



# On the monodromy of irreducible symplectic manifolds

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## ABSTRACT

Exploiting recent results on the ample cone of irreducible symplectic manifolds, we provide a different point of view for the computation of their monodromy groups. In particular, we give the final step in the computation of the monodromy group for generalised Kummer manifolds and we prove that the monodromy of O’Grady’s ten-dimensional manifold is smaller than what was expected.

## 1. Introduction

Recently, there have been several results on the ample (or Kähler) cone of symplectic manifolds, generalising the well-known case of  $K3$  surfaces. This was mainly done through the study of stability conditions on derived categories of surfaces, which has been completed in several cases. This yielded a wall-and-chamber decomposition of the positive cone of moduli spaces of stable objects on those derived categories. Basically, there is a set of divisors whose orthogonals divide the positive cone into chambers and one such chamber is the ample cone. Moreover, all other chambers can be reached by elementary birational transformations and reflections on contractible divisors. Such a result was obtained by Yoshioka [Yos12] for smooth moduli spaces on abelian surfaces, by Bayer and Macrì [BM14] for smooth moduli spaces on  $K3$  surfaces and by Meachan and Zhang [MZ16] for some well-behaved singular moduli spaces on  $K3$  surfaces. It was implicitly expected that such a wall-and-chamber decomposition exists for all irreducible symplectic manifolds, as several results pointed out; see for example [Huy03b] for the general behaviour of birational maps between irreducible symplectic manifolds and [HT09] for a detailed analysis of one of the known fourfolds. The wall-and-chamber decomposition obtained from the categorical point of view was also recently extended by Bayer, Hassett and Tschinkel [BHT15] for all projective manifolds of  $K3^{[n]}$ -type and independently also for the other deformation classes by the author [Mon15] and by Amerik and Verbitsky [AV14]. In this paper we will exploit an interesting byproduct of [Mon15] to compute the monodromy group of irreducible symplectic manifolds. The key idea is that parallel-transport Hodge isometries preserve the wall-and-chamber decomposition of the positive cone. Knowing such a decomposition, it is possible to compute the group of isometries preserving it. This gives an upper bound for the group of parallel-transport Hodge isometries and therefore also for the monodromy group. We will apply this idea to two of the known ex-

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amples: deformations of generalised Kummer manifolds and of O’Grady’s ten-dimensional manifold [O’G99]. In the first case, thanks to previous computations by Markman [MM12], we have a lower bound for the monodromy group and the upper bound we obtain coincides with it; therefore we obtain the expected monodromy group. In the second case, as stated in [Mar11, Conjecture 10.7], the monodromy group is expected to coincide with all orientation-preserving isometries. However, we obtain an orientation-preserving isometry which does not preserve a wall of the decomposition of the positive cone, therefore disproving the conjecture.

## 2. Notation and preliminaries

Let  $X$  be an irreducible symplectic manifold, that is, a simply connected Kähler manifold such that  $H^{2,0}(X)$  is generated by a symplectic form. For a detailed description of the properties of irreducible symplectic manifolds we refer to Huybrechts’ survey [Huy03a]. We denote by  $A_X$  the discriminant group of its second cohomology, that is,  $H^2(X, \mathbb{Z})^\vee / H^2(X, \mathbb{Z})$ , where the lattice structure is given by the Beauville–Bogomolov form  $b(\cdot, \cdot)$ . We also identify  $H_2(X, \mathbb{Z})$  with a subset of  $H^2(X, \mathbb{Q})$  by the natural embedding of the dual of a lattice  $L$  inside  $L \otimes \mathbb{Q}$ . For any element  $D$  of  $H^2(X, \mathbb{Z})$ , we denote by  $\text{div}(D)$  a positive generator of the ideal  $b(D, H^2(X, \mathbb{Z}))$ . We remark that  $D/\text{div}(D)$  is primitive in  $H_2(X, \mathbb{Z})$ .

We denote by  $\text{Mon}^2(X)$  the group of parallel-transport operators on the second cohomology of  $X$ , that is, the group of isometries of  $H^2(X, \mathbb{Z})$  obtained by parallel transport. We call it the monodromy group. Likewise,  $\text{Mon}^2(X, Y)$  denotes parallel-transport operators between two irreducible symplectic manifolds. We remind the reader that  $\text{Mon}^2(X)$  is a deformation invariant. By  $\text{Hdg}(X, Y)$  and  $\text{Hdg}(X)$  we denote respectively Hodge isometries between  $H^2(X, \mathbb{Z})$  and  $H^2(Y, \mathbb{Z})$  and Hodge isometries of  $H^2(X, \mathbb{Z})$ . The group  $O^+(H^2(X, \mathbb{Z}))$  denotes orientation-preserving isometries. For known results on the monodromy group and related statements, we refer to Markman’s survey [Mar11].

The positive cone  $\mathcal{C}_X$  of an irreducible symplectic manifold  $X$  is the connected component of the cone of positive (with respect to the Beauville–Bogomolov form) real  $(1, 1)$  classes containing the Kähler cone  $\mathcal{K}_X$ . The birational Kähler cone  $\mathcal{BK}_X$  is the pullback of the Kähler cones of all irreducible symplectic manifolds  $X'$  birational to  $X$ . It is a disjoint union of convex cones and its closure coincides with the movable cone if  $X$  is projective. Given a symplectic surface  $S$  and a polarisation  $H$ , we denote by  $M_v(S, H)$  the moduli space of  $H$ -stable sheaves on  $S$  with Mukai vector  $v \in H^{2*}(S)$ . Here the Mukai vector of a sheaf  $\mathcal{F}$  is  $(\text{rk}(\mathcal{F}), c_1(\mathcal{F}), c_1(\mathcal{F})^2/2 - c_2(\mathcal{F}) + \text{rk}(\mathcal{F}))$  if  $S$  is a  $K3$  surface and  $(\text{rk}(\mathcal{F}), c_1(\mathcal{F}), c_1(\mathcal{F})^2/2 - c_2(\mathcal{F}))$  if  $S$  is abelian. If  $S$  is a  $K3$  surface and  $M_v(S, H)$  is smooth, then the latter is an irreducible symplectic manifold and its second cohomology is isometric to  $v^\perp \subset H^{2*}(S, \mathbb{Z})$ . The same holds also for abelian surfaces after replacing  $M_v(S, H)$  with its Albanese fibre  $K_v(S, H)$ . A similar construction can be extended to objects in the derived category of such surfaces, giving more deformations of the above-mentioned manifolds; see [BM14] and [Yos12].

We call manifold of Kummer  $n$ -type any deformation of the Albanese fibre of  $\text{Hilb}^{n+1}(S)$ , where  $S$  is an abelian surface. Such a fibre is  $K_{(1,0,-n-1)}(S, H)$ . These are irreducible symplectic manifolds of dimension  $2n$ . The second example we consider is a symplectic resolution (and its smooth deformations) of a moduli space of sheaves on a  $K3$  surface with Mukai vector  $v = 2w = (2, 0, -2)$ . Such an example was constructed by O’Grady [O’G99]. Finally, we denote by  $\Lambda_{24}$  the lattice isometric to  $H^{2*}(K3, \mathbb{Z})$  and by  $\Lambda_8$  the lattice  $H^{2*}(A, \mathbb{Z})$  for an abelian surface  $A$ . They are respectively isometric to  $U^4 \oplus E_8(-1)^2$  and  $U^4$ , where  $U$  is the hyperbolic lattice and  $E_8$  is

the unique positive even unimodular lattice of rank eight.

### 3. Decomposition of the positive cone and monodromy

The decomposition of the positive cone we are concerned with is a set of open chambers whose boundary walls are given by the orthogonals to the following divisors.

**DEFINITION 3.1.** Let  $X$  be an irreducible symplectic manifold, and let  $D$  be a divisor on  $X$ . Then  $D$  is called a wall divisor if  $D^2 < 0$  and  $h(D^\perp) \cap \mathcal{BK}_X = \emptyset$  for all parallel-transport Hodge isometries  $h$ .

The most important fact about such a decomposition is that one of the open chambers is the Kähler cone. Indeed, let  $R$  be an extremal ray of the Mori cone of  $X$ , and let  $D$  be a primitive divisor such that  $D/\text{div}(D) = R$ . Then in [Mon15, Lemma 1.4] it is proven that  $D$  is a wall divisor. The second fundamental property of wall divisors is that they are preserved under parallel-transport Hodge isometries.

**THEOREM 3.2** ([Mon15, Theorem 1.3] and [AV14, Theorem 1.17]). *Let  $X$  and  $Y$  be irreducible symplectic manifolds, and let  $\mathcal{D}_X$  and  $\mathcal{D}_Y$  be the sets of wall divisors of  $X$  and  $Y$ . Then  $\text{Mon}^2(X, Y) \cap \text{Hdg}^2(X, Y)$  sends  $\mathcal{D}_X$  into  $\mathcal{D}_Y$ .*

The positive cone  $\mathcal{C}_X$  of an irreducible symplectic manifold is therefore divided in walls and chambers by the orthogonal to any element of  $\mathcal{D}_X$ . Using stability conditions on the derived category of a symplectic surface, this wall-and-chamber decomposition was determined for several irreducible symplectic manifolds.

Let  $M_v(S, H)$  be the moduli space of stable sheaves with primitive Mukai vector  $v$  on the  $K3$  surface  $S$  with respect to a given  $v$ -generic polarization  $H$ . Let  $D$  be a divisor of  $M_v(S, H)$  with  $D^2 < 0$ . Let  $T$  be the rank two hyperbolic primitive sublattice of  $\Lambda_{24}$  containing  $v$  and  $D$ .

**THEOREM 3.3** ([BM14, Theorems 5.7 and 12.1]). *Let  $D$ ,  $v$  and  $T$  be as above, then  $D$  is a wall divisor if and only if one of the following holds:*

- *There exists  $w \in T$  such that  $w^2 = -2$  and  $0 \leq (w, v) \leq v^2/2$ .*
- *There exists  $w \in T$  such that  $w^2 \geq 0$  and  $w^2 < (w, v) \leq v^2/2$ .*

Similarly, a decomposition of the ample cone for the Albanese fibre of moduli spaces of stable objects on the derived category of an abelian surface has been obtained by Yoshioka.

Let  $K_v(S, H)$  be the Albanese fibre of the moduli space of stable sheaves with primitive Mukai vector  $v$  on the abelian surface  $S$  with respect to a given  $v$ -generic polarization  $H$ . Let  $D$  be a divisor of  $K_v(S, H)$  with  $D^2 < 0$ . Let  $T$  be the rank two hyperbolic primitive sublattice of  $\Lambda_8$  containing  $v$  and  $D$ .

**THEOREM 3.4** ([Yos12, Proposition 1.3 and Section 3.2]). *Let  $v$ ,  $D$  and  $T$  be as above, then  $D$  is a wall divisor of  $K_v(S, H)$  if and only if there exists  $w \in T$  such that  $w^2 \geq 0$  and  $w^2 < (w, v) \leq v^2/2$ .*

Meachan and Zhang extended these results to special singular moduli spaces (and their symplectic resolutions), obtaining certain wall divisors for O’Grady’s ten-dimensional example. Let  $M_{2w}(S, H)$  be a ten-dimensional singular moduli space of stable sheaves on a  $K3$  surface  $S$  with respect to a  $2w$ -generic polarisation  $H$  and let  $X$  be its symplectic resolution of singularities. Let  $D$  be a divisor on  $X$  obtained by pullback from a divisor  $D'$  on  $M_{2w}(S, H)$ . Let  $T$  be the rank two hyperbolic primitive sublattice of  $\Lambda_{24}$  containing  $w$  and  $D'$ .

**THEOREM 3.5** ([MZ16, Theorems 5.3 and 5.4]). *We keep the notation as above. Then  $D$  is a wall divisor on  $X$  if and only if one of the following holds:*

- *There exists  $s \in T$  such that  $s^2 = -2$  and  $(s, w) = 0$ .*
- *There exists  $s \in T$  such that  $s^2 = -2$  and  $(s, w) = 1$ .*

*Let  $D$  be a primitive generator of  $\langle w, s \rangle \cap w^\perp$ . Then  $D$  is a wall divisor on  $X$ .*

We remark that these are the wall divisors for the moduli space  $M_w(S, H)$ , which sits naturally inside the singular locus of  $M_{2w}(S, H)$ .

#### 4. Kummer $n$ -type manifolds

In this section we compute the monodromy group of manifolds of Kummer  $n$ -type using two results. The first is Markman’s computation of the intersection of the monodromy group of a Kummer  $n$ -type manifold  $X$  and the group of isometries acting as  $\pm 1$  on  $A_X$ . The second is Yoshioka’s result on the Kähler cone of generalised Kummer manifolds. We use the notation of [MM12]: let  $\mathcal{W}(X)$  be the subgroup of  $O^+(H^2(X, \mathbb{Z}))$  acting as  $\pm 1$  on  $A_X$ . It is an order  $2^a$  subgroup of the group of orientation-preserving isometries, where  $a + 1$  is the number of prime factors of  $n + 1$ . Let  $\chi$  denote the character corresponding to the action on  $A_X$ . Let  $\mathcal{N}_X$  be the kernel of  $\det \circ \chi: \mathcal{W}_X \rightarrow \{\pm 1\}$ .

**PROPOSITION 4.1** ([MM12, Corollary 4.8]). *Let  $X$  be a manifold of Kummer  $n$ -type. Then  $\text{Mon}^2(X) \cap \mathcal{W}_X = \mathcal{N}_X$ .*

**PROPOSITION 4.2** ([Yos12]). *Let  $X = K_v(S, H)$  for an abelian surface  $S$ . Then the movable cone of  $X$  is cut out by all primitive isotropic elements  $w \in H^{2*}(S, \mathbb{Z})$  of type  $(1, 1)$  such that  $(v, w) = 1$  or  $2$ .*

*Proof.* Let  $w$  be in the above set of isotropic classes. Then  $v$  and  $w$  generate a lattice  $T$  which satisfies the hypothesis of Theorem 3.4, so any generator of  $v^\perp \cap T$  is a wall divisor. Notice that such a divisor  $D$  is  $v - (2n + 2)w$  if  $(v, w) = 1$  and  $v - (n + 1)w$  in the second case. By [Yos12, Theorem 3.31], there is a divisorial contraction of some birational model of  $X$  associated with such elements and those elements cut out the movable cone.  $\square$

We remark that the two cases above belong to two different orbits of the isometry group of  $H^2(X, \mathbb{Z})$ , since the associated wall divisors have different divisibility. We will call them divisorial contractions of type I when  $(v, w) = 1$  and type II otherwise. In particular, this type of subdivision is preserved for all wall divisors in the same isometry orbit.

**THEOREM 4.3.** *Let  $X$  be a manifold of Kummer  $n$ -type. Then  $\text{Mon}^2(X) = \mathcal{N}_X$ .*

*Proof.* To prove this statement, it is enough to prove that a wall-preserving isometry must act as  $\pm 1$  on the discriminant group  $A_X$ . Without loss of generality, we can do this computation on a generalised Kummer manifold  $X$ . We denote by  $2\delta = (2, 0, 2n + 2)$  the class of the exceptional divisor of the Hilbert–Chow morphism and set  $v = (1, 0, -n - 1)$ . Note that  $\delta^\perp \subset H^2(X, \mathbb{Z})$  is unimodular. We let  $s = (v - \delta)/(2n + 2) \in H^{2*}(S, \mathbb{Z})$ . The lattice  $T = \langle v, s \rangle$  is isometric to  $\begin{pmatrix} 2n+2 & 1 \\ 1 & 0 \end{pmatrix}$ , which means that  $\delta$  is a divisorial contraction of type I. Let  $g$  be a monodromy operator in  $\text{Mon}^2(X)$  sending  $\delta$  to another wall divisor. We have  $g(\delta) = k\delta + (2n + 2)l$ , where  $l \in \delta^\perp$ . Let  $[k] \neq \pm[1]$  in  $A_X$ ; up to a sign change we can freely suppose  $2 \leq k < n + 1$ . Let  $T_g$  be the saturation of the lattice generated by  $v$  and  $g(\delta)$  in  $H^{2*}(S, \mathbb{Z})$ . As  $w := (kv - g(\delta))/(2n + 2)$

is an element of  $T_g$ , this lattice is still isometric to  $T \cong U$ . However, we have changed the pairing of the set of isotropic elements with  $v$ . Indeed, for any  $2 \leq k < n + 1$ , we have that  $v - g(\delta) = -(k - 1)v + (2n + 2)w$  and  $v + g(\delta) = (k + 1)v - (2n + 2)w$  are isotropic and divisible by  $2r = \text{GCD}(k - 1, 2n + 2)$  and  $2t = \text{GCD}(k + 1, 2n + 2)$  respectively. This yields two isotropic vectors which have pairing respectively  $n+1/t$  and  $n+1/r$  with  $v$ . Moreover, all isotropic elements of  $T_g$  are multiples of  $v + g(\delta)$  or  $v - g(\delta)$ , hence there are no isotropic elements with pairing 1 with  $v$ . By [Yos12, Proposition 3.27], this implies that the divisor  $g(\delta)$  is not of type I, hence  $g \notin \text{Mon}^2(X)$ .  $\square$

As an explanatory example, consider the case  $n = 5$  and  $k = [5]$ . We then have  $r = 2$  and  $t = 3$ .

*Remark 4.4.* The above computation implies in particular that a manifold  $X$  whose second cohomology is Hodge isometric to that of  $K_v(S, H)$ , for some  $S, v$  and  $H$ , is itself a moduli space of stable objects on the derived category of the same abelian surface  $S$ . This was conjectured by Wandel and the author in [MW15]. The missing ingredient to extend the proof of [MW15, Proposition 2.4] is precisely the above computation of the monodromy group.

### 5. O’Grady’s ten-dimensional manifold

In this section we focus on the deformation class of the ten-dimensional sporadic irreducible symplectic manifold constructed by O’Grady [O’G99]. We cook up a peculiar example and we combine it with Meachan and Zhang’s results to obtain a restriction on the monodromy group. In this section,  $S$  is a  $K3$  surface,  $v = 2w$  is a non-primitive Mukai vector of square 8, and  $H$  is a  $v$ -generic polarisation on  $S$ . Furthermore,  $X$  is the symplectic resolution of  $M_v(S, H)$ . The manifold  $X$  is deformation equivalent to O’Grady’s ten-dimensional manifold. We will also use a natural embedding given by Perego and Rapagnetta [PR13] for the second cohomology of such manifolds.

**THEOREM 5.1** ([PR13, Theorem 1.7]). *We keep the notation as above. There exist a pure weight two Hodge structure on  $H^2(M_v(S, H), \mathbb{Z})$  and a Hodge isometry  $v^\perp = w^\perp \rightarrow H^2(M_v(S, H), \mathbb{Z})$ . Moreover, the pullback map  $H^2(M_v(S, H), \mathbb{Z}) \rightarrow H^2(X, \mathbb{Z})$  is injective.*

*Example 5.2.* Let  $S$  be a projective  $K3$  surface with a symplectic automorphism  $\varphi$  of order three, and let  $X$  be a symplectic resolution of the moduli space of stable sheaves  $M_v(S, H)$  for a  $\varphi$ -invariant ample class  $H$ . Here  $v = 2w$ ,  $v^2 = 8$  and  $\varphi(v) = v$ . As remarked in [MW15, Proposition 4.3], the morphism  $\varphi$  induces a symplectic automorphism  $\widehat{\varphi}$  of  $X$ . Let  $T_{\widehat{\varphi}}(X)$  be the invariant sublattice for the induced action of  $\widehat{\varphi}$  on  $H^2(X, \mathbb{Z})$ , and let  $S_{\widehat{\varphi}}(X)$  be its orthogonal. Again in [MW15, Proposition 4.3] it is proven that  $S_{\widehat{\varphi}}(X) \cong S_\varphi(S)$ . Notice in particular that  $S_{\widehat{\varphi}}(X) \subset H^{1,1}(X)$ , since the symplectic form is invariant, and that there exists a  $\widehat{\varphi}$ -invariant Kähler class  $\omega$ . The lattice  $S_\varphi(S)$  has been explicitly computed by Garbagnati and Sarti [GS07]. In particular, it contains an element  $F$  of square  $-10$ , obviously orthogonal to the Kähler class  $\omega$ . Therefore  $F$  is not a wall divisor on  $X$ . Note that, as  $F$  lies in the cohomology of  $X$  canonically isometric to the cohomology of the underlying  $K3$  surface,  $\text{div}(F) = 1$ . (It is contained in a unimodular sublattice of  $H^2(X, \mathbb{Z})$ .)

**THEOREM 5.3.** *Let  $X$  be a manifold deformation equivalent to O’Grady’s ten-dimensional irreducible symplectic manifold. Then  $\text{Mon}^2(X)$  is strictly smaller than  $O^+(H^2(X, \mathbb{Z}))$ .*

*Proof.* Let  $S$  be a very general  $K3$  surface having a symplectic automorphism of order three and an invariant polarisation  $H$  of degree two. Let  $X$  be the resolution of singularities of a moduli space of stable objects in the derived category of  $S$  with Mukai vector  $v = 2w = (2, 0, -2)$ . Let  $s = (2, H, 1) \in H^*(S, \mathbb{Z})$ . We have  $s^2 = -2$  and  $(s, w) = 1$ . Let  $T = \langle w, s \rangle$ . Let  $D := (3, 2H, 3)$ , then  $D$  is a generator of  $T \cap v^\perp$ . Thus,  $D$  is a wall divisor on  $X$ , as follows from Theorem 3.5. Notice that  $D$  has divisibility 2 when considered as an element of  $H^2(M_v(S, H), \mathbb{Z})$ , but it has divisibility 1 inside  $H^2(X, \mathbb{Z})$ . Let now  $F$  be (as in the previous example) a class of square  $-10$  inside the coinvariant lattice of the induced order three symplectic automorphism on  $X$ . We want to apply Eichler's criterion [GHS10, Lemma 3.5] to find an orientation-preserving isometry of  $H^2(X, \mathbb{Z})$  sending  $D$  to  $F$ . The lattice  $H^2(X, \mathbb{Z})$  contains at least two copies of  $U$ , and the elements  $D$  and  $F$  have the same square; moreover they both have divisibility 1, thus  $[D/\text{div}(D)] = [F/\text{div}(F)] = 0$  in the discriminant group  $H^2(X, \mathbb{Z})^\vee/H^2(X, \mathbb{Z})^\vee$  and Eichler's criterion applies. This implies that the group of isometries preserving wall divisors is strictly smaller than the group of isometries. Hence, the monodromy group is also strictly smaller than  $O^+(H^2(X, \mathbb{Z}))$ .  $\square$

As said before, this differs from what was previously expected; see [Mar11, Conjecture 10.7].

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