how to consider the role played by analogy in model-based reasoning (§ 5). I draw general conclusions of these connections in the last section (§ 6).

2 Analogy and models

Two preliminary questions seem to be relevant and strictly related to our discussion: how is analogy connected to models? How is analogy connected to concepts? They imply, however, a more general question: how to define analogy?

There are many answers to such a question, according to different ways to see analogy. We may say that analogy is: a cognitive process, a transfer of (semantic) information, an inference – actually an inductive inference – from a formal point of view, and a type of argument – argument by analogy – from an informal viewpoint. But, from a conceptual standpoint, analogy can also be regarded as a similarity relation, or a structure alignment, or an abstraction process. Finally, we can also say that analogy is a particular kind of model-based reasoning, if we consider the known situation as the model for the analogical unknown situation; or even, an analogy can be regarded as a model itself, insofar as it involves a model structure rooted in the traditional, ancient meaning of analogy, that is, the meaning of proportion, stemming from a mathematical view of analogy as the relationship between ratios. All these elements concur to structure the concept of analogy, which makes it very hard to find a single definition – we could call it the hard problem of analogy.

A main distinction is between analogy as a fact and analogy as an act. In the former case, we may consider every analogy as an individual *fact* – an inference, an argument, an established similarity relation – that we can analyze *ex post* in order to evaluate its accuracy or goodness, or likewise to justify it, especially in the scientific context. In the latter case, we may see every analogy as an individual *act* (of thought, of reasoning), whose rising and happening we have to explain, so as to understand the *ex ante* aspects leading to the specific analogy. Usually, an analogy as a fact is mostly a matter of logic and it is studied as the outcome of analogical reasoning, in an attempt to provide a justification of it or to formalize it[3]. On the other hand, analogy as an act has been studied as a cognitive process, to model

[3] A recent study on such a topic is Bartha (2010).

it or to explain the creative process of analogy building[4]. What's more, these two ways of dealing with analogy are usually rather separate, because they have distinct methods and aims, and they seem to be two different things and not one and the same thing. The result is that it appears unavoidable to sacrifice the understand of analogy as a fact if we want to explain the act of analogy, and vice versa – we could call it the entanglement of analogy.

How can we combine them? Maybe, by connecting analogy and models through concept creation so as to encompass the process and the outcome of analogy within a single perspective. We will see in the following sections in which sense analogy may be a support for model-based reasoning, but we first need to consider the relation between analogy and models.

For our purposes, a model is a simplified reproduction of a real phenomenon, in which relevant elements are structured to study the phenomenon itself by manipulating its elements and parameters. The key point is the *relevance* of features constituting the model. Analogy involves relevance as well. In fact, one or more relevant features of a known situation or domain are connected to corresponding features of an unknown or less known situation or domain to establish new features in it. So, the known situation or domain is modeled – i.e. becomes the model – for the unknown one, producing new knowledge through the transfer of the old knowledge. This is true both in logic and in the computational/cognitive approach to analogy. So, the informal argument by analogy (Baronett 2012) is:

A is similar to *B* in certain respects
A also has the feature P
 $\Rightarrow$ *B* has the feature P
(plausibly or with some degree of support)

where the "certain (relevant) respects" plus P of A are the model for B. In a more formal fashion, the structure of analogy as inductive inference is:

$$
\begin{aligned}
&P_1(x) \wedge P_2(x) \wedge P_3(x) \wedge \ldots\ldots \wedge P_n(x)\\
&P_1(y) \wedge P_2(y) \wedge P_3(y) \wedge \ldots\ldots \wedge P_n(y)\\
&Q(x)\\
&\quad \Rightarrow \; Q(y)
\end{aligned}
$$

where x and y are objects and P and Q are properties. For example, this kind of inference is used to ascribe properties to a species, or a medical substance, starting from the known properties. Generally speaking and with reference to a logical standpoint, we may characterize the *analogy as the use of a model to extend the application domain of a specific property*.

From another perspective, analogy itself is a model, whose structure involves four elements. In the classical meaning of analogy as a proportion, two ratios are compared. The typical scheme of analogy of ancient and medieval thought is: A : B =

[4] See for example Holyoak and Thagard (1995); Gentner et al. (2001).

C : D[5]. In modern terms, analogical problems of this type require that D be found. So, for instance, problems like “Book : Reader = Instrument : ?” or “Sun : Summer = Snow : ?” most likely have solutions such as “Player” and “Winter” respectively[6]. The solution is just likely and not certain, because it depends on the context in which the analogy problem is solved, assuming that the context of four elements, or three plus a fourth to be found, can be indeterminately complex. The fact that the solution is not certain is the reason why analogy has become a kind of inductive inference in modern logic, whereas the four elements pattern underlies contemporary computational and cognitive approaches to analogy, especially for issues raised by the difficulty in establishing the relevance of features involved in an analogical cognitive process.

A model of analogy based on the four elements structure is provided by Hesse (1966) in her tabular representation. In this model, there is a distinction between a source domain (S) and a target domain (T), which subsequently became typical of cognitive approaches as well. S and T are in a horizontal relation, whereas the list of traits of each domain is the vertical part of the pattern. A new correspondence can be inferred between a feature of S and a feature of T from the known similarities between traits. This model can be described as being tabular in shape, or as a four-element pattern whose vertical and horizontal relations are arranged in a square form. The corresponding elements of the two domains are objects, properties, relations, functions, roles, etc.

Tabular representation can be regarded as a model of analogical inference, but also of analogy as a cognitive process. For example, in French (1995) this model is meant to capture the process of analogy building from a dynamic standpoint. So, from S an abstract scheme is drawn and then its variables are replaced in the process of conceptual slipping leading to a new abstract scheme, whose variables are re-constrained to attain the situation in T. In the model, the process of abstraction and its opposite are vertical, the process of conceptual slipping is horizontal and corresponds to semantic information transportation, producing new knowledge. The process of abstraction and the relation between abstract and concrete are typical in cognitive approaches to analogy. One of their purposes is to explain how an analogy is created or built and how the contextual pressures cause the emerging of analogy relevant elements[7]. And all these processes involving abstraction involve *concepts* as well.

Models also involve concepts, but analogy seems to involve other features shared with models: representations, relational structures and their systematicity, relations of different kinds within the S and T domains (logical, causal, explanatory, functional, and/or mathematical relations), the salience of features upon which both of them – a model and an analogy – are built. For example, the SME traditional cog-

[5] See Prior Analytics by Aristotle 69a1 (Aristotle 1984).
[6] For one of the first computational approach to this kind of analogy problems see Evans (1968).
[7] For a list of different cognitive approaches to analogy, see Kokinov and French (2003).

nitive model of Gentner finds analogy in different domains (such as the atom and solar system) by exploiting the abstract relations of two domains (“greater”, “cause”, “attracts”, etc.), which are in this way modeled and aligned by the program (Falkenhainer et al. 1989). The SME model has been criticized because it uses pre-built (by a programmer) representations. Subsequent connectionist and hybrid models have been designed to capture the dynamicity of analogical cognitive process, in particular by building their representations of elements of two domains (Chalmers et al. 1992), or by exploiting the dynamical activation of the nodes in a semantic network, as occurs in the multiconstraints satisfaction theory of analogy of Holyoak and Thagard (1995)[8].

These two main computational approaches to analogy stress different traits of the analogy phenomenon. On the one hand, computational models based on prebuilt representation want to capture the structured, deep-relation-rooted nature of analogy, its connection with inferential process, the justification of the analogy as the outcome of the process itself, and the definition of relevant and essential features of situations involved in an analogy. On the other hand, computational models based on dynamical and autonomous – by the program – representation building try to capture the spontaneity of analogy, the pressures of context, the abstraction processes and the analogy creation, without considering involved domain structures alone, but the duality of superficial and deep features involved in the analogy building. If we consider models as being involved in analogies, we may say that they share the same condition and that both are subjected to the same dichotomies. So, the opposition, in different approaches, between a given analogy and analogy building corresponds to the one between a given model and model building. In the former case, we may have an *ex post* explanation of an analogy phenomenon as well as models involved in the analogy; in the latter case, we may have an *ex ante* explanation of an analogy phenomenon as well as of the building of the models involved. The latter case is more relevant to our purposes because it is connected with concepts and concept formation, even because finding a model in an analogy overlaps using a model to make an analogy. Below, we will see an interesting proposal that illustrates this point.

3 Analogy, models and concept formation

A noteworthy theory on analogy, model-based reasoning and concept formation (in scientific domain) is the one formulated by Nersessian (2008). The main idea is that the traditional S-T scheme of analogy as a cognitive process has to be supplemented as follows: the transfer of information is not direct from source (S) to target (T), but there is a model mediation between S and T. This model mediation is due to a *hybrid model* in which connection crossing of the S and T elements takes place (Nersessian 2008; Nersessian and Chandrasekharan 2009). In this way,

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[8] There are many computational models of analogy. A close examination is beyond the scope of this paper. For an introduction and a discussion of the cognitive processes involved see Hall (1989) and Kokinov and French (2003).

the hybrid model, comprised of constraints from the S and T domains, can be exploited for further refining of the model itself, on the basis of the S and T constraints *and* the constraints provided by the hybrid model. The process is dynamical because it is grounded on an increase in the number of constraints that leads to a clarification of the S-T relationship and to the solutions of the analogy problem. From this point of view, analogy is seen and explained as a mechanism of conceptual innovation.

Nersessian and Chandrasekharan (2009) describe the process that led to conceptual innovation during a research aimed at understanding undesired spontaneous "bursts" in an *in vitro* model of cortical neural network activity, which are phenomena that do not occur in *in vivo* properly functioning animal brains. The *in vitro* model, named "the dish", was stimulated using different electric signals and was connected to robot devices and visualized animats moving around in simulated computational words. The aim of the experiment is to control this embodied cortical neural network. Since the *in vitro* model had constraints that were relevant features of real neurons and the neuron network, the dish was already an analogy, but was not enough to attain the goal. Thus, an *in silico* model, a computational neural network, was built by using constraints from the neuroscience domain and from a generic dish: it was a hybrid model. Different and more refined versions of the computational model were built until a visualized activity of a version allowed the novel notion of "center of activity trajectory" to be developed. At that point, this last version of the computational model was able to replicate the activity of the dish. The greater controllability of the computational model allowed many experiments to be conducted with the dish, thanks to the potential transfer from the *in silico* model to the *in vitro* model. So, what was first developed for the computational model was then developed, by analogy, for the *in vitro* model.

One of the most important outcomes of the whole process was the emergence of a new concept, the "center of activity trajectory". It was a consequence of the mapping between the two models (an analogy of analogies, if we consider the two models according to their analogical nature) and the visualization process. In particular, the hybrid models, the dynamical and incremental process of (re)-building the computational model that integrates constraints from sources, targets and models themselves, are what allows the emergence of new structures, behaviors and, eventually, new concepts.

The hybrid model theory is very interesting for several reasons. First of all, it does not reject the traditional scheme of analogy and its standard cognitive explanation (Gentner 1983). Second, even though Gentner's structure-mapping model is based on the formal connection of structures of relations, which are considered unavoidable for analogy, hybrid analogy theory also assumes that semantic and pragmatic aspects are equally important, just as they are in the multiconstraints satisfaction theory of analogy of Holyoak and Thagard (1995)[9]. In the hybrid analogy theory,

[9] See the description of the computational model based on the simultaneous satisfaction of a set of semantic, structural and pragmatic constraints, and the description of the ARCS program, in Thagard et al. (1990).

the understanding, interpretation and goals of an analogy problem drive the selection of relevant constraints in the incremental process of hybrid model development, thereby contributing, together with the syntactical aspect, to the emergence of solutions as well as of new concepts. Third, the stress on representation building (Nersessian 2008) implies that the way in which the model representations are built is a fundamental issue that needs to be dealt with, as the constraints of T determine which constraints of S are (potentially) relevant to analogical comparison. This is the big issue of creative analogies: pinpointing relevance. The incremental process of model representation building is an attempt to deal with such a fundamental problem. As we said above, although the problem of representation building was not central in the literature on analogy and analogical reasoning in past decades, we should bear in mind that there are some hints of it in computational modeling of analogy. Hofstdadter *et al.* (1995) provide some interesting reflections on this topic in their criticism of ready-made representations of the SME cognitive model (Falkenhainer et al. 1989) based on the structured mapping theory by Gentner, which is nevertheless valid for the systematicity principle of relation interconnection in analogy explanation. Representation-building and transfer are two further processes that Kokinov and French (2003) added to the four standard approaches to the computational modelling of analogy: recognition, elaboration, evaluation, consolidation (Hall 1989). Chalmers *et al.* (1992) addressed the problem from the artificial intelligence methodology point of view, arguing that the only way to understand cognitive process is to consider representation as the outcome of a continuous, dynamical process of high-level perception, concerning both modal and amodal aspects in relation with concept representation. This is especially true for analogy explanation.

To summarize the important features of the hybrid models theory, which in Nersessian and Chandrasekharan give rise to hybrid analogies, we may say that hybrid models a) allow creative analogies – i.e. analogies between analogies, which are the models used in experimental laboratory processes; b) are used for reasoning purposes; c) are only models and not real world entities; d) allow visual, imagistic, simulative and manipulative processes; e) and lead to conceptual innovation and to new concepts. For the purposes of our investigation, a final remark by the authors is very noteworthy: “although our case might be considered extraordinarily creative, our intuition is that if analogy use ‘in the wild’ were to be studied systematically, the construction of such intermediary hybrid representations, making use of visualization and mental simulation, would be seen to be significant dimensions of *mundane* usage as well” (Nersessian and Chandrasekharan 2009: 187 [emphasis added]). We will now consider another proposal that connects analogy, concepts and categorization.

4 Analogy as a kind of categorization

When considering the relation between analogy and categorization, two elements from the previous sections are needed to establish to what extent hybrid models are connected to the categorization process: i) the idea that a dynamical and purpose-oriented representation building process is fundamental in the model-based reasoning involving analogy, and ii) the fact that concept innovation is strictly linked to concept formation, and thus to categorization as a sort of concept formation. The idea of analogy as categorization is not new. For example, Glucksberg and Keysar (1990) see metaphors as class inclusion statements and argue that understanding metaphors means understanding such statements, which are categorizations or, rather, category attributions. Research on teaching science subjects and the role of analogy shows that analogy creation is different from analogy interpretation and that analogies and analogy creation can be seen as categorization phenomena (Atkins 2004).

Starting from the psychological evidence that mechanisms underlying analogy and conceptual processes, especially categorization, are very similar[10], Dietrich (2010) tries to unify analogy-making and categorization by showing that analogy-making is based on construing, which is a kind of categorization. He claims that, besides the usual incremental representation building process, which is typical of the reasoning phase, two further representational processes are present in the analogy-making phase. The two steps of the analogy-making process are rapid abstraction and construing, the latter being a type of categorization. In relevant and insightful cases of analogy construing involves very different semantic analogs.

An example of the rapid abstraction process is modeled by STRANG, a computational model that makes analogies in the letter string domain (Dietrich et al. 2003)[11]. The STRANG program uses a grammar to pack strings of letters so as to find an abstract structure that is equivalent in two strings: Target (T) and Base (B). Target is the input string and Base is the string in the long-term memory. For example, if T is ababccc and B is mnopqrhijhijhij, the program produces the following outcome: (((ab)(ab))(ccc)) and (((mno)(pqr))((hij)(hij)(hij))), putting together the two strings according to the abstract description "two same-length sequences followed by a 3-item repeating string" (Dietrich 2010). The process modeled by STRANG is an example of rapid abstraction, which is particularly interesting insofar as the program associated with this process can violate the grammar rules to create packages of letters that are not the results of a direct application of a rule, but rather the continuous application of rules until the program finds a general representational abstraction connecting T and B. Significantly, B is a *model* in the long-term memory.

What is not modeled in this version of STRANG is the second step, based on semantic distance. According to Liberman and Trope's theory on the relation between psychological and semantic distance and the abstraction process[12], psychological distance induces abstraction[13]. So, if we start from a point of origin, a situation we are dealing with here and now, and we are provided with a *relevant*

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[10] See, among others, Ramscar and Yarlett (2003).

[11] For a discussion on letter-string domain and COPYCAT, one of most important computational models of analogy in this domain, see Mitchell (1993).

[12] See Liberman and Trope (2008) and Liberman and Förster (2009).

[13] Even though this aspect of theory is problematic and presents some weaknesses. For a discussion see Dietrich (2010).

psychological distance, we can make an analogy or, in some circumstances, analogies may arise spontaneously. The relevant psychological distance enables the second rapid representational abstraction, according to a dynamical process involving a passage from concreteness to abstraction: "*concrete* representations are less structured, more contextualized, and contain more information in the form of incidental features. Higher level *abstract* representations are schematic, decontextualized, and tend to represent the *gist* of an object or event by focusing on core features and omitting incidental information" (Dietrich 2010: 338). For example, if I am thinking of my home, an analogy may arise with my country because of the psychological distance between them, which is nonetheless based on relevant abstract shared features, such as organization or place-where-I-live. The analogy stems from a representational change from concrete to abstract. In the domain of scientific discovery, this kind of explanation should hold also as regards, for example, an analogy between the Rutherford-atom model and solar-system model because of their semantic and psychological distance.

There are some problems in this view. For instance, how can we explain semantic distance without being too vague? And what makes what is relevant in an analogy relevant? If we consider the point of origin alone, we do not obtain an explanation for why some specific concepts are retrieved and other semantically distant concepts are not. To solve the semantic problem, Dietrich inverts the perspective and considers analogy-making as categorization, stating that the construing process is a process leading to meaningful categorization, which in two steps produces an analogy; and this is true for every analogy. A first construal is attained through a process from an initial visual stimulus to integration, by means of a mapping process, between perceptual elements and a retrieved category. The construal turns out to be the meaningful categorization of a perceived object. This is a transition from a semanticless stimulus to a semantic meaningful mental representation (in the preceding example, my home). The first construal is the base – the point of origin – for another (meta-)construal, attained by retrieving a semantic distant category (in the example, my country). Only at this point is the analogy complete and is it possible a work on its details, which, according to Dietrich, can be properly called "analogical reasoning".

Even if this attempt to unify analogy-making and categorization has some problems, it is interesting because it connects analogy-making and new knowledge production through concept exploitation. The construal is a form of categorization. It is clear in the first step of the construing process. However, if semantic connection between distant categories is also a construal, we have to conclude that this is another case of categorization, and consequently that analogy in the proper meaning of connection of abstract features between two distant domains – whose gift is what is relevant – is a sort of categorization. So, categorization turns out to be the combination of shared features within a new conceptual structure at an abstract level. In other terms, it is a dynamical building of a representation, which is, in fact, the analogy and eventually leads to the detailed analogical reasoning work.

5 Analogy as categorization, and its consequences

Dietrich's view is noteworthy because it is an attempt to hold together cognitive and, at least in part, logical aspects of analogy. Another theory that goes in depth in dealing with analogy and categorization is based on the idea that "the spotting of analogies pervades every moment of our thought, thus constituting thought's core" (Hofstadter and Sander 2013: 18). In Hofstadter's perspective, it stems from seeing analogy as deeply intertwined with the process of high-level perception and representation building. It is also part of a general theory according to which analogy is the core and the essence of cognition (Hofstadter 2001). In other terms, analogy as analogy-making is what allows the general dynamics of cognition, by being an integral part of perceptual and representational processes, reasoning, learning, memory and language. It also underlies what is usually and standardly considered as analogy, the correspondence between an S and a T domain, and it is very present in the creativity process, scientific discovery, decision making, concept formation and categorization – in fact, it itself is categorization. This main thesis can be divided in two sub-theses: 1) analogy (making) is categorization; 2) categorization is analogy (making). Both of them are implied in the general idea that concepts are (formed by) analogies. In the rest of this section, I will try to explain this claim, proposing a theory of conceptual extension that is consistent with Hofstadter and Sander's view.

Analogy-making implies memory retrieval as a fundamental part. Every cognitive process is, at its core, due to a central cognitive loop that works this way: "a long-term memory node is accessed, transferred to short-term memory and there unpacked to some degree, which yields new structures to be perceived, and the high-level perceptual act activates yet further nodes, which are then in turn accessed, transferred, unpacked, etc., etc." (Hofstadter 2001: 517). Emphasis on memory and memory retrieval is found in subcognitive models[14], and it is in line with other general cognitive architectures. So, one root of this idea clearly lies in cognitive modelling, especially in the traditional symbolic approach, such as the total cognitive system scheme used by Allen Newell to explore his attempt to find his own unified theory of cognition (Newell 1990). The main features of the total cognitive system are a long-term memory, with different sub-processes, connected to a working memory interacting with the external environment by means of perceptual systems (i.e. the input of the systems) and motor systems (i.e. the whole system behavioral output), which can also be part of the input.

More interestingly, a second root of Hofstadter's theory lies in concept theories. For example, Barsalou asserts that "Rather than being retrieved as static units from memory to represent categories, concepts originate in a highly flexible process that retrieves generic information and episodic information from *long term memory* to construct temporary concepts in *working memory* [...] This concept construction process is highly constrained by goals. . .[and]. . .context..." (Barsalou 1987: 101). There is a huge body of literature on the central role of con-

[14] Such as Copycat, Metacat, Tabletop, Letter Spirit; see Hofstadter et al. (1995), Mitchell (1993), French (1995), Marshall (2006).

text, similarity and dynamical concept development dating from the 1980s and 1990s[15].

Starting from subthesis 1, which asserts that analogy is categorization, and according to Hofstadter and Sander's arguments, I will try to show that analogies involve conceptual extension, and two kinds of conceptual extensions in particular: vertical extension (VE) and horizontal extension (HE). Hofstadter and Sander refer to them as vertical category leap and horizontal categorical broadening, respectively, but this distinction is not so clear-cut in their theory because in many cases "we see that there is no sharp line of demarcation between vertical category leaps and horizontal category extensions" (Hofstadter and Sander 2013: 468). It seems to depend on the interpretation of analogy and on the context pressures considered in the explanation of an analogy. So, as contextual pressures are connected to the problem of relevance, I will propose a dynamical pattern through which analogy is produced in human mundane and scientific thought. My argument from sub-thesis 1 will proceed in this way:

1. Categorization is concept formation
2. Concept formation is concept extension (or broadening)
3. Concept extension is on a horizontal or vertical level
4. Analogy is categorization

A first conclusion is that:

5. Analogy is *both* on a horizontal *and* vertical level

from which we can draw a second conclusion:

6. Analogy involving *abstraction* (VE move) is always analogy between analogies (HE move).

Let's start with an example to show how concept extension, and thus concept formation, works. The development of concepts from childhood to adulthood is an enrichment process that usually leads a concept from a single-member to a cloud of concepts (Hofstadter and Sander 2013: 37)[16]. Consider a very common concept: father. First a child get to know her/his Daddy – with the capital letter, as there is just *one* daddy as far as that child knows. Upon finding out that other children have their own daddy, the child develops the concept of daddy – with the small letter, as the concept refers to many different people, i.e. daddies. Then the child can learn that there is a more objective sense of the concept, and finds out that her/his daddy belongs to the category "father". Afterwards, the concept may be extended to embrace other forms or kinds of fatherhood, which are more distant from the initial core of the concept, such as adoptive father, father-in-law, father-to-be, father of mathematics, Fathers of the Nation, Fathers of the Church, etc., in a hierarchical level from the center to the periphery of the cloud[17]. During life our concept of father extends without ever reaching a final boundary, as it can always

[15] See, among others, Barsalou and Medin (1986) and Goldstone (1994).

[16] On conceptual development, see Rakison and Oakes (2008), Carey (2009).

[17] Semantic and local neural networks can be used to model hierarchical and heterarchical structures of concept clouds in their dynamical activation pattern; see, among others, Mitchell (1993).

be extended further in an increasingly metaphorical way. Indeed, we place within the same category an increasing number of different categories and instances by analogy, that is by exploiting similarities at levels of varying abstraction. This is why analogy produces categorization and, in the end, analogy is categorization. Such an endless process is the vertical extension, which enlarges a concept and gives rise to new concepts by adding parts of its meaning.

Now, let's consider another concept, which is related to the concept of father: mother. We may imagine the same conceptual development and, consequently, a similar (vertical) extension: from Mommy to mommy (because there are mommies), then to mother, and afterwards to adoptive mother, nursing mother, surrogate mother, mother-in-law, mother-to-be, mother earth, mother country, Mother Nature, Mother Church, mother tongue, etc. If we compare these two concepts, we make another kind of conceptual extension, which leads to conceptual innovation or formation. For example, we can connect mother and father and discover the concept of parent (it is very likely something that happened many years ago in our life, when we were six or seven). And it is highly likely that we have, at a certain point later in our life, made another horizontal extension, connecting adoptive father and adoptive mother, discovering the concept of adoptive parent, under the pressure of context of our familiar and social environment, as in the first case. The extending process has no an end point, and we may imagine that, for instance, the concepts of Fathers of the Nation and mother country, which are a long way from the core terms "mother" and father" in our hierarchical cloud structure, may sooner or later be connected, yielding the new concept of "Parents of the Land", or rather, an *unlabeled-by-a-single-word concept* that is subsequently named "Parents of the Land". This is how horizontal extension works, i.e. relating things that are considered at the same level by analogy so as to produce new knowledge, new concepts, and sometimes new words or phrases (even though concepts do not need single word denotation to be concepts). In short, horizontal extension is a conjunction that produces conceptual innovation by unification, and thus new concepts that may be vertically extended[18].

By combining the two kinds of extension, we have another version of the starting analogy model, the one based on four elements. VE is analogy as categorization. In the dynamical process of analogy-building and supposing we do not have two parallel concepts, but we have to find an analogical one, or rather, another concept/domain/situation that is analogous to the initial one, we choose a superordinate category in which we want to include something (a fact, a situation, an element). We see it as a member of that particular category, that is, as analogous to other members of the same category for features that are relevant in the context of analogy we are making. After this first step, features enable other features that guide the search for something parallel (a fact, a situation, an element) in another domain inside the general category we have chosen. The HE give rises to concept innovation or a new concept, which becomes the concrete base for other VE. This

[18] The notion of "unification" in language and semantic context has been emphasized, among others, by Jackendoff (1997).

is the sense in which HE is an analogy between analogies, an analogy between different things that are categorized in the same way. The HE allows the emerging of a new category core, which becomes the concrete level for new abstractions (new VEs).

So, the process of analogy-making, based on the four-element model, proceeds in a dynamical way with an alternation of VE and HE (VE – HE – VE – HE – VE – HE …), which mirrors the alternation between concrete and abstract elements. Both are required to build the analogical correspondence leading to concept formation or innovation, as every new conceptual correspondence, which is relevant to the analogy-making process, is a new concept, in which the analogy, so to speak, introduces "old", known concepts by categorizing them in a new way. In the dynamical interaction between concreteness and abstraction, abstraction lies on the second level and requires a first level of concreteness. The four elements involved in the process generate an overall model with all the relevant concepts involved, in which some of them are the concrete for the others concepts involved. In the supervised analogy-making process, as occurs in those in scientific discoveries, establishing what is concrete and what is abstract depends on the constraints chosen each time for the overall model building. In mundane contexts, the relative concrete and abstract features are an outcome of perceptual and memory retrieval processes.

Although the way in which Hofstadter and Sander develop their theory is not comparable, they provide support for the dynamical explanation of the analogy-making process, especially in scientific discovery. They stress the prominence of concrete/abstract relation and alternation in mathematical progress, especially as regards complex and imaginary numbers: "it would be hard to overstate the importance of geometrical *visualization* in mathematics in general, which is to say of attaching geometrical interpretations to entities whose existence would otherwise seem counterintuitive, if not self-contradictory. The acceptance of abstract mathematical entities is always facilitated if a geometrical way of envisioning them is discovered; any such mapping confers on these entities a *concreteness* that makes them seem much more plausible." (Hofstadter and Sander 2013: 443 [emphasis added])[19]. They extend these remarks to mathematical discovery in general: "The modus operandi of mathematical abstraction is [...]: you begin with a "familiar" idea (that is, familiar to a sophisticated mathematician but most likely totally alien to an outsider), you try to distill its *essence*, and then you try to find, in some other area of mathematics, something that shares this same distilled essence. An alternative pathway towards *abstraction* involves recognizing an analogy between two structures in different domains, which then focuses one's attention on the *abstract structure* that they share. *This new abstraction then becomes a "concrete" concept that one can study*, and this goes on until someone realizes that this is far from the end of the line, and that *one can further generalize the new concept* in one of the two ways just described. And thus it goes…" (Hofstadter and Sander 2013: 449

[19] For a similar treatment of this topic from the point of view of conceptual blending see Fauconnier and Turner (2002: 270-274).

[emphasis added]). The "two ways just described" can be seen as two ways of regarding the dynamical VE-HE-VE… described above. Thus, the formation of the concrete concept appears to be closely related to a sort of "affordance" of the abstraction process in category alignment, as if the abstraction almost spontaneously emerges from the situation we are faced with when we apply our four-element model of analogy.

This model also guides the choice of level structure that is relevant to the analogical process. HE is on the same level while VE is on two different levels, but the level selection is guided by context pressures and dynamical model building. For example, we may place a leg and an arm on the same level and consider them as playing the same role in an analogical situation (for instance, a diagnosis process); by contrast, a leg and a limb are on a different level because a leg is a member of the category limb, as it is an arm, and the step from a leg to a limb is a VE, an abstraction process due to context pressures. But nothing prevents us from conceiving a situation in which an arm and a leg are on different levels and the leg is a general category to which the arm and other things belong (for instance, a tale about people that move on all fours).

Hofstadter and Sander provide many examples of such analogies in physics by trying to reconstruct the Einsteinian analogies in his processes of discovery. For example, in the extension of the Galileian principle of relativity to special relativity, Einstein made vertical and horizontal mental moves (Hofstadter and Sander 2013: 465-468), which can be schematized as follows:

1. Principle of Relativity in Mechanics (from Galileo)
2. Mechanics <==> Electromagnetism (HE)
3. Mechanics $\wedge$ Electromagnetism
4. Physics
5. Principle of Relativity in Physics (VE by unification)

where the HE step, the correspondence between mechanics and electromagnetism, leads to their unification, which is equivalent to physics (the step from 3 to 4), which in turn gives rise to the result of extending the principle of special relativity to any kind of physical experiment, for example by asking oneself how optical and electromagnetic phenomena behave in motion (actually, in the same way as in rest). And the analogy was made explicit by Einstein himself[20]. Likewise, the step from special relativity to general relativity can be seen as the outcome of another analogous extension, involving the indistinguishability of an accelerating reference frame from a non-accelerating reference frame as regards, first, any kind of mechanical experiments, and then any kind of physical experiments[21]. Another extension – actually, two VEs from mechanics to physics and from non-

[20] "That a principle of such broad generality should hold with such exactness in one domain of phenomena, and yet should be invalid for another is *a priori* not very probable" (Einstein 1920: 17).

[21] Through some thoughts experiments, such as the space lab pulled by a rocket and the ray of light crossing an accelerating lab in a gravitational field. For a discussion see Hofstadter and Sander (2013: 490-495).

accelerating reference frames to every reference frame – gives rise to another analogy as categorization, which forms the basis of a new discovery and a new theory.

6 Conclusion

In this paper, I have put together some ideas about models, analogy and concepts in an attempt to show how the model of analogy works and can be understood in the logic and cognitive fields, in what way models are part of analogy, and how analogy-making and analogical reasoning are consequently a kind of model-based reasoning.

In the first part, I discussed the distinction of analogy in logic and cognitive science, showing that these different fields of research share the same model, which is based on four elements. But while logic deals with analogy from a static viewpoint, cognitive science has become increasingly interested in the dynamical processes underlying analogy-making. In the second part, I discussed Nersessian's proposal of hybrid analogies involving incremental model building, which leads to conceptual innovation. Models exploited by analogy in scientific experiments are, in fact, conceptual structures built by means of constraints stemming from different domains involved in the analogy process. In the third part, I debated the idea that analogy-making is based on a construal process, a type of categorization, which has the consequence of unifying, at least partially, analogy and categorization. In the fourth part, I discussed Hofstadter and Sander's theory that analogy is always categorization and vice versa, a very general cognitive process concerning every conceptual cognitive process, from high-level perception to scientific discovery. I have tried to show how this theory is based on concept extension in two different perspectives, VE and HE, each of which is a different form of analogy as categorization. I have also tried to provide a model of dynamical development of relation between concreteness and abstraction, which is involved in the analogy-making process as well as in concept innovation or formation. This dynamical development is not described in the same way in Hofstadter and Sander's theory, though it is consistent with it.

The two kinds of conceptual extensions are consistent with the general four-element model of analogy, involving horizontal and vertical levels of correspondence designed to capture the relationship between concrete and abstract, which is unavoidable in an analogy, even in relative terms. They are also involved in dynamical process of concept innovation and concept creation. This is especially true for HE, which follows the VE process of abstraction and is how new concepts are formed, concepts which in turn yield new abstraction processes. This is why I have claimed that HE produces an analogy between analogies, namely categorizations. Conceptual structures involved in the two kinds of extensions are based on a hybrid model, a conceptual representation built from two different domains involved in the analogy-making process. Model mediation by hybrid models is consequently always categorization (concept innovation or formation) involving analogy between analogies of different domains in the HE-VE-HE… dynamics.

Many things have yet to be understood regarding analogical processes and analogy, the connection between analogy as inference and analogy as dynamical/representational/semantic process, and the way in which concepts and conceptual structures are involved in this capability of reasoning and thought. Difficulties are also encountered when trying to conceive suitable experiments to get an insight into the range of problems raised by analogy. This may, along with the construction of cognitive models and architectures that include the problem of categorization, represent an interesting challenge for further research. Finally, it could be successful dealing with this set of problems in the framework of situated cognition and external representations (Magnani 2009), at the same time trying to explain how we use stored knowledge for producing freshly knowledge, understanding physical world situations and transferring external entities in internal representations to have new abstract models, which nevertheless we use in interaction with external perceptions and representations.

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