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Quality, Distance and Trade: A Strategic Approach*

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Abstract

This paper contributes to the literature on distance and quality by identifying a firm- based force contributing to explain the observed increase of the quality of shipped goods with the distance of their destination market. This force originates from the influence of distance on firms' strategic behavior in the presence of consumer heterogeneity, when the quality level of goods is a choice variable for them. Our approach differs from the extant literature because it does not rely on technology or preference/income differentials to identify the determinants and drivers of trade flows. We find that distance has an unambiguously positive effect on the average quality of traded goods. Our results contribute to the analysis of the determinants of firms' trade performance.

Keywords: Product Quality, Distance, Strategic Interaction.
JEL Codes: F10, L13

1 Introduction

In the discussion of the determinants of trade flows, the focus of economists has gradually shifted from features such as comparative advantage, increasing returns to scale and consumer preferences to factors operating at the firm level (see Bernard *et al.* 2007, for a discussion). In particular, firm heterogeneity has been emphasized as a fundamental element to understand the drivers of trade flows. In this respect, the literature recognizes two main dimensions along which firms may be heterogeneous (see, for instance, Hallak and Sivadasan 2009). The first relates to productivity (see, among others, Melitz 2003; Chaney 2008; Melitz and Ottaviano 2008). The other dimension of firm heterogeneity is connected with the quality level of the output (see, e.g., Baldwin and Harrigan 2011;

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Johnson 2010; Kugler and Verhoogen 2012).[1] The present paper is concerned with the second type of heterogeneity. In particular, we explore the relationship between the quality level of traded goods and the distance of trading partners. Indeed, recent analyses have unveiled empirical regularities concerning the relationship between the quality of exported goods and the distance of the country of destination. Specifically, they show that unit values (free on board prices) of exported goods increase with the distance of the trading partner, which suggests that firms upgrade the quality level of the goods they export to more distant markets compared to closer ones.[2] This evidence is robust both at the product and firm levels, see, for example, Baldwin and Harrigan (2011); Bastos and Silva (2010); Helble and Okubo (2008); Manova and Zhang (2012).

In this paper, we identify a firm-based force, originating from strategic behavior, that contributes to explain this stylized fact. For this purpose, we modify the classical model of vertical product differentiation with oligopolistic competition (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982) to account for distance between trading partners. In such a model, consumers are heterogeneous in their quality appreciation, which allows both high and low quality to exist in the market. We use this model to investigate the effect of distance on the quality level of traded goods, when firms are free to set the level of quality of their products to meet consumers' heterogeneous preferences. To make our analysis sharper, we abstract from any supply-side (productivity) differences between trading partners and neutralize any income effect, so as to focus on the pure role of strategic interaction among firms. This approach, therefore, assumes that firms are not negligible with respect to the market. This is consistent with the observation that exporting firms are on average bigger than non-exporters (Bernard *et al.* 2003, using U.S. data, find that they ship on average 5.6 times more).

Our work directly relates to the flourishing literature analyzing the effects of distance on the quality level of traded goods, as measured by unit values. Many works tackle this issue by using monopolistic competition with homogeneous firms (Helpman and Krugman, 1985), comparative advantage (Eaton and Kortum, 2002) or monopolistic competition with heterogeneous firms (Melitz, 2003).[3] Recently, useful insights on the behavior of firms exposed to trade competition have been drawn by addressing to the Industrial Organization literature. This strand of literature usually combines consumer and/or firm heterogeneity with partial equilibrium analysis (see, e.g., Verhoogen 2008 and Khandelwal 2010). In particular, Verhoogen (2008) works out a model of North-South trade in which the exporting firms of the poor country ship to the rich country commodities of higher quality relative to those produced for the domestic market, "to appeal to richer northern consumers" (Verhoogen, 2008, p. 489). In general, this literature delivers a mapping between *ex-ante* characteristics of trading partners - productivity, factor endowments, capability, consumer preferences or income - and the features of trade. More productive firms supply higher-quality products, earn larger profits and are more likely to be exporters, whereas richer countries tend to consume and import commodities of higher quality. Our paper is also related to two other strands of literature. The first investigates trade with quality-differentiated products (see, e.g., Eaton and Kierzkowski 1984; Shaked and Sutton 1984; Flam and

[1] The role of product quality in international trade has been receiving a growing theoretical and empirical attention since the seminal contributions by Linder (1961) and Alchian and Allen (1964).

[2] It is common, in the literature, to measure the quality level of a commodity by its unit value (see e.g. Greenaway *et al.* 1995 and Wooldridge 2002). Noticeable exceptions are Hallak and Schott (2011) and Khandelwal (2010) which disentangle unit values into quality and quality-related price components.

[3] See Baldwin and Harrigan (2011) for an excellent recapitulation of these approaches and an exposition of their empirical implications.

Helpman 1987; Motta *et al.* 1997; Frascatore 2001; Cabrales and Motta 2001; De Fraja and Norman 2004; Schott 2004; Hallak 2006; Choi *et al.* 2006; Sutton 2007). A second stream of literature analyzes the optimal trade policy under trade with quality-differentiated products, and includes papers such as Herguera *et al.* (2000, 2002), Zhou *et al.* (2002), Boccard and Wauthy (2005) and Saggi and Sara (2008). With respect to the extant literature on quality, trade, and distance, the two distinguishing features of our approach are (i) the focus on firm's strategic behavior and (ii) the elimination of any supply-side differences among trading partners.[4]

In a nutshell, the mechanism we identify is the following. Distance, through transport costs, imposes an anticompetitive burden on exporting firms, which is heavier the farther away the destination market is. Exporting firms strategically react by increasing the quality level of their goods -thus making them more attractive to consumers- so as to (partially) recover the competitive edge eroded by distance. Our modeling choice proves to be useful in two respects. First, it identifies a new force shaping trade flows, which is based on the effect exerted by distance on firms' strategic behavior both in price and non-price competition. Second, it allows us to clearly distinguish between the price-setting and the quality choice (product design) activities, so that empirical implications for prices and qualities are drawn separately. As a last remark, we would like to stress that we are fully convinced that supply- and demand-side drivers play a crucial role in shaping trade flows. Nonetheless, we believe that our work sheds light on a novel, complementary mechanism that has been neglected thus far.

To delve into the mechanism we have highlighted, we consider a model of one-way trade with two firms producing variants of a vertically differentiated commodity at zero costs. One firm is located "away" from consumers so that a transport cost has to be paid to consume the good sold by that firm. We set up and solve a two-stage game in which firms first simultaneously and at no cost select the quality level of the good they produce and then simultaneously set prices. We consider the location where the consumers reside as the "destination market", and focus on the prices set as well as on the quality levels of the goods available there. We show that trade costs and strategic behavior determine the role of "high-" versus "low-quality" producer for domestic and foreign firms, and that they influence the average quality level of goods in the "destination" country for given "roles" assigned to firms. In particular, the main outcomes of our model are the following. First, we identify the conditions under which pure-strategy Nash equilibria with trade exist. Second, we show that distance always increases the average quality level of the traded good. Later, we extend our basic model to account for two-way trade flows and show that our result is confirmed in this scenario as well. From an empirical standpoint, our results are in accordance both with the acquired evidence on the positive effect of distance on the quality level of traded goods, and, also, with a higher likelihood of "zeros" in trade patterns as distance (measured by trade costs) increases.

The paper is organized as follows. Section 2 presents the basic one-way trade model and Section 3 solves it. Section 4 deals with two-way trade. Section 5 discusses our results with reference both to the empirical and theoretical literature. Finally, Section 6 provides a short conclusion. All of the proofs are relegated to the Appendix.

[4]While the present paper analyzes the quality choices of firms for given locations, Bacchiega and Minniti (2009) analyzes the location choices of firms producing goods of given qualities

2 The Model

Preliminaries. We develop a model of one-way trade with two countries: one is foreign (F) and one is domestic (D). Two firms, one located in each country, produce a vertically differentiated good to sell in the domestic country. To import the foreign good, consumers must pay a transport cost. We investigate the subgame-perfect Nash equilibria in a game where firms simultaneously select the quality level of their goods and subsequently set prices.

Consumers. The domestic country is inhabited by a continuum of consumers that are heterogeneous in their willingness to pay for quality. Consumers are uniformly distributed with unit density over the interval $[0,1]$ according to their appreciation for quality, θ.[5] Consumers purchase either one or zero units of the good. The utility derived by consumer θ when purchasing one unit of variant j is $U(\theta) = \theta u_j$, with $u_j \in]0, \bar{u}]$ being the (commonly perceived) quality level of variant j. Similarly, let ρ_j denote the total price paid by consumers to buy one unit of good j. We define ρ_j as $p_j + t$ with $t \geq 0$ representing the unit transport cost paid by consumers, clearly $t = 0$ if the purchased variant is produced locally, otherwise $t > 0$.[6] It is reasonable to assume that transport costs increase with the distance of the trading partner. Thus, here, we will refer to t simply as the "distance" from the foreign firm/country to the destination market. Assume that the utility of no consumption is zero. Following Mussa and Rosen (1978) the surplus of consumer θ is

$$U(\theta) = \begin{cases} \theta u_j - \rho_j & \text{when buying one unit of good } j \text{ ;} \\ 0 & \text{when abstaining from consumption.} \end{cases}$$

Demands are obtained through the standard marginal consumer approach. Consumers choose the version of the good providing them with the largest surplus as long as this is positive, else they do not buy. Let h identify the firm selling the high-quality variant, and, similarly, let l label the firm supplying the low-quality one.[7] Standard computations return the value of θ identifying the consumer indifferent between purchasing one unit of the high- and low-quality good and that indifferent between purchasing one unit of the low-quality good and abstaining from consumption. Their expressions are reported in the following.

$$\theta_1 = \frac{\rho_h - \rho_l}{u_h - u_l}; \qquad \theta_0 = \frac{\rho_l}{u_l}. \tag{1}$$

Once the marginal consumers expressions are obtained, the demand system under duopoly is easily derived.

$$\mathcal{D}_h = (1 - \theta_1); \qquad \mathcal{D}_l = (\theta_1 - \theta_0). \tag{2}$$

Notice that θ_1 and θ_0 define the demands' bounds only if they lie within the interval $[0,1]$ and $\theta_1 > \theta_0$. Yet, they may not do for some combinations of prices, qualities and transport costs. In this case,

[5] The parameter θ is equivalently interpreted as an income index, with higher θ associated with higher income (see Tirole 1988, page 97). Under this interpretation, thus, consumers are heterogeneous in terms of income.

[6] With this specification the burden of transport costs is on consumers, such that the present is a shopping model. Our results, however, do not change if firms pay per unit transport costs to deliver the good to customers (shipping model). Calculations are available upon request.

[7] By anticipating an equilibrium argument, we assume that qualities are different at an equilibrium with two active firms to avoid price competition with a homogeneous good. See Section 3 and Appendices A-D for a detailed proof.

the demand for the imported good vanishes.[8] Imagine, for example, high levels of t (when transport costs are prohibitively high the foreign good is not traded), or a domestic low-quality good with a quality level "close enough" to the imported high-quality one (all consumers prefer to patronize a –slightly– lower quality domestic producer but save on transport costs). In this case, the demand the domestic firm faces needs to be re-defined accordingly. Furthermore, notice that θ_1 is not defined for $u_h = u_l$. Because in our model both prices and qualities are endogenously determined, this behavior of demand has to be carefully examined when characterizing the possible Nash equilibria of the game. We refer the reader to the equilibrium analysis in the Appendices for an accurate discussion on this point.

Firms. The aim of our paper is to delve into the effects of the distance on the strategic behavior in the determination of the direction and characteristics of trade flows. Accordingly, we abstract from any supply-side issue by (a) normalizing firms' production costs to zero and (b) assuming that product design in terms of vertical differentiation is costless as in Choi and Shin (1992). This choice allows us to eliminate from quality setting any effects that are not linked to trade costs to the destination market, and thus to focus on the pure effect of distance on the quality of the shipped good. Firm's profits are, thus

$$\pi_j^i = \mathcal{D}_j^i p_j^i, \tag{3}$$

where $j \in \{h, l\}$ is the quality level of firm $i = D, F$.

Timing. The game we analyze has two stages. In the first stage firms simultaneously select the quality levels of their variants, in the second stage they simultaneously set prices.

Our model then takes the form of a game Γ where the players are the two firms $i = D, F$, their strategies are price-quality vectors (p_i, u_i) and their payoffs are their profits $\pi_i(\cdot)$.

3 Equilibrium

The game is solved by backward induction, addressing the price stage first.

3.1 Price stage

In this section we fully develop the pricing stage of the standard duopoly case only, namely when both expressions in (1) lie within the $[0, 1]$ interval. We refer to the equilibrium analysis developed in the Appendices for the cases where they do not. The solution to this case involves standard calculations in the class of models of vertical product differentiation (see e.g. Gabszewicz and Thisse, 1979), thus it will be quickly dealt with. The pricing decisions of firms depend upon their roles as high- or low-quality suppliers, which, in turn, are determined in the first stage. Thus, we consider the two cases that correspond to the two different branches of game Γ, in the first (i), the high-quality producer is the domestic firm, in the second (ii), the high-quality producer is the foreign firm.

[8] The fact that some demands may be driven down to zero following parameter changes is not new in the literature on vertical differentiation, both with and without trade, see for example Wauthy (1996) and Frascatore (2001).

(i) Domestic high-quality producer

In this case the D-firm has selected the high-quality version of the good at the first stage. Therefore profits accruing to firms are:

$$\pi_h^D = \mathcal{D}_h^D p_h^D, \qquad \pi_l^F = \mathcal{D}_l^F p_l^F. \tag{4}$$

Simultaneous maximization of (4) w.r.t. p_h^D and p_l^F respectively yield the following expressions for the optimal first-stage prices.[9]

$$\hat{p}_h^D(u_h^D, u_l^F) = \frac{u_h^D[2(u_h^D - u_l^F) + t]}{4u_h^D - u_l^F}, \quad \hat{p}_l^F(u_h^D, u_l^F) = \frac{u_l^F(u_h^D - u_l^F) - t(2u_h^D - u_l^F)}{4u_h^D - u_l^F}. \tag{5}$$

By plugging (5) back into (4) we obtain firms' profits in the second stage:

$$\hat{\pi}_h^D(u_h^D, u_l^F) = \frac{{u_h^D}^2[2(u_h^D - u_l^F) + t]^2}{(4u_h^D - u_l^F)^2(u_h^D - u_l^F)}, \quad \hat{\pi}_l^F(u_h^D, u_l^F) = \frac{u_h^D[(u_h^D - u_l^F)u_l^F - t(2u_h^D - u_l^F)]^2}{u_l^F(u_h^D - u_l^F)(4u_h^D - u_l^F)^2}. \tag{6}$$

As noted above, the duopoly demand system $\mathcal{D}_h^D$, $\mathcal{D}_l^F$ involves non-negative prices and quantities for firm F if and only if the difference between firms' quality levels is large enough. The precise condition is reported in the following.

Remark 1. *The foreign firm's price and demand are non-negative under duopoly pricing if and only if* $u_h^D \geq \frac{u_l^F(u_l^F - t)}{u_l^F - 2t} > u_l^F$.

If qualities are too similar, the low-quality firm (which is disadvantaged because of transport costs) cannot enjoy a positive market share under duopolistic competition. In other terms, if $u_h^D < \frac{u_l^F(u_l^F - t)}{u_l^F - 2t}$ entry in the market is "blockaded". We will expand on this point in Section 5.

(ii) Foreign high-quality producer

This case mirrors the previous, in that the foreign firm is now offering the high-quality variant of the good. Accordingly, firms' profits now are

$$\pi_h^F = \mathcal{D}_h^F p_h^F, \qquad \pi_l^D = \mathcal{D}_l^D p_l^D. \tag{7}$$

As in the previous case, simultaneous maximization of profits in (7) gives the optimal prices, which are reported hereafter.[10]

$$\hat{p}_h^F(u_h^F, u_l^D) = \frac{2u_h^F(u_h^F - u_l^D) - t(2u_h^F - u_l^D)}{4u_h^F - u_l^D}, \qquad \hat{p}_l^D(u_h^F, u_l^D) = \frac{(u_h^F - u_l^D + t)u_l^D}{4u_h^F - u_l^D}. \tag{8}$$

Substitution of (8) back into (7) yields the following expressions for firms' profits.

$$\pi_h^F(u_h^F, u_l^D) = \frac{[2u_h^F(u_h^F - u_l^D) - t(2u_h^F - u_l^D)]^2}{(u_h^F - u_l^D)(4u_h^F - u_l^D)^2}, \quad \pi_l^D(u_h^F, u_l^D) = \frac{u_h^F u_l^D(u_h^F - u_l^D + t)}{(u_h^F - u_l^D)(4u_h^F - u_l^D)^2}. \tag{9}$$

Inspection of (8) reveals that the price of the domestic low-quality producer is always positive, whereas the foreign high-quality producer's optimal price (and consequently demand) is positive if and only

[9] Easy calculations show that second order conditions are met as long as $u_h^D > u_l^F$, which is true by assumption in case (i).

[10] Again second order conditions are satisfied if $u_h^F > u_l^D$.

if u_h^F is large enough. More precisely:

Remark 2. *The foreign firm's price and demand are non-negative under duopoly pricing if and only if* $u_h^F \geq \frac{1}{2}\left(t + u_l^D + \sqrt{t^2 + (u_l^D)^2}\right) > u_l^D$.

Similarly to case (i), if qualities are too similar, a profit-dissipating price competition prevails, and the disadvantaged firm is the foreign one, which in this case produces the high-quality good. Again, we refer to Section 5 for a discussion on this point.

In the ensuing analysis, we will focus on the situation where both firms enjoy strictly positive market shares, so as to rule out duopoly equilibria where one firm has a zero market share even if the price set is nil. With a slight abuse of terminology, we will refer to these "subgame-perfect Nash equilibria with two active firms" simply as "subgame-perfect Nash equilibria".

3.2 Quality choice

Let us move now to the core of our paper, namely, quality choice.[11] We will tackle separately cases (i) and (ii), proving the possible existence of subgame-perfect Nash equilibria in the two cases (Propositions 1 and 3) and performing comparative statics analysis on the equilibrium quality levels (Propositions 2 and 4). We will thus report our main economic results in Theorems 1 and 2 and Corollary 1. We start with case (i). In the ensuing analysis, for the sake of readability, we will omit the first-stage optimal prices when describing firms' strategies, to focus on quality levels instead.[12] We provide the economic intuition of our results at the end of this section, and we refer to Section 5 for a more articulated discussion on them.

(i) Domestic high-quality producer

The main result in this case is the following.

Proposition 1. *In game Γ, there exists a unique cutoff value for t, $\bar{t} \in (0, u_l^F)$ such that for all $t \in [0, \bar{t})$ there is one and only one SPNE where the high-quality producer is domestic and the low-quality one is foreign. At this equilibrium, $u_h^{D*} = \bar{u}$ and $u_l^{F*}(u_h^{D*}) = \underline{u}^F \in (0, \bar{u})$.*

Proof. See Appendix A. □

We can proceed further by delving into the behavior of the equilibrium strategy $\underline{u}^F$ as a function of t. Our findings are reported in what follows.

Proposition 2. *For every $t < \frac{\underline{u}^F}{2}$,*

(i) $\underline{u}^F \in (\frac{4}{7}\bar{u},\ \frac{29}{49}\bar{u})$;

(ii) $\dfrac{\partial \underline{u}^F}{\partial t} > 0$ *in the whole interval* $(\frac{4}{7}\bar{u},\ \frac{29}{49}\bar{u})$.

Proof. See Appendix B. □

[11] One natural reference for this analysis is the article by Choi and Shin (1992). Their main result is that with costless quality choice and firms located in the same market (no transport costs), the firm producing high quality selects the upper bound in the quality space, $\bar{u}$ in our notation, while its low-quality rival chooses a quality that is $\frac{4}{7}\bar{u}$. Choi and Shin (1992) develop a sequential-move game, while ours is a simultaneous-move one. By setting $t = 0$, our model boils down to the simultaneous-move version of Choi and Shin (1992).

[12] According to our choice, firm i's strategy $(\hat{p}_j^i, u_j^i)$ will be reported as (u_j^i).

(ii) Foreign high-quality producer

In case (ii), the main result is summarized in the following.

Proposition 3. *In game* Γ*, (a) when* $t < \tilde{t} \approx \frac{3\bar{u}}{126}$ *and* $\bar{u} > \frac{1}{2}\left(t + u_l^D + \sqrt{t^2 + (u_l^D)^2}\right) \equiv \hat{u}$*, there exists one and only one SPNE where the high-quality producer is foreign, and the low-quality one is local, in this case,* $u_h^{F*} = \bar{u}$ *and* $u_l^{D*} \equiv \underline{u}^D \in (0, \bar{u})$*; (b) when* $t \geq \tilde{t}$*, there exists no SPNE with a domestic low-quality producer and a foreign high-quality producer.*

Proof. See Appendix C. □

As in case (i) we analyze the characteristics of the optimal quality level for the domestic firm $\underline{u}^D$. The next Proposition summarizes our results.

Proposition 4. *For every* $t \in [0, \tilde{t}]$

(i) $\underline{u}^D \in (\frac{4}{7}\bar{u}, \frac{4}{5}\bar{u}]$*;*

(ii) $\dfrac{\partial \underline{u}^D}{\partial t} > 0.$

Proof. See Appendix D. □

Propositions 1 and 3 characterize the subgame-perfect Nash equilibria of the two branches of game Γ. They define parameter regions in the $(t, \bar{u})$ space where the game Γ has pure strategy SPNE with both firms selling positive quantities at positive prices.

To finalize equilibrium analysis, however, we need to analyze the relative size of $\tilde{t}$ and $\bar{t}$. The next Lemma tackles this point.

Lemma 1. *At every equilibrium of game* Γ*,* $\tilde{t} < \bar{t}$*.*

Proof. By direct comparison it is easy to obtain that $\tilde{t} < \bar{t}$ as long as $\frac{27}{25}u_l^F < \bar{u} < \frac{164}{25}u_l^F$. This condition is satisfied at every equilibrium of the game because from Propositions 2 and 4 we know that $\underline{u}^F \in (\frac{4}{7}\bar{u}, \frac{29}{49}\bar{u})$ and $\underline{u}^D \in (\frac{4}{7}\bar{u}, \frac{4}{5}\bar{u}]$. □

We then complete the equilibrium analysis of this game by putting our results together in what follows.

Theorem 1 (Equilibrium existence)**.** *Let* $\bar{u} > \hat{u}$ *and* $t < \bar{t}$*.*

(i) If $0 \leq t < \tilde{t}$ *then game* Γ *has two subgame-perfect Nash equilibria. At the first one the high-quality (low-quality) producer is the domestic (foreign) firm, and at the second one, the high-quality (low-quality) producer is the foreign (domestic) firm.*

(ii) If $\tilde{t} \leq t < \bar{t}$ *then game* Γ *has one and only one subgame-perfect Nash equilibrium, at which the high-quality producer is the domestic firm, and the low-quality producer is the foreign one.*

Part (i) of Theorem 1 states that when distance is "small", there are two equilibria: at the first one, the domestic firm produces high quality whereas the foreign firm produces low quality, and at the second one, the opposite happens. The result that all quality configurations may arise at equilibrium is in accordance with intuition: when distance is small firms compete in "almost" the same country, thus there is no reason for one firm to be more likely the high- (or low-) quality producer.[13] By contrast, when distance is "large" (ii), our analysis predicts that one configuration only arises at equilibrium. Specifically, in the unique equilibrium of the game, the high-quality version of the good is locally produced, while the low-quality one is imported. Transport costs induced by distance impose a lower bound *above* marginal cost (zero in our case) to the price paid by consumers for the imported good, the domestic producer can exploit this asymmetry and "blockade high-quality entry" on the market by credibly committing to begin a price war with a (almost) homogeneous good. In this case, a foreign firm (producing high quality in the parameter region $0 \leq t < \tilde{t}$), anticipating the aggressive behavior by the local firm in the second stage of the game, switches position in the quality ladder and produces the low-quality good.[14] Thus, distance acts as a credible aggressive-behavior commitment device for the domestic producer.

Next, we enlighten the effect of distance on quality. As we know from the previous analysis, the high-quality firm always chooses the best available quality $\bar{u}$ independently on the extent of trade costs.[15] Thus, quality does not change with distance for the high-quality producer. The impact of trade costs on the *quality level* of the low-quality variant is studied in Propositions 2 and 4. The following theorem summarizes our result.

Theorem 2 (Comparative statics)**.** *Let $\bar{u} > \hat{u}$ and $t < \bar{t}$. In the two parameter regions $t \leq \tilde{t}$ and $t \geq \tilde{t}$, the quality level of the low-quality good increases with distance.*

Theorem 2 delivers a clear-cut result. Distance increases the quality level of the low-quality good, and the increase in quality is larger the higher the distance, as long as it is small enough ($t < \bar{t}$) to make trade itself viable.

Although the result is the same whether the low-quality producer is domestic or foreign, the economic intuition underlying the two cases is different. Consider first the case of a foreign low-quality producer. Trade costs harm the low-quality foreign firm because they increase the total price paid by consumers for the variant. Then the reaction of firm F is twofold. On the one hand, it reduces the optimal price it charges on consumers, so as to dampen the effect of distance on consumers' choice (it is easily ascertained that $\frac{\partial \hat{p}_l^F(\cdot)}{\partial t} < 0$). On the other hand, firm F attempts to recover the competitive edge eroded by (increased) distance by increasing the utility consumers derive from low-quality consumption through an increase in quality. On top of this, notice that the quality increase makes the domestic and foreign products more similar, intensifying further price competition. Consider now the case where the low-quality producer is domestic. An increase in the transport costs causes an increase in the total price consumers have to pay for the high-quality variant, making it less attractive and dampening price competition. Consequently, the domestic low-quality producer can increase the quality level of its good (and extract more surplus from consumers), and still avoid engagement in a fiercer price war due to more homogeneous goods.

We may combine the results of Theorems 1 and 2 in the following.

[13] Theorem 1 raises the issue of equilibrium multiplicity, on which we come back in Section 5 when we discuss the relationship between quality and distance.

[14] Contrarily, quality switching does not occur when a foreign firm producing low quality keeps producing the same good as distance increases.

[15] This result derives from the fact that, in our model, there are no costs of quality improvement (Choi and Shin, 1992, Wauthy, 1996). All else equal, by selecting the best feasible quality the high-quality producer enjoys two advantages. First, it extracts the maximum amount of surplus from the consumers and, second, it differentiates the most from the rival, relaxing price competition.

Corollary 1 (Quality and distance). *Let $\bar{u} > \hat{u}$ and $t < \bar{t}$. In the two parameter regions $t \leq \tilde{t}$ and $t \geq \tilde{t}$, the expected quality level of the traded good increases with distance.*

This last result relies on the fact that, within each interval $[0, \tilde{t}]$ and $[\tilde{t}, \bar{t})$, the quality level of the low-quality good increases with distance. The term "expected" can obviously be dropped for $t > \tilde{t}$, while it comes from equilibrium multiplicity when $t \in [0, \tilde{t}]$, because each firm has a positive probability to be the quality leader. Equilibrium multiplicity may potentially pose an issue to our result because, by starting with $t < \tilde{t}$ in the equilibrium where the traded good is high-quality, and crossing $\tilde{t}$ one ends up in the equilibrium where the traded good is low-quality. This may result in a sudden decrease in the average quality of the traded good. In Section 4, however, we prove that this potential outcome is a technical artifact of the discrete change in the number of equilibria in $\tilde{t}$, and disappears if analyzing two-way trade.[16]

4 Two-way trade

In this section we extend our basic model to account for two-way trade flows. Our main result that the quality level of the low-quality good increases with the distance of the trading partners is confirmed. In order to avoid confusion with the one-way trade scenario, in this Section we label the two countries involved in trade A and B.

Assume that a unit mass of consumers, identical to those described above lives in each of A and B. One firm is settled in each country. We analyze a new game, Γ', similar to Γ, where firms simultaneously select the quality level of their goods and subsequently set prices. We assume that firms are single-product, so that the variant each of them offers is the same in the two markets. We also assume that markets are segmented, so that price discrimination is possible. Similarly to the one-way trade scenario, a per-unit transport cost t has to be borne by consumers when purchasing one unit of a good produced abroad.[17] In Appendix E we prove the following.

Theorem 3. *Let $0 < t < \frac{31}{800}\bar{u}$, then*

(i) (Equilibrium existence) Game Γ' has a unique (up to a permutation in the firms' indexes) subgame-perfect Nash equilibrium, at which (a) trade flows are positive and (b) the high-quality producer selects the top quality level $\bar{u}$ and the low-quality producer an optimal level $\underline{u} \in (\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u})$.

(ii) (Comparative statics) For all $(\underline{u}, t) \in (\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u}) \times (0, \frac{31}{800}\overline{u})$, $\frac{\partial \underline{u}}{\partial t} > 0$.

This immediately entails

Corollary 2 (Quality and distance). *The average quality level of the traded good increases with the distance of the trading partners.*

As in the one-way scenario, two-way trade emerges if transport costs are low enough, otherwise the low-quality producer has an incentive to switch to the high-quality production and deter the sales of the foreign high-quality good in its own market. Under two-way trade, each firm enjoys an advantage in its home market –local consumers do not have to pay the transport cost to purchase

[16] We are grateful to a Referee that suggested us how to deal with this point.

[17] Under two-way trade, the whole game has two symmetric branches. In one the high-quality good is produced in A and the low-quality one in B, in the other the specular configuration obtains. However, the symmetry of the model guarantees that these branches are identical up to a swap in the firms' indexes.

the local good– but, symmetrically, suffers from a disadvantage in the export market. As there are no costs of quality improvement, the high-quality producer still selects the highest possible quality level. The low-quality producer selects a quality level that is increasing with the distance from the trading partner, which confirms the one-way trade result. The intuition for this result is obtained by combining the explanations provided for the two separate cases of domestic and foreign low-quality producer under one-way trade. Indeed, following an increase in the distance from the trading partner, the low-quality producer finds it profitable to increase the quality supplied in its home market, because it can extract more surplus without increasing the degree of price competition. Similarly this firm needs to increase the quality on the export market, in order to recoup a part of the competitiveness eroded by transport costs. Since the two effects have the same sign, they drive the quality level in the same direction (for more details, see page 10). In this case the "expected" term follows from the observation that both the high-and low-quality good are simultaneously traded.

The foregoing analysis may raise the legitimate question whether our result holds in the presence of markets of different size. To deal with this point, we modify the setup of Section 2 by letting the mass of the consumers in country B be $M > 0$, while that in A is normalized to one. Clearly, when $M \to 0$ we fall into the case of one-way trade, while $M = 1$ corresponds to the symmetric two-way trade just analyzed. It may be shown that an increase in the distance between trading partners still increases the quality level of the low-quality good (the high-quality one being yet set to the maximal level), which confirms our main result, and that, if distance is "high", trade does not take place.[18] Furthermore, this new scenario allows us to draw some interesting insights on the effects of a variation in market size on the quality level of the low-quality good. For t such that two-way trade emerges at equilibrium, we find that an increase in M *increases* the quality level of the low-quality good, and vice versa. The intuition goes as follows. The parameter M is a demand shifter that does not affect the preferences of consumers, and, hence, the equilibrium prices neither. Accordingly, for each firm, M enters the profit function only multiplicatively, in the term relative to the sales in market B. Keeping this in mind, imagine, for one moment, that firms set different qualities in the two markets, without swapping their respective high- versus low-quality producer roles between markets. Firms would set their quality levels as in the one-way analysis. In particular, the high-quality producer would pick the top qualities levels in each market, while the low-quality one would set them as predicted by Proposition 1 in its export market and by Proposition 2 in its home market. Both these values depend on t but not on M. Numerical examples show that the quality of the "home" variant is always larger than that of the "export" one. Now, recall that each firm is actually restricted to produce one single variant. This has obviously no influence on the high-quality producer. By contrast, this induces the low-quality one to select a quality level that is a "weighted combination" the previous two, the "weight" being dependent on the size of M. If M is initially zero, we are in the case of one-way trade with foreign low-quality producer and quality defined in Proposition 1. Consider an increase in M from zero. The home market for the low-quality firm acquires sales importance, and, to serve it, the low-quality firm raises the quality of its product to take into account that the optimal quality in the domestic market is larger than that in the export one. Clearly, a larger M increases the importance of the domestic market and makes the actual quality more similar to the quality optimally designed for this market. Similarly, by starting from a consumer mass $M > 0$ and reducing it, the home market

[18] Given the complexity of the extension, the analysis has been carried out with the software `Mathematica` v. 9.0.1.0, the source code is available from the authors.

loses sales importance and the product is more similar to that optimally designed for export.

5 Discussion

In this section we discuss our results of the one-way and symmetric two-way trade models in the light of the existing empirical literature.

Quality and distance. The main message conveyed by our paper is that distance increases the average quality level of goods available for consumption in the country of destination. This is in a direct relationship with the literature highlighting the increase of the quality level of traded goods, the greater the distance of their destination market (Baldwin and Harrigan, 2011; Bastos and Silva, 2010; Johnson, 2010; Helble and Okubo, 2008; Manova and Zhang, 2012).

Under one-way trade with "distant" trading partners -case (ii) of Theorem 1- the interpretation is straightforward because the quality level of the imported (low-quality) good increases with distance. When distance is "small" -case (i) of Theorem 1- the interpretation refers to the "expected quality level of import". Indeed, an increase in the distance results in an increase of the quality level of the imported good only when the low-quality producer is foreign, while it has no effect on the import quality level when the low-quality firm is local.[19] As explained above, however, equilibrium multiplicity makes both configurations *ex ante* possible. Thus, we conclude that distance causes the *expected* quality level of import to increase also when trade partners are "close".[20] Finally, it is worth noticing that part (ii) of Theorem 1 points out that for a "large distance" the imported good is of low-quality. This should not be considered in contrast with empirical evidence for the following reason. This evidence highlights that the quality level of goods increases with the distance of the shipping destination, but does not claim that exporters become the quality leaders (i.e. the producers of the top-quality available variant) in their destination markets.

Under two-way trade, the symmetry of the model results in two equilibria co-existing in all the parameter space, which are, however, identical up to the swapping of firms' indexes. In these equilibria *both the high- and low-quality goods are traded.* Since a higher distance of the trading partners unambiguously increases the quality level of the low-quality good –that of the high-quality one being kept at its maximum level– we conclude that distance increases the average quality of the traded good.

Corollary 1 may be interpreted with reference to the "Alchian-Allen Conjecture" (AAC), which predicts that per unit trade costs increase the consumption share of high-quality goods relative to that of low-quality ones in the country of destination with respect to the country of origin (Alchian and Allen, 1964, p. 64). Per unit trade costs, indeed, increase the relative price of the low-quality goods

[19] Recall that the high-quality firm always selects the highest feasible quality level because of costless product design.

[20] In models of vertical product differentiation, equilibrium multiplicity at the quality-choice stage is usually dealt with the "risk dominance" criterion (Harsanyi and Selten, 1988). In our model, the complexity of equilibrium values does not allow for a direct application of this criterion. Yet, numerical simulations show that in the parameter constellation defined by Theorem 1, the risk-dominant equilibrium is that with the domestic high-quality producer and the foreign low-quality one. Another possible way out of equilibrium multiplicity is to assume that firms choose qualities sequentially, and then set prices simultaneously. In this case, the quality leader would choose the high-quality variant, and the follower would be forced to fill the low-quality product niche. Thus, the equilibrium outcome would depend on how the leader and follower roles are assigned to the firms. However, unless there were a strong argument to assign these roles, both cases would be possible, therefore the actual outcome of the interaction for $t \in [0, \tilde{t})$ would still be undetermined.

at destination.[21] The AAC compares consumption shares, thus its mechanism cannot be directly compared to ours. This notwithstanding, the present analysis suggests that per unit trade costs should also cause an increase in the average quality of consumption because *the average quality of the goods itself increases due to trade*, not only because the relative consumption of goods of *given* quality is modified. In particular, the quality level of the imported low-quality goods is higher, the farther away the trading partners are; and the farther the high-quality foreign producer is, the higher the quality of the domestic production. The same message can be drawn also from Corollary 2, which, as already explained, combines the effects of the two possible one-way trade configurations.

Zeros and distance An issue, which is empirically relevant, is that of trade zeros. Following the definition of (Baldwin and Harrigan, 2011, p. 16), a zero is "a trade flow which could have occurred but did not". Our model is in accordance with the factual evidence reported above on the positive relationship between the likelihood of zeros and the distance to the destination market. Under one-way trade, Theorem 1 states that a duopoly equilibrium with two active firms exists only when trade costs are low enough ($t < \bar{t}$). When t is large there is no duopoly equilibrium with two active firms and thus no trade. Furthermore, as noted above, for given qualities, the higher t is, the less likely the condition ensuring that second-stage prices are positive (see Remarks 1 and 2.) Although comparative statics cannot be performed on the value of $\bar{t}$, numerical exercises suggest that this cutoff value is increasing with $\bar{u}$, the upper bound of the feasible quality spectrum. This is in accordance with common sense: the wider the range is over which qualities can be selected, the easier it is to "relax price competition through product differentiation",[22] and thus the larger the distance compatible with viable trade. Stated differently, the wider the technical capacity to differentiate products within the industry, the more distant the trading partners can be. Finally, notice that the capacity to differentiate, as summarized by $\bar{u}$ also influences the equilibrium configuration. It is easy to ascertain that $\tilde{t}$ is increasing in $\bar{u}$: for given t, the wider the feasible quality range, the larger the parameter region is where the foreign firm may be the quality leader. The two-way trade analysis provides a similar intuition. For large enough t the low-quality firm prefers to match the quality level of its rival, completely giving up sales in the export market but simultaneously "deterring entry" in its own.

Trade policy Recent trade literature has started to explore strategic trade policies in markets with vertical differentiation (see, e.g., Herguera *et al.* 2000, 2002; Boccard and Wauthy 2005). This literature has focused on the phenomenon of *quality reversal* which occurs when, in a set-up of vertical product differentiation, a trade policy reverses the equilibrium quality choices of domestic versus foreign producers. These papers show that, in trade models with endogenous quality, there are large incentives for lagging industries to reverse the situation to their advantage by increasing trade barriers. A trade policy may, in fact, induce a quality reversal that turns the domestic producer into a quality leader in the market: in such a case, the domestic firm's profits and, consequently, the domestic welfare increase. As shown in Theorem 1, in our model distance may lead to a reversal of the quality ranking. Because of transport costs, in fact, the domestic firm can credibly commit to begin a price-war with the foreign firm, even if the latter sells initially the best available quality.

[21]For a recent treatment of the AAC see Hummels and Skiba (2004).
[22]Gabszewicz and Thisse (1979); Shaked and Sutton (1982).

When $\tilde{t} \leq t < \bar{t}$, distance makes the domestic firm be more aggressive at the quality stage, with the foreign firm that can only accommodate with a low-quality good. Our one-way trade analysis thus suggests that trade policy has a scope that is limited geographically to neighboring countries; when trade partners are distant, it is strategic interaction itself that induces the domestic producer to become of high quality.[23] The two-way trade extension provides a somehow more nuanced message. In this scenario as well a high t induces the firms to set the maximal quality level, which implies that every producer is the quality leader in its own market. Yet, in this case, no actual trade takes place because transport costs with a homogeneous product make export (and import) not profitable. This certainly increases the producer's profits, but it may increase or reduce consumer surplus, depending on the values of $\overline{u}$ and t.

6 Conclusion

This paper contributes to the vast and growing literature on trade and quality by identifying a mechanism explaining the observed increase in unit values (and thus quality) with the distance of the country of shipping. This mechanism, which complements the ones that are usually reckoned as shaping quality-differentiated trade flows, acts through the effect of distance on the strategic behavior of firms. In fact, the literature usually seeks the determinants of firms' specialization, trade viability and trade flow characteristics among ex ante productivity (broadly speaking) and/or preference differences between firms and countries. In contrast, by removing any ex ante asymmetry (except location) between trading partners, we have delved into the interaction of distance and strategic behavior in determining both the roles (high- versus low-quality producer) of domestic and foreign firms and the effects of distance on quality-differentiated trade flows. In particular, we have analyzed the relationship between trade, quality and distance by means of a parsimonious trade model with endogenous quality choice and oligopolistic competition. We have modeled the interaction between firms as a two-stage game (quality design, then price-setting) with simultaneous moves at each stage. We have identified the conditions under which trade is viable and have characterized the subgame-perfect Nash equilibrium of such a game both under one-way and two-way trade. Our main result –distance unambiguously increases the average quality level of the traded good– holds in both set-ups.

[23]One could argue that a high level of protection against foreign competition might reduce the incentive for domestic firms to produce high quality goods as in Herguera *et al.* (2002). Yet, in our model quality is costless, and, all else equal, for the high-quality producer an increase in quality yields a higher surplus extraction and a higher degree of product differentiation at zero cost. Therefore, this firm can charge higher prices earning higher profits and, hence, has no incentives to reduce its quality level.

A Proof of Proposition 1

We prove Proposition 1 through a series of lemmata.

Lemma 2. *Let $u_h^D \geq u_l^F$, then the unique maximizer of $\hat{\pi}(u_h^D)$ is $u_h^{D*} = \bar{u}$.*

Proof. (i) Consider the range $u_h^D \in (\frac{u_l^F(u_l^F-t)}{u_l^F-2t}, \bar{u}]$ first. In this case the market structure is duopolistic and second stage profits are the same as in (4). The partial derivative of $\hat{\pi}_h(u_h^D)$ w.r.t. u_h^D is

$$\frac{\partial \hat{\pi}_h^D}{\partial u_h^D} = \frac{u_h^D(t + 2u_h^D - 2u_l^F)f(u_h^D)}{(u_h^D - u_l^F)^2(4u_h^D - u_l^F)^3} \tag{A.1}$$

where

$$f(u_h^D) = \left[8(u_h^D)^3 - (u_h^D)^2(4t + 14u_l^F) - u_h^D u_l^F t - 10u_h^D(u_l^F)^2 - 4(u_l^F)^3 + 2(u_l^F)^2 t\right] \tag{A.2}$$

The denominator and the first two terms of the numerator are positive. Our task is therefore to prove that the polynomial $f(u_h^D)$ is positive. Notice that $f(\frac{u_l^F(u_l^F-t)}{u_l^F-2t}) > 0$. Consider now the first-order derivative of (A.2)

$$f'(u_h^D) = 24(u_h^D)^2 - 4(2t + 7u_l^F)u_h^D - (t - 10u_l^F)u_l^F.$$

It can be proved that for all $u_h^D \geq \frac{u_l^F(u_l^F-t)}{u_l^F-2t}$ this expression is positive, and thus so is $f(u_h^D)$. This implies that (A.1) is positive for all $u_h \in (\frac{u_l^F(u_l^F-t)}{u_l^F-2t}, \bar{u}]$ and ultimately that the profit-maximizing quality is $\bar{u}$.

(ii) Consider now the interval $[u_l^F, \frac{u_l^F(u_l^F-t)}{u_l^F-2t}]$. In this case, both the price and the demand of the foreign low-quality firm are zero under the pricing rules (5), thus (2) no longer defines the demand system. The price charged by the domestic high-quality firm is then its best reply at the second stage against the foreign low-quality firm setting a zero price, namely, $\tilde{p}_h^D(u_h^D, u_l^F) = \frac{1}{2}(u_h^D - u_l^F + t)$, and its profits are $\tilde{\pi}_h^D(u_h^D, u_l^F) = \tilde{p}_h^D(\cdot)(1 - \frac{\tilde{p}_h^D(\cdot)-t}{u_h^D-u_l^F})$. It is then a matter of simple calculations to ascertain that $\tilde{\pi}_h^D$ is maximized for $u_h^D = \frac{u_l^F(u_l^F-t)}{u_l^F-2t}$.

(iii) It is easy to show that for all $\bar{u}$

$$\tilde{\pi}_h^D\left[\frac{u_l^F(u_l^F-t)}{u_l^F-2t}\right] < \hat{\pi}_h^D(\bar{u}).$$

(iv) Notice that if $u_h^D = u_l^F$, the good is homogeneous and thus equations in (2) no longer represent demands. In this case the price of the domestic high-quality good is $t - \varepsilon$, while that of the foreign low-quality one is zero. At these prices the demand of the foreign good is zero, while the one of the domestic firm is $1 - \frac{t-\varepsilon}{u_l^F}$. Accordingly, its profits are $\left(1 - \frac{t-\varepsilon}{u_l^F}\right)(t - \varepsilon)$. It is easily ascertained that this level of profits falls short of $\hat{\pi}_h^D(\bar{u})$.

□

We now consider the low-quality firm. We state the following:

Lemma 3. *Let $u_h^D \geq u_l^F$, then there exists a unique $\bar{t} \in (0, u_l^F)$ such that for all $t < \bar{t}$ the unique maximizer of $\hat{\pi}_l^F(\cdot)$, $u_l^{F*}(u_h^D) \in (0, u_h^D)$.*

Proof. (i) Assume that $u_l^F < u_h^D$ first and consider the derivative of $\hat{\pi}_l^F$ w.r.t. u_l^F.

$$\frac{\partial \hat{\pi}_l^F}{\partial u_l^F} = -\frac{u_h^D \left[{u_l^F}^2 - u_l^F(t + u_h^D) + 2tu_h^D \right] g(u_l^F)}{{u_l^F}^2 (u_h^D - u_l^F)^2 (4u_h^D - u_l^F)^3}, \tag{A.3}$$

where

$$g(u_l^F) \equiv \left[(u_l^F)^3 (7u_h^D - 2t) + (u_l^F)^2 \left[9tu_h^D - 11(u_h^D)^2 \right] + u_l^F \left(4(u_h^D)^3 - 18t(u_h^D)^2 \right) + 8t{u_h^D}^3 \right].$$

First of all notice, that over the relevant interval $[0, u_h^D)$ the derivative (A.3) is continuous in u_l^F. Second, notice that concavity of (A.3) with respect to u_l^F requires that $t < \bar{t}$, with $0 < \bar{t} < u_l^F$.[24] We now proceed by finding the zeros of (A.3). This function has five roots, namely, the two zeros of the first term at the numerator, and three zeros of $g(u_l^F)$. The roots of the first term are real, but they can be disregarded as candidate maximizers because, although lying in the interval $[0, u_h^D]$, they do not fulfill local second-order conditions and in correspondence to these values $\hat{\pi}_l^F(\cdot) = 0$.[25] Consider now the remaining factor, the polynomial function $g(u_l^F)$, which is a one-parameter family of cubics depending on $t > 0$. First, note that $g(0) = 8t(u_h^D)^3 > 0$. Since $\lim_{u_l^F \to -\infty} g(\cdot) = -\infty$, this implies that $g(u_l^F)$ and consequently (A.3) admits a negative real root. In the following, we prove that $g(\cdot)$ admits two further real roots for every value of $t > 0$, but only one of them belongs to the relevant interval $[0, \bar{u}_h^D)$, whereas the other one is necessarily larger than u_h^D and thus not acceptable. To show this, notice that $g(u_h^D) = -3t(u_h^D)^3 < 0$ and $\lim_{u_l^F \to \infty} g(\cdot) = \infty$. By continuity, the function $g(\cdot)$ (and thus (A.3)) must cross the real axis at a value larger than u_h^D. As noted above this root is not acceptable. Consequently there exists another value $u_l^{F*} \in (0, u_h^D)$ such that $g(u_l^{F*}) = \frac{\partial \hat{\pi}_l^F (u_l^{F*}, u_h^D)}{\partial u_l^F} = 0$. Since the solutions to the first term of (A.3) are internal to $[0, u_h^D]$ and correspond to local minima, the solution $u_l^{F*}(u_h^D)$ lies between them and is a local maximum for all $t \in [0, \bar{t}]$.

(ii) Assume now that $u_l^F = u_h^D$. In this case, (2) is no longer the demand system because price competition with a homogeneous good triggers a Bertrand war. It is straightforward to ascertain that, in this case, the optimal price of the high-quality firm is $t - \varepsilon$ and that of the low-quality one is 0. Consequently, no consumer patronizes the foreign producer, and thus its profits are zero.

□

Lemma 4. *There exists a unique cutoff $\tilde{u} > u_l$ such that for all the $\bar{u} > \tilde{u}$, the pair $(\bar{u}, \underline{u}^F)$, where $\underline{u}^F \equiv u_l^{F*}(\bar{u})$, is a couple of mutual best replies at the quality-choice stage of game Γ for the domestic and foreign firms, respectively, and thus they are the quality levels chosen at a subgame-perfect Nash equilibrium of this game.*

Proof. The proof of this lemma requires that no firm has a profitable deviation from the candidate equilibrium strategies $\bar{u}$ for firm D and $\underline{u}^F$ for firm F.[26] Note that the robustness of these strategies has to be checked against unilateral deviations in the whole strategy space, not only in that defined

[24] The cumbersome expression of $\bar{t}$ is available upon request.
[25] Calculations are available upon request.
[26] Recall that we summarize firms' strategies by reporting optimal qualities only, $(\hat{p}_j^i, u_j^{i*})$ is represented by $(u_j^{i*}(\bar{u}))$.

by case (i). In other words, to demonstrate Lemma 3 we need to show that no firm wants to leapfrog its rival (see, for example, Motta *et al.* 1997).

(i) Consider firm D first. We need to prove that this firm has no profitable deviations from $(\bar{u})$ when its rival plays $(\underline{u}^F)$. Clearly, the only strategy (sub-)space where deviations have to be looked for is $[0, \underline{u}^F]$. In this case, the actual high-quality producer is the foreign firm, while the domestic one plays the role of the low-quality producer. Thus we need to re-define firm D's profits to take into account this fact. Let $u_{\mathcal{L}}^D \in [0, \underline{u}^F]$ and $p_{\mathcal{L}}^D$ be the quality level and the price firm D deviates to. Accordingly, let its deviation demand be[27]

$$\mathcal{D}_{\mathcal{L}}^D = \frac{\hat{p}_l^F(\cdot) + t - p_{\mathcal{L}}^D}{\underline{u}^F - u_{\mathcal{L}}^D} - \frac{p_{\mathcal{L}}^D}{u_{\mathcal{L}}^D}. \tag{A.4}$$

Thus the deviation profits are

$$\pi_{\mathcal{L}}^D = p_{\mathcal{L}}^D \left[\frac{\hat{p}_l^F(\cdot) + t - p_{\mathcal{L}}^D}{\underline{u}^F - u_{\mathcal{L}}^D} - \frac{p_{\mathcal{L}}^D}{u_{\mathcal{L}}^D} \right]. \tag{A.5}$$

Simple calculations show that there exists a unique price that maximizes (A.5), namely, $\hat{p}_{\mathcal{L}}^D = \frac{u_{\mathcal{L}}^D[2t\bar{u} + \bar{u}\underline{u}^F - (\underline{u}^F)^2]}{2\underline{u}^F(4\bar{u} - \underline{u}^F)}$, which can be plugged back into (A.5) to obtain the expression for the deviation profits as a function of quality levels only:

$$\hat{\pi}_{\mathcal{L}}^D(u_{\mathcal{L}}^D, \bar{u}, \underline{u}^F) = \frac{[2t\bar{u} + (\bar{u} - \underline{u}^F)\underline{u}^F]^2 u_{\mathcal{L}}^D}{4\underline{u}^F(4\bar{u} - \underline{u}^F)^2(\underline{u}^F - u_{\mathcal{L}}^D)}. \tag{A.6}$$

It can be proved that the demand $\mathcal{D}_{\mathcal{L}}^D$ is always increasing in $u_{\mathcal{L}}^D$, but its upper bound stops growing as $u_{\mathcal{L}}^D$ hits $\hat{u}_{\mathcal{L}}^D = \frac{2\bar{u}\underline{u}^F[3\underline{u}^F - 2t]}{\underline{u}^F[7\bar{u} - \underline{u}^F] - 2t\bar{u}} < \underline{u}^F$. For all $u_{\mathcal{L}}^D \in [\hat{u}_{\mathcal{L}}^D, \underline{u}^F]$, the deviating firm's profit is $(1 - \frac{\hat{p}_{\mathcal{L}}^D}{u_{\mathcal{L}}^D})p_{\mathcal{L}}^D$, which is always increasing in $u_{\mathcal{L}}^D$, and thus the profit-maximizing quality level is $\underline{u}^F$.[28] The corresponding profit is:

$$\hat{\pi}_{\mathcal{L}}^D(u_{\mathcal{L}}^{D*}, \bar{u}, \underline{u}^F) = \pi_{\mathcal{L}}^{D*}(\bar{u}, \underline{u}^F) = \frac{\bar{u}[\underline{u}^F(7\bar{u} - \underline{u}^F) - 2t\bar{u}][\underline{u}^F(\bar{u} - \underline{u}^F) + 2t\bar{u}]}{4\underline{u}^F(4\bar{u} - \underline{u}^F)^2}. \tag{A.7}$$

Direct comparison of $\hat{\pi}_h^D(\bar{u}, \underline{u}^F)$ and $\pi_{\mathcal{L}}^{D*}(\bar{u}, \underline{u}^F)$ indicates that

$$\hat{\pi}_h^D(\bar{u}, \underline{u}^F) > \pi_{\mathcal{L}}^{D*}(\bar{u}, \underline{u}^F). \tag{A.8}$$

In principle, there is another possible deviation available to the domestic firm, namely, to set $u_{\mathcal{L}}^D = \underline{u}^F$ and $p_{\mathcal{L}}^D = \hat{p}_l^F(\bar{u}, \underline{u}^F) + t - \varepsilon$. It can be proved, however, that the profit earned in this case never exceeds $\hat{\pi}_h^D(\bar{u}, \underline{u}^F)$ as long as $\bar{u} > \tilde{u}$. Thus, we conclude that there exists no profitable deviation from $\bar{u}$ for firm D when firm F selects $\underline{u}^F$.

(ii) Firm F has no profitable deviation for $u_l^F < \bar{u}$, because $\underline{u}^F$ is the unique profit maximizer for

[27] The demand is as in (A.5) as long as $p_{\mathcal{L}}^D$ and $u_{\mathcal{L}}^D$ are such that $\frac{\hat{p}_l^F(\cdot)+t-p_{\mathcal{L}}^D}{\underline{u}^F - u_{\mathcal{L}}^D} < 1$. If $\frac{\hat{p}_l^F(\cdot)+t-p_{\mathcal{L}}^D}{\underline{u}^F - u_{\mathcal{L}}^D} \geq 1$ then the demand is $1 - \frac{p_{\mathcal{L}}^D}{u_{\mathcal{L}}^D}$.

[28] Notice that a Bertrand war is avoided here because the lower bound to the foreign good price is equal to t, the transport cost. Furthermore, at the optimal deviation quality the condition defined by Remark 1 is not met. Thus the demand for the foreign good goes down to zero, and the market is solely served by the deviating firm.

all $u_l^F < u_h^D$, and, by construction, it cannot deviate to a quality higher than $\bar{u}$, the maximum level of the quality spectrum. Finally, a deviation to $\bar{u}$ is not profitable because it would entail a price war over a homogeneous good.

$\square$

Proposition 1 is obtained by combining the results of Lemmata 2-4.

B Proof of Proposition 2

Although we are able to compute explicitly the equilibrium value for u_l^F, this turns out to be exceedingly cumbersome and thus not very informative.[29] We shall therefore perform comparative statics analysis by means of indirect methods.

As a general remark, notice that $\bar{t} < \frac{\underline{u}^F}{2}$.

(i) Consider the function $g(u_l^F)$ defined in Lemma 3:

$$g(u_l^F) = (u_l^F)^3(7\overline{u} - 2t) + (u_l^F)^2\left[9t\overline{u} - 11(\overline{u})^2\right] + u_l^F\left(4(\overline{u})^3 - 18t(\overline{u})^2\right) + 8t\overline{u}^3,$$

within the interval $\left[\frac{4}{7}\overline{u}, \frac{29}{49}\overline{u}\right]$. At the left-end boundary of the interval the function's value is $g\left(\frac{4}{7}\overline{u}\right) = \frac{96t\overline{u}^3}{343} > 0$, while at the right-end its value is

$$g\left(\frac{29}{49}\overline{u}\right) = \frac{(9973t - 4060\overline{u})\overline{u}^3}{117649}.$$

Notice, however, that the restriction $t < \frac{u_l^F}{2}$ implies $t < \frac{29\overline{u}}{98}$, and thus the value of $g\left(\frac{29}{49}\overline{u}\right)$ can be evaluated as

$$\frac{(9973t - 4060\overline{u})\overline{u}^3}{117649} < -\frac{108663}{11529602}\overline{u}^4 < 0.$$

By continuity, the relevant root of $g(u_l^F)$, that is, the optimal strategy for the foreign firm, belongs to the interval $\left(\frac{4}{7}\overline{u}, \frac{29}{49}\overline{u}\right)$.

(ii) Consider $g(u_l^F, t)$ as a two-variable function of u_l^F and t, restricted to the domain $\left[\frac{4}{7}\overline{u}, \frac{29}{49}\overline{u}\right] \times \left[0, \frac{29}{98}\overline{u}\right]$. By the Implicit Function Theorem, in this domain there exists a C^1 function $\underline{u}^F(t)$ such that:

$$\underline{u}^{F\prime}(t) = -\frac{\frac{\partial g}{\partial t}}{\frac{\partial g}{\partial \underline{u}^F}} = \frac{2(\underline{u}^F)^3 - 9\overline{u}(\underline{u}^F)^2 + 18\overline{u}^2\underline{u}^F - 8\overline{u}^3}{3(7\overline{u} - 2t)(\underline{u}^F)^2 + 2(9t\overline{u} - 11\overline{u}^2)\underline{u}^F + 4\overline{u}^3 - 18t\overline{u}^2}. \tag{B.1}$$

To evaluate the sign of (B.1) consider first the function at the numerator

$$N(\underline{u}^F) = 2(\underline{u}^F)^3 - 9\overline{u}(\underline{u}^F)^2 + 18\overline{u}^2\underline{u}^F - 8\overline{u}^3.$$

Notice that $N\left(\frac{4}{7}\overline{u}\right) = -\frac{96\overline{u}^3}{343}$, $N\left(\frac{29}{49}\overline{u}\right) = -\frac{3084\overline{u}^3}{15625}$, and since

$$N'(\underline{u}^F) = 6[(\underline{u}^F)^2 - 3\overline{u}\underline{u}^F + 6\overline{u}^2] > 0,$$

[29] The value of $\underline{u}^F$ is available upon request from the authors.

we conclude that $N(\underline{u}^F)$ is monotonically increasing and negative in the whole domain $\left[\frac{4}{7}\overline{u}, \frac{29}{49}\overline{u}\right]$.
Move now to the function at the denominator

$$D(\underline{u}^F, t) = 3(7\overline{u} - 2t)(\underline{u}^F)^2 + 2(9t\overline{u} - 11\overline{u}^2)\underline{u}^F + 4\overline{u}^3 - 18t\overline{u}^2.$$

Since $\dfrac{\partial D}{\partial t} < 0$ for every admissible $\underline{u}^F$, the gradient of $D(\cdot)$ never vanishes in the rectangle under scrutiny, hence we evaluate the function at its boundary to establish whether it may change its sign.

Firstly the evaluation of $D(\cdot)$ at the boundaries for the choice variable yields:

$$\begin{aligned} D\left(\frac{4}{7}\overline{u}, t\right) &= -42(79t + 14\overline{u})\overline{u}^2 < 0, \\ D\left(\frac{29}{49}\overline{u}, t\right) &= -49(22686t + 3997\overline{u})\overline{u}^2 < 0, \end{aligned}$$

for all values of t.

Secondly the evaluation of $D(\cdot)$ at the boundaries for the unit transport cost returns:

$$D\left(\underline{u}^F, 0\right) = \overline{u}[21(\underline{u}^F)^2 - 22\underline{u}^F\overline{u} + 4\overline{u}^2] < -\frac{417}{343}\overline{u}^3 < 0,$$

and

$$\begin{aligned} D\left(\underline{u}^F, \frac{29}{98}\overline{u}\right) &= -\frac{\overline{u}}{49}(65\overline{u}^2 + 817\overline{u}\underline{u}^F - 942(\underline{u}^F)^2) < \\ &< -\frac{\overline{u}}{49}\left(65\overline{u}^2 + \frac{13237}{29}(\underline{u}^F)^2\right) < 0. \end{aligned}$$

Since $D(\cdot)$ is negative on the boundary of the rectangle and has no stationary points inside it, its negative sign over the whole rectangle is ensured. Then, we conclude that the sign of (B.1) is positive.

C Proof of Proposition 3

Similarly to the proof of Proposition 1, we proceed by demonstrating a series of Lemmata.

Lemma 5. *Let $u_l^D \leq u_h^F$ and $\bar{u} > \frac{1}{2}\left(t + u_l^D + \sqrt{t^2 + (u_l^D)^2}\right) \equiv \hat{u}$, then the unique maximizer of π_h^F is $\bar{u}$.*

Proof. First of all recall (see Remark 2) that, if $\bar{u} \leq \hat{u}$, there is no quality level along the feasible quality spectrum for the foreign firm compatible with a positive demand. Remark 2 implies also that if $\bar{u} < \hat{u}$, the foreign firm cannot optimally select any quality level below $\hat{u}$ because this would entail a zero demand. Focus therefore on the interval $u_h^F \in (\hat{u}, \bar{u}]$ and consider now the partial derivative of the foreign firm's profits w.r.t. u_h^F:

$$\frac{\partial \pi_h^F}{\partial u_h^F} = \frac{h(u_h^F)[2u_h^F(u_h^F - u_l^D) - t(2u_h^F - u_l^D)]}{(u_h^F - u_l^D)^2(4u_h^F - u_l^D)^3}, \tag{C.1}$$

where

$$h(u_h^F) = 8(u_h^F)^2(t + u_h^F) + 5(u_l^D)^2(t + 2u_h^F) - 2u_h^F u_l^D(5t + 7u_h^F) - 4(u_l^D)^3. \tag{C.2}$$

Notice that the denominator of (C.1) is positive for all $u_l^D < u_h^F$ and that the term within square brackets at the numerator is positive when $u_h^F > \hat{u}$. Moving to the polynomial (C.2), it can be ascertained that $h(u_h^F) > 0$. Consider now the partial derivative $h'(u_h^F)$. Standard computations show that $h'(\cdot) > 0 \; \forall u_h^F > u_l^D$. Thus, maximization of $\hat{\pi}_l^F(\cdot)$ requires the foreign firm to hit the upper bound of the quality spectrum. □

Lemma 6. *Let $u_l^D \leq u_h^F$, then (i) if $t < \tilde{t}$ there exists a unique maximizer of $\hat{\pi}_l^D$, $u_l^{D*}(u_h^F) \in (0, u_h^F)$; (ii) if $t \geq \tilde{t}$ the unique maximizer of $\hat{\pi}_l^D$ is u_h^F.*

Proof. (i) Assume $u_l^D < u_h^F$ and $t < \frac{u_h^F}{11}$. Consider the partial derivative

$$\frac{\partial \hat{\pi}_l^D(\cdot)}{\partial u_l^D} = \frac{u_h^F(t + u_h^F - u_l^D)m(u_l^D)}{(u_h^F - u_l^D)^2(4u_h^F - u_l^D)^3}, \tag{C.3}$$

where $m(u_l^D) \equiv (u_l^D)^2(7u_h^F - 2t) + u_h^F u_l^D(t - 11u_h^F) + 4(u_h^F)^2(t + u_h^F)$.

The denominator and the first term of the numerator are positive as long as $u_l^D < u_h^F$. Thus we focus on the polynomial $m(\cdot)$. This function has two real roots within the interval $(0, u_h^F)$ as long as $t < \frac{u_h^F}{11}$. Furthermore, only one of these roots satisfies the second-order condition for a maximum. Label this solution $u_h^{F*}(u_l^D)$.

(ii) Keep assuming that $u_l^D < u_h^F$ but move now to the case $t \geq \frac{u_h^F}{11}$. In this case, (C.3) has no real roots within $(0, u_h^F)$, and it is always increasing over this interval. Thus the domestic firm finds it profitable to increase its product's quality as much as possible, in principle, up to u_h^F. This would eventually violate the condition reported in Remark 2. However it can be easily ascertained that as the quality of the domestic firm rises to u_h^F, its demand increases while the optimal price decreases. The demand stops growing as the condition in Remark 2 is met. Any further increase in u_l^D would entail no increase in the demand but a reduction in the optimal price, resulting in a profit loss. These observations allow us to conclude that, in this case, the profit-maximizing quality level is $u_l^D = \frac{2u_h^F(u_h^F - t)}{2u_h^F - t} < u_h^F$.

Finally consider the situation $u_l^D = u_h^F$. In this case the good is homogeneous and a price war arises. Again (2) is no longer the demand system. Optimal prices are thus $t - \varepsilon$, with ε positive and arbitrarily small for the domestic firm and 0 for the foreign one. At these prices, the foreign producer has no demand, while that of the domestic firm is $1 - \frac{t-\varepsilon}{u_h^F}$, such that its profits amount to $\tilde{\pi}_l^D \equiv \left(1 - \frac{t-\varepsilon}{u_h^F}\right)(t - \varepsilon)$. It can be proved that, while $\tilde{\pi}_l^D > \hat{\pi}_l^D \left[\frac{2u_h^F(u_h^F - t)}{2u_h^F - t}\right]$, $\tilde{\pi}_l^D \leq \hat{\pi}_l^D(u_l^{D*}) \Leftrightarrow t \in [0, \tilde{t}]$, where $\tilde{t} = \dfrac{u_h^F}{229}\left[137 - \dfrac{1143^{2/3}}{\sqrt[3]{1131 - 458\sqrt{6}}} - 6\sqrt[3]{3\left(1131 - 458\sqrt{6}\right)}\right] \approx \dfrac{3u_h^F}{126}$.

□

Lemma 7. *(i) Let $t < \tilde{t}$ and $\bar{u} > \hat{u}$. Then, the pair $(\bar{u}, \underline{u}^D)$, where $\underline{u}^D \equiv u_l^{D*}(\bar{u})$ is a couple of mutual best replies at the quality-choice stage of game Γ for the domestic and foreign firm respectively. Thus they are the quality levels chosen at a subgame-perfect Nash equilibrium of this game. (ii) Let $t > \tilde{t}$ or*

$\bar{u} < \hat{u}$. Then there is no subgame-perfect Nash equilibrium of game Γ with the domestic firm producing the low-quality good and the foreign firm supplying the high-quality one.

Proof. (i) It is necessary to show that there are no profitable unilateral deviations from $(\bar{u}, \underline{u}^D)$ in the strategy space. Because this proof parallels that of Lemma 4, it will be just sketched. If the domestic low-quality producer leapfrogs downwards its rival, it finds it profitable to increase the quality of its variant up to $\underline{u}^D$. In this case, the good would be homogeneous, and thus the optimal price of the deviating firm would be $\hat{p}_l^D(\bar{u}, \underline{u}^D) - t - \varepsilon$. Accordingly, all of the demand would be served by the foreign firm. It is a matter of calculations to prove, however, that although positive, the deviation payoff is lower than that of the strategy under scrutiny. Thus the deviation is not profitable.

Consider now the domestic high-quality producer. From Lemma 6 we know that as long as $t < \tilde{t}$, the unique maximizer of the domestic firm's profits is $\underline{u}^D \in [0, \bar{u}]$. Deviations strictly above $\bar{u}$ are not admissible by construction.

(ii) From Lemma 6 (ii)-(iii) it follows that if $t > \tilde{t}$, the domestic firm wants to deviate from the prescribed strategy $\underline{u}^D$ and set $u_l^D = \bar{u}$. Furthermore, if $\bar{u} < \hat{u}$, there is no quality level for the foreign firm compatible with a positive demand under duopoly pricing. This firm may therefore undercut its rival and become the low-quality producer earning positive profits (see Lemma 7 (i)). In both cases, $(\bar{u}, \underline{u}^D)$ cannot be an equilibrium.

□

The results of Lemmata 5 to 7 prove Proposition 3.

D Proof of Proposition 4

The optimal quality level of the domestic firm is:

$$\underline{u}^D(t) = \frac{\bar{u}\left(11\bar{u} - t - \sqrt{9\bar{u}^2 - 102\bar{u}t + 33t^2}\right)}{14\bar{u} - 4t}.$$

It is easily ascertained that $\underline{u}^D(0) = \frac{4}{7}\bar{u}$ and $\underline{u}^D(\frac{\bar{u}}{11}) = \frac{4}{5}\bar{u}$. Moreover,

$$\frac{\partial \underline{u}^D}{\partial t} = \frac{\bar{u}^2\left[15\bar{u}\sqrt{(3\bar{u} - t)(\bar{u} - 11t)} + \sqrt{3}\bar{u}(113\bar{u} - 43t)\right]}{2\bar{u}(7\bar{u} - 2t)^2\sqrt{(3\bar{u} - t)(\bar{u} - 11t)}} > 0 \;\forall t < \frac{\bar{u}}{11}.$$

E Proof of Section 4

Equilibrium existence. As a preliminary observation, notice that at any equilibrium of Γ' with positive trade flows the two firms select different quality levels. Indeed, if firms selected the same quality level, the Bertrand paradox would lead them to set the price $t - \varepsilon$ and serve its home market only. Since firms are symmetric at the outset (each has a disadvantage relative to the rival due to the transport cost in its export market, and, symmetrically, an advantage in the home one) the game with positive trade flows has two symmetric branches. In the first the firm in A produces the high-quality good and that in B the low-quality, in the second the symmetric configuration arises.

In the following, we shall limit ourselves to the analysis of the first case, because the second may be obtained with an appropriate change in the indexes. Since firms are single-product, we drop the indexes identifying the country in the quality levels.

The demands in country A are

$$\mathcal{D}_h^A = 1 - \theta_1^A, \quad \mathcal{D}_l^A = \theta_1^A - \theta_0^A, \tag{E.1}$$

with $\theta_1^A = \frac{p_h^A - p_l^A - t}{u_h - u_l}$ and $\theta_0^A = \frac{p_l^A + t}{u_l}$. Similarly, those in country B write

$$\mathcal{D}_h^B = 1 - \theta_1^B, \quad \mathcal{D}_l^B = \theta_1^B - \theta_0^B, \tag{E.2}$$

with $\theta_1^B = \frac{p_h^B + t - p_l^B}{u_h - u_l}$ and $\theta_0^B = \frac{p_l^B}{u_l}$. The total profits accruing to firms are $\Pi_h = \mathcal{D}_h^A p_h^A + \mathcal{D}_h^B p_h^B$ and $\Pi_l = \mathcal{D}_l^A p_l^A + \mathcal{D}_l^B p_l^B$. Simultaneous profit maximization yields the optimal prices $\hat{p}_h^A(u_h, u_l) = \frac{u_h[2(u_h - u_l) + t]}{4u_h - u_l}, \hat{p}_l^A(u_h, u_l) = \frac{(u_h - u_l + t)u_l}{4u_h - u_l}, \hat{p}_h^B(u_h, u_l) = \frac{2u_h(u_h - u_l) - t(2u_h - u_l)}{4u_h - u_l}, \hat{p}_l^B(u_h, u_l) = \frac{u_l(u_h - u_l) - t(2u_h - u_l)}{4u_h - u_l}$.[30] By plugging back these values into the profit functions we obtain

$$\hat{\Pi}_h(u_h, u_l) = \frac{8u_h^2(u_h - u_l)^2 + t^2(5u_h^2 - 4u_h u_l + u_l^2) - 4tu_h(u_h - u_l)^2}{(4u_h - u_l)^2(u_h - u_l)}, \tag{E.3}$$

$$\hat{\Pi}_l(u_h, u_l) = \frac{2u_h[u_l^2(u_h - u_l)^2 + t^2(2u_h - 2u_h u_l + u_l^2) - 2tu_l(u_h - u_l)^2]}{u_l(u_h - u_l)(4u_h - u_l)^2}. \tag{E.4}$$

Similarly, optimal prices may be plugged back into demands to observe that all demands are positive as long as $t < \frac{u_h u_l - u_l^2}{2u_h - u_l}$.

Consider quality choice now. The first-order derivative of $\hat{\Pi}_h(u_h, u_l)$ w.r.t. u_h is

$$\frac{\partial \hat{\Pi}_h(\cdot)}{\partial u_h} = \frac{32u_h^5 - 88u_l u_h^4 + (96u_l^2 - 8tu_l - 20t^2)u_h^3 - (56u_l^3 - 12tu_l^2 - 27t^2 u_l)u_h^2 + (16u_l^4 - 18t^2 u_l^2)u_h + 5t^2 u_l^3 - 4tu_l^4}{(u_h - u_l)(4u_h - u_l)^3}. \tag{E.5}$$

Under the assumptions that $0 < u_l < u_h$ the denominator is positive, so we focus on the numerator, which we denote $z(u_h)$. It may be easily ascertained that

$$\begin{gathered}\lim_{u_h \to -\infty} z(u_h) = -\infty, \quad \lim_{u_h \to 0} z(u_h) = t(5t - 4u_l) < 0, \\ \lim_{u_h \to u_l/4} z(u_h) = \frac{3}{16} u_l^3 (9u_l^2 - 18tu_l + 10t^2) > 0, \\ \lim_{u_h \to u_l} z(u_h) = -6t^2 u_l^3, \quad \lim_{u_h \to \infty} z(u_h) = \infty. \end{gathered} \tag{E.6}$$

The signs of the finite terms come from the assumptions on t that guarantee the existence of a duopoly equilibrium with positive trade flows, namely $t < \frac{31}{800} u_h$.[31] This implies that the fifth-degree polynomial has at least three real roots. Among these, the only maximum lies in $(\frac{u_l}{4}, u_l)$, which is clearly not acceptable. To ascertain that these are the only real roots, and therefore that the profit is increasing for all $u_h > u_l$, consider the second-order derivative $z''(u_h) = 640u_h^3 - 1056u_h^2 u_l + 2(288u_l^2 - 24tu_l - 60t^2)u_h + 2(27t^2 u_l + 12tu_l^2 - 56u_l^3)$. Its discriminant is $7077888(5t^3 - 2u_l^3)(125t^2 + 150t^2 u_l + 30tu_l^2 + 52u_l^3)$, which is negative under the assumptions for the existence of a duopoly equilibrium. This implies that $z''(\cdot)$ has only one real root, which, in turn entails that the first-order condition has

[30] Second-order conditions are fulfilled for any $u_h > u_l$.

[31] See Lemma 8 below.

only one inflection point and ultimately that it can have no more than three real roots. We conclude that the optimal value for u_h is $\overline{u}$. As for the low-quality producer, consider the derivative of (E.4) w.r.t. u_l:

$$\frac{\partial \hat{\Pi}_l(\cdot)}{\partial u_l} = \frac{2u_h(2t-7u_h)u_l^5+2u_h(18u_h^2-2t^2)u_l^4+2u_h(7t^2u_h-6tu_h^2-15u_h^3)u_l^3-2u_h(16t^2u_h^2-4tu_h^3-4u_h^4)u_l^2+44t^2u_h^4u_l-16t^2u_h^5}{(u_h-u_l)^2(4u_h-u_l)^3u_l} \tag{E.7}$$

Since by assumption $0 < u_l < u_h$ the denominator of (E.7) is always positive. So we focus on the numerator, which we call $k(u_l)$, to assess the existence of a best reply by the low-quality firm. The numerator is a fifth-degree polynomial, thus it has at most five real roots. Simple computations show the following.

$$\begin{aligned} \lim_{u_l \to -\infty} k(u_l) &= \infty, \quad k(0) = -15u_h^5 < 0; \\ k(u_h) = 6t^2u_h^5 &> 0, \quad \lim_{u_l \to \infty} k(u_l) = -\infty. \end{aligned} \tag{E.8}$$

Because $k(u_l)$ is continuous, a direct consequence of (E.8) is that it has one negative real root and another real root larger than u_h, which are both not acceptable. Now, two cases may emerge, namely either the remaining roots are all real, or two of them are complex and the remaining one is real. If one root only is real, it must lie between zero and u_h, and it corresponds to a minimum of the profit function because, in this case, $k(u_l)$ crosses the real axes from below. We disregard this case as it would imply that there is no internal solution to our maximization problem. Now, consider the case where the three remaining roots are real. The constraints defined by (E.8) imply that three sub-cases only may emerge, namely the three roots are (i) negative, (ii) larger than u_h or they lie in $(0, u_h)$. It is easy to ascertain that cases (i) and (ii) imply that the -only- root in $(0, u_h)$ corresponds to a minimum. In case (iii), by contrast, the first and third root correspond to minima, whereas the second root corresponds to a maximum. The existence of such a maximum relies on the relative size of u_h, u_l and t. To continue the analysis we shall therefore find sufficient conditions that guarantee the existence of such a maximum.

When $t = 0$ the only maximizer to (E.7) is $u_l = \frac{4u_h}{7}$. Consider now $t > 0$, direct inspection reveals that $k(\frac{4u_h}{7}) = \frac{192tu_h^5(54u_h+77)}{16807} > 0$, which means that the maximizer of $k(\cdot)$, if it exists, lies to the right of $\frac{4u_h}{7}$. Consider then $k(\frac{30u_h}{49}) = \frac{8u_h^5(56307517t^2+20793600tu_h+-1137150u_h^2)}{282475249}$, which is negative if $0 < t < 0.0652u_h$. This implies that $t < 0.0483u_h$ is sufficient to have an interior solution to the maximization problem.

Let us now label the two candidate maximizers $\overline{u}$ and $\underline{u}$ respectively. To ascertain that these are indeed the qualities chosen at a subgame-perfect Nash equilibrium of the game there remains to check the absence of profitable deviations. It is easy to prove that the high-quality producer does not want to leapfrog downwards its rival. Intuitively, it would lose its competitive edge in both markets, without any cost savings (quality improvement is costless). By contrast, it may want to match the quality of its rival, by setting its quality to $u_h = \underline{u}$, and its price to $t - \varepsilon$. In this way, it deters entry in market A at the cost of not entering market B. It is a matter of easy calculations to show that, in the region where two-way trade is possible, this deviation is not profitable. Consider now the low-quality producer. Any quality deviation must be such that $u_l = \overline{u}$. In this case, the low-quality firm matches the quality of its rival and sets a price equal to $t - \varepsilon$ in market B, giving up sales in market A. Its profit in this case is $(1 - \frac{t-\varepsilon}{\overline{u}})(t - \varepsilon)$. By comparing this profit to (E.4) it may be ascertained that such a deviation is not profitable for

$$t \leq \frac{u_h u_l (u_h - u_l)(20u_h^2 - 12u_h u_l + u_l^2 - \sqrt{368u_h^4 - 576u_h^3 u_l + 360u_h^2 u_l^2 - 96u_h u_l^3 + 9u_l^4})}{8u_h^4 + 24u_h^3 u_l - 44u_h^2 u_l^2 + 18u_h u_l^3 - 2u_l^4} \equiv \hat{t}. \tag{E.9}$$

Now, remember that the existence of a maximum of $k(\cdot)$ requires that $t < 0.0652\overline{u}$, thus we have to compare this value to $\hat{t}$. Direct comparison does not yield clear results, so we proceed by indirect methods. At the boundaries of relevant interval $(\frac{4}{7}\overline{u}, \frac{45}{49}\overline{u})$ the values of $\hat{t}$ are $0.387\overline{u}$ and $0.0389\overline{u}$. Similarly, the derivative of $\hat{t}$ w.r.t. u_l is positive at the lower bound of the interval and negative at the upper bound. Finally, within that interval, this derivative has one zero only ($u_l = 0.603\overline{u}$). This guarantees that $\hat{t}$ as a function of u_l is always larger than $0.0387\overline{u}$. We summarize the above observations in the following.

Lemma 8. *Let $0 < t < 0.0387\overline{u} \approx \frac{31}{800}\overline{u}$, then game Γ' has a unique subgame-perfet Nash equilibrium (up to a permutation in the firms' indexes). At this equilibrium the high-quality producer selects the top quality level $\overline{u}$ and the low-quality producer sets $\underline{u} \in (\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u})$.*

Effects of distance on quality. As in the main analysis, we shall rely on indirect methods. We restrict ourselves to the rectangle $[\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u}] \times [0, \frac{31}{800}\overline{u}]$ in the $(\underline{u}, t)$ space. By the implicit function theorem:

$$\underline{u}'(t) = -\frac{\dfrac{\partial k}{\partial t}}{\dfrac{\partial k}{\partial \underline{u}}} = \frac{-2\underline{u}^5 + 4t\underline{u}^4 + (6\overline{u}^2 - 14t\overline{u})\underline{u}^3 - (4\overline{u}^3 - 32t\overline{u}^2)u_l^2 - 44t\overline{u}^3\underline{u} + 16t\overline{u}^4}{-(35\overline{u} - 10t)\underline{u}^4 + (72\overline{u}^2 - 8t^2) + (21t^2\overline{u} - 18t\overline{u}^2 - 45\overline{u}h^3)\underline{u}^2 + (8\overline{u}^4 + 8t\overline{u}^3 - 32t^2\overline{u}^2) + 22t^2\overline{u}^3}. \tag{E.10}$$

Consider the numerator of (E.10) and, in particular, its behavior relative to t. Simple calculations show that the numerator is negative at $t = 0$ and $t = \frac{31}{800}\overline{u}$ for all $\underline{u} \in [\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u}]$. Furthermore, it is increasing in t for $0 < t < \frac{\underline{u}(4\overline{u}^3 - 9\overline{u}^2\underline{u} + 5\overline{u}^3)}{-22\overline{u} + 32\overline{u}^2\underline{u} - 21\overline{u}\underline{u}^2 + 8\underline{u}^3}$, and decreasing for $\frac{\underline{u}(4\overline{u}^3 - 9\overline{u}^2\underline{u} + 5\overline{u}^3)}{-22\overline{u} + 32\overline{u}^2\underline{u} - 21\overline{u}\underline{u}^2 + 8\underline{u}^3} < t < \bar{t}$. It is a matter of calculations to show that, at $t = \frac{\underline{u}(4\overline{u}^3 - 9\overline{u}^2\underline{u} + 5\overline{u}^3)}{-22\overline{u} + 32\overline{u}^2\underline{u} - 21\overline{u}\underline{u}^2 + 8\underline{u}^3}$, the numerator is negative, thus we conclude that so it is for all $t \in [0, \frac{31}{800}\bar{u})$. By the same token, consider the behavior of the numerator of (E.10) relative to $\underline{u}$. For all $t \in [0, \frac{31}{800}\overline{u})$, the numerator is negative at $\underline{u} = \frac{4}{7}\overline{u}$ and $\underline{u} = \frac{30}{49}\overline{u}$. Consider the second derivative of the numerator w.r.t. $\underline{u}$: $v(\underline{u}) = -40\underline{u}^3 + 48t\underline{u}^2 - 12\overline{u}\underline{u}(3\overline{u} - 7t) - 8\overline{u}^2(\overline{u} - 8t)$. The discriminant of this cubic is $55296\overline{u}(85\overline{u}^4 + 35t\overline{u}^3 - 2801t^2\overline{u}^2 + 1457t^3\overline{u} - 218t^4)$ which is positive. This implies that $v(\underline{u})$ has three real distinct roots. By noticing that $\lim_{\underline{u} \to -\infty} v(\underline{u}) = \infty$, $v(0) = -8\overline{u}^2(\overline{u} - t)$, $v(\frac{4}{7}\overline{u}) = \frac{8}{343}(1358t + 219\overline{u}) > 0$, $v(\frac{30}{49}\overline{u}) = \frac{8\overline{u}(449477t + 71486\overline{u})}{117649} > 0$ and $\lim_{\underline{u} \to \infty} v(\underline{u}) = \infty$, we conclude that $v(\cdot)$ is positive for all $\underline{u} \in [\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u}]$, which implies that the numerator of (E.10) is convex over that interval. This, together with the negative sign at the boundaries, ensures that the numerator is negative over the whole interval. Thus, we conclude that the numerator of (E.10) is negative over the rectangle $[\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u}] \times [0, \frac{31}{800}\overline{u}]$. Next, move to the denominator and consider its behavior relative to t. At $t = 0$ and $t = \frac{31}{800}\overline{u}$ the denominator is negative for all $\underline{u} \in [\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u}]$, furthermore, its second-order derivative w.r.t. t is $44\overline{u}^3 - 64\overline{u}^2\underline{u} + 42\overline{u}\underline{u}^2 - 16\underline{u}^3$, which is positive in the rectangle under scrutiny. This ensures that the denominator is negative for all $\underline{u} \in [\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u}]$ over $t \in [0, \frac{31}{800}\overline{u})$. It remains to check the negative sign of the denominator w.r.t. $\underline{u}$. This function is negative at $\underline{u} = \frac{4}{7}\overline{u}$ and $\underline{u} = \frac{30}{49}\overline{u}$ for $t \in [0, \frac{31}{800}\overline{u}]$. To prove that it is negative over this entire interval it is enough to observe that its second-order partial derivative $-60(7\overline{u} - 2t)\underline{u}^2 + 48(9\overline{u}^2 - t^2) - 6\overline{u}(15\overline{u} + 6t\overline{u} - 7t^2)$ is positive between its roots

$\frac{36\overline{u}^2-4t\pm\sqrt{2(123\overline{u}^4-60t\overline{u}^3+161t^2\overline{u}-70t^3\overline{u}+8t^4)}}{70\overline{u}^2-20t}$. The smallest of these roots lies in the interval $(0, \frac{4}{7}\overline{u})$ and the other is larger than $\frac{30}{49}\overline{u}$. This yields that the denominator is convex over the interval under scrutiny and, together with its negative sign at the boundaries, that it is negative over the whole interval itself.

The foregoing analysis ultimately entails the following.

Lemma 9. *Let* $(\underline{u}, t) \in (\frac{4}{7}\overline{u}, \frac{30}{49}\overline{u}) \times (0, \frac{31}{800}\overline{u})$*, then* $\frac{\partial \underline{u}}{\partial t} > 0$.

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