Competitive Customer Poaching with Asymmetric Firms

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Abstract

Conditioning the pricing policies on purchase history is proven to generate a cutthroat price competition enhancing consumer surplus. This result typically relies on a framework where competitors are assumed to be symmetric. This paper demonstrates that under significant asymmetries of competing firms, the strong firm trades off current market share for future market share and the weak firm does the opposite. This inter-temporal market sharing agreement generates unidirectional poaching and entails new and distinctive welfare implications. In particular, if consumers are sufficiently myopic, price discrimination softens price competition in relation to uniform pricing, overturning the conclusion of previous studies.

Keywords: Asymmetric price discrimination, Customer poaching, Price discrimination based on purchase history, Privacy
JEL classification: L11,L13, D43

1. Introduction

Recently, the understanding of the consequences of customer recognition has been an important topic that stimulated public debate. On the one hand, the development of big data and the availability of consumers' sensible information to firms have raised issues concerning consumers' privacy. On the other hand, the improvements in obtaining and processing such information enable firms to infer preferences of consumers and to discriminate prices based on their past purchase behaviour. For this reason, a strategy of dynamical price discrimination is being used frequently in many markets, such as the market of telecommunication and those of the internet, storage, streaming and payment services, where consumers are often rewarded with advantageous deals in order to switch providers.

These practices have captured the attention of many economists,[1] whose main concern has been the understanding of the consequences of such strategies on firms' profits, consumer surplus and price levels. Essentially, the economic literature agrees on the conclusion that this behaviour-based price discrimination (BBPD) reduces firms' profitability, as firms tend to

[1] Starting from Chen (1997), Villas-Boas (1999) and Fudenberg and Tirole (2000). Esteves (2009) provides an extensive and up-to-date survey on the existing literature in this field.

compete fiercely for switchers, with a consequent benefit for consumers in relation to uniform pricing. However, most of these works rely on the assumption that sellers are perfectly symmetric. Symmetry implies that competing firms show identical incentives both to attack the rival's territory and to defend their own. As a result, they end up attracting the same number of new consumers, who are offered a lower price.

As a matter of fact, markets where BBPD is used are often characterized by some level of asymmetry. In many instances, some firm enjoys an advantage given by either having been a public monopoly or exploiting some brand name developed in other related markets in the past. In the telecommunication market, for example, the reforms of privatization and the consequent opening to competition started in the late 80's in all OECD countries generated an asymmetric contention between incumbent and new entrants in both traditional land-line segment and in the emerging mobile & ISP ones. After many years of competition, former incumbents often maintain a leading position or still play a very important role in the aforesaid industries, even though competitors were to some extent accommodated. More recently, the markets of online content providers are initially dominated by some players already known in other markets. Google and Amazon powerfully entered the market of digital books, exploiting their brand names and a consumers' favor. Similarly, Netflix's successful transformation from a DVD rental company to the leading streaming video firm is evidence of positive spillovers from the first to the second market.

The present paper contributes to the ongoing debate by putting forward the following argument: if contenders are sufficiently asymmetric, the inter-temporal incentives they have are divergent, as strong firms can somehow temporize whereas weak firms need to compete strongly immediately to recover the competitive disadvantage. This leads to the endogenous emergence of an inter-temporal agreement that takes the shape of a *de facto* pseudo-collusive conduct. In the two-firm model proposed by the paper, the strong firm trades current market share for future market share and the weak firm opts for the opposite strategy.

This finding results from a two-period Hotelling model where two firms set prices and consumers buy their preferred good. The observation of consumers' first-period buying choices let firms identify old and new consumers and discriminate second-period prices accordingly. The firms take into account how the price competition today affects the "cost" of attracting switchers tomorrow. Clearly, the trade-off is given by the fact that a high market share today makes the attraction of the residual consumers very costly, as the latter would ask for a very small price in order to switch. Consumers evaluate firms' products differently, and are therefore willing to pay a price premium for the good offered by one of the two firms. Hereafter, we will name the firm enjoying this premium as the strong/big firm, and the rival as the weak/small firm.

A higher price premium induces the strong firm to use extreme pricing strategies. For a given price set by the weak rival, the best reply is either (i) to attack, focusing only on current market share, or (ii) to accommodate the competitor, focusing on margins and postponing an attack until tomorrow. Clearly, when the small rival sets a low price, the attack turns out to be too demanding. For this reason, the big firm prefers to lay down arms today, bearing in mind the fact that this brings about cheap switching tomorrow. Meanwhile, the weak firm anticipates that the price premium the rival enjoys is associated with a more arduous attraction of the switcher tomorrow. This pushes it to pursue a market-share focusing strategy.

Unsurprisingly, if firms are assumed to be sufficiently symmetric, the inter-temporal trade-off is solved by both firms in a balanced way, and the paper accords with the previous literature of BBPD with symmetric competitors.[2] As soon as a sufficient level of asymmetry is reached, the equilibrium takes the following form: the strong seller adopts a margin-focusing strategy and the weak firm does the opposite. As a consequence, when discriminating prices are offered, buyers move only from the weak to the strong firm (One-Direction Switching, ODS). In addition, price discrimination may cause the exit of the small firm, which would have been active under uniform pricing.

With these mechanisms in mind, the implications on profits and consumer surplus of the inter-temporally unbalanced equilibrium are straightforward and depend ultimately on the level of sophistication of consumers. If the latter are sufficiently myopic, BBPD becomes a very powerful tool for the strong seller, which is given the opportunity to decide on the destiny of the rival. At equilibrium, asymmetry helps the strong firm to make high margins, and *lets the weak firm enjoy a lessened competition.* Oppositely, when customers are sufficiently sophisticated to anticipate the future offers of firms, the need of the weak firm to attack the market in the first period prevails over that the intention of the strong firm to accommodate, so to strengthen first-period competition. The final outcome will thus favor consumers since the weak firm sets a very low price and the *strong firm suffers the increased competition.*

If one accepts the not-too-stringent view that consumers are sufficiently myopic, the paper offers a new device to antitrust authorities about the consequences of BBPD. In particular, BBPD may represent a way for firms to engage in an inter-temporal market sharing agreement, which allocates the surplus over time: the big seller trades today's market share for that of tomorrow and the weak firm does exactly the opposite. This turns out to be ex-post preferred to the uniform pricing as it relaxes price competition in the first period. On the one hand, BBPD diminishes the gains from undercutting the rival because a high market share in the first period is accompanied by a disadvantageous position in the late competition. On the other hand, the observation of consumers' identity enlarges the set of available strategies of the strong firm, which leads therefore the market more severely than if price discrimination was not at hand. As a result, the leading firm first accommodates the rival and then attacks the latter on its turf, whereas the small firm first makes profits and then accommodates the attack of the strong rival. This helps firms keep prices higher than the uniform price, with a benefit on industry profits.[3]

[2]The model replicates the results of Fudenberg and Tirole (2000) if the price premium is assumed to be zero.

[3]The complex relation between current and future profits is the key to understand how firms can tacitly collude. Among other factors, Ivaldi et al. (2003) mention two aspects as crucial to understand tacit collusion,

The paper evidently draws on the literature studying price discrimination in oligopolies, which generally agrees on a negative effect on firms' profits compared to uniform pricing. This is because the typical positive effect in the monopoly case (the so-called *Surplus Extraction* effect) is accompanied and often overturned by an intensification of competition in oligopolistic markets (*Business Stealing* effect). As a matter of fact, the information about brand preferences of consumers can be used in two different ways when markets are duopolistic. On the one hand, each firm aims to charge consumers belonging to its "strong" market (i.e., exhibiting relatively strong brand preference) with a high price, thus exploiting information in order to extract their surplus. On the other hand, a given seller also seeks to set a low price in its "weak" market to steal the rival's business. In the jargon used by Corts (1998), the market exhibits best-response asymmetry, as the"strong" market for a firm is "weak" for the competitor. In these cases, firms' dominant strategy is to charge low prices in the rival's "strong" market and this, in turns, prevents the latter to fully extract surplus. In a very influential article, Thisse and Vives (1988) showed that if firms know the precise location of each consumer and can engage in perfect price discrimination accordingly, then all prices might fall in relation to uniform pricing as the more distant firm is very aggressive in each location. For given prices offered by the rival, both firms find it profitable to discriminate, but this leads to a reduction in prices in the style of a prisoner's dilemma situation.

The paper is more specifically linked to the literature on BBPD, in which firms learn consumers' preferences by observing their purchase behaviour in the past rather than have full information about their locations. In Chen (1997),Villas-Boas (1999), Fudenberg and Tirole (2000) and Esteves (2010), the observation of consumers' identities allows sellers to distinguish between "strong" market (previous buyers) and "weak" market (rival's inherited consumers), as purchase reveals how much a consumer is inclined to buy a product or another. The loss of firms and consequent gain of consumers remain: as the latter can be identified and price discrimination is permitted, both sellers have incentives to steal each other's consumers and prices fall. More recent articles have demonstrated how results may slightly or substantially differ under different settings. In a very recent paper, Colombo (2015) studies the incentives to price discrimination shown by a firm facing a discriminating competitor. He demonstrates that if consumers are myopic enough, the optimal choice is to commit to uniform prices even if the access to information about purchases of consumers is completely costless. Furthermore, Esteves and Reggiani (2014) show how increasing the demand elasticity reduces the negative impact of BBPD on firms' profits, while Chen and Pearcy (2010) demonstrate that when a weak correlation between preferences of consumers is assumed over time, BBPD will actually be beneficial to firms and detrimental to consumers.

The intuition behind the present paper is that the welfare effect of BBPD depends cru-

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which is said to be easier to sustain when "gains from undercutting are lower ", "there is significant chance that undercutting leads to rival's reaction". In the context of the present paper, both conditions are fulfilled, given that the possibility of discriminating prices and enjoying switching in the future changes completely the way firms compete in the present.

cially on the symmetry of the market: if firms are identical ex-ante and compete fiercely for switchers, they end up poaching the same number of consumers with the consequence of a lower level of prices and profits. In the analysis of their two-period model, for example, Fudenberg and Tirole (2000) need specifically to eliminate asymmetric subgames in order to provide their SPNE.[4] Namely, they do not take into account the fact that switching may occur only from the dominant to the dominated firm if inherited market shares are unbalanced in favor of the former.[5]

Other articles dealing with price discrimination in asymmetric duopolies have results directly comparable with the ones of this paper. As pointed out by Chen (2008), the effects of dynamic price discrimination change substantially from symmetric to asymmetric markets. In a considerably different approach from the present paper with regard to time horizon and consumers' preferences, he finds that price discrimination can be a tool for a low-cost firm to eliminate the less efficient competitor, and, if exit happens, consumers are worse-off compared to uniform pricing. Shaffer and Zhang (2002) propose a model where vertically and horizontally differentiated firms are allowed to (costly) target consumers with one-to-one promotions (perfect price discrimination). They find that even though promotional offers intensify price competition, these can result in a benefit in terms of market share and profits for the strong firm. In Liu and Serfes (2005), firms can costly acquire information about consumer-specific characteristics. They show that when information is not too costly, only the strong firm will buy it and engage in price discrimination, with the weak firm opting for a uniform price strategy at equilibrium. Unlike the last two articles, in the present model information cannot be acquired and price discrimination is only based on past purchase behaviour and, for strong firms' differentiation, price discrimination benefits the weak firm, as price competition is relaxed in the early stage. Gehrig et al. (2011, 2012) propose models in which the asymmetry of the firms is given by some inherited market dominance and firms are allowed to discriminate prices according to the (exogenous) purchase history of consumers.[6] Roughly speaking, their analysis is similar and allows for similar switching behaviours in relation to the subgames of the model presented hereafter, which endogenizes the purchase history of consumers.

The rest of the paper is organised as follows. The next section presents the principal elements of the model. After, sections 3 and 4 are devoted to the analysis of the two benchmarks of uniform and discriminatory pricing. The two regimes are then compared in order to provide a welfare analysis on the effects of BBPD in Section 5. Finally, Section 6 contains some concluding remarks.

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[4]From the article at page 639: "We will show that, provided that $|\theta^*|$ is not too large, the second-period equilibrium has this form: Both firms poach some of their rival's first-period customers, so that some consumers do switch providers". In their model $|\theta^*|$ represents the location of the time 1 indifferent agent in a Hotelling with firms symmetrically located around zero.

[5]See Gehrig et al. (2007) for an analysis of Fudenberg and Tirole (2000) second period with the past taken as given.

[6]In particular, Gehrig et al. (2011) provides the limit case of an entry model.

2. Description of the model

Two competing firms $i = H, L$ aim to sell a good to a population of customers assumed to be uniformly distributed along a unit segment. The firms' locations are kept fixed at the end-points of this segment: firm H is located at $l^H = 0$ and L at $l^L = 1$. The sellers' products are evaluated differently by consumers, who give a price premium for the good sold by firm H. Formally, it is assumed that $u^H \geq u^L$, where u^i denotes the utility that any consumer enjoys from buying product (sold by firm) i.

Consumers face a transportation cost normalised to 1 per unit of distance covered to reach the location of each firm. According to these assumptions, the per-period utility of an agent located at x who buys good i will be given by:

$$U(x, i) = u^i - p^i - |x - l^i|. \tag{1}$$

The firms set prices in order to maximize profits, facing a unitary cost normalised to 0 in each time period. They discount the future at a factor δ_f normalized to 1,[7] whereas consumers give value $\delta_c \in [0, 1]$ to future utilities. We refer throughout the paper to δ_c as the level of farsightedness/sophistication or myopia of consumers: the lower (respectively higher) δ_c, the more myopic (sophisticated/farsighted) consumers are with respect to firms. In the first period, the firms simultaneously set prices p_1^H and p_1^L and the consumers decide upon purchase. In time 2, the firms simultaneously set prices, knowing who bought which good in period 1: p_2^{iH} is defined as the price set by firm i for a consumer who bought good H in period 1, while p_2^{iL} is offered to L's inherited clients. Finally, consumers observe the new prices and buy the preferred good again.

The following sections provide a complete analysis of the model. In particular, the next section introduces a benchmark case in which customer recognition is not allowed. This benchmark is used to isolate the effects of BBPD. The subsequent section describes the possible equilibria when firms are allowed to engage in BBPD.

3. Uniform Pricing

Assume there exists a ban on price discrimination or that customers' purchases cannot be observed. In this scenario, the utility of an agent buying good H and good L will be respectively:

$$U(x, H) = u^H - p^H - x, \quad U(x, L) = u^L - p^L - (1 - x).$$

Accordingly, the indifferent consumer is located at:

[7] This normalization is mainly done for the sake of exposition of the results. The main results of the paper hold also with an arbitrary δ_f. The author can provide a version with arbitrary discount factor upon request.

$$\bar{x} = \frac{1}{2} + \frac{\Delta + p^L - p^H}{2}, \tag{2}$$

where $\Delta \equiv u^H - u^L$ represents the price premium enjoyed by firm H. We assume hereafter that u^H and u^L are high enough so that consumers of all locations prefer to buy at least one of the two products (full market coverage), and that the prices chosen by the two firms are not too different in order not to consider situations in which one firm corners the market. Accordingly, the cut-off $\bar{x}$ determines a demand of $\bar{x}$ for firm H and $1-\bar{x}$ for firm L. Moreover, the attention is restricted only to cases in which the price premium Δ is not too large to eject the weak firm out of the market. As it can be clearly seen below, the necessary and sufficient condition for this to be the case is $\Delta < 3$, which allows firm L to charge an above-marginal-cost price at equilibrium. This assumption is maintained hereafter.

Anticipating the reaction of consumers, the firms set prices in order to maximize the following static profits:

$$\pi^H = p^H \left(\frac{1}{2} + \frac{\Delta + p^L - p^H}{2} \right), \quad \pi^L = p^L \left(\frac{1}{2} - \frac{\Delta + p^L - p^H}{2} \right).$$

It is worth noticing that, in comparison with the standard Hotelling with $\Delta = 0$, firm H can charge higher prices and the opposite happens to the weak firm. Indeed, the equilibrium prices are the following:

$$p_u^H = 1 + \frac{\Delta}{3}, \quad p_u^L = 1 - \frac{\Delta}{3}.$$

They take into account two aspects. Specifically, 1 represents the market power that both firms enjoy on consumers, whereas $\Delta/3$ is the result of the competitive advantage that firm H enjoys due to the price premium that the consumers are willing to pay for the product it sells. The prices above result in the following static equilibrium profits:

$$\pi_u^H = \frac{(3+\Delta)^2}{18}, \quad \pi_u^L = \frac{(3-\Delta)^2}{18}.$$

Under uniform price in both periods, subgame perfect Nash equilibrium gives a replication of the static equilibrium, with the following overall profits:

$$\pi_u^H = \frac{(3+\Delta)^2}{9} \text{ and } \pi_u^L = \frac{(3-\Delta)^2}{9}. \tag{3}$$

The consumer surplus will be:

$$CS_u = \int_0^{\bar{x}} U^{HH}(x)dx + \int_{\bar{x}}^1 U^{LL}(x)dx = \frac{(1+\delta_c)(18u^H + 18u^L + \Delta^2 - 45)}{36},$$

where $U_u^{ii}(x) = (1+\delta_c)\left(u^i - p_u^i - |x - l^i|\right)$ represents the utility of buying in the two period good i paying the non-discriminatory price.

4. Observation of Consumer Purchases and BBPD

In this section, first-period prices as well as the behaviour of first-period consumers are assumed to be observable to both firms when they choose second-period discriminatory prices. Subgame perfection is used as an equilibrium concept.

4.1. Second-Period Subgames

The consumers observe prices for loyalists and for switchers offered by both firms. On the inherited turf of firm H, a consumer prefers to buy again good H rather than switch seller when $u^H - p_2^{HH} - x > u^L - p_2^{LH} - (1-x)$, which gives the following indifferent location:

$$x_2^H = \frac{1}{2} + \frac{\Delta + p_2^{LH} - p_2^{HH}}{2}, \tag{4}$$

so that x_2^H agents buy again good H. Defining x_1 as the inherited market share of firm H, $x_1 - x_2^H$ agents will instead switch towards firm L. Concerning the turf of firm L, consumers compare $u^H - p_2^{HL} - x$ with $u^L - p_2^{LL} - (1-x)$. It means that all agents located on the right of

$$x_2^L = \frac{1}{2} + \frac{\Delta + p_2^{LL} - p_2^{HL}}{2} \tag{5}$$

will buy again good L, whereas agents located in the interval $\left[x_1, x_2^L\right]$ will switch to firm H.

The firms anticipate this reaction of consumers in term of purchase and set prices. The analysis at this stage depends on the market shares $(x_1, 1-x_1)$ inherited from the first period, which determine the actual chances to switch from one firm to the other one, and the other way around. Unlike Fudenberg and Tirole (2000), who assume the inherited markets to be symmetric enough, here all possible subgames are analysed in the backward-induction analysis of the model. In particular, we have subgames with two-direction switching (TDS) and subgames with switching only towards one of the two firms (one-direction switching or ODS).

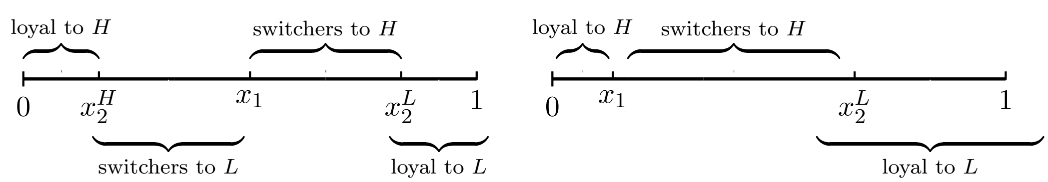


Figure 1: Different Switching Scenarios

When the firms expect switching to occur in both directions, the thresholds described by equations (4) and (5) are located in such a way that prices can be found on both turfs such that $x_2^H < x_1 < x_2^L$. When instead firms expect switching to occur only towards the strong firm (H), the thresholds above are located in such a way that $x_1 \leq x_2^H$ and $x_1 < x_2^L$. These two examples are depicted in Figure 1.

Following the proof provided by Esteves (2014) for the case of $\Delta = 0$, three different second-period market configurations may arise in the second period.

Proposition 1. *When the firms are allowed to price discriminate between old and new consumers, the second-period equilibrium prices are:*

$$\text{(i)} \quad \left.\begin{array}{ll} p_2^{HH} = 1 + \Delta - 2x_1, & p_2^{LH} = 0, \\ p_2^{HL} = 1 + \frac{\Delta}{3} - \frac{4}{3}x_1, & p_2^{LL} = 1 - \frac{\Delta+2}{3} + \frac{2}{3}(1 - x_1) \end{array}\right\} \text{ when } x_1 \leq \frac{1+\Delta}{4}$$

$$\text{(ii)} \quad \left.\begin{array}{ll} p_2^{HH} = \frac{1}{3} + \frac{\Delta}{3} + \frac{2}{3}x_1, & p_2^{LH} = 1 - \frac{\Delta}{3} - \frac{4}{3}(1 - x_1), \\ p_2^{HL} = 1 + \frac{\Delta}{3} - \frac{4}{3}x_1, & p_2^{LL} = 1 - \frac{\Delta+2}{3} + \frac{2}{3}(1 - x_1) \end{array}\right\} \text{ when } x_1 \in \left(\frac{1+\Delta}{4}, \frac{3+\Delta}{4}\right)$$

$$\text{(iii)} \quad \left.\begin{array}{ll} p_2^{HH} = \frac{1}{3} + \frac{\Delta}{3} + \frac{2}{3}x_1, & p_2^{LH} = 1 - \frac{\Delta}{3} - \frac{4}{3}(1 - x_1), \\ p_2^{HL} = 0, & p_2^{LL} = 1 - \Delta - 2(1 - x_1) \end{array}\right\} \text{ when } x_1 \geq \frac{3+\Delta}{4}$$

Proof. See Appendix 7.1. ∎

In order to better grasp the intuition behind Proposition 1 let us consider the equilibrium prices in point (ii). Unsurprisingly, a stronger asymmetry is associated with a competitive advantage in favor of the strong firm, whose equilibrium prices for old and new consumers are both increasing with Δ. Exactly the opposite relation exists between the prices of the weak firm and the price premium.

Nevertheless, the own inherited market share affects positively the price a given firm charges to the old loyal consumers and negatively the one offered to the switchers. Intuitively, the relation between prices and market share follows directly from the effective power that the size of the first-period market creates on each turf for the "attacking" (else turf) and the "defending" firm (own turf). Clearly, the attack on the rival turf turns out to be more costly as the size of the market already conquered in the first period becomes larger. In other words, the price offered to the switchers should be lower when many consumers were attracted earlier, since the non-conquered portion is really distant in the Hotelling line. For extreme levels of the market share,[8] attracting new consumers is not profitable as it would require a below-marginal-cost price. These cases are presented in points (i) and (iii), where the dominating strategy of one firm will be to set the price equal to the marginal cost (i.e., 0) on the rival's turf.

Therefore, from the point of view of the defending firm, the higher the market share inherited from the past, the weaker the price competition on its own turf, as the rival becomes less aggressive. For this reason, the equilibrium price for loyalists is increasing in the inherited market share. In the extreme cases in which the attacking rival sets the price equal to the marginal cost (points (i) and (iii) in the proposition), the optimal response of the defending firm is to offer to past consumers a price just sufficient not to lose any of them.

These equilibrium prices will determine peculiar switching behaviours of consumers. If the first-period market is balanced enough, then both firms succeed in finding profitable prices to offer to the rival's consumers and both are able to attract (and consequently suffer

[8] According to the proposition, this level will be $\frac{3+\Delta}{4}$ for firm H and $1 - \frac{1+\Delta}{4}$ for firm L.

the loss of) some new (old) consumers. If instead the market is strongly dominated by a firm in the first period, the dominating firm does not attract any rival consumers, even though it charges a price equal to the marginal cost. For this reason, switching will occur uniquely towards the dominated firm. These results are formally presented in the following corollary:

Corollary 1. *Given the equilibrium prices in Proposition 1: (i) when $x_1 \leq \frac{\Delta+1}{4}$, consumers only switch to firm H (*ODS*); (ii) when $x_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, consumers switch from H to L and vice-versa (*TDS*); and (iii) when $x_1 \geq \max\{\frac{\Delta+3}{4}, 1\}$, consumers only switch to firm L (*ODS*).*

Proof. Plugging the equilibrium prices in proposition 1, it is easy to find the following cut-offs: (i) when $x_1 \leq \frac{\Delta+1}{4}$, $x_2^H = x_1$ and $x_2^L = \frac{\Delta+2x_1+3}{6}$; (ii) when $x_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, $x_2^H = \frac{\Delta+2x_1+1}{6}$ and $x_2^L = \frac{\Delta+2x_1+3}{6}$; (iii) when $x_1 \geq \max\{\frac{\Delta+3}{4}, 1\}$, $x_2^H = \frac{\Delta+2x_1+1}{6}$ and $x_2^L = 1 - x_1$. ■

4.2. First Period

Similarly to the second period, the consumers observe prices and buy the good that gives them the highest utility. We let the consumers have an arbitrary level of sophistication, measured by δ_c. Therefore, the consumers take into account the possibility of tomorrow's switching, so that the utility is given by: $U^{ij}(x) = u^i - p_1^i - |x - l^i| + \delta_c(u^j - p_2^{ij} - |x - l^j|)$, with j possibly different from i in case of second-period switching.

If the consumers expect $x_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, the rational consumer who is indifferent in period 1 anticipates that if she buys product H in period 1, she will switch to product L in period 2, whereas if she chooses product L in period 1 she will switch to product H in period 2. Thus, the indifferent consumer is located in the x_1 such that

$$u^H - p_1^H - x_1 + \delta_c\left[u^L - p_2^{LH} - (1 - x_1)\right] = u^L - p_1^L - (1 - x_1) + \delta_c(u^H - p_2^{HL} - x_1).$$

Rewriting and plugging the second-period prices, we get the following cut-off:

$$x_{1TDS} = \frac{1}{2} + \frac{(3 - \delta_c)\Delta + 3(p_1^L - p_1^H)}{2\delta_c + 6}, \tag{6}$$

so that all agents to the left of the cut-off above buy good H, and all agents to the right buy good L. Differently, if the consumers expect $x_1 \leq \frac{\Delta+1}{4}$, the indifferent rational consumer anticipates that if she buys product H in period 1, she will buy it again in period 2, whereas if she chooses product L in period 1 she will switch to product H in period 2. Thus, the indifferent consumer is located at:

$$x_{1H} = \Delta - \frac{3(p_1^H - p_1^L + \Delta - 1)}{2(3 - \delta_c)}. \tag{7}$$

Following similar reasonings, if $x_1 \geq \frac{\Delta+3}{4}$ the indifferent consumer is located at $x_{1L} = \Delta + 1 - \frac{3\left(p_1^H - p_1^L + \Delta + 1\right)}{2(3-\delta_c)}$. Notice that $x_{1TDS} = x_{1H} = x_{1L} = \bar{x} = \frac{1}{2} + \frac{\Delta + p^L - p^H}{2}$ if the consumers are perfectly myopic ($\delta_c = 0$). It is easy to verify that the location of the indifferent consumer is more sensible to price changes when tomorrow's switching is expected to be uni-directional.

Lemma 1. *For $i, j \in \{H, L\}$ it holds that:*

$$|\frac{\partial x_{1j}}{\partial p_1^i}| = \frac{3}{6 - 2\delta_c} > \frac{1}{2} = |\frac{\partial \bar{x}}{\partial p_1^i}| > \frac{3}{6 + 2\delta_c} = |\frac{\partial x_{1TDS}}{\partial p_1^i}|.$$

Compared to the non-discriminatory regime, forward-looking consumers change their "elasticity" because they take into account not only the direct impact of a price variation,[9] but also the indirect effect of a variation over the second-period prices. Colombo (2015) provides a very accurate and precise explanation of this effect in the TDS case with symmetric firms and points out how the demand "elasticity" is lower under BBPD.[10] Oppositely, when ODS is assumed to be the case, consumers anticipate that tomorrow's discounted prices will be less attractive as firm H will not need to lower the price too much to attract switchers.[11] As a result, the first-period benefit from switching after a price decrease is higher than in the uniform case.

Following a backward induction reasoning, at the beginning of the game the firms correctly anticipate both first-period purchase decisions and all possible subgames. Hence, firm H and L maximize the following inter-temporal profits:

$$\pi_1^H + \pi_2^H = p_1^H x_1 + \left[p_2^{HH} \min\left\{x_2^H, x_1\right\} + p_2^{HL} \max\left\{x_2^L - x_1, 0\right\}\right],$$
$$\pi_1^L + \pi_2^L = p_1^L(1 - x_1) + \left[p_2^{LL} \min\left\{1 - x_2^L, 1 - x_1\right\} + p_2^{LH} \max\left\{x_1 - x_2^H, 0\right\}\right].$$

Clearly, the future profits depend on the expectations the firms have about tomorrow's movements of consumers. The following paragraph discusses the main features and the method used to build the best responses, which exhibit distinctive features due to the fact that the firms can choose very different pricing strategies according to the inter-temporal objective they want to pursue.[12] Subsequently, the equilibria of the model are presented, giving also some insights into the main characteristics of prices and switching behaviour.

Best Responses and Price Premium. The best-reply price will inter-temporally trade-off today's profits (market share and per-consumer margin) for tomorrow's cost of poaching consumers. In particular, attacking fiercely the market today makes the attraction of new consumers very costly tomorrow. On the contrary, accommodating the opponent today makes

[9]Notice that they would consider only this direct effect both in the uniform pricing regime and in the myopic-consumers case.

[10]Studying an increase in the price of firm i, he concludes the following: "It follows that the first-period benefit from shifting from firm i to firm j is lower when future is taken into account. Hence, the higher δ_c is, the lower is the benefit from shifting after a first-period price decrease."

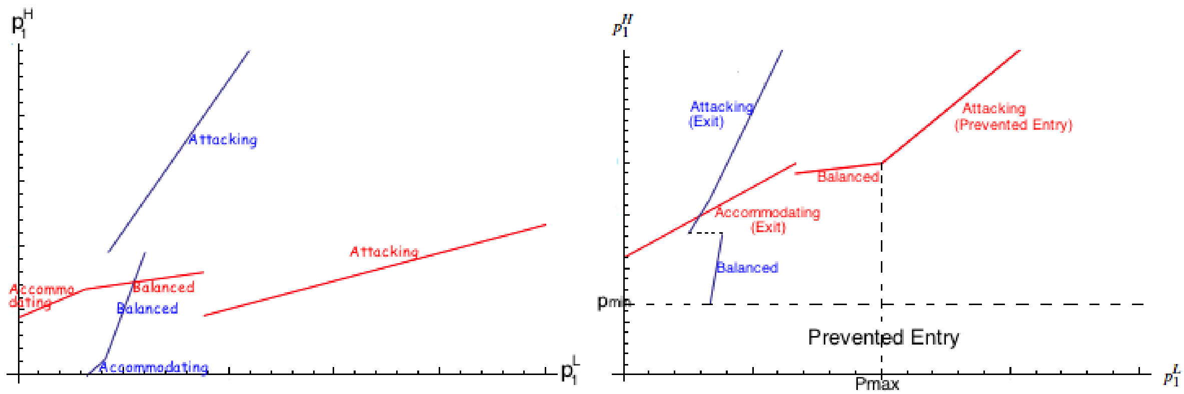
[11]We will come back to this discussion on the "attractiveness" of the second period offers at the end of this section.

[12]A complete construction of the best replies can be found in Appendix 7.2.

an invasion of its turf tomorrow very cheap to be carried out. How firms trade these two effects off is explained below.

For a given price chosen by the competitor, each firm has three different alternatives: an attacking, a balanced, and an accommodating strategy. The first strategy consists of setting a relatively low price in response to the one chosen by the rival, thus focusing on the conquest of a wide market. More precisely, this strategy will yield a market share larger than $\frac{3+\Delta}{4}$, with the drawback that no new consumer will be attracted in the subsequent period. The accommodating strategy follows the opposite argument: setting a relatively high price, conquering a market share smaller than $\frac{1+\Delta}{4}$, and focusing on attracting consumers in the future. Finally, a balanced strategy splits market share relatively symmetrically and tomorrow poaching will be bi-directional. Each strategy would result in some second-period and overall profits, for a given price chosen by the rival. Therefore the resulting inter-temporal profits are compared and the strategy yielding the highest profits will be picked as the best response.

Intuitively, attacking (accommodating) today is preferred when the rival accommodates (attacks). Indeed, if the competitor sets a high price, a firm is given the opportunity to make high margins out of a large market today so that it does not care at all about tomorrow's switching. On the contrary, when the rival is very aggressive and charges a relatively low price, the seller lays down arms today when the fight becomes too hard, being aware of the fact that this brings about a cheap conquest of rival territory tomorrow.



$\Delta = .8$ $\Delta = 2$

Figure 2: Best Responses for Increasing Δ (fixing δ_c to 1)

This optimal behaviour can be evidently observed in the two graphics depicted in Figure 2, which show the best responses of firms (coloured red for firm H and blue for firm L) for different levels of Δ fixing the discount factor to 1.[13] It is worth noticing that for intermediate

[13]The graphs aim at giving some general description of the best responses. They would be very similar if one takes any level of customers' myopia.

levels of the rival's price, the balanced strategy is preferred. In this region, the best-response curves are less steep as they involve an inter-temporal balance of incentives compared to the more extreme ones, where either the present (attacking) or the future (accommodating) only matter when deciding the optimal pricing policy.

The difference between the two graphs of Figure 2 is that increasing the asymmetry (right figure) gives the strong firm the chance to prevent the entry of the weak opponent by choosing a sufficiently low price (i.e., $p_1^H \leq p_{min} \equiv \frac{7\Delta+1-3\delta_c\Delta-3\delta_c}{9}$). As long as the firms' asymmetry gets more severe, two effects are at work. On the one hand, firm-L entry is prevented for gradually higher levels of firm-H prices, as p_{min} moves upward in response to increasing levels of Δ. On the other hand, if the weak firm offers a price $p_1^L \geq p_{max} \equiv \frac{17+3\delta_c\Delta+9\delta_c-7\Delta}{9}$, firm H would occupy the entire market line. Therefore, the balanced strategy is always dominated by the attacking strategy, which by definition is the best pricing rule a firm can follow if the objective is to conquer a large market (*a fortiori* the entire market) in the first period. It is easy to see that p_{max} decreases as asymmetry increases, so that the balanced strategy tends to fade out from firm H's best response as the level of asymmetry gets bigger.

Existence and Uniqueness of Equilibria. The framework proposed here presents some specificities which are not present when firms are symmetric.[14] Indeed, under firm symmetry, the marginal consumer on the equilibrium path is always in the middle of the Hotelling line. Therefore, the cutoff being centered, a slight change in a firm's price does not entail any change in demand structure, i.e., still two-directional switching will happen. Under firm asymmetry, the marginal consumer in a symmetric market configuration might be close to a threshold between two demand structures, i.e., TDS and ODS. Correspondingly, a slight change in one price can lead to different demand structures in the subsequent period. In turns, a movement in the demand structure may entail a discrete jump in terms of second-period profits going from just below and just above the aforesaid threshold.

As an example, let us consider a situation where firm H takes the rival's price as given. What is the optimal response of firm H choosing x_1 when it has to decide between TDS and ODS? Its first-period profit is continuously differentiable in x_1, so let us focus on the second-period profit, which is $\pi^H_{2TDS} = p_2^{HH} x_2^H + p_2^{HL}(x_2^L - x_1) = \frac{\Delta^2+5(2x_1^2-2x_1+1)-2\Delta(x_1-2)}{9}$ if TDS and $\pi^H_{2H} = p_2^{HH} x_1 + p_2^{HL}(x_2^L - x_1) = \frac{\Delta^2+(9-2x_1(10x_1+3))+2\Delta(5x_1+3)}{18}$ if ODS to H. Regardless of p_1^H, it can be demonstrated that:

$$\frac{\partial \pi^H_{2TDS}}{\partial x_1} \Big|_{x_1=\frac{1+\Delta}{4}} - \frac{\partial \pi^H_{2H}}{\partial x_1} \Big|_{x_1=\frac{1+\Delta}{4}} = \frac{\Delta}{3}$$

Let us consider a situation where x_1 is slightly above $\frac{1+\Delta}{4}$ and $\Delta > 1$. Within TDS, it is easy to verify that firm H has incentives to decrease x_1, because the profit π_{2TDS} is decreasing in x_1. However, once $\frac{1+\Delta}{4}$ is reached, there is a discrete jump in $\frac{\partial \pi_2^H}{\partial x_1}$(measured

[14] The result of Fudenberg and Tirole (2000) can be replicated in the present paper just setting $\Delta = 0$.

by $\frac{\Delta}{3}$), so that firm H has no incentive to raise x_1 when it is higher than $\frac{1+\Delta}{4}$. Same reasoning can be applied if we start with x_1 just above $\frac{1+\Delta}{4}$.[15] The potential "jump" between demand structures does not compromise the existence and uniqueness of an equilibrium if the market is either sufficiently symmetric (TDS emerges as a unique equilibrium) or sufficiently asymmetric (ODS unique equilibrium). However, it is the reason behind the non-existence of an equilibrium and of the multiplicity of equilibria for intermediate levels of Δ and extreme levels of δ_c.

For the sake of expositive clarity, prices that can emerge at equilibrium are reported here. As it is demonstrated in the appendix, if the strong firm sets its price following the accommodating strategy and the weak firm follows the attacking strategy, the prices will be

$$(p_{1NE}^{H*}, p_{1NE}^{L*}) = \left(p_u^H + \frac{44+\Delta(6(\delta_c-5)\delta_c+8)+6\delta_c^2-38\delta_c}{105-27\delta_c}, p_u^L + \frac{2(22+3\Delta(13-(8-\delta_c)\delta_c)-\delta_c(19-3\delta_c))}{3(35-9\delta_c)}\right).$$

This couple of prices will generate ODS towards the strong firm in the second period, when the small firm survives in the market with a tiny but positive market share. Differently, if unidirectional switching will cause the exit of firm L in time 2, the first-period prices will be:

$$(p_{1E}^{H*}, p_{1E}^{L*}) = \left(p_u^H + \frac{18+2\left(\delta_c^2+(\delta_c-4)(\delta_c+1)\Delta-9\delta_c\right)}{33-9\delta_c}, p_u^L + \frac{9-2(\delta(\delta(\Delta-2)-4\Delta+10)-2\Delta)}{33-9\delta_c}\right).$$

However, if both firms adopt a balanced strategy, the resulting prices will be

$$(p_{1TDS}^{H*}, p_{1TDS}^{L*}) = \left(p_u^H + \frac{\delta_c}{3} - \frac{\Delta(4-3(1-\delta_c)\delta_c)}{3(9\delta_c+7)}, p_u^L + \frac{\delta_c}{3} + \frac{\Delta(4-3(1-\delta_c)\delta_c)}{3(9\delta_c+7)}\right),$$

which are going to generate bi-directional movements of consumers in the second period. The following proposition outlines all possible scenarios that can emerge as equilibria, specifying also the conditions under which those equilibria exist and are unique.

Proposition 2. *Let* $\underline{\Delta}_E = \frac{5}{3}$,

$$\bar{\Delta}_{TDS} = \begin{cases} \frac{9\delta_c+7}{5-3\delta_c} & \text{if } \delta_c \leq 2/21 \\ \frac{5}{3} & \text{if } \delta_c \in (2/21, 0.95121) \\ \frac{(9\delta_c+7)\left((59-3\delta_c(\delta_c(13-27\delta_c)+71))-4\sqrt{(1-\delta_c)^2(14-3\delta_c)(3\delta_c+4)(6\delta_c+1)^2}\right)}{3(\delta_c(\delta_c(9\delta_c(29\delta_c-68)-694)+644)-47)} & \text{if } \delta_c > 0.95121 \end{cases}$$

and

$$\underline{\Delta}_{NE} = \begin{cases} \frac{4(3\delta_c-14)(3\delta_c(5\delta_c-16)+25)+4\sqrt{(35-9\delta_c)^2(\delta_c-1)^2(14-3\delta_c)(3\delta_c+4)}}{3(\delta_c(\delta_c(9\delta_c(17\delta_c-148)+3422)-2452)+385)} - 1 & \text{if } \delta_c \in (0.86286, 867653) \\ \frac{3\delta_c(\delta_c(9\delta_c(31\delta_c-234)+3659)+892)+32-4\sqrt{\delta_c^2(83\delta_c)(3\delta_c+4)(3\delta_c(29-9\delta_c)+70)^2}}{3(\delta_c(\delta_c(9\delta_c(17\delta_c-106)+1361)+564)+32)} & \text{if } \delta_c > 0.867653 \end{cases}$$

[15] This intuitive explanation of the non-existence/multiplicity of equilibria comes from the very insightful suggestions of an anonymous referee.

Then:

(i) if $\Delta < \bar{\Delta}_{TDS}$*, there exists a unique equilibrium where the prices are* $(p^{H*}_{1TDS}, p^{L*}_{1TDS})$*,*

(ii) if $\Delta \geq \underline{\Delta}_{E}$*, there exists a unique equilibrium where the prices are* $(p^{H*}_{1E}, p^{L*}_{1E})$*,*

(iii) if $\Delta \in [\underline{\Delta}_{NE}, \underline{\Delta}_{E})$*, there exists an equilibrium with prices* $(p^{H*}_{1NE}, p^{L*}_{1NE})$*,*

(iii) Equilibria in (i) and in (iii) may both arise when $\delta_c \in (0.86286, 0.9783)$ *and* $\Delta \in (\underline{\Delta}_{NE}, \bar{\Delta}_{TDS})$

(iv) No equilibrium exists if $\delta_c < 2/21$ *and* $\Delta \in (\bar{\Delta}_{TDS}, \underline{\Delta}_{E})$ *or if* $\delta_c > 0.9783$ *and* $\Delta \in (\bar{\Delta}_{TDS}, \underline{\Delta}_{NE})$*.*

Proof. See Appendix 7.3 for a complete proof. ■

The conditions in Proposition 2 can be easily interpreted if one looks at Figure 3 below, which describes the emergence of equilibria for each possible value of δ_c and Δ. The proposition describes that we can observe two different types of equilibrium. In the first one, described in point (i) in the proposition, both firms choose an inter-temporally balanced pricing strategy (hereafter, we refer to this equilibrium with TDS or balanced equilibrium). This is an equilibrium state for sufficiently weak asymmetry. As the price premium enjoyed by firm H is sufficiently important, inter-temporally unbalanced behaviours arise (points (ii) and (iii)). Specifically, the strong firm finds it profitable to accommodate the opponent which, in turns, wants to attack the market in the first period. We will call this equilibrium as the unbalanced equilibrium and, so highlighting the exit/non-exit of the small firm, as ODS_E or ODS_{NE}.

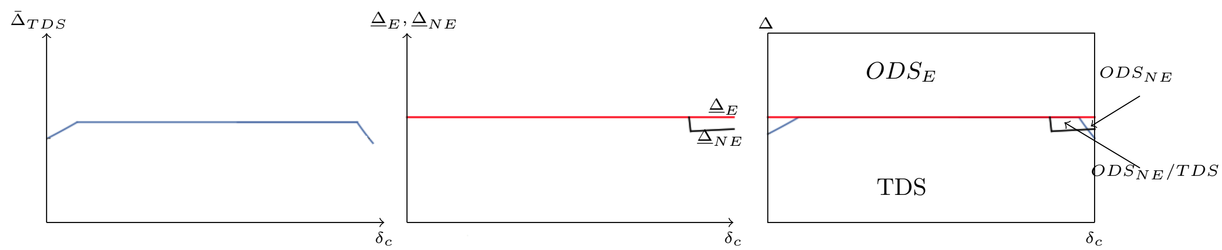


Figure 3: $\bar{\Delta}$,$\underline{\Delta}_{NE}$, $\underline{\Delta}_{E}$ and Characterization of Equilibria. The left graph depicts the maximal level of Δ compatible with the emergence of a TDS equilibrium, the central graph represents the minimal level of Δ compatible with a ODS equilibrium, respectively with the exit (red line) and with the survival (black curve) of the weak firm in the second period. Right graph plots all possible scenarios: non-labeled areas represent combinations of δ_c and Δ where no equilibrium can be found.

In the balanced equilibrium (areas TDS and ODS_{NE}/TDS), the firms share the first-period market in a relatively balanced way and both succeed in stealing rival consumers in the second period. In the unbalanced one, reached when the price premium is important enough, we observe a type of market-sharing agreement, according to which the firms allocate market shares and surplus over time in a distinctive way. In particular, firm H pursues an accommodating pricing strategy, consisting of being inoffensive today in order to induce a favorable response of the weak rival. This strategy allows firm H for the opportunity of a large market to be conquered cheaply afterwards. For most of the combinations of δ_c and Δ, the inter-temporally unbalanced pricing behaviour entails the exit of the small firm, which cannot find any profitable way to compete in the second period (area labeled E in right panel of Figure 3). When this is not the case (small areas labeled ODS_{NE} and ODS_{NE}/TDS corresponding to high levels of the discount factor and intermediate levels of the price premium), still the weak firm will be relegated to a tiny corner of the market. This is due to the fact that the large firm conquers a small market in the first period, with the consequence that competing in the rival's territory in the second period becomes very easy. This gives a way to profitably attack the rival on its turf and to conquer the entire (or most of the) Hotelling segment.

Moreover, first-period prices might be higher than the uniform price. This result is formally stated in the following corollary:

Corollary 2. *For what concerns first-period equilibrium prices, the following holds:*

(a) firm H: $p_{1TDS}^{H*} < p_u^H$ *if* $\Delta \geq \frac{9\delta_c^2+7\delta_c}{3\delta_c^2-3\delta_c+4}$, *the opposite is true otherwise,*

$p_{1NE}^{H*} < p_u^H$ *for all* $\Delta \in (\underline{\Delta}_{NE}, \underline{\Delta}_E)$,

$p_{1E}^{H*} > p_u^H$ *for all* $\Delta > \underline{\Delta}_E$ *if* $\delta_c < 0.362541$ *and, for higher* δ_c, *when* $\Delta \in \left(\frac{5}{3}, \frac{18+\delta_c^2-9\delta_c}{4+3\delta_c-\delta_c^2}\right)$.

(b) firm L: $p_{1TDS}^{L*} > p_u^L$ *if* $\Delta < \bar{\Delta}_{TDS}$,

$p_{1NE}^{L*} > p_u^L$ *for all* $\Delta \in (\underline{\Delta}_{NE}, \underline{\Delta}_E)$,

$p_{1E}^{L*} > p_u^L$ *for all* $\Delta > \underline{\Delta}_E$.

First of all, it is worth noticing how the first-period price of the small firm is always higher than the uniform price, no matter if the equilibrium is balanced or unbalanced and no matter the level of sophistication of consumers. This follows from the fact that BBPD in association with the enjoyed price premium gives to the strong firm the further possibility to attract consumers relatively cheaply tomorrow. This possibility is present in the TDS as well as in the ODS_E and in the ODS_{NE} equilibria and it makes the strong firm more prone to reduce the intensity of competition in the first period compared to the case of uniform pricing. The comparison between p_{1E}^{H*} with p_u^{H*} does not give these unambiguous results and ultimately depends on the level of consumers' farsightedness, as explained below.

Myopia vs. farsightedness of consumers. The interpretation of Proposition 2 and its corollaries turns out to be easier if one looks at the specific cases of perfectly myopic ($\delta_c = 0$) and perfectly farsighted ($\delta_c = 1$) consumers. Moreover, as it will be explained in Section 5, the level of sophistication of consumers turns out to be the pivotal element to understand the welfare consequences of BBPD, since the more sophisticated consumers are, the more they are able to understand how firms compete inter-temporally and, consequently, change their first-period response to prices.

		Uniform	BBPD Equilibrium	BBPD ($\delta_c = 0$)	BBPD ($\delta_c = 1$)
$\Delta = 1$	p_1^H	$4/3$	TDS	$4/3 - 4/21$	$4/3 + 1/4$
	p_1^L	$2/3$		$2/3 + 4/21$	$2/3 + 5/12$
$\Delta = 8/5$	p_1^H	$23/15$	ODS_{NE} ($\delta_c = 1$)	No eq.	$23/15 - 34/195$
	p_1^L	$7/15$			$7/15 + 58/65$
$\Delta = 2$	p_1^H	$5/3$	ODS_E	$5/3 + 20/33$	$5/3 - 1/6$
	p_1^L	$1/3$		$1/3 + 10/33$	$1/3 + 1/4$

Table 1: Example of equilibrium prices for $\delta_c = 0$ and $\delta_c = 1$. Three levels of Δ are reported: one leading to TDS ($\Delta = 1$), one to ODS without exit ($\Delta = 48/30$) and one to ODS with exit ($\Delta = 2$).

Table 1 reports the uniform price and the BBPD price in the two extreme cases of myopic and farsighted consumers, fixing a specific level of Δ. It is worth considering three cases. One with TDS ($\Delta = 1$), one with ODS_{NE} ($\Delta = 8/5$) and one with ODS_E ($\Delta = 2$).[16] First notice that, no matter the equilibrium that emerges under BBPD, the first-period price of the small firm is always higher than the uniform price. Therefore, let us focus on the strong-firm price.

When TDS emerges as an equilibrium ($\Delta = 1$), the price of the strong firm reduces to $p^{H*}_{1TDS}(\delta_c = 0, \Delta = 1) = 4/3 - 4/21$ when consumers are myopic and to $p^{H*}_{1TDS}(\delta_c = 1, \Delta = 1) = 4/3 + 1/4$ when they are sophisticated. This is a standard result of the symmetric BBPD literature. If consumers do not anticipate tomorrow switching, their response to first-period price is the same as the case of uniform pricing and this pushes first-period price to decrease. As soon as consumers anticipate tomorrow offers, their response to first-period price becomes weaker, thus reducing price competition.

It turns out this mechanism to be reversed when the equilibrium is unbalanced, both in the ODS_{NE} and in the ODS_E case. Under ODS_{NE} ($\Delta = 8/5$), only the case of forward-looking consumers can be reported since for $\delta_c = 0$ we fall in the region where no equilibrium exists. In that case, $p^{H*}_{1NE}(1, 8/5) = p^H_u - \frac{2(4\Delta-3)}{39} = 23/15 - 34/195$ is clearly lower-than-uniform. When ODS leads to the exit of the small firm ($\Delta = 2$), the price of the big firm becomes higher-than-uniform if consumers are myopic ($p^{H*}_{1E}(0, 2) = 5/3 + 20/33$) and lower-than-uniform ($p^{H*}_{1E}(1, 2) = 5/3 - 1/6$) if consumers are sophisticated. Therefore, the strategy of accommodation that firm H pursues in the ODS_E scenario turns out to be effective in

[16] These values of Δ take into account all possible equilibria. Notice that the TDS equilibrium arises for $\Delta < 1.4$ when $\delta_c = 0$ and for $\Delta < 1.33$ when $\delta_c = 1$. Firm L always exits the market (ODS_E) when $\Delta > 5/3$, no matter the discount factor of consumers. Finally, an equilibrium with ODS_{NE} where firm L survives in the second period is possible only when $\delta_c = 1$ and $\Delta \in (1.47856, 5/3)$.

reducing first-period price competition only if consumers are sufficiently myopic.

This difference between equilibrium scenarios on the effect of customers' myopia on firms' desirability of BBPD must not surprise. Indeed, as stated in Lemma 1, first-period elasticity moves upwards (respectively downwards) in the inter-temporally balanced (unbalanced) equilibrium, since consumers anticipate a strong (weak) competition for "poached" individuals tomorrow. To better understand this point and conclude the discussion introduced by Lemma 1, it is sufficient to see what happens in the turf of firm L in the second period under the TDS and the ODS_E scenarios.[17] Plugging p_{1TDS}^{H*} and p_{1TDS}^{L*} into x_{1TDS} and, in turns, into the prices offered by firm H to switchers, we get:

$$p_2^{HL}(TDS) = \frac{1}{3} - (23 + 2\delta(8 - 9\delta))\Delta, \tag{8}$$

which is evidently lower than the uniform price. Therefore, as consumers anticipate future advantageous offers, their first-period "elasticity" to price decreases. Exactly the opposite occurs in the ODS_E case. Indeed, the strong firm attacks the rival's turf with a relatively high price. Plugging p_{1E}^{H*} and p_{1E}^{L*} into x_{1H} and, in turns, into the prices offered by firm H to switchers, we get:

$$p_2^{HL}(ODS_E) = 1 + \frac{\Delta}{3} - \frac{4(8\Delta + 9 - 3\delta(\Delta - 2))}{3(9\delta + 35)}. \tag{9}$$

It is easy to notice that this offer to switchers is still lower-than-uniform, but higher than the price in equation (8). Therefore, when consumers anticipate the not-so-appealing offer for tomorrow switching, they become more sensitive to first-period price and the "elasticity" increases. For this reason, firms would prefer to face forward-looking consumers if the asymmetry is moderate and TDS emerges as an equilibrium, whereas consumers myopia is positive to firms in the unbalanced equilibrium (high asymmetry).

5. Welfare Analysis

The current section presents the effects of BBPD on both firms' and consumers' welfare. In order to provide this analysis, profits and consumer surplus resulting under customer recognition are compared with the benchmark case of uniform pricing. Clearly, we are able to provide an analysis for the regions of parameters for which we have an equilibrium, i.e., $\Delta \in (\underline{\Delta}_{NE}, \underline{\Delta}_E)$, $\Delta > \underline{\Delta}_E$ and $\Delta < \bar{\Delta}_{TDS}$ only. Indeed, in the remaining regions, the existence of equilibria cannot be guaranteed, not even in mixed strategies.[18]

[17] Clearly, only L turf is considered because no switching occurs in the turf of firm H.

[18] A similar issue of non-existence of pure-strategy Nash equilibria due to discontinuities in the best responses can be found in Belleflamme and Picard (2007). They are able to find mixed-strategy equilibria, thanks to the fact that the continuity of profits is sufficient for the existence of a mixed strategy equilibrium, by Glicksberg (1952). Unfortunately, the present model exhibits jumps in second-period profits that prevent to find any equilibrium (pure- or mixed-strategy) for some regions of parameters.

Let us consider first the case where Δ is sufficiently low ($\Delta < \bar{\Delta}_{TDS}$), so that the equilibrium is symmetric. In this case it is easy to verify the following proposition.

Proposition 3. *If $\Delta < \bar{\Delta}_{TDS}$, price discrimination according to past purchase behaviour will be*

(i) detrimental to both firms if $\Delta < \tilde{\Delta} \equiv \frac{(9\delta_c+7)\left(3\delta_c+39-\sqrt{3\delta_c(3\delta_c(9\delta_c^2-96\delta_c+230)-400)+1561}\right)}{(5-3\delta_c)(69\delta_c-9\delta_c^2-2)}$

(ii) detrimental to the strong firm firm and beneficial to the weak firm if $\Delta \in (\tilde{\Delta}, \bar{\Delta}_{TDS})$.

(iii) beneficial for consumers.

Proof. See Appendix. ■

Proposition 3 outlines that low levels of the price premium yield the firm-damaging scenario shown in the traditional literature of BBPD.[19] Indeed, BBPD brings about an intensification of competition benefiting consumers in terms of lower prices. It is important to notice that the the weak firm is not always damaged by BBPD when Δ is sufficiently high.[20] In that case, first-period demand is less elastic to price, since consumers anticipate tomorrow offers, and first-period prices are above the uniform price. This effect on the first period results in a benefit in terms of higher profits for the weak firm, which is better off together with the consumers at the expenses of the strong firm.

Things change radically when the firms are assumed to be sufficiently asymmetric. In particular, as the price premium reaches a certain level, the strong firm is given the choice to decide the destiny of the competitor. Hence, the strong firm optimally opts for a fat-cat strategy, consisting of being inoffensive today in order to induce a favorable response of the rival, and the weak seller adapts to that strategy simply taking what is left. The following proposition shows how the welfare effects of BBPD in an asymmetric market ultimately depend on the level of sophistication of consumers.

Proposition 4. *If $\Delta \in (\underline{\Delta}_{NE}, \underline{\Delta}_{E})$, price discrimination according to past purchase behaviour will be detrimental for firm H and beneficial for firm L and consumers. If $\Delta > \underline{\Delta}_{E}$, then BBPD:*

(i) benefits both firms if firms if $\delta_c < 0.18$;

(ii) benefits the weak firms and damages the strong firm if $\delta_c > 0.235$

(iii) harms consumers if $\delta_c < 0.43$, whereas it makes them better-off if $\delta_c > 0.77$.

Proof. See Appendix. ■

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[19] In particular, assuming Δ to be zero replicates the results of Fudenberg and Tirole (2000), demonstrating de facto that their assumption of symmetry is not needed, as firms reach the symmetric equilibrium endogenously.

[20] Notice that $\tilde{\Delta}$ takes value 0.356645 when consumers are fully myopic and 0.193715 when they are fully sophisticated.

In the unbalanced equilibria, the strong firm accommodates the weak firm which, in turns, aims at attacking the market anticipating that competing tomorrow will be impossible. This will have the effect of a reduced competition in the poaching phase compared to the inter-temporally balanced equilibrium, since the weak firm will not be able to profitably attract new consumers and the strong firm will easily invade rival's turf. The welfare consequences of this strategy essentially depend on the ability of consumers to anticipate the effect on second-period competition.[21]

If consumers are (sufficiently) myopic, they do not (fully) internalize the reduced competition in the poaching stage. The final effect is that the strong firm makes high margins and *lets the weak firm enjoy a lessened competition*. This strategy gives the small rival the opportunity to obtain a relatively large market share, without the need to offer an extremely low price. This is clearly beneficial in the early competition, but it becomes harmful in the second period. Indeed, the strong firm steals many consumers and the weak firm only loses market share. The balance between these two opposite effects is always positive for the small seller, no matter whether it survives or exits the market.[22]

Oppositely, when consumers are sufficiently forward-looking, the negative push that firm L's attack exerts on final prices will prevail over the positive impact of firm H's accommodation. Indeed, being they are able to identify the inter-temporal market sharing agreement the firms are engaging in, consumers act accordingly. Namely, first-period demand elasticity to price moves upward when consumers expect unidirectional switching to occur. This will push the weak firm to compete very fiercely in the first period, and the final outcome turns out to be that the *strong firm suffers the increased competition*.

6. Conclusion

Despite the issues in terms of privacy created by the access of firms to consumer-specific information, BBPD literature has been in favor of consumer recognition due to the fact that consumers benefit from it in terms of lower prices and increased competition. In particular, the main message of Fudenberg and Tirole (2000) is the same as the one sent by the traditional price discrimination literature on oligopolistic markets: once firms can discriminate, they suffer a more intense competition, leading to lower prices and a positive effect for consumer surplus.

This paper argues that this result does not necessarily hold anymore if firms are somehow

[21]The threshold values of δ_c in points (i), (ii) and (iii) of Proposition 4 give sufficient conditions. In the intervals $(0.18, 0.235)$ and $(0.43, 0.77)$, the cutoffs of δ_c are functions of Δ. These cutoffs can be found in appendix.

[22]At first sight, the fact that firm L is better off out of the market can be counterintuitive. Nevertheless, when the price premium is very pronounced, a situation without customer recognition is not so appealing for this firm, which would create a niche in the market nonetheless. Therefore, the ODS scenario gives to the weak firm the possibility to get a level of profits that would not be reached in the benchmark case, not even in two periods.

asymmetric and consumers are less sophisticated than firms. Indeed, if consumers are willing to pay a price premium for the good sold by one competitor, inter-temporally unbalanced pricing behaviours arise. Specifically, the strong firm optimally chooses to accommodate the rival, serving a relatively small number of consumers at a high price. This outcome turns out to be ex-post preferred to uniform pricing as the sacrifice in terms of first-period demand is offset by the gain in margin and the cheap attraction of many switchers.

The weak seller adapts to the strategy of the rival simply taking what is left by the latter. This equilibrium turns out to be favorable as opposed to uniform pricing, due to the fact that the small firm can enjoy a large part of the demand, charging an above-uniform price in the early competition. The positive effect is not neutralized even if this unbalanced scenario led to the exit of the weak firm from the market. In sum, the recognition of consumers at the end of the first period helps the strong firm to make high margins and *lets the weak firm enjoy a lessened competition* if consumers are sufficiently myopic. Oppositely, if consumers are sufficiently forward-looking, they change their first-period elasticity to price upward when they expect unidirectional switching in the second period. This will lead to a situation in which the weak firm will compete fiercely in the first period, so that *the strong firm suffers the increased competition.*

The paper proves to be an original contribution to the literature concerned with the welfare consequences of BBPD and provides a new tool to analyze pricing under customer recognition when a firm benefits from a competitive advantage. The emerging online subscription markets as well as the telecommunication market often see a strong player and BBPD is clearly a tool in the hands of companies, who have much information about their subscribers. From the antitrust viewpoint, if one accepts the idea that consumers are not able to strategize as much as firms, BBPD becomes a very powerful instrument hold by those strong competitors, helping them limit price competition over time. It is by now conventional wisdom that the more information firms have access to, the more consumers benefit from it, due to an intensification of competition. This argument supports, to some extent, the idea that the invasion of consumers' privacy is offset by the increase of their surplus. In contrast with this argument, the present paper argues that the privacy invasion may be utterly unacceptable from the point of view of customers, as it may help a pseudo-collusive pricing behaviour and may give a strong competitive advantage to players that are already favored for many other reasons. Therefore, the paper raises awareness of competition authorities about the fact that the use of consumers' data might become a device to weaken price competition, at the detriment of consumers.

Moreover, the equilibrium analysis of the paper might be tested empirically in the telecommunication market. Data on the telecommunication market can be used to test the emergence of temporal movements in the relative market shares of former monopolists and new entrants. Whenever the monopolist kept a leading position after a (some) period (periods) of accommodation, this would suggest that BBPD had been one among other tools to somehow control the leadership of the market, damaging consumers. In point of fact, the emergence of the inter-temporal market sharing agreement might be sensitive to the assumption of a two-period model rather than a longer-lasting market. However, one can also interpret the equilibrium as a long-run phenomenon. Let us assume the two-period game to be repeated and all the consumers (or a sufficient share of them) to change from one repetition to the subsequent. Read this way, the model predicts inter-temporal "jumps" in proportion to market shares, as some consumers move from one firm to the other and back, as in ping-pong match, which helps firms to maintain relatively high prices.

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7. Appendix

7.1. Proof of Proposition 1

Let us analyze second-period pricing decisions. Given the cutoffs x_2^H and x_2^L in equations (4) and (5), firms solve the following problem in A's turf:

$$\max_{p_2^{HH}} p_2^{HH} x_2^H = p_2^{HH}\left(\tfrac{1}{2} + \tfrac{\Delta + p_2^{LH} - p_2^{HH}}{2}\right),$$
$$\max_{p_2^{LH}} p_2^{LH}\left(x_1 - x_2^H\right) = p_2^{LH}\left(x_1 - \tfrac{1}{2} - \tfrac{\Delta + p_2^{LH} - p_2^{HH}}{2}\right).$$

Solving the maximization problem, firm H's best response turns out to be $p_2^{HH} = \frac{1+\Delta+p_2^{LH}}{2}$, and firm L best response is to set a price $p_2^{LH} = x_1 + \frac{p_2^{HH}-1-\Delta}{2}$, which give the following equilibrium prices:

$$p_2^{HH} = \frac{\Delta + 2x_1 + 1}{3} \text{ and } p_2^{LH} = \frac{4x_1 - 1 - \Delta}{3}.$$

Doing the same in L's turf yields:

$$p_2^{HL} = \frac{\Delta + 3 - 4x_1}{3} \text{ and } p_2^{LL} = \frac{3 - 2x_1 - \Delta}{3}.$$

Notice that charging a price $p_2^{LH} < 0$ is a dominated strategy for firm L. Therefore, whenever the equilibrium price $p_2^{LH} < 0$ then the best option for this firm is to set $p_2^{LH} = 0$. This will happen when $\frac{4x_1-1-\Delta}{3} \leq 0 \Leftrightarrow x_1 \leq \frac{\Delta+1}{4}$.

When $x_1 \leq \frac{\Delta+1}{4}$ it follows that $p_2^{LH} = 0$ and thus the best response of firm H is to charge the maximal possible price compatible with not to lose the marginal consumer located at x_1, i.e., a price p_2^{HH} such that $u^H - p_2^{HH} - x_1 = u^L - 0 - (1 - x_1)$, which gives $p_2^{HH} = \Delta + 1 - 2x_1$. This will give equilibrium prices when $x_1 \leq \frac{\Delta+1}{4}$:

$$\begin{array}{ccc} p_2^{HH} = \Delta + 1 - 2x_1 & \text{and} & p_2^{LH} = 0; \\ p_2^{HL} = \frac{\Delta+(3-4x_1)}{3} & \text{and} & p_2^{LL} = \frac{3-2x_1-\Delta}{3}. \end{array}$$

In this case switching is one direction towards firm H. Similarly, firm H will set $p_1^{HL} = 0$ when $\frac{\Delta+3-4x_1}{3} \leq 0 \Leftrightarrow x_1 \geq \frac{\Delta+3}{4}$.

When $x_1 \geq \frac{\Delta+3}{4}$ it follows that $p_2^{HL} = 0$ and thus the best response of firm L is to charge the maximal possible price compatible with not to lose the marginal consumer located at x_1, i.e., a price p_1^{LL} such that $u^H - 0 - x_1 = u^L - p_2^{LL} - (1 - x_1)$, which gives $p_2^{LL} = 2x_1 - 1 - \Delta$. This will give equilibrium prices when $x_1 \leq \frac{\Delta+3}{4}$:

$$\begin{array}{ccc} p_2^{HH} = \frac{\Delta+2x_1+1}{3} & \text{and} & p_2^{LH} = \frac{4x_1-1-\Delta}{3}; \\ p_2^{HL} = 0 & \text{and} & p_2^{LL} = 2x_1 - 1 - \Delta. \end{array}$$

Notice that if $\Delta > 1$, this scenario with ODS to firm L cannot be reached unless $x_1 = 1$.

7.2. Construction of the Best Replies.

Firm H best response.

(i) If $x_{1TDS} \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, TDS occurs and firm H enjoys the following second-period profits:

$$\pi_{2TDS}^H = p_2^{HH} x_2^H + p_2^{HL}(x_2^L - x_1) = \frac{\Delta^2 + 5(2x_1^2 - 2x_1 + 1) - 2\Delta(x_1 - 2)}{9}.$$

Accordingly, firm H solves $\max_{p_1^H} \pi_{TDS}^H = \max_{p_1^H} p_1^H \hat{x}_1 + \pi_{2TDS}^H$ under the constraints $\frac{\Delta+1}{4} < x_{1TDS} < \frac{\Delta+3}{4}$. The first order condition of this problem gives:

$$p_{1TDS}^H = \frac{(3\delta_c - 1)p_1^L + 9 + \delta_c(\delta_c + 4\Delta + 6 - \delta_c\Delta) + \Delta}{6\delta_c + 8},$$

with correspondent $x_{1TDS} = \frac{9p_1^L+7\Delta-3\delta_c(\Delta-1)-1}{4(3\delta_c+4)}$. If $\Delta \leq 1$, then $x_{1TDS} \in (0, 1)$ and constraints are met if $\frac{5+6\delta_c\Delta-3\Delta}{9} \equiv \hat{p}_{HC} < p_1^L < \hat{p}_{LC} \equiv \frac{13+6\delta_c\Delta+6\delta_c-3\Delta}{9}$. The correspondent profit will be:

$$\pi_{TDS}^H = \frac{9\delta_c^2(\Delta-1)^2+6\delta_c(19-3(\Delta-8)\Delta)+3\Delta(27\Delta+38)+81(p_1^L)^2-18p_1^L(3\delta_c(\Delta-1)-7\Delta+1)+161}{72(3\delta_c+4)}.$$

We have three more cases to consider.

- When $\Delta > 1$, the constraint $x_{1TDS} < \frac{\Delta+3}{4}$ is non-binding. In this case, whenever p_1^L is such that $\hat{x}_1(p_{TDS}^H, p_1^L) \geq 1$, i.e. $p_1^L \geq p_{max} \equiv \frac{17+3\delta_c\Delta+9\delta_c-7\Delta}{9}$, TDS cannot occur tomorrow and firm H becomes a monopolist setting the price p_1^H such that $\hat{x} = 1$ or $p_{1M}^H = \Delta + p_1^L - 1$ and resulting profit of $\pi_M^H = \frac{\Delta(\Delta+11)+9p_1^L-3\delta_c(\Delta+1)-4}{9}$, where the subscript M stays for monopoly of firm H.
- If $\Delta \leq 1$, the constraint $x_{1TDS} < \frac{\Delta+3}{4}$ turns out to be binding when $p_1^L \geq \hat{p}_{LC}$, p_1^H is such that $x_{1TDS} = \frac{\Delta+3}{4}$, or $p_{1LC}^H = \frac{6p_1^L-3-3\delta_c\Delta-\delta_c+3\Delta}{6}$, leading to ODS to L. The profit overall will be:

$$\pi_{LC}^H = \frac{1}{72}\left(6\Delta(3\Delta+8) + 18(\Delta+3)p_1^L - 3\delta_c(3\Delta+1)(\Delta+3) - 2\right).$$

- Finally, if $p_1^L \leq \hat{p}_{HC}$, then $x_{1TDS} \leq \frac{\Delta+1}{4}$. It means that p_1^H is such that the constraint is binding, i.e., $p_{1HC}^H = \frac{6p_1^L+3-3\delta_c\Delta+\delta_c+3\Delta}{6}$, leading to ODS to H. The correspondent profit will be:

$$\pi_{HC}^H = \frac{1}{72}\left(18(\Delta+1)p_1^L + 34 - 3\delta_c(3\Delta-1)(\Delta+1) + 18\Delta(\Delta+2)\right).$$

(ii) If $x_{1H} \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm H, which receives profit

$$\pi_{2H}^H = p_2^{HH}x_1 + p_2^{HL}(x_2^L - x_1) = \frac{\Delta^2 + (9 - 2x_1(10x_1+3)) + 2\Delta(5x_1+3)}{18}.$$

The maximisation problem will be the following $\max_{p_1^H} \pi_H^H = \max_{p_1^H} p_1^H\hat{x}_1 + \pi_{2H}^H$ under the constraints $x_{1H} \leq \frac{\Delta+1}{4}$ and $x_2^L(x_{1H}) = \frac{\Delta+2x_{1H}+3}{6} < 1$ (otherwise the strong firm becomes monopolist in the second period).
The first order condition of this problem gives:

$$p_{1NE}^H = \frac{(19-3\delta_c)p_1^L + 22 + 2(7-(7-\delta_c)\delta_c)\Delta - 4\delta_c}{28-6\delta_c},$$

with correspondent $x_{1H} = \frac{14\Delta+9p_1^L+6-6\delta_c\Delta}{56-12\delta_c}$. The first constraint is met if $0 < p_1^L < \hat{p}_H \equiv \frac{3\delta_c\Delta-3\delta_c+8}{9}$, while the second constraint requires $p_1^L < \frac{26+4\delta_c\Delta-6\delta_c-14\Delta}{3} \equiv \hat{p}_E$, where E stands for exit of the small firm in second period. The correspondent profit will be:

$$\pi_{NE}^H = \frac{4(7-(5-\delta_c)\delta_c)\Delta^2+3\left(3(p_1^L)^2+4p_1^L+20-4\delta_c\right)+4\Delta(7(p_1^L+2)-\delta_c(3p_1^L+4))}{8(14-3\delta_c)}.$$

If $p_1^L < \hat{p}_H$, $x_{1H} \geq \frac{\Delta+1}{4}$ and thus firm H sets a price such that $\hat{x}_1 = \frac{\Delta+1}{4}$, i.e., p_{1HC}^H. Moreover, since $\pi_{2H}^H(x_1 = \frac{\Delta+1}{4}) = \pi_{2TDS}^H(x_1 = \frac{\Delta+1}{4})$, this profit turns out to be π_{HC}. If $\hat{p}_E < p_1^L < \hat{p}_H$ (which is a nonempty interval only provided that $\Delta > \frac{5}{3}$), then the firm solves:

$$\max_{p_1^H} p_1^H x_1 + p_2^{HH} x_1 + p_2^{HL}(1-x_1) = \max_{p_1^H} p_1^H x_1 + (1+\Delta-2x_1)x_1 + \left(1 + \frac{\Delta}{3} - \frac{4}{3}x_1\right)(1-x_1)$$

The first order condition of this problem gives $p_{1E}^H = \frac{3(5-\delta)p_1^L + 9(\Delta+3) - \delta_c((11-2\delta_c)\Delta+7)}{6(4-\delta_c)}$, with correspondent profit of:

$$\pi_E^H = \frac{(5-2\delta_c)^2\Delta^2 + (22-4\delta_c)\Delta - 24\delta_c + 9(p_1^L)^2 - 6p_1^L((2\delta_c-5)\Delta+1) + 97}{24(4-\delta_c)}.$$

Notice that the resulting cutoff $x_1 = \frac{(5-2\delta)\Delta + 3p_1^L - 1}{4(4-\delta)} < \frac{\Delta+1}{4}$ only provided that $p_1^L < \bar{p}_E \equiv \frac{5+\delta\Delta-\delta-\Delta}{3}$.

(iii) If $x_{1L} \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm L. This case can exist only if $\Delta \leq 1$ or, when $\Delta > 1$, if $x_1 = 1$. ODS to L would give firm H a second-period profit of

$$\pi_{2L}^H = p_2^{HH} x_2^H = \frac{(\Delta + 2x_1 + 1)^2}{18}.$$

- If $\Delta \leq 1$, then firm H solves $\max_{p_1^H} \pi_L^H = \max_{p_1^H} p_1^H \hat{x}_1 + \pi_{2L}^H$ under the constraint $x_{1L} \geq \frac{\Delta+3}{4}$. The first order condition of this problem gives:

$$p_{1L}^H = \frac{3(9\delta_c+7)p_1^L + \delta_c(36\delta_c + 21\Delta + 49) + 15(\Delta+1)}{6(9\delta_c+8)},$$

with correspondent $x_{1L} = \frac{12\delta_c + 11\Delta + 9p_1^L + 11}{36\delta_c + 32}$. The constraint is met if $p_1^L \geq \hat{p}_L \equiv \frac{9\delta_c\Delta + 15\delta_c - 3\Delta + 13}{9}$, for other prices ODL cannot occur. The resulting profit will be:

$$\pi_L^H = \frac{48\delta_c^2 + 4\delta_c(\Delta+1)(3\Delta+25) + 51(\Delta+1)^2 + 27(p_1^L)^2 + p_1^L(72\delta_c + 66(\Delta+1))}{24(9\delta_c+8)}.$$

If $p_1^L \leq \hat{p}_L$, $x_{1L} \leq \frac{\Delta+3}{4}$ and thus firm H sets a price such that $\hat{x}_1 = \frac{\Delta+1}{4}$, i.e., p_{1LC}^L. Moreover, since $\pi_{2L}^H(x_1 = \frac{\Delta+3}{4}) = \pi_{2TDS}^H(x_1 = \frac{\Delta+3}{4})$, this profit turns out to be π_{LC}^H. The case in which $x_1 = 1$ has been already discussed in the TDS case.

Up to now, we obtained all possible best responses of firm H within each regime. In order to build up the global best response, we must compare profits across regimes in each segment. We have 3 possible cases:

1. If $\Delta < 1$, then we have the following segments:
   (a) $p_1^L \leq \hat{p}_{HC}$. $\implies$ best response p_{1NE}^H.

(b) $p_1^L \in (\hat{p}_{HC}, \hat{p}_H)$ If $p_1^L > \frac{(1-\delta_c)(\Delta+1)\sqrt{-9\delta_c^2+30\delta_c+56}}{6(5-3\delta_c)} + \frac{\Delta+1}{5-3\delta_c} + \frac{\delta_c(3\Delta-1)}{6} - \frac{2\Delta}{3} + \frac{2}{9} \equiv \hat{p}$, then $\pi_{TDS}^H > \pi_H^H$. Otherwise, $\pi_H^H > \pi_{TDS}^H$. Notice that $\hat{p} > \hat{p}_{HC}$ for all $\delta_c < 1$, whereas $\hat{p} = \hat{p}_{HC} = \hat{p}_H$ when consumers are perfectly farsighted ($\delta_c = 1$).

(c) $p_1^L \in [\hat{p}_H, \hat{p}_{LC})$. $\Longrightarrow$ best response p_{1TDS}^H.

(d) $p_1^L \geq\in [\hat{p}_{LC}, \hat{p}_L]$. $\Longrightarrow$ best response p_{1LC}^H. Notice that $\hat{p}_L = p_{LC}$ if consumers are myopic ($\delta_c = 0$), so that this segment will disappear.

(e) $p_1^L \geq \hat{p}_L$. $\Longrightarrow$ best response p_{1L}^H.

2. $\Delta \in [1, \frac{5}{3}]$ and $\Delta < \frac{12\delta_c+9}{7}$, then $\hat{p}_H < p_{max}$. In segments (a) and (b) nothing changes. Above $\hat{p}_H$ we have:

(c.i) $p_1^L \in [\hat{p}_H, p_{max}]$. $\Longrightarrow$ best response p_{1TDS}^H.

(d.i) $p_1^L \geq p_{max}$. $\Longrightarrow$ best response p_{1M}^H. Notice that if $\Delta > \frac{12\delta_c+9}{7}$, then $\hat{p}_H > p_{max}$, but the best response does not change.

3. If $\Delta > \frac{5}{3}$. We have:

(a.ii) $p_1^L < \hat{p}_E$. The best response is p_{1H}^H.

(b.ii) $p_1^L \in (\hat{p}_E, \bar{p}_E)$ The best response is p_{1E}^H.

(c.ii) $p_1^L \in [\bar{p}_E, \hat{p}_{max}]$. The best response is p_{1TDS}^H.

(d.ii) $p_1^L \geq p_{max}$. The best response p_{1M}^H.

Putting together all the results above, the best response will depend on the size of Δ. Indeed, the best response of firm H will be the following:

$$p_1^H(p_1^L) = \begin{cases} p_{1NE}^H & \text{if } p_1^L \leq \min\{\hat{p}, \hat{p}_{HC}\} \\ p_{1TDS}^H & \text{if } p_1^L \in (\min\{\hat{p}, \hat{p}_{HC}\}, \hat{p}_{LC}), \\ p_{1LC}^H & \text{if } p_1^L \in [\min\{\hat{p}_{LC}, \hat{p}_L\}, \max\{\hat{p}_{LC}, \hat{p}_L\}], \\ p_{1L}^H & \text{if } p_1^L > \max\{\hat{p}_{LC}, \hat{p}_L\}, \end{cases}$$
$$\text{when } \Delta < 1$$

$$p_1^H(p_1^L) = \begin{cases} p_{1NE}^H & \text{if } p_1^L \leq \min\{\hat{p}, \hat{p}_{HC}\}, \\ p_{1TDS}^H & \text{if } p_1^L \in (\min\{\hat{p}, \hat{p}_{HC}\}, p_{max}), \\ p_{1M}^H & \text{if } p_1^L \geq p_{max}, \end{cases} \quad \text{when } \Delta \in [1, \tfrac{5}{3}],$$

$$p_1^H(p_1^L) = \begin{cases} p_{1NE}^H & \text{if } p_1^L \leq \hat{p}_E, \\ p_{1E}^H & \text{if } p_1^L \in (\hat{p}_E, \bar{p}_E) \\ p_{1TDS}^H & \text{if } p_1^L \in (\bar{p}_E, p_{max}), \\ p_{1M}^H & \text{if } p_1^L \geq p_{max}, \end{cases} \quad \text{when } \Delta > \tfrac{5}{3}.$$

Firm L best response.

(i) If $x_{1TDS} \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, *TDS* occurs and firm L enjoys a second-period profit of

$$\pi_{2TDS}^L = p_2^{LL}(1 - x_2^L) + p_2^{LH}(x_1 - x_2^H) = \frac{\Delta^2 + 5(2x_1^2 - 2x_1 + 1) - 2\Delta(x_1 + 1)}{9}.$$

Accordingly, firm L solves $\max_{p_1^L} \pi_{TDS}^L = \max_{p_1^L} p_1^L(1-\hat{x}_1) + \pi_{2TDS}^L$ under the constraints $\frac{\Delta+1}{4} < x_1 < \frac{\Delta+3}{4}$. The first order condition of this problem gives:

$$p_{1TDS}^L = \frac{((\delta_c-4)\delta_c-1)\Delta + (\delta_c+3)^2 + (3\delta_c-1)p_1^A}{8+6\delta_c},$$

with resulting $x_{1TDS} = \frac{-3\delta_c\Delta+9\delta_c+7\Delta-9p_1^A+17}{12\delta_c+16}$. If $\Delta \leq 1$, then constraints are met if $\frac{-6\delta_c\Delta+3\Delta+5}{9} \equiv \tilde{p}_{LC} < p_1^H < \tilde{p}_{HC} \equiv \frac{-6\delta_c\Delta+6\delta_c+3\Delta+13}{9}$. The correspondent profit will be:

$$\pi_{TDS}^L = \frac{114\delta_c - 114\Delta + 9\left(\delta_c^2(\Delta+1)^2 - 2\delta_c\Delta(\Delta+8) + 9\Delta^2 + 9(p_1^A)^2 + 2p_1^A(3\delta_c(\Delta+1) - 7\Delta - 1)\right) + 161}{72(3\delta_c+4)}.$$

We have three more cases to consider.

- If $\Delta > 1$ the constraint $x_1 < \frac{\Delta+3}{4}$ is non-binding. Whenever $p_1^H < p_{min} \equiv \frac{7\Delta+1-3\delta_c\Delta-3\delta_c}{9}$, $x_{1TDS} \geq 1$ firm L cannot enter the market.
- If $p_1^H \geq \tilde{p}_{HC}$, then first constraint is not satisfied and thus p_1^L will be such that $x_1 = \frac{\Delta+1}{4}$ or equivalently $p_{1HC}^L = \frac{3(\delta_c-1)\Delta-\delta_c+6p_1^A-3}{6}$. In this case the profit will be:

$$\pi_{HC}^L = \frac{1}{72}\left(-9(\delta_c-2)\Delta^2 - 9\delta_c - 6\Delta(-5\delta_c+3p_1^A+8) + 54p_1^A - 2\right).$$

- When $\Delta \leq 1$ and $p_1^H \leq \tilde{p}_{LC}$, then the second constraint is not satisfied and thus p_1^L will be such that $x_1 = \frac{\Delta+3}{4}$ or equivalently $p_{1LC}^L = \frac{3(\delta_c-1)\Delta+\delta_c+6p_1^A+3}{6}$. The correspondent profit will be:

$$\pi_{LC}^L = \frac{1}{72}\left(-9(\delta_c-2)\Delta^2 + 3\delta_c + 6\Delta(\delta_c-3p_1^A-6) + 18p_1^A + 34\right).$$

(ii) If $x_{1H} \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm H and firm L gets

$$\pi_{2H}^L = p_2^{LL}(1-x_2^L) = \frac{(\Delta+(2x_1-3))^2}{18}.$$

The maximisation problem will be $\max_{p_1^L} \pi_H^L = \max_{p_1^L} p_1^L(1-\hat{x}_1) + \pi_{2H}^L$ under the constraints that $x_1 \leq \frac{\Delta+1}{4}$ and $x_2^L(x_{1H}) = \frac{\Delta+2x_{1H}+3}{6} < 1$ (otherwise the strong firm becomes monopolist in the second period). . The first order condition of this problem gives:

$$p_{1NE}^L = \frac{(\delta_c-1)(2\delta_c-5)(\Delta-1) + (3\delta_c-7)p_1^A}{6\delta_c-16},$$

with correspondent $x_1 = \frac{6\delta_c(\Delta+1)-11\Delta+9p_1^A-21}{4(3\delta_c-8)}$. The first onstraint is met if $p_1^H \geq \frac{13-3\delta_c\Delta-3\delta_c+3\Delta}{9} \equiv \tilde{p}_H$ and the second one if $p_1^H > \tilde{p}_E \equiv \frac{4\delta+9\Delta-4\delta\Delta-9}{3}$. The correspondent profit will be:

$$\pi_{NE}^L = \frac{(4(\delta_c-4)\delta_c+17)(\Delta-1)^2 + 9(p_1^A)^2 + 2(6\delta_c-11)(\Delta-1)p_1^A}{8(8-3\delta_c)}.$$

If $p_1^H \leq \tilde{p}_H$, then $x_1 \geq \frac{\Delta+1}{4}$ and thus firm L sets a price such that $\hat{x}_1 = \frac{\Delta+1}{4}$, i.e., p_{1HC}^H. Moreover, since $\pi_{2H}^L(x_1 = \frac{\Delta+1}{4}) = \pi_{2TDS}^L(x_1 = \frac{\Delta+1}{4})$, the profit will be π_{HC}^L.
If $\tilde{p}_E > p_1^L > \tilde{p}_H$ (which is possible only provided that $\Delta > \frac{5}{3}$), then the firm solves $\max_{p_1^L} p_1^L(1-x_{1H})$. The first order condition of this problem gives $p_{1E}^L = \frac{3p_1^H+3+2\delta_c\Delta-2\delta_c-3\Delta}{6}$, with correspondent profit of $\pi_E^L = \frac{(3p_1^H-(3-2\delta_c)(\Delta-1))^2}{24(3-\delta_c)}$.

(iii) If $x_{1L} \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm L. This case can exist only if $\Delta < 1$ or, when $\Delta > 1$, if $x_1 = 1$. ODS to L would give firm L a second-period profit of

$$\pi_{2L}^L = p_2^{LL}(1-x_1) + p_2^{LH}(x_1 - x_2^H) = \frac{\Delta^2 + \left(46x_1 - 20x_1^2 - 17\right) + 2\Delta(5x_1 - 8)}{18}.$$

If $\Delta < 1$, then firm L maximizes $\max_{p_1^L} \pi_L^L = \max_{p_1^L} p_1^L(1-\hat{x}_1) + \pi_{2L}^L$ under the constraint $x_1 \geq \frac{\Delta+3}{4}$. The first order condition of this problem gives:

$$p_{1L}^L = \frac{18\delta_c^2 - 6(2\delta_c+7)\Delta + 74\delta_c + 27\delta_c p_1^A + 57p_1^A + 66}{54\delta_c + 84},$$

with correspondent $x_1 = \frac{30\delta_c+14\Delta-9p_1^A+50}{36\delta_c+56}$. Constraint is met if $p_1^H \leq \tilde{p}_L \equiv \frac{8\delta_c}{9}$ and the correspondent profit will be:

$$\pi_L^L = \frac{12(\delta_c+7)\Delta^2+3\left(4(\delta_c(\delta_c+11)+15)+9(p_1^A)^2+12(\delta_c+1)p_1^A\right)-4\Delta(32\delta_c+21p_1^A+42)}{24(9\delta_c+14)}.$$

If the constraint is not satisfied (i.e., $p_1^H \geq \tilde{p}_L$), then we are back to the case with price p_{1LC}^L and profit π_{LC}^L. If instead $x_1 = 1$, firm L entry is prevented.

Up to now, we obtained all possible best responses of firm L within each regime. In order to build up the global best response, we must compare profits across regimes in each segment. We have two cases:

1. $\Delta < 1$, then $\tilde{p}_{HC} > \tilde{p}_H > \tilde{p}_L > \tilde{p}_{LC}$. We will have four segments:
   (a) $p_1^H < \tilde{p}_{LC} \implies p_{1L}^L$.
   (b) $p_1^H \in (\tilde{p}_{LC}, \tilde{p}_L)$ If $p_1^H > \frac{3(\delta_c+1)\sqrt{(3\delta_c+4)(9\delta_c+14)}(1-\Delta)-3\delta_c(\delta_c(9\Delta+3)+7\Delta-3)+42\Delta+38}{54\delta_c+90} \equiv \tilde{p}$, then $\pi_{TDS}^L > \pi_L^L$. Otherwise, $\pi_L^L > \pi_{TDS}^L$.
   (c) $p_1^H \in (\tilde{p}_L, \tilde{p}_H)$. $\implies p_{1TDS}^L$.
   (d) $p_1^H \in (\tilde{p}_H, \tilde{p}_{HC})$. If $p_1^H < \tilde{\tilde{p}}$ with:

$$\tilde{\tilde{p}} \equiv \begin{cases} \frac{1}{18}\left(8 + \frac{12(\Delta-3)}{3\delta_c-2} + \delta_c(3-9\Delta) + 12\Delta - 3\sqrt{-\frac{\delta_c^2(3\delta_c-8)(3\delta_c+4)(\Delta-3)^2}{(2-3\delta_c)^2}}\right) & \text{if } \delta_c < 2/3, \\ \frac{14}{9} & \text{if } \delta_c = 2/3, \\ \frac{1}{18}\left(8 + \frac{12(\Delta-3)}{3\delta_c-2} + \delta_c(3-9\Delta) + 12\Delta + 3\sqrt{-\frac{\delta_c^2(3\delta_c-8)(3\delta_c+4)(\Delta-3)^2}{(2-3\delta_c)^2}}\right) & \text{if } \delta_c > 2/3, \end{cases}$$

   then $\pi_{TDS}^L > \pi_{NE}^L \implies \pi_{1H}^L$. When $p_1^H > \tilde{\tilde{p}} \implies \pi_{TDS}^L < \pi_{1H}^L$. Notice that if $\delta_c = 0$, then $\tilde{\tilde{p}} = \tilde{p}_{HC} = \tilde{p}_H$, therefore this segment will disappear.

(e) $p_1^H \geq \tilde{p}_{HC} \implies p_{1NE}^L$.

2. $\Delta \in [1, \frac{5}{3}]$, then in the last two segments nothing changes compared to the case with $\Delta < 1$. For $p_1^H \leq \tilde{p}_H$:

   (a.i) $p_1^H < p_{min}$. Firm L is out of the market.

   (b.i) $p_1^H \in (p_{min}, \tilde{p}_H)$. p_{1TDS}^L.

3. $\Delta > \frac{5}{3}$, then (a.i),(b.i),(c) and (d) remain the same, while for $p_1^L > \tilde{p}_{HC}$, we have:

   (e.i) $p_1^H \in (\tilde{p}_{HC}, \tilde{p}_E) \implies p_{1NE}^L$.

   (f.i) $p_1^H \geq \tilde{p}_E \implies p_{1E}^L$.

Putting together all the results above, the best response of firm L is

$$p_1^L(p_1^H) = \begin{cases} p_{1L}^L & \text{if } p_1^H \leq \tilde{p} \\ p_{1TDS}^L & \text{if } p_1^H \in \left[\tilde{p}, \min\{\tilde{\tilde{p}}, \tilde{p}_{HC}\}\right], \\ p_{1NE}^L & \text{if } p_1^H > \tilde{\tilde{p}}, \end{cases} \quad \Delta < 1,$$

$$p_1^L(p_1^H) = \begin{cases} p_{1TDS}^L & \text{if } p_1^H \in \left(p_{min}, \min\{\tilde{\tilde{p}}, \tilde{p}_{HC}\}\right), \\ p_{1NE}^L & \text{if } p_1^H > \min\{\tilde{\tilde{p}}, \tilde{p}_{HC}\}, \end{cases} \quad \Delta \in [1, \underline{\Delta}_E]$$

$$p_1^L(p_1^H) = \begin{cases} p_{1TDS}^L & \text{if } p_1^H \in \left(p_{min}, \min\{\tilde{\tilde{p}}, \tilde{p}_{HC}\}\right), \\ p_{1E}^L & \text{if } p_1^H \in (\min\{\tilde{\tilde{p}}, \tilde{p}_{HC}\}, \tilde{p}_E), \\ p_{1NE}^L & \text{if } p_1^H > \tilde{p}_E. \end{cases} \quad \Delta > \underline{\Delta}_E$$

7.3. Proof of Proposition 2

Existence and uniqueness of the equilibria.

- **TDS scenario**. Assume an equilibrium generating two-direction switching, i.e., in which both firms i, j use $p_{1TDS}^i(p_{1TDS}^j)$. In this equilibrium prices will be:

$$(p_{1TDS}^{H*}, p_{1TDS}^{L*}) = \left(1 + \frac{\Delta}{3} + \frac{\delta_c}{3} - \frac{\Delta(4 - 3(1-\delta_c)\delta_c)}{3(9\delta_c + 7)}, 1 - \frac{\Delta}{3} + \frac{\delta_c}{3} + \frac{\Delta(4 - 3(1-\delta_c)\delta_c)}{3(9\delta_c + 7)}\right),$$

  Firms are both on their best responses when $\Delta < 1$. If $\Delta > 1$, firm H will deviate from this equilibrium when $\Delta > \frac{9\delta_c+7}{5-3\delta_c}$, since $p_{1TDS}^L > p_{max}$. Moreover, $p_{1TDS}^L < \hat{p}$, if

$$\Delta > \begin{cases} \Delta_1 \equiv \frac{(9\delta_c+7)\left((59-3\delta_c(\delta_c(13-27\delta_c)+71))+4\sqrt{(1-\delta_c)^2(14-3\delta_c)(3\delta_c+4)(6\delta_c+1)^2}\right)}{3(\delta_c(\delta_c(9\delta_c(29\delta_c-68)-694)+644)-47)} & \text{if } \delta_c \in (0.32374, 0.58548) \\ \Delta_2 \equiv \frac{(9\delta_c+7)\left((59-3\delta_c(\delta_c(13-27\delta_c)+71))-4\sqrt{(1-\delta_c)^2(14-3\delta_c)(3\delta_c+4)(6\delta_c+1)^2}\right)}{3(\delta_c(\delta_c(9\delta_c(29\delta_c-68)-694)+644)-47)} & \text{if } \delta_c > 0.58548 \end{cases}$$

  Moreover, firm H always deviates if $\Delta > \underline{\Delta}_E$, since $p_{1TDS}^L < \bar{p}_E$. Notice that $\underline{\Delta}_E$ is higher than $\frac{9\delta_c+7}{5-3\delta_c}$ if $\delta < \frac{2}{21}$, higher than Δ_1 if $\delta < 0.58548$ and always higher than Δ_2

if $\delta > 0.951209$. Whenever firm H does not deviate, firm L will never deviate from this equilibrium. Hence, summarizing, this equilibrium exists for all Δ lower than:

$$\bar{\Delta}_{TDS} = \begin{cases} \frac{9\delta_c+7}{5-3\delta_c} & \text{if } \delta_c \leq 2/21 \\ \frac{5}{3} & \text{if } \delta_c \in (2/21, 0.951209) \\ \Delta_2 & \text{if } \delta_c > 0.951209 \end{cases}$$

- **ODH scenario**. We can have two cases:
  1. Equilibria with firm positive marker share for firm L in the second period. Both firms i, j use $p^i_{1NE}(p^j_{1NE})$. In this equilibrium prices will be:

     $$(p^{H*}_{1NE}, p^{L*}_{1NE}) = \left(1 + \frac{\Delta}{3} + \frac{44+\Delta(6(\delta_c-5)\delta_c+8)+6\delta_c^2-38\delta_c}{105-27\delta_c}, 1 - \frac{\Delta}{3} + \frac{2(22+3\Delta(13-(8-\delta_c)\delta_c)-\delta_c(19-3\delta_c))}{3(35-9\delta_c)}\right).$$

     If $\delta_c < 2/3$, firm L always deviates from this equilibrium since $p^H_{1NE} < \tilde{\tilde{p}}$. Above $\delta_c = 2/3$, firm L does not deviate provided that Δ is higher than Δ_{1NE}, with

     $$\Delta_{1NE} \equiv \frac{3\delta_c(\delta_c(9\delta_c(31\delta_c-234)+3659)+892)+32-4\sqrt{\delta_c^2(83\delta_c)(3\delta_c+4)(3\delta_c(29-9\delta_c)+70)^2}}{3(\delta_c(\delta_c(9\delta_c(17\delta_c-106)+1361)+564)+32)},$$

     and firm H does not deviate provided that $\Delta > \Delta_{2H}$, where:

     $$\Delta_{2H} \equiv \frac{4(3\delta_c-14)(3\delta_c(5\delta_c-16)+25)+4\sqrt{(35-9\delta_c)^2(\delta_c-1)^2(14-3\delta_c)(3\delta_c+4)}}{3(\delta_c(\delta_c(9\delta_c(17\delta_c-148)+3422)-2452)+385)} - 1,$$

     Notice that $\Delta_{2H} < \underline{\Delta}_E$ for all $\delta_c > 2/3$ and $\Delta_{1NE} = \underline{\Delta}_E$ when $\delta_c = 0.86286$. Moreover, $\Delta_{1NE} > \Delta_{2H}$ when $\delta_c < 0.867653$, and the opposite is true above 0.867653.
  2. Equilibria with the exit of firm L in the second period. Both firms i, j use $p^i_{1E}(p^j_{1E})$. In this equilibrium, the prices will be:

     $$(p^{H*}_{1E}, p^{L*}_{1E}) = \left(1 + \frac{\Delta}{3} + \frac{18+2\left(\delta_c^2+(\delta_c-4)(\delta_c+1)\Delta-9\delta_c\right)}{33-9\delta_c}, 1 - \frac{\Delta}{3} + \frac{9-2(\delta(\delta(\Delta-2)-4\Delta+10)-2\Delta)}{33-9\delta_c}\right).$$

  Both firms are at their best response, i.e. $p^{L*}_{1E} \in (\hat{p}, \bar{p}_E)$ and $p^{H*}_{1E} \in (\tilde{\tilde{p}}, \tilde{p}_E)$, when $\Delta > \underline{\Delta}_E$.

- **ODL scenario**. Two cases:
  1. When $\Delta < 1$, only one couple of prices can lead to this scenario, i.e., $(p^H_{1L}, p^L_{1L}) = \left(\frac{30\Delta\delta_c+14\Delta+90\delta_c^2+208\delta_c+98}{81\delta_c+105}, = \frac{\delta_c(72\delta_c+233)+149-\Delta(3\delta_c+43)}{81\delta_c+105}\right)$.
     This cannot be an equilibrium because the best response of firm H to p^L_{1L} is p^H_{1TDS}.
  2. If $\Delta \geq 1$, the only possibility is to have a monopoly of firm H in the first period, choosing price $p^H_{1M} = p^L_1 + \Delta - 1$. This strategy is effective (i.e., firm L cannot enter the market) only if $p^L_1 + \Delta - 1 < p_{min} = \frac{7\Delta+1-3\delta_c\Delta-3\delta_c}{9} \Leftrightarrow p^L_1 < \frac{10-3\delta_c\Delta-3\delta_c-2\Delta}{9}$. Firm H always deviates to ODH when $\Delta > \frac{9}{7}$ because $\frac{10-3\delta_c\Delta-3\delta_c-2\Delta}{9} < \hat{p}_H$.

Proof of Proposition 3

When $\Delta < \bar{\Delta}_{TDS}$, the first-period equilibrium prices are p^{H*}_{1TDS} and p^{L*}_{1TDS}. The market splitting cutoffs are $x_{1TDS} = \frac{1}{2} + \frac{\Delta(5-3\delta_c)}{2(9\delta_c+7)}$, $x^H_2 = \frac{1}{3} + \frac{(\delta_c+2)\Delta}{9\delta_c+7}$ and $x^L_2 = \frac{2}{3} + \frac{(\delta_c+2)\Delta}{9\delta_c+7}$. Plugging the first-period cutoff in the second-period prices in point (ii) of Proposition 1 and computing profits of the two firms, we get:

$$\begin{array}{rl} \pi^H_{TDS} = & \frac{9(\delta+2)(3\delta(\delta+2)+11)\Delta^2-6(\delta+1)(3\delta-16)(9\delta+7)\Delta+(3\delta+14)(9\delta+7)^2}{18(9\delta+7)^2} \\ \text{and} & \\ \pi^L_{TDS} = & \frac{9(\delta+2)(3\delta(\delta+2)+11)\Delta^2+6(\delta+1)(3\delta-16)(9\delta+7)\Delta+(3\delta+14)(9\delta+7)^2}{18(9\delta+7)^2} \end{array} \tag{10}$$

Comparing the profits in (10) with the ones in (3), it holds that $\pi^H_{TDS} < \pi^H_u$ when $\Delta < \bar{\Delta}_{TDS}$ and $\pi^L_{TDS} < \pi^L_u$ if $\Delta < \tilde{\Delta}$. For point (ii), the consumer surplus under BBPD is equal to:

$$\begin{aligned} CS_{TDS} &= \int_0^{x^H_2} U^{HH}_{TDS}(x)dx + \int_{x^H_2}^{x_1} U^{LH}_{TDS}(x)dx + \int_{x_1}^{x^L_2} U^{HL}_{TDS}(x)dx + \int_{x^L_2}^{1} U^{LL}_{TDS}(x)dx \\ &= \frac{18(\delta_c+1)(9\delta_c+7)^2(u^A+u^B)-(9\delta_c+7)^2(43\delta_c+45)+9(\delta_c((71-\delta_c)\delta_c-23)+25)\Delta^2}{36(9\delta_c+7)^2} \end{aligned}$$

where $U^{ij}_{TDS}(x) = u^i - p^i_1 - |x-l^i| + \delta_c(u^j - p^{ij}_2 - |x-l^j|)$ refers to the inter-temporal utility of a consumer located at x who buys good j in the first period and good i in the second one, with possibly $i \neq j$ in case of switching in the second period. It is easy to verify that $CS_u < CS_{TDS}$ for all values of parameters δ_c and Δ.

Proof of Proposition 4

- When $\Delta \in (\underline{\Delta}_{NE}, \underline{\Delta}_E)$, the first-period equilibrium prices are p^{H*}_{1NE} and p^{L*}_{1NE}. The market splitting cutoffs are $x_{1NE} = \frac{1}{2} - \frac{17+\delta_c(6\Delta-3)-16\Delta}{70-18\delta_c}$,$x^H_2 = x_{1NE}$ and $x^L_2 = \frac{41-5\delta_c\Delta-11\delta_c+17\Delta}{70-18\delta_c}$. If $x^L_2 = 1$, plugging the first-period cutoff in the second-period prices in point (ii) of Proposition 1 and computing profits of the two firms, we get:

$$\begin{array}{rl} \pi^H_{NE} = & \frac{-36\delta^3(\Delta+1)^2+\delta^2\left(387\Delta^2+906\Delta+627\right)-2\delta(\Delta(681\Delta+1870)+1611)+7(\Delta(229\Delta+734)+741)}{6(35-9\delta)^2} \\ \text{and} & \\ \pi^L_{NE} = & \frac{-36\delta^3(\Delta-2)^2+3\delta^2(3\Delta-7)(35\Delta-79)+\delta\left(-978\Delta^2+4772\Delta-6242\right)+\Delta(1091\Delta-5254)+7619}{6(35-9\delta)^2} \end{array} \tag{11}$$

  Comparing the profits in (12) with the ones in (3), it holds that $\pi^H_{NE} < \pi^H_u$ and $\pi^L_{NE} > \pi^L_u$ if $\Delta \in (\underline{\Delta}_{NE}, \underline{\Delta}_E)$. Moreover, the consumer surplus under BBPD is equal to:

$$CS_{NE} = \int_0^{\hat{x}_1} U_H^{HH}(x)dx + \int_{\hat{x}_1}^{\hat{x}_2^L} U_H^{HL}(x)dx + \int_{\hat{x}_2^L}^{1} U_H^{LL}(x)dx$$
$$= \frac{1}{486}\left(582u^L + 147u^H - (16+81u^H)\Delta^2 - 954\right),$$

which is always higher than CS_u if $\Delta \in (\underline{\Delta}_H, \underline{\Delta}_E)$.

- When $\Delta > \underline{\Delta}_E$, the first-period equilibrium prices are p_{1E}^{H*} and p_{1E}^{L*}. The market splitting cutoffs are $x_{1NE} = \frac{1}{2} - \frac{17+\delta_c(6\Delta-3)-16\Delta}{70-18\delta_c}$,$x_2^H = x_{1NE}$ and $x_2^L = \max\left\{\frac{41-5\delta_c\Delta-11\delta_c+17\Delta}{70-18\delta_c}, 1\right\}$. If $x_2^L = 1$, plugging the first-period cutoff in the second-period prices in point (ii) of Proposition 1 and computing profits of the two firms, we get:

$$\pi_E^H = \frac{18\delta_c^2(\Delta(2\Delta+5)+5)-4\delta_c^3(\Delta+1)^2-3\delta_c(\Delta(35\Delta+114)+167)+2\Delta(50\Delta+221)+826}{6(11-3\delta_c)^2}$$
$$\text{and} \tag{12}$$
$$\pi_E^L = \frac{(3-\delta_c)(2\delta_c(\Delta-2)-5\Delta+17)^2}{6(11-3\delta_c)^2}$$

It holds that $\pi_E^L > \pi_u^L$ when $\Delta > \underline{\Delta}_E$. Moreover, $\pi_E^H > \pi_u^H$ whenever $\delta_c < 0.18$ and $\pi_E^H > \pi_u^H$ if $\delta_c > 0.235$. When $\delta_c \in (0.18, 0.235)$, the stricter condition $\delta_c < \hat{\delta}_c$, where
$\hat{\delta}_c^3\left(12\Delta^2 + 24\Delta + 12\right) + \hat{\delta}_c^2\left(-90\Delta^2 - 162\Delta - 108\right) + \hat{\delta}_c\left(183\Delta^2 + 234\Delta + 315\right) - 58\Delta^2 + 126\Delta + 300 = 0$
is needed to have $\pi_E^H > \pi_u^H$, and the opposite is true otherwise. To prove point (ii) of Proposition 4, we have:

$$CS_E = \int_0^{x_2^H} U_H^{HH}(x)dx + \int_{x_1}^{1} U_H^{HL}(x)dx$$
$$=$$
$$\frac{9373(11\Delta+8)(\Delta+1)}{15066(35-9\delta_c)} + \frac{7(22-93\Delta)(\Delta+1)}{540(11-3\delta_c)} - \frac{3(17\Delta+142)(\Delta+1)}{620(\delta_c+3)} - \frac{(11\Delta+8)^2}{9(35-9\delta)^2}$$
$$+ \frac{(32u^H+22u^L-47-2\Delta^2)\delta_c}{54} + \frac{2\Delta^2+280u^H+206u^L-937}{486}$$

It holds that $CS_E > CS_u$ when $\delta_c \geq 0.0.77$ and $CS_E < CS_u$ when $\delta_c \leq 0.43$ When $\delta_c \in (0.43, 0.77)$, the stricter condition $\delta_c < \underline{\delta}_c$, where
$34743\Delta^2 + 84078\Delta - 1083753\left(567\Delta^2 - 810\Delta - 3321\right)\underline{\delta}_c^5 + \left(-4581\Delta^2 + 6174\Delta + 33975\right)\underline{\delta}_c^4$
$+ \left(7386\Delta^2 + 13620\Delta - 69318\right)\underline{\delta}_c^3 + \left(24430\Delta^2 - 181764\Delta - 289470\right)\underline{\delta}_c^2$
$+ \left(302238\Delta + 1214415 - 75329\Delta^2\right)\underline{\delta}_c = 0$ is needed to have $CS_E < CS_u$, and the opposite is true otherwise.

References

Belleflamme, P., Picard, P. M., 2007. Piracy and competition. Journal of Economics & Management Strategy 16 (2), 351–383.

Chen, Y., December 1997. Paying customers to switch. Journal of Economics & Management Strategy 6 (4), 877–897.

Chen, Y., December 2008. Dynamic price discrimination with asymmetric firms. Journal of Industrial Economics 56 (4), 729–751.

Chen, Y., Pearcy, J., 2010. Dynamic pricing: when to entice brand switching and when to reward consumer loyalty. The RAND Journal of Economics 41 (4), 674–685.

Colombo, S., 2015. Should a firm engage in behaviour-based price discrimination when facing a price discriminating rival? a game-theory analysis. Information Economics and Policy 30 (0), 6 – 18.

Corts, K. S., 1998. Third-degree price discrimination in oligopoly: All-out competition and strategic commitment. The RAND Journal of Economics 29 (2), pp. 306–323.

Esteves, R. B., 2009. A survey on the economics of behaviour-based price discrimination. NIPE Working Papers 5/2009, NIPE - Universidade do Minho.

Esteves, R.-B., 2010. Pricing with customer recognition. International Journal of Industrial Organization 28 (6), 669 – 681.

Esteves, R.-B., 2014. Behavior-based price discrimination with retention offers. Information Economics and Policy 27, 39 – 51.
URL http://www.sciencedirect.com/science/article/pii/S0167624514000146

Esteves, R.-B., Reggiani, C., 2014. Elasticity of demand and behaviour-based price discrimination. International Journal of Industrial Organization 32 (0), 46 – 56.

Fudenberg, D., Tirole, J., Winter 2000. Customer poaching and brand switching. RAND Journal of Economics 31 (4), 634–657.

Gehrig, T., Shy, O., Stenbacka, R., 2007. Market Dominance and Behaviour-based Pricing Under Horizontal and Vertical Differentiation. Centre for Economic Policy Research.

Gehrig, T., Shy, O., Stenbacka, R., 2011. History-based price discrimination and entry in markets with switching costs: A welfare analysis. European Economic Review 55 (5), 732 – 739.

Gehrig, T., Shy, O., Stenbacka, R., 2012. A welfare evaluation of history-based price discrimination. Journal of Industry, Competition and Trade 12 (4), 373–393.

Glicksberg, I. L., 1952. A further generalization of the kakutani fixed point theorem, with application to nash equilibrium points. Proceedings of the American Mathematical Society 3 (1), 170–174.
URL `http://www.jstor.org/stable/2032478`

Ivaldi, M., Jullien, B., Rey, P., Seabright, P., Tirole, J., 2003. The economics of tacit collusion. Final Report for DG Competition, European Commission.

Liu, Q., Serfes, K., June 2005. Imperfect price discrimination in a vertical differentiation model. International Journal of Industrial Organization 23 (5-6), 341–354.

Shaffer, G., Zhang, Z. J., September 2002. Competitive One-to-One Promotions. Management Science 48 (9), 1143–1160.

Thisse, J.-F., Vives, X., 1988. On the strategic choice of spatial price policy. American Economic Review 78 (1), 122–137.

Villas-Boas, J. M., Winter 1999. Dynamic competition with customer recognition. RAND Journal of Economics 30 (4), 604–631.