Misspecification and Expectations Correction in New Keynesian DSGE Models

Published Version:
Misspecification and Expectations Correction in New Keynesian DSGE Models / Angelini, Giovanni; Fanelli, Luca. - In: OXFORD BULLETIN OF ECONOMICS AND STATISTICS. - ISSN 1468-0084. - STAMPA. - 78:(2016), pp. 623-649. [10.1111/obes.12126]

Availability:
This version is available at: https://hdl.handle.net/11585/562089 since: 2022-04-12

Published:
DOI: http://doi.org/10.1111/obes.12126

Angelini, G., & Fanelli, L. (2016). Misspecification and Expectations Correction in New Keynesian DSGE Models. Oxford Bulletin of Economics and Statistics, 78(5), 623-649.

The final published version is available online at:

https://doi.org/10.1111/obes.12126

Misspecification and Expectations Correction in New Keynesian DSGE Models*

Giovanni Angelini† and Luca Fanelli‡

† *Department of Statistical Sciences and School of Economics, Management and Statistics, University of Bologna, Italy (email: giovanni.angelini3@unibo.it)*

‡ *Department of Statistical Sciences and School of Economics, Management and Statistics, University of Bologna, Italy (email: luca.fanelli@unibo.it)*

Abstract

This paper focuses on the dynamic misspecification that characterizes the class of small-scale New-Keynesian models currently used in monetary and business cycle analysis, and provides a remedy for the typical difficulties these models have in accounting for the rich contemporaneous and dynamic correlation structure of the data. We suggest using a statistical model for the data as a device through which it is possible to adapt the econometric specification of the New-Keynesian model such that the risk of omitting important propagation mechanisms is kept under control. A pseudo-structural form is built from the baseline system of Euler equations by forcing the state vector of the system to have the same dimension as the state vector characterizing the statistical model. The pseudo-structural form gives rise to a set of cross-equation restrictions that do not penalize the autocorrelation structure and persistence of the data. Standard estimation and evaluation methods can be used. We provide an empirical illustration based on U.S. quarterly data and a small-scale monetary New Keynesian model.

JEL Classification numbers: C22; C51; C52; E32; E52.

Keywords: Dynamic stochastic general equilibrium model, Expectations, New Keynesian models, State space model.

* This paper is inspired by Chapter 3 of the first author's PhD Thesis. The authors wish to thank two anonymous referees, Francesco Zanetti and the following people for usefull comments on previous versions of this article: Piergiorgio Alessandri, Martin M. Andreasen, Emanuele Bacchiocchi, Gunnar Bårdsen, Fabio Busetti, Efrem Castelnuovo, Giuseppe Cavaliere, Michele Costa, Marco Del Negro, Davide Delle Monache, Massimo Franchi, Attilio Gardini, Søren Johansen, Riccardo 'Jack' Lucchetti, Helmut Lütkepohl, Ragnar Nymoen, Alessandro Notarpietro, Alessia Paccagnini, Paolo Paruolo, Massimiliano Pisani, Majid Al Sadoom and Carlo Trivisano. We also thank conference participants at the 'Ninth International Conference on Computational and Financial Econometrics', London (December 2015), 'Second Annual Conference of the International Association for Applied

Econometrics', Thessaloniki (June 2015), the 'Sixth Italian Congress of Econometrics and Empirical Economics', Salerno (January 2015), and workshop participants at the 'Padova Macro Talks', Padova (July 2015), 'Second CIdE Workshop for PhD students in Econometrics and Empirical Economics', Perugia (August 2014). Both authors gratefully acknowledge partial financial support from the Italian MIUR Grant PRIN-2010/2011, prot. 2010RHAHPL 003. The second author also acknowledges RFO grants from the University of Bologna.

1 Introduction

Small-scale dynamic stochastic general equilibrium models developed within the New Keynesian tradition (henceforth NK-DSGE models) have been treated as the benchmark of much of the monetary policy literature, given their ability to explain the impact of monetary policy on output and inflation. A recent generation of NK-DSGE models that feature financial frictions and the fiscal/monetary policy mix are currently used to evaluate macroeconomic scenarios and to predict economic activity. It is well recognized, however, that these models capture only stylized features of the business cycle and the monetary policy stance and display a limited time series performance (Henry and Pagan, 2004; An and Schorfheide, 2007). Assessing the correspondence between what these models imply and what the data tell us is a crucial step in the process of analyzing policy options and their effects.

One important source of misspecification can be ascribed to the difficulties NK-DSGE models display in generating sufficient endogenous persistence and propagation mechanisms to match the persistence and propagation mechanisms observed in quarterly data. NK-DSGE models are built upon the rational expectations (RE) paradigm. Under RE, agents are assumed to know the data generating process and form their expectations consistently. Two types of restrictions arise on the model's reduced form solution: (i) parametric nonlinear cross-equation restrictions (CER) that map the structural to the reduced form parameters; (ii) constraints on the lag order and correlation structure of the variables. The restrictions in (i) are the Hansen and Sargent's (1980, 1981) traditional 'metric' for the evaluation of models based on forward-looking behaviour and RE, see also Hansen (2014). Instead, the restrictions in (ii) are 'implicit', and very often, practitioners are not aware of their role and importance in the empirical performance of NK-DSGE models.

The unique stable solution associated with NK-DSGE models can be represented as a state space model, possibly expressed in minimal form (Komunjer and Ng, 2011), or as finite-order vector autoregressive (VAR) systems in the special case in which all endogenous variables are observed. These solutions generally involve one (two) lag(s) of the endogenous variables, giving rise to what we call throughout the paper an 'omitted dynamics' issue. By this term, we denote the situation that occurs when the constraints in (ii) conflict with the propagation mechanisms one detects from the data using a statistical model that does not embody all parametric constraints implied by the theory. Testing the validity of the NK-DSGE model through the CER when the restrictions in (ii) conflict with the actual autocorrelation structure of the data might distort the overall evaluation process.

What should investigators do? The natural and obvious fix in these cases would require the estimation of a theoretically micro-founded model with less restrictive dynamics than the original New Keynesian model. An excellent example is provided in, e.g., Lubik and Schorfheide (2004), Section 5.D. These authors estimate a dynamically less restrictive version of their NK-DSGE model as a robustness check, introducing a consumption Euler equation which features habit formation that generalizes the previously specified purely forward-looking consumption equation, and an 'hybrid' Phillips curve, as opposed to its purely forward-looking version. Examples like this, nevertheless, are rare, because it is not always practical to microfound all propagation mechanisms that characterize quarterly (or monthly) time series. What do practitioners typically do? They generally follow two approaches. Either they endow the shocks of the NK-DSGE model with more elaborate and persistent time series models like, e.g., AR or ARMA-type processes (Smets and Wouters, 2007; Cúrdia and Reis, 2010), without (apparently) changing the specification of their structural equations, or they enrich the dynamics of the system by adding measurement errors in the associated state space representation, see e.g. Ireland (2004) and Zanetti (2008).

The aim of this paper is to provide an alternative approach to the dynamic misspecification of NK-DSGE models that neutralizes the extent of the restrictions in (ii). More specifically, we pursue the idea that only the CER in (i) should be considered and tested to evaluate the model, while the restrictions in (ii) should not be binding when clearly at odds with the data. Our solution requires the use of a statistical state space model for the data which is used as the actual agents' expectations generating mechanism, without the need to resort to the adaptive learning framework (Evans and Honkapohja, 1999; 2001; Branch and Evans, 2006; Milani, 2007). This leads to the definition of a 'pseudo-structural' model that combines the structural information subsumed by the NK-DSGE model with features of the data, as captured by the statistical model. The pseudo-structural form is specified by augmenting the original system of Euler equations with a given number of additional lags of the variables, such that the gap between the dimension of the state vector of the structural model and the dimension of the state vector of the statistical model is filled up. We denote these additional components 'expectations correction' (ExC) terms and call the so-built pseudo-structural form the 'NK-DSGE model under ExC'.[1] By construction, the unique stable solution associated with the NK-DSGE model under ExC has the same time series representation as the statistical model for the data, and the implied set of CER involves only restrictions of type (i).

[1] Our idea is broadly related to the concept of Quasi Rational Expectations (QRE) used by Nerlove *et al.* (1979), Nelson and Blessler (1992), Nerlove and Fornari (1998) and Holt and McKenzie (2003) in different fields of research. Strictly speaking, QRE would require replacing expectational variables in the structural equations with their values calculated from the 'best fitting' statistical model for them. See e.g., Fanelli (2009) for an early example in the context of NK-DSGE models.

Frequentist and Bayesian estimation and evaluation methods can easily be applied. The NK-DSGE model under RE is nested within the pseudo-structural model under ExC, hence, likelihood-ratio tests can be used to select the 'best' specification. More generally, information criteria or any other evaluation method can be exploited. We propose an empirical illustration based on U.S. quarterly data, where we use the monetary business cycle model discussed in Benati and Surico (2009) as the reference structural model. We compare and evaluate the results obtained under RE and ExC.

It is worth stressing that we do not propose the active use of a statistical model to rectify the specification of the NK-DSGE model as an end in itself. Rather, we see our approach as providing a useful specification check for NK-DSGE models, allowing a researcher to robustify inferences against one important dimension about the misspecification of the model, while capturing some important 'stylized facts'. In this respect, our approach shares the viewpoint also adopted in Franchi and Juselius (2007) and, to some extent, in Consolo *et al.* (2009).[2] However, a number of alternative approaches that address the omitted dynamics issue from a 'theory-based' perspective, while preserving the agents' rationality, could also be applied. For instance, one might consider alternative timing of expectations along the lines discussed in Woodford (2003), Ch. 3. In this respect, Mankiw and Reis (2002) propose a new way to model sluggish macroeconomic adjustment based on the concept of 'information stickiness', which is extended to the case of NK-DSGE models in Mankiw and Reis (2007), while Sims (2003, 2006) argues that the concept of 'rational inattention' can explain the smooth and delayed cross-variable relationships observed among most macroeconomic time series.

Our paper has several connections with the existing literature. As already mentioned, ways to address the poor time series performance of structural forward-looking models have been recently popularized by Smets and Wouters (2007) and Cúrdia and Reis (2010) on the one hand, and by Ireland (2004), among others, on the other hand. Cúrdia and Reis (2010) suggest augmenting the overall dynamics of macro business cycle models by allowing for disturbances that have a rich contemporaneous and dynamic correlation structure. In practice, they suggest replacing the usual unsatisfactory autoregressive specification of order one (AR(1)) of the model's dis-

[2] Broadly speaking, the statistical model might potentially (but not necessarily) also exploit information 'external' to the structural model, possibly derived from large datasets, think e.g., about factor models. For instance, Beyer *et al.* (2008) propose to combine factor analysis for information extraction from large data sets and generalized method of moments to estimate the parameters of systems of forward-looking equations. In principle, factor-augmented VAR models, as in Consolo *et al.* (2009), might be used as the agents' expectations generating system. In this paper, we stick to the concept of model-consistent expectations; hence, it is assumed that the agents exploit only the information 'internally' recoverable from the structural model.

turbances with more general AR or ARMA-type processes, allowing for possible cross-equation dependence, so as to maximize the best time series performance of the model. Similarly, Smets and Wouters (2007) specify ARMA(1,1)-type processes for the price mark-up and the wage mark-up disturbances in their medium-sized estimated DSGE model, observing that for these shocks the inclusion of the moving average terms is designed to capture the high-frequency fluctuations in inflation and wages. Instead, Ireland (2004) suggests adding measurement errors in the measurement equations of the system in order to capture all comovement in the data not accounted for by the structural model, see also Zanetti (2008). As is know, adding measurement errors is also a possible remedy to the 'stochastic singularity' issue, see e.g. Sargent (1989) and DeJong and Dave (2007). Admittedly, these ways of tackling the omitted dynamics issue appear more general than our ExC approach which is, as it stands, confined to linear(ized) approximations and small-scale systems. Like Ireland (2004) and Cúrdia and Reis (2010), we let the data speak freely about the dimension of the dynamic misspecification of the system, but unlike Ireland (2004) and Cúrdia and Reis (2010), our starting point is a statistical model which is anchored to the theoretical model to make expectations consistent with the persistence and propagation mechanisms found in the data. With our ExC approach, practitioners are forced to shift their attention from modeling unobserved shocks to modeling observed time series. Moreover, Franchi and Juselius's (2007) concerns about the practice of adding shocks to the measurement system do not apply in our framework.

The most common and known alternative to RE is the adaptive learning hypothesis, see Evans and Honkapohja (1999; 2001), Branch and Evans (2006) and Milani (2007) for details. Under adaptive learning, agents are assumed to form and update their beliefs by using forecast models with time-varying coefficients and recursive updating rules. The postulated agents' forecasting model, or perceived law of motion, is typically (albeit not necessarily) the reduced form solution of the system under RE. Although the adaptive learning hypothesis can induce more persistence in the data (Branch and Evans 2006; Milani 2007; Chevillon *et al.* 2010), and it permits a substantial statistical relaxation of the strength of the CER (Fanelli 2008; Fanelli and Palomba 2011), a typical learning model focuses on the dynamic interaction between beliefs and observed data and is not designed to solve the misspecification issue with which we are concerned in this paper.[3] Our approach does not deviate from the concept of model-consistent expectations. 'Consistency', however, refers to the statistical model that approximates the data.

Recently, Cole and Milani (2014) have investigated the ability of popular New Keynesian models to match the data in terms of their interaction between macroeconomic variables and

[3] Cho and Kasa (2015) have recently proposed a model validation approach to learning, where agents operating in a self-referential environment are aware of potential model misspecification and try to detect it in real-time, using econometric specification tests.

their corresponding expectations. First, they report the failure of NK-DSGE models under RE to account for the dynamic interaction between macroeconomic expectations and macroeconomic realizations. Second, they observe that alternative models of expectations formation, including, e.g., extrapolative and heterogeneous expectations, can reconcile the NK-DSGE models with the data. Our approach represents another contribution towards the idea of reconciling the time series performance of NK-DSGE models with the data.

We also have some points in common with the DSGE-VAR approach of Del Negro *et al.* (2007). The DSGE-VAR approach is driven by the idea of assessing how far/close a dynamic macro model based on RE is from the data. Del Negro *et al.* (2007) propose a Bayesian evaluation method. They use a VAR system for the observed variables as the statistical model for the data, and centre the prior distribution for the VAR parameters on the CER implied by the structural model. The dispersion of these priors from the CER is governed by a scalar (hyper)parameter: small values of such a (hyper)parameter indicate that the VAR is far from the theoretical model, while large values of this (hyper)parameter indicate that the theoretical model is supported by the data. In our setup, the statistical model that describes the data is either a VAR system or a state space model, depending on whether one can observe/proxy all endogenous variables or not. The statistical model determines the dynamic structure of the pseudo-structural model that is confronted with the data. Like Del Negro *et al.* (2007), we are motivated by the idea of relaxing the tightness of the restrictions implied by the RE hypothesis, without renouncing to the concept of model-consistent expectations.

The rest of the paper is organized as follows. In Section 2 we present our main idea through a simple uni-equational example. In Section 3 we introduce our prototype structural NK-DSGE model and discuss the omitted dynamics issue that arises under RE, and in Section 4 we present our approach. In Section 5 we estimate a NK-DSGE model for the U.S. economy using quarterly data: In Sub-section 5.1 we discuss the reference structural model, in Sub-section 5.2 we deal with the statistical model for the data and, finally, in Sub-sections 5.3 and 5.4 we address the frequentist and Bayesian estimation and evaluation results. Section 6 contains some concluding remarks. An Appendix S1 complements the results of the paper in several dimensions.[4]

2 Background

Consider a simple economy described by the uni-equational linear RE model

$$Z_t = \gamma_f E_t Z_{t+1} + \gamma_b Z_{t-1} + \omega_t \ , \ \omega_t \sim \mathrm{WN}(0,1) \ \ , \ \ t = 1, ..., T. \tag{1}$$

[4] Appendix S1 is available at http://www.rimini.unibo.it/fanelli/TS_Angelini_Fanelli_DSGE.pdf

Z_t is an observable scalar generated by a covariance stationary process, Z_0 is given, $E_t Z_{t+1}:=E(Z_{t+1} \mid \mathcal{F}_t)$ is the expectation operator conditional on the information set $\mathcal{F}_t$, and ω_t is a scalar white noise process with variance 1, called structural (or fundamental) disturbance (or structural shock). We call the model in Eq. (1) 'structural model'. The structural parameters are $\gamma_f > 0$, $\gamma_b > 0$, and are collected in the vector $\theta:=(\gamma_f, \gamma_b)'$.

Assuming that $\gamma_f + \gamma_b < 1$, the unique stable RE solution to the model in Eq. (1) is given by the autoregressive model of order one (AR(1)):

$$Z_t = \tilde{\phi} Z_{t-1} + \tilde{\psi}\omega_t \quad , \quad t = 1, ..., T \tag{2}$$

where $\tilde{\phi} = \phi(\theta)$ and $\tilde{\psi} = \psi(\theta)$ are reduced form parameters that depend nonlinearly on θ. A 'tilde' over ϕ and ψ is used to stress the fact that these parameters are forced to depend on θ under RE. In particular, $\tilde{\phi}$ is the real stable root (i.e. $\tilde{\phi} \in (0,\, 1)$) of the second-order equation $\gamma_f \phi^2 - \phi + \gamma_b = 0$, and $\tilde{\psi}=(1-\gamma_f\tilde{\phi})^{-1}$.

Under RE, the data generating process belongs to the class of models described by Eq. (2). Consistent estimates of θ can be obtained from the autoregressive parameter ϕ and the variance σ_ε^2 of $\varepsilon_t:=\tilde{\psi}\omega_t$, by imposing the CER: $\phi=\tilde{\phi}$, $\sigma_\varepsilon^2=\tilde{\sigma}_\varepsilon^2$, where $\tilde{\phi}$ is the real stable solution to $\gamma_f\phi^2 - \phi + \gamma_b = 0$ and $\tilde{\sigma}_\varepsilon^2 = (1-\gamma_f\tilde{\phi})^{-2}$. Moreover, the autocorrelation structure of the time series Z_1, Z_2, ..., Z_T should conform to that of AR(1)-type processes.

Assume now that based on his/her specification analysis, the econometrician believes the data generating process belongs to the class of covariance stationary AR(2) processes of the form

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \varepsilon_t \quad , \quad \varepsilon_t \sim \mathrm{WN}(0, \sigma_\varepsilon^2) \quad , \, t = 1, ..., T, \tag{3}$$

where the autoregressive coefficient associated with the second lag, ϕ_2, is such that $\phi_2 \neq 0$. We call the model in Eq. (3) the statistical model for the data. The parameters of the statistical model are $\tau:=(\phi_1, \phi_2, \sigma_\varepsilon^2)'$. Compared to the reduced form solution in Eq. (2), the AR(2) model in Eq. (3) involves an additional lag of the state variable Z_t. For the econometrician, the best forecast of Z_{t+1} conditional on the information set available at time t will be $E(Z_{t+1} \mid \mathcal{F}_t) = \phi_1 Z_t + \phi_2 Z_{t-1}$, not $E(Z_{t+1} \mid \mathcal{F}_t) = \phi_1 Z_t$ as predicted by the structural model under RE.

Since the model in Eq. (2) is nested in Eq. (3), the AR(2) model might be interpreted as the reduced form solution associated with the structural model in Eq. (1) if the following set of restrictions hold:

$$\begin{array}{ll} \text{res-I:} & \begin{array}{l} \phi_1=\tilde{\phi}, \text{ where } \tilde{\phi} \text{ is the stable root of } \gamma_f\phi^2 - \phi + \gamma_b = 0 \\ \sigma_\varepsilon^2=\tilde{\sigma}_\varepsilon^2, \text{ where } \tilde{\sigma}_\varepsilon^2 = (1-\gamma_f\tilde{\phi})^{-2} \end{array} \\ & \\ \text{res-II:} & \phi_2=\tilde{\phi}_2=0. \end{array} \tag{4}$$

In principle, the structural parameters θ might be estimated consistently from the model in Eq. (3) by imposing the restrictions in Eq. (4). It is clear, however, that the restrictions res-II in Eq. (4) conflict with the econometrician's finding that $\phi_2 \neq 0$. If the data generating process belongs to the class of models in Eq. (3) based on $\phi_2 \neq 0$, the estimator of θ recovered from model Eq. (2) imposing the CER in Eq. (4) will be distorted because of the omission of a relevant regressor.

The natural fix to this shortcoming should be the re-specification of a theory-based structural model implying a time series representation for Z_t featuring Z_{t-2}, other than Z_{t-1}. Yet only seldom is that feasible. We discuss two solutions to the 'omitted dynamics' issue. One is the 'conventional' approach, the other our solution.

Conventional approach

The 'conventional' approach works by endowing the structural model in Eq. (1) with an AR(1) process for the shocks, now denoted with ω_t^*, i.e.

$$\begin{aligned} Z_t &= \gamma_f E_t Z_{t+1} + \gamma_b Z_{t-1} + \omega_t^* \quad , \quad t = 1, ..., T \\ \omega_t^* &= \rho \omega_{t-1}^* + v_t \quad , \quad |\rho| < 1 \; , \quad v_t \sim \mathrm{WN}(0, 1 - \rho^2). \end{aligned} \tag{5}$$

In this specification, ρ is an autoregressive parameter and v_t is the structural shock (which is normalized such that the variance of ω_t^* is still equal to 1). The autoregressive equation for ω_t^* and the associated autoregressive parameter, ρ, are not generally derived from first principles but from the practical purpose of improving the statistical fit of the model. Apparently, the theoretical structural model in Eq. (1) has not been changed. Actually, by exploiting the autoregressive structure of ω_t^* and using simple algebra, we obtain:

$$Z_t = \frac{\gamma_f}{1+\rho\gamma_f} E_t Z_{t+1} + \frac{\gamma_b + \rho}{1+\rho\gamma_f} Z_{t-1} - \frac{\rho\gamma_b}{1+\rho\gamma_f} Z_{t-2} + \frac{1}{1+\rho\gamma_f} v_t^* \quad , \quad t = 1, ..., T \tag{6}$$

where $v_t^* {:=} \rho\gamma_f \eta_t + v_t$, and $\eta_t {:=} Z_t - E_{t-1} Z_t$ is a martingale difference sequence forecast error ($E_{t-1}\eta_t = 0$). The representation in Eq. (6) recalls a well known fact from textbook econometrics: Autoregressive disturbances amount to additional lagged regressors of the endogenous variable. The unique stable RE solution associated with Eq. (6) is given by the AR(2) process in Eq. (3), with τ subject to the following set of CER: $\phi_1 = \tilde{\phi}_1$, $\phi_2 = \tilde{\phi}_2$, $\sigma_\varepsilon^2 = \tilde{\sigma}_\varepsilon^2$, where

$$\begin{aligned} (1 - \gamma_f \tilde{\phi}_1 + \rho\gamma_f)\tilde{\phi}_1 &= (\gamma_f \tilde{\phi}_2 + \gamma_b + \rho) \\ (1 - \gamma_f \tilde{\phi}_1 + \rho\gamma_f)\tilde{\phi}_2 &= -\rho\gamma_b \\ \tilde{\sigma}_\varepsilon^2 &= \left(\frac{1-\rho}{1-\gamma_f\tilde{\phi}_1} \right)^2 . \end{aligned} \tag{7}$$

It can be noticed that $\rho \neq 0$ implies $\phi_2 \neq 0$. Instead, if $\rho = 0$, the restrictions above collapse to those in Eq. (4).

Suggested approach

Our suggested approach is based on a slight change of perspective. We assume that the AR(2) model in Eq. (3) is the agents' forecast model, and introduce a 'pseudo-structural' form that combines the information provided by the structural model in Eq. (1) with the information provided by the statistical model in Eq. (3). Our main requirement is that the reduced form solution associated with the pseudo-structural form has time series representation consistent with Eq. (3). The pseudo-structural form is given by

$$Z_t = \gamma_f E_t Z_{t+1} + \gamma_b Z_{t-1} + \zeta Z_{t-2} + \omega_t^{**} \ , \ \omega_t^{**} \sim \mathrm{WN}(0,1), \ \ t = 1, ..., T, \tag{8}$$

and is obtained from Eq. (1) by adding the term ζZ_{t-2}. The disturbance ω_t^{**} is still a white noise term with variance 1.

The crucial question here is: How do we interpret the ζZ_{t-2} term in Eq. (1)? In principle, ζZ_{t-2} might be interpreted as a term capturing propagation mechanisms that are not directly explained by the theoretical model, because of the omission of adjustment costs, information delays, time-to-build effects, etc. These effects, however, should be modelled endogenously in the structural specification, if present. In our setup, ζZ_{t-2} plays the role of an 'expectations correction' (ExC) term, in a sense that will be qualified below.

The vector of parameters associated with the pseudo-structural form is $\theta^{*}:=(\theta', \zeta)'=(\gamma_f, \gamma_b, \zeta)'$, hence the determinacy of Eq. (8) will depend also on the auxiliary parameter ζ, other than γ_f and γ_b. To leave the determinacy conditions implied by theoretical model ($\gamma_f + \gamma_b < 1$) unchanged, we can restrict ζ imposing e.g. that if for a given $\theta = \breve{\theta}$ the solution to the theoretical model in Eq. (1) is unique and stable, the solution to the pseudo-structural model in Eq. (8) also must be unique and stable. For instance, it is possible to prove that given $\gamma_f + \gamma_b < 1$, e.g. the inequality $-1/2 < \zeta < 1 - (\gamma_f + \gamma_b)$ is sufficient for determinacy in Eq. (8).

The specification in Eq. (8) should be no more disturbing than that in Eq. (6) obtained by adding the autoregressive disturbance to the structural theoretical model. The unique stable solution associated with the model in Eq. (8), if it exists, is given by the AR(2) process in Eq. (3) with parameters τ subject to the following set of CER: $\phi_1 = \tilde{\phi}_1$, $\phi_2 = \tilde{\phi}_2$, $\sigma_\varepsilon^2 = \tilde{\sigma}_\varepsilon^2$, where

$$\begin{gathered}(1 - \gamma_f \tilde{\phi}_1)\tilde{\phi}_1 = (\gamma_f \tilde{\phi}_2 + \gamma_b) \\ (1 - \gamma_f \tilde{\phi}_1)\tilde{\phi}_2 = \zeta \\ \tilde{\sigma}_\varepsilon^2 = (1 - \gamma_f \tilde{\phi}_1)^{-2}.\end{gathered} \tag{9}$$

It can be noticed that, in this case, $\zeta \neq 0$ also implies $\phi_2 \neq 0$. Instead, if $\zeta = 0$, the restrictions above collapse to those in Eq. (4).

Although the likelihoods associated with the AR(2) model under the restrictions in Eq. (7)

and in Eq. (9) may be numerically different (recall that ρ lies in the (-1,1) interval, while ζ is subject to a different requirement), at first glance, the two approaches seem to be equivalent. Yet they are conceptually different. With the 'conventional' approach, the practitioner does not need to specify any statistical model for the data. He/she will specify a time series process for the disturbance ω_t with the aim of improving the overall empirical fit of the model. Our approach is instead based on the idea of treating the statistical model for the data like the 'unconstrained' version of the actual agents' expectations generating system. The term ζZ_{t-2} in Eq. (8) is an ExC term that gives rise to an 'higher-order' hybrid LRE model that fits the data better than the original structural equation. The term ζZ_{t-2} guarantees that the unique stable solution associated with the pseudo-structural form has the same time series representation as the agents' statistical model for the data, and that the differences between these two models is only due to the CER.

3 The NK-DSGE model under Rational Expectations and the omitted dynamics issue

Let $Z_t{:=}(Z_{1,t}, Z_{2,t}, \cdots, Z_{n,t})'$ be a $n \times 1$ vector of endogenous variables and assume that after log-linearization, the structural form of the NK-DSGE model can be represented in the form

$$\Gamma_0 Z_t = \Gamma_f E_t Z_{t+1} + \Gamma_b Z_{t-1} + C + \eta_t \quad , \quad t = 1, ..., T \tag{10}$$

where, $\Gamma_i{:=}\Gamma_i(\theta)$, $i \in \{0, f, b\}$ are $n \times n$ matrices whose elements depend on the vector of structural parameters θ, $C{:=}C(\theta)$ is a $n \times 1$ constant which can be non-zero when it is intended to capture steady state values of the variables of the system, and η_t is a $n \times 1$ vector of disturbances which is assumed to be adapted to the sigma-field $\mathcal{F}_t$, where $\mathcal{F}_t$ represents the agents' information set at time t, $E_t Z_{t+1}{:=}E(Z_{t+1} \mid \mathcal{F}_t)$. Without any loss of generality, Γ_0 is assumed non-singular. When a direct link between the process generating η_t and a set of 'forcing variables' is not provided by the theory, a typical completion of system (10) is obtained through the autoregressive specification

$$\eta_t = R\eta_{t-1} + \omega_t \quad , \quad \omega_t \sim \mathrm{WN}(0, \Sigma_\omega) \tag{11}$$

where R is a $p \times p$ diagonal stable matrix (i.e. with its eigenvalues inside the unit disk), and ω_t is a white noise term with covariance matrix Σ_ω that can be diagonal or non-diagonal. The true value of θ, θ_0, is assumed to be an interior point of the parameter space Θ.

The multivariate linear RE model in Eq.s (10)-(11) nests a large class of small-scale linearized NK-DSGE models used in monetary policy analysis. There exists many solution methods available in the literature by which a reduced form solution of system (10)-(11) can be computed under RE. A solution of system (10)-(11) is any stochastic process $\{Z_t^*\}_{t=0}^{\infty}$ such that, for $\theta \in \Theta$, $E_t Z_{t+1}^* = E(Z_{t+1}^* \mid \mathcal{F}_t)$ exists and if Z_t^* is substituted for Z_t into the structural equations, the model is verified for each t, for fixed initial conditions. A reduced form solution is a member of the solution set whose time series representation is such that Z_t can be expressed as a function of ω_t, lags of Z_t and ω_t and, possibly, other arbitrary martingale difference sequences (MDS) with respect to $\mathcal{F}_t$, independent of ω_t, called 'sunspot shocks', see Fanelli (2012) and Castelnuovo and Fanelli (2015).

Assuming that θ_0 lies in the determinacy region of Θ, the unique stable reduced form solution associated with system (10)-(11) can be represented in the form (see Binder and Pesaran, 1995; Uhlig, 1999; Klein, 2000)

$$\underset{x_t}{\begin{pmatrix} Z_t - \tilde{\upsilon} \\ Z_{t-1} - \tilde{\upsilon} \end{pmatrix}} = \underset{A(\theta)}{\begin{pmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 \\ I_n & 0_{n\times n} \end{pmatrix}} \underset{x_{t-1}}{\begin{pmatrix} Z_{t-1} - \tilde{\upsilon} \\ Z_{t-2} - \tilde{\upsilon} \end{pmatrix}} + \underset{G(\theta)}{\begin{pmatrix} \tilde{\Psi} \\ 0_{n\times n} \end{pmatrix}} \omega_t \tag{12}$$

where $\tilde{\upsilon} := (I_n - \tilde{\Phi}_1 - \tilde{\Phi}_2)^{-1}\tilde{\mu}$, and we use 'tildes' over the matrices of parameters to remark the fact that Φ_1, Φ_2, Ψ and μ depend on θ through the set of CER:

$$\begin{gathered} (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1)\tilde{\Phi}_1 - \Gamma_f \tilde{\Phi}_2 + \Gamma_{b,1} = 0_{n\times n} \\ (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1)\tilde{\Phi}_2 - \Gamma_{b,2} = 0_{n\times n} \end{gathered} \tag{13}$$

$$\begin{gathered} C - (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1 - \Gamma_f)\tilde{\mu} = 0_{n\times 1} \\ \tilde{\Sigma}_\varepsilon = \tilde{\Psi}\ \Sigma_\omega\ \tilde{\Psi}' \quad , \quad \tilde{\Psi} = \left(\Gamma_0 - \Gamma_f \tilde{\Phi}_1\right)^{-1}. \end{gathered} \tag{14}$$

In the expressions in Eq.s (13)-(14), $\Gamma_0^R = (\Gamma_0 + R\Gamma_f)$, $\Gamma_{b,1} = (\Gamma_b + R\Gamma_0)$, $\Gamma_{b,2} = -R\Gamma_b$, and $\tilde{\Sigma}_\varepsilon$ is the covariance matrix of the reduced form disturbance $\varepsilon_t = \tilde{\Psi}\omega_t$, see Bårdsen and Fanelli (2015) and Castelnuovo and Fanelli (2015) for details. A convenient way to summarize the equilibrium in Eq.s (12)-(14) is to refer to the representation

$$\underset{2n\times 1}{x_t} = \underset{2n\times 2n}{A(\theta)}\ \underset{2n\times 1}{x_{t-1}} + \underset{2n\times n}{G(\theta)}\ \underset{n\times 1}{\omega_t}. \tag{15}$$

Let $y_t := (y_{1,t}, y_{2,t}, \cdots, y_{p,t})'$ be the $p \times 1$ vector of observable variables. When all variables in Z_t are observed, $y_t = Z_t$, and the transition system in Eq. (15) along with the measurement system: $y_t = Hx_t$, $H := (I_n : 0_{n\times n})$, collapse to a VAR representation for Z_t in which the VAR coefficients depend on θ through the CER in Eq.s (13)-(14). In general, not all variables in Z_t are observed, hence we consider the measurement system

$$y_t = Hx_t + Vv_t \tag{16}$$

where H is a $p\times 2n$ matrix, v_t a $b\times 1$ vector ($b\leq p$) of measurement errors with covariance matrix Σ_v, and V is a $p\times b$ selection matrix. Let u_t:=$(\omega_t', v_t')'$ be the $(n+b)$-dimensional 'complete' vector of innovations. By substituting Eq. (15) into Eq. (16) and using some algebra, one obtains the so-called ABCD representation

$$\begin{aligned} \underset{2n\times 1}{x_t} &= \underset{2n\times 2n}{A(\theta)}\ \underset{2n\times 1}{x_{t-1}} + \underset{2n\times(n+b)}{B(\theta)}\ \underset{(n+b)\times 1}{u_t} \\ \underset{p\times 1}{y_t} &= \underset{p\times 2n}{C(\theta)}\ \underset{2n\times 1}{x_{t-1}} + \underset{2n\times(n+b)}{D(\theta)}\ \underset{(n+b)\times 1}{u_t} \end{aligned} \tag{17}$$

where $B(\theta)$:=$(G(\theta) : 0_{2n\times b})$, $C(\theta)$:=$HA(\theta)$ and $D(\theta)$:=$(HG(\theta) : V)$.[5]

The state space system (17) summarizes the determinate (unique and stable) equilibrium associated with the NK-DSGE model under RE. Provided θ is locally identifiable, the state space model defined by Eq.s (15)-(16) can be taken to the data using different estimation methods, see e.g. Ruge-Murcia (2007). When instead θ is unidentified, identification can be restored by imposing suitable restrictions along the lines discussed in e.g. Iskrev (2010) and Komunjer and Ng (2011). Estimation procedures, however, can fail to deliver consistent estimates of the structural parameters when important propagation mechanisms are omitted from the system, see e.g. Jondeau and Le Bihan (2008) and Fanelli (2012).

To characterize our approach, we assume that there exists a statistical model for the data represented by the state space system

$$\underset{x_t^*}{\underbrace{\begin{pmatrix} Z_t - \upsilon \\ Z_{t-1} - \upsilon \\ \vdots \\ Z_{t-k+1} - \upsilon \end{pmatrix}}} = \underset{A^*(\tau)}{\underbrace{\begin{pmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_{k-1} & \Phi_k \\ I_n & 0_{n\times n} & \cdots & 0_{n\times n} & 0_{n\times n} \\ & \ddots & & \vdots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & I_n & 0_{n\times n} \end{pmatrix}}} \underset{x_{t-1}^*}{\underbrace{\begin{pmatrix} Z_{t-1} - \upsilon \\ Z_{t-2} - \upsilon \\ \vdots \\ Z_{t-k} - \upsilon \end{pmatrix}}} + \underset{G^*(\tau)}{\underbrace{\begin{pmatrix} I_n \\ 0_{n\times n} \\ \vdots \\ 0_{n\times n} \end{pmatrix}}} \varepsilon_t \tag{18}$$

$$y_t = H^* x_t^* + V^* v_t \tag{19}$$

where $\Phi_1, \Phi_2, ..., \Phi_k$, μ and Σ_ε:=$E(\varepsilon_t\varepsilon_t')$ are matrices of coefficients in which no theoretical restriction is placed, υ:=$(I_n - \Phi_1 - \Phi_2 - ... - \Phi_k)^{-1}\mu$, and H^* and V^* are matrices of suitable dimensions. It is assumed that $\Phi_k \neq 0_{n\times n}$ and that the only restriction on the covariance matrix Σ_ε is symmetry. Collecting the parameters of system (18)-(19) in the vector τ:=$(vec(\Phi_1)', ..., vec(\Phi_k)', \mu', vech(\Sigma_\varepsilon)', vech(\Sigma_v)')'$ and defining the vector of innovations

[5] We refer to Fernández-Villaverde et al. (2007), Ravenna (2007), Franchi and Vidotto (2013) and Franchi and Paruolo (2015) for a detailed analysis of the cases in which y_t can be given a fundamental and finite-order VAR representation when the $D = D(\theta)$ matrix in Eq. (17) is square. More generally, the state space model in Eq. (17) will give rise to VARMA-type representations for y_t, see e.g. Hannan and Deistler (1988).

$u_t{:=}(\omega_t', v_t')'$, the statistical model above can also be summarized in the representation:

$$\begin{aligned} x_t^* &= A^*(\tau)\ x_{t-1}^* + \ B^*(\tau)\ u_t \\ y_t &= C^*(\tau)\ x_{t-1}^* + \ D^*(\tau)\ u_t \end{aligned} \tag{20}$$

that is postulated to be in 'minimal form' and such that τ is locally identifiable, see Komunjer and Ng (2011).[6] The state space model in Eq. (20) collapses to a stationary VAR for Z_t when $y_t = Z_t$.

Some remarks are in order. First, using the language of simultaneous system of equations, the statistical model in Eq. (20) is intended to play the role of 'unrestricted reduced form' associated with the NK-DSGE model, i.e. the associated state space representation of the data before any restriction stemming from the theory is imposed. Second, as is known, finding the 'unrestricted reduced form' is not a trivial task in the context of NK-DSGE models, because of the difficulties associated with finding an identified minimal form, see e.g. Schorfheide (2010), Komunjer and Ng. (2011), Guerron-Quintana *et al.* (2013) and Andrews and Mikusheva (2015) for discussions. Despite these difficulties, Guerron-Quintana *et al.* (2013) have shown that it is generally possible to couple the ABCD form associated with a NK-DSGE model with a state space representation that nests it. Third, the estimation of 'unrestricted' (and identified) state space models in minimal form has its 'natural' counterpart in the estimation problem of identified VARMA processes for y_t that feature the left-coprime condition (Hannan and Deistler, 1988).

The simple comparison of systems (17) and (20) reveals that the dimension of the state vector in Eq. (20) will be generally larger than the dimension of the state vector in system (17), i.e. $\dim(x_t^*){\geq}\dim(x_t)$. The condition $\dim(x_t^*){=}\dim(x_t)$ is obtained with $k = 2$.

The CER that the NK-DSGE model in Eq.s (10)-(11) places on τ can be represented in the form

$$\begin{aligned} (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1)\tilde{\Phi}_1 - \Gamma_f \tilde{\Phi}_2 + \Gamma_{b,1} &= 0_{n\times n} \\ (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1)\tilde{\Phi}_2 - \Gamma_{b,2} &= 0_{n\times n} \end{aligned} \tag{21}$$

$$C - (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1 - \Gamma_f)\tilde{\mu} = 0_{n\times 1}$$

$$\tilde{\Sigma}_\varepsilon = \tilde{\Psi}\ \Sigma_\omega\ \tilde{\Psi}' \ \ ,\ \tilde{\Psi}{:=}(\Gamma_0 - \Gamma_f \tilde{\Phi}_1)^{-1} \tag{22}$$

$$\Phi_j{=}\breve{\Phi}_j = 0_{n\times n} \ \ ,\ \ j = 3, 4, ..., k. \tag{23}$$

While the restrictions in Eq.s (21)-(22) coincide with those in Eq.s (13)-(14), now we have the additional set of $n^2(k-2)$ zero restrictions, summarized in Eq. (23), that force the dimension of

[6] Minimality means that the model involves a minimum (non redundant) number of state variables. In practice, this conditions corresponds to ruling out common (cancelling) roots from VARMA-type systems.

the state vector x_t^* of the statistical model to match the dimension of the state vector x_t in the structural model. While the CER in Eq.s (21)-(22) define a nonlinear mapping from θ to τ (res-I), say $\tau = g(\theta)$, where $g(\cdot)$ is a nonlinear differentiable vector function, the zero restrictions in Eq. (23) imply that $\dim(x_t^*)$=$\dim(x_t)$ (res-II). When in particular the data generating process belongs to the class of models defined by system (18)-(19) (or equivalently system (20)) and $k \geq 3$, $\dim(x_t^*)$>$\dim(x_t)$ and the CER in Eq.s (13)-(22) lead to the omitted dynamics issue.

4 The pseudo-structural form

Consider the NK-DSGE model in Eq.s (10)-(11) and the statistical model in Eq.s (18)-(19). Our objective is to combine the information provided by both models, circumventing the zero restrictions in Eq. (23). We consider the following assumptions.

Assumption 1 [Data generating process] The data generating process belongs to the class of models in Eq.s (18)-(19) with k=k^{op}, $\dim(x_t^*) = nk^{op}$ and $\Phi_{k^{op}} \neq 0_{n\times n}$; for τ=τ^{op}:=$(vec(\Phi_1)', ..., vec(\Phi_{k^{op}})', \mu', vech(\Sigma_\varepsilon)', vech(\Sigma_v)')'$ the associated state space representation in Eq. (20) is in minimal form, and such that τ^{op} is locally identified and not affected by the 'weak identification' issues we qualify below.

Assumption 2 [Stationarity] The matrix $A^*(\tau^{op})$ is stable.

Assumption 3 [Parameters invariance] The parameters in τ^{op} does not vary for $t = 1, 2, ..., T$.

Assumption 1 maintains that the data generating process belongs to the specified statistical model, and that such a model involves the minimum number of state variables necessary to capture the propagation mechanisms at work in the data. The possibility that τ^{op} be 'weakly identified' in the sense discussed in Canova and Sala (2009) is ruled out. The hypothesis of 'strong identification' for τ^{op} is necessary in our setup to compute likelihood-ratio (LR) tests for the CER. In principle, the strong identification of τ^{op} can coexist with the weak identification of θ, see Guerron-Quintana *et al.* (2013). We come back on this at the end of this section.

Assumption 2 implies that the statistical model is asymptotically stable. When $y_t = Z_t$, the analysis can be easily extended to the case of unit roots and cointegration along the lines discussed in e.g. Fanelli (2009), Fukač and Pagan (2009) and Bårdsen and Fanelli (2015).

Assumption 3 postulates that the parameters of the statistical model are time-invariant. This assumption is debatable but is consistent with the hypothesis that the vector of structural parameters, θ, is a fixed point in our reference NK-DSGE model. Although the framework we discuss below is general enough to cover the large majority of small-scale NK-DSGE models currently used in the monetary and business cycle literature, our ExC approach is not consistent, as it stands, with structural specifications in which θ and τ^{op}, or sub-vectors of θ and τ^{op}, are time-varying as in e.g. Cogley and Sbordone (2008) or Cagliarini and Kulish (2013), just to mention a few. In principle, a sound econometric analysis of system Eq.s (18)-(19) might lead the practitioner to discover possible breaks in τ^{op} (and possibly in θ under the CER, see below). In these cases, the method we discuss in this paper can be applied by considering the sample periods in which τ^{op} is found to be time-invariant.

Given the structural form in Eq.s (10)-(11) and the statistical model in Eq.s (18)-(19), we build a 'pseudo-structural' form given by:

$$\left\{\begin{array}{c}\Gamma_0 Z_t = \Gamma_f E_t Z_{t+1} + \Gamma_b Z_{t-1} + \left(\sum_{j=2}^{k^{op}-1} \Upsilon_j Z_{t-j}\right) \mathbb{I}_{\{k^{op}\geq 3\}} + C + \eta_t \\ \eta_t = R\eta_{t-1} + \omega_t \end{array}\right. \tag{24}$$

where $\mathbb{I}_{\{\cdot\}}$ is the indicator function, and the $n \times n$ matrices Υ_j, $j = 2, ..., k^{op} - 1$ contain, when $k^{op} \geq 3$, additional auxiliary parameters associated with $k^{op} - 2$ additional lags of Z_t that we denote ExC terms. Let ζ be the vector collecting the ExC parameters contained in the matrices Υ_j, $j = 2, ..., k^{op} - 1$, and $\theta^* = (\theta', \zeta')'$ the vector containing all parameters associated with the pseudo-structural form in Eq. (24). The true value of θ^*, θ_0^*, is assumed to be an interior point of the parameter space Θ^*. When $k^{op} \leq 2$, there are no ExC terms and the pseudo-structural form coincides with the 'conventional' NK-DSGE model in Eq.s (10)-(11).[7] It turns out that the NK-DSGE model in Eq.s (10)-(11) is nested within system (24). To keep the number of auxiliary parameters as small as possible, the matrices Υ_js can be specified diagonal.

It is tempting to interpret the ExC terms $\left(\sum_{j=2}^{k^{op}-1} \Upsilon_j Z_{t-j}\right) \mathbb{I}_{\{k^{op}\geq 3\}}$ in Eq. (24) as a component summarizing the effects of propagation mechanisms that are present in the data but are omitted by the baseline structural specification, such as length of real contracts, adjustment costs, delays in information flows, decision lags, etc., see e.g. Kozicki and Tinsley (1999), Rudebusch (2002*a*, 2002*b*) and Fuhrer and Rudebusch (2004) for examples. However, if actually important, these effects should be micro-founded and incorporated directly in the structural specification. In our setup, the quantity $\left(\sum_{j=2}^{k^{op}-1} \Upsilon_j Z_{t-j}\right) \mathbb{I}_{\{k^{op}\geq 3\}}$ in Eq. (24) defines an 'higher-order' hybrid structural model and forces the reduced form solution associated with system (24) to be consistent with Assumptions 1, as Proposition 1 below will clarify.

To fully understand the nature of system (24), we focus on its i-th Euler equation for $k^{op} \geq 3$, which is given by

$$Z_{i,t} = \gamma_{i,0}' Z_{i,t}^* + \gamma_{i,f}' E_t Z_{t+1} + \gamma_{i,b}' Z_{t-1} + \left(\sum_{j=2}^{k^{op}-1} \zeta_{i,j}' Z_{i,t-j}\right) + C_i + \eta_{i,t}$$

[7] In our setup the case $k^{op} = 1$ coincides with the situation where $R = 0_{n\times n}$ in Eq. (11).

$$\eta_{i,t} = R_i \eta_{i,t-1} + \omega_{i,t} \quad , \ i = 1, ..., n.$$

In this equation, the $(n-1)\times 1$ vector $Z_{i,t}^{*}$ denotes Z_t with its i-th entry suppressed, the $(n-1)\times 1$ vector $\gamma_{i,0}$ collects the structural parameters that enter the i-th row of Γ_0, the $n \times 1$ vector $\gamma_{i,f}$ collects the structural parameters that enter the i-th row of Γ_f, the $n \times 1$ vector $\gamma_{i,b}$ contains the structural parameters that enter the i-th row of Γ_b, $\zeta_{i,j}^{\prime}$ is the i-th diagonal element of Υ_j, $j = 1, ..., k-1$, C_i is the i-th element of C and, finally, $\eta_{i,t}$ and $\omega_{i,t}$ are the i-th elements of the vectors η_t and ω_t, respectively, where the autoregressive parameter $-1<R_i<1$ is the i-th diagonal component of R.

The determinacy of system (24) depends on whether θ_0^{*} lies in the determinacy region of Θ^{*}, therefore it also depends on the auxiliary parameters ζ, other than θ. Our final assumption ensures that the presence of the ExC terms in system (24) does not alter the determinacy conditions that characterize θ in the baseline structural model in Eq.s (10)-(11).

Assumption 4 [Determinacy] Given the pseudo-structural form in Eq. (24), the ExC parameters in ζ are restricted such that for any $\theta = \breve{\theta} \in \Theta$ for which a determinate solution for the NK-DSGE model in Eq.s (10)-(11) exists, it is possible to find a $\theta^{*} = \breve{\theta}^{*} = (\breve{\theta}^{\prime}, \breve{\zeta}^{\prime})^{\prime} \in \Theta^{*}$ such that a determinate solution to system (24) also exists.

We do not have a formal proof that it always exists a ζ that satisfies the condition in Assumption 4. A practical way to check that Assumption 4 is respected in empirical analysis is discussed in the estimation procedure presented next.

The proposition that follows derives the model-consistent reduced form solution and the CER implied by the pseudo-structural form in Eq. (24).

Proposition 1 [The CER under ExC] Under Assumptions 1-4, if a unique stable reduced form solution exists for the model in Eq. (24), it can be represented as in the form (18)-(19) with the parameters in τ^{op} subject to the following set of CER: $\Phi_j = \tilde{\Phi}_j$, $j = 1, ..., k^{op}$, $\mu = \tilde{\mu}$, $\Sigma_{\varepsilon} = \tilde{\Sigma}_{\varepsilon}$, where

$$\begin{cases} (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1)\tilde{\Phi}_1 - (\Gamma_f \tilde{\Phi}_2 + \Gamma_{b,1}) = 0_{n\times n} \\ (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1)\tilde{\Phi}_2 - (\Gamma_f \tilde{\Phi}_3 + \Gamma_{b,2} + \Upsilon_2) = 0_{n\times n} \\ (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1)\tilde{\Phi}_3 - (\Gamma_f \tilde{\Phi}_4 + \Upsilon_3 - R\Upsilon_2) = 0_{n\times n} \\ \quad \vdots \\ (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1)\tilde{\Phi}_{k^{op}} + R\Upsilon_{k^{op}-1} = 0_{n\times n} \\ (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1 - \Gamma_f)\tilde{\mu} - (I_n - R)C = 0_{n\times 1} \\ \tilde{\Sigma}_{\varepsilon} - \tilde{\Psi}\ \Sigma_{\omega}\ \tilde{\Psi}^{\prime} = 0_{n\times n} \ , \ \tilde{\Psi} := (\Gamma_0^R - \Gamma_f \tilde{\Phi}_1)^{-1}. \end{cases} \tag{25}$$

and $\Gamma_0^R := (\Gamma_0 + R\Gamma_f)$.

Proof: Technical Supplement.

The interesting feature of the restrictions in Eq. (25) is that there are no zero restrictions that reduce the length of the vector x_t^* in system (18)-(19).[8]

A frequentist econometric procedure for the estimation and testing of the NK-DSGE under ExC can be based on the following steps:

Step 1 Fit the statistical model in Eq.s (18)-(19) to the data, and use information criteria or likelihood-ratio tests to determine the dimension of the state vector, $\dim(x_t^*) = nk^{op}$. This can be done by estimating the state space model through maximum likelihood and Kalman filtering. The specific procedure we use to find a global maximum for τ^{op} is the CMAES algorithm, see Andreasen (2010). For each estimated model, we check whether the minimality (controllability and observability) and local identification conditions discussed in Komunjer and Ng (2011) are satisfied in correspondence of the maximum likelihood estimate $\hat{\tau}^{op}$. If it is found that $k^{op} \leq 2$, the NK-DSGE model is estimated and evaluated in the 'conventional' way, i.e. under RE. If it is found that $k^{op} \geq 3$, consider the next step;

Step 2 Given k=k^{op} and $\dim(x_t^*) = nk^{op}$, estimate $\theta^* = (\theta', \zeta')'$ from system (18)-(19) under a numerical approximation of the CER in Eq. (25), and verify that Assumption 4 is respected in correspondence of the point estimate $\hat{\theta}^* = (\hat{\theta}', \hat{\zeta}')'$.[9] Then test the CER through a likelihood ratio test that compares the likelihood obtained in the previous step, $\log L_T(\hat{\tau}^{op})$, and the likelihood associated with θ^*, $\log L_T(\hat{\theta}^*)$, obtaining LR_T^{CER}:=$-2(\log L_T(\hat{\theta}^*) - \log L_T(\hat{\tau}^{op}))$. The log-likelihood maximization is also achieved through Kalman filtering and CMAES algorithm.

Under standard regularity conditions, the estimator of θ^* (hence the estimator of θ) derived in the Step 2 is consistent and asymptotically Gaussian, and LR_T^{CER} is asymptotically $\chi^2(d)$, with

[8] A natural concern here is whether the CER in Eq. (25) allow to identify θ^*. A convenient way to summarize the CER derived in Proposition 1 is by the distance function $f(\tau^{op}, \theta^*) = 0_{a\times 1}$, where $f(\cdot,\cdot)$ is a nonlinear continuos vector differentiable function and $a = n^2 k^{op} + n + \frac{1}{2}n(n+1)$. By the implicit function theorem, the CER can be represented in explicit form $\tau^{op} = g(\theta^*)$, where $g(\cdot)$ is a nonlinear continuos differentiable vector function. Although an analytic expression for the function $g(\cdot)$ is not generally available, the Jacobian of the relationship can be computed with minor adaptations either analytically or numerically.

[9] This can be done by verifying the stability of the matrix $S(\hat{\theta})$ estimated under RE, where $\hat{\theta}$ is the maximum likelihood estimate of θ obtained with the 'conventional' approach, and the stability of the matrix $S(\hat{\theta}^*)$ estimated under ExC, where $\hat{\theta}^*$ is the maximum likelihood estimate of θ^*. The matrix $S(\theta^*)$ is introduced in the proof of Proposition 1, see the Technical Supplement and Binder and Pesaran (1995) and Bårdsen and Fanelli (2015). The matrix $S(\theta)$ is the RE analogue of $S(\theta^*)$. If both $S(\theta)$ and $S(\theta^*)$ are stable, the condition in Assumption 4 is respected. If $S(\hat{\theta})$ is stable but $S(\hat{\theta}^*)$ has unstable eigenvalues, it is necessary to impose proper restrictions on the auxiliary parameters ζ to restore stability.

d =dim(τ^{op})-dim(θ^*). In this situation, the possible rejection of the CER can not be ascribed to the omitted dynamics issue. Standard regularity conditions might not hold in the NK-DSGE model when θ (and θ^*) are weakly identified, see e.g. Andrews and Mikusheva (2015). In these cases, it is in principle possible to adapt the Step 2 of the procedure by using the 'full-information' identification-robust methods discussed in e.g. Dufour *et al.* (2013), Guerron-Quintana *et al.* (2013) and Castelnuovo and Fanelli (2015). This requires inverting (numerically) the LR_T^{CER} test, obtaining asymptotically valid identification-robust confidence sets for θ^*. As explained in Guerron-Quintana *et al.* (2013), the computation and inversion of the test LR_T^{CER} requires the strong identifiability of the state space model estimated in the Step 1, which motivates our Assumption 1. It turns out that a sound (mis)specification analysis of the statistical model estimated in the Step 1 is a key aspect of approach.

The Step 2 of the procedure can easily be adapted to the Bayesian approach. Given the statistical model built in Step 1, it is possible to specify a prior distribution for θ^*, $p(\theta^*)$, and then compute the posterior given the observations $y_1, ..., y_T$, $p(\theta^* \mid y_1, ..., y_T)$. This can be done by using e.g. the Random Walk Metropolis (RWM) algorithm along the lines of An and Schorfheide (2007). In our framework, it seems 'natural' to specify priors for the expectations correction parameters ζ that are centered on zero, i.e. on the RE solution, such that the extent of the misspecification, if any, is determined by the data, see the next sections. Moreover, the selection between the NK-DSGE model under RE and ExC can be based on Bayesian information criteria or odds-ratios, etc. More details are provided in Sub-section 5.4.

5 Empirical analysis

In this section, we estimate and empirically evaluate a small-scale monetary NK-DSGE model on U.S. quarterly data, applying the ExC methodology discussed in the previous section. We also compare our approach with the 'conventional' RE case. In Sub-section 5.1 we introduce the reference structural model. In Sub-section 5.2 we describe the data and discuss the specification of the statistical model. In Sub-section 5.3, we estimate and evaluate the NK-DSGE model under ExC using a frequentist maximum likelihood approach, while in Sub-section 5.4 we repeat the same exercise using the Bayesian approach.

5.1 Structural model

Our reference NK-DSGE model is taken from Benati and Surico (2009) and is based on the following three equations:

$$\tilde{o}_t = \gamma E_t \tilde{o}_{t+1} + (1-\gamma)\tilde{o}_{t-1} - \delta(R_t - E_t\pi_{t+1}) + \eta_{\tilde{y},t} \tag{26}$$

$$\pi_t = \frac{\beta}{1+\beta\alpha} E_t\pi_{t+1} + \frac{\alpha}{1+\beta\alpha}\pi_{t-1} + \kappa\tilde{o}_t + \eta_{\pi,t} \tag{27}$$

$$R_t = \rho R_{t-1} + (1-\rho)(\varphi_\pi \pi_t + \varphi_{\tilde{o}}\tilde{o}_t) + \eta_{R,t} \tag{28}$$

where

$$\eta_{x,t} = \rho_x \eta_{x,t-1} + \omega_{x,t} \quad , \text{-}1<\rho_x<1 \quad , \omega_{x,t} \sim \mathrm{WN}(0, \sigma_x^2) \quad , \quad x = \tilde{o}, \pi, R. \tag{29}$$

The variables $\tilde{o}_t{:=}o_t - o_t^p$, π_t, and R_t stand for the output gap (o_t is output and o_t^p the natural rate of output), inflation, and the nominal interest rate, respectively; γ is the weight of the forward-looking component in the intertemporal IS curve; α is the price setters' extent of indexation to past inflation; δ is households' intertemporal elasticity of substitution; β is a discount factor which is fixed at the value $\beta{:=}0.99$ and treated as known; κ is the slope of the Phillips curve; ρ, φ_π, and $\varphi_{\tilde{y}}$ are the interest rate smoothing coefficient, the long-run coefficient on inflation, and that on the output gap in the monetary policy rule, respectively; finally, $\eta_{\tilde{o},t}$, $\eta_{\pi,t}$ and $\eta_{R,t}$ in Eq. (29) are the mutually independent, autoregressive of order one disturbances and $\omega_{\tilde{o},t}$, $\omega_{\pi,t}$ and $\omega_{R,t}$ are the structural (fundamental) shocks with variances σ_x^2, $x = \tilde{o}, \pi, R$.

This and similar small-scale models have successfully been employed to conduct empirical analyses concerning the U.S. economy. Clarida *et al.* (2000) and Lubik and Schorfheide (2004) have investigated the influence of systematic monetary policy over the U.S. macroeconomic dynamics; Boivin and Giannoni (2006) and Benati and Surico (2009) have replicated the U.S. Great Moderation, while Castelnuovo and Fanelli (2015) have tested the determinacy/indeterminacy properties of the implied equilibria controlling for identification failure. It is worth noting that Benati and Surico's (2009) model is 'hybrid', in the sense that given the policy rule, both the IS curve and the NKPC feature lags of $\tilde{o}_t$ and π_t other than future expectations. In this respect, it seems particularly suited to serve as a reference structural model in the estimation/evaluation exercise with which we are concerned in this paper.

The three-equation system (26)-(29) can be cast in the form in Eq.s (10)-(11) by setting $Z_t{:=}(\tilde{o}_t, \pi_t, R_t)'$, $(n = 3)$, $\eta_t{:=}(\eta_{\tilde{o},t}, \eta_{\pi,t}, \eta_{R,t})'$, $\omega_t{:=}(\omega_{\tilde{o},t}, \omega_{\pi,t}, \omega_{R,t})'$ and

$$\Gamma_0{:=}\begin{pmatrix} 1 & 0 & \delta \\ -\kappa & 1 & 0 \\ -(1-\rho)\varphi_{\tilde{o}} & -(1-\rho)\varphi_\pi & 1 \end{pmatrix}, \Gamma_f{:=}\begin{pmatrix} \gamma & \delta & 0 \\ 0 & \frac{\beta}{1+\beta\alpha} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_b{:=}\begin{pmatrix} 1-\gamma & 0 & 0 \\ 0 & \frac{\alpha}{1+\beta\alpha} & 0 \\ 0 & 0 & \rho \end{pmatrix}.$$

$$R{:=}dg(\rho_{\tilde{o}}, \rho_{\pi}, \rho_{R}) \quad , \quad \Sigma_{\omega}{:=}dg(\sigma^2_{\tilde{o}}, \sigma^2_{\pi}, \sigma^2_{R}),$$

where the operator $dg(\cdot)$ denotes a diagonal matrix and the entries are in the argument. $\theta{:=}(\gamma, \delta, \alpha, \kappa, \rho, \varphi_{\tilde{y}}, \varphi_{\pi}, \rho_{\tilde{o}},$ is the $13{\times}1$ vector of structural parameters. The constant C is, in this case, set to zero because estimation is based on demeaned variables, see below.

As in Bårdsen and Fanelli (2015), we complete the specification of the model in Eq.s (26)-(29) by assuming that the natural rate of output o^p_t captures the effects of technology shocks through the Random Walk process:

$$o^p_t = o^p_{t-1} + \eta_{o^p,t} \quad , \quad \eta_{o^p,t} \sim \mathrm{WN}(0, \sigma^2_{o^p}). \tag{30}$$

Using Eq. (30) and the definition of $\tilde{o}_t{:=}o_t - o^p_t$, we obtain the relationship

$$\tilde{o}_t - \tilde{o}_{t-1} = \Delta o_t - \eta_{o^p,t} \tag{31}$$

where $\Delta o_t{:=}o_t - o_{t-1}$, which will be exploited in the measurement system below.

5.2 Data and statistical model

We employ quarterly data relative to the 'Great Moderation' sample 1984q2-2008q3. The starting date of our estimation and evaluation sample, 1984q2, is justified by McConnell and Pérez-Quirós (2000), who find a break in the variance of the U.S. output growth in 1984q1. The ending date is instead motivated by the fact that, with data after 2008q3, it would be hard to identify a 'conventional' monetary policy shock with our structural model during the well known zero lower bound (ZLB) episodes. We have three observable variables, $y_t{:=}(\Delta o_t, \pi_t, R_t)'$ $(p = 3)$, where Δo_t is related to the unobservable output-gap $\tilde{o}_t$ through Eq. (31).[10] Output, o_t, is the log of real GDP. The inflation rate, π_t, is the quarterly growth rate of the GDP deflator. For the short-term nominal interest rate, R_t, we consider the effective Federal funds rate expressed in quarterly terms (averages of monthly values). The source of the data is the website of the Federal Reserve Bank of St. Louis. The three variables are demeaned.[11]

[10] We have also considered the case in which $\tilde{o}_t$ is proxied by a measure of the output-gap computed by using the measure of potential output released by the Congressional Budget Office (CBO). In that case, estimation does not necessarily require the use of Eq. (31). Results are available upon request to the authors.

[11] Before demeaning and estimating the model, we run a preliminary check for stationarity of $y_t{:=}(\Delta o_t, \pi_t, R_t)'$. We compute Johansen's (1996) cointegration rank test using a VAR model for y_t with restricted (to the cointegration space) and unrestricted constants, respectively. Results does not clearly rule out the possible presence of unit roots in the system. On economic grounds, it is difficult to justify the occurrence of unit roots in $y_t{:=}(\Delta o_t, \pi_t, R_t)'$. The poor finite sample power of the employed cointegration rank test is a reasonable explanation of our findings. We therefore treat the vector y_t as generated by an highly persistent, covariance stationary process.

Step 1 of our estimation and evaluation procedure (Section 4) requires fitting the state-space model in Eq.s (18)-(19) to the data, and selecting the optimal length of the state vector, i.e. $k{=}k^{op}$and $\dim(x_t^*){=}nk^{op}$. Starting from a maximum lag order of $k^{\max} = 6$, the 'largest' statistical model which is taken to the data is given by

$$\underbrace{\begin{pmatrix} Z_t \\ Z_{t-1} \\ \vdots \\ Z_{t-(k^{\max}-1)} \end{pmatrix}}_{x_t^*} = \underbrace{\begin{pmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_{k^{\max}-1} & \Phi_{k^{\max}} \\ I_3 & 0_{3\times 3} & \cdots & 0_{n\times n} & 0_{3\times 3} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0_{3\times 3} & 0_{3\times 3} & & I_3 & 0_{3\times 3} \end{pmatrix}}_{A^*(\tau)} \underbrace{\begin{pmatrix} Z_{t-1} \\ Z_{t-2} \\ \vdots \\ Z_{t-k^{\max}} \end{pmatrix}}_{x_{t-1}^*} + \underbrace{\begin{pmatrix} \varepsilon_t \\ 0_{3\times 3} \\ \vdots \\ 0_{3\times 3} \end{pmatrix}}_{G^*(\tau)} \qquad (32)$$

$$\underbrace{\begin{pmatrix} \Delta o_t \\ \pi_t \\ R_t \end{pmatrix}}_{y_t} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} \tilde{o}_t \\ \pi_t \\ R_t \\ \vdots \\ \tilde{o}_{t-(k^{\max}-1)} \\ \pi_{t-(k^{\max}-1)} \\ R_{t-(k^{\max}-1)} \end{pmatrix}}_{x_t^*} + \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{V} \underbrace{v_{1,t}}_{v_t} \qquad (33)$$

where $Z_t{:=}(\tilde{o}_t, \pi_t, R_t)'$, $\omega_t{:=}(\omega_{\tilde{o},t}, \omega_{\pi,t}, \omega_{R,t})'$, and $v_t{:=}v_{1,t}{=}\eta_{o^p,t}$ from Eq. (31).

We estimate the space state model in Eq.s (32)-(33) on the period 1984q2-2008q3, varying k from 2 to $6{=}{:}k^{\max}$, using a Kalman filter-based maximum likelihood approach in conjunction with the CMAES algorithm (see Andreasen, 2010). For each estimated model, we check whether the minimality (controllability and observability) and local identification conditions discussed in Komunjer and Ng (2011) are satisfied in correspondence of the parameter values delivered by the likelihood maximization algorithm. We then select the optimal lag k^{op} computing the Akaike, Hannan-Quinn and Schwarz information criteria and the LR test. The results of this specification analysis are summarized in Table 1.

Table 1 shows that using the 5% nominal level of significance, the LR tests selects the model based on $k = k^{op}{=}4$ lags. The Akaike information criterion selects 5 lags, while Schwarz and Hannan-Quinn select 2 lags. That different criteria lead to different lag orders in small samples is not surprising; see e.g. Lütkepohl (1993) for the case of finite-order VARs. In the Technical Supplement, we run a Monte Carlo experiment to envisage to what extent the uncertainty that characterizes the lag length selection in Table 1 is 'admissible'. Monte Carlo evidence suggests that the specification analysis summarized in Table 1 is perfectly consistent with what a practitioner can obtain with samples of size $T{=}100$ when the data generating process is given by our pseudo-structural form. Moreover, the LR test proves to be the 'best' selection criterion in small samples relative to the Akaike, Hannan-Quinn and Schwarz criteria, and is not biased towards the case of RE. Driven by these facts, we select k^{op}=4 lags ($\dim(x_t^*)$=nk^{op} = 12) as suggested by the LR test. In this case, the vector of parameters associated with the statistical model is given by τ^{op}:=$(vec(\Phi_1)', vec(\Phi_2)', vec(\Phi_3)', vec(\Phi_4)', vech(\Sigma_\varepsilon)', \sigma^2_{o^p})'$.

5.3 Frequentist estimation and empirical evaluation

Assuming that the statistical model that fits the data is based on k^{op}=4 lags, i.e. $\dim(x_t^*)$=nk^{op} = $12 > \dim(x_t)$=$n2 = 6$, the pseudo-structural form associated with our NK-DSGE model is given by

$$\begin{aligned}
\tilde{o}_t &= \gamma E_t\tilde{o}_{t+1} + (1-\gamma)\tilde{o}_{t-1} - \delta(R_t - E_t\pi_{t+1}) + \zeta_{\tilde{o},2}\tilde{o}_{t-2} + \zeta_{\tilde{o},3}\tilde{o}_{t-3} + \eta_{\tilde{y},t} \\
\pi_t &= \frac{\beta}{1+\beta\alpha}E_t\pi_{t+1} + \frac{\alpha}{1+\beta\alpha}\pi_{t-1} + \kappa\tilde{o}_t + \zeta_{\pi,2}\pi_{t-2} + \zeta_{\pi,3}\pi_{t-3} + \eta_{\pi,t} \\
R_t &= \rho R_{t-1} + (1-\rho)(\varphi_\pi\pi_t + \varphi_{\tilde{o}}\tilde{o}_t) + \zeta_{R,2}R_{t-2} + \zeta_{R,3}R_{t-3} + \eta_{R,t}
\end{aligned} \tag{34}$$

$$\eta_{x,t} = \rho_x\eta_{x,t-1} + \omega_{x,t} \quad , \text{-}1<\rho_x<1 \quad , \omega_{x,t} \sim \mathrm{WN}(0,\sigma_x^2) \quad , \quad x = \tilde{o}, \pi, R \tag{35}$$

where $\zeta_{\tilde{o},2}$, $\zeta_{\pi,2}$,$\zeta_{R,2}$,$\zeta_{\tilde{o},3}$, $\zeta_{\pi,3}$ and $\zeta_{R,3}$ are the expectations correction parameters that enter the (supposed diagonal) matrices Υ_2 and Υ_3, see Eq. (24). System (34)-(35) defines an higher order 'hybrid' NK-DSGE model that rectifies the baseline structural specification. Thus, ζ:=$(\zeta_{\tilde{o},2}, \zeta_{\pi,2}, \zeta_{R,2}, \zeta_{\tilde{o},3}, \zeta_{\pi,3}, \zeta_{R,3})' = (diag(\Upsilon_2)', diag(\Upsilon_3)')'$ and $\theta^* = (\theta', \zeta')'$:=$(\gamma, \delta, \alpha, \kappa, \rho, \varphi_{\tilde{o}}, \varphi_\pi, \rho_{\tilde{o}}, \rho_\pi, \rho_R, \sigma^2_{\tilde{o}}, \sigma^2_\pi, \sigma^2_R, \zeta_{\tilde{o},2}, \zeta_{\pi,2}, \zeta_{R,2}, \zeta_{\tilde{o},3}, \zeta_{\pi,3}, \zeta_{R,3})'$ is the 19×1 vector containing the truly structural and ExC parameters.

Step 2 of the procedure summarized in Section 4 requires estimating $\theta^* = (\theta', \zeta')'$ from the state space model (32)-(33) by imposing the CER derived in Proposition 1. We obtain: $\Phi_i = \tilde{\Phi}_i$, $i = 1, 2, 3, 4$, Σ_ε=$\tilde{\Sigma}_\varepsilon$, where

$$\begin{cases}
(\Gamma_0^R - \Gamma_f\tilde{\Phi}_1)\tilde{\Phi}_1 - (\Gamma_f\tilde{\Phi}_2 + \Gamma_{b,1}) = 0_{3\times3} \\
(\Gamma_0^R - \Gamma_f\tilde{\Phi}_1)\tilde{\Phi}_2 - (\Gamma_f\tilde{\Phi}_3 + \Upsilon_2 - R\Gamma_b) = 0_{3\times3} \\
(\Gamma_0^R - \Gamma_f\tilde{\Phi}_1)\tilde{\Phi}_3 - (\Gamma_f\tilde{\Phi}_4 + \Upsilon_3 - R\Upsilon_2) = 0_{3\times3} \\
(\Gamma_0^R - \Gamma_f\tilde{\Phi}_1)\tilde{\Phi}_4 + R\Upsilon_3 = 0_{3\times3} \\
\tilde{\Sigma}_\varepsilon - \tilde{\Psi}\ \Sigma_\omega\ \tilde{\Psi}' = 0_{n\times n} \quad , \quad \tilde{\Psi}:=(\Gamma_0^R - \Gamma_f\tilde{\Phi}_1)^{-1}.
\end{cases} \tag{36}$$

Estimation results for $\theta^* = (\theta', \zeta')'$ are reported in Table 2. In the upper panel of Table 2, we summarize the estimate of θ obtained under RE, i.e. taking the structural model in Eq.s (26)-(29) to the data in the 'conventional way', and the corresponding estimate obtained from the pseudo-structural form (34)-(35). We label the estimates obtained from the pseudo-structural form with the acronym 'ExC', see fourth column of Table 2. Observe that in order to obtain the CER under RE (k^{op}=2), it is sufficient to set the matrices Υ_2 and Υ_3 to zero in Eq. (36), leading to $\tilde{\Phi}_3 = \tilde{\Phi}_4 = 0_{3\times 3}$ (under these restrictions agents' expectations coincide with RE).

The lower panel of Table 2 summarizes the Akaike, Hannan-Quinn and Schwarz information criteria and a battery of LR tests through which it is possible to select the 'best' specification. All three information criteria favour the model estimated under ExC. The LR test for the null ζ=$0_{6\times 1}$ (RE) against the alternative $\zeta \neq 0_{6\times 1}$ (ExC) strongly rejects the null hypothesis. In both cases, the CER are strongly rejected.

Coming back to the estimated parameters in the upper panel of Table 2, we notice that the large majority (four out of six) of the ExC parameters ζ:=$(\zeta_{\tilde{o},2}, \zeta_{\pi,2}, \zeta_{R,2}, \zeta_{\tilde{o},3}, \zeta_{\pi,3}, \zeta_{R,3})'$ (forth column) are significant at conventional significance levels. This confirms that there is a mismatch between agents' expectations as implied by the statistical model and the case of RE. Focusing on the truly structural parameters θ, we notice that the main differences between the estimates obtained under RE and ExC involve the intertemporal elasticity of substitution δ and the forward-looking parameter γ in the IS curve, the slope κ and shock persistence parameter ρ_{π} in the NKPC, and the Fed's long run response to output gap $\varphi_{\tilde{o}}$ and shock persistence parameter ρ_{π} in the policy reaction function.

The magnitude and precision of the estimated δ is considerably higher under ExC, whereas δ does not seem to be empirically identified under RE. Conversely, the magnitude and precision of the estimated γ is lower under ExC relative to RE, suggesting a lesser extent of forward-looking behaviour once we account for the whole dynamics of the system. This result can be clearly explained in light of the more 'flexible' expectations generating system we assume. The slope parameter of the NKPC is poorly estimated in both cases, confirming a traditional difficulty in its empirical identification. The magnitude of the estimated indexation parameter of the NKPC, α, is the same in the two cases; we observe that precision is considerably higher under ExC relative to RE. However, the estimated α obtained under ExC is comparatively more precise than the estimate obtained under RE. Overall, our maximum likelihood estimates seems to suggest that the NKPC can be more precisely empirically identified by relaxing some constraints on the autocorrelation structure of the data.

As concerns the policy rule, we notice that the Fed's long run response to output gap is remarkably higher relative to the case of RE (1.5 as opposed to 0.336) and more precisely estimated under ExC. As it known, the empirical literature on the identifiability of the policy parameters $\varphi_{\tilde{o}}$ and φ_{π} in New Keynesian models is huge and has not yet reached a consensus. The recent empirical literature, which makes increasing use of identification-robust methods, suggests that it is difficult to estimate $\varphi_{\tilde{o}}$ (and φ_{π}) precisely on the Great Moderation era, see, among others, Mavroeidis (2010) and Castelnuovo and Fanelli (2015) and references therein. As already explained in Section 4, if the researcher suspects that $\theta^* = (\theta', \zeta')'$ or some of its components are weakly identified, our approach can be potentially adapted to the case of identification-robust methods. We do not pursue this important check here. The lesson we learn from Table 2 is that a sound dynamic specification analysis of the New Keynesian model can aid the empirical identification process of monetary policy parameters.

5.4 Bayesian estimation and empirical evaluation

In the Bayesian approach, Step 1 is exactly as in Sub-section 5.2; hence, the estimated pseudo-structural form in Step 2 is given by system (34)-(35). The priors used for the truly structural parameters, θ, are taken from Benati and Surico (2009), while the priors used for the ExC parameters, ζ, are centered at the RE equilibrium. More precisely, for each $\zeta_{i,j}$, $i = \tilde{o}, \pi, R$, $j = 2, 3$ in Eq.s (34)-(35), we use a Gaussian distribution centered on 0 with variance 0.25. Table 3 summarizes the modes and standard deviations of the prior distributions for all structural parameters. The RWM algorithm delivers the posterior distributions reported in Table 4.

As expected, the DIC information criterion favours the NK-DSGE model estimated under ExC, relative to the case of RE. The estimates in Table 4 are quantitatively different from their counterparts in Table 2 obtained with the frequentist maximum likelihood approach. Similarly to the frequentist estimation approach, we observe that the mismatch between agents' expectations and RE seems to be relevant. The magnitude of estimated persistence parameters, $\rho_{\tilde{o}}$, ρ_{π} and ρ_R, is considerably larger in the pseudo-structural form compared to the case of RE, suggesting that other than capturing omitted propagation mechanisms, the pseudo-structural model does not penalize the persistence of the data, given the chosen priors.

The main differences between the estimates obtained under RE and by the pseudo-structural form involve the forward-looking parameter of the IS curve, γ, and the policy reaction of the Fed to the output gap, $\varphi_{\tilde{o}}$. Contrary to what is reported in Table 2, the magnitude of the estimated γ is considerably larger under ExC, pointing towards a greater extent of forward-looking behaviour. Obviously, the difference in the estimates of γ in Table 2 and Table 4 can solely be ascribed to the role of the prior distributions. On the other hand, we notice that in the Bayesian approach as well, the Fed's long run response to output gap is remarkably higher relative to the case of RE (1.054 as opposed to 0.449) and more precisely estimated under ExC. This evidence confirms the finding obtained with the frequentist approach in Sub-section 5.3. All other estimates in Table 4 are roughly the same as in Table 2.

6 Concluding remarks

In this paper, we have focused on the poor time series performance that characterizes the class of small-scale NK-DSGE models currently used in monetary policy and business cycle analysis. Under RE, NK-DSGE models give rise to a set of nonlinear CER and constraints on the lag order of the system that may conflict with the actual autocorrelation structure that characterizes quarterly (or monthly) time series. In these cases, the investigator should reformulate the structural model by specifying a less restrictive, possibly microfounded, dynamic structure that accounts for the previously omitted propagation mechanisms. This is not always feasible. Practitioners typically react to this type of misspecification by postulating *ad hoc* time series models for the shocks, or adding measurement errors in the associated state space representation. We rationalize these practices by using a data-driven procedure based on a statistical model for the data, which is combined with the original structural form without abandoning the logic and concept of model-consistent expectations. Our approach is illustrated empirically by focusing on the 'hybrid' NK-DSGE monetary model by Benati and Surico (2009) as the reference system.

References

An, S. and Schorfheide, F. (2007). 'Bayesian analysis of DSGE models', *Econometric Reviews*, Vol. 26, pp. 113-172.

Andreasen, M. M. (2010). 'How to maximize the likelihood function for a DSGE model', *Computational Economics*, Vol. 35, pp. 127-154.

Andrews, I. and Mikusheva, A. (2015). 'Maximum likelihood inference in weakly identified dynamic stochastic general equilibrium models', *Quantitative Economics*, Vol. 6, pp. 123-152.

Bårdsen, G. and Fanelli, L. (2015). 'Frequentist evaluation of small DSGE models', *Journal of Business and Economic Statistics*, Vol. 33, pp. 307-322.

Benati, L. and Surico, P. (2009). 'VAR analysis and the Great Moderation', *American Economic Review*, Vol. 99, pp. 1636-1652.

Beyer, A., Farmer, R., Henry, J. and Marcellino, M. (2008). 'Factor analysis in a model with rational expectations', *The Econometrics Journal*, Vol. 11, pp. 271–286.

Binder, M. and Pesaran, M. H. (1995). 'Multivariate rational expectations models and macroeconomic modelling: a review and some new results', in M. H. Pesaran and M. Wickens (eds.), *Handbook of Applied Econometrics*, pp. 139-187 (Ch. 3). Oxford: Blackwell.

Boivin, J. and Giannoni, M.P. (2006). 'Has monetary policy become more effective ?', *Review of Economics and Statistics*, Vol. 88, pp. 445-462.

Branch, W.A. and Evans, G.W. (2006). 'A simple recursive forecasting model', *Economic Letters*, Vol. 91, pp. 158-166.

Cagliarini, A. and Kulish, M. (2013). 'Solving linear rational expectations models with predictable structural changes', *Review of Economics and Statistics*, Vol. 95, pp. 328-336.

Canova, F. and Sala, L. (2009). 'Back to square one: identification issues in DSGE models', *Journal of Monetary Economics*, Vol. 56, pp. 431-449.

Castelnuovo, E. and Fanelli, L. (2015). 'Monetary policy indeterminacy and identification failures in the U.S.: Results from a robust test', *Journal of Applied Econometrics*, Vol. 30, pp. 857-1010.

Chevillon, G., Massmann, M. and Mavroeidis, S. (2010). 'Inference in models with adaptive learning', *Journal of Monetary Economics*, Vol. 57, pp. 341-351.

Cho, I-K and Kasa, K. (2015). 'Learning and model validation', *Review of Economic Studies*, Vol. 82, pp. 45-82.

Clarida R.J., Galì, J. and Gertler, M. (2000). 'Monetary policy rules and macroeconomic stability: evidence and some theory', *Quarterly Journal of Economics*, Vol. 115, pp. 147-180.

Cogley, T. and Sbordone, A. M. (2008). 'Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips Curve', *The American Economic Review*, Vol. 98, pp. 2101-2126.

Cole, S. J. and Milani, F. (2014). 'The misspecification of expectations in New Keynesian Models: a DSGE-VAR approach', Working Paper, Department of Economics, University of California, Irvine.

Consolo, A., Favero, C.A. and Paccagnini, A. (2009). 'On the statistical identification of DSGE models', *Journal of Econometrics*, Vol. 150, pp. 99-115.

Cúrdia,V. and Reis, R. (2010). ‘Correlated disturbances and U.S. business cycle’, *NBER* Working Paper 15774.

DeJong, D.N. and Dave, C. (2007). *Structural Macroeconometrics*, Princeton University Press, Princeton.

Del Negro, M., Schorfheide, F., Smets, F. and Wouters, R. (2007). ‘On the fit of New Keynesian models’, *Journal of Business and Economic Statistics*, Vol. 25, pp. 123-143.

Dufour, J-M, Khalaf, L. and Kichian, M. (2013). ‘Identification-robust analysis of DSGE and structural macroeconomic models’, *Journal of Monetary Economics*, Vol. 60, pp. 340-350.

Evans, G. W. and Honkapohja, S. (1999). *Learning dynamics*, in Handbook of Macroeconomics 1A, Chap. 7

Evans, G. W. and Honkapohja, S. (2001). *Learning and expectations in macroeconomics*, Princeton University Press.

Fanelli, L. (2008). ‘Evaluating the New Keynesian Phillips Curve under VAR-based Learning’, *Economics: The Open-Access, Open-Assessment E-Journal*, 2, 2008-33.

Fanelli, L. (2009). ‘Estimation of Quasi-Rational DSGE monetary models’, Dipartimento di Scienze Statistiche, Quaderno di Dipartimento Serie Ricerche 2009 n.3.

Fanelli, L. (2012). ‘Determinacy, indeterminacy and dynamic misspecification in Linear Rational Expectations models’, *Journal of Econometrics*, Vol. 170, pp. 153-163.

Fanelli, L. and Palomba, G. (2011). ‘Simulation-based tests of forward-looking models under VAR learning dynamics’, *Journal of Applied Econometrics*, Vol. 26, pp. 762-782.

Fernández-Villaverde, J., Rubio-Ramírez, J., Sargent, T. and Watson, M. (2007). ‘ABCs (and Ds) of understanding VARs’, *American Economic Review*, Vol. 97, pp. 1021-1026.

Franchi, M. and Vidotto, A. (2013). ‘A check for finite order VAR representations of DSGE models’, *Economic Letters*, Vol. 120, pp. 100-103.

Franchi, M. and Paruolo, P. (2015). ‘Minimality of state space solutions of DSGE models and existence conditions for their VAR representation’, *Computational Economics*, Vol. 46, pp. 613-626.

Franchi, M. and Juselius, K. (2007). ‘Taking a DSGE model to the data meaningfully’, *Economics: The Open-Access, Open-Assessment E-Journal*, 1, 2007-4.

Fuhrer, J. and Rudebusch, G.D. (2004). 'Estimating the Euler equation for output', *Journal of Monetary Economics*, Vol. 51, pp. 1133-1153.

Fukač, M. and Pagan, A. (2009). 'Limited information estimation and evaluation of DSGE models', *Journal of Applied Econometrics*, Vol. 25, pp. 55-70.

Guerron-Quintana, P., Inoue, A. and Kilian, L. (2013). 'Frequentist inference in weakly identified DSGE models', *Quantitative Economics*, Vol. 4, pp. 197-229.

Hannan, E.J. and Deistler, M. (1988), *The Statistical Theory of Linear Systems*, SIAM ed. Wiley, New York.

Hansen, L.P. (2014). 'Nobel Lecture: Uncertainty outside and inside economic models', *Journal of Political Economy*, Vol. 122, pp. 945-987.

Hansen, L. P. and Sargent, T. J. (1980). 'Formulating and estimating dynamic linear rational expectations models', *Journal of Economic Dynamics and Control*, Vol. 2, pp. 7-46.

Hansen, L. P. and Sargent, T. J. (1981). 'Linear rational expectations models for dynamically interrelated variables', in: R.E.Jr. Lucas and T. J. Sargent, (Eds.), *Rational Expectations and Econometric Practise*. University of Minnesota Press, Minneapolis, pp. 127-156.

Henry, S.G.B. and Pagan, A.R. (2004). 'The econometrics of the New Keynesian policy model: Introduction', *Oxford Bulletin of Economics and Statistics*, Vol. 66, Supplement, 581-607.

Holt, M.T. and Mckenzie, A. M. (2003). 'Quasi-rational and ex ante price expectations in commodity supply models: an empirical analysis of the broiler market', *Journal of Applied Econometrics*, Vol. 18, pp. 407-426.

Ireland, P. (2004). 'A method for taking models to the data', *Journal of Economic Dynamics and Control*, Vol. 28, pp. 1205-1226.

Iskrev, N. (2010). 'Local identification in DSGE models', *Journal of Monetary Economics*, Vol. 57, pp. 189–202.

Johansen, S. (1996). *Likelihood-based inference in cointegrated vector auto-regressive models*, Oxford University Press, Oxford.

Jondeau, E. and Le Bihan, H. (2008). 'Examining bias in estimators of linear rational expectations models under misspecification', *Journal of Econometrics*, Vol. 143, pp. 375-395.

Klein, P. (2000). 'Using the generalized Schur form to solve a multivariate linear rational expectations model', *Journal of Economic Dynamics and Control*, Vol. 24, pp. 1405-1423.

Komunjer, S. and Ng, S. (2011). 'Dynamic identification in DSGE models'. *Econometrica*, Vol. 79, pp. 1995-2032.

Kozicki, S. and Tinsley, P. (1999). 'Vector rational error correction', *Journal of Economic Dynamics and Control*, Vol. 23, pp. 1299-1327.

Lubik, T. A. and Schorfheide, F. (2004). 'Testing for indeterminacy: an application to U.S. monetary policy', *American Economic Review*, Vol. 94, pp. 190-217.

Mankiw, N.G. and Reis, R. (2002). 'Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips Curve', *Quarterly Journal of Economics*, vol. 117, pp. 1295-1328.

Mankiw, N.G. and Reis, R. (2007). 'Sticky information in general equilibrium', *Journal of the European Economic Association*, Vol. 5, pp. 603-613.

Mavroeidis, S. (2010). 'Monetary policy rules and macroeconomic stability: some new evidence', *American Economic Review*, Vol. 100, pp. 491-503.

McConnell, M.M. and Pérez-Quirós, G. (2000). 'Output fluctuations in the United States: what has changed since the early 1980's?', *American Economic Review*, Vol. 90, pp. 1464-1476.

Milani, F. (2007). 'Expectations, learning and macroeconomic persistence', *Journal of Monetary Economics*, Vol. 54, pp. 2065-2082.

Nelson, R. G. and Bessler, D. A. (1992). 'Quasi-rational expectations: Experimental evidence', *Journal of Forecasting*, Vol. 11, pp. 141-156.

Nerlove, M. and Fornari, I. (1998). 'Quasi-rational expectations, an alternative to fully rational expectations: An application to US beef cattle supply', *Journal of Econometrics*, Vol. 83, pp. 129-161.

Nerlove, M., Grether, D. and Carvalho, J.L. (1979). *Analysis of Economic Time Series.* Academic Press, New York.

Ravenna, F. (2007). 'Vector autoregressions and reduced form representations of DSGE models', *Journal of Monetary Economics*, Vol. 54, pp. 2048-2064.

Rudebusch, G.D. (2002*a*). 'Assessing nominal income rules for monetary policy with model and data uncertainty', *Economic Journal*, Vol. 112, pp. 402-432.

Rudebusch, G.D. (2002*b*). 'Term structure evidence on interest rate smoothing and monetary policy inertia', *Journal of Monetary Economics*, Vol. 49, pp. 1161-1187.

Ruge-Murcia, F.J. (2007). 'Methods to estimate dynamic stochastic general equilibrium models', *Journal of Economic Dynamics and Control*, Vol. 31, pp. 2599-2636.

Sargent, T. J. (1989). 'Two models of measurements and the investment accelerator', *Journal of Political Economy*, Vol. 97, pp. 251-287.

Schorfheide, F. (2010). 'Estimation and evaluation of DSGE models: Progress and challenges', *NBER* Working Paper n. 141.

Sims, C. A. (2003). 'Implications of rational inattention', *Journal of Monetary Economics*, Vol. 50, pp. 665–690.

Sims, C. A. (2006). 'Rational inattention: Beyond the Linear-Quadratic case', *American Economic Review*, Vol. 96, pp. 158-163.

Smets, F. and Wouters, R. (2007). 'Shocks and frictions in U.S. business cycles', *American economic Review*, Vol. 97, pp. 586-606.

Uhlig, H. (1999). 'A Toolkit for analyzing nonlinear dynamic stochastic models easily', in R. Marimon and A. Scott (eds.), *Computational methods for the study of dynamics economies*, pp. 30-61, Oxford University Press, New York.

Zanetti, F. (2008). 'Labor and investment frictions in a real business cycle model', *Journal of Economic Dynamics and Control*, Vol. 32, pp. 3294-3314.

Woodford, M. (2003). *Interest and prices*, Princeton University Press, Princeton.

TABLE 1. Lag length selection in the statistical model in Eq.s (32)-(33).

Estimation sample: 1984q2 - 2008q3						
	LR tests			Information criteria		
lag	Likelihood	LR	p-value	Akaike	Hannan-Quinn	Schwarz
2	151.42	74.71	0.000	-240.84	-208.57*	-161.03*
3	161.08	55.40	0.001	-242.15	-200.69	-139.58
4	176.99	23.57	0.167*	-255.98	-205.42	-130.84
5	186.44	4.68	0.861	-256.87*	-197.29	-109.36
6	188.78	-	-	-243.55	-175.04	-73.87

NOTES: The log-likelihood is maximized by a Kalman-filtering approach and the CMAES algorithm (Andreasen, 2010). The LR tests are computed by comparing the log-likelihoods obtained with $k = 2, .., 5 =: k^{\max} - 1$ lags with the log-likelihood obtained with $k^{\max} = 6$. Asterisks denote the optimal lag selection according to the test/information criterion.

TABLE 2. Estimated structural parameters of the model in Eq.s (26)-(29).

Estimation sample: 1984q2 - 2008q3

Parameters	Interpretation	RE	ExC
δ	IS: inter. elast. of substitution	0.010(0.057)	0.079(0.055)
γ	IS: forward looking term	0.572(0.062)	0.269(0.207)
α	NKPC: indexation past inflation	0.035(0.230)	0.035(0.039)
κ	NKPC: slope	0.041(0.121)	0.0267(0.043)
ρ	Policy rule: smoothing term	0.908(0.054)	0.889(0.034)
$\varphi_{\tilde{o}}$	Policy rule: reaction to output gap	0.336(0.963)	1.500(0.248)
φ_{π}	Policy rule: reaction to inflation	1.650(0.974)	1.650(0.803)
$\rho_{\tilde{o}}$	IS: shock persistence	0.908(0.034)	0.801(0.190)
ρ_{π}	NKPC: shock persistence	0.100(0.342)	0.775(0.082)
ρ_R	Policy rule: shock persistence	0.539(0.080)	0.192(0.157)
$\sigma^2_{\tilde{o}}$	IS: variance of shock	0.001(0.001)	0.006(0.002)
σ^2_{π}	NKPC: variance of shock	0.025(0.003)	0.053(0.010)
σ^2_R	Policy rule: variance of shock	0.011(0.002)	0.006(0.001)
$\sigma^2_{o^p}$	Variance of potential output	0.045(0.009)	0.031(0.006)
$\zeta_{\tilde{o},2}$	IS: ExC (Υ_2)	-	-0.061(0.191)
$\zeta_{\pi,2}$	NKPC: ExC (Υ_2)	-	-0.444(0.176)
$\zeta_{R,2}$	Policy rule: ExC (Υ_2)	-	0.057(0.061)
$\zeta_{\tilde{o},3}$	IS: ExC (Υ_3)	-	0.047(0.016)
$\zeta_{\pi,3}$	NKPC: ExC (Υ_3)	-	0.065(0.131)
$\zeta_{R,3}$	Policy rule: ExC (Υ_3)	-	-0.192(0.058)
Likelihood		115.09	129.79
Akaike		-202.18	-219.57*
Hannan-Quinn		-187.54	-198.66*
Schwarz		-165.99	-167.88*

LR(RE vs ExC)=29.40 [0.00] ; LR(CER model with RE)=72.66 [0.00] ; LR(CER model with ExC)=94.40 [0.00]

NOTES: The log-likelihood is maximized by a Kalman-filtering approach and the CMAES algorithm (Andreasen, 2010), using the bounds: [0.010-0.200] for δ; [0.100-0.999] for γ; [0.035-0.100] for α; [0.025-∞] for κ; [0.001-0.999] for ρ; [0.001-1.500] for $\varphi_{\tilde{o}}$; [1.650-5.500] for φ_{π}; [0.001-0.999] for $\rho_{\tilde{o}}$, ρ_{π} and ρ_R, leaving all remaining parameters, including ζ:=($\zeta_{\tilde{o},2}, \zeta_{\pi,2}, \zeta_{R,2}, \zeta_{\tilde{o},3}, \zeta_{\pi,3}, \zeta_{R,3}$)', free on condition that model's determinacy is met (see footnote 8). Standard errors in parentheses have been calculated by adapting the 'hessian.m' function available in Matlab. P-values in brackets. Asterisks denote the selected models..

TABLE 3. Bayesian approach, prior distributions used for the structural parameters of the model in Eq.s (26)-(29).

Parameter	Interpretation	Density	Mode	Standard Deviation
δ	IS: inter. elast. of substitution	*Inverse Gamma*	0.06	0.04
γ	IS: forward looking term	*Beta*	0.25	0.20
α	NKPC: indexation past inflation	*Beta*	0.75	0.20
κ	NKPC: slope	*Gamma*	0.05	0.01
ρ	Policy rule: smoothing term	*Beta*	0.75	0.20
$\varphi_{\tilde{o}}$	Policy rule: reaction to output gap	*Gamma*	0.15	0.25
φ_{π}	Policy rule: reaction to inflation	*Gamma*	1.00	0.50
$\rho_{\tilde{o}}$	IS: shock persistence	*Beta*	0.25	0.20
ρ_{π}	NKPC: shock persistence	*Beta*	0.25	0.20
ρ_R	Policy rule: shock persistence	*Beta*	0.25	0.20
$\sigma^2_{\tilde{o}}$	IS: variance of shock	*Inverse Gamma*	0.25	0.25
σ^2_{π}	NKPC: variance of shock	*Inverse Gamma*	0.50	0.50
σ^2_R	Policy rule: variance of shock	*Inverse Gamma*	0.25	0.25
σ^2_{τ}	Variance of potential output	*Inverse Gamma*	0.25	0.25
$\zeta_{i,j}$	Auxiliary, $i = \tilde{o}, \pi, R$; $j = 2, 3$	*Normal*	0	0.25

NOTES: The prior distributions for the truly structural parameters, θ, are taken from Table 1 in Benati and Surico (2009). The parameter δ corresponds to σ^{-1} in Benati and Surico (2009), hence we use an Inverse-Gamma distribution in place of a Gamma.

TABLE 4. Bayesian approach, estimated structural parameters of the model in Eq.s (26)-(29).

Estimation sample: 1984q2 - 2008q3			
Parameters	Interpretation	Posterior RE Mean [5%, 95%]	Posterior ExC Mean [5%, 95%]
δ	IS: inter. elast. of substitution	0.183[0.156,0.199]	0.185[0.158,0.199]
γ	IS: forward looking term	0.136[0.102,0.200]	0.829[0.649,0.951]
α	NKPC: indexation past inflation	0.056[0.036,0.088]	0.062[0.037,0.093]
κ	NKPC: slope	0.053[0.038,0.072]	0.053[0.038.0.071]
ρ	Policy rule: smoothing term	0.783[0.682,0.870]	0.733[0.584,0.902]
$\varphi_{\tilde{o}}$	Policy rule: reaction to output gap	0.449[0.116,0.805]	1.054[0.167,1.478]
φ_{π}	Policy rule: reaction to inflation	2.107[1.682,3.043]	1.801[1.658,2.120]
$\rho_{\tilde{o}}$	IS: shock persistence	0.529[0.341,0.714]	0.845[0.681,0.952]
ρ_{π}	NKPC: shock persistence	0.484[0.175,0.771]	0.783[0.637,0.903]
ρ_{R}	Policy rule: shock persistence	0.470[0.204,0.717]	0.641[0.233,0.916]
$\sigma^2_{\tilde{o}}$	IS: variance of shock	0.041[0.030,0.054]	0.047[0.034,0.063]
σ^2_{π}	NKPC: variance of shock	0.232[0.193,0.278]	0.249[0.204,0.301]
σ^2_{R}	Policy rule: variance of shock	0.112[0.092,0.135]	0.115[0.094,0.139]
$\sigma^2_{o^p}$	Variance of potential output	0.048[0.035,0.063]	0.054[0.039,0.073]
$\zeta_{\tilde{o},2}$	IS: ExC (Υ_2)	-	-0.082[-0.364,0.209]
$\zeta_{\pi,2}$	NKPC: ExC (Υ_2)	-	-0.638[-0.913,-0.370]
$\zeta_{R,2}$	Policy rule: ExC (Υ_2)	-	0.125[-0.176,0.439]
$\zeta_{\tilde{o},3}$	IS: ExC (Υ_3)	-	0.011[-0.249,0.263]
$\zeta_{\pi,3}$	NKPC: ExC (Υ_3)	-	-0.247[-0.538,0.055]
$\zeta_{R,3}$	Policy rule: ExC (Υ_3)	-	-0.053[-0.307,0.213]
	DIC	42.41	3.26*

NOTES: 'RE' means that the model in Eq.s (26)-(29) is estimated in the 'conventional' way under rational expectations; 'ExC' means that the estimated model is the pseudo-structural form in Eq.s (34)-(35). The prior distributions are reported in Table 3. Posterior distributions are computed using the Random Walk Metropolis algorithm. 'Mean [5%, 95%]' denotes a 90% credible set. The posteriors satisfy the standard convergence criteria and the acceptance ratio is 22.94% for the model estimated under RE and 36.79 for the model estimated under ExC. DIC is the Deviance Information Criterion.