Online Appendix for "Screening Workers for Ability and Motivation"

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A Constraints

For worker AM the constraints are

$$w_{AM} - \frac{1}{2}e_{AM}^2 + \gamma e_{AM} \geq 0 \qquad (PC_{AM})$$

and

$$w_{AM} - \frac{1}{2}e_{AM}^2 + \gamma e_{AM} \geq w_{Am} - \frac{1}{2}e_{Am}^2 + \gamma e_{Am} \qquad (IC_{AM/Am})$$

$$w_{AM} - \frac{1}{2}e_{AM}^2 + \gamma e_{AM} \geq w_{aM} - \frac{1}{2}e_{aM}^2 + \gamma e_{aM} \qquad (IC_{AM/aM})$$

$$w_{AM} - \frac{1}{2}e_{AM}^2 + \gamma e_{AM} \geq w_{am} - \frac{1}{2}e_{am}^2 + \gamma e_{am}. \qquad (IC_{AM/am})$$

For type Am

$$w_{Am} - \frac{1}{2}e_{Am}^2 \geq 0 \qquad (PC_{Am})$$

and

$$w_{Am} - \frac{1}{2}e_{Am}^2 \geq w_{AM} - \frac{1}{2}e_{AM}^2 \qquad (IC_{Am/AM})$$

$$w_{Am} - \frac{1}{2}e_{Am}^2 \geq w_{aM} - \frac{1}{2}e_{aM}^2 \qquad (IC_{Am/aM})$$

$$w_{Am} - \frac{1}{2}e_{Am}^2 \geq w_{am} - \frac{1}{2}e_{am}^2. \qquad (IC_{Am/am})$$

For type aM

$$w_{aM} - \frac{1}{2}\theta e_{aM}^2 + \gamma e_{aM} \geq 0 \qquad (PC_{aM})$$

and

$$w_{aM} - \frac{1}{2}\theta e_{aM}^2 + \gamma e_{aM} \geq w_{AM} - \frac{1}{2}\theta e_{AM}^2 + \gamma e_{AM} \qquad (IC_{aM/AM})$$

$$w_{aM} - \frac{1}{2}\theta e_{aM}^2 + \gamma e_{aM} \geq w_{Am} - \frac{1}{2}\theta e_{Am}^2 + \gamma e_{Am} \qquad (IC_{aM/Am})$$

$$w_{aM} - \frac{1}{2}\theta e_{aM}^2 + \gamma e_{aM} \geq w_{am} - \frac{1}{2}\theta e_{am}^2 + \gamma e_{am}. \qquad (IC_{aM/am})$$

Finally, for worker am one has

$$w_{am} - \frac{1}{2}\theta e_{am}^2 \geq 0 \qquad (PC_{am})$$

and

$$w_{am} - \frac{1}{2}\theta e_{am}^2 \geq w_{AM} - \frac{1}{2}\theta e_{AM}^2 \qquad (IC_{am/AM})$$

$$w_{am} - \frac{1}{2}\theta e_{am}^2 \geq w_{Am} - \frac{1}{2}\theta e_{Am}^2 \qquad (IC_{am/Am})$$

$$w_{am} - \frac{1}{2}\theta e_{am}^2 \geq w_{aM} - \frac{1}{2}\theta e_{aM}^2. \qquad (IC_{am/aM})$$

Considering participation constraints, one can show that participation constraint PC_{aM} is automatically satisfied when PC_{am} and $IC_{aM/am}$ both hold. Also, participation constraint PC_{AM} is automatically satisfied when PC_{Am} and $IC_{AM/Am}$ are. Finally, once incentive constraint $IC_{Am/am}$ and participation constraint PC_{am} hold, then also participation constraint PC_{Am} is satisfied. So, when all worker types are expected to be hired by the firm and when there is truthful revelation (that is under full participation and full separation of types), it is only necessary to consider the participation constraint of the worst type am.

As for the incentive compatibility constraints, one can sum them two by two yielding a partial ranking of effort levels. In particular, adding $IC_{Am/am}$ to $IC_{am/Am}$ and summing $IC_{aM/AM}$ to $IC_{AM/aM}$ one has $e_{Aj} \geq e_{aj}$ $\forall j = M, m$, meaning that, given motivation, effort required must be higher the higher worker's ability. In the same way, adding $IC_{aM/am}$ to $IC_{am/aM}$ on the one hand and adding $IC_{AM/Am}$ to $IC_{Am/AM}$ on the other hand yields $e_{iM} \geq e_{im}$ $\forall i = A, a$. Namely, given ability, effort is higher the higher the motivation. Hence the monotonicity condition

$$e_{AM} \geq \max\{e_{aM}, e_{Am}\} \geq \min\{e_{aM}, e_{Am}\} \geq e_{am} \qquad (1)$$

holds. Condition (1) also allows us to eliminate some 'global' downward incentive constraints and focus on 'local' ones. Indeed, adding $IC_{AM/aM}$ and $IC_{aM/am}$ one obtains

$$w_{AM} - \frac{1}{2}e_{AM}^2 + \gamma e_{AM} \geq w_{am} - \frac{1}{2}\theta e_{am}^2 + \gamma e_{am} + \frac{1}{2}\Delta\theta e_{aM}^2.$$

But, when $e_{aM} \geq e_{am}$, the right-hand side of the above inequality is greater than $w_{am} - \frac{1}{2}e_{am}^2 + \gamma e_{am}$, which in turn implies that the global downward incentive constraint $IC_{AM/am}$ is satisfied when the two local incentives constraints $IC_{AM/aM}$ and $IC_{aM/am}$ are.[1]

What about intermediate types of workers aM and Am? Adding $IC_{Am/aM}$ and $IC_{aM/Am}$ one has

$$\frac{1}{2}\Delta\theta\left(e_{Am} - e_{aM}\right)\left(e_{Am} + e_{aM}\right) - \gamma\left(e_{Am} - e_{aM}\right) \geq 0,$$

[1] The same conclusion holds taking the two local incentive constraints $IC_{AM/Am}$ and $IC_{Am/am}$.

which is satisfied for all $e_{aM} = e_{Am}$ or for $e_{aM} \neq e_{Am}$ when either

$$e_{aM} > e_{Am} \quad \text{and} \quad e_{Am} + e_{aM} \leq \frac{2\gamma}{\Delta\theta}, \tag{2}$$

or

$$e_{Am} > e_{aM} \quad \text{and} \quad e_{Am} + e_{aM} \geq \frac{2\gamma}{\Delta\theta} \tag{3}$$

hold.

Using the same arguments as before, one can get rid of other global constraints. Suppose that condition (2) is verified: then, the sum of the local constraints $IC_{AM/aM}$ and $IC_{aM/Am}$ implies that the global constraint $IC_{AM/Am}$ is satisfied as well. Moreover, $IC_{aM/Am}$ and $IC_{Am/am}$ imply $IC_{aM/am}$. By the same token, suppose that condition (3) holds: then, one can prove that constraints $IC_{AM/Am}$ and $IC_{Am/aM}$ imply constraint $IC_{AM/aM}$ and also that $IC_{Am/aM}$ and $IC_{aM/am}$ can be used to eliminate $IC_{Am/am}$.

B Motivation prevails (Case $\mathcal{M}$)

B.1 Full separation and full participation

In Case $\mathcal{M}$, a separating equilibrium occurs if condition (2) holds, that is if $e_{aM} > e_{Am}$ and $e_{Am} + e_{aM} \leq \frac{2\gamma}{\Delta\theta}$ both hold. The implementability condition (1) thus becomes $e_{AM} > e_{aM} > e_{Am} > e_{am}$. At the solution with full participation, the binding constraints are the downward local incentive constraints $IC_{AM/aM}$, $IC_{aM/Am}$, $IC_{Am/am}$ and PC_{am}. Given the binding constraints, one can solve for the wage schedules and isolate information rents, whereby

$$w_{am} = \frac{1}{2}\theta e_{am}^2, \tag{4}$$

$$w_{Am} = \frac{1}{2}e_{Am}^2 \quad \underbrace{+\frac{1}{2}\Delta\theta e_{am}^2}_{\text{Info rent worker } Am} \quad , \tag{5}$$

$$w_{aM} = \frac{1}{2}\theta e_{aM}^2 - \gamma e_{aM} \underbrace{-\frac{1}{2}\Delta\theta e_{Am}^2 + \gamma e_{Am} + \frac{1}{2}\Delta\theta e_{am}^2}_{\text{Info rent worker } aM} \tag{6}$$

and finally

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma e_{AM} \underbrace{+\frac{1}{2}\Delta\theta e_{aM}^2 - \frac{1}{2}\Delta\theta e_{Am}^2 + \gamma e_{Am} + \frac{1}{2}\Delta\theta e_{am}^2}_{\text{Info rent worker } AM}. \tag{7}$$

All information rents include at least one term depending on $\Delta\theta$, as in Benchmark $U\mathcal{A}$. Only information rents of motivated types aM and AM contain a term which depends on motivation γ and which comes from incentive constraint $IC_{aM/Am}$ linking the two unidimensional screening problems of Benchmark $U\mathcal{A}$.

We then substitute the expressions for the wages into the firm's objective function (i.e. into program SB) and maximise with respect to effort levels only. This yields optimal effort levels

$$e_{AM}^{SB\mathcal{M}} = 1 + \gamma = e_{AM}^{FB}, \tag{8}$$

$$e_{aM}^{SB\mathcal{M}} = \frac{(1-\nu)(1+\gamma)}{(\theta-\nu)} = e_{aM}^{U\mathcal{A}}, \tag{9}$$

$$e_{Am}^{SB\mathcal{M}} = \frac{\nu(1-\mu) - \mu\gamma}{(1-(1-\nu)(1-\mu)) - \mu\theta} \tag{10}$$

and

$$e_{am}^{SB\mathcal{M}} = \frac{(1-\nu)(1-\mu)}{\theta - (1-(1-\nu)(1-\mu))}. \tag{11}$$

All effort levels are always strictly positive, except for $e_{Am}^{SB\mathcal{M}}$. In order for $e_{Am}^{SB\mathcal{M}}$ to be positive and to be a maximum of the firm's expected profits, it is necessary to impose that both the numerator and the denominator in its expression be positive;[2] it must be that both

$$\gamma < \frac{\nu(1-\mu)}{\mu} = \gamma_0, \tag{12}$$

where $\gamma_0 > 1$ for $\mu > \frac{\nu}{1+\nu} = \mu_0$ (thus $\mu > \mu_0$ implies that $\gamma < \gamma_0$ is always verified), and that

$$\theta < \frac{(1-(1-\nu)(1-\mu))}{\mu} = \overline{\theta}_1^{\mathcal{M}},$$

with $\overline{\theta}_1^{\mathcal{M}} > 1$, hold.

As far as the monotonicity conditions are concerned, $e_{aM}^{SB\mathcal{M}} > e_{Am}^{SB\mathcal{M}}$ is satisfied if and only if

$$\gamma > \frac{(\mu(1-\nu) + \nu(1-\mu))(\theta-1)}{\nu\mu(\theta-1) + (1-\nu)(1-(1-\nu)(1-\mu))} = \underline{\gamma}^{SB\mathcal{M}},$$

where $\underline{\gamma}^{SB\mathcal{M}} < 1$ is always the case for $(3\mu\nu - \nu - \mu) \geq 0$, that is for $\nu > \frac{1}{3}$ and $\mu \geq \frac{\nu}{(3\nu-1)}$, whereas, for $(3\mu\nu - \nu - \mu) < 0$, inequality $\underline{\gamma}^{SB\mathcal{M}} < 1$ is true when

$$\theta < \frac{\mu + \nu - 3\mu\nu + (1-\nu)(1-(1-\nu)(1-\mu))}{\mu + \nu - 3\mu\nu} = \overline{\theta}_2^{\mathcal{M}},$$

with $\overline{\theta}_2^{\mathcal{M}} > \overline{\theta}_1^{\mathcal{M}}$ if and only if $\mu > \mu_0$ (with $\mu_0 < \frac{1}{2}$). Moreover, $e_{am}^{SB\mathcal{M}} < e_{Am}^{SB\mathcal{M}}$ holds for

$$\gamma < \frac{(1-\mu)(1-(1-\nu)(1-\mu))(\theta-1)}{\mu(\theta-(1-(1-\nu)(1-\mu)))} = \overline{\gamma}^{SB\mathcal{M}},$$

with $\overline{\gamma}^{SB\mathcal{M}} < 1$ being always true for $\mu \geq \mu_0$.

Recall that condition (2), which amounts to $e_{Am}^{SB\mathcal{M}} + e_{aM}^{SB\mathcal{M}} \leq \frac{2\gamma}{\Delta\theta}$, must be satisfied and this is equivalent to

$$\gamma \geq \frac{(\theta-1)(2\nu(1-\mu)(1-\nu) + (\nu-\mu)(\theta-1))}{2\nu(1-\nu)(1-\mu) + (\theta-1)\nu(2-\mu(\theta+1)) - (\theta-1)(1-\nu)(1-(1-\nu)(1-\mu))} = \gamma_1^{SB\mathcal{M}},$$

[2] This can easily be seen by collecting e_{Am} in the principal's objective function, once the wage schedules have been substituted, and observing the sign of the coefficient of e_{Am}^2.

where $\gamma_1^{SB\mathcal{M}} < \underline{\gamma}^{SB\mathcal{M}}$ if and only if $\theta < \overline{\theta}_1^{\mathcal{M}}$. Finally, note that the chain of inequalities $\gamma_1^{SB\mathcal{M}} < \Delta\theta < \underline{\gamma}^{SB\mathcal{M}} < \overline{\gamma}^{SB\mathcal{M}} < \gamma_0$ holds provided that the denominator of $e_{Am}^{SB\mathcal{M}}$ is positive, that is provided that $\theta < \overline{\theta}_1^{\mathcal{M}}$.

Below, we summarize what we have found so far.

Result 1 ***Full screening when motivation prevails.*** *A solution to the firm's program SB which entails full participation and full separation of worker's types, which satisfies the monotonicity condition* $e_{AM} > e_{aM} > e_{Am} > e_{am} > 0$, *and which is such that effort levels are given by expressions from (8) to (11) exists if and only if* $\theta < \min\left\{\overline{\theta}_1^{\mathcal{M}}, \overline{\theta}_2^{\mathcal{M}}\right\}$ *and* $\underline{\gamma}^{SB\mathcal{M}} < \gamma < \overline{\gamma}^{SB\mathcal{M}}$ *with*

$$\begin{aligned} \underline{\gamma}^{SB\mathcal{M}} &\equiv \frac{(\mu(1-\nu)+\nu(1-\mu))(\theta-1)}{(\nu\mu(\theta-1)+(1-\nu)(1-(1-\nu)(1-\mu)))} \\ \overline{\gamma}^{SB\mathcal{M}} &\equiv \frac{(1-\mu)(1-(1-\nu)(1-\mu))(\theta-1)}{\mu(\theta-(1-(1-\nu)(1-\mu)))} \\ \overline{\theta}_1^{\mathcal{M}} &\equiv \frac{(1-(1-\nu)(1-\mu))}{\mu} \\ \overline{\theta}_2^{\mathcal{M}} &\equiv \frac{((\mu+\nu-3\mu\nu)+(1-\nu)(1-(1-\nu)(1-\mu)))}{(\mu+\nu-3\mu\nu)} \end{aligned}.$$

Interestingly, both $\Delta\theta < \underline{\gamma}^{SB\mathcal{M}}$ and $\min\left\{\overline{\theta}_1^{\mathcal{M}}, \overline{\theta}_2^{\mathcal{M}}\right\} < 2$ hold, so that the alignment of second-best effort levels with the ranking obtained at first-best necessarily holds.

B.2 Pooling and exclusion

The firm can also resort to contracts involving pooling of worker's types and eventually exclusion of some worker's types.

Observe that the full screening solution is characterised by the lower bound $\underline{\gamma}^{SB\mathcal{M}}$, which comes from the condition $e_{aM}^{SB\mathcal{M}} > e_{Am}^{SB\mathcal{M}}$. Therefore, if $\gamma \leq \underline{\gamma}^{SB\mathcal{M}}$, the principal is forced to offer the same contract to both workers aM and Am. Likewise, the full screening solution is characterised by the upper bound $\overline{\gamma}^{SB\mathcal{M}}$, which corresponds to $e_{Am}^{SB\mathcal{M}} > e_{am}^{SB\mathcal{M}}$. And if $\gamma \geq \overline{\gamma}^{SB\mathcal{M}}$ we always expect a pooling contract where workers am and Am receive the same offer. We refer the reader to Appendix D.2 for the detailed analysis of the first situation, while we consider the second one in what follows.

Suppose that there's *pooling* between non motivated workers and that $e_{Am} = e_{am} = e_{\overline{p}}$. Then the ordering of effort levels is $e_{AM} > e_{aM} > e_{Am} = e_{am} = e_{\overline{p}}$ and the relevant downward incentive constraints that one expects to be binding are $IC_{AM/aM}$ and $IC_{aM/Am}$ (or $IC_{aM/am}$, which is equivalent) together with participation constraint PC_{am}. Since here worker types Am and am receive the same wage and provide the same effort, $u_{Am} > u_{am}$ necessarily holds. The wages are

$$w_{Am} = w_{am} = w_{\overline{p}} = \frac{1}{2}\theta e_{\overline{p}}^2, \tag{13}$$

$$w_{aM} = \frac{1}{2}\theta e_{aM}^2 - \gamma e_{aM} \qquad \underbrace{+\gamma e_{\overline{p}}}_{\text{Info rent worker } aM}$$

and

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma e_{AM} + \underbrace{\frac{1}{2}\Delta\theta e_{aM}^2 + \gamma e_{\overline{p}}}_{\text{Info-rent worker } AM}.$$

Substituting the wages into the objective function of the principal and maximizing yields

$$e_{AM} = e_{AM}^{SB\mathcal{M}} = e_{AM}^{FB} = 1 + \gamma,$$

$$e_{aM} = e_{aM}^{SB\mathcal{M}} = \frac{(1-\nu)(1+\gamma)}{(\theta - \nu)}$$

and

$$e_{Am} = e_{am} = e_{\overline{p}}^{SB\mathcal{M}} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)\theta} = e_{am}^{U\mathcal{M}} \tag{14}$$

Note that the expressions for e_{AM} and e_{aM} are the same as in Case $\mathcal{M}$, meaning that no-distortion-at-the-top is verified and that the effort of individual aM is lower than the corresponding first-best level. Moreover, $e_{AM} > e_{aM}$ still holds. Concerning $e_{\overline{p}}^{SB\mathcal{M}}$, it is the same as in Benchmark $U\mathcal{M}$, it is strictly positive for $\gamma < \gamma^{U\mathcal{M}}$ and such that $e_{aM}^{SB\mathcal{M}} > e_{\overline{p}}^{SB\mathcal{M}}$ holds if and only if

$$\gamma > \frac{\nu(1-\mu)\Delta\theta}{\theta(1-\nu) + \mu\nu\Delta\theta} = \gamma_{\overline{p}1}$$

where $\gamma_{\overline{p}1} < \underline{\gamma}^{SB\mathcal{M}}$ always holds. Finally, condition (2) holds whenever $e_{\overline{p}}^{SB\mathcal{M}} + e_{aM}^{SB\mathcal{M}} \leq \frac{2\gamma}{\Delta\theta}$, which is equivalent to

$$\gamma > \frac{\Delta\theta(1-\mu)((\theta-\nu) + \theta(1-\nu))}{\theta(1-\nu) + \mu\nu - 2\theta(1-(1-\mu)(1-\nu)) + \theta^2(1+\nu(1-\mu))} = \gamma_{\overline{p}2}$$

with $\gamma_{\overline{p}2} < \underline{\gamma}^{SB\mathcal{M}}$.

We are then able to state the following result.

Result 2 ***Full participation and pooling between worker types*** Am ***and*** am ***when motivation prevails.*** *The solution to the firm's program SB which entails full participation and pooling between workers Am and am, which satisfies the monotonicity condition* $e_{AM} > e_{aM} > e_{Am} = e_{am} > 0$, *and which is such that effort levels are given by expressions (8), (9) and (14) exists if and only if* $\gamma_{\overline{p}} < \gamma < \min\{\gamma^{U\mathcal{M}}, 1\}$ *with*

$$\gamma_{\overline{p}} \equiv \max\{\gamma_{\overline{p}1}, \gamma_{\overline{p}2}\} = \max\left\{\frac{\nu(1-\mu)\Delta\theta}{\theta(1-\nu)+\mu\nu\Delta\theta}, \frac{\Delta\theta(1-\mu)((\theta-\nu)+\theta(1-\nu))}{\theta(1-\nu)+\mu\nu-2\theta(1-(1-\mu)(1-\nu))+\theta^2(1+\nu(1-\mu))}\right\}$$
$$\gamma^{U\mathcal{M}} \equiv \frac{(1-\mu)}{\mu}$$

being $\gamma^{U\mathcal{M}} < 1$ *for* $\mu > \frac{1}{2}$.

The conditions of existence of a solution with full participation and pooling of workers am and Am are less restrictive than the ones we obtained in Result 1 because the requirement $e_{am}^{SB\mathcal{M}} < e_{Am}^{SB\mathcal{M}}$ is no longer relevant. Also, the pooled effort $e_{\overline{p}}^{SB\mathcal{M}}$ is always in-between expressions (10) and (11): in particular, $e_{am}^{SB\mathcal{M}} > e_{\overline{p}}^{SB\mathcal{M}} > e_{Am}^{SB\mathcal{M}}$ holds if and only if $\gamma > \overline{\gamma}^{SB\mathcal{M}}$.

Note that $\gamma^{U\mathcal{M}} \geq 1$ if and only if $\mu \leq \frac{1}{2}$, therefore the employer always proposes a pooling contract to workers Am and am when motivation is sufficiently high (i.e. for $\gamma \geq \overline{\gamma}^{SB\mathcal{M}}$) and the probability of workers being motivated is sufficiently low (i.e. for $\mu \leq \frac{1}{2}$). Conversely, when $\mu > \frac{1}{2}$ and $\gamma^{U\mathcal{M}} < 1$, then for $\gamma \geq \gamma^{U\mathcal{M}}$ the firm is expected to exclude worker am since the probability of having motivated workers is high and the productivity loss from worker am is low.

As for *exclusion*, the necessary and sufficient condition for full participation requires in general that, for any worker type ij, the expected profit from employing type ij be higher than the expected information rents that have to be paid to her mimickers; this condition is satisfied as long as type ij's effort is strictly positive. However, the condition $e_{ij} > 0$ might call for some restrictions on the parameter space, as occurred in Benchmark $U\mathcal{M}$.

In order to derive the conditions for existence and to characterise the contract with exclusion of worker am, we proceed as in the case with full participation, but we obviously drop worker am from the firm's maximisation program SB and omit the monotonicity condition $e_{Am} > e_{am}$. Since the upper bound $\overline{\gamma}^{SB\mathcal{M}}$ of the existence range for a solution with full participation comes precisely from the constraint $e^{SB\mathcal{M}}_{Am} > e^{SB\mathcal{M}}_{am}$, the range for the existence of a separating equilibrium with exclusion of am is broader on the right hand side with respect to the interval $\left[\underline{\gamma}^{SB\mathcal{M}}, \overline{\gamma}^{SB\mathcal{M}}\right]$. Moreover, the optimal effort levels of the remaining workers are given by the same expressions from (8) to (10), even with exclusion. Instead, the optimal wages of the remaining workers will be lower than under full screening because the portions of the three information rents that depend on e_{am} disappear.

Result 3 ***Exclusion of worker*** *am* ***when motivation prevails.*** *The solution to the principal's program SB, which entails separation and exclusion of worker type am, which satisfies the monotonicity condition* $e_{AM} > e_{aM} > e_{Am} > e_{am} = 0$ *and which is such that effort levels are given by expressions from (8) to (10), exists if and only if* $\theta < \min\left\{\overline{\theta}^{\mathcal{M}}_1, \overline{\theta}^{\mathcal{M}}_2\right\}$ *and* $\underline{\gamma}^{SB\mathcal{M}} < \gamma < \gamma_0 \equiv \frac{\nu(1-\mu)}{\mu}$.

Up to now, we have identified all possible classes of solutions to the firm's program SB, when motivation prevails, and their corresponding (possibly overlapping) existence regions. Actually, there still remains to characterise the solution when there is bunching between intermediate types aM and Am and we refer the reader to Appendix D.2 for the analysis of such a situation. In order to single out the optimal contract chosen by the employer, we still have one step to go: when different solutions coexist, we must pick the one that yields the highest profits to the firm and discard the others. This is precisely what we do next.

B.3 Proof of Proposition 1

We want to show that the solution entailing full participation and full separation of types dominates (meaning that it provides higher profits to the employer) both full separation but exclusion of at least worker am and full participation but pooling of two workers' types. Moreover, we prove that full participation and pooling of two different types dominates full separation and exclusion of worker am, whenever the two solutions coexist. We consider the situation in which motivation prevails (Case $\mathcal{M}$); the same line of reasoning applies to Case $\mathcal{A}$ as well, which is therefore omitted.

Start with the comparison between full participation and full separation of types and exclusion of worker am. We must evaluate the costs and benefits from participation of the worst worker type am. The principal's benefit from employing worker am is the expected profit

$$(1-\mu)(1-\nu)(e_{am}-w_{am}), \tag{15}$$

whereas the cost from participation of am is represented by the information rents paid to the three remaining workers' types, which add up to

$$\frac{1}{2}(1-(1-\mu)(1-\nu))\Delta\theta e_{am}^{2} \tag{16}$$

Thus, the firm prefers full participation to exclusion of type am if and only if (15) is strictly greater than (16). Taking into account the expression for the wage w_{am} in Case $\mathcal{M}$ (given in the main text), the inequality reduces to $2e_{am}^{SB\mathcal{M}} > e_{am}^{SB\mathcal{M}}$, which is obviously satisfied as long as $e_{am}^{SB\mathcal{M}} > 0$. Alternatively, substituting both the expression for the wage w_{am} and expression (11) for e_{am} in (15) and (16), we obtain that full separation and full participation dominates full separation and exclusion of worker am if and only if $\theta-(1-(1-\mu)(1-\nu)) > 0$. This is the denominator of e_{am} in expression (11). The previous inequality says that the effort cost of a low ability type θ must be larger than the joint probability of hiring types that are overall more efficient than am. This condition always holds in our setting, given that $\theta > 1$. Notice that the above requirements are exactly equivalent to the ones we would find by direct comparison the firm's profits in case of full participation and full separation (whose expression is given in 17 below) and in the case of full separation but exclusion of type am worker.

Consider now the comparison between full separation and full participation of types and full participation but pooling of workers aM and Am (as said, this solution is considered in detail in Appendix D.2 but we anticipate some findings here for expositional convenience). Now the trade-off between costs and benefits from full separation becomes less clear, so let us resort directly to the comparison between the firm's profits under the two solutions. The principal's payoffs under full separation and full participation of types are

$$\pi_{FS,FP}^{SB\mathcal{M}} = \frac{1}{2}\left(\nu\mu(1+\gamma)^2+\mu\frac{(1-\nu)^2(1+\gamma)^2}{\theta-\nu}+\frac{(\nu(1-\mu)-\mu\gamma)^2}{(1-(1-\nu)(1-\mu))-\mu\theta}+\frac{(1-\nu)^2(1-\mu)^2}{\theta-(1-(1-\nu)(1-\mu))}\right) \tag{17}$$

while, under full participation but pooling of workers aM and Am, profits amount to

$$\pi_{FP,aM=Am}^{SB\mathcal{M}} = \frac{1}{2}\left(\nu\mu(1+\gamma)^2+\frac{(\nu(1-\mu)+\mu(1-\nu)-\gamma\mu\nu)^2}{\nu(1-\mu)+\mu(1-\nu)}+\frac{(1-\nu)^2(1-\mu)^2}{\theta-(1-(1-\nu)(1-\mu))}\right)$$

It is immediate to check that $\pi^{SB\mathcal{M}}_{FS,FP} > \pi^{SB\mathcal{M}}_{FP,aM=Am}$ always holds.

Consider now the comparison between full separation and full participation of types and full participation but pooling of workers am and Am (see Appendix B.2). The firm's payoffs under full participation but pooling of workers am and Am are given by

$$\pi^{SB\mathcal{M}}_{FP,am=Am} = \frac{1}{2}\left(\nu\mu\left(1+\gamma\right)^2 + \frac{\mu(1-\nu)^2(1+\gamma)^2}{(\theta-\nu)} + \frac{((1-\mu)-\mu\gamma)^2}{\theta(1-\mu)}\right)$$

and, again, it is straightforward to check that $\pi^{SB\mathcal{M}}_{FS,FP} > \pi^{SB\mathcal{M}}_{FP,am=Am}$ always holds.

Finally, consider the comparison between full participation but pooling of workers am and Am and full separation but exclusion of worker am. Since both solutions are dominated by full separation and full participation, they can be candidate optimal contracts only above $\overline{\gamma}^{SB\mathcal{M}}$. The principal's profits at the latter solution are

$$\pi^{SB\mathcal{M}}_{FS,am=0} = \frac{1}{2}\left(\nu\mu\left(1+\gamma\right)^2 + \frac{\mu(1-\nu)^2(1+\gamma)^2}{(\theta-\nu)} + \frac{(\nu(1-\mu)-\mu\gamma)^2}{\nu(1-\mu)-\mu(\theta-1)}\right)$$

and $\pi^{SB\mathcal{M}}_{FP,am=Am} > \pi^{SB\mathcal{M}}_{FS,am=0}$ if and only if

$$\left((1-\mu)-\mu\gamma\right)e^{SB\mathcal{M}}_{\overline{p}} > \left(\nu\left(1-\mu\right)-\mu\gamma\right)e^{SB\mathcal{M}}_{Am}.$$

The above inequality is always verified since $((1-\mu)-\mu\gamma) > (\nu(1-\mu)-\mu\gamma)$ always holds and $e^{SB\mathcal{M}}_{\overline{p}} > e^{SB\mathcal{M}}_{Am}$ is true above $\overline{\gamma}^{SB\mathcal{M}}$.

Note that the comparison between full participation but pooling of workers aM and Am and full separation but exclusion of worker am is meaningless because, below $\underline{\gamma}^{SB\mathcal{M}}$, it is never feasible to separate types aM and Am. So we are done.

B.4 Optimal contracts when motivation prevails

Considering Proposition 1 (in the main text), Results from 1 to 3 and Result 13 in Appendix D.2, it is now possible to characterise the optimal contracts when motivation prevails.

Result 4 *When motivation prevails, the optimal contracts proposed by the employer are as follows:*
(i) Full participation and pooling between types aM and Am and $IC_{Am/am}$ binding (characterised in Result 13) is implemented if and only if $\theta < \overline{\theta}^{\mathcal{M}}_1$ and $\Delta\theta \leq \gamma < \underline{\gamma}^{SB\mathcal{M}}$.
(ii) Full participation and full separation of types (characterised in Result 1) is implemented if and only if $\theta < \min\left\{\overline{\theta}^{\mathcal{M}}_1, \overline{\theta}^{\mathcal{M}}_2\right\}$ and $\underline{\gamma}^{SB\mathcal{M}} \leq \gamma \leq \overline{\gamma}^{SB\mathcal{M}}$.
(iii) Full participation and pooling between types Am and am (characterised in Result 2) is implemented if and only if $\theta < \overline{\theta}^{\mathcal{M}}_1$ and $\overline{\gamma}^{SB\mathcal{M}} \leq \gamma \leq \min\left\{\gamma^{U\mathcal{M}}, 1\right\}$.
(iv) Full separation and exclusion of type am (characterised in Result 3) is implemented if and only if $\mu > \frac{1}{2}$, $\theta < \overline{\theta}^{\mathcal{M}}_1$ and $\gamma^{U\mathcal{M}} < \gamma \leq 1$.

Observe that, if $\theta \geq \overline{\theta}_1^{\mathcal{M}}$, then the fully separating solutions do not exist and the only candidate solutions are the ones with pooling of intermediate types and $IC_{Am/am}$ binding or pooling of non-motivated types.

C Ability prevails (Case $\mathcal{A}$)

When ability prevails, condition (3) holds and $e_{Am} > e_{aM}$ together with $e_{Am}+ e_{aM} \geq \frac{2\gamma}{\Delta\theta}$ must be satisfied. In order to find a full screening solution to the firm's problem SB, one has to consider the participation constraint PC_{am} and the following incentive constraints: $IC_{AM/Am}$, $IC_{Am/aM}$ or eventually $IC_{Am/am}$ (whichever one binds first), $IC_{aM/am}$ or $IC_{aM/Am}$ (again whichever one binds first). Since $IC_{Am/aM}$ and $IC_{aM/Am}$ cannot be simultaneously binding at a separating equilibrium, then the possible situations are the following: ($\mathcal{A}$.1) all downward local ICs are binding and thus $IC_{AM/Am}$, $IC_{Am/aM}$ and $IC_{aM/am}$ hold with equality; ($\mathcal{A}$.2) the downward local constraints $IC_{AM/Am}$ and $IC_{aM/am}$ and the global downward constraint $IC_{Am/am}$ are all binding; ($\mathcal{A}$.3) constraints $IC_{AM/Am}$, $IC_{Am/am}$ and the upward $IC_{aM/Am}$ hold with equality.

Such three possible cases will be analysed in detail in what follows.

C.1 Case $\mathcal{A}$.1

C.1.1 Full separation and full participation

Suppose that all downward local ICs are binding. Solving the binding constraints for salaries, one obtains the following wage schedules and information rents

$$w_{am} = \frac{1}{2}\theta e_{am}^2, \tag{18}$$

$$w_{aM} = \frac{1}{2}\theta e_{aM}^2 - \gamma e_{aM} \underbrace{+\gamma e_{am}}_{\text{Info rent worker } aM}, \tag{19}$$

$$w_{Am} = \frac{1}{2}e_{Am}^2 \underbrace{+\frac{1}{2}\Delta\theta e_{aM}^2 - \gamma e_{aM} + \gamma e_{am}}_{\text{Info rent worker } Am} \tag{20}$$

and

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma e_{AM} \underbrace{+\gamma e_{Am} + \frac{1}{2}\Delta\theta e_{aM}^2 - \gamma e_{aM} + \gamma e_{am}}_{\text{Info rent worker } AM}. \tag{21}$$

All information rents are positive and have the usual cumulative structure. They all include at least one term depending on γ as in Benchmark $U\mathcal{M}$, where asymmetric information concerns motivation only. Only type Am receives an information rent which also depends on the difference in ability $\Delta\theta$: this comes from the fact that this program embeds the two problems in Benchmark $U\mathcal{M}$ and links them through constraint $IC_{Am/aM}$. Type AM cumulates this rent too when trying to mimic Am.

Substituting the wage schedules into the objective function and differentiating with respect to effort levels we obtain

$$e_{AM}^{SB\mathcal{A}1} = 1 + \gamma \tag{22}$$

$$e_{Am}^{SB\mathcal{A}1} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)} = e_{Am}^{U\mathcal{M}}, \tag{23}$$

$$e_{aM}^{SB\mathcal{A}1} = \frac{(1-\nu)\,\mu + (1-(1-\nu)(1-\mu))\,\gamma}{(1-(1-\nu)(1-\mu))\,\theta - \nu} \tag{24}$$

and

$$e_{am}^{SB\mathcal{A}1} = \frac{(1-\nu)(1-\mu) - (1-(1-\nu)(1-\mu))\,\gamma}{(1-\nu)(1-\mu)\,\theta}. \tag{25}$$

Observe that $e_{AM}^{SB\mathcal{A}1}$ and $e_{aM}^{SB\mathcal{A}1}$ are strictly positive, while $e_{Am}^{SB\mathcal{A}1} > 0$ if and only if $\gamma < \gamma^{U\mathcal{M}}$, and $e_{am}^{SB\mathcal{A}1} > 0$ if and only if

$$\gamma < \frac{(1-\nu)(1-\mu)}{(1-(1-\nu)(1-\mu))} = \gamma_1^{SB\mathcal{A}1}.$$

Actually, $e_{Am}^{SB\mathcal{A}1} > 0$ always holds when $\mu \leq \frac{1}{2}$ or when $e_{am}^{SB\mathcal{A}1}$ is strictly positive, since $e_{am}^{SB\mathcal{A}1} > 0$ implies $e_{Am}^{SB\mathcal{A}1} > 0$ (being $\gamma^{U\mathcal{M}} > \gamma_1^{SB\mathcal{A}1}$).

As for the monotonicity conditions, it can be checked that $e_{AM}^{SB\mathcal{A}1} > e_{Am}^{SB\mathcal{A}1}$ always holds, that $e_{Am}^{SB\mathcal{A}1} > e_{aM}^{SB\mathcal{A}1}$ is true for

$$\gamma < \frac{(1-\mu)(1-(1-\nu)(1-\mu))(\theta-1)}{\mu(1-(1-\nu)(1-\mu))(\theta-1)+(\nu(1-\mu)+\mu(1-\nu))} = \gamma_2^{SB\mathcal{A}1}$$

and that inequalities $e_{AM}^{SB\mathcal{A}1} > e_{aM}^{SB\mathcal{A}1}$, $e_{Am}^{SB\mathcal{A}1} > e_{am}^{SB\mathcal{A}1}$ and $e_{aM}^{SB\mathcal{A}1} + e_{Am}^{SB\mathcal{A}1} > \frac{2\gamma}{\Delta\theta}$ all hold when $\gamma < \gamma_2^{SB\mathcal{A}1}$. Note that $\gamma_2^{SB\mathcal{A}1} < \gamma_1^{SB\mathcal{A}1}$ if and only if

$$\theta < \frac{\mu\,(1-\nu\,(1-\nu)) + \nu\,(1-\mu)}{\nu\,(1-(1-\nu)(1-\mu))} \equiv \overline{\theta}^{\mathcal{A}1}.$$

Finally, $e_{aM}^{SB\mathcal{A}1} > e_{am}^{SB\mathcal{A}1}$ for

$$\gamma > \frac{\nu\,(1-\nu)(1-\mu)(\theta-1)}{(1-(1-\nu)(1-\mu))(\theta-\nu)} = \underline{\gamma}^{SB\mathcal{A}1}$$

where it is always the case that $\underline{\gamma}^{SB\mathcal{A}1} < \min\left\{\gamma_1^{SB\mathcal{A}1}, \gamma_2^{SB\mathcal{A}1}\right\}$.

We are then able to state the following result.

Result 5 ***Full screening when ability prevails and Case $\mathcal{A}.1$ holds.*** *The solution to the firm's program SB, which entails full participation, full separation of worker's types and constraints $IC_{AM/Am}$, $IC_{Am/aM}$ and $IC_{aM/am}$ binding, which satisfies the monotonicity condition $e_{AM} > e_{Am} > e_{aM} > e_{am} > 0$ and which is such that effort levels are given by expressions from (22) to (25) exists and represents the optimal contract if and only if $\underline{\gamma}^{SB\mathcal{A}1} < \gamma < \overline{\gamma}^{SB\mathcal{A}1}$ with*

$$\underline{\gamma}^{SB\mathcal{A}1} \equiv \frac{\nu(1-\nu)(1-\mu)(\theta-1)}{(1-(1-\nu)(1-\mu))(\theta-\nu)}$$
$$\overline{\gamma}^{SB\mathcal{A}1} = \min\left\{\gamma_1^{SB\mathcal{A}1}, \gamma_2^{SB\mathcal{A}1}\right\}$$

and

$$\begin{array}{rcl}\gamma_1^{SB\mathcal{A}1} & \equiv & \frac{(1-\nu)(1-\mu)}{(1-(1-\nu)(1-\mu))} \\ \gamma_2^{SB\mathcal{A}1} & \equiv & \frac{(1-\mu)(1-(1-\nu)(1-\mu))(\theta-1)}{\mu(1-(1-\nu)(1-\mu))(\theta-1)+(\nu(1-\mu)+\mu(1-\nu))}\end{array}.$$

Finally note that $\Delta\theta > \max\left\{\gamma_1^{SB\mathcal{A}1}, \gamma_2^{SB\mathcal{A}1}\right\}$ is always true, therefore Case $\mathcal{A}.1$ with full participation and full separation is always a subset of the first-best state of the world in which $e_{Am}^{FB} > e_{aM}^{FB}$ holds.

C.1.2 Pooling and exclusion

In light of Proposition 1 (in the main text), the solution will always be characterised by full participation and full separation of types, except when the former solution is not viable, in which case pooling and possibly exclusion will also be part of the optimal contract.

First of all consider *pooling.* Observe that the lower bound $\underline{\gamma}^{SB\mathcal{A}1}$ corresponds to condition $e_{aM}^{SB\mathcal{A}1} > e_{am}^{SB\mathcal{A}1}$. Thus, if $\gamma \leq \underline{\gamma}^{SB\mathcal{A}1}$, we expect a pooling equilibrium where workers aM and am receive the same contract. Suppose that there's pooling between the less able workers and that $e_{aM} = e_{am} = e_{\underline{p}}$ holds. Then the ordering of effort levels is $e_{AM} > e_{Am} > e_{\underline{p}} > 0$ and the relevant downward incentive constraints that one assumes to be binding are $IC_{AM/Am}$ and $IC_{Am/am}$ (or $IC_{Am/aM}$, which is equivalent) with participation constraint PC_{am}. Since here the incentive constraints $IC_{Am/aM}$ and $IC_{Am/am}$ are both binding by construction, we do not need any condition on the sum of e_{aM} and e_{am}. Moreover, since the two types of workers receive the same wage and provide the same effort, $u_{aM} > u_{am}$ necessarily holds. The wages are

$$w_{aM} = w_{am} = w_{\underline{p}} = \frac{1}{2}\theta e_{\underline{p}}^2,$$

$$w_{Am} = \frac{1}{2}e_{Am}^2 \quad \underbrace{+\frac{1}{2}\Delta\theta e_{\underline{p}}^2}_{\text{Info rent worker } Am}$$

and

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma e_{AM} \underbrace{+\gamma e_{Am} + \frac{1}{2}\Delta\theta e_{\underline{p}}^2}_{\text{Info rent worker } AM}.$$

Substituting the salaries into the objective function of the principal and maximising with respect to effort levels yields

$$e_{AM}^{SB\mathcal{A}1} = 1 + \gamma,$$

$$e_{Am}^{SB\mathcal{A}1} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)} = e_{Am}^{U\mathcal{M}}$$

and

$$e_{aM} = e_{am} = e_{\underline{p}}^{SB\mathcal{A}1} = \frac{(1-\nu)}{(\theta-\nu)} = e_{am}^{U\mathcal{A}} \tag{26}$$

Note that the expressions for e_{AM} and e_{Am} are the same as in Case $\mathcal{A}.1$ (and $\mathcal{A}.2$ that follows) with full separation, meaning that no-distortion-at-the-top is verified and that the effort of individual Am

is lower than the corresponding first-best level. Moreover, $e_{AM} > e_{Am}$ still holds. Concerning $e_{\underline{p}}^{SB\mathcal{A}1}$, which is strictly positive, we expect that this effort lies in-between the effort exerted by workers aM and am in Case $\mathcal{A}$.1 with full separation. One can easily check that $e_{aM}^{SB\mathcal{A}1} < e_{\underline{p}}^{SB\mathcal{A}1} < e_{am}^{SB\mathcal{A}1}$ if and only if $\gamma < \underline{\gamma}^{SB\mathcal{A}1}$. Finally, $e_{Am}^{SB\mathcal{A}1} > e_{\underline{p}}^{SB\mathcal{A}1}$ if and only if

$$\gamma < \frac{(1-\mu)(\theta-1)}{\mu(\theta-\nu)} = \gamma_{\underline{p}}$$

where $\gamma_{\underline{p}} > \underline{\gamma}^{SB\mathcal{A}1}$ always holds.

Now consider the upper bounds (recall that condition $\gamma < \gamma_1^{SB\mathcal{A}1}$ is equivalent to $e_{am}^{SB\mathcal{A}1} > 0$ and that inequality $\gamma < \gamma_2^{SB\mathcal{A}1}$ is equivalent to $e_{Am}^{SB\mathcal{A}1} > e_{aM}^{SB\mathcal{A}1}$): if $\gamma \geq \overline{\gamma}^{SB\mathcal{A}1}$, we expect a solution in which either workers aM and Am are pooled together or exclusion occurs or both.[3]

Result 6 (i) ***Full participation and pooling between workers*** aM ***and*** am ***when ability prevails.*** *The solution to the firm's program SB which entails full participation, pooling between workers aM and am, which satisfies the monotonicity condition* $e_{AM} > e_{Am} > e_{aM} = e_{am} = e_{\underline{p}} > 0$ *and which is such that effort levels are given by expressions (22), (23) and (26) exists if and only if* $0 < \gamma \leq \gamma_{\underline{p}} \equiv \frac{(1-\mu)\Delta\theta}{\mu(\theta-\nu)}$ *and represents the optimal contract when* $0 < \gamma \leq \underline{\gamma}^{SB\mathcal{A}1}$.

(ii) ***Full participation and pooling between workers*** aM ***and*** Am ***when ability prevails.*** *The solution to the firm's program SB which entails full participation, pooling between workers aM and Am and* $IC_{aM/am}$ *binding, which satisfies the monotonicity condition* $e_{AM} > e_{aM} = e_{Am} = e_{\overline{p}} > e_{am} > 0$ *and which is such that effort levels are given by expressions (22), (25) and*

$$e_{aM} = e_{Am} \equiv e_{\overline{p}}^{SB\mathcal{A}1} = \frac{(\nu(1-\mu)+\mu(1-\nu))(1+\gamma)}{\nu\mu(\theta-1)+(\nu(1-\mu)+\mu(1-\nu))\theta},$$

represents the optimal contract only if $\overline{\gamma}^{SB\mathcal{A}1} \neq \gamma_1^{SB\mathcal{A}1}$ *and* $\overline{\gamma}^{SB\mathcal{A}1} < \gamma < \min\left\{\overline{\gamma}^{SB\mathcal{P}1}, \gamma_1^{SB\mathcal{A}1}\right\}$ *with*

$$\overline{\gamma}^{SB\mathcal{P}1} \equiv \frac{(\theta-1)(1-\mu)(1-\nu)(\mu\nu(\theta-1)+2\theta(\mu(1-\nu)+\nu(1-\mu)))}{\nu\mu(1-(1-\nu)(1-\mu))((4-\theta)\theta+1)+\theta(\theta+1)(\nu+\mu)-2\theta(4\mu\nu+\mu^2(1-\nu)+\nu^2(1-\mu))}.$$

Note that when $\overline{\gamma}^{SB\mathcal{A}1} = \gamma_1^{SB\mathcal{A}1}$ and $\gamma_1^{SB\mathcal{A}1} < \gamma < \gamma_2^{SB\mathcal{A}1}$, the employer will necessarily exclude worker am. This would lead us to consider alternative solutions where either full separation but exclusion of type am (and where $IC_{Am/aM}$ and PC_{aM} are binding), or pooling of workers aM and Am and exclusion of type am, or else exclusion of both types am and aM are implemented.[4]

[3] We refer the reader to Appendix D.1 for the detailed analysis of this situation.

[4] In the region $\gamma \geq \overline{\gamma}^{SB\mathcal{A}1}$, we do not provide the full characterization of the solution (available upon request though) because different cases might arise and the analysis becomes cumbersome without being very insightful.

C.2 Case $\mathcal{A}.2$

C.2.1 Full separation and full participation

Given the pattern of binding constraints characterising Case $\mathcal{A}.2$, the salaries of workers aM and am are the same as in Case $\mathcal{A}.1$, and given by expressions (19) and (18) respectively, whereas the salary of type Am has the same expression as in Case $\mathcal{M}$ (see equation 5) and Case $\mathcal{A}.3$; the other relevant wage level is now

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma e_{AM} \underbrace{+\gamma e_{Am} + \frac{1}{2}\Delta\theta e_{am}^2}_{\text{Info rent worker } AM}.$$

The information rent of worker Am is formed by one term only, $\frac{1}{2}\Delta\theta e_{am}^2$ (as in Benchmark $U\mathcal{A}$, Case $\mathcal{M}$ and Case $\mathcal{A}.3$ that follows) which depends on the effort exerted by worker am, while no rent depending on e_{aM} appears: this is because worker Am mimics type am directly, without "going through" type aM. For the same reason, information rents accruing to both workers AM and Am are "shorter" than in Case $\mathcal{A}.1$, as the paths of binding incentive constraints in Figure 5b show. Also the information rent of worker aM only depends on the effort provided by of worker am, however in w_{aM} the rent is γe_{am} (as the one Benchmark $U\mathcal{M}$). Thus, we can interpret this Case $\mathcal{A}.2$ as a program that is in-between Case $\mathcal{A}.1$ and Case $\mathcal{A}.3$.

Substituting the wage functions into the firm's expected profits and differentiating with respect to effort levels, we obtain

$$e_{AM}^{SB\mathcal{A}2} = 1 + \gamma, \tag{27}$$

$$e_{Am}^{SB\mathcal{A}2} = \frac{(1-\mu) - \gamma\mu}{(1-\mu)} = e_{Am}^{SB\mathcal{A}1} = e_{Am}^{U\mathcal{M}}, \tag{28}$$

$$e_{aM}^{SB\mathcal{A}2} = \frac{1+\gamma}{\theta} = e_{aM}^{FB} \tag{29}$$

and

$$e_{am}^{SB\mathcal{A}2} = \frac{(1-\nu)\left((1-\mu) - \gamma\mu\right)}{\nu\Delta\theta + \theta(1-\mu)(1-\nu)}. \tag{30}$$

Observe that both $e_{am}^{SB\mathcal{A}2} > 0$ and $e_{Am}^{SB\mathcal{A}2} > 0$ hold provided that $\gamma < \gamma^{U\mathcal{M}}$, that $e_{AM}^{SB\mathcal{A}2} > e_{Am}^{SB\mathcal{A}2} > e_{am}^{SB\mathcal{A}2}$ and $e_{aM}^{SB\mathcal{A}2} > e_{am}^{SB\mathcal{A}2}$ always hold, while $e_{Am}^{SB\mathcal{A}2} > e_{aM}^{SB\mathcal{A}2}$ if and only if

$$\gamma < \frac{(1-\mu)\Delta\theta}{1+\mu\Delta\theta} = \gamma_1^{SB\mathcal{A}2}.$$

It is easy to check that the condition $\gamma < \gamma_1^{SB\mathcal{A}2}$ implies both $e_{am}^{SB\mathcal{A}2} > 0$ and $e_{Am}^{SB\mathcal{A}2} > 0$, being $\gamma_1^{SB\mathcal{A}2} < \gamma^{U\mathcal{M}}$, and also that $\gamma_1^{SB\mathcal{A}2} < \gamma_2^{SB\mathcal{A}1}$ always holds, being the requirement $e_{Am}^{SB\mathcal{A}2} > e_{aM}^{SB\mathcal{A}2} = e_{aM}^{FB}$ more restrictive than $e_{Am}^{SB\mathcal{A}1} > e_{aM}^{SB\mathcal{A}1}$, the corresponding requisite in Case $\mathcal{A}.1$. Then, all monotonicity conditions are satisfied provided that $\gamma < \gamma_1^{SB\mathcal{A}2}$. Moreover, condition $\gamma < \gamma_1^{SB\mathcal{A}2}$ suffices for $e_{aM}^{SB\mathcal{A}2} + e_{Am}^{SB\mathcal{A}2} \geq \frac{2\gamma}{\Delta\theta}$.

There remains to check that incentive constraint $IC_{Am/am}$ is binding rather than $IC_{Am/aM}$ and that $IC_{aM/am}$ is binding rather than $IC_{aM/Am}$, which amounts to $e_{am}+e_{aM}\le\frac{2\gamma}{\Delta\theta}\le e_{am}+e_{Am}$. As for inequality $e_{am}^{SB\mathcal{A}2}+e_{Am}^{SB\mathcal{A}2}\ge\frac{2\gamma}{\Delta\theta}$, it holds if and only if

$$\gamma\le\frac{\Delta\theta(1-\mu)(\Delta\theta(1-\mu(1-\nu))+2(1-\mu)(1-\nu))}{2(1-\mu)^2(1-\nu)+(\theta-1)^2\mu(1-\mu(1-\nu))+2\Delta\theta(1-\mu)}=\gamma_2^{SB\mathcal{A}2}\ ,$$

conversely $e_{aM}^{SB\mathcal{A}2}+e_{am}^{SB\mathcal{A}2}\le\frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma\ge\frac{\Delta\theta(2\theta(1-\mu)(1-\nu)+\nu\Delta\theta)}{(\nu\Delta\theta+\theta(1-\mu)(1-\nu))(\theta+1)+\theta\Delta\theta(1-\nu)\mu}=\underline{\gamma}^{SB\mathcal{A}2}\ ,$$

whereby a solution exists for $\underline{\gamma}^{SB\mathcal{A}2}\le\gamma<\min\left\{\gamma_1^{SB\mathcal{A}2},\gamma_2^{SB\mathcal{A}2}\right\}\equiv\overline{\gamma}^{SB\mathcal{A}2}$. Now, $\underline{\gamma}^{SB\mathcal{A}2}<\gamma_2^{SB\mathcal{A}2}<\gamma_1^{SB\mathcal{A}2}$ is true if and only if $\mu<\frac{1}{2}$ and

$$\theta>\frac{(1-\mu(1+\nu))}{(1-2\mu)(1-\mu(1-\nu))}=\underline{\theta}^{\mathcal{A}2},$$

with $\underline{\theta}^{\mathcal{A}2}<2$ if and only if

$$\mu<\frac{(5-3\nu)-\sqrt{\left((5-3\nu)^2-16(1-\nu)\right)}}{8(1-\nu)}=\mu_0<\frac{1}{2}\ .$$

Hence a solution with full separation and full participation under Case $\mathcal{A}.2$ does not exist for $\mu\ge\mu_0$.

We are then able to state the following result.

Result 7 ***Full screening when ability prevails and Case $\mathcal{A}.2$ holds.*** *A solution to the firm's program SB, which entails full participation, full separation of types and $IC_{AM/Am}$, $IC_{Am/am}$ and $IC_{aM/am}$ binding, which satisfies the monotonicity condition $e_{AM}>e_{Am}>e_{aM}>e_{am}>0$ and which is such that effort levels are given by expressions from (27) to (30) exists and represents the optimal contract if and only if $\mu<\mu_0$, $\theta>\underline{\theta}^{\mathcal{A}2}$ and $\underline{\gamma}^{SB\mathcal{A}2}\le\gamma<\overline{\gamma}^{SB\mathcal{A}2}$, with*

$$\begin{aligned}\underline{\gamma}^{SB\mathcal{A}2}&\equiv\frac{\Delta\theta(2\theta(1-\mu)(1-\nu)+\nu\Delta\theta)}{(\nu\Delta\theta+\theta(1-\mu)(1-\nu))(\theta+1)+\theta\Delta\theta(1-\nu)\mu}\\ \overline{\gamma}^{SB\mathcal{A}2}&\equiv\frac{\Delta\theta(1-\mu)(\Delta\theta(1-\mu(1-\nu))+2(1-\mu)(1-\nu))}{2(1-\mu)^2(1-\nu)+(\theta-1)^2\mu(1-\mu(1-\nu))+2\Delta\theta(1-\mu)}\\ \mu_0&\equiv\frac{(5-3\nu)-\sqrt{\left((5-3\nu)^2-16(1-\nu)\right)}}{8(1-\nu)}<\frac{1}{2}\\ \underline{\theta}^{\mathcal{A}2}&\equiv\frac{(1-\mu(1+\nu))}{(1-2\mu)(1-\mu(1-\nu))}\end{aligned}\ .$$

Observe that $\gamma_2^{SB\mathcal{A}2}=\overline{\gamma}^{SB\mathcal{A}2}<\Delta\theta$ always holds, thus implying that this solution is attained when, at the first-best, $e_{Am}^{FB}>e_{aM}^{FB}$ holds.

C.2.2 Pooling and exclusion

What happens when full screening is not viable? Below $\underline{\gamma}^{SB\mathcal{A}2}$, one expects the employer to exclude less skilled workers, namely am and possibly aM too, while above $\overline{\gamma}^{SB\mathcal{A}2}$, one expects to have a pooling equilibrium where worker Am and aM are given the same contract and, eventually, the worst worker am

is excluded. Again, we refer the reader to Appendix D.2 for the detailed analysis of the latter situation and we concentrate here on the first one, exclusion.

Suppose that the employer excludes worker am and offers her the null contract. The firm's program must be slightly modified with respect to full participation, the main differences being that monotonicity constraint $e_{aM} > e_{am}$ is omitted and that PC_{aM} (rather than PC_{am}) is assumed to be binding. Moreover, the requirement that incentive constraint $IC_{Am/am}$ be binding and $IC_{Am/aM}$ be slack reduces to the need that PC_{Am} binds and that $e_{aM}^{SB\mathcal{A}2} \leq \frac{2\gamma}{\Delta\theta}$ holds, which is true if and only if

$$\gamma \geq \frac{\Delta\theta}{\theta+1} = \underline{\underline{\gamma}}^{SB\mathcal{A}2},$$

where $\underline{\underline{\gamma}}^{SB\mathcal{A}2} < \underline{\gamma}^{SB\mathcal{A}2}$ always holds when $\mu < \frac{1}{2}$. Furthermore, the requirement that incentive constraint $IC_{aM/am}$ be binding and $IC_{aM/Am}$ be slack reduces to PC_{aM} being binding and to $e_{Am}^{SB\mathcal{A}2} \geq \frac{2\gamma}{\Delta\theta}$, which is true for

$$\gamma \leq \frac{(1-\mu)\,\Delta\theta}{2\,(1-\mu)+\mu\Delta\theta} = \overline{\overline{\gamma}}^{SB\mathcal{A}2},$$

with $\underline{\underline{\gamma}}^{SB\mathcal{A}2} < \min\left\{\overline{\overline{\gamma}}^{SB\mathcal{A}2}, \underline{\gamma}^{SB\mathcal{A}2}\right\}$. Hence a solution characterised by exclusion of type am, separation of the remaining types and both PC_{Am} and PC_{aM} binding exists for $\underline{\underline{\gamma}}^{SB\mathcal{A}2} \leq \gamma < \min\left\{\overline{\overline{\gamma}}^{SB\mathcal{A}2}, \underline{\gamma}^{SB\mathcal{A}2}\right\}$.

Result 8 (i) ***Separation and exclusion of (at least) worker*** am ***when ability prevails.*** *The solution to the principal's program SB, which entails separation but exclusion of worker am, both* PC_{aM} *and* PC_{Am} *binding, which satisfies the monotonicity condition* $e_{AM} > e_{Am} > e_{aM} > e_{am} = 0$ *and which is such that effort levels are given by expressions from (27) to (29) represents the optimal contract when* $\mu < \frac{1}{2}$ *and* $\underline{\underline{\gamma}}^{SB\mathcal{A}2} \leq \gamma \leq \min\left\{\underline{\gamma}^{SB\mathcal{A}2}, \overline{\overline{\gamma}}^{SB\mathcal{A}2}\right\}$, *where*

$$\begin{array}{lll} \underline{\underline{\gamma}}^{SB\mathcal{A}2} & \equiv & \frac{\Delta\theta}{(\theta+1)} \\ \overline{\overline{\gamma}}^{SB\mathcal{A}2} & \equiv & \frac{(1-\mu)\Delta\theta}{2(1-\mu)+\mu\Delta\theta} \end{array}.$$

The solution characterised by exclusion of both worker types am and aM represents the optimal contract either when $\gamma < \underline{\underline{\gamma}}^{SB\mathcal{A}2}$ *or when* $\overline{\overline{\gamma}}^{SB\mathcal{A}2} < \gamma < \underline{\gamma}^{SB\mathcal{A}2}$.

(ii) ***Full participation and pooling between workers*** aM ***and*** Am ***when ability prevails and*** $IC_{Am/am}$ ***is binding.*** *The solution to the firm's program SB which entails full participation and pooling between workers Am and aM and* $IC_{Am/am}$ *binding, which is such that effort levels are given by expressions (27), (30) and*

$$e_{Am} = e_{aM} \equiv e_{\underline{p}}^{SB\mathcal{A}2} = \frac{(\nu\,(1-\mu)+\mu\,(1-\nu)) - \gamma\mu\nu}{(\nu\,(1-\mu)+\mu\,(1-\nu))} = e_{\underline{p}}^{SB\mathcal{M}} \tag{31}$$

represents the optimal contract when $\gamma \geq \underline{\gamma}^{SB\mathcal{P}2}$, *where*

$$\underline{\gamma}^{SB\mathcal{P}2} \equiv \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta(\Delta\theta+2(1-\nu)(1-\mu))}{(\theta-(1-(1-\nu)(1-\mu)))(2(\nu(1-\mu)+\mu(1-\nu))+\mu\nu\Delta\theta)} > \overline{\gamma}^{SB\mathcal{A}2}.$$

(iii) ***Pooling between workers*** aM ***and*** Am ***and exclusion of*** am ***when ability prevails.*** *The solution to the firm's program SB which entails pooling between types* Am *and* aM*, exclusion of worker* am *and* PC_{Am} *binding, which is such that effort levels are given by expressions (27) and (31) represents the optimal contract when* $\overline{\gamma}^{SB\mathcal{A}2} \leq \gamma < \underline{\gamma}^{SB\mathcal{P}2}$.

Observe that Result 8(ii) describes precisely the same pooling equilibrium obtained in Case $\mathcal{M}$ for motivation levels below the threshold $\underline{\gamma}^{SB\mathcal{M}}$.

C.3 Case $\mathcal{A}.3$

C.3.1 Full separation and full participation

Suppose that the binding incentive constraints are $IC_{AM/Am}$, $IC_{Am/am}$ and the *upward* incentive constraint $IC_{aM/Am}$, together with participation constraint PC_{am}. This results in inequality $e_{am} + e_{Am} \leq \frac{2\gamma}{\Delta\theta} \leq e_{aM} + e_{Am}$.

The relevant wage levels and information rents are now

$$w_{am} = \frac{1}{2}\theta e_{am}^2, \tag{32}$$

$$w_{aM} = \frac{1}{2}\theta e_{aM}^2 - \gamma e_{aM} \underbrace{-\frac{1}{2}\Delta\theta e_{Am}^2 + \gamma e_{Am} + \frac{1}{2}\Delta\theta e_{am}^2}_{\text{Info rent worker } aM}, \tag{33}$$

$$w_{Am} = \frac{1}{2}e_{Am}^2 \quad \underbrace{+\frac{1}{2}\Delta\theta e_{am}^2}_{\text{Info rent worker } Am} \tag{34}$$

and

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma e_{AM} \underbrace{+\gamma e_{Am} + \frac{1}{2}\Delta\theta e_{am}^2}_{\text{Info rent worker } AM}. \tag{35}$$

The information rent of worker aM is composed of two terms: one is $\frac{1}{2}\Delta\theta e_{am}^2$ and represents the rent cumulated from worker Am mimicking am (which accrues to all types except am); the other one is $-\frac{1}{2}\Delta\theta e_{Am}^2 + \gamma e_{Am}$ and represents the portion of the rent specific to type aM mimicking Am. The latter term already appeared in Case $\mathcal{M}$ and it is strictly positive in this case as well. Also note that motivated workers receive an information rent which depends both on the difference in ability and on motivation, so that this case shares some features both with Benchmark $U\mathcal{A}$ and with Benchmark $U\mathcal{M}$.

Substituting the wage functions into the firm's expected profits and maximising, one obtains optimal effort levels

$$e_{AM}^{SB\mathcal{A}3} = 1 + \gamma, \tag{36}$$

$$e_{Am}^{SB\mathcal{A}3} = \frac{\nu(1-\mu) - \mu\gamma}{(\mu(1-\nu) + \nu(1-\mu)) - \mu(1-\nu)\theta}, \tag{37}$$

$$e_{aM}^{SB\mathcal{A}3} = \frac{1+\gamma}{\theta} = e_{aM}^{SB\mathcal{A}2} = e_{aM}^{FB} \tag{38}$$

and

$$e_{am}^{SB\mathcal{A}3} = \frac{(1-\mu)(1-\nu)}{\theta - (1-(1-\mu)(1-\nu))} = e_{am}^{SB\mathcal{M}}. \tag{39}$$

Observe that $e_{am}^{SB\mathcal{A}3}$ has the same expression as $e_{am}^{SB\mathcal{M}}$ and, more importantly, that both $e_{AM}^{SB\mathcal{A}3}$ and $e_{aM}^{SB\mathcal{A}3}$ are equal to their first-best levels. Moreover, the usual downward distortion holds for worker Am, despite the upward incentive constraint $IC_{aM/Am}$ being binding. All effort levels are always strictly positive, except for $e_{Am}^{SB\mathcal{A}3}$. In order for $e_{Am}^{SB\mathcal{A}3}$ to be positive and to be a maximum of the principal's expected profits, it is necessary to impose that both the numerator and the denominator of its expression be positive: the numerator of $e_{Am}^{SB\mathcal{A}3}$ is positive for $\gamma < \gamma_0$ (see expression 12) and the denominator of $e_{Am}^{SB\mathcal{A}3}$ is positive when

$$\theta < \frac{(\mu(1-\nu) + \nu(1-\mu))}{\mu(1-\nu)} = \overline{\theta}^{\mathcal{A}3}.$$

Note that $\overline{\theta}^{\mathcal{A}3} > 2$ if and only if $\mu < \nu$, thus, under the assumption that $\theta \leq 2$, the requirement $\theta < \overline{\theta}^{\mathcal{A}3}$ is always satisfied when $\mu < \nu$.

As for the monotonicity conditions, it must be that $e_{Am}^{SB\mathcal{A}3} > e_{aM}^{SB\mathcal{A}3}$, which holds if and only if

$$\gamma < \frac{(\mu(1-\nu) + \nu(1-\mu))\,\Delta\theta}{\mu\nu\theta + (\mu(1-\nu) + \nu(1-\mu))} = \overline{\gamma}^{SB\mathcal{A}3}$$

where $\overline{\gamma}^{SB\mathcal{A}3} < \Delta\theta$ and $\overline{\gamma}^{SB\mathcal{A}3} < \gamma_0$ are always true. Moreover, $e_{aM}^{SB\mathcal{A}3} > e_{am}^{SB\mathcal{A}3}$ always holds and $e_{Am}^{SB\mathcal{A}3} > e_{am}^{SB\mathcal{A}3}$ is always satisfied when $e_{Am}^{SB\mathcal{A}3} > e_{aM}^{SB\mathcal{A}3}$ is (namely when $\gamma < \overline{\gamma}^{SB\mathcal{A}3}$). Notice that $e_{Am}^{SB\mathcal{A}3}$ is distorted downwards if and only if

$$\gamma > (1-\nu)\,\Delta\theta = \gamma_1^{SB\mathcal{A}3}$$

where $\gamma_1^{SB\mathcal{A}3} < \overline{\gamma}^{SB\mathcal{A}3}$. Hence if motivation is not too high, Case $\mathcal{A}.3$ could be compatible with an upward distortion in the effort of the skilled but non-motivated worker Am.

Consider now the additional constraints $e_{Am} + e_{am} \leq \frac{2\gamma}{\Delta\theta} \leq e_{Am} + e_{aM}$. As for $\frac{2\gamma}{\Delta\theta} \leq e_{Am}^{SB\mathcal{A}3} + e_{aM}^{SB\mathcal{A}3}$, it is always satisfied provided that $\gamma < \overline{\gamma}^{SB\mathcal{A}3}$, while $e_{Am}^{SB\mathcal{A}3} + e_{am}^{SB\mathcal{A}3} \leq \frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma \geq \frac{\Delta\theta(1-\mu)\left(2\nu(1-\nu)(1-\mu) + \left(\nu - \mu(1-\nu)^2\right)\Delta\theta\right)}{(\theta-(1-(1-\mu)(1-\nu)))(2\nu(1-\mu) - \mu(1-2\nu)\Delta\theta)} = \underline{\gamma}^{SB\mathcal{A}3}$$

where $\underline{\gamma}^{SB\mathcal{A}3} > \gamma_1^{SB\mathcal{A}3}$ (implying that $e_{Am}^{SB\mathcal{A}3}$ is always distorted downwards when full participation and full separation is possible) and $\underline{\gamma}^{SB\mathcal{A}3} < \overline{\gamma}^{SB\mathcal{A}3}$ when

$$\theta > \frac{\mu(1-\nu) + \nu(1-\mu) - \nu\mu((1-(1-\mu)(1-\nu)))}{\mu(1-\nu) + \nu(1-\mu) - \nu\mu((1+(1-\mu)(1-\nu)))} = \underline{\theta}^{\mathcal{A}3},$$

where $\underline{\theta}^{\mathcal{A}3} < 2$ is always true when $\nu < 1 - \frac{1}{3}\sqrt{6} = 0.1835$ and otherwise $\underline{\theta}^{\mathcal{A}3} < 2$ holds if and only if

$$\mu < \frac{\left(6\nu - 3\nu^2 - 1\right) - \sqrt{((6\nu - 3\nu^2 - 1))^2 - 12\nu^2(1-\nu)}}{6\nu(1-\nu)} = \mu_1$$

Moreover, $\underline{\theta}^{\mathcal{A}3} < \overline{\theta}^{\mathcal{A}3}$ if and only if

$$\mu < \frac{\left(4\nu-\nu^2-1\right)-\sqrt{((4\nu-\nu^2-1))^2-4\nu(3\nu-2)(1-\nu)}}{2(3\nu-2)(1-\nu)} = \mu_2 > \frac{1}{2}$$

(for $\nu \neq \frac{2}{3}$ or if and only if $\mu < \frac{\nu}{4\nu-\nu^2-1}$ for $\nu = \frac{2}{3}$).

We are thus able to provide the conditions under which the optimal contract with full separation and full participation is implemented.

Result 9 ***Full screening when ability prevails and Case $\mathcal{A}.3$ holds.*** *The solution to the firm's program SB, which entails full participation, full separation of workers' types and $IC_{AM/Am}$, $IC_{aM/Am}$ and $IC_{Am/am}$ binding, which satisfies the monotonicity condition $e_{AM} > e_{Am} > e_{aM} > e_{am} > 0$ and which is such that effort levels are given by expressions from (36) to (39), exists and represents the optimal contract if and only if $\mu < \min\{\mu_1, \mu_2\}$, $\underline{\theta}^{\mathcal{A}3} < \theta < \overline{\theta}^{\mathcal{A}3}$ and $\underline{\gamma}^{SB\mathcal{A}3} \leq \gamma < \overline{\gamma}^{SB\mathcal{A}3}$, with*

$$\begin{aligned}
\underline{\gamma}^{SB\mathcal{A}3} &\equiv \frac{\Delta\theta(1-\mu)\left(2\nu(1-\nu)(1-\mu)+\left(\nu-\mu(1-\nu)^2\right)\Delta\theta\right)}{(\theta-(1-(1-\mu)(1-\nu)))(2\nu(1-\mu)-\mu(1-2\nu)\Delta\theta)} \\
\overline{\gamma}^{SB\mathcal{A}3} &\equiv \frac{\Delta\theta(\mu(1-\nu)+\nu(1-\mu))}{\mu\nu\theta+(\mu(1-\nu)+\nu(1-\mu))} \\
\mu_1 &\equiv \frac{\left(6\nu-3\nu^2-1\right)-\sqrt{((6\nu-3\nu^2-1))^2-12\nu^2(1-\nu)}}{6\nu(1-\nu)} \\
\mu_2 &\equiv \frac{\left(4\nu-\nu^2-1\right)-\sqrt{((4\nu-\nu^2-1))^2-4\nu(3\nu-2)(1-\nu)}}{2(3\nu-2)(1-\nu)} > \frac{1}{2} \\
\overline{\theta}^{\mathcal{A}3} &\equiv \frac{(\mu(1-\nu)+\nu(1-\mu))}{\mu(1-\nu)} \\
\underline{\theta}^{\mathcal{A}3} &\equiv \frac{(\mu(1-\nu)+\nu(1-\mu)-\nu\mu((1-(1-\mu)(1-\nu))))}{(\mu(1-\nu)+\nu(1-\mu)-\nu\mu((1+(1-\mu)(1-\nu))))}
\end{aligned}.$$

C.3.2 Pooling and exclusion

What happens when full participation and full separation is not viable? Above $\overline{\gamma}^{SB\mathcal{A}3}$, one expects to have a pooling equilibrium where types Am and aM are given the same contract. And also below $\underline{\gamma}^{SB\mathcal{A}3}$ one still finds that this solution is relevant. Again, we refer the reader to Appendix D.2 for the conditions of existence of a pooling equilibrium and we focus attention here on optimal contracts.

Result 10 ***Full participation and pooling between workers aM and Am when ability prevails and $IC_{Am/am}$ is binding.*** *The solution to the firm's program SB which is characterised by full participation and pooling between types Am and aM and $IC_{Am/am}$ binding, by effort levels described by expressions (36), (39) and (31) represents the optimal contract when $\overline{\gamma}^{SB\mathcal{A}3} \leq \gamma \leq \Delta\theta$ and when $\underline{\gamma}^{SB\mathcal{P}2} \leq \gamma \leq \underline{\gamma}^{SB\mathcal{A}3}$.*

Below $\underline{\gamma}^{SB\mathcal{A}3}$ one also finds pooling between workers aM and Am and exclusion of type am and (eventually) a solution with separation but exclusion of type am. Interestingly, in the latter case, it is possible to have an *upward distortion* of the effort required to type Am, but not so important as to allow for a pooling equilibrium where types AM and Am are given the same contract.

Suppose that worker am is left out. In this circumstance, the optimal levels of effort are the same as under full participation, except for $e_{am} = 0$, and all relevant constraints are satisfied whenever the chain of inequalities $e_{Am} \leq \frac{2\gamma}{\Delta\theta} \leq e_{Am} + e_{aM}$ holds.

Now, $\frac{2\gamma}{\Delta\theta} \leq e_{Am}^{SB\mathcal{A}3} + e_{aM}^{SB\mathcal{A}3}$ is always satisfied when $\gamma < \overline{\gamma}^{SB\mathcal{A}3}$, whereas $e_{Am}^{SB\mathcal{A}3} \leq \frac{2\gamma}{\Delta\theta}$ is true if and only if

$$\gamma \geq \frac{\nu(1-\mu)\Delta\theta}{(2\nu(1-\mu) - \mu\Delta\theta(1-2\nu))} = \underline{\underline{\gamma}}^{SB\mathcal{A}3}$$

where $\underline{\underline{\gamma}}^{SB\mathcal{A}3} < \underline{\gamma}^{SB\mathcal{A}3}$ always holds and where $\underline{\underline{\gamma}}^{SB\mathcal{A}3} > \gamma_1^{SB\mathcal{A}3}$ if and only if $\nu > \frac{1}{2}$. Hence a solution with exclusion of type am under Case $\mathcal{A}.3$ exists for $\underline{\underline{\gamma}}^{SB\mathcal{A}3} \leq \gamma < \overline{\gamma}^{SB\mathcal{A}3}$ and $\theta < \overline{\theta}^{\mathcal{A}3}$. Observe that, when $\nu < \frac{1}{2}$ and $\underline{\underline{\gamma}}^{SB\mathcal{A}3} \leq \gamma < \gamma_1^{SB\mathcal{A}3}$, the solution entails an upward distortion in the level of effort provided by worker Am.

Result 11 (i) ***Pooling between workers** aM **and** Am **and exclusion of worker** am **when ability prevails and** PC_{Am} **is binding.*** *A solution to the firm's program SB with pooling between workers Am and aM and exclusion of type am, with PC_{Am} binding and with effort levels described by expressions (36) and (31) represents the optimal contract when $\underline{\underline{\gamma}}^{SB\mathcal{P}2} < \gamma < \min\left\{\underline{\underline{\gamma}}^{SB\mathcal{A}3}, \underline{\gamma}^{SB\mathcal{P}2}\right\}$, where*

$$\begin{array}{ll} \underline{\underline{\gamma}}^{SB\mathcal{A}3} \equiv & \frac{\nu(1-\mu)\Delta\theta}{(2\nu(1-\mu)-\mu\Delta\theta(1-2\nu))} \\ \underline{\underline{\gamma}}^{SB\mathcal{P}2} \equiv & \frac{\Delta\theta(\nu(1-\mu)+\mu(1-\nu))}{(\nu\mu\Delta\theta+2(\nu(1-\mu)+\mu(1-\nu)))} \end{array}.$$

(ii) ***Separation and exclusion of worker** am **when ability prevails and** $IC_{aM/Am}$ **and** PC_{Am} **are binding.*** *A solution to the firm's program SB with exclusion of worker am and $IC_{aM/Am}$ and PC_{Am} binding and with effort levels described by expressions from (36) to (38) represents the optimal contract only if $\underline{\underline{\gamma}}^{SB\mathcal{A}3} < \underline{\gamma}^{SB\mathcal{P}2}$ and $\underline{\underline{\gamma}}^{SB\mathcal{A}3} \leq \gamma < \underline{\gamma}^{SB\mathcal{P}2}$.*

Result 11(i) describes precisely the same pooling equilibrium obtained in Case $\mathcal{M}$ and Case $\mathcal{A}.2$.

C.4 Proof of Remark 5

Consider the contracts with full separation and full participation in Cases $\mathcal{M}$ and $\mathcal{A}.3$. Since γ is always higher in Case $\mathcal{M}$ than in Case $\mathcal{A}.3$, let $\gamma^{\mathcal{M}}$ and $\gamma^{\mathcal{A}3}$, with $\gamma^{\mathcal{M}} > \gamma^{\mathcal{A}3}$, denote two levels of motivation supporting the two solutions; instead, let θ be the same in the two situations.[5]

Expected information rents paid by the employer in Case $\mathcal{M}$ are higher than in Case $\mathcal{A}.3$ if and only if

$$\mu\nu u_{AM}^{SB\mathcal{M}} + \mu(1-\nu) u_{aM}^{SB\mathcal{M}} + \nu(1-\mu) u_{Am}^{SB\mathcal{M}} > \mu\nu u_{AM}^{SB\mathcal{A}3} + \mu(1-\nu) u_{aM}^{SB\mathcal{A}3} + \nu(1-\mu) u_{Am}^{SB\mathcal{A}3},$$

[5] It is always the case that $\min\left\{\overline{\theta}_1^{\mathcal{M}}, \overline{\theta}_2^{\mathcal{M}}\right\} < \overline{\theta}^{\mathcal{A}3}$ but there is a wide range of probabilities $\mu < \frac{1}{2}$ and ν such that $\underline{\theta}^{\mathcal{A}3} < \min\left\{\overline{\theta}_1^{\mathcal{M}}, \overline{\theta}_2^{\mathcal{M}}\right\}$, meaning that the two subsets of θ are at least partially overlapping.

where u_{am} is omitted from both sides because it is equal to zero. A sufficient condition for the above inequality to hold is that $u_{ij}^{SB\mathcal{M}} \geq u_{ij}^{SB\mathcal{A}3}$ for every type of worker ij, with at least one strict inequality, where the actual expressions for information rents are given by (4) to (7) for Case $\mathcal{M}$ and by (33) to (35) for Case $\mathcal{A}.3$.

Now, $u_{AM}^{SB\mathcal{M}} > u_{AM}^{SB\mathcal{A}3}$ holds if and only if

$$\tfrac{1}{2}(\theta-1)\left(\left(e_{aM}^{SB\mathcal{M}}\right)^2 - \left(e_{Am}^{SB\mathcal{M}}\right)^2\right) + \gamma^{\mathcal{M}} e_{Am}^{SB\mathcal{M}} + \tfrac{1}{2}(\theta-1)\left(e_{am}^{SB\mathcal{M}}\right)^2 \quad > \quad \gamma^{\mathcal{A}3} e_{Am}^{SB\mathcal{A}3} + \tfrac{1}{2}(\theta-1)\left(e_{am}^{SB\mathcal{A}3}\right)^2 \ .$$

Given that $\left(\left(e_{aM}^{SB\mathcal{M}}\right)^2 - \left(e_{Am}^{SB\mathcal{M}}\right)^2\right)$ is always positive in Case $\mathcal{M}$ and that $e_{am}^{SB\mathcal{M}} = e_{am}^{SB\mathcal{A}3}$ for fixed θ, a sufficient condition for the above inequality to hold is simply that $\gamma^{\mathcal{M}} e_{Am}^{SB\mathcal{M}} > \gamma^{\mathcal{A}3} e_{Am}^{SB\mathcal{A}3}$ which is indeed the case.

Moreover, $u_{aM}^{SB\mathcal{M}} > u_{aM}^{SB\mathcal{A}3}$ if and only if

$$\begin{array}{c} -\tfrac{1}{2}(\theta-1)\left(e_{Am}^{SB\mathcal{M}}\right)^2 + \gamma^{\mathcal{M}} e_{Am}^{SB\mathcal{M}} + \tfrac{1}{2}(\theta-1)\left(e_{am}^{SB\mathcal{M}}\right)^2 > \\ -\tfrac{1}{2}(\theta-1)\left(e_{Am}^{SB\mathcal{A}3}\right)^2 + \gamma^{\mathcal{A}3} e_{Am}^{SB\mathcal{A}3} + \tfrac{1}{2}(\theta-1)\left(e_{am}^{SB\mathcal{A}3}\right)^2 \end{array} . \tag{40}$$

For the time being, suppose that $\gamma^{\mathcal{M}} = \gamma^{\mathcal{A}3} = \gamma$; since $e_{am}^{SB\mathcal{M}} = e_{am}^{SB\mathcal{A}3}$, one can simplify the above inequality as

$$-\tfrac{1}{2}(\theta-1)\left(e_{Am}^{SB\mathcal{M}} + e_{Am}^{SB\mathcal{A}3}\right)\left(e_{Am}^{SB\mathcal{M}} - e_{Am}^{SB\mathcal{A}3}\right) + \gamma\left(e_{Am}^{SB\mathcal{M}} - e_{Am}^{SB\mathcal{A}3}\right) \quad > \quad 0$$

and, being $e_{Am}^{SB\mathcal{M}} > e_{Am}^{SB\mathcal{A}3}$ for the same γ, one can further simplify it as

$$e_{Am}^{SB\mathcal{M}} + e_{Am}^{SB\mathcal{A}3} < \frac{2\gamma}{\Delta\theta}.$$

Substituting for the expressions of $e_{Am}^{SB\mathcal{M}}$ and $e_{Am}^{SB\mathcal{A}3}$, the latter condition is equivalent to

$$\gamma > \quad \frac{(\theta-1)(1-\mu)(2\mu+2\nu-2\theta\mu-3\mu\nu+\theta\mu\nu)}{(2\mu+2\nu-2\theta\mu-6\mu\nu+2\theta\mu\nu-3\mu^2+4\theta\mu^2+4\mu^2\nu-2\theta\mu^2\nu-\theta^2\mu^2)} \quad \equiv \quad \gamma' \ .$$

Note that $\gamma' < \underline{\gamma}^{SB\mathcal{A}3}$ always holds so $e_{Am}^{SB\mathcal{M}} + e_{Am}^{SB\mathcal{A}3} < \frac{2\gamma}{\Delta\theta}$ is always satisfied when both Cases $\mathcal{M}$ and $\mathcal{A}.3$ are relevant. A fortiori, inequality (40) also holds for $\gamma^{\mathcal{M}} > \gamma^{\mathcal{A}3}$.

Finally, $u_{Am}^{SB\mathcal{M}} = u_{Am}^{SB\mathcal{A}3}$ because $u_{Am} = \frac{1}{2}(\theta-1)\, e_{am}^2$ and again $e_{am}^{SB\mathcal{M}} = e_{am}^{SB\mathcal{A}3}$.

C.5 Proof of Proposition 6

Step 1. Let us first show that profits to the principal in Case $\mathcal{M}$ are strictly increasing in γ. Consider profits to the principal, whose expression is the objective function in program SB, and substitute all wages for their expressions, which are given by the binding IC constraints that are relevant in Case $\mathcal{M}$. This yields profits as a function of effort levels only

$$\begin{aligned} \pi_{FS,FP}^{\mathcal{M}} = \quad & \nu\mu\left((1+\gamma)\, e_{AM} - \tfrac{1}{2}e_{AM}^2\right) + (1-\nu)\,\mu\left((1+\gamma)\, e_{aM} - \tfrac{1}{2}\theta e_{aM}^2\right) - \nu\mu\left(\tfrac{1}{2}(\theta-1)\, e_{aM}^2\right) \\ & + \nu(1-\mu)\left(e_{Am} - \tfrac{1}{2}e_{Am}^2\right) + \mu\left(\tfrac{1}{2}(\theta-1)\, e_{Am}^2 - \gamma e_{Am}\right) + (1-\nu)(1-\mu)\left(e_{am} - \tfrac{1}{2}\theta e_{am}^2\right) \\ & - (1-(1-\mu)(1-\nu))\left(\tfrac{1}{2}(\theta-1)\, e_{am}^2\right) \end{aligned}$$

Applying the envelope theorem, which allows to disregard the indirect effect of parameters on optimal effort levels, and differentiating the profit function $\pi^{\mathcal{M}}$ with respect to γ, yields

$$\frac{\partial \pi^{\mathcal{M}}}{\partial \gamma} = \mu \left(\nu e_{AM} + (1-\nu) e_{aM} - e_{Am} \right).$$

Given that $e_{AM} > e_{aM}$ and that $e_{aM} > e_{Am}$ in Case $\mathcal{M}$, it can easily be checked that the above expression is positive, so that profits are strictly increasing in γ in Case $\mathcal{M}$.

Step 2. Let us then show that profits to the principal in Case $\mathcal{A}.3$ are strictly increasing in γ. Now the expression for profits is given by

$$\begin{aligned} \pi^{\mathcal{A}3}_{FS,FP} = \quad & \nu\mu \left(e_{AM} - \left(\tfrac{1}{2} e_{AM}^2 - \gamma e_{AM} \right) \right) + (1-\nu) \mu \left(e_{aM} - \left(\tfrac{1}{2} \theta e_{aM}^2 - \gamma e_{aM} \right) \right) \\ & + (\nu(1-\mu) - \gamma\mu) e_{Am} - \tfrac{1}{2} (e_{Am})^2 \left((\mu(1-\nu) + \nu(1-\mu)) - \mu(1-\nu)\theta \right) \\ & - \tfrac{1}{2} (e_{am})^2 (\theta - \mu - \nu + \mu\nu) + (1-\nu)(1-\mu) e_{am} \end{aligned}$$

Applying the envelope theorem, again, one can differentiate the profit function with respect to γ and obtain

$$\frac{\partial \pi^{\mathcal{A}3}}{\partial \gamma} = \mu \left(\nu e_{AM} + (1-\nu) e_{aM} - e_{Am} \right)$$

But now the derivative cannot be signed clearly because, in Case $\mathcal{A}.3$, inequality $e_{Am} > e_{aM}$ holds. Substituting optimal effort levels into the above expression yields

$$\tfrac{\partial \pi^{\mathcal{A}3}}{\partial \gamma} > 0 \quad \Leftrightarrow \quad \nu(1+\gamma) + (1-\nu) \left(\tfrac{1+\gamma}{\theta} \right) - \tfrac{\nu(1-\mu) - \mu\gamma}{(\mu(1-\nu) + \nu(1-\mu)) - \mu(1-\nu)\theta} > 0$$

and solving the inequality for γ one obtains

$$\tfrac{\partial \pi^{\mathcal{A}3}}{\partial \gamma} > 0 \quad \Leftrightarrow \quad \gamma > \tfrac{(\theta-1)(1-\nu)(\mu(1-\nu)+\nu(1-\mu)+\theta\mu\nu)}{((\theta-1)\nu+1)(\mu(1-\nu)+\nu(1-\mu))+\nu\mu\theta((\theta-1)\nu+2-\theta)} = \widehat{\gamma} \ ,$$

where $\widehat{\gamma} < \overline{\gamma}^{SB\mathcal{A}3}$. A sufficient condition for profits in Case $\mathcal{A}.3$ to be always strictly increasing in γ is that $\widehat{\gamma} < \underline{\gamma}^{SB\mathcal{A}3}$. The latter condition is verified for

$$\begin{aligned} & (1-\nu) \left((\mu(1-\nu) + \nu(1-\mu)) - \nu\mu(\mu + \nu - \mu\nu) \right) + \theta^2 \nu \left(\nu - 2\mu + 3\mu\nu + \mu^2 - \mu\nu^2 - 2\mu^2\nu + \mu^2\nu^2 \right) \\ & -\theta \left(((1-(1-\mu)(1-\nu))) - 2\nu(1-\nu) \left(2((1-(1-\mu)(1-\nu))) - (1-\mu)(\nu - \mu(1-\nu)) \right) \right) > 0 \end{aligned} \tag{41}$$

It can be checked that condition (41) is always satisfied above the locus of points (μ, ν) such that

$$\begin{aligned} & 2\mu^3 - \mu^2 - \mu\nu + \nu^3 + 3\mu\nu^2 + 6\mu^2\nu - 4\mu\nu^3 - 8\mu^3\nu + \mu\nu^4 + \\ & -10\mu^2\nu^2 + 7\mu^2\nu^3 + 11\mu^3\nu^2 - 2\mu^2\nu^4 - 6\mu^3\nu^3 + \mu^3\nu^4 \qquad = 0 \end{aligned} \tag{42}$$

(in particular it is always satisfied for $\nu > 0.28$). Below it, condition (41) is verified for $\theta < \widetilde{\theta}$, with

$$\widetilde{\theta} = \quad \tfrac{\left(\mu + \nu - 7\mu\nu - 2\nu^2 + 2\nu^3 + 10\mu\nu^2 + 2\mu^2\nu - 4\mu\nu^3 - 4\mu^2\nu^2 + 2\mu^2\nu^3 \right) - \sqrt{C}}{2\nu(\nu - 2\mu + 3\mu\nu + \mu^2 - \mu\nu^2 - 2\mu^2\nu + \mu^2\nu^2)}$$

where

$$C = \left(\left(\mu+\nu-7\mu\nu-2\nu^2+2\nu^3+10\mu\nu^2+2\mu^2\nu-4\mu\nu^3-4\mu^2\nu^2+2\mu^2\nu^3\right)\right)^2 \\ -4\nu\left(\nu-2\mu+3\mu\nu+\mu^2-\mu\nu^2-2\mu^2\nu+\mu^2\nu^2\right)(1-\nu)\left(\left(\mu(1-\nu)+\nu(1-\mu)\right)-\nu\mu\left(1-(1-\mu)(1-\nu)\right)\right)$$

Note that $\underline{\theta}^{\mathcal{A}3} < \widetilde{\theta} < \overline{\theta}^{\mathcal{A}3}$, so when Case $\mathcal{A}.3$ exists, and when μ and ν are such that we are below the locus of points where expression (42) holds, profits to the principal are increasing in γ for $\underline{\theta}^{\mathcal{A}3} < \theta < \widetilde{\theta}$.

Step 3. Finally, let us then show that, fixing θ and dropping the terms depending on e_{am} (because they are the same in the two cases and do not depend on γ), the lowest possible profits attainable under Case $\mathcal{M}$, that is $\pi^{\mathcal{M}}\left(\underline{\gamma}^{SB\mathcal{M}}\right)$, are always higher than the highest profits attainable under case $A.3$, that is $\pi^{\mathcal{A}3}\left(\overline{\gamma}^{SB\mathcal{A}3}\right)$. We have

$$\pi^{\mathcal{M}}\left(\underline{\gamma}^{SB\mathcal{M}}\right) = \frac{(1-(1-\mu)(1-\nu))^2\left(\nu^2(1-(1-\mu)(1-\nu))+(1-2\nu)(\mu(1-\nu)+\nu(1-\mu))+\nu\mu\theta(\theta-2\nu)\right)}{(\mu\nu(\theta-1)+(1-\nu)(1-(1-\mu)(1-\nu)))^2}$$

and

$$\pi^{\mathcal{A}3}\left(\overline{\gamma}^{SB\mathcal{A}3}\right) = \frac{\left((\mu(1-\nu)+\nu(1-\mu))+\theta^2\mu\nu\right)((1-(1-\mu)(1-\nu)))^2}{((\mu(1-\nu)+\nu(1-\mu))+\theta\mu\nu)^2}$$

and $\pi^{\mathcal{M}}\left(\underline{\gamma}^{SB\mathcal{M}}\right) > \pi^{\mathcal{A}3}\left(\overline{\gamma}^{SB\mathcal{A}3}\right)$ holds if and only if

$$\left(2\mu+2\nu-5\mu\nu-\nu^2+\mu\nu^2+2\theta\mu\nu\right) > 0$$

or else for

$$\theta > \frac{5\mu\nu-2\nu-2\mu+\nu^2-\mu\nu^2}{2\mu\nu} = \widehat{\theta}.$$

But the condition $\theta > \widehat{\theta}$ is always satisfied because $\widehat{\theta} < 1$, so we are done.

D Pooling between intermediate types aM and Am

Suppose that the principal offers a single contract to both agents Am and aM. Then one has $e_{Am} = e_{aM} = e_p$ and $w_{Am} = w_{aM} = w_p$. The relevant constraints are

$$w_{AM} - \frac{1}{2}e_{AM}^2 + \gamma e_{AM} \geq w_p - \frac{1}{2}e_p^2 + \gamma e_p$$

for type AM,

$$w_p - \frac{1}{2}e_p^2 \geq w_{am} - \frac{1}{2}e_{am}^2 \tag{43}$$

for type Am or

$$w_p - \frac{1}{2}\theta e_p^2 + \gamma e_p \geq w_{am} - \frac{1}{2}\theta e_{am}^2 + \gamma e_{am} \tag{44}$$

for type aM. Finally, for type am

$$w_{am} - \frac{1}{2}\theta e_{am}^2 \geq 0.$$

The binding participation constraint is the one of type am above, while all other participation constraints are satisfied provided that PC_{am} is. The monotonicity condition $e_{AM} \geq e_p \geq e_{am}$ holds; but which incentive compatibility constraint between (43), that is $IC_{Am/am}$, and (44), or else $IC_{aM/am}$, binds first? Taking into account the binding participation constraint of type am, it must be that

$$w_p \geq \max\left\{\frac{1}{2}\theta e_p^2 - \gamma e_p + \gamma e_{am}; \frac{1}{2}e_p^2 + \frac{1}{2}\Delta\theta e_{am}^2\right\}.$$

Thus, $IC_{aM/am}$ is binding first when

$$\frac{1}{2}\theta e_p^2 - \gamma e_p + \gamma e_{am} \geq \frac{1}{2}e_p^2 + \frac{1}{2}\Delta\theta e_{am}^2 \Longleftrightarrow e_p + e_{am} \geq \frac{2\gamma}{\Delta\theta},$$

whereas $IC_{Am/am}$ is binding when

$$\frac{1}{2}\theta e_p^2 - \gamma e_p + \gamma e_{am} \leq \frac{1}{2}e_p^2 + \frac{1}{2}\Delta\theta e_{am}^2 \Longleftrightarrow e_p + e_{am} \leq \frac{2\gamma}{\Delta\theta}$$

In what follows we study the two sub-cases separately.

D.1 Pooling between intermediate types with $IC_{aM/am}$ binding

Suppose that when pooling occurs, $IC_{aM/am}$ is binding while $IC_{Am/am}$ is slack. We call this situation Case $\mathcal{P}.1$. Then one has $e_p + e_{am} \geq \frac{2\gamma}{\Delta\theta}$. Wages must satisfy

$$w_{am} = \frac{1}{2}\theta e_{am}^2, \tag{45}$$

$$w_p = \frac{1}{2}\theta e_p^2 - \gamma e_p \underbrace{+\gamma e_{am}}_{\text{Info rent worker } aM}, \tag{46}$$

and

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma e_{AM} \underbrace{+\frac{1}{2}\Delta\theta e_p^2 + \gamma e_{am}}_{\text{Info rent worker } AM}. \tag{47}$$

The wage w_p has the same expression as w_{aM} in Cases $\mathcal{A}.1$ and $\mathcal{A}.2$ (see equation 19).

Substituting again the wage schedules into the principal's program we find

$$\begin{aligned} e_{AM}^{SB\mathcal{P}1} &= 1+\gamma, \\ e_p^{SB\mathcal{P}1} \equiv e_{\overline{p}}^{SB\mathcal{A}1} &= \frac{(\nu(1-\mu)+\mu(1-\nu))(1+\gamma)}{\nu\mu\Delta\theta + (\nu(1-\mu)+\mu(1-\nu))\theta} \end{aligned} \tag{48}$$

and

$$e_{am}^{SB\mathcal{P}1} = \frac{(1-\nu)(1-\mu) - \gamma(1-(1-\nu)(1-\mu))}{(1-\nu)(1-\mu)\theta} = e_{am}^{SB\mathcal{A}1}.$$

Note that $e_{AM}^{SB\mathcal{P}1} > e_p^{SB\mathcal{P}1}$ and $e_{AM}^{SB\mathcal{P}1} > e_{am}^{SB\mathcal{P}1}$ always hold. Moreover $e_{am}^{SB\mathcal{P}1}$ is the same as $e_{am}^{SB\mathcal{A}1}$ since in both cases participation constraint of worker am is binding. Also observe that $e_{am}^{SB\mathcal{P}1}$ is strictly positive if and only if $\gamma < \gamma_1^{SB\mathcal{A}1}$, and $e_p^{SB\mathcal{P}1} > e_{am}^{SB\mathcal{P}1}$ if and only if

$$\gamma > \frac{\nu\mu(1-\nu)(1-\mu)\Delta\theta}{\nu\mu(1-(1-\nu)(1-\mu))\Delta\theta + \theta(\mu(1-\nu)+\nu(1-\mu))} = \underline{\gamma}^{SB\mathcal{P}1},$$

where $\underline{\gamma}^{SB\mathcal{P}1} < \underline{\gamma}^{SB\mathcal{A}1}$ always holds. Moreover, $e_{Am}^{SB\mathcal{A}1} > e_p^{SB\mathcal{P}1} > e_{aM}^{SB\mathcal{A}1}$ if and only if $\gamma < \gamma_2^{SB\mathcal{A}1}$ and the condition $e_p + e_{am} > \frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma < \frac{\Delta\theta(1-\mu)(1-\nu)(\mu\nu\Delta\theta+2\theta(\mu(1-\nu)+\nu(1-\mu)))}{\nu\mu(1-(1-\nu)(1-\mu))((4-\theta)\theta+1)+\theta(\theta+1)(\nu+\mu)-2\theta(4\mu\nu+\mu^2(1-\nu)+\nu^2(1-\mu))} = \overline{\gamma}^{SB\mathcal{P}1}$$

where $\overline{\gamma}^{SB\mathcal{P}1} > \underline{\gamma}^{SB\mathcal{P}1}$ is always true, $\overline{\gamma}^{SB\mathcal{P}1} < \gamma_1^{SB\mathcal{A}1}$ if and only if

$$\theta < \frac{(\nu(1-\mu)(1-\mu(1-\nu))+\mu(1-\nu)(1-\nu(1-\mu)))}{\left((2\nu-1)(\nu(1-\mu)+\mu(1-\nu))+2(1-\nu)^2\mu^2\right)} = \overline{\theta}^{\mathcal{P}1}$$

(always for $\nu < \frac{1}{2}$ and $\mu < \frac{(1-2\nu)^2+\sqrt{(1-2\nu)(1+2\nu-4\nu^2)}}{4(1-\nu)^2} \equiv \mu_3 < \frac{1}{2}$), where $\overline{\theta}^{\mathcal{A}1} < \overline{\theta}^{\mathcal{P}1}$ if and only if $\mu < \frac{(1-2\nu)+\sqrt{1+4\nu(1-\nu)}}{4(1-\nu)} \equiv \mu_4$, with $\mu_4 > \frac{1}{2}$, and $\overline{\gamma}^{SB\mathcal{P}1} < \gamma_2^{SB\mathcal{A}1}$ if and only if

$$\theta > \frac{(\mu(1-\nu)+\nu(1-\mu))+\mu\nu^2}{(\mu(1-\nu)+\nu(1-\mu))+\mu\nu^2-2\mu^2(1-\nu)^2} = \underline{\theta}^{\mathcal{P}1}$$

with $\overline{\theta}^{\mathcal{P}1} > \underline{\theta}^{\mathcal{P}1}$ if and only if $\mu < \mu_2$.

For further reference, note that, should type am be excluded, then the condition $e_p + e_{am} > \frac{2\gamma}{\Delta\theta}$ ensuring that $IC_{aM/am}$ is binding while $IC_{Am/am}$ is slack would become $e_p > \frac{2\gamma}{\Delta\theta}$ which is equivalent to the requirement that PC_{aM} be binding while PC_{Am} be slack. Then $e_p > \frac{2\gamma}{\Delta\theta}$ if and only if

$$\gamma < \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta}{2\nu\mu\Delta\theta+(\nu(1-\mu)+\mu(1-\nu))(\theta+1)} = \overline{\overline{\gamma}}^{SB\mathcal{P}1}$$

with $\underline{\gamma}^{SB\mathcal{P}1} < \overline{\overline{\gamma}}^{SB\mathcal{P}1} < \overline{\gamma}^{SB\mathcal{P}1}$.

It is now possible to state the following result.

Result 12 *(i)* ***Full participation and pooling between types*** aM ***and*** Am ***with*** $IC_{aM/am}$ ***binding.*** *A solution to the principal's program SB which entails full participation and pooling between types aM and Am and* $IC_{aM/am}$ *binding, which satisfies the monotonicity condition* $e_{AM} > e_{aM} = e_{Am} > e_{am} > 0$, *and which is such that effort levels are given by expressions (22), (25) and (48) exists if and only if* $\underline{\gamma}^{SB\mathcal{P}1} < \gamma < \min\left\{\gamma_1^{SB\mathcal{A}1}, \overline{\gamma}^{SB\mathcal{P}1}\right\}$ *with*

$$\underline{\gamma}^{SB\mathcal{P}1} \equiv \frac{\nu\mu(1-\nu)(1-\mu)\Delta\theta}{\nu\mu(1-(1-\nu)(1-\mu))\Delta\theta+\theta(\mu(1-\nu)+\nu(1-\mu))}$$

$$\overline{\gamma}^{SB\mathcal{P}1} \equiv \frac{\Delta\theta(1-\mu)(1-\nu)(\mu\nu\Delta\theta+2\theta(\mu(1-\nu)+\nu(1-\mu)))}{\nu\mu(1-(1-\nu)(1-\mu))((4-\theta)\theta+1)+\theta(\theta+1)(\nu+\mu)-2\theta(4\mu\nu+\mu^2(1-\nu)+\nu^2(1-\mu))}$$

$$\gamma_1^{SB\mathcal{A}1} \equiv \frac{(1-\nu)(1-\mu)}{(1-(1-\nu)(1-\mu))}$$

(ii) ***Pooling between types*** aM ***and*** Am ***with*** $IC_{aM/am}$ ***binding and exclusion of type*** am. *A solution to the principal's program SB which entails pooling between types aM and Am and* PC_{aM} *binding, exclusion of type am and which satisfies the monotonicity condition* $e_{AM} > e_{aM} = e_{Am} > 0$, *and which is such that effort levels are given by expressions (22) and (48) exists if and only if* $\gamma < \overline{\overline{\gamma}}^{SB\mathcal{P}1}$.

Note that in this Case $\mathcal{P}.1$ it never happens that type aM is asked to provide an effort which falls in the range where her utility is increasing in effort, namely it is never the case that $e_{aM} = e_{Am} = e_p^{SB\mathcal{P}1} < \frac{\gamma}{\theta}$. This might occur in the subsequent Case $\mathcal{P}.2$.

D.2 Pooling between intermediate types with $IC_{Am/am}$ binding

Suppose now that when pooling occurs, $IC_{Am/am}$ is binding while $IC_{aM/am}$ is slack. We call this situation Case $\mathcal{P}.2$, in which $e_p + e_{am} \leq \frac{2\gamma}{\Delta\theta}$. Wages must satisfy

$$w_{am} = \frac{1}{2}\theta e_{am}^2,$$
$$w_p = \frac{1}{2}e_p^2 \quad \underbrace{+\frac{1}{2}\Delta\theta e_{am}^2}_{\text{Info rent worker } Am} \tag{49}$$

and

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma e_{AM} \underbrace{+\gamma e_p + \frac{1}{2}\Delta\theta e_{am}^2}_{\text{Info rent worker } AM}.$$

Note that the wage w_p now has the same expression as w_{Am} in Case $\mathcal{M}$, Case $\mathcal{A}.2$ and Case $\mathcal{A}.3$.

Substituting the wage schedules into the program and deriving yields

$$e_{AM}^{SB\mathcal{P}2} = 1 + \gamma,$$
$$e_p^{SB\mathcal{P}2} \equiv e_{\underline{p}}^{SB\mathcal{M}} = e_{\overline{p}}^{SB\mathcal{A}2} = \frac{(\nu(1-\mu) + \mu(1-\nu)) - \gamma\mu\nu}{(\nu(1-\mu) + \mu(1-\nu))} \tag{50}$$

and

$$e_{am}^{SB\mathcal{P}2} = \frac{(1-\nu)(1-\mu)}{\theta - (1 - (1-\nu)(1-\mu))} = e_{am}^{SB\mathcal{M}} = e_{am}^{SB\mathcal{A}3},$$

where $e_{am}^{SB\mathcal{P}2}$ is equal to $e_{am}^{SB\mathcal{M}}$ and $e_{am}^{SB\mathcal{A}3}$ since in all cases the incentive constraint $IC_{Am/am}$ is binding. Note that $e_{AM}^{SB\mathcal{P}2} > e_p^{SB\mathcal{P}2}$ and $e_{AM}^{SB\mathcal{P}2} > e_{am}^{SB\mathcal{P}2}$ always hold, while $e_p^{SB\mathcal{P}2} > e_{am}^{SB\mathcal{P}2}$ holds if and only if

$$\gamma < \frac{(\nu(1-\mu) + \mu(1-\nu))\Delta\theta}{\nu\mu(\theta - (1-(1-\nu)(1-\mu)))} = \overline{\gamma}^{SB\mathcal{P}2}$$

which is such that $\overline{\gamma}^{SB\mathcal{P}2} > \Delta\theta$ whenever $\theta < \overline{\theta}_1^{\mathcal{M}}$ and such that $\overline{\gamma}^{SB\mathcal{P}2} > \underline{\gamma}^{SB\mathcal{M}}$ and $\overline{\gamma}^{SB\mathcal{P}2} > \overline{\gamma}^{SB\mathcal{A}2}$ always hold. Furthermore, observe that $e_p^{SB\mathcal{P}2} > 0$ always holds whenever $\gamma < \overline{\gamma}^{SB\mathcal{P}2}$ is true. Finally, the condition $e_p^{SB\mathcal{P}2} + e_{am}^{SB\mathcal{P}2} \leq \frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma \geq \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta(\Delta\theta+2(1-\nu)(1-\mu))}{(\theta-(1-(1-\nu)(1-\mu)))(2(\nu(1-\mu)+\mu(1-\nu))+\mu\nu\Delta\theta)} = \underline{\gamma}^{SB\mathcal{P}2}$$

where $\underline{\gamma}^{SB\mathcal{P}2} < \min\left\{\Delta\theta, \overline{\gamma}^{SB\mathcal{P}2}\right\}$ is always true and where $\overline{\gamma}^{SB\mathcal{A}2} < \underline{\gamma}^{SB\mathcal{P}2}$ and $\underline{\gamma}^{SB\mathcal{A}3} < \underline{\gamma}^{SB\mathcal{P}2} < \overline{\gamma}^{SB\mathcal{A}3}$ are also true.

Result 13 ***Full participation and pooling between types*** aM ***and*** Am ***with*** $IC_{Am/am}$ ***binding.*** *A solution to the principal's program SB which entails full participation and pooling between types aM and Am and* $IC_{Am/am}$ *binding, which satisfies the monotonicity condition* $e_{AM} > e_{aM} = e_{Am} > e_{am} > 0$*, and which is such that effort levels are given by expressions (8), (11) and (50) exists if and only if* $\underline{\gamma}^{SB\mathcal{P}2} \leq \gamma < \overline{\gamma}^{SB\mathcal{P}2}$ *with*

$$\underline{\gamma}^{SB\mathcal{P}2} \equiv \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta(\Delta\theta+2(1-\nu)(1-\mu))}{(\theta-(1-(1-\nu)(1-\mu)))(2(\nu(1-\mu)+\mu(1-\nu))+\mu\nu\Delta\theta)}$$
$$\overline{\gamma}^{SB\mathcal{P}2} \equiv \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta}{\nu\mu(\theta-(1-(1-\nu)(1-\mu)))}$$

Concerning exclusion of the worst type, we need to consider a similar program where, instead of having $IC_{Am/am}$ binding and $IC_{aM/am}$ slack, we need PC_{Am} to be binding and PC_{aM} to be slack. In this case, the requirement $e_p^{SB\mathcal{P}2}+ e_{am} \leq \frac{2\gamma}{\Delta\theta}$ reduces to the more general condition $e_p^{SB\mathcal{P}2} \leq \frac{2\gamma}{\Delta\theta}$, which is satisfied if and only if

$$\gamma \geq \frac{\Delta\theta\left(\nu\left(1-\mu\right)+\mu\left(1-\nu\right)\right)}{\left(\nu\mu\Delta\theta+2\left(\nu\left(1-\mu\right)+\mu\left(1-\nu\right)\right)\right)} = \underline{\underline{\gamma}}^{SB\mathcal{P}2}$$

where $\underline{\underline{\gamma}}^{SB\mathcal{P}2} < \underline{\gamma}^{SB\mathcal{P}2}$.

Result 14 ***Pooling between types aM and Am with PC_{Am} binding and exclusion of type am.*** *A solution to the principal's program SB which entails pooling between types aM and Am with PC_{Am} binding and exclusion of type am, which satisfies the monotonicity condition $e_{AM} > e_{aM} = e_{Am} > 0$, and which is such that effort levels are given by expressions (8) and (50) exists if and only if $\underline{\underline{\gamma}}^{SB\mathcal{P}2} \leq \gamma < \overline{\gamma}^{SB\mathcal{P}2}$ with*

$$\underline{\underline{\gamma}}^{SB\mathcal{P}2} \equiv \frac{\Delta\theta(\nu(1-\mu)+\mu(1-\nu))}{(\nu\mu\Delta\theta+2(\nu(1-\mu)+\mu(1-\nu)))}\;.$$

For further reference note that $\underline{\underline{\gamma}}^{SB\mathcal{P}2}$ is smaller than $\overline{\gamma}^{SB\mathcal{A}2}$ provided that $\theta \leq 2$.

Also note that it might eventually be the case that $e_{aM} = e_{Am} = e_p^{SB\mathcal{P}2} < \frac{\gamma}{\theta}$ in which situation type aM would have incentive to provide more effort than the one required by her optimal contract since the required effort falls in the range in which her indifference curve is downward sloping in the space (e, w). Nonetheless, the wage received by type aM would always be positive.

E Example

Let $\gamma_m = 0$ and $\gamma_M = \gamma \in (0, 1]$ and let $\theta_A = 1$ and $\theta_a = \theta \in (1, 2]$. Assume that motivation and skills are uniformly distributed across workers, so that $\mu = \nu = \frac{1}{2}$. Case $\mathcal{M}$ with full screening is attained for $1 < \theta < \frac{3}{2}$, Case $\mathcal{A}.2$ does not exist, while Case $\mathcal{A}.3$ with full screening holds for $\frac{5}{3} < \theta \leq 2$. Hence one can have three classes of problems: (i) the difference in ability is low and $1 < \theta < \frac{3}{2}$, and either motivation prevails and Case $\mathcal{M}$ is attained or ability prevails and Case $\mathcal{A}.1$ holds; (ii) the difference in ability is high and $\frac{5}{3} < \theta \leq 2$, ability always prevails and either Case $\mathcal{A}.1$ or Case $\mathcal{A}.3$ holds depending on the value taken by γ; (iii) the difference in ability is intermediate so that $\frac{3}{2} \leq \theta \leq \frac{5}{3}$, ability prevails and only Case $\mathcal{A}.1$ holds.

In situation (i), one observes the following optimal contracts: when $0 < \gamma \leq \frac{\Delta\theta}{3(2\theta-1)} = \underline{\gamma}^{SB\mathcal{A}1}$ the principal offers a pooling contract to low-skilled types aM and am, when $\underline{\gamma}^{SB\mathcal{A}1} < \gamma < \overline{\gamma}^{SB\mathcal{A}1} = \gamma_2^{SB\mathcal{A}1} = \frac{3\Delta\theta}{3\theta+1}$ full participation and full separation under Case $\mathcal{A}.1$ is implemented, when $\overline{\gamma}^{SB\mathcal{A}1} \leq \gamma < \overline{\gamma}^{SB\mathcal{P}1} = \frac{(5\theta-1)\Delta\theta}{13\theta^2-12\theta+3}$ the principal offers a pooling contract to intermediate types aM and Am, which is such that $IC_{aM/am}$ is binding. Notice that $\overline{\gamma}^{SB\mathcal{A}1} < \overline{\gamma}^{SB\mathcal{P}1}$ if and only if $\theta < \frac{5}{4}$, hence for $1 < \theta < \frac{5}{4}$

the latter pooling equilibrium exists, while for $\frac{5}{4} \leq \theta < \frac{3}{2}$ it does not. When $\max\left\{\overline{\gamma}^{SB\mathcal{P}1}, \overline{\gamma}^{SB\mathcal{A}1}\right\} \leq \gamma < \underline{\gamma}^{SB\mathcal{P}2} = \frac{4(2\theta-1)\Delta\theta}{(4\theta-3)(\theta+3)}$ there is pooling between intermediate types aM and Am with the constraint $IC_{Am/am}$ binding and exclusion of type am. Note that $\underline{\gamma}^{SB\mathcal{P}2} < \Delta\theta$ so that we still are in the domain in which ability prevails and $e_{Am} > e_{aM}$. When $\underline{\gamma}^{SB\mathcal{P}2} \leq \gamma \leq \underline{\gamma}^{SB\mathcal{M}} = \frac{4\Delta\theta}{2\theta+1}$ we have pooling between intermediate types aM and Am with the constraint $IC_{Am/am}$ binding but full participation is attained, and we cross $\Delta\theta$ so that motivation prevails and $e_{aM} > e_{Am}$. When $\underline{\gamma}^{SB\mathcal{M}} < \gamma < \frac{3\Delta\theta}{4\theta-3} = \overline{\gamma}^{SB\mathcal{M}} < \frac{1}{2}$, full separation and full participation is attained under Case $\mathcal{M}$. When $\overline{\gamma}^{SB\mathcal{M}} \leq \gamma < 1$ the principal offers a pooling contract to non-motivated types Am and am.

In situation (ii), one observes the following: when $0 < \gamma < \underline{\gamma}^{SB\mathcal{P}2}$ there are the same optimal contracts as in (i), when $\underline{\gamma}^{SB\mathcal{P}2} \leq \gamma < \underline{\gamma}^{SB\mathcal{A}3} = \frac{(3\theta-1)\Delta\theta}{2(4\theta-3)}$ we have pooling between intermediate types aM and Am with the constraint $IC_{Am/am}$ binding and full participation, when $\underline{\gamma}^{SB\mathcal{A}3} < \gamma < \overline{\gamma}^{SB\mathcal{A}3} = \frac{2\Delta\theta}{\theta+2}$ there is full participation and full separation under Case $\mathcal{A}.3$. When $\gamma \geq \overline{\gamma}^{SB\mathcal{A}3}$, two optimal contracts coexists: full participation and pooling between intermediate types aM and Am with the constraint $IC_{Am/am}$ binding, and full participation and pooling between non-motivated types Am and am. The former yields higher payoffs to the principal when $\gamma < \frac{-4(4\theta-3)(\theta-1)^2+10(\theta-1)\sqrt{\theta(2\theta-1)(4\theta-3)}}{(4\theta-3)(11\theta-2\theta^2-4)} = \widetilde{\gamma}$. Hence, when $\overline{\gamma}^{SB\mathcal{A}3} \leq \gamma < \widetilde{\gamma}$, there is full participation and pooling between intermediate types aM and Am with the constraint $IC_{Am/am}$ binding, while when $\widetilde{\gamma} \leq \gamma \leq 1$ there is full participation and pooling between non-motivated workers.

In situation (iii), one observes the following optimal contracts: when $0 < \gamma < \underline{\gamma}^{SB\mathcal{P}2}$ there are the same solutions as in (i) and (ii), when $\underline{\gamma}^{SB\mathcal{P}2} \leq \gamma < \widetilde{\gamma}$ we have full participation and pooling between intermediate types aM and Am with the constraint $IC_{Am/am}$ binding, and finally when $\widetilde{\gamma} \leq \gamma \leq 1$ there is full participation and pooling between non-motivated workers.