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Research Article

Analytical evaluation of the Stress Intensity Factor in stiffened sheets with multiple side damage

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Abstract: A closed form solution of the Stress Intensity Factor (SIF) for stiffened flat sheets, typically used in aircraft construction, in Multiple Site Damage (MSD) conditions, has been developed. The well-known theory of complex variable functions has been used, through the application of functions specifically developed for the case of cracks equally spaced and of equal length. Moreover, the superposition principle has been applied to evaluate the compression loads transmitted by the stringers through the rivets, by imposing the equilibrium on the crack free surfaces and the compatibility of displacements between sheet and stringers at the rivets location. The results have been compared with solutions available in the literature, obtained by combination of various analytical techniques and experimental methodologies, showing a good agreement. The proposed method is a reference for the validation of other numerical or analytical methods and effectively can replace the Finite Element Method for simple geometries.

Keywords: Multiple Site Damage; Stress Intensity Factor; closed-form solution

1. Introduction

In 1988, an Aloha Airlines Boeing 737 lost a large portion of the fuselage when the aircraft reached its cruising altitude of 24,000 ft. The aircraft in question was one with the highest number of ground-air-ground cycles. The collapse was produced by the simultaneous presence of several cracks in a fuselage riveted joint (Multiple Site Damage: MSD).

The Damage Tolerance philosophy, recommended for the design of aircraft structures, involves the ability of a component to sustain anticipated loads in the presence of fatigue, corrosion or accidental damage until such damage is detected [1]. However, standard DT analyses focus only on the evolution of singular or isolated flaws [2]. The occurrence of Widespread Fatigue Damage (WFD), as in the case of the Aloha Airlines flight, poses therefore a risk that the analyses can be invalidated over time due to the simultaneous presence of flaws within a structure. This problem produced the introduction of revised requirements to take into account in the fatigue life evaluation, among others, the effect of corrosion or repairs, as may happen in aging aircraft [3]. The new rules [4] included the definition of a Limit of Validity of the engineering data in order to support the continuing structural maintenance of an aircraft. As a result, there is a renewed interest in robust and efficient analysis methods for predicting WFD and its effects [5].

The time to the onset of WFD in a structure can be determined by deterministic or by statistical methods [6]. In both cases, the growth rate of a set of variously arranged cracks in a structure has to be evaluated. The interaction between the cracks, propagating under fatigue loads, may indeed reduce the residual static strength of the structure, when the dimensions of the cracks become significant. This reduction can be significant and depends on a number of factors such as: material properties, geometry of the joint, size of the cracks and its distribution [7].

The growth rate of the individual cracks is calculated by means of the Fracture Mechanics criteria, such as the Stress Intensity Factor (SIF). In such calculations, most of the life of the structure is spent in the presence of very small cracks, thus poorly interacting. As long as the size of the cracks remains small, the problem can be approximated as linear elastic; conversely, for longer cracks, the effects of reciprocal interactions and the nonlinearity of materials become significant. The numerical evaluation of the SIF has already been developed for many different geometrical configurations [8,9,10]; however, the comparison of these results with the corresponding closed form to date has never been carried out, since the only available solution is related to the isotropic flat sheets with multiple collinear cracks.

This paper therefore develops a closed form solutions of SIF for a riveted stiffened flat sheet with a collinear array of cracks of equal length. The problem is solved by the theory of complex variables functions, referring to literature solutions for the calculation of the state of stress, while developing the tools to calculate the displacement field in the flat sheet; the knowledge of the displacements allows the evaluation of the influence functions in the body equilibrium equations. Furthermore, a sample calculation on a stiffened flat sheet was conducted, confirming the effectiveness of the results.

2. SIF Evaluation by the Complex Variable Functions Method

An infinite thin plate of metallic material (elastic homogeneous isotropic, plane stress conditions), riveted to equally spaced stiffeners (Figure 1) has been taken into account. The plate and the stringers are subject respectively to the uniaxial tension σ and $\sigma E_s / E$, so as to obtain the same deformation at a great distance from the cracking line.

A multiple damage configuration with a series of collinear cracks originated in correspondence of the housing holes of the rivets and interacting between them will be analysed. This configuration is representative of actual fatigue problems in aircraft skin and the results in [11] indicate that this is the most critical case of real interest. In Figure 2, the forces shared by the rivets and the stiffened flat sheet are shown.

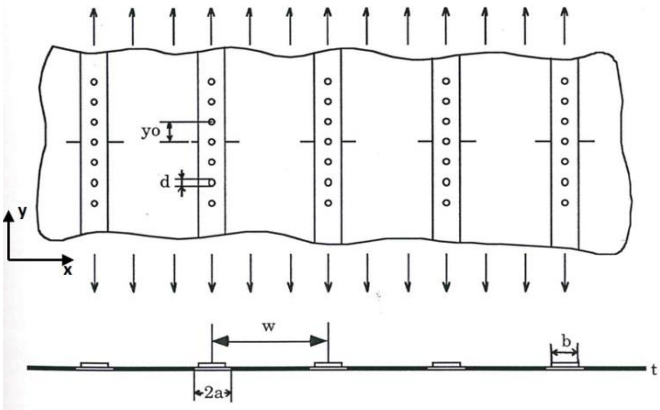

Figure 1. Stiffened sheet geometrical configuration.

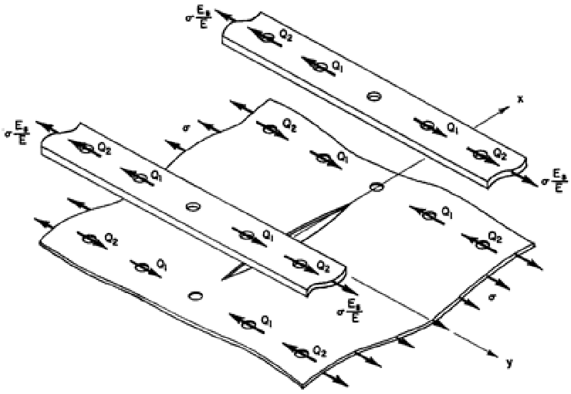

Figure 2. Forces shared by rivets and sheet.

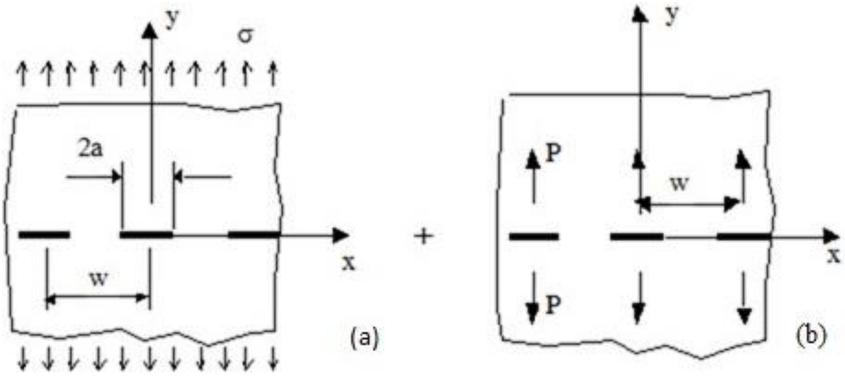

Figure 3. Superposition principle: (a) infinite cracked plate with uniform load; (b) infinite cracked plate with row of concentrated forces.

The SIF at each crack tip for the sheet of Figure 1 can be calculated using the superposition principle, as shown in Figure 3:

$$K_I = \overline{K} + \sum_i K_i \cdot P_i \tag{1}$$

where:

$$\bar{K} = \sigma\sqrt{\pi a} \cdot \sqrt{\frac{w}{\pi a} tan\left(\frac{\pi a}{w}\right)} \tag{2}$$

is the SIF for the infinite cracked plate with uniform load (Figure 3a), and:

$$K_i = \frac{1}{\pi}\sqrt{\pi a} \cdot \left\{k(a, y_0) - \frac{y_o}{2(1-\upsilon)} \cdot \frac{\partial k(a, y_0)}{\partial y_0}\right\} \tag{3}$$

is the SIF for the infinite cracked plate with unknown loads P_i, transferred by the i-th rivets row at y_0 distance from the crack line; $k(a, y_0)$ is a function from literature [12]:

$$k(a, y_0) = \left(\frac{\pi}{w} cosh\left(\frac{\pi y_0}{w}\right)\right)\sqrt{\frac{w}{\pi a} tan\left(\frac{\pi a}{w}\right)} \cdot \frac{1}{\sqrt{\left(sin\left(\frac{\pi a}{w}\right)\right)^2 + \left(sinh\left(\frac{\pi y_0}{w}\right)\right)^2}} \tag{4}$$

In the case of multi cracked stiffened sheets, no explicit solutions are available in the literature. The SIF calculation is then considerably more complicated than in the case of a simple plate. In fact, the stress field around the crack tips is influenced not only by the tensile load applied to the sheet but also by the compression loads transmitted by the stringers through the rivets. The first step is the evaluation of rivet loads P_i in Eqn. (1).

The magnitude of the concentrated forces P_i is calculated applying the displacement compatibility method whereby, as suggested by Poe [13], the equality between skin and stiffeners displacements under the investigated loading condition is imposed.

The displacement components for symmetrical problems with respect to the x axis (Figure 1) are given by [14,15]:

$$\begin{cases} u = \frac{1-\upsilon}{E} \cdot Re\tilde{Z} - \frac{1+\upsilon}{E} \cdot y \cdot ImZ - \frac{1+\upsilon}{E} \cdot A \cdot x \\ v = \frac{2}{E} \cdot Im\tilde{Z} - \frac{1+\upsilon}{E} \cdot y \cdot ReZ + \frac{1+\upsilon}{E} \cdot A \cdot y \end{cases} \tag{5}$$

where: υ and E are respectively the Poisson ratio and the Young's modulus of the sheet's material; Z(z) is an analytical function of the complex variable z = x + iy; $\tilde{Z}$(z) is the first integral of Z(z); A is a constant depending on the boundary conditions. The displacements calculation is therefore linked to the evaluation of Z(z) and A.

By imposing the equivalence of the displacements of the plate v_i and the stringers v_i^s in the i-th row of rivets, a system of algebraic equations in the unknowns P_i can be written. The displacements can be expressed as a function of coefficients of influence A_{ij}, B_i, A_{ij}^s, B_j^s [13]:

$$v_i = -\sum_{j=1}^{N} A_{ij} \cdot P_j + B_i \cdot \sigma \tag{6}$$

$$v_i^s = \sum_{j=1}^{N} A_{ij}^s \cdot P_j + B_i^s \cdot \sigma \frac{E_s}{E} \tag{7}$$

being E_s and E respectively the elastic moduli of the stringer and the plate. Matching the Eqn. (6) and Eqn. (7) for the i-th row, with N indicating the number of rivets rows and L the number of the stringers considered in the calculation, the following linear system in the unknowns P_j can be obtained:

$$\sum_{j=1}^{N}(A_{ij}^{S}+A_{ij})\cdot P_j-\left(B_i-B_i^{S}\frac{E_S}{E}\right)\cdot\sigma=0 \qquad i=1,L \tag{8}$$

2.1. Coefficient of Influence B_i

For an infinite plate loaded by uniform asymptotic traction, with multiple damages (Figure 3a), the v displacement is given by Eqn. (5), assuming A = σ/2 [9] and, for $Z(z)$ and $\tilde{Z}(z)$, the following expressions:

$$Z(z)=\sigma\cdot\left[1-\left(\sin\frac{\pi a}{w}/\sin\frac{\pi z}{w}\right)^2\right]^{-\frac{1}{2}}-\frac{\sigma}{2} \tag{9}$$

$$\tilde{Z}(z)=i\cdot\frac{\sigma\cdot w}{\pi}\cdot\log_e\left[\left(\cos\frac{\pi z}{w}/\sin\frac{\pi a}{w}\right)+\sqrt{1-\left(\sin\frac{\pi z}{w}/\sin\frac{\pi a}{w}\right)^2}\right]-\frac{\sigma}{2}\cdot z \tag{10}$$

Finally, the coefficient of influence B_i, i.e. the displacement of a point in the plate coincident with the i-th rivet position under unitary uniform asymptotic traction, is obtained considering $\sigma=1$, being $z=x_i+iy_i$ the spatial coordinate of the i-th row of rivets.

2.2. Coefficient of Influence A_{ij}

The displacements along the y-axis of the cracked plate loaded by a row of concentrated forces P_i placed at a distance y_0 from the crack row (Figure 3b), are given by Eqn. (5) assuming for $Z(z)$ and $\tilde{Z}(z)$ the following expressions [12]:

$$Z(z)=\frac{P_i}{\pi}\cdot\left\{[F(z,y_0,a)-F(z,y_0,0)]-\frac{y_0}{2(1-\upsilon)}\cdot\frac{\partial[F(z,y_0,a)-F(z,y_0,0)]}{\partial y_0}\right\} \tag{11}$$

$$\tilde{Z}(z)=\frac{P_i}{\pi}\cdot\left\{[\tilde{F}(z,y_0,a)-\tilde{F}(z,y_0,0)]-\frac{y_0}{2(1-\upsilon)}\cdot\frac{\partial[\tilde{F}(z,y_0,a)-\tilde{F}(z,y_0,0)]}{\partial y_0}\right\} \tag{12}$$

where F is a stress function and $\tilde{F}$ its first integral [12]. The coefficient of influence A_{ij}, i.e., the displacement of a point in the plate coincident with the i-th rivet position under unitary concentrated force coincident with the j-th rivet position ($P_i=1$), is obtained by replacing in Eqn. (5), with a = 0, the Eqn. (11) and (12).

2.3. Coefficient of Influence B_j^s

The v displacement of a generic point of a stringer loaded by an axial asymptotic direct stress $\sigma\, E_s/E$ is given by:

$$v_{S\sigma}=\frac{\sigma\cdot y}{E} \tag{13}$$

Finally, the coefficient of influence B_j^s, i.e. the displacement of a point in the stringer coincident with the i-th rivet position under unitary uniform asymptotic traction, is obtained from the Eqn. (13) using $\sigma = 1$.

2.4. Coefficient of Influence A_{ij}^{s}

The v displacement of a generic point of a stringer loaded by a pair of concentrated forces arranged symmetrically with respect to the x axis is given by [13]:

$$v_{SP} = \frac{\sigma(1+\upsilon)\cdot(3-\upsilon)\cdot P}{8\pi E_s t_{eq}} \cdot \sum_{n=0}^{\infty} \Psi_n \tag{14}$$

where t_{eq} is the stringer equivalent thickness and Ψ_n is a stress function. The A_{ij}^{s} coefficients, i.e. the displacement of a point in the plate coincident with the i-th rivet position under unitary concentrated force coincident with the j-th rivet position, can be obtained by using $P_i = 1$ into the Eqn. (14).

3. Example of SIF Calculation

A Fortran software has been developed for the calculation of the SIF in a stiffened sheet with multi-site damage by means of the complex variable functions theory. The results have been compared with literature data and analytical solutions.

The test geometry consists of an aluminum stiffened sheet, Figure 1, loaded with a uniform tension σ = 10 MPa, with the following geometric characteristics: w = 120 mm, y_0 = 20 mm, d = 5 mm, t = 1 mm, b = 25 mm, t_{eq} = 4.8 mm. The SIF has been calculated for crack half-length values of 12 mm, 24 mm, 36 mm, 48 mm, corresponding to a/w ratios respectively of 0.1, 0.2, 0.3, 0.4. The values of K/K_0 ratio ($K_0 = \sigma \cdot \sqrt{\pi a}$) at the tip of the central crack are compared in Figure 4 with the well-known results on the same stiffened panel containing a single crack [13] and the data on the MSD configuration obtained by applying the compounding method [16].

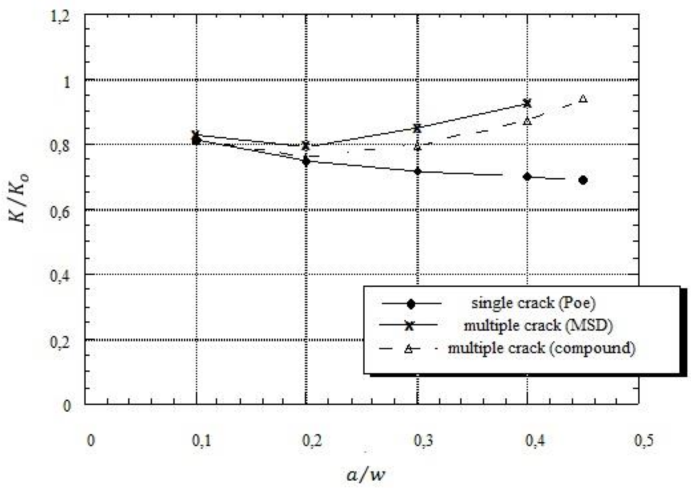

Figure 4. Comparison of the single crack, complex variables MSD and compound solutions for FIS in a stiffened sheet.

As long as the size of the slots is small in comparison to the spacing of the current (a/w < 0.2), the K/K_0 ratio value in multiple damage condition is equal to the value for a single crack while, with increasing the size of the cracks, the interaction effects become relevant, producing a rapid rise of the values of the parameter K. Again, the difference between the SIF values evaluated with the complex variable functions respect those obtained by applying the method of composition is always lower than 7%.

4. Conclusion

An original method based on the analytical solution of the complex variable function has been developed for the evaluation of the Stress Intensity Factor in a stiffened panel in the presence of an infinite array of cracks. Some examples of SIF calculation were performed in function of crack parameters (length, position and number of the slots); the results were compared with solutions available in the literature, obtained by combination of various analytical techniques and experimental methodologies, showing a good agreement. Therefore, the presented method can be advantageously used for a theoretical prediction of the SIF values, the key parameter in Fracture Mechanics for the determination of the residual static strength of a component with different damage conditions.

Conflict of Interest

The author declares no conflicts of interest in this paper

References

1. Goranson U (2007) Damage tolerance facts and fiction. International conference on damage tolerance of aircraft structures, Delft University of Technology, Delft, Netherlands.
2. Nesterenko B, Nesterenko G (2011) Analysis of requirements on fatigue and damage tolerance for civil transport airplanes. In: Komorowski J editor, Proceedings of the 26th Symposium of the international committee on aeronautical fatigue 39–59.
3. Bellinger NC, Komorowski JP (1997) Corrosion pillowing stresses in fuselage lap joints. *AIAA J* 35: 317–320.
4. Federal Aviation Administration (2011) Damage Tolerance and Fatigue Evaluation of Structure. Advisory Circular AC 25.571-1D
5. Wang W, Rans C, Alderliesten RC, et al (2015) Predicting the influence of discretely notched layers on fatigue crack growth in fiber metal laminates. *Eng Fract Mech* 145: 1–14.
6. Wang L, Chow WT, Kawai H, et al. (1996) Prediction of widespread fatigue damage threshold. Proceeding of the FAA/NASA Symposium on Continued Airworthiness of Aircraft Structures, Atlanta (GA), USA.
7. Gruber ML, Wilkins KE, Worden RE (1996) Investigations of fuselage structure subject to widespread fatigue damage. In: Bigelow C editor. Proceedings from the FAA/NASA Symposium on the Continued Airworthiness of Aircraft Structures 439–460.
8. Galatolo R, Nilsson KF (2001) An experimental and numerical analysis of residual strength of butt-joints panels with multiple site damage. *Eng Fract Mech* 68: 1437–1461.

9. Schijve J (1994) Some calculation on the stress distribution in an infinite sheet with a single crack and with periodic collinear cracks. Report LR-770, TUDelft, Netherlands.
10. Molinari G, Meneghin I, Melega M, et al. (2012) Parametric damage tolerance design of metallic aeronautical stiffened panels. *Aeronaut J* 1182: 815–831.
11. Bloom JM, Sanders JL (1966) The effect of a riveted stringer on the stress of a cracked sheet. *ASME J Appl Mech* 33: 561–570.
12. Tada H (1970) Westergaard stress functions for several periodic crack problems. *Eng Fract Mech* 2: 177–180.
13. Poe CC (1971) Stress intensity factor for a cracked sheet with riveted and uniformly spaced stringers. NASA TR R-358, Langley Research Center, Hampton (VA), USA.
14. Westergaard HM (1939) Bearing pressures and cracks. *J Appl Mech* 6: 49–53.
15. Eftis J, Leibowitz H (1972) On the modified Westergaard equation for certain plane crack problems. *Int J Fracture* 8: 383–392
16. Carthwrigth DJ, Rooke DP (1976) *Compendium of Stress Intensity Factors*, HMSO, London, UK.