Leader-Following Coordination of Nonlinear Agents under Time-varying Communication Topologies

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Leader-Following Coordination of Nonlinear Agents under Time-varying Communication Topologies

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***Abstract*— The paper deals with the consensus problem between nonlinear agents exchanging information through a time-varying communication network in a "leader-follower" configuration. Under a minimum-phase assumption on the follower dynamics, conditions under which the outputs of the followers track the output of the leader are presented in presence of not necessarily connected communication topologies. The theory of output regulation for nonlinear systems is adopted in order to design decentralised controllers embedding an internal model of the leader dynamics securing robust consensus between the agents.**

I. Introduction

The problem of achieving consensus (among states and/or outputs) in a (homogeneous or heterogenous) network of systems has attracted a major attention in the past fifteen years. This area of research is now pretty well established and a rather complete coverage of the original literature, which begins with a series of seminal contributions such as those of [1], [2], [6], [7], [8], [9] can be found, e.g., in the excellent dissertation [10]. In what follows, we limit ourselves to quote more recent contributions, in particular those that are closer to and/or have substantially influenced our own approach. The case of a network of linear systems connected through a time-invariant graph has been fully addressed in the papers [11], [12], [13], [14], [15], [16]. In particular, [16] for linear systems and [10] for nonlinear systems, have shown that if the outputs of the agents of a heterogenous network achieve consensus on a nontrivial trajectory, the trajectory in question is necessarily the output of some autonomous (linear or, respectively, nonlinear) system. This is the equivalent, in the context of the consensus problem, of the celebrated internal model principle of control theory. Motivated by this, [15] and [16], have proposed a two-layer control structure for achieving consensus in heterogenous network of linear systems connected through a time-invariant graph. In their approach, a network of identical local reference generators is synchronized and the theory of output regulation is used to guarantee that the outputs of the (non-identical) agents follow the (synchronized) outputs of each local generators. This approach has been recently extended in [18] to nonlinear systems connected through a time-invariant graph. Consensus problems for a heterogeneous network of nonlinear systems have also been successfully addressed in the very recent paper [19]. The approach of [15] has also been extended in [17] to the case of a switched topology.

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The consensus problem in the case of systems connected through time-varying communication graph has been successfully addressed in the milestone paper [7], who fully solves the problem in the case of a network of integrator systems, under very mild connectivity conditions. This approach, though, has not been extended yet to the case of higher-dimensional linear agents, exchanging relative (full-state and/or partial state) information, let alone the case of higher-dimensional nonlinear agents. The problem is marginally easier in the so-called "leader-follower" configuration, where states (or outputs) of the agents are required to asymptotically track the state (or the output) of a single leader. The pattern of communication still consists in exchange of relative information, as in the case of standard consensus problems, with the only difference that the leader receives no information from the followers. This is reflected in the fact that the entries of one row of the so-called adjacency matrix of the graph (the one whose index is the index which identifies the leader) are all zero. The problem of consensus in such special communication setup has been successfully addressed in [20] and in [21] for linear systems exchanging relative full-state information.

The purpose of this paper is to extend, to the case of nonlinear systems exchanging relative output information, part of the results of [20] and [21]. This will be done appealing, as in [18], to some relevant results concerning the theory of output regulation of nonlinear systems and taking advantage of some interesting consequences of the approach of [21].

Notations. In the paper $\mathbb{R}$ and $\mathbb{R}_{\geq 0}$ denote respectively the set of real and nonnegative real numbers. The symbol $\mathbb{N}$ denotes the set of nonnegative integers. With $\mathbb{R}^n$ we indicate the n-dimensional Euclidean space. For $\mathcal{C}$ a closed subset of $\mathbb{R}^n$, $|x|_{\mathcal{C}} = \min_{y \in \mathcal{C}} |x - y|$ denotes the distance of x from $\mathcal{C}$. For a continuous function $V : \mathbb{R}^n \to \mathbb{R}$ and a vector $f : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the upper directional derivative of V at (x, t) along $f(x, t)$ is defined as

$$D^+_{f(x,t)} V(x) = \limsup_{h \to 0^+} \frac{1}{h}[V(x + hf(x, t)) - V(x)]$$

In some parts of the paper we will use tools developed in the context of hybrid dynamics systems. In those parts, this work uses the framework and results of [3] from which also the notation is taken.

II. Preliminaries

In what follows we consider N nonlinear agents, all having relative degree r, modeled in normal form as

$$\begin{array}{rcl} \dot{z}_k &=& f_k(z_k,\xi_k) \\ \dot{\xi}_{k1} &=& \xi_{k2} \\ && \cdots \\ \dot{\xi}_{k,r-1} &=& \xi_{kr} \\ \dot{\xi}_{kr} &=& q_k(z_k,\xi_k)+b_k(z_k,\xi_k)u_k \\ y_k &=& \xi_{k1} \end{array} \tag{1}$$

in which $z_k \in \mathbb{R}^{n_k}$, $\xi_k \in \mathbb{R}^r$ denotes the vector $\xi_k = \mathrm{col}(\xi_{k1},\xi_{k2},\dots,\xi_{kr})$ and $u_k \in \mathbb{R}$. Note that the assumption that all agents have the same relative degree is not restrictive, since it is always possible to achieve such property by adding a suitable number of integrators on the input channel of each agent.

The outputs y_k of all such agents are requested to asymptotically track the output y_0 of a (single) leader

$$\begin{array}{rcl} \dot{w} &=& s(w) \\ y_0 &=& h_0(w) \end{array} \tag{2}$$

in which $w \in W$, with W a compact set, invariant for the dynamics of (2).

The (decentralized) control structure consists of a set of k local feedback controllers of the form

$$\begin{array}{rcl} \dot{\varsigma}_k &=& \varphi_k(\varsigma_k,\nu_k) \\ u_k &=& \varrho_k(\varsigma_k,\nu_k) \end{array} \tag{3}$$

exchanging information through a time-varying communication graph. Specifically, the input ν_k of each of such controllers, which represents exchange of relative information between the leader and the individual agents, is assumed to be the form

$$\nu_k(t) = a_{k0}(t)(\vartheta_0(t)-\vartheta_k(t)) + \sum_{\substack{j=1\\ j\neq k}}^{N} a_{kj}(t)\,(\vartheta_j(t)-\vartheta_k(t)) \tag{4}$$

$k = 1,\dots,N$, in which ϑ_0 and the ϑ_j's, $j = 1,\dots,N$ represent information taken at the leader and, respectively, at each agent, while the $a_{kj}(t)$ are positive functions modelling the weight of the communication link between the k-th and the i-th agents. In the simple case in which $r = 1$,

$$\vartheta_i = y_i \qquad \text{for all } i = 0,1,\dots,N.$$

All functions/maps considered in these models are assumed to be smooth. It is also assumed that, for some fixed pair of real numbers $0 < \underline{b} \le \bar{b}$ the so-called "high-frequency gain" coefficient $b_k(z_k,\xi_k)$ of the k-th agent satisfies

$$0 < \underline{b} \;\le\; b_k(z_k,\xi_k) \;\le\; \bar{b}\,. \tag{5}$$

The basic assumption on each of the agents (1) is that of being *strongly minimum phase*, formally specified as follows. Define

$$\xi_{\rm ss}(w) = \begin{pmatrix} h_0(w) \\ L_s h_0(w) \\ \cdots \\ L_s^{r-1} h_0(w) \end{pmatrix}$$

and observe that, if perfect tracking is achieved,

$$\xi_k(t) = \xi_{\rm ss}(w(t))\,.$$

Assumption 1. *There exists a smooth map $\pi_k : W \to \mathbb{R}^{n_k}$ satisfying*

$$L_s\pi_k(w) = f_k(\pi_k(w),\xi_{\rm ss}(w)) \qquad \forall w \in W\,,$$

and the system

$$\begin{array}{rcl} \dot{w} &=& s(w) \\ \dot{z}_k &=& f_k(z_k,\xi_{\rm ss}(w)+u) \end{array} \tag{6}$$

is input-to-state stable (ISS) to the invariant set [1]

$$\mathcal{A}_k = \{(w,z_k) \in W\times\mathbb{R}^{n_k} : z_k = \pi_k(w)\}\,.$$

with a linear gain function and with an exponential decay rate. In particular, there exists a locally Lipschitz function $V_k : W\times\mathbb{R}^{n_k} \to \mathbb{R}$ such that the following holds:

- *there exist positive $\underline{a}_k$ and $\bar{a}_k$ such that*

$$\underline{a}_k\|(w,z_k)\|_{\mathcal{A}_k} \le V_k(w,z_k) \le \bar{a}_k\|(w,z_k)\|_{\mathcal{A}_k}$$

  for all $(w,z_k,u) \in W\times\mathbb{R}^{n_k}\times\mathbb{R}$;
- *there exist positive c_k and d_k such that for all $(w,z_k,u) \in W\times\mathbb{R}^{n_k}\times\mathbb{R}$*

$$D^+_{\mathrm{col}(s(w),f(w,z_k,u))}V_k(w,z_k) \le -c_kV_k(w,z_k) + d_k|u|$$

Remark. The existence of ISS Lyapunov function with the properties detailed in the previous assumption implies that for all $(w(0),z(0)) \in W\times\mathbb{R}^{n_k}$ and all bounded $u(t)$, the resulting trajectory $(w(t),z(t))$ of (6) satisfies

$$\|(w(t),z_k(t))\|_{\mathcal{A}_k} \le \max\{\lambda_k e^{-c_kt}\|(w(0),z_k(0))\|_{\mathcal{A}_k}\,,\; g_k^\circ\|u(\cdot)\|_\infty\} \tag{7}$$

with $\lambda_k = 2\bar{a}_k/\underline{a}_k$ and $g_k^\circ = 2d_k\int_0^\infty e^{-c(t-s)}ds/\underline{a}_k$, for all $t \ge 0$. ◁

Remark. The assumption that the gain function of (6) is linear could be weakened, requiring only linearity in a neighborhood of the origin, in which case a bound similar to the bound (7) would hold, so long as it can be guaranteed that – for some compact set U – the input function $u(\cdot)$ of (6) satisfies $|u(t)| \le U$ for all $t \ge 0$, with g_k° a parameter depending on the set U. This would entail weaker convergence results, as remarked later in the paper. ◁

[1] See [22] for an introduction to the concept of input-to-state stability.

III. Standard results on asymptotic tracking

A. Reduction to relative degree 1

It is well known that - if $r > 1$ - the output of system (1) can be redefined, so as to lower the relative degree down to 1 while keeping the property of being strongly minimum phase. This is achieved by picking as "new output" the function

$$\vartheta_k = \xi_{kr} + \sum_{j=1}^{r-1} c_j \xi_{kj} \tag{8}$$

in which the c_j's are such that the polynomial $p(\lambda) = \lambda^{r-1} + c_{r-1}\lambda^{r-2} + \cdots + c_2\lambda + c_0$ is Hurwitz. A trivial calculation shows that the dynamics of (1), with ξ_{kr} replaced by ϑ_k, can be seen as a system in normal form having relative degree 1 between input u and output ϑ_k

$$\begin{array}{rcl}\dot{z}_k &=& f_k(z_k, \ell(\xi_{k1}, \ldots, \xi_{k,r-1}, \vartheta_k)) \\ \dot{\xi}_{k1} &=& \xi_{k2} \\ && \cdots \\ \dot{\xi}_{k,r-1} &=& -\sum_{j=1}^{r-1} c_j \xi_{kj} + \vartheta_k \\ \dot{\vartheta}_k &=& q_k(z_k, \ell(\xi_{k1}, \ldots, \xi_{k,r-1}, \vartheta_k)) \\ && + \sum_{j=1}^{r-2} c_j \xi_{k,j+1} + c_{r-1}[-\sum_{j=1}^{r-1} c_j \xi_{kj} + \vartheta_k] \\ && + b_k(z_k, \ell(\xi_{k1}, \ldots, \xi_{k,r-1}, \vartheta_k))u_k\end{array}$$

in which

$$\ell(\xi_{k1}, \ldots, \xi_{k,r-1}, \vartheta_k) = \mathrm{col}(\xi_{k1}, \ldots, \xi_{k,r-1}, -\sum_{j=1}^{r-1} c_j \xi_{kj} + \vartheta_k).$$

Having set

$$\vartheta_{\mathrm{ss}}(w) = L_s^{r-1} h_0(w) + \sum_{j=1}^{r-1} c_j L_s^{j-1} h_0(w),$$

it is readily seen, as a standard consequence of the property that the cascade of two ISS systems is an ISS system, that the system

$$\begin{array}{rcl}\dot{w} &=& s(w) \\ \dot{z}_k &=& f_k(z_k, \ell(\xi_{k1}, \ldots, \xi_{k,r-1}, \vartheta_{\mathrm{ss}}(w) + u)) \\ \dot{\xi}_{k1} &=& \xi_{k2} \\ && \cdots \\ \dot{\xi}_{k,r-1} &=& -\sum_{j=1}^{r-1} c_j \xi_{kj} + \vartheta_{\mathrm{ss}}(w) + u\end{array} \tag{9}$$

is ISS to the invariant set

$$\begin{array}{l}\mathcal{A}'_k = \{(w, z_k, \xi_{i1}, \ldots, \xi_{i,r-1}) : w \in W, \\ \quad z_k = \pi_k(w), \xi_{i1} = h_0(w), \ldots, \xi_{i,r-1} = L_s^{r-2} h_0(w)\}.\end{array}$$

Hence, if (1) is strongly minimum phase (in the sense of Assumption 1), so is the system obtained after the replacement of the original output y_k by means of the re-defined output ϑ_k. Note also that, since system (6) is ISS, to the set $\mathcal{A}_k$, with a linear gain function, then (9) is ISS, to the set $\mathcal{A}'_k$, with a linear gain function.

In view of this, from now on we restrict our analysis to the case in which all agents have relative degree 1. In this respect, it should also be observed that $\vartheta_k(t)$ is a linear combination of higher derivatives of y_k, i.e.

$$\vartheta_k(t) = y_k^{r-1}(t) + \sum_{j=1}^{r-1} c_j y_k^{j-1}(t).$$

Classical results (see [24]) can be used to prove that a "partial state information" such as ϑ_k can be replaced – with appropriate precautions – to the purpose of establishing the desired tracking results, by a "rough" approximation provided by a high-gain observer, driven by the actual output y_k. We will return on this issue at the end of the paper.

B. The standard internal model for each agent

Consider now the case of agents having relative degree 1. To simplify matters, we also assume that the "high-frequency gain" coefficient is *independent* of the state variables. In other words, we consider the case of agents modeled by equations of the form

$$\begin{array}{rcl}\dot{z}_k &=& f_k(z_k, y_k) \\ \dot{y}_k &=& q_k(z_k, y_k) + b_k u_k\end{array} \tag{10}$$

in which b_k is a (possibly unknown) positive number.

Define $\psi_k : W \to \mathbb{R}$ via

$$L_s h_0(w) = q_k(\pi_k(w), h_0(w)) + b_k \psi_k(w).$$

Based on the results of [23] it is known that there exist an integer m_k, a Hurwitz matrix $F \in \mathbb{R}^{m_k \times m_k}$, a vector $G \in \mathbb{R}^{m_k \times 1}$ such that the pair F, G is controllable, a function $\gamma_k : \mathbb{R}^{m_k} \to \mathbb{R}$ and a map $\sigma_k : W \to \mathbb{R}^{m_k}$, satisfying

$$\begin{array}{rcl}L_s\sigma_k(w) &=& F\sigma_k(w) + G\gamma_k(\sigma_k(w)) \\ \psi_k(w) &=& \gamma_k(\sigma_k(w))\end{array} \qquad \forall w \in W.$$

This makes it possible to design, for the k-th agent, an "internal model" of the form[2]

$$\begin{array}{rcl}\dot{\eta}_k &=& F\eta_k + G\gamma_k(\eta_k) + G_k v_k \\ u_k &=& \gamma_k(\eta_k) + v_k.\end{array} \tag{11}$$

Note that the function $\gamma_k(\cdot)$ is only known to be continuous. However, in what follows, for convenience it will be *assumed* that the function in question is locally Lipschitz.

Define a *tracking error* at the k-th agent as

$$e_k = y_k - y_0.$$

The composition of (10) and (11), viewed as a system with input v_k and output e_k having relative degree 1, can be put in normal form by changing η_k into

$$\zeta_k = \eta_k - \frac{1}{b_k} G e_k.$$

[2] It follows from the results of [23], since the number of agents is finite, it is possible to pick a single pair F, G for all agents, as the notation suggests.

The normal form in question is

$$\begin{aligned}\dot z_k &= f_k(z_k, h_0(w)+e_k)\\ \dot\zeta_k &= F\zeta_k + G\gamma_k(\sigma_k(w)) - \frac{1}{b_k}[q_k(z_k,h_0(w)+e_k)\\ &\quad -q_k(\pi_k(w),h_0(w)) - FGe_k]\\ \dot e_k &= q_k(z_k,h_0(w)+e_k) - q_k(\pi_k(w),h_0(w))\\ &\quad +b_k[\gamma_k(\zeta_k+\frac{1}{b_k}Ge_k) - \gamma_k(\sigma_k(w)] + b_k v_k\,.\end{aligned} \tag{12}$$

Having assumed that the agent is strongly minimum-phase, and bearing in mind the fact that F is a Hurwitz matrix, it is easy to check (using again the property that the cascade of two ISS systems is an ISS system) the system

$$\begin{aligned}\dot w &= s(w)\\ \dot z_k &= f_k(z_k, h_0(w)+e_k)\\ \dot\zeta_k &= F\zeta_k + G\gamma_k(\sigma_k(w)) - \frac{1}{b_k}[q_k(z_k,h_0(w)+e_k)\\ &\quad -q_k(\pi_k(w),h_0(w)) - FGe_k]\end{aligned} \tag{13}$$

viewed as a system with input e_k, is input-to-state stable to the invariant set

$$\mathcal{A}^{\rm a}_k = \{(w,z_k,\zeta_k) : w\in W, z_k = \pi_k(w_k), \zeta_k = \sigma_k(w_k)\}\,.$$

If, in addition, it is assumed that the function $[q_k(z_k,h_0(w)+e_k) - q_k(\pi_k(w),h_0(w))]$, which vanishes if $\|(w,z_k)\|_{\mathcal{A}_k} = 0$ and $e_k = 0$, satisfies a bound of the form

$$\begin{aligned}|q_k(z_k,h_0(w)+e_k) - q_k(\pi_k(w),h_0(w))| \le\\ c_z\|(w,z_k)\|_{\mathcal{A}_k} + c_e|e_k|\end{aligned} \tag{14}$$

for some pair (c_z, c_e) of positive numbers independent of ω, it can be concluded that (13) is ISS to the invariant set $\mathcal{A}^{\rm a}_k$ with a linear gain function and and exponential decay rate. In particular, there exists a locally Lipschitz function $V^{\rm a}_k : W\times\mathbb{R}^{n_k}\times\mathbb{R}^{m_k}\to\mathbb{R}$ such that the following holds:

- there exist positive $\underline{a}^{\rm a}_k$ and $\bar a^{\rm a}_k$ such that

  $$\underline{a}^{\rm a}_k\|(w,z_k,\zeta_k)\|_{\mathcal{A}^{\rm a}_k} \le V^{\rm a}_k(w,z_z,\zeta_k) \le \bar a^{\rm a}_k\|(w,z_z,\zeta_k)\|_{\mathcal{A}^{\rm a}_k}$$

  for all $(w,z_z,\zeta_k)\in W\times\mathbb{R}^{n_k}\times\mathbb{R}^{m_k}$;
- there exists positive $c^{\rm a}_k$ and $d^{\rm a}_k$ such that for all $(w,z_z,\zeta_k)\in W\times\mathbb{R}^{n_k}\times\mathbb{R}^{m_k}$

  $$D^+_{(13)}V^{\rm a}_k(w,z_z,\zeta_k) \le -c^{\rm a}_k V^{\rm a}_k(w,z_z,\zeta_k) + d^{\rm a}_k|e_k|\,.$$

Finally, note the the "coupling term" in the last equation of (12), namely

$$\begin{aligned}q_k(z_k,h_0(w)+e_k) - q_k(\pi_k(w),h_0(w))\\ +b_k[\gamma_k(\zeta_k+\frac{1}{b_k}Ge_k) - \gamma_k(\sigma_k(w)]\end{aligned}$$

vanishes if $\|(w,z_k,\zeta_k)\|_{\mathcal{A}^{\rm a}_k} = 0$ and $e_k = 0$. Thus, if it were possible to pick $v_k = -\gamma e_k$, the problem of steering e_k to zero would be trivially solved by taking a large enough γ, as the theory of output regulation predicts (see e.g. [23]).

This mode of control may not be the feasible, though, because the k-th agent may not have access to the k-th tracking error. Thus the whole structure of exchange of information must be taken into account.

C. The overall control structure

As seen in the previous sub-section, the k-th control loop can be seen as a SISO system having relative degree 1, modeled by equations of the form

$$\begin{aligned}\dot z^{\rm a}_k &= f^{\rm a}_k(z^{\rm a}_k, e_k, w)\\ \dot e_k &= q^{\rm a}_k(z^{\rm a}_k, e_k, w) + b_k v_k\,.\end{aligned}$$

Stacking all such systems together, we obtain a system with N inputs and N output modeled by equations of the form

$$\begin{aligned}\dot z &= f(z,e,w)\\ \dot e &= q(z,e,w) + Bv\end{aligned} \tag{15}$$

in which

$$\begin{aligned}z &= {\rm col}(z^{\rm a}_1, z^{\rm a}_2, \ldots, z^{\rm a}_N)\\ e &= {\rm col}(e_1,e_2,\ldots,e_N)\\ v &= {\rm col}(v_1,v_2,\ldots,v_N)\\ f(z,e,w) &= {\rm col}(f^{\rm a}_1(z^{\rm a}_1,e_1,w),\ldots,f^{\rm a}_N(z^{\rm a}_N,e_N,w))\\ q(z,e,w) &= {\rm col}(q^{\rm a}_1(z^{\rm a}_1,e_1,w),\ldots,q^{\rm a}_N(z^{\rm a}_N,e_N,w))\\ B &= {\rm diag}(b_1,b_2,\ldots,b_N)\end{aligned}$$

Note that, in view of the whole construction, if – for all $k = 1,\ldots,N$ – Assumption 1 and the bound (14) hold and the function $\gamma_k(\cdot)$ is locally Lipschitz, then:

(i) the system

$$\begin{aligned}\dot w &= s(w)\\ \dot z &= f(z,e,w)\,,\end{aligned} \tag{16}$$

viewed as a system with input e, is input-to-state stable, with a linear gain function and exponential decay rate, to a compact invariant set $\mathcal{A}^*$. In particular, there exist λ_z, c_z, $g_z > 0$ such that,

$$\begin{aligned}&\|(w(t),z(t))\|_{\mathcal{A}^*}\\ &\quad\le \max\{\lambda_z e^{-c_z t}\|(w(0),z(0))\|_{\mathcal{A}^*}\,,\ g_z\|e(\cdot)\|_\infty\}\end{aligned} \tag{17}$$

for all $t\ge 0$;

(ii) there exists a pair (K_1,K_2) of positive numbers such that

$$|q(z,e,w)| \le K_1|e| + K_2\|(w,z)\|_{\mathcal{A}^*} \tag{18}$$

for all w, z, e.

Item (i) above, in particular, implies the existence of a locally Lipschitz function $V_z : W\times\mathbb{R}^n\to\mathbb{R}$, $n = \sum_{k=1}^N(n_k+m_k)$, such that the following holds:

- there exist positive $\underline{a}$ and $\bar a$ such that

  $$\underline{a}\|(w,z)\|_{\mathcal{A}^*} \le V_z(w,z) \le \bar a\|(w,z)\|_{\mathcal{A}^*}$$

  for all $(w,z)\in W\times\mathbb{R}^n$;
- there exists positive c and d such that for all $(w,z)\in W\times\mathbb{R}^n$

  $$D^+_{{\rm col}(s(w),f(z,e,w))}V_z(w,z) \le -cV_z(w,z) + d|e|\,.$$

IV. THE COMMUNICATION PROTOCOL

A. The setup

We assume the reader is familiar with the major results about consensus of networked systems exchanging information over communication graphs and, therefore, we refrain from repeating well established definitions concerning graphs. As anticipated in section II, the exchange of information between leader and followers has the expression (4), which in the present context (agents having relative degree 1) takes the form

$$\nu_k(t) = a_{k0}(t)(y_0(t) - y_k(t)) + \sum_{\substack{j=1 \\ j\neq k}}^{N} a_{kj}(t)\,(y_j(t) - y_k(t))\,, \tag{19}$$

for $k = 1, \dots, N$, where $a_{kj}(t)$ is the element on the k-th row and j-th column of the so-called *adjacency matrix* $A(t)$ of the underlying communication *digraph*. All $a_{kj}(t)$'s are piecewise-continuous and bounded functions of time, $a_{kj}(t) \geq 0$ and $a_{kk}(t) = 0$, for all $t \in \mathbb{R}$. Note that, in this specific case of a leader-followers configuration, $a_{0j}(t) \equiv 0$ for all $j = 1, \dots, N$.

Recalling the definition of tracking errors, the information ν_k can be expressed as

$$\nu_k(t) = \sum_{\substack{j=1 \\ j\neq k}}^{N} a_{kj}(t)e_j - [a_{k0}(t) + \sum_{\substack{j=1 \\ j\neq k}}^{N} a_{kj}(t)]e_k\,,$$

and, in compact form, as

$$\nu(t) = M(t)e(t) \tag{20}$$

in which

$$\nu = \mathrm{col}(\nu_1, \nu_2, \dots, \nu_N)$$

and $M(t) \in \mathbb{R}^{N\times N}$ is a matrix defined as

$$\begin{aligned} m_{kj}(t) &= a_{kj}(t) \qquad \text{for } k \neq j \\ m_{kk}(t) &= -\sum_{\substack{i=0 \\ j\neq k}}^{N} a_{kj}(t)\,. \end{aligned} \tag{21}$$

Remark. Note that the off-diagonal elements of $M(t)$ are non-negative, and, for each $k = 1, \dots, N$, the sum of all elements of the k-th row is equal to $-a_{k0}(t)$. As a matter of fact, the negative of $M(t)$ coincides with the lower-right $N \times N$ block of the so-called *Laplacian matrix* $L(t)$ of the graph induced by the matrix $A(t)$. ◁

The purpose of this paper is to show that the target of asymptotic tracking can be achieved by means of a control law of the form

$$v(t) = \gamma\nu(t)\,,$$

in which $\gamma > 0$ is a gain parameter. This choice, in view of (20), yields an overall controlled network which, augmented with the dynamics of the leader, reads as

$$\begin{aligned} \dot{w} &= s(w) \\ \dot{z} &= f(z, e, w) \\ \dot{e} &= q(z, e, w) + \gamma BM(t)e\,. \end{aligned} \tag{22}$$

Of course, the possibility of achieving this goal depends on the connectivity properties of the communication graph, which are reflected in properties of the matrix $M(t)$ which, in turn, influences the asymptotic properties of the time-varying linear system

$$\dot{e} = BM(t)e\,. \tag{23}$$

B. A digression on a Theorem of Moreau

In order to analyze the asymptotic properties of system (23), it is convenient to recall a fundamental result of L. Moreau, who has determined connectivity assumptions under which the state $x \in \mathbb{R}^{N+1}$ of a network of $N+1$ first-order agents

$$\dot{x}_k = u_k \qquad k = 0, \dots, N \tag{24}$$

controlled by

$$u_k = \sum_{\substack{j=0 \\ j\neq k}}^{N} a_{kj}(t)(x_j - x_k) \tag{25}$$

asymptotically converges to the equilibrium subspace $\mathcal{A} = \{x \in \mathbb{R}^{N+1} : x_0 = x_1 = \ldots = x_N\}$.

In the present context of a leader-followers configuration, $u_0 = 0$ and hence

$$\dot{x}_0 = 0\,.$$

Thus, without loss of generality, one can assume $x_0(t) = 0$ for all $t \in \mathbb{R}$ and describe the network in equivalent form in terms of the relative differences $e_k = x_k - x_0$ as

$$\dot{e}_k = \sum_{j=1}^{N} m_{kj}(t)e_j \qquad k = 1, \dots, N, \tag{26}$$

in which the $m_{kj}(t)$ are the coefficients defined in (21).

The connectivity property, determined in [6], under which the convergence of (24) – (25) to the equilibrium subspace takes place, can be described – in the present context of a leader-followers configuration – as follows.

Definition. The digraph associated with the adjacency matrix $A(t)$ is *uniformly connected* if there is a threshold value θ and an interval length $T > 0$ such that, for all $t \in \mathbb{R}$, in the θ-digraph [3] associated with the adjacency matrix

$$\int_t^{t+T} A(s)ds$$

all nodes may be reached from node 0. ◁

Theorem 1 of [6] proves that, if the digraph associated with the adjacency matrix $A(t)$ is uniformly connected, the equilibrium $e = 0$ of (26) is exponentially stable. Such result can be easily used also to determine the asymptotic properties

[3]The θ-digraph associated to an adjacency matrix $A_0(t)$ is a digraph with an arc from j to k ($k \neq j$) if and only if the element (k, j) of $A_0(t)$ is strictly larger than θ for all $t \in \mathbb{R}$.

of system (23). In fact, it suffices to observe that the k-th row of system (23) reads as

$$\dot{e}_k = b_k \sum_{j=1}^{N} m_{kj}(t) e_j \,,$$

and hence system (23) can be interpreted as a system of the form (26) corresponding to an adjacency matrix $\tilde{A}(t)$ in which $\tilde{a}_{kj}(t) = b_k a_{kj}(t)$. Since b_k is bounded as in (5), it is readily seen that if the digraph associated with the adjacency matrix $A(t)$ is uniformly connected so is the digraph associated with the adjacency matrix $\tilde{A}(t)$. Thus, as an immediate Corollary of Theorem 1 of [6], it is observed that if the digraph associated with the adjacency matrix $A(t)$ is uniformly connected, the equilibrium $e = 0$ of (23) is exponentially stable.

Theorem 1 of [6] is proven by showing the existence of a (time-independent) positive definite function of e which asymptotically decreases along trajectories. The function in question, in the present context of a leader-followers configuration and hence of a system described as in (26), is the function

$$V(e) = \max\{e_1, \ldots, e_N, 0\} - \min\{e_1, \ldots, e_N, 0\} \,. \quad (27)$$

This function is continuous but not continuously differentiable. However, it can be seen that this function can be bounded as

$$\underline{a}_e |e| \le V(e) \le \bar{a}_e |e| \qquad \forall e \in \mathbb{R}^N \,, \quad (28)$$

from which it is also seen that $V(e)$ is globally Lipschitz.

The proof of Theorem 1 of [6] shows that, if the digraph associated with the adjacency matrix $A(t)$ is uniformly connected, along any trajectory $e(t)$ of (26):

(i) the function $V(e(t))$ is non-increasing,

(ii) for some class $\mathcal{K}_\infty$ function $\gamma(\cdot)$

$$V(e(t_0 + NT)) - V(e(t_0)) \le -\gamma(|e(t_0)|) \quad (29)$$

for any t_0 (where the number T is the parameter appearing in the definition of uniform connectivity).

C. Asymptotic Coordination

Motivated by the result of [6], we assume in what follows that the adjacency matrix $A(t)$ which characterizes the communication between agents is such that the following assumption holds.

Assumption 2. *There exists a globally Lipschitz function $V_e : \mathbb{R}^N \to \mathbb{R}$, bounded as in*

$$\underline{a}_e |e| \le V_e(e) \le \bar{a}_e |e| \qquad \forall\, e \in \mathbb{R}^N$$

for some positive $\underline{a}_e$, $\bar{a}_e$, such that

$$D^+_{BM(t)e} V_e(e) \le 0 \qquad \forall\, (e,t) \in \mathbb{R}^N \times \mathbb{R}_{\ge 0} \,. \quad (30)$$

Moreover, there exist a time T_0, a number $a > 0$ and a countable sequence of closed intervals $\{I_k\}_{k\in\mathbb{N}} \subset \mathbb{R}_{\ge 0}$ of the form $I_k = [t_{k,1}, t_{k,2}]$, with $t_{k,1} \le t_{k,2} \le t_{k+1,1}$ and $t_{k+1,1} - t_{k,2} \le T_0$, such that

$$D^+_{BM(t)e} V_e(e) \le -a V_e(e) \quad \forall\, (e,t) \in \mathbb{R}^N \times I_k \,. \quad (31)$$

As a matter of fact, using the results of [6], it is possible to check the following result.

Proposition 1. *Suppose that the digraph associated with the adjacency matrix $A(t)$ is uniformly connected. Then, Assumption 2 holds.*

Proof. As observed above, the digraph associated with the adjacency matrix $BA(t)$ is uniformly connected. Therefore, along the trajectories of (23), the function $V(e)$ defined in (27) is non-increasing and property (29) holds. The fact that $V(e)$ is non-increasing implies (30). From the inequality (29), it is easy to deduce the existence of a closed interval $I_{t_0} \subset [t_0, t_0 + NT]$ of positive measure such that

$$D^+_{BM(t)e} V(e) \le -\frac{1}{2NT}\gamma(|e(t_0)|) \qquad \forall t \in I_{t_0} .$$

This inequality, in turn, using the estimate (28) for $V(e)$ and the property that $V(e(t))$ is non-increasing, can be further elaborated as

$$D^+_{BM(t)e} V(e) \le -\frac{1}{2NT}\gamma(\frac{V(e(t_0))}{\bar{a}_e}) \le -\frac{1}{2NT}\gamma(\frac{V(e(t))}{\bar{a}_e}) \,.$$

Finally, is it observed that the estimates provided in [6] show that the function $\gamma(\cdot)$ on the left-hand-side of (29) can be bounded as $a_0|e| \le \gamma(|e|)$ for some $a_0 > 0$. As a consequence, it is seen that

$$D^+_{BM(t)e} V(e) \le -\frac{a_0}{2NT\bar{a}_e} V(e) \quad \forall\, (e,t) \in \mathbb{R}^N \times I_{t_0} \,,$$

from which it is seen that also property (31) holds. □

It is seen from this Proposition that Assumption 2 is actually weaker than the assumption of uniform connectivity. As such, Assumption 2 may not be strong enough to guarantee exponential stability of (23), for the simple reason that no lower bound is prescribed on the measure of the intervals I_k. In view of this, it is convenient to strengthen this Assumption by requiring, for instance, that the I_k's (which, we recall, are intervals of the form $[t_{k,1}, t_{k,2}]$) satisfy, for some $n_0 \in \mathbb{N}$, $n_0 \ge 1$, and some $\tau \in \mathbb{R}_{>0}$,

$$\sum_{k=j}^{i-1} (t_{k,2} - t_{k,1}) \ge (i - j - n_0)\tau \,.$$

This inequality essentially expresses the property that, in the average, the intervals I_k have a guaranteed length, so as to secure – in view of (31) – that the solutions of (23) asymptotically decay to zero. The time τ, in particular, can be seen as an average length of the intervals of the intervals I_k, while n_0 represents the number of interval I_k of zero duration that can occur in a row. As a whole, the condition can be regarded as a "average dwell-time" condition (see [4]). If this condition holds for *some* τ and n_0, then the solutions of (23) exponentially decay to zero. Moreover, as it will be shown in a moment, if τ is sufficiently large, then also the solutions of (22) are such that $e(t)$ exponentially decays to zero, provided that the value of γ is large enough.

Proposition Consider system (22) under Assumption 1 and 2. There exist $\gamma^\star > 0$ and $\tau^\star > 0$ such that for all $\gamma \geq \gamma^\star$ and $\tau \geq \tau^\star$ the set $\mathcal{A}^\star \times \{0\}$ is globally asymptotically stable for system (22).

Proof. With $V_z(w,z)$ and $V_e(e)$ the Lyapunov functions introduced respectively at the end of Section III-C and in Assumption 2, let $V_{\rm cl} : W \times \mathbb{R}^n \to \mathbb{R}$ be the candidate Lyapunov function for the closed-loop defined as $V_{\rm cl}(w,z,e) = V_z(w,z) + \beta V_e(e)$ with $\beta > 0$ yet to be chosen. By taking the upper directional derivative of $V_{\rm cl}(\cdot)$ along (22), one obtains

$$D^+_{{\rm col}(\dot w,\dot z,\dot e)} V_{\rm cl}(w,z,e) = D^+_{{\rm col}(\dot w,\dot z)} V_z(w,z) + \beta D^+_{\dot e} V_e(e)$$

We develop separately the two terms. Regarding the derivative of $V_z(\cdot)$ we have

$$\begin{aligned} D^+_{{\rm col}(\dot w,\dot z)} V_z(w,z) &= -cV_z(w,z) + d|e| \\ &\leq -cV_z(w,z) + \frac{d}{\underline{a}_e} V_e(e)\,. \end{aligned}$$

Regarding $V_e(\cdot)$, we have

$$\begin{aligned} & D^+_{\dot e} V_e(e) = \\ & \limsup_{h\to 0^+} \frac{1}{h}[V_e(e+h\dot e) - V_e(e)] \leq \\ & \limsup_{h\to 0^+} \frac{1}{h}[V_e(e+h\dot e) - V_e(e + h\gamma B M(t) e) \\ & + \limsup_{h\to 0^+} \frac{1}{h}[V_e(e + h\gamma B M(t) e) - V_e(e)] \leq \\ & \gamma D^+_{BM(t)e} V_e(e) + \limsup_{h\to 0^+} \frac{1}{h} L|hq(\cdot)| \leq \\ & \gamma D^+_{BM(t)e} V_e(e) + LK_1|e| + LK_2\|(w,z)\|_{\mathcal{A}^\star} \leq \\ & \gamma D^+_{BM(t)e} V_e(e) + L\frac{K_1}{\underline{a}_e} V_e(e,t) + L\frac{K_2}{\underline{a}_z} V_z(w,z) \end{aligned}$$

having denoted by L the Lipschitz constant of $V_e(\cdot)$, namely, by bearing in mind (30) and (31),

$$D^+_{\dot e} V_e(e) \leq -(\gamma a - L\frac{K_1}{\underline{a}_e}) V_e(e) + L\frac{K_2}{\underline{a}_z} V_z(w,z)$$

for all $t \in \{I_k\}$ and for all $(w,z,e) \in W \times \mathbb{R}^n \times \mathbb{R}$, and

$$D^+_{\dot e} V_e(e) \leq L\frac{K_1}{\underline{a}_e} V_e(e) + L\frac{K_2}{\underline{a}_z} V_z(w,z)$$

for all $t \notin \{I_k\}$ and and for all $(w,z,e) \in W \times \mathbb{R}^n \times \mathbb{R}$. Thus, choosing β so that

$$c - \beta L\frac{K_2}{\underline{a}_z} \geq \frac{c}{2}$$

and $\gamma^\star$ so that

$$\beta\gamma^\star a - \beta\frac{LK_1}{a_e} - \frac{d}{\underline{a}_e} \geq \frac{c}{2}\beta$$

we have that, for all $t \in \{I_k\}$, for all $(w,z,e) \in W \times \mathbb{R}^n \times \mathbb{R}$, and for all $\gamma \geq \gamma^\star$,

$$\begin{aligned} & D^+_{{\rm col}(\dot w,\dot z,\dot e)} V_{\rm cl}(w,z,e) \leq \\ & -cV_z(w,z) + \frac{d}{\underline{a}_e} V_e(e) - \beta(\gamma a - \frac{LK_1}{a_e}) V_e(e) + \\ & \qquad \beta\frac{LK_2}{\underline{a}_z} V_z(w,z) \leq \\ & -(c - \beta\frac{LK_2}{\underline{a}_z}) V_z(w,z) - (\beta\gamma a - \beta\frac{LK_1}{\underline{a}_e} - \frac{d}{\underline{a}_e}) V_e(e) \leq \\ & -(c - \beta\frac{LK_2}{\underline{a}_z}) V_z(w,z) - \frac{(\beta\gamma a - \beta LK_1/\underline{a}_e - d/\underline{a}_e)}{\beta} \beta V_e(e) \leq \\ & -\frac{c}{2}(V_z(w,z) + \beta V_e(e)) = \\ & -\frac{c}{2} V_{\rm cl}(w,z,e) := \\ & -\alpha_{\rm c} V_{\rm cl}(w,z,e) \end{aligned}$$

Similarly, for all $t \notin \{I_k\}$, for all $(w,z,e) \in W \times \mathbb{R}^n \times \mathbb{R}$, and for all $\gamma \geq 0$

$$\begin{aligned} & D^+_{{\rm col}(\dot w,\dot z,\dot e)} V_{\rm cl}(w,z,e) \leq \\ & -cV_z(w,z) + \frac{d}{\underline{a}_e} V_e(e) + \beta L\frac{K_1}{\underline{a}_e} V_e(e) + \beta L\frac{K_2}{\underline{a}_z} V_z(w,z) \leq \\ & (\beta L\frac{K_2}{\underline{a}_z} - c) V_z(w,z) + (\frac{d}{\underline{a}_e} + \beta L\frac{K_1}{\underline{a}_e}) V_e(e) \leq \\ & \alpha_{\rm d} V_{\rm cl}(w,z,e) \end{aligned}$$

with $\alpha_{\rm d} := \max\{(\beta LK_2/\underline{a}_z - c)\,,\ (d/\underline{a}_e + \beta LK_1/\underline{a}_e)/\beta\}$. In the time intervals in which the topology is not connected, namely $t \in [t_{k,2}, t_{k+1,1}]$, we have that the growth of the Lyapunov function can be estimated as

$$V_{\rm cl}(t_{k+1,1}) \leq e^{\alpha_{\rm d}(t_{k+1,1} - t_{k,2})} V_{\rm cl}(t_{k,2}) \leq e^{\alpha_{\rm d} T_0} V_{\rm cl}(t_{k,2})$$

We will continue the analysis by considering the closed-loop system as an hybrid system flowing during the time intervals in which the topology is connected, and "instantaneously" jumping in the intervals in which the topology is disconnected. During flows the closed-loop Lyapunov function satisfies $D^+_{{\rm col}(\dot w,\dot z,\dot e)} V_{\rm cl}(\cdot) \leq -\alpha_{\rm c} V_{\rm cl}(\cdot)$, while, during jumps, the jump of the Lyapunov function satisfies $V_{\rm cl}(\cdot)^+ \leq e^{\alpha_{\rm d} T_0} V_{\rm cl}(\cdot)$. The fact that the intervals I_k satisfy an average dwell-time condition expressed above allows one to say (see [5]) that flow and jump times of the hybrid system can be thought of as governed by a clock variable $\upsilon_{\rm c}$ flowing according to $\dot\upsilon_{\rm c} \in [0, 1/\tau]$ when $\upsilon_{\rm c} \in [0, n_0]$, and jumping according to $\upsilon_{\rm c}^+ = \upsilon_{\rm c} - 1$ when $\upsilon_{\rm c} \in [1, n_0]$. We thus endow the closed-loop system with the clock variable and study the resulting hybrid system whose Lyapunov function flows and jumps according to the following rules

$$\left.\begin{array}{rcl} \dot\upsilon_{\rm c} & \in & [0, 1/\tau] \\ D^+_{{\rm col}(\dot w,\dot z,\dot e)} V_{\rm cl} & \leq & -\alpha_{\rm c} V_{\rm cl} \end{array}\right\} \quad (\upsilon_{\rm c}, V_{\rm cl}) \in [0, n_0] \times \mathbb{R}$$

$$\left.\begin{array}{rcl} \upsilon_{\rm c}^+ & = & \upsilon_{\rm c} - 1 \\ V_{\rm cl}^+ & \leq & e^{\alpha_{\rm d} T_0} V_{\rm cl} \end{array}\right\} \quad (\upsilon_{\rm c}, V_{\rm cl}) \in [1, n_0] \times \mathbb{R}$$

For this hybrid system we consider the Lyapunov function

$$V_{\rm h}(\upsilon_{\rm c}, w, z, e) = e^{N\upsilon_{\rm c}} V_{\rm cl}(w,z,e)$$

with $N \in (\alpha_{\rm d} T_0, \alpha_{\rm c}\tau)$, by taking

$$\tau^\star = \frac{\alpha_{\rm d} T_0}{\alpha_{\rm c}}\,.$$

During flows we have that

$$\begin{array}{rcl} D^+_{{\rm col}(\dot{\upsilon}_{\rm c},\dot{V}_{\rm cl})} V_{\rm h} & = & N\dot{\upsilon}_{\rm c} e^{N\upsilon_{\rm c}} V_{\rm cl} + e^{N\upsilon_{\rm c}} D^+_{\dot{V}_{\rm cl}} V_{\rm cl} \\ & \le & \dfrac{N}{\tau} e^{N\upsilon_{\rm c}} V_{\rm cl} - \alpha_{\rm c} e^{N\upsilon_{\rm c}} V_{\rm cl} \\ & \le & \dfrac{N}{\tau} V_{\rm h} - \alpha_{\rm c} V_{\rm h} \\ & \le & -\alpha'_{\rm c} V_{\rm h} \end{array}$$

where $\alpha'_{\rm c} = \alpha_{\rm c} - N/\tau > 0$. On the other hand, during jumps,

$$\begin{array}{rcl} V_{\rm h}^+ & = & e^{N\upsilon_{\rm c}^+} V_{\rm cl}^+ \\ & \le & e^{N(\upsilon_{\rm c}-1)} e^{\alpha_{\rm d} T_0} V_{\rm cl} \\ & = & e^{-(N-\alpha_{\rm d} T_0)} e^{N\upsilon_{\rm c}} V_{\rm cl} \\ & = & \epsilon V_{\rm h} \end{array}$$

with $\epsilon := e^{-(N-\alpha_{\rm d}T_0)} < 1$. This Lyapunov function is thus decreasing both during flows and during jumps and it is positive definite with respect to the set $[0, n_0] \times \mathcal{A}^\star \times \{0\}$. From this the result follows.

V. Conclusions

In this paper, we have extended to the case of *nonlinear* agents, exchanging only *output* information (as opposite to the case in which agents exchange full state information), the coordination result proven in Theorem 2 of [21] for a network of linear agents exchanging full state information, in the presence of *time-varying* communications between agents. Our result reposes on a connectivity property expressing the guaranteed decay, on fixed time intervals, of a candidate Lyapunov function associated to an auxiliary network of integrator systems. In this context, our result encompasses not only the connectivity properties discussed in [21], but can also be used to analyze leader-followers coordination problems in the presence of time-varying communication protocol that satisfy the (weaker) connectivity properties considered in [6].

References

[1] A. Jadbabaie, J. Lin, A.S. Morse, Coordination of Groups of Mobile Autonomous Agents Using Nearest Neighbor Rules, *IEEE Trans. Aut. Contr.*, **48**(6), pp. 988-1001, 2003.

[2] R. Olfati-Saber, R. M. Murray, Consensus Problems in Networks of Agents With Switching Topology and Time-Delays, *IEEE Trans. Aut. Contr.*, **49**(9), pp. 1520-1533, 2004.

[3] R. Goebel, R. Sanfelice, A. R. Teel, *Hybrid Dynamical Systems: Modeling, Stability, and Robustness*, Princeton University Press, 2012.

[4] J. P. Hespanha, A. S. Morse, Stability of switched systems with average dwell time. Proc. 38th IEEE Conference on Decision and Control, pp. 2655-2660, 1999.

[5] C. Cai, A.R. Teel, R. Goebe, Smooth Lyapunov functions for hybrid systems, Part II: (Pre)-asymptotically stable compact sets, *IEEE Trans. Automat. Contr.*, **53**, pp. 734-748, 2008.

[6] L. Moreau Stability of continuous-time distributed consensus algorithms. http://arxiv.org/abs/math/0409010v1,2004a. arXiv:math/0409010v1[math.OC].

[7] L. Moreau, Stability of multi-agent systems with time-dependent communication links, *IEEE Trans. Automat. Contr.*, **50**(2), pp. 169-182, 2005.

[8] M. Arcak. Passivity as a design tool for group coordination", *IEEE Trans. Aut. Contr.*, **52**(8), pp. 1380-1390, 2007.

[9] Z. Lin, B. Francis, M. Maggiore, State agreement for continuous-time coupled nonlinear systems, *SIAM J. Contr. Optimiz.* **46**(1), pp. 288-307. 2007

[10] P. Wieland, *From Static to Dynamic Couplings in Consensus and Synchronization among Identical and Non-Identical Systems*, PhD thesis, Universität Stuttgart, 2010.

[11] L. Scardovi, R. Sepulchre, Synchronization in networks of identical linear systems, *Automatica*, **45**(10), pp. 2557-2562, 2009.

[12] J. H. Seo, H. Shima, J. Back, Consensus of high-order linear systems using dynamic output feedback compensator: low gain approach, *Automatica*, **45**(11), pp. 2659-2664, 2009.

[13] Z. Li, Z. Duan, G. Chen, Consensus of Multiagent Systems and Synchronization of Complex Networks: A Unified Viewpoint, *IEEE Trans on Circuits and Systems-I*, **57**(1), pp. 213-224, 2010.

[14] C. Ma, J. Zhang, Necessary and Sufficient Conditions for Consensusability of Linear Multi-Agent Systems, *IEEE Trans. Automat. Contr.*, **55**(5), pp. 1263-1268, 2010.

[15] H. Kim, H. Shim, and J.H. Seo. Output consensus of heterogeneous uncertain linear multi-agent systems, *IEEE Trans. Aut. Contr.*, **56**(1): 200-206, 2011.

[16] P. Wieland, R.Sepulchre, and F. Allgöwer. "An internal model principle is necessary and sufficient for linear output synchronization". *Automatica*, **47**(), 1068-1074, 2011.

[17] J.H. Seo, J. Back, H. Kim, H. Shim, Output feedback consensus for high-order linear systems having uniform ranks under switching topology, *IET Control Theory Appl.*, **6**(8), pp. 1118-1124, 2012.

[18] A. Isidori, L. Marconi, G. Casadei, Robust output synchronization of a network of heterogeneous nonlinear agents via nonlinear regulation theory, *Proc. of the 52th IEEE Conf. Decision Control*, Florence, 2013.

[19] G. Wen, Z. Duan, G. Chen, W. Yu, Consensus Tracking of Multi-Agent Systems With Lipschitz-Type Node Dynamics and Switching Topologies, *IEEE Trans on Circuits and Systems-I*, to appear.

[20] W. Ni, D. Cheng, Leader-following consensus of multi-agent systems under fixed and switching topologies, *Syst. and Control Lett.* **59**, pp. 209-217, 2010.

[21] J. Qin, C. Yu, H. Gao, Coordination for Linear Multiagent Systems With Dynamic Interaction Topology in the Leader-Following Framework, *IEEE Trans. Industr. Electronics*, **61**(5), pp.2412-2422, 2014.

[22] E.D. Sontag, On the input-to-state stability property, *European Journal of Control*, **1**(1), pp. 24–36 1995.

[23] L. Marconi, L. Praly, A. Isidori, Output Stabilization via Nonlinear Luenberger Observers. *SIAM J. on Contr. and Optimiz.*, **45**(6), pp. 2277-2298, 2007.

[24] F. Esfandiari and H. Khalil, Output feedback stabilization of fully linearizable systems, *International Journal of Control*, **56**, pp. 1007–1037, 1992.

[25] A.R. Teel and L. Praly, Tools for semiglobal stabilization by partial state and output feedback. *SIAM J. Control Optim.*, **33**, pp. 1443–1485, 1995.

[26] J.P. Gauthier, I. Kupka, *Deterministic Observation Theory and Applications*, Cambridge University Press, Cambridge (2001).

[27] A. Isidori, *Nonlinear Control Systems: volume II*, Springer Verlag, London (1999).