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Training and Product Quality in Unionized Oligopolies

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Abstract

In this paper we analyze the private and public incentives towards skill acquisition when the skill level of workers determines the quality level of goods, and both labor and product markets are non competitive. We delve into the mechanisms that determine the equilibrium skill acquisition outcomes and show that both “pure” (training set by either firms or unions only) and “mixed” (training set by firms and unions) training scenarios may emerge at equilibrium. We show that firms have generally greater training incentives than unions, resulting in a higher product quality. In line with empirical evidence, we also find that the wage differential between high-skill workers and low-skill workers is lower when the training levels of the workforce are selected by unions than by firms. Finally, we analyze the welfare properties of our framework at the possible equilibria as well as the optimal public training skill levels, and conclude that both unions and firms under-invest in training in comparison with the social optimum. Yet, at the welfare-maximizing training levels, the skill premium is the lowest.

Keywords: Workers’ Skills, Product Quality, Unionized Oligopoly, Training.

JEL Codes: L11, L13, J51

INTRODUCTION

Product quality is one of the main dimensions through which firms may gain a competitive edge over their rivals. The empirical literature has emphasized that the extent to which firms can quality-differentiate their products crucially hinges on the availability of quality-differentiated inputs. Stated differently, there is a *quality transfer* from input to output. In particular, the role of the skill level of the workforce employed in quality-differentiated industries has been recognized as a key driver of differences in the quality of final products (see Courakis, 1991; Webster, 1993; Maskus *et al.*, 1994; Oulton, 1996; Greenaway and Torstensson, 2000; Martín-Montaner and Ríos, 2002; Schott, 2004 among others).

Despite the empirical relevance of skill availability in determining the performance of an industry with (potentially) quality-differentiated products, little effort has been devoted to a formal analysis of markets

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where the availability of skilled labor determines the possibility, for oligopolistic firms, of differentiating their products. Relevant exceptions are [Gabszewicz and Turrini \(1999, 2000\)](#). These papers, however, consider the *skill level* of the workforce to be exogenous. To the best of our knowledge, no paper addresses skill acquisition in a framework where the skill level of the workforce directly determines the quality level of the available products.

The present paper is an attempt to bridge this gap. Indeed, this is a relevant field of investigation, as the skill acquisition decisions of the workforce influence final market competition through the degree of product differentiation. In turn, the extent of market competition determines the revenues of firms, and therefore, the remuneration of inputs and labor in particular as well. Finally, the (expected) earnings of firms and workers influence their willingness to invest in skill training. In such a framework, the existence of skilled labor requirements for the production of a specific variant of a differentiated good provides workers with bargaining power relative to their employers. Consequently, the wage earned by workers does not coincide, in general, with their marginal productivity.

In this paper, we analyze both incentives for skill acquisition and their effects on product market competition when both labor and product markets are not competitive.¹ In particular, we model an industry where each firm is in a relationship with a workplace-specific union. We assume that labor is the only input used in the production process, and each union is the only labor supplier with which a firm is paired. Furthermore, we assume that skills are worthless outside the industry under scrutiny. Accordingly, the remuneration of skilled labor is the result of bargaining between the union representing the workers and the firm that employs the workers only. Prior to bargaining over the wage, costly skill acquisition occurs. The skill level of workers determines the quality of the product of the firm that employs them. We first analyze, as a benchmark, the case of a single firm-union pair active on the market. Next, we develop the analysis for the duopoly case, which is the main contribution of our paper. Different scenarios may emerge according to which agent decides on the skill level to be obtained. In principle, training may be provided by the firm or acquired by the workers themselves. The former case is typically referred to as *firm-sponsored training* (see, e.g., [Booth and Chatterji, 1998](#); [Acemoglu and Pischke, 1998, 1999a,b](#)). The latter case is the so-called *union-sponsored training* (see, e.g., [Takahashi and Meléndez, 2004](#)): unions directly engage in workforce-development activities. This second possibility is consistent with the *rent-creator* role of the union (see [Freeman and Medoff, 1979](#)), whereby unions can directly undertake initiatives that increase the productivity of workplace-specific labor. One of these activities is skill creation/enhancement (see [Sutherland and Rainbird, 2000](#); [Rigby, 2002](#) and the references therein). Finally, we assess the welfare performance of the possible configurations and consider the case of public training to obtain the socially optimal training levels and put into perspective our result by analyzing alternative modeling assumptions.

In our analysis skill acquisition is costly. Accordingly, the agents investing in training have to carefully trade off the marginal cost of training and its marginal benefit. The latter, in turn, is determined by the effects of a change in the skill levels both on the degree of product differentiation (and hence of market competition) and on the variable production costs of the firms. In fact, a crucial feature of the present paper is that, under duopoly, the training level of the workforce directly influences the degree of market competition through the *quality levels* of products. An initial, obvious consequence is that, to

avoid product homogeneity and a harsh price war, training levels (and consequently product qualities) differ. This phenomenon, in turn, has a feedback effect on firm performance and thus on the workers' wages. Regarding competition in the product market, all else being equal, an increase of the quality level of the high-quality good results both in a higher surplus extraction and reduced price competition due to less homogeneous products. However, a quality increase requires costly training. By contrast, a decrease in the quality level of the low-quality good allows for savings in training costs and relaxes price competition but also results in reduced surplus extraction. In the labor market, a higher skill level for the high-skilled workers always increases their wage, whereas the wage of the low-skilled workers increases in their skill level only if product differentiation is sufficiently large. In sum, at equilibrium, training has a twofold effect on firms. On the one hand, training may improve firms' market performance by increasing the degree of product differentiation, leading to an enhanced ability to extract consumer surplus through more sophisticated products. On the other hand, more highly trained workers are paid higher wages, which increases production costs.

For the unions the forces at work are similar. An increase in training always increases the wage of high-skilled workers and may also increase that of low-skilled workers. However, the higher production costs generated by higher wages result in higher prices. This, in turn, reduces the market demands for the two goods and, ultimately, labor demand.²

However, there is a relevant difference between the firms and the unions in the incentives for making training investments. The firms control the goods prices, whereas unions do not. This gives firms an additional tool to extract surplus from consumers, and thus it turns out that the returns to training are higher for firms than for unions.

Our main results as for the equilibrium wage levels are as follows. First, the high-skill workers' equilibrium wage is always higher than that of the low-skill workers, reflecting the higher value-added given to the product. Second, we find that the highest skill (and product quality) levels are set in the case where the firms make the training decisions, while the lowest levels are obtained when the unions train the workers. This is due to the fact that firms, by choosing prices, are able to appropriate higher returns to training than unions do. Third, the wage differential between high-skill workers and low-skill workers is lower when the training levels of the workforce are selected by the union than by the firm. This prediction is consistent with empirical evidence on the determinants of the wage mark-up that suggests that unions contribute to reduce wage dispersion within the unionized sectors of the economy (see, e.g., [Freeman, 1980](#), [Gosling and Machin, 1994](#) and [Card, 1996](#)). Fourth, we find that mixed training patterns (training provided by one firm and one union) may emerge. In this case, the equilibrium skill levels are intermediate relative to the "pure" cases. Fifth, our model also provides predictions about the employment levels. Those "mirror" the ones for the skill levels, indeed the highest employment levels obtain in the case of union-sponsored training, whereas the lowest in the case of firm-sponsored training. The "mixed training cases" are associated with intermediate levels of employment.

Finally, we assess the welfare properties of this model and show that the optimal public training levels are higher than the ones obtained in all the "private-training" scenarios, but interestingly, the difference between the skill levels is the least at the socially optimal skill levels. In this case, market competition is tough and the skill premium is low. The results of the social welfare analysis suggest two general

conclusions. First, both unions and firms under-invest in training in comparison with the social optimum. This under-investment problem arises from the fact that, by increasing the training levels of the workforce, the quality levels of the products are increased, and thus, the surplus that is generated by consumption. Yet, when setting the training levels, neither firms nor unions take into account that consumers benefit from higher quality goods, which results in under-investment in training from a social perspective. Second, private training leads to a larger wage inequality due to higher skill differences. Intuitively, more similar skill levels lead to more homogeneous products, and ultimately to a harsher price war between firms. It follows that tougher competition benefits consumers through lower prices.

Our paper is related to a vast body of work that, departing from traditional human capital theory, has called into question the belief that the free market provides adequate incentives to train. This strand of literature argues that, since the markets for training are often characterized by imperfect competition and imperfect information, people generally do not receive adequate compensation for the training they acquire and provide. Within this body of research, [Stevens \(1994\)](#) explores the role of investment externalities due to the imperfect appropriability of transferable training by the firms. In her paper, when firms are imperfect competitors for labor, the benefits from training go not only to employers that provide the training and employees that acquire it but also to firms that poach. The author shows that this “poaching” externality may lead to under-investment in training (see, e.g., [Booth and Snower, 1995](#), [Acemoglu, 1997](#) and [Leuven, 2005](#)). Contrasting this result, [Moen \(1998\)](#) and [Moen and Rosén \(2004\)](#) argue that, even in the presence of frictions in the labor market, firms incentives to train are optimal. We explicitly contribute to this strand of literature by showing that, even if training is not transferable, the interaction between product quality and skills may represent another important source of under-provision in training. Our paper is also related to [Aidt and Sena \(2005\)](#), who study the process of skill acquisition in a unionized oligopoly. This paper analyzes the union’s choice between the activities of rent extraction and rent creation and considers the degree of market competition as exogenous, whereas in our paper it is endogenously determined by the training decisions. Finally, our paper relates to [Oosterbeek *et al.* \(2007\)](#) who find that, experimentally, employers have weakly greater incentives for making specific training investments than workers in an outside option bargaining game. Our theoretical analysis confirms this experimental finding. In a recent paper, [Shintoyo \(2010\)](#) develops a search and matching model of general human capital to contrast firm-sponsored training with worker-financed training and evaluate which type of training prevails in the economy. The author shows that firm-sponsored training is more likely to occur when there are greater frictions in the labor market, whereas worker-financed training prevails in a less frictional market. Unlike this paper, we focus on the comparison of the training outcomes in the two different scenarios, rather than determining when one prevails over the other.

The remainder of the paper is organized as follows. Section [I](#). presents the model. Section [II](#). determines the possible training equilibria and characterizes them when training is private, while Section [III](#). addresses optimal public training. Section [IV](#). discusses our main results and explores some extensions of the analysis. Finally, Section [V](#). provides a brief conclusion. All proofs are relegated in the Appendices.

I. THE MODEL

Consider a market where each active firm bargains with a workforce-specific union. All workers are unionized and immobile across unions. Unions are the only suppliers of labor, which is, in turn, the only production factor. We assume that each worker is endowed with one unit of labor that is inelastically supplied. We further assume that one unit of labor is required to produce one unit of a good.³ Our model is equivalent to one where each firm only hires one worker (see, e.g., [Hashimoto, 1981](#)) who supplies the amount of labor required by the firm. Firms may produce vertically differentiated goods depending on the skill level of their workforce. The skill of the workers is fully determined by the amount of training they receive, which is costly and set either by the union the workers belong to or by the firm that hires them. Training is not transferable so that poaching is not allowed. The workforce skill level translates into product quality; consequently, a more highly trained worker allows the firm to offer the market a higher quality product.⁴ Firms aim at profit maximization, while the unions' objective function is the sum of their members' wages.⁵

On the consumption side, we assume that there is a continuum of unit mass of consumers heterogeneous with respect to quality appreciation.⁶ Label this quality appreciation θ and let θ be uniformly distributed over $[0, \bar{\theta}]$ with density $1/\bar{\theta}$. Consumers purchase at most one unit of the good. The indirect utility they derive from consumption is standardly defined *à la* [Mussa and Rosen \(1978\)](#), so that their surplus is:

$$U(\theta, s_i, p_i) = \begin{cases} \theta s_i - p_i & \text{if buying one unit of good } i, \\ 0 & \text{if abstaining from consumption.} \end{cases} \quad (1)$$

In (1) s_i is the quality of good i , and p_i is its price.

MONOPOLY. When one firm only is active, its demand is obtained by identifying the consumer that is indifferent between purchasing the good and abstaining from consumption, that is to say $U(\theta, s_m, p_m) = 0 \Leftrightarrow \theta_0 = p_m/s_m$, where p_m is the price of the good under monopoly and s_m is the skill level of the workforce, which directly defines the quality level of the supplied good. All consumers to the left of θ_0 prefer not to consume, whereas those to the right purchase the good.⁷ Accordingly, the demand under monopoly writes

$$D_m = \frac{1}{\bar{\theta}}(\bar{\theta} - \theta_0). \quad (2)$$

Let ν be the workers' wage under monopoly, so that the profit of the firm and the objective of the union (see [Sørensen, 1992](#)), which we will refer to as the “union surplus”, *gross of training costs*, are

$$\pi_m = (p_m - \nu)D_m \quad \text{and} \quad E_m = D_m\nu. \quad (3)$$

DUOPOLY. Consider now two firms and two unions, and each union is assigned to a firm. Firm-union pairs are ex-ante symmetric; however, anticipating an equilibrium argument, we shall assume that one of them provides the workers with more training than the other. Accordingly, we will label the former “high-skill workers” and the latter “low-skill workers”. The reason for this is that if both train the workers

to the same skill level, the quality of the goods produced by the firms would be the same, making the good produced by the two firms homogeneous. This outcome, combined with price competition, would result in a profit-dissipating price war.⁸ Let s_h be the skill level of the high-skilled workers and s_l that of the low-skilled workers, with $s_h > s_l$. As under monopoly, we assume that the skill level of the workforce determines the quality of the product; accordingly, $s_i, i = h, l$, also represent the quality levels of the goods. This entails that one firm produces the high- and the other the low-quality product. We will label the first firm “firm h ” and the second “firm l ”.

Demands for the high- and low-quality goods are again easily derived through the marginal consumer approach. Label $\theta_{hl} = \theta_{hl} = \frac{p_h - p_l}{s_h - s_l}$ the solution to $U(\theta, s_h, p_h) = U(\theta, s_l, p_l)$ and $\theta_{l0} = p_l/s_l$ the solution to $U(\theta, s_l, p_l) = 0$, then the demands for the high- and low-quality goods, D_h and D_l respectively, are:

$$D_h = \frac{1}{\theta} (\bar{\theta} - \theta_{hl}), \quad D_l = \frac{1}{\theta} (\theta_{hl} - \theta_{l0}). \quad (4)$$

The profits of the high- and low-quality firms, *gross* of training costs, are therefore:

$$\pi_h = D_h(p_h - w), \quad \pi_l = D_l(p_l - r), \quad (5)$$

where w and r are the wages paid to the high-skilled and low-skilled workers, respectively, which are determined by the bargaining taking place within each firm-union pair.

Each union comprises the same number of workers, for the sake of clarity, we will refer to the union of the high-skilled workers as the “high-skill union” and to the other as the “low-skill union”. The surplus of the unions (gross of training costs) are:

$$E_h = D_h w, \quad E_l = D_l r. \quad (6)$$

WAGE BARGAINING AND TRAINING COSTS. Both under monopoly and duopoly, the wages of the workers are determined by bargaining that takes place within the firm-union pair. We use the Nash Bargaining solution to obtain the bargaining outcome (see, e.g., [Naylor, 1998, 1999, 2000](#) and the references therein). Training generates a cost that is increasing and convex in the skill level itself. What we have in mind is that, the higher is the skill level of the workforce, the more difficult it is to train it to higher skill levels. Accordingly, the marginal cost of training increases in the skill level. To be more concrete, to train the workers from their initial skill level (which we normalize to 0) to, say, s_i , costs $C(s_i) = s_i^2/2$. The training technology reflects the basic idea that training mostly involves fixed costs, so that once borne the set-up cost (hiring a trainer, creating a training division), the marginal cost of training is negligible.⁹ In our setting, explicit cost sharing is not possible.¹⁰ This amounts to assuming that either the firm or the union takes the decision of how much to invest in training and bears the full costs of it.

TIMING. The sequence of choices under monopoly is as follows. At the first stage, either the monopolist or the union decides on the amount of training provided to workers. At the second stage, bargaining over the workers’ wage takes place within the firm-union pair. At the third stage, the monopolist sets the price and employment is therefore determined. We model the timing of the duopoly case as close as possible

to the monopoly scenario. Under duopoly, we keep the same ordering of choices but we also assume that actions both between and within each firm-union pair are simultaneous at each stage. We solve the game through backward induction to obtain subgame-perfect Nash equilibria.

II. EQUILIBRIUM

We first study the benchmark case of a single-product monopoly bargaining over the workers' wage with one union.¹¹ Next, we delve into the analysis of duopoly.

Monopoly

Pricing stage The maximization of monopoly profits π_m with respect to the price p_m (second order conditions are always satisfied) yields $\hat{p}_m = \frac{\bar{\theta}s_m + \nu}{2}$, which may be plugged back into (2) and (3) to obtain $\hat{D}_m = \frac{(\bar{\theta}s_m - \nu)}{2\theta s_m}$, $\hat{\pi}_m = \frac{(\bar{\theta}s_m - \nu)^2}{4\theta s_m}$ and $\hat{E}_m = \frac{\nu(\bar{\theta}s_m - \nu)}{2\theta s_m}$.

Wage bargaining The bargaining stage is tackled through the Nash Bargaining solution. We assume that if no agreement is reached no worker is hired elsewhere in the economy and no production takes place, therefore the outside options of both the union and the monopolist are zero. The firm and the union maximize the product $\hat{E}_m \hat{\pi}_m = \frac{\nu(\bar{\theta}s_m - \nu)^3}{8\theta^2 s_m^2}$ with respect to ν , which returns $\nu^* = \frac{\bar{\theta}s_m}{4}$. By substituting this value back into profits and union's utility we get $\pi_m^* = \frac{9\nu^*}{16}$ and $E_m^* = \frac{3\nu^*}{8}$. The wage level is increasing in the skill level of the workers. Indeed a higher skill level allows to produce higher quality goods, which, *ceteris paribus*, are sold at a higher price. The firm's increased sales revenue is then shared with the workers at the wage bargaining stage.

Training Two cases have to be analyzed, namely firm-sponsored training and union-sponsored training. In the first case, the monopolist maximizes $\pi_m^f = \pi_m^* - C(s_m)$; in the second case, instead, the union maximizes $E_m^f = E_m^* - C(s_m)$.¹² With respect to each maximization problem, we can state the following:

Lemma 1. *Under monopoly:*

- (i) *the firm-sponsored training level is $s_m^f = 0.1406\bar{\theta}$. This yields $\pi_m^f = 0.0099\bar{\theta}^2$ and $E_m^f = 0.0132\bar{\theta}^2$ as equilibrium profits (net of training costs) and total wage respectively; $D_m^f = 0.375$ as equilibrium demand and employment, $\nu_m^f = 0.0351\bar{\theta}^2$ and $p_m^f = 0.0879\bar{\theta}^2$ as equilibrium wage and price;*
- (ii) *the union-sponsored training level is $s_m^u = 0.09375\bar{\theta}$. This yields $\pi_m^u = 0.0132\bar{\theta}^2$ and $E_m^u = 0.0044\bar{\theta}^2$ as equilibrium profits and total wage (net of training costs) respectively; $D_m^u = 0.375$ as equilibrium demand and employment, $\nu_m^u = 0.0234\bar{\theta}^2$ and $p_m^u = 0.0586\bar{\theta}^2$ as equilibrium wage and price.*

Under monopoly, the firm invests more in training than the union. The intuition for this is that the monopolist sets the market price, whereas the union does not. Furthermore, the firm sets the prices *after* the wage is bargained over, trading off a higher mark-up with a lower quantity demanded. The price, in turn, determines demand and employment, and thus the union's utility level. Yet, the union has no direct control over this variable (though the wage level indirectly influences it). This results into a profit level *gross of training costs* that is larger than the total wage *gross of training costs*. In particular, it is easy to ascertain that the gross-of-training-costs profit ($\pi_m^* = \frac{9\bar{\theta}s_m}{64}$) is 1.5 times larger than the

gross-of-training-costs union's utility ($E_m^* = \frac{3\bar{\theta}s_m}{32}$). Since the training technology is the same for the firm and the union, it follows that the firm sets a training level 1.5 times larger than the union. This, in turn, implies that the wage of the workers is higher when the training level is chosen by the firm. The ultimate consequence is that the price is also higher when the firm sets the training level because a higher training level, on the one hand, increases the wage, which is the marginal production cost of the good, and on the other hand, increases the quality of the good, and therefore the price that the firm may charge. It is instructive that both under firm-sponsored training and union-sponsored training the equilibrium demand is the same under monopoly pricing. To understand this, notice that the behavior of the demand is determined by the behavior of $\theta_0 = \frac{p_m}{s_m}$ (see eq. 2), where $p_m(\nu^*(s_m), s_m) = \frac{5}{8}\bar{\theta}s_m$, which implies that θ_0 does not depend on s_m . Stated differently, an increase of s_m generates the same percentage increase in $p_m(\nu^*(s_m), s_m)$ so that demand and employment are unaffected.

Duopoly

Pricing stage In the last stage, firms simultaneously set prices. By taking the first-order derivative of π_i with respect to p_i , $i = h, l$ and solving the system so defined yields the following optimal prices:

$$\hat{p}_h = \frac{s_h [2\bar{\theta}(s_h - s_l) + r + 2w]}{4s_h - s_l}, \quad \hat{p}_l = \frac{s_l [\bar{\theta}(s_h - s_l) + w] + 2rs_h}{4s_h - s_l}. \quad (7)$$

Second order conditions are fulfilled so long as $s_h > s_l$. By plugging (7) back into (4), (5) and (6) we obtain:

$$\hat{D}_h = \frac{2\bar{\theta}s_h(s_h - s_l) - s_h(2w - r) + ws_l}{\bar{\theta}(4s_h - s_l)(s_h - s_l)}, \quad \hat{D}_l = \frac{s_h [\bar{\theta}s_l(s_h - s_l) + (w + r)s_l - 2rs_h]}{\bar{\theta}s_l(s_h - s_l)(4s_h - s_l)}; \quad (8)$$

$$\hat{\pi}_h = \hat{D}_h^2 \bar{\theta}(s_h - s_l), \quad \hat{\pi}_l = \hat{D}_l^2 \bar{\theta}(s_h - s_l) \frac{s_l}{s_h}; \quad (9)$$

and

$$\hat{E}_h = \hat{D}_h w, \quad \hat{E}_l = \hat{D}_l r. \quad (10)$$

Wage bargaining We again assume that the outside options for the firm and the union at the bargaining stage are zero. Accordingly, the maximization problems determining the wages of the high- and low-skilled workers are, respectively:

$$\max_w \hat{E}_h \hat{\pi}_h, \quad \max_r \hat{E}_l \hat{\pi}_l. \quad (11)$$

Like under monopoly, the problems defined in (11) assume that the bargaining power of firms and unions is the same. This keeps the model tractable and yields analytical results for all the model variables. Numerical exercises, however, reveal that the use of the weighted Nash product, with weights that may differ across firm-union pairs, leaves our results qualitatively unchanged for a relevant parameter constellation.¹³

The solutions to the problems in (11) are the following.¹⁴

$$\hat{w} = \frac{s_h [2\bar{\theta}(s_h - s_l) + \hat{r}]}{4(2s_h - s_l)}, \quad \hat{r} = \frac{s_l [\bar{\theta}(s_h - s_l) + \hat{w}]}{4(2s_h - s_l)}. \quad (12)$$

The solution to the system defined by (12) is easy to obtain:

$$w^* = \frac{\bar{\theta}s_h(16s_h - 7s_l)(s_h - s_l)}{64s_h^2 - 65s_h s_l + 16s_l^2}, \quad r^* = \frac{2\bar{\theta}s_l(5s_h - 2s_l)(s_h - s_l)}{64s_h^2 - 65s_h s_l + 16s_l^2}. \quad (13)$$

Inspection of (13) allows us to state the following:

Lemma 2. $w^* > r^*$ for all $s_h > s_l$.

At equilibrium, the high-skilled workers receive a higher wage than the low-skilled workers. It is worth noting that the skill premium does not originate from a greater productivity among high-skilled workers, as the *physical* productivities of high- and low-skilled labor are the same. It also does not emerge from greater bargaining power on the part of the high-skilled union.¹⁵ Rather, the skill premium stems from the value in terms of quality that the skilled workers add to the good.

Substituting (13) back into (7)-(10) yields:

$$p_h^* = \frac{2(5s_h - 2s_l)w^*}{(4s_h - s_l)}, \quad p_l^* = \frac{2(5s_h - 2s_l)r^*}{(4s_h - s_l)}; \quad (14)$$

$$D_h^* = \frac{3w^*(2s_h - s_l)}{\zeta}, \quad D_l^* = \frac{3r^*(2s_h - s_l)s_h}{\zeta s_l}; \quad (15)$$

$$\pi_h^* = (p_h^* - w^*)D_h^* = \frac{9(w^*)^2(2s_h - s_l)^2}{\zeta(4s_h - s_l)}, \quad \pi_l^* = (p_l^* - r^*)D_l^* = \frac{9(r^*)^2 s_h(2s_h - s_l)^2}{\zeta s_l(4s_h - s_l)}; \quad (16)$$

and

$$E_h^* = D_h^* w^* = \frac{3(w^*)^2(2s_h - s_l)}{\zeta}, \quad E_l^* = D_l^* r^* = \frac{3(r^*)^2(2s_h - s_l)s_h}{\zeta s_l}, \quad (17)$$

where $\zeta \equiv \bar{\theta}(4s_h - s_l)(s_h - s_l)$.

Training We are now in a position to address the training stage under duopoly. As mentioned previously, training in our case takes the form of a fixed cost that is paid to increase the skill level of the workers from the initial level (normalized to zero) to the desired one. Four alternative scenarios will be analyzed depending on who makes the training investment decision. Training may be set by firms alone, by unions alone, or by one union and one firm, and in this case either may be the actor electing to train the workforce to the high-skill level. As stressed above, explicit cost sharing is not possible as in [MacLeod and Malcomson \(1993\)](#). Rather, it is assumed that the actor who takes the decision of how much to invest in training bears the full costs of it.¹⁶ We begin with the case where the firms set the training levels of their workforce; for the convenience of the reader, we will label this case “*f*”. The profit functions of the firms, net of training costs, are:

$$\pi_h^* - C(s_h), \quad \pi_l^* - C(s_l). \quad (18)$$

Firms simultaneously and non-cooperatively maximize profits as in (18). We can state the following:

Lemma 3. *The skill (and quality) levels chosen by the firms at the unique subgame-perfect Nash equilibrium are $s_h^f = 0.1453\bar{\theta}$ and $s_l^f = 0.0416\bar{\theta}$. The profits and total wages are $\pi_h^f = 0.0073\bar{\theta}^2$, $\pi_l^f = 0.0012\bar{\theta}^2$; $E_h^f = 0.0129\bar{\theta}^2$ and $E_l^f = 0.0015\bar{\theta}^2$. The wages paid to workers are $w^f = 0.031\bar{\theta}^2$ and $r^f = 0.0056\bar{\theta}^2$ and the equilibrium prices are $p_h^f = 0.0741\bar{\theta}^2$ and $p_l^f = 0.0134\bar{\theta}^2$. Finally, the demand and employment levels*

are, respectively, $D_h^f = 0.414$ and $D_l^f = 0.262$.

Proof. See Appendix A. □

Now consider the case where the unions decide the training level of the workers, and label it case “u”. The unions simultaneously and non-cooperatively set the workers’ training levels to maximize their objective functions:

$$E_h^* - C(s_h), \quad E_l^* - C(s_l). \quad (19)$$

We state the following:

Lemma 4. *The skill (and quality) levels chosen by the unions at the unique subgame-perfect Nash equilibrium are $s_h^u = 0.0983\bar{\theta}$ and $s_l^u = 0.032\bar{\theta}$. The profits and total wages are $\pi_h^u = 0.0117\bar{\theta}^2$, $\pi_l^u = 0.0015\bar{\theta}^2$; $E_h^u = 0.0038\bar{\theta}^2$ and $E_l^u = 0.0006\bar{\theta}^2$. The wages paid to workers are $w^u = 0.0204\bar{\theta}^2$ and $r^u = 0.0042\bar{\theta}^2$ and the equilibrium prices are $p_h^u = 0.0483\bar{\theta}^2$ and $p_l^u = 0.01\bar{\theta}^2$. Finally, the demand and employment levels are, respectively, $D_h^u = 0.421$ and $D_l^u = 0.267$.*

Proof. See Appendix B. □

Lemmas 3 and 4 describe the possible outcomes of the training decisions. Particularly striking is that the highest skill levels are obtained in the case of firm-sponsored training. In a nutshell, this may be explained by the fact that, in each firm-union pair, the firm controls the goods prices and sets them so as to maximize profits. By contrast, the union does not have at its disposal such a lever to directly extract the surplus of consumers. It follows that the marginal benefit from investing in skills is larger for the firms than for the unions. This, together with the fact that the training cost function is the same for all agents, drives the result. A marginal benefit/marginal cost analysis of the training investment will help clarifying this point by disentangling all the forces that lead to this outcome. As argued above, the training cost function is the same for the firm and the union, thus we concentrate on the marginal benefit side. To ascertain that the firms have larger training incentives than the unions, we need to find out under which conditions $\frac{\partial \pi_h^*}{\partial s_h} > \frac{\partial E_h^*}{\partial s_h}$ for the high-skill level, and $\frac{\partial \pi_l^*}{\partial s_l} > \frac{\partial E_l^*}{\partial s_l}$ for the low-skill one.

TRAINING INCENTIVES: HIGH-SKILL. Let us start from the high-quality, high-skill firm-union pair. From eqs. (16) and (17), condition $\frac{\partial \pi_h^*}{\partial s_h} > \frac{\partial E_h^*}{\partial s_h}$ can be rearranged as:

$$\frac{\partial(D_h^* p_h^*)}{\partial s_h} - \frac{\partial(D_h^* w^*)}{\partial s_h} > \frac{\partial(D_h^* w^*)}{\partial s_h} \Leftrightarrow D_h^* \frac{\partial p_h^*}{\partial s_h} + p_h^* \frac{\partial D_h^*}{\partial s_h} - D_h^* \frac{\partial w^*}{\partial s_h} - w^* \frac{\partial D_h^*}{\partial s_h} > D_h^* \frac{\partial w^*}{\partial s_h} + w^* \frac{\partial D_h^*}{\partial s_h}. \quad (20)$$

Denoting by $\epsilon_{x,s_h} = \frac{\partial x}{\partial s_h} \frac{s_h}{x}$ the elasticity of variable x with respect to the high-skill level s_h , this condition may be re-written as

$$p_h^* (\epsilon_{p_h^*, s_h} + \epsilon_{D_h^*, s_h}) - w^* (\epsilon_{w^*, s_h} + \epsilon_{D_h^*, s_h}) > w^* (\epsilon_{w^*, s_h} + \epsilon_{D_h^*, s_h}). \quad (21)$$

The left-hand side term in inequality (21) is the marginal benefit of training investment for the firm and the right-hand side one is the marginal benefit for the union, both expressed as functions of the skill-

elasticities of the high-quality good price, the skilled workers' wage rate and the demand/employment level of the high-quality firm.

Let us first focus on the left-hand side. Its first element is the increase in revenue due to an infinitesimal increase in the quality level, and combines the percent infinitesimal increase in the price with the percent infinitesimal reduction in the demand due to a unit percent increase in the quality level of the good, multiplied by the price level. The second term is the marginal cost for the firm of an increase in the skill level of the workforce, and combines the effect of a percent infinitesimal increase in the wage with the percent infinitesimal reduction in the employment of high-skill workers, multiplied by the wage level. This term appears to the right-hand side of the inequality as well, but in this case it represents the marginal benefit for the union, which mirrors the marginal cost for the firm.¹⁷

Condition (21) is always verified, which implies that the net marginal benefit from training for the high-quality firm is larger than that of the skilled-workers union. Intuitively, this is so because an increase in the skill level positively affects the firm's profit through the increase in price, and negatively through the increase in the wage and the reduction in demand. It also positively affects the union through the increase in the wage and negatively through the reduction in employment. Yet, the high-quality firm directly controls the price, which is highly reactive to increases in the skill level. By contrast, the wage level is only indirectly affected by the skill level, namely through the expansion of the sum of profits and union's surplus. When deciding how much to invest in skill creation, the high-quality firm knows that it may charge higher prices even though it has to pay higher wages and serve a lower demand. Similarly, the union knows that it will obtain a higher wage, at the cost of a lower employment. Given the technological assumptions of our model, the demand reduction coincides with the reduction in employment. Yet, because of the high skill-elasticity of price, the firm may increase the price by a larger amount than the increase in the wage. This ultimately entails greater incentives to invest in high-skill training for the firm than for the union.

TRAINING INCENTIVES: LOW-SKILL. Similarly to the preceding case, by using eqs. (16) and (17) and defining $\epsilon_{x,s_l} = \frac{\partial x}{\partial s_l} \frac{s_l}{x}$ as the elasticity of variable x to the low-skill level s_l , condition $\frac{\partial \pi_l^*}{\partial s_l} > \frac{\partial E_l^*}{\partial s_l}$ may be rearranged as follows:

$$p_l^*(\epsilon_{p_l^*,s_l} + \epsilon_{D_l^*,s_l}) - r^*(\epsilon_{r^*,s_l} + \epsilon_{D_l^*,s_l}) > r^*(\epsilon_{r^*,s_l} + \epsilon_{D_l^*,s_l}). \quad (22)$$

The interpretation of (22) parallels that of (21). Yet, differently from the high-skill training case, this condition is met for $s_l < 0.3819s_h$ (which is true at the optimal skill levels in all the training configurations). The reason for this is that ϵ_{r^*,s_l} and $\epsilon_{p_l^*,s_l}$ do not have the same sign for all admissible low-skill levels, and $\epsilon_{D_l^*,s_l}$ is positive rather than negative.¹⁸ Like for the high-quality product, an increase in the low-skill level s_l triggers an increase in the quality level of the low-quality good which, all else equal, increases the willingness to pay for it and consequently the price the firm can charge and the sum of the low-quality firm profits and union surplus, we call this "quality appreciation effect". However, unlike that case, an increase of s_l also makes the goods more homogeneous. This harshens price competition, which leads to a reduction in prices and the sum of profits and union surplus, we label this "competitive effect".

The closer are the skill levels, the stronger is the competitive effect over the quality appreciation one.

At the optimal training levels of Lemmas 3 and 4, condition (22) is fulfilled. Furthermore, the elasticities of r^* and p_l^* to s_l are positive as well at these values, therefore their interpretation parallels the one for the high-quality good price and skilled workers wage. By contrast, the elasticity of the low-quality demand/low-skill workers employment level is negative, which is explained by the prevalence of the competitive effect over the quality appreciation one.

Notice that these different training incentives have an indirect effect on the employment levels. Indeed, lower training levels with respect to those chosen by the firms translate in lower wages, which reduces the variable production costs of the firms and, in turn, the prices they set. This leads to an increase in the sales of the goods and, ultimately, in the level of employment of both skilled and unskilled workers. This effect is also reinforced by the fact that the degree of product differentiation determined by the skill levels is lower under union-sponsored training ($s_h^u/s_l^u = 3.0706 < 3.496 = s_h^f/s_l^f$). This implies that the goods are more homogeneous under union-sponsored training, resulting in a harsher price war that yields lower price levels ($p_h^u < p_h^f$ and $p_l^u < p_l^f$) in this case. Clearly, this makes the quantities demanded, and consequently the employment level, increase further.

As a final remark, let us expand on the individual wages w and r paid to workers and the wage mark-up. In general, the presence of a training-setting firm increases the wage paid to the workers. This phenomenon can be easily ascertained by an inspection of the wages reported in Lemmas 3 and 4. The intuition for this is to be found in the higher training level selected by firms which translates to higher product quality and ultimately in value-added to the product. As concerns the skill premium, from Lemma 3 and Lemma 4 it easily follows that the wage differential between workers with different skills is lower when the training levels of the workforce are selected by the union than by the firm. This result is in accordance with several studies on the wage mark-up for different skill groups that indicate that unions contribute to reduce wage dispersion within the unionized sectors of the economy (see, e.g., Freeman 1980).¹⁹

To conclude the analysis, we will consider the last two “mixed” cases where in one pair the firm sets the training level and the union does in the other. In this case, let the apex identify the agent setting the high and low training level, respectively.²⁰

Lemma 5. (i) *There exists a unique subgame-perfect equilibrium where in one pair the union selects the high level of training and in the other pair the firm selects the low level of training. At this equilibrium, the training levels are $s_h^{uf} = 0.0997\bar{\theta}$ and $s_l^{uf} = 0.0359\bar{\theta}$. The profits and total wages are $\pi_h^{uf} = 0.0116\bar{\theta}^2$, $\pi_l^{uf} = 0.001\bar{\theta}^2$; $E_h^{uf} = 0.0036\bar{\theta}^2$ and $E_l^{uf} = 0.0012\bar{\theta}^2$. The wages paid to workers are $w^{uf} = 0.0201\bar{\theta}^2$ and $r^{uf} = 0.0046\bar{\theta}^2$ and the equilibrium prices are $p_h^{uf} = 0.0474\bar{\theta}^2$ and $p_l^{uf} = 0.0108\bar{\theta}^2$. Finally, the demand and employment levels are, respectively, $D_h^{uf} = 0.4269$ and $D_l^{uf} = 0.2711$.*

(ii) *There exists a unique subgame-perfect equilibrium where in one pair the firm selects the high level of training and in the other pair the union selects the low level of training. At this equilibrium, the training levels are $s_h^{fu} = 0.1437\bar{\theta}$ and $s_l^{fu} = 0.0343\bar{\theta}$. The profits and total wages are $\pi_h^{fu} = 0.0078\bar{\theta}^2$, $\pi_l^{fu} = 0.0017\bar{\theta}^2$; $E_h^{fu} = 0.013\bar{\theta}^2$ and $E_l^{fu} = 0.0006\bar{\theta}^2$. The wages paid to workers are $w^{fu} = 0.0317\bar{\theta}^2$*

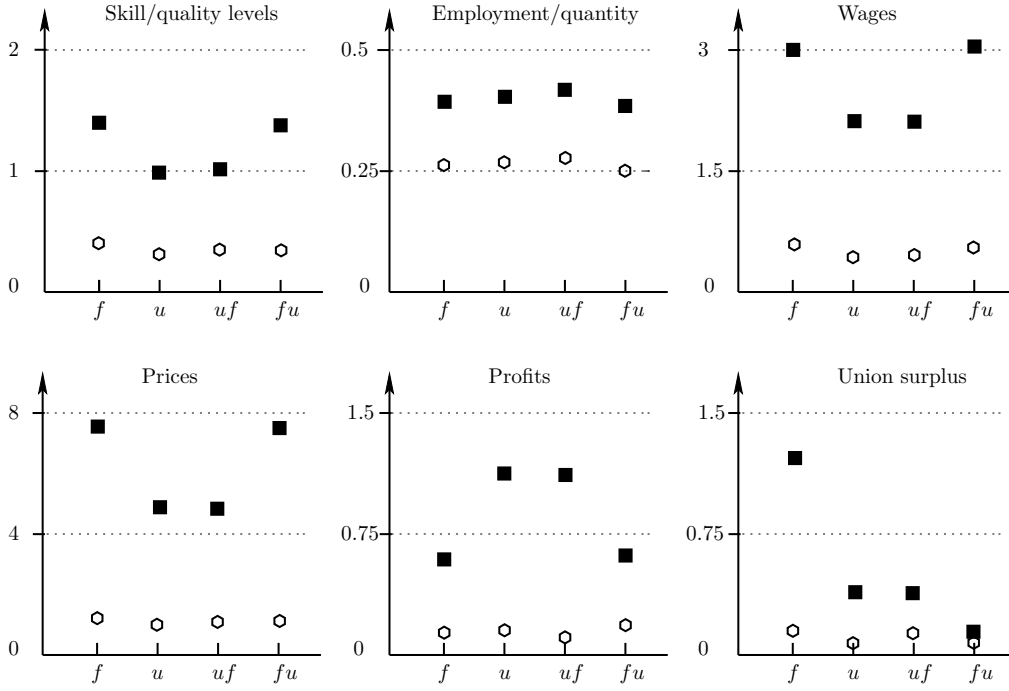


FIGURE 1. Equilibrium values in the possible training configurations for $\bar{\theta} = 10$; ■ represents high skill/quality values, ○ low skill/quality values.

and $r^{fu} = 0.0048\bar{\theta}^2$ and the equilibrium prices are $p_h^{fu} = 0.0763\bar{\theta}^2$ and $p_l^{fu} = 0.0115\bar{\theta}^2$. Finally, the demand and employment levels are, respectively, $D_h^{fu} = 0.4075$ and $D_l^{fu} = 0.2573$.

Proof. See Appendix D. □

The previous discussion is helpful in understanding the results reported in Lemma 5, consider part (i) of the Lemma first. In the firm-union pair producing the high-quality good, the union sets the training level, while in the other pair, the low-level of training is selected by the firm. Recall that the union has less incentives to make training investments than the firm because it cannot set the final price, which explains the reduced level of training provided to high-skilled workers with respect to the case where the two firms make the training decision (case f). In contrast, the low-quality firm can appropriate consumer surplus through both training and price setting, which fosters its incentives to provide training. This yields a training (and quality) level that is larger than that obtained in the case where training levels were set by the two unions (case u). A similar rationale applies, specularly, to part (ii) of the Lemma. In this case, a high level of training is selected by the firm, whereas the low-level is selected by the union. The union has “low” training incentives, therefore it sets a training level below that in the two-firm case f . In contrast, the firm can also set the market price; hence it invests more than in the two-union case u . However, the training and quality levels selected by the firm are lower than in the two-firm case, which may be explained by referring to strategic interaction. In this case, the firm knows that the union will limit its investment in (low-level) training, which will result in a “low” product quality level. Consequently, the training investment required for the firm to achieve an optimal product differentiation level is lower, allowing the firm to save on training costs. In sum, when the union selects the high level of training and the firm the low one, the optimal training levels are “close”, resulting in “low” product differentiation

($s_h^{uf}/s_l^{uf} = 2.7787$). In the opposite case the training levels are distant, yielding to a high degree of product differentiation ($s_h^{fu}/s_l^{fu} = 4.1846$). These observations are helpful in explaining the employment levels in the two scenarios because the training levels, as argued above, determine the quantities demanded through their influence on the final prices. To this regard, it is instructive that the highest employment levels are obtained in the case where in one pair the union sets the high training level and, in the other, the firm sets the low training level. As pointed out above, this case is characterized by a low degree of product differentiation, which increases the intensity of price competition, resulting in “low” prices. Ultimately, this generates high demands for both products and, consequently high employment levels. The symmetric reasoning applies to the case where the firm trains the workers to the higher skill level and the union to the lower one. In this case the degree of product differentiation is “high” and the price competition “mild”, resulting in high prices and low quantities and employment levels.

By comparing the pure configurations u and f with the mixed ones uf and fu we state the following:

Proposition 1. (i) *The highest workers’ skill and product quality levels are obtained in case f , whereas the lowest are obtained in case u . Cases uf and fu entail intermediate training and quality levels.*
(ii) *The highest employment levels are obtained in case uf , whereas the lowest are obtained in case fu . Cases f and u entail intermediate employment levels.*

Proof. Follows from direct comparison. □

Figure 1 illustrates the equilibrium values of variables in all the possible training configurations.

Our discussion highlights that the training decisions determine the remuneration of workers and contribute to shaping competition in the product market. Consequently, they also have effects on consumer surplus and total welfare. We address the issue of the welfare effects of training in the following section.

III. WELFARE AND PUBLIC TRAINING

In this section, we assess the welfare properties of our model. We first compute the welfare levels at equilibrium. Next, we explore the case where the training levels of the workers are determined by a benevolent social planner. In line with the preceding analysis, we will adopt a partial-equilibrium perspective here. Thus, as is standard in this approach, we define the welfare of the industry as the sum of consumer surplus, firm profits and total wages minus training costs. Under monopoly, social welfare is given by

$$W(s_m, \bar{\theta}) = \frac{1}{\bar{\theta}} \left[\int_{\theta_0}^{\bar{\theta}} (\theta s_m - p_m) d\theta \right] + \pi_m + E_m - C(s_m) = \frac{1}{128} s_m (39\bar{\theta} - 64s_m). \quad (23)$$

Substituting the optimal training levels of Lemma 1 into (23) gives the following:

Lemma 6. *The welfare level amounts to $W^{mf} = 0.03295\bar{\theta}^2$ under firm-sponsored training, and $W^{mu} = 0.02416\bar{\theta}^2$ under union-sponsored training.*

Not surprisingly, the firm-sponsored training configuration results in a higher level of social welfare than the union-sponsored training one. Under the former configuration, in fact, the higher level of product

quality generates a larger gross surplus among consumers because the higher training level translates into a higher product quality that fosters the consumers' willingness to pay, and thus their surplus.

Let us turn to the duopoly case. Social welfare is given by

$$\begin{aligned}
W(s_h, s_l, \bar{\theta}) &= \frac{1}{\bar{\theta}} \left[\int_{\theta_{l0}}^{\theta_{hl}} (\theta s_l - p_l) d\theta + \int_{\theta_{hl}}^{\bar{\theta}} (\theta s_h - p_h) d\theta \right] + \pi_h + \pi_l + E_h + E_l - C(s_h) - C(s_l) = \\
&= \frac{1}{\bar{\theta}} \left(\int_{\theta_{l0}}^{\theta_{hl}} \theta s_l d\theta + \int_{\theta_{hl}}^{\bar{\theta}} \theta s_h d\theta \right) - C(s_h) - C(s_l) = \\
&= \frac{3\bar{\theta}s_h(2s_h - s_l) (6656s_h^5 - 9240s_h^4s_l + 2518s_h^3s_l^2 + 1525s_h^2s_l^3 - 912s_hs_l^4 + 128s_l^5)}{2(4s_h - s_l)^2 (64s_h^2 - 65s_hs_l + 16s_l^2)^2} - \frac{1}{2} (s_h^2 + s_l^2). \quad (24)
\end{aligned}$$

By substituting the optimal training levels of Lemmas 3-5 into (24), it is immediate to obtain the following:

Lemma 7. *The welfare levels in the possible training configurations are as follows: $W^f = 0.0413\bar{\theta}^2$ under firm-sponsored training, $W^u = 0.0311\bar{\theta}^2$ under union-sponsored training, $W^{fu} = 0.0398\bar{\theta}^2$ and $W^{uf} = 0.0321\bar{\theta}^2$ in the mixed-cases.*

The highest welfare is obtained when firms set the training levels. Like under monopoly, the qualities of the products are the highest in this case. Symmetrically, the lowest welfare obtains under union-sponsored training. The lower training incentives of the unions result in a low quality level of the goods, which leads to a limited creation of gross surplus. In the “mixed” cases the higher training incentives of the firms are combined with the lower ones of the unions, resulting in “intermediate” welfare levels.

We now turn to the analysis of the socially optimal training levels. The relevance of the public policies aimed to foster skill acquisition and a clear understanding of the mechanisms that drive it, is stressed, for example, in OECD (2011). We assume that a benevolent social planner may set the skill levels of the workforce by taking pricing as given. Furthermore, the social planner cannot (directly) modify the number of competing firms or unions, nor it can change the balance of bargaining power between them. Moreover, we assume that the training cost of the social planner is equal to that of the firms and unions and therefore rule out the possibility that public training is more efficient than its private counterpart. Under monopoly, the planner's optimization problem is $\max_{s_m} W(s_m, \bar{\theta})$ and the following proposition provides its solution.

Proposition 2. *The socially optimal training (and quality) level is $s_m^W = 0.30468\bar{\theta}$. The corresponding price and wage are respectively $p_m^W = 0.19043\bar{\theta}^2$, $u^W = 0.07617\bar{\theta}^2$. Finally, the equilibrium employment level is $D_m^W = 0.375$.*

Proof. The values for the price, wage and quantity/employment follow from direct substitution. \square

The socially optimal training level is higher than both the firm-sponsored and union-sponsored training levels. The reason why the social planner increases the general level of training is related to the positive link between product quality and workers' skills. By increasing the training levels of the workforce, the quality level of the product is increased, and thus, so is the surplus that is generated by consumption. Because both the firm and the union do not take into account the increase in net consumer surplus due to higher quality goods, they under-invest in training.

Under duopoly, the planner's optimization problem writes as $\max_{s_h, s_l} W(s_h, s_l, \bar{\theta})$. The following proposition provides a summary of the results.

Proposition 3. *The socially optimal training (and quality) levels are $s_h^W = 0.3056\bar{\theta}$ and $s_l^W = 0.2077\bar{\theta}$. The corresponding prices and wages are respectively $p_h^W = 0.0886\bar{\theta}^2$, $p_l^W = 0.039\bar{\theta}^2$, $w^W = 0.0404\bar{\theta}^2$ and $r^W = 0.0178\bar{\theta}^2$. Finally, the equilibrium employment levels are $D_h^W = 0.4929$ and $D_l^W = 0.3192$.*

Proof. See Appendix E. □

The socially optimal training levels have two relevant features relative to those set by firms and/or unions. First, their *absolute values* are the highest among all the possible training levels and, second, their *relative difference* is the lowest, resulting in low product differentiation ($s_h^W/s_l^W = 1.471$). On the one hand, as in monopoly, the social planner increases the general level of training. On the other hand, it reduces the skill differences, because more similar skill levels lead to more homogeneous products, and ultimately to a harsher price war between firms, which results in lower prices. The effect is twofold. First, this increases the surplus consumers enjoy; second, it increases the total demand for the two products. It is simple to determine that the equilibrium prices corrected for quality (p_i/s_i) under the socially optimal training levels are the lowest among all possible scenarios, while the equilibrium demands and employment levels are the highest.

Our welfare results contrast with Moen (1998) and Moen and Rosén (2004), who show that even with frictions in the labor markets, workers and firms may have socially efficient training incentives. In our model both firm-sponsored and union-sponsored training are suboptimal, for two reasons. On the one hand, training investments are not completely appropriable by agents because of the Nash bargaining over the wage rate. On the other hand, neither the firms nor the unions take into account the positive effect of training on the surplus of consumers when setting the training levels, which results in inefficiently low skills and product qualities. Furthermore, our analysis argues that whenever the unions take part in the training process, the equilibrium welfare is *lower* than in the case where all the training is firm-sponsored. Thus, our results cast some doubts on the potentially welfare increasing role of the unions (see, for example, Donado and Wälde, 2012).²¹

IV. DISCUSSION AND EXTENSIONS

In this section, we put our paper into perspective by comparing the monopoly and duopoly outcomes and discussing some extensions to the model. We first analyze the consequences of asymmetric bargaining powers between the firms and the unions. Then, we discuss the robustness of our results to alternative training technologies and investigate the role of the normalization of the consumer mass. Finally, we develop the case of efficient bargaining and extend the analysis to contractible investment.

Monopoly versus duopoly Under monopoly the firm sets the final price given the elasticity of the demand and the bargained wage, but without being constrained by the competitive pressure of the rivals, since it is by assumption alone on the market. This has two distinct effects on the training incentives. The first one concerns the *absolute training incentives*. By comparing the optimal skill levels of Lemmas 1 point (i) and 3, and of Lemmas 1 point (ii) and 4, it is clear that, under monopoly, both the firm

and the union set skill levels that are lower than the maximum attained under duopoly ($s_m^f < s_h^f$ and $s_m^u < s_h^u$). This is due to the fact that, under monopoly, the desire to relax price competition through product differentiation is lower (absent altogether, indeed) than under oligopoly. Consequently, both the firm and the union save on training costs and, not surprisingly, end up earning more than under duopoly, where profits and union surplus are eroded both by price competition and higher training costs. The second effect is on the *relative training incentives between the firms and the unions*. Under monopoly, the absence of competition makes the price an extremely effective tool to extract consumer surplus. Under duopoly, by contrast, each firm is also constrained by the competitive pressure of the rival, which exerts a downward force on prices.²² This reduces the effectiveness of the price as an instrument to extract consumer surplus and, therefore, lowers the incentives of the firms to invest in training. In fact, under monopoly the firm sets a training level that is 1.5 times that of the union (see section II.1.3, whereas, under duopoly $s_h^f/s_h^u = 1.4786$ and $s_l^f/s_l^u = 1.2987$. Namely, for each product, the training investment incentive of the firm *relative* to that of the union is lower under duopoly than under monopoly. As argued above, the only difference between duopoly and monopoly here is the degree of market competition and, hence, the ability to extract consumer surplus through the price. This intuition is further confirmed by the extension of the model to efficient bargaining. In that case, in fact, wage and employment levels are set to maximize the joint surplus of the firm and the union (namely the gross profits), which are then shared according to the bargaining weights. With symmetric bargaining, the post-bargaining profit and total wage in each firm-union pair are the same, which results in the same incentives for training between the firm and the union (see Section IV..5 for details).

Generalized Nash bargaining Our analysis is carried out under the assumption of symmetric Nash bargaining. A natural question that may arise concerns the effect of different bargaining weights in the bargaining process. Unfortunately, a model featuring the generalized Nash bargaining solution is not tractable. However, some insights on the characteristics of such a model can be drawn by analyzing the effects of letting the relative weights in the bargaining process change. In particular, tables 1-4 in Appendix H report the equilibrium values for the model's variables under the assumptions of all bargaining power going to the workers (monopoly union), 75% of the bargaining power going to the workers and 25% to the firms, the symmetric case of 75% to the workers and 25% to the firms and, finally, the case of all bargaining power attributed to the firms. Three remarks are worth making here.

First, as could be expected, an increase in the bargaining power of an agent increases its payoff at the expense of the other party involved in the bargaining. As a consequence, an increase (decrease) in the bargaining power of the agent setting the training level results in an increase (decrease) in the amount of training provided, because this increases (decreases) the appropriability of the returns on the investments in skills.

Second, in the case of asymmetric bargaining powers, some training configurations may not be a subgame-perfect Nash equilibrium of the game. In particular, when the workers have “most of” the bargaining power, no mixed-configuration with one firm training to the high-skill level and one union training to the low-skill level exists. In this case, the union has incentives to deviate and “leapfrog upwards” to provide a high skill level. This outcome originates from the fact that, in general, higher

profits and higher total wages are earned by the pair producing the high-quality good. When unions are stronger than firms at the bargaining stage, the “high-skill” training level set by a firm is low because it has low bargaining power (see the above discussion). Consequently, for the union, the increase in training required to overtake this training level –and, thus, to reap a higher surplus– is “low”. A similar reasoning applies for the case where the firms have greater bargaining power than the unions. The most relevant difference between the two cases is that when *all* of the bargaining power is held by the firms, *no* equilibrium exists with at least one union setting the quality level (see table 4). The intuition for this result is straightforward. When all of the bargaining power is held by the firms, the wage paid to the workers is equal to the outside option, which does not depend on the quality level of the good. This makes any investment in training completely non-appropriable by the unions and ultimately entails that no union is willing to provide training. Finally, it is worth noticing that in the polar case where all of the bargaining power is held by the unions, the firms still have positive incentives to invest in training. Again, this outcome occurs because the firms set prices, and, therefore, may impose a wedge between the bargained wage (their average production cost) and the price.

Third, from a social welfare perspective, any positive bargaining power to the workers reduces the aggregate welfare because it increases marginal production costs of the firms, thereby increasing prices and depressing demands.

Training technology The training cost function that we have adopted in the paper is quadratic. This is a common assumption in the literature on product innovation in markets for vertically differentiated goods (see, e.g. [Motta, 1993](#); [Aoki and Prusa, 1997](#); [Hoppe and Lehmann-Grube, 2001](#); [Herguera et al., 2002](#)). One straightforward extension of our analysis consists of letting the inefficiency of the training technology vary following a parameter $\gamma > 0$, so that the training cost is $C(s_i) = \gamma \frac{s_i^2}{2}$, and the marginal training cost is γs_i . A larger γ implies, all else equal, higher total and marginal training costs, and the contrary. This extension slightly modifies the equilibrium values of the games. The intuition is that, if the marginal training cost increases, all else equal, the optimal training levels are proportionally reduced. In fact the first-order condition for the agent setting the training level writes, in general terms, $B'(s_i, s_j) - \gamma s_i = 0, i, j = h, l; i \neq j$, where $B'(\cdot)$ is the marginal benefit of the training ($\frac{\partial \pi_i^*}{\partial s_i}$ for the firm and $\frac{\partial E_i^*}{\partial s_i}$ for the union). The first-order condition implies that $s_i = \frac{B'(\cdot)}{\gamma}$, the optimal training level is scaled by $\frac{1}{\gamma}$ relative to the case where $\gamma = 1$. It is immediate to observe that, consequently, the equilibrium wages, prices, profits and union surplus are scaled by the same factor. By contrast, the demands are not affected by the training technology efficiency, because this parameter appears to the same power, both at the numerator of the demand (in the prices) and at the denominator (in the quality levels), thus canceling out.

One legitimate question concerns to what extent our analysis is robust to more general cost functions. It is well known that with general, although convex, cost functions a pure-strategy Nash equilibrium in games of quality choice may not exist, even in the simplest case of zero marginal production costs ([Lehmann-Grube, 1997](#)). Furthermore, in the case of existence of an equilibrium, the complexity of the functions of the first stage of the game seriously limits the readability of the results, as far as the analysis of the optimal training levels is concerned. Nonetheless, some insights on the *relative* training incentives

between firms and unions may be drawn by using conditions (21) and (22). These conditions compare the net marginal training incentives of the union and of the firm. Since the marginal cost is the same for both, it does not enter these conditions. As a consequence, (21) and (22) describe the relative training incentives of firms and unions for all cost functions, as long as they are the same for the firm and the union *within* each firm and union pair. As a consequence, for any pair of such cost functions for which a pure-strategy Nash equilibrium exists, the firm always has larger marginal training incentives than the union in the high-skill training case. Instead, the union may have larger training incentives than the firm depending on the optimal relative training levels in the low-skill case ($s_l \lesseqgtr 2.168s_h$).

Mass of consumers The analysis has been developed under the assumption of a unit mass of consumers. Instead, if we abandon this normalization by letting $M > 0$ be the mass of consumers, the direct effect is a scaling in the demands of size M , so that if the demand in the unit-mass case is $D_i(p_h, p_l)$, $i = h, l$, it becomes $MD_i(\cdot)$ in the M -mass case. Because the shift in demand does not affect the preferences of consumers, it has no direct effect on the optimal prices set by firms, nor on the bargained wages. This implies that the M -shift is transmitted unchanged to the profits and union surplus gross of training costs, so that, in this case, they write $M\pi_i^*$ and ME_i^* , for $i = h, l$. A further consequence is that, at the training stage, the mass of consumers exerts a positive effect on the optimal training levels. To see this, it is enough to notice that the first-order condition for setting the training levels is $MB'_i(\cdot) = s_i$ (B'_i is, as before, the marginal benefit of agent i) which implies that, relative to the unit mass case, the optimal training levels are scaled by M as well. The intuition is clear. Since the training cost is fixed with respect to the quantity produced, the “thickness” of the market determines profits and union surplus, thereby affecting the marginal benefit of training. Interestingly, in turn this has an indirect effect on prices and wages that are, at equilibrium, scaled by M as well, because of the variation in the quality of the products due to the variation in the training levels. The ultimate consequence is that the equilibrium profits and union surplus are scaled by a factor M^2 , because both the demand and the mark-up are positively affected by M .

Efficient Bargaining Under efficient bargaining the firm and the union in each pair simultaneously bargain over employment and wage rate (see, e.g., McDonald and Solow, 1981; Dhillon and Petrakis, 2002). It is worth noting that, in our framework, setting the employment level amounts to determining the level of output of the firm. Indeed, each worker inelastically supplies one unit of labor that, in turn, produces one unit of output.

The first step to tackle the case of efficient bargaining is to invert the demand system (4) to obtain the inverse demands (see, e.g. Motta, 1993):

$$p_h = \bar{\theta}[(1 - D_h)s_h - s_l D_l], \quad p_l = \bar{\theta}(1 - D_h - D_l)s_l; \quad (25)$$

in this case D_i , $i = h, l$ is the quantity supplied by firm i . The profits of the firms and the objective functions of the unions, gross of training costs, are, thus

$$\pi_h = (p_h - w)D_h, \quad \pi_l = (p_l - r)D_l, \quad (26)$$

and

$$E_h = D_h w, \quad E_l = D_l r. \quad (27)$$

Wage and employment/production setting As argued above, under efficient bargaining D_h and D_l are not unilaterally set by the firms with the objective to maximize profits, but result from the bargaining between the firm and the union within each pair. As a consequence, the game unravels along two stages only, namely investment in training in the first stage and wage and employment/output setting in the second stage. Proceeding by backward induction, we tackle the last stage first. The wage and employment levels are obtained by simultaneously solving the following problems.²³

$$\max_{w, D_h} E_h \pi_h, \quad \max_{r, D_l} E_l \pi_l. \quad (28)$$

The solution of the system defined by the first-order conditions for the first maximization problem (second-order conditions are fulfilled) yields

$$\hat{w} = \frac{\bar{\theta}(s_h - D_l s_l)}{4}, \quad \hat{D}_h = \frac{s_h - D_l s_l}{2s_h}. \quad (29)$$

By the same token, for the second maximization problem (SOCs are satisfied as well)

$$\hat{r} = \frac{\bar{\theta}(1 - D_h)s_l}{4}, \quad \hat{D}_l = \frac{1 - D_h}{2}. \quad (30)$$

Finally, the solution to the system defined by (29) and (30) is

$$w^* = \frac{\bar{\theta}s_h(2s_h - s_l)}{2(4s_h - s_l)}, \quad r^* = \frac{\bar{\theta}s_h s_l}{2(4s_h - s_l)}, \quad (31)$$

and

$$D_h^* = \frac{2s_h - s_l}{4s_h - s_l}, \quad D_l^* = \frac{s_h}{4s_h - s_l}. \quad (32)$$

It is worth noting that the bargained employment (and output) levels in (32) coincide with the profit-maximizing quantities by Motta (1993, p. 119). This should not be surprising for the following reason. It is easy to show that the first-order conditions (29) and (30) imply the maximization of the joint surplus of each firm-union pair with respect to the quantity supplied to the market, which coincides with the employment level. Now, the joint surplus of each firm-union pair is the gross profit of the firm, namely $q_i p_i, i = h, l$. This would be the profit of a firm with zero production costs, which indeed is the case discussed in Motta (1993). In our setup, the joint profit, once maximized with respect to employment/output, is apportioned between the firm and the union through the wage level. Since we assume equal the bargaining weight between the two, the price for each single unit of product is shared equally between the union and the firm. In fact, it is a simple matter of algebra to ascertain that the equilibrium prices are twice the wage rate:

$$p_h^* = \frac{\bar{\theta}s_h(2s_h - s_l)}{4s_h - s_l} = 2w^*, \quad p_l^* = \frac{\bar{\theta}s_h s_l}{4s_h - s_l} = 2r^*. \quad (33)$$

A direct consequence of this is that the profit of the firm and the surplus of the union, gross of training costs, *coincide* within each firm-union pair. In fact:

$$\pi_h^* = E_h^* = \frac{\bar{\theta}^2 s_h (2s_h - s_l)^2}{2(4s_h - s_l)^2}, \quad \pi_l^* = E_l^* = \frac{\bar{\theta}^2 s_l^2 s_l}{2(4s_h - s_l)^2}. \quad (34)$$

Training The above results have important consequences on the solution of the first stage of the game. Indeed, because the profits and union surplus are the same within each firm-union pair, to determine the optimal training level, it is immaterial to establish which agent actually takes the training decision and bears its cost. Accordingly, though in principle the four cases of Section II. are possible (namely pure firm-sponsored training, pure union-sponsored training and the two mixed cases), they all lead to the same optimal training (and wage and employment) levels (but clearly not to the same post-investment profit/union surplus levels). We summarize the main outcomes in the following

Lemma 8. *Under efficient bargaining a unique subgame-perfect equilibrium exists in all four training configurations. At all these equilibria, the equilibrium training levels are $s_h^e = 0.126\bar{\theta}$ and $s_l^e = 0.0451\bar{\theta}$, the equilibrium wages are $w^e = 0.0284\bar{\theta}^2$ and $r^e = 0.0062\bar{\theta}^2$ and the equilibrium employment (and output) levels are $D_h^e = 0.4508$ and $D_l^e = 0.2746$. Finally, the equilibrium prices are $p_h^e = 0.0568\bar{\theta}^2$ and $p_l^e = 0.0124\bar{\theta}^2$.*

Proof. See Appendix F. □

As mentioned earlier, with efficient bargaining the training incentives of the firms and unions coincide within each firm-union pair. As for the training levels themselves, they are "intermediate" with respect to the right-to-manage scenario. This is due to the fact that, as pointed out above, in our paper efficient bargaining implies output setting, which relaxes the degree of market competition. This reduces the incentive of firms and unions to differentiate the workers' skills and product qualities, allowing for savings in training costs.²⁴

Contractible investment So far, the analysis has been developed under the assumption that only one of the parties at a time engages in training. In this section, we allow for the possibility that the investment is contractible and thus the firm and the union jointly choose the training level of the workforce and share its cost. This outcome is possible only when the investment in training is fully verifiable and perfectly observable. To keep the analysis simple, we assume that at the training stage the firm and the union within each pair sign an agreement to maximize the sum of their payoffs. More precisely, the high-quality firm and the high-skilled union choose s_h in order to maximize $\pi_h^* + E_h^* - C(s_h)$, whereas the low-quality firm and the low-skilled union choose s_l in order to maximize $\pi_l^* + E_l^* - C(s_l)$. The solution to this maximization problem gives $s_h^c = 0.2433\bar{\theta}$ and $s_l^c = 0.0732\bar{\theta}$.²⁵ By direct substitution, we easily get that the sum of the payoffs of the high-quality firm and high-skilled union is equal to $0.0214\bar{\theta}^2$, whereas the sum of the payoffs of the low-quality firm and low-skilled union amounts to $0.0034\bar{\theta}^2$. As can be seen, the training levels set in this case are higher than in the cases of pure firm- and union-sponsored training. This should not come as a surprise, because the (marginal) training cost is the same, but the (marginal) benefit is now larger, as it is the sum of the firms profit and union surplus. Indeed, within each firm-union

pair, the sums of the payoffs are larger than the corresponding ones under firm- and union-sponsored training. This means that if the investment in training were contractible, and set to maximize the joint payoffs within each union-firm pair, both the firms and the unions would find it profitable to reach such an agreement.

V. CONCLUSION

In this paper we have analyzed the private and public incentives towards skill acquisition when the skill level of workers determines the quality level of goods, and both labor and product markets are imperfectly competitive. We have shown that both “pure” training scenarios (training levels are set by either firms or unions) and “mixed” ones (training simultaneously set by firms and unions) may emerge. Furthermore, in all scenarios, firms have greater incentives to engage in training than unions and, therefore, the skill levels they set are generally higher, resulting in a higher product quality. In line with empirical evidence, we have also found that the skill premium is lower when the training levels of the workforce are selected by unions than by firms. Finally, we have analyzed the optimal public training skill levels and showed that both unions and firms under-invest in training in comparison with the social optimum. Yet, in this case, the skill premium is the lowest.

APPENDIX A PROOF OF LEMMA 3

We follow the approach developed by [Motta \(1993\)](#). The first-order conditions with respect to the quality levels are as follows.²⁶

$$\frac{\partial[\pi_h^* - C(s_h)]}{\partial s_h} = 0 \Leftrightarrow (16s_h - 7s_l) \Phi \Gamma = s_h, \quad (\text{A-1})$$

$$\frac{\partial[\pi_l^* - C(s_l)]}{\partial s_l} = 0 \Leftrightarrow 4s_h (3s_h - s_l) \Psi \Gamma = s_l, \quad (\text{A-2})$$

where $\Phi \equiv (8192s_h^6 - 23424s_h^5s_l + 30360s_h^4s_l^2 - 22270s_h^3s_l^3 + 9579s_h^2s_l^4 - 2256s_hs_l^5 + 224s_l^6)$, $\Psi \equiv (2560s_h^5 + -8792s_h^4s_l + 10822s_h^3s_l^2 - 6307s_h^2s_l^3 + 1778s_hs_l^4 - 196s_l^5)$ and $\Gamma \equiv \frac{9\bar{\theta}s_h(2s_h - s_l)}{(4s_h - s_l)(64s_h^2 - 65s_hs_l + 16s_l^2)^3}$. By allowing $s_h = \alpha s_l$, with $\alpha > 1$, dividing (A-1) by (A-2) and rearranging terms we obtain

$$\frac{-51200\alpha^8 + 327392\alpha^7 - 718904\alpha^6 + 862444\alpha^5 - 654856\alpha^4 + 327298\alpha^3 - 104717\alpha^2 + 19376\alpha - 1568}{51200\alpha^7 - 196320\alpha^6 + 286776\alpha^5 - 212716\alpha^4 + 86016\alpha^3 - 18144\alpha^2 + 1568\alpha} = 0. \quad (\text{A-3})$$

The only solution of (A-3) larger than one is $\alpha = 3.496$. By plugging this value back into the first-order condition, we obtain $s_h^f = 0.1453\bar{\theta}$ and $s_l^f = 0.0416\bar{\theta}$. To complete the proof and show that these values are indeed part of the equilibrium strategies of the game, we need to ascertain that no firm has an incentive to “leapfrog” the rival. We begin by confirming the absence of such a deviation for the low-quality firm. If this firm “leapfrogs” its rival, it sets a training level for the workforce equal to $s_l^{Df} > s_h^f$, earning thus a profit equal to

$$\pi_l^D(s_h^f, s_l^{Df}) = \frac{1}{2}s_h^2 \left[\frac{18\bar{\theta}(s_h - 0.1453\bar{\theta})(32s_h^2 - 4.3588\bar{\theta}s_h + 0.1478\bar{\theta}^2)^2}{(256s_h^3 - 47.0753\bar{\theta}s_h^2 + 2.7232\bar{\theta}^2s_h - 0.0491\bar{\theta}^3)^2} - 1 \right]. \quad (\text{A-4})$$

It may be determined that (A-4) is negative for all $s_l^{Df} > s_h^f$ and $\bar{\theta} > 0$. We now turn to the high-quality firm. If this firm sets a training level equal to $s_h^{fD} < s_l^f$, it earns profits equal to

$$\pi_h^D(s_h^{Df}, s_l^f) = \frac{1.4962\bar{\theta}^2 s_l (2s_l - 0.2078\bar{\theta})^2 (0.0416\bar{\theta} - s_l) (s_l - 0.0831\bar{\theta})^2}{(s_l - 0.1662\bar{\theta})^2 (16s_l^2 - 2.7014\bar{\theta}s_l + 0.1105\bar{\theta}^2)^2} - \frac{s_l^2}{2}. \quad (\text{A-5})$$

Expression (A-5) has one local maximum for $s_h^{Df} < s_l$, namely $s_h^{Df} = 0.0208\bar{\theta} \equiv \underline{s}_h^{Df}$. The deviation, however, is not profitable, in fact $\pi_h^D(\underline{s}_h^{Df}, s_l^f) = 0.0006\bar{\theta}^2 < 0.0073\bar{\theta}^2 = \pi_h(s_h^f, s_l^f)$.

The remaining equilibrium values are obtained by direct substitution.

APPENDIX B PROOF OF LEMMA 4

To prove Lemma 4, we follow the steps of the preceding proof. The first-order conditions for the unions are the following.²⁷

$$\frac{\partial[E_h^* - C(s_h)]}{\partial s_h} = 0 \Leftrightarrow (16s_h - 7s_l) \rho \sigma = s_h, \quad (\text{B-1})$$

$$\frac{\partial[E_l^* - C(s_l)]}{\partial s_l} = 0 \Leftrightarrow 4s_h (5s_h - 2s_l) \rho \tau = s_l, \quad (\text{B-2})$$

where $\sigma \equiv (8192s_h^6 - 25472s_h^5 s_l + 35384s_h^4 s_l^2 - 26668s_h^3 s_l^3 + 11225s_h^2 s_l^4 - 2480s_h s_l^5 + 224s_l^6)$, $\tau \equiv (2560s_h^5 + -8152s_h^4 s_l + 9276s_h^3 s_l^2 - 4981s_h^2 s_l^3 + 1294s_h s_l^4 - 132s_l^5)$ and $\rho \equiv \frac{4\bar{\theta}s_h}{(4s_h - s_l)^2 (64s_h^2 - 65s_h s_l + 16s_l^2)^3}$.

Again, by allowing $s_h = \alpha s_l$, $\alpha > 1$, and dividing (B-1) by (B-2) we obtain

$$\frac{-4608\alpha^8 + 23264\alpha^7 - 49072\alpha^6 + 60276\alpha^5 - 46136\alpha^4 + 22120\alpha^3 - 6447\alpha^2 + 1044\alpha - 72}{4608\alpha^7 - 15072\alpha^6 + 18864\alpha^5 - 12020\alpha^4 + 4176\alpha^3 - 756\alpha^2 + 56\alpha} = 0. \quad (\text{B-3})$$

Equation (B-3) has only one root that is larger than one, namely $\alpha = 3.0706$. Substitution back into (B-1) and (B-2) returns $s_h^u = 0.0983\bar{\theta}$ and $s_l^u = 0.032\bar{\theta}$.

As in the previous case, it remains to confirm the non-existence of profitable deviations for the union. We will begin with the low-skill union. If it leapfrogs the other union in the training level of its members, by setting $s_l^{Du} > s_h^u$, it obtains a total wage equal to

$$E(u_u, s_l^{Du}) = \frac{1}{2}(s_l^{Du})^2 \left\{ \frac{6\bar{\theta} [0.6879\bar{\theta} - 16s_l^{Du}]^2 [-0.2948\bar{\theta}s_l^{Du} + 0.0097\bar{\theta}^2 + 2(s_l^{Du})^2]}{[4s_l^{Du} - 0.0983\bar{\theta}] [-6.3873\bar{\theta}s_l^{Du} + 0.1545\bar{\theta}^2 + 64(s_l^{Du})^2]^2} - 1 \right\}. \quad (\text{B-4})$$

Expression (B-4) is negative for all $s_l^{Du} > s_h^u$ and $\bar{\theta} > 0$, implying that there is no profitable leapfrogging for the low-skill union.

Let us now turn to the high-skill union. If it deviates to a utility level $s_h^{Du} < s_l^u$, the total wage is

$$E_h(s_h^{Du}, s_l^u) = \frac{0.384\bar{\theta}^2 s_h^{Du} (0.16\bar{\theta} - 2s_h^{Du})^2 [0.002\bar{\theta}^2 - 0.096\bar{\theta}s_h^{Du} + (s_h^{Du})^2]}{(0.128\bar{\theta} - s_h^{Du}) [0.0655\bar{\theta}^2 - 2.08\bar{\theta}s_h^{Du} + 16(s_h^{Du})^2]^2} - \frac{(s_h^{Du})^2}{2}. \quad (\text{B-5})$$

Expression (B-5) has a unique maximizer over $[0, s_l^u]$, namely $\underline{s}_h^{Du} = 0.0177$, but it is easy to ascertain that $E_h(s_h^u, s_l^u) = 0.0038\bar{\theta}^2 > 0.0004\bar{\theta}^2 = E_h(\underline{s}_h^{Du}, s_l^u)$.

The remaining equilibrium values are obtained by direct substitution.

APPENDIX C TRAINING INCENTIVES

In this Appendix, we analyze thoroughly the forces underlying conditions (21) and (22). We start from the high-quality, high-skill firm-union pair and, then, we turn to the low-quality, low-skill one.

High skill

Decomposition of the effects The elasticity of the skilled workers wage rate to the skill level is positive ($\epsilon_{w^*, s_h} > 0$) because an increase in the skill level leads to an increase in the quality level of the good. This, in turn, allows for a higher surplus extraction from consumers, which increases the sum of the firm profits and union surplus. The ultimate consequence is that the workers' remuneration increases.

The elasticity of p_h to s_h is also positive:

$$\epsilon_{p_h^*, s_h} = \frac{\partial p_h^*}{\partial s_h} \frac{s_h}{p_h^*} = \frac{\partial \hat{p}_h(s_h, s_l, w^*, r^*)}{\partial s_h} \frac{s_h}{p_h^*} = \underbrace{\left(\frac{\partial \hat{p}_h}{\partial s_h} \Big|_{\substack{w=w^* \\ r=r^*}} \right)}_{\text{Direct eff.}} + \underbrace{\frac{\partial \hat{p}_h}{\partial w} \frac{\partial w^*}{\partial s_h}}_{\text{Own wage eff.}} + \underbrace{\frac{\partial \hat{p}_h}{\partial r} \frac{\partial r^*}{\partial s_h}}_{\text{Rival wage eff.}} \frac{s_h}{p_h^*} > 0. \quad (\text{C-1})$$

In fact, in (C-1) it may be ascertained that the direct effect is positive because an increase in the quality level of the good both increases the consumers' willingness to pay for it and reduces the degree of price competition by making goods less homogeneous; both forces push the price of the high-quality good up. The own wage effect is positive too, as argued before. Finally, the rival wage effect is also positive because an increase in s_h reduces market competition and thus increases the low-quality firm-union pair gross profits ($\frac{\partial D_l^* p_l^*}{\partial s_h} > 0$), leading to an increase in the unskilled workers' remuneration r^* . Consequently, the low-quality firm marginal production cost increases, which entails an increase in p_h^* .

Last, the elasticity of the high-quality good demand to the skill level s_h is negative. This elasticity writes as follows

$$\epsilon_{D_h^*, s_h} = \frac{\partial D_h^*}{\partial s_h} \frac{s_h}{D_h^*} = \frac{\partial D_h(s_h, s_l, p_h^*, p_l^*)}{\partial s_h} \frac{s_h}{D_h^*} = \underbrace{\left(\frac{\partial D_h}{\partial s_h} \Big|_{\substack{p_h=p_h^* \\ p_l=p_l^*}} \right)}_{\text{Direct eff.}} + \underbrace{\frac{\partial D_h}{\partial p_h} \frac{\partial p_h^*}{\partial s_h}}_{\text{Own price eff.}} + \underbrace{\frac{\partial D_h}{\partial p_l} \frac{\partial p_l^*}{\partial s_h}}_{\text{Rival price eff.}} \frac{s_h}{D_h^*} < 0. \quad (\text{C-2})$$

The direct effect in (C-2) is positive, indeed an increase in the skill level increases the quality of the good and thus, all else equal, the quantity demanded/employment level. The own price effect is negative because the good is normal $\frac{\partial D_h}{\partial p_h} < 0$, and, as argued above, $\frac{\partial p_h^*}{\partial s_h} > 0$ (see C-1). Finally, the rival price effect is positive as well because the high- and low- quality goods are substitutes, hence $\frac{\partial D_h}{\partial p_l} > 0$, and the effect of an increase of the high-skill level on the low-quality good price is positive:

$$\frac{\partial p_l^*}{\partial s_h} = \frac{\partial \hat{p}_l(s_h, s_l, w^*, r^*)}{\partial s_h} = \underbrace{\left(\frac{\partial \hat{p}_l}{\partial s_h} \Big|_{\substack{w=w^* \\ r=r^*}} \right)}_{\text{Direct eff.}} + \underbrace{\frac{\partial \hat{p}_l}{\partial w} \frac{\partial w^*}{\partial s_h}}_{\text{Rival wage eff.}} + \underbrace{\frac{\partial \hat{p}_l}{\partial r} \frac{\partial r^*}{\partial s_h}}_{\text{Own wage eff.}} > 0. \quad (\text{C-3})$$

In (C-3), the direct effect is positive for $s_h < 3.2455s_l$ and negative otherwise. Intuitively, starting from a "low" s_h and increasing it leads to more differentiated goods and thus milder price competition, allowing the low-quality price to increase. Yet, when s_h is "high", the high-quality good becomes extremely desirable for consumers, hence the low-quality firm must lower its price to attract consumers. The rival

wage effect is positive, $\frac{\partial w^*}{\partial s_h} > 0$ as argued before, furthermore $\frac{\hat{p}_l}{\partial w} > 0$ because an increase in the rival's marginal production cost triggers an increase in own price. Last, the own wage effect is positive too because $\frac{\partial \hat{p}_l}{\partial r} > 0$, an increase in own marginal production cost leads to an increase in the final price, and $\frac{\partial r^*}{\partial s_h} > 0$ because an increase in the high-skill level yields to an increase in the quality level of the high-quality good that relaxes price competition, hence increasing the gross profits of the low-quality firm-union pair. This yields an increase in the bargained remuneration of the low-skill workers r^* . It is a matter of algebra to ascertain that the overall balance of the effects results in a negative elasticity in eq. (C-2).

Algebraic expressions We now report the values of the skill-elasticities as well as of the forces that compose them. The elasticity of the wage with respect to the skill level amounts to

$$\epsilon_{w^*, s_h} = \frac{\partial w^*}{\partial s_h} \frac{s_h}{w^*} = \frac{1024s_h^4 - 2080s_h^3s_l + 1815s_h^2s_l^2 - 736s_h s_l^3 + 112s_l^4}{(16s_h - 7s_l)(s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)} > 0,$$

where

$$\frac{\partial w^*}{\partial s_h} = \frac{\bar{\theta}(1024s_h^4 - 2080s_h^3s_l + 1815s_h^2s_l^2 - 736s_h s_l^3 + 112s_l^4)}{(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0.$$

The elasticity of the price is equal to

$$\epsilon_{p_h^*, s_h} = \frac{\partial p_h^*}{\partial s_h} \frac{s_h}{p_h^*} = \frac{20480s_h^6 - 51840s_h^5s_l + 57852s_h^4s_l^2 - 35878s_h^3s_l^3 + 12969s_h^2s_l^4 - 2592s_h s_l^5 + 224s_l^6}{(16s_h - 7s_l)(5s_h - 2s_l)(s_h - s_l)(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)} > 0,$$

where

$$\frac{\partial p_h^*}{\partial s_h} = \frac{2\bar{\theta}(20480s_h^6 - 51840s_h^5s_l + 57852s_h^4s_l^2 - 35878s_h^3s_l^3 + 12969s_h^2s_l^4 - 2592s_h s_l^5 + 224s_l^6)}{(s_l - 4s_h)^2(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0.$$

The derivative $\frac{\partial p_h^*}{\partial s_h}$ could be decomposed into three effects (see C-1):

1. Direct effect

$$\left. \frac{\partial \hat{p}_h}{\partial s_h} \right|_{\substack{w=w^* \\ r=r^*}} = \frac{2\bar{\theta}(2s_h - s_l)(128s_h^3 - 138s_h^2s_l + 69s_h s_l^2 - 14s_l^3)}{(4s_h - s_l)^2(64s_h^2 - 65s_h s_l + 16s_l^2)} > 0.$$

2. Own wage effect

$$\frac{\partial \hat{p}_h}{\partial w} \frac{\partial w^*}{\partial s_h} = \frac{2\bar{\theta}s_h(1024s_h^4 - 2080s_h^3s_l + 1815s_h^2s_l^2 - 736s_h s_l^3 + 112s_l^4)}{(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0,$$

where $\frac{\partial \hat{p}_h}{\partial w} = \frac{2s_h}{4s_h - s_l} > 0$ and $\frac{\partial w^*}{\partial s_h}$ is as above.

3. Rival wage effect

$$\frac{\partial \hat{p}_h}{\partial r} \frac{\partial r^*}{\partial s_h} = \frac{6\bar{\theta}s_h s_l^2(41s_h^2 - 32s_h s_l + 6s_l^2)}{(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0,$$

where

$$\frac{\partial \hat{p}_h}{\partial r} = \frac{s_h}{4s_h - s_l} > 0, \quad \frac{\partial r^*}{\partial s_h} = \frac{6\bar{\theta}s_l^2(41s_h^2 - 32s_h s_l + 6s_l^2)}{(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0.$$

Finally, the elasticity of the demand amounts to

$$\epsilon_{D_h^*, s_h} = \frac{\partial D_h^*}{\partial s_h} \frac{s_h}{D_h^*} = -\frac{2s_l(1344s_h^4 - 2336s_h^3s_l + 1569s_h^2s_l^2 - 480s_h s_l^3 + 56s_l^4)}{(16s_h - 7s_l)(2s_h - s_l)(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)} < 0.$$

The partial derivative $\frac{\partial D_h^*}{\partial s_h}$ could be decomposed into three effects (see C-2):

1. Direct effect

$$\left. \frac{\partial D_h}{\partial s_h} \right|_{p_h=p_h^*, p_l=p_l^*} = \frac{2(5s_h - 2s_l)(16s_h^2 - 17s_h s_l + 4s_l^2)}{(s_h - s_l)(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)} > 0.$$

2. Own price effect

$$\frac{\partial D_h}{\partial p_h} \frac{\partial p_h^*}{\partial s_h} = -\frac{2(20480s_h^6 - 51840s_h^5s_l + 57852s_h^4s_l^2 - 35878s_h^3s_l^3 + 12969s_h^2s_l^4 - 2592s_h s_l^5 + 224s_l^6)}{(s_h - s_l)(4s_h - s_l)^2(64s_h^2 - 65s_h s_l + 16s_l^2)^2} < 0,$$

where $\frac{\partial D_h}{\partial p_h} = -\frac{1}{\theta(s_h - s_l)} < 0$ and $\frac{\partial p_h^*}{\partial s_h}$ is as above.

3. Rival price effect

$$\frac{\partial D_h}{\partial p_l} \frac{\partial p_l^*}{\partial s_h} = \frac{12s_l^2(5s_h - 2s_l)(228s_h^3 - 298s_h^2s_l + 137s_h s_l^2 - 22s_l^3)}{(s_h - s_l)(s_l - 4s_h)^2(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0.$$

where $\frac{\partial D_h}{\partial p_l} = \frac{1}{\theta(s_h - s_l)} > 0$ and $\frac{\partial p_l^*}{\partial s_h}$ is decomposed as follows (see C-3):

(a) Direct effect

$$\left. \frac{\partial \hat{p}_l}{\partial s_h} \right|_{w=w^*, r=r^*} = \frac{\bar{\theta}s_l(-64s_h^3 + 264s_h^2s_l - 195s_h s_l^2 + 40s_l^3)}{(s_l - 4s_h)^2(64s_h^2 - 65s_h s_l + 16s_l^2)} \geq 0 \Leftrightarrow s_h \leq 3.2455s_l.$$

(b) Rival wage effect

$$\frac{\partial \hat{p}_l}{\partial w} \frac{\partial w^*}{\partial s_h} = \frac{\bar{\theta}s_l(1024s_h^4 - 2080s_h^3s_l + 1815s_h^2s_l^2 - 736s_h s_l^3 + 112s_l^4)}{(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0,$$

where $\frac{\partial \hat{p}_l}{\partial w} = \frac{s_l}{4s_h - s_l} > 0$ and $\frac{\partial w^*}{\partial s_h}$ is as above.

(c) Own wage effect

$$\frac{\partial \hat{p}_l}{\partial r} \frac{\partial r^*}{\partial s_h} = \frac{12\bar{\theta}s_h s_l^2(41s_h^2 - 32s_h s_l + 6s_l^2)}{(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0,$$

where

$$\frac{\partial \hat{p}_l}{\partial r} = \frac{2s_h}{4s_h - s_l} > 0, \quad \frac{\partial r^*}{\partial s_h} = \frac{6\bar{\theta}s_l^2(41s_h^2 - 32s_h s_l + 6s_l^2)}{(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0.$$

Low-skill

Decomposition of the effects Differently from the high-skill training case, ϵ_{r^*, s_l} and $\epsilon_{p_l^*, s_l}$ do not have the same sign for all admissible low-skill levels, and $\epsilon_{D_l^*, s_l}$ is positive rather than negative. The reason for this is that, like for the high-quality product, an increase in the low-skill level s_l triggers an increase in the quality level of the low-quality good which, all else equal, increases the willingness to pay for it and consequently the price the firm can charge and the sum of the low-quality firm profits and union surplus, we call this “quality appreciation effect”. However, unlike the previous case, an increase of s_l also makes

the goods more homogeneous. This makes price competition harsher, which leads to a reduction in prices and the sum of profits and union surplus, we label this “competitive effect”.

Consider the elasticity $\epsilon_{r^*, s_l} = \frac{\partial r^*}{\partial s_l} \frac{s_l}{r^*}$ first. It is a matter of computations to ascertain that it is positive for $s_l < 0.6156s_h$ and negative for larger values. Intuitively, when s_l is “low”, the quality level of the good is “low” as well, hence the price and demand of the good are low. Consequently, the sum of profits and union surplus is low, resulting in a “low” remuneration of the low-skilled workers. An increase in s_l triggers the two effects highlighted above, yet, initially, the quality appreciation effect is stronger than the competitive one. Consequently, the surplus extracted from consumers increases and the remuneration of the low-skill workers increases with it. As s_l further increases, the goods become more and more homogeneous, thus the competitive effect becomes more relevant and, eventually more than offsets the quality appreciation one, ultimately resulting in a reduction of r^* .

Let us now move to the elasticity of p_l^* to s_l :

$$\epsilon_{p_l, s_l} = \frac{\partial p_l^*}{\partial s_l} \frac{s_l}{p_l^*} = \frac{\partial \hat{p}_l(s_h, s_l, w^*, r^*)}{\partial s_l} \frac{s_l}{p_l^*} = \underbrace{\left(\frac{\partial \hat{p}_l}{\partial s_l} \Big|_{\substack{w=w^* \\ r=r^*}} \right)}_{\text{Direct eff.}} + \underbrace{\frac{\partial \hat{p}_l}{\partial r} \frac{\partial r^*}{\partial s_l}}_{\text{Own wage eff.}} + \underbrace{\frac{\partial \hat{p}_l}{\partial w} \frac{\partial w^*}{\partial s_l}}_{\text{Rival wage eff.}} \frac{s_l}{p_l^*} \stackrel{?}{\geq} 0 \Leftrightarrow s_l \stackrel{?}{\leq} 0.5874s_h. \quad (\text{C-4})$$

This elasticity is positive for “low” s_l and negative otherwise. Notice, nonetheless, that at the optimal training levels, this elasticity is positive as for the high-skill training. By focusing on the set of forces driving the result, calculations show that the direct effect in (C-4) is positive for $s_l < 0.6375s_h$ and negative otherwise. The intuition is as before, namely for low s_l , the quality appreciation effect prevails over the competitive one, leading to an increase in the price. The balance of forces reverses for values of s_l closer and closer to s_h . The own wage effect also is non-monotonic because $\frac{\partial \hat{p}_l}{\partial r}$ is positive (see C-3), but, as discussed above, $\frac{\partial r^*}{\partial s_l} \stackrel{?}{\geq} 0 \Leftrightarrow s_l \stackrel{?}{\leq} 0.6156s_h$. Finally, the rival wage effect is negative because $\frac{\partial \hat{p}_l}{\partial w} > 0$ (an increase in the rival’s marginal cost increases the own price) but $\frac{\partial w^*}{\partial s_l} < 0$ because an increase in s_l only entails a negative effect on the high-quality firm/union pair due to more homogeneous goods, which reduces the pair gross profits and hence the bargained wage w^* .

Last, let us delve into the elasticity of the demand for the low-quality good to s_l :

$$\epsilon_{D_l^*, s_l} = \frac{\partial D_l^*}{\partial s_l} \frac{s_l}{D_l^*} = \frac{\partial D_l(s_h, s_l, p_h^*, p_l^*)}{\partial s_l} \frac{s_l}{D_l^*} = \underbrace{\left(\frac{\partial D_l}{\partial s_l} \Big|_{\substack{p_h=p_h^* \\ p_l=p_l^*}} \right)}_{\text{Direct eff.}} + \underbrace{\frac{\partial D_l}{\partial p_l} \frac{\partial p_l^*}{\partial s_l}}_{\text{Own price eff.}} + \underbrace{\frac{\partial D_l}{\partial p_h} \frac{\partial p_h^*}{\partial s_l}}_{\text{Rival price eff.}} \frac{s_l}{D_l^*} > 0. \quad (\text{C-5})$$

The direct effect is positive because, all else equal, an increase in s_l raises the quality of the good, therefore the demand for it. The own price effect is negative for $s_l < 0.5874s_h$ and negative otherwise because $\frac{\partial D_l}{\partial p_l} < 0$ (good l is normal) and, as pointed out above $\frac{\partial p_l^*}{\partial s_l}$ is non monotonic (see C-4). The rival price effect is negative, indeed, $\frac{\partial D_l}{\partial p_h} > 0$ and $\frac{\partial p_h^*}{\partial s_l} < 0$. This latter partial derivative may be decomposed as follows:

$$\frac{\partial p_h^*}{\partial s_l} = \frac{\partial \hat{p}_h(s_h, s_l, w^*, r^*)}{\partial s_l} = \underbrace{\left(\frac{\partial \hat{p}_h}{\partial s_l} \Big|_{\substack{w=w^* \\ r=r^*}} \right)}_{\text{Direct eff.}} + \underbrace{\frac{\partial \hat{p}_h}{\partial r} \frac{\partial r^*}{\partial s_l}}_{\text{Rival wage eff.}} + \underbrace{\frac{\partial \hat{p}_h}{\partial w} \frac{\partial w^*}{\partial s_l}}_{\text{Own wage eff.}} < 0. \quad (\text{C-6})$$

The direct effect is negative because, *ceteris paribus*, an increase of s_l makes goods more homogeneous and, thus, fosters price competition. Last, from the above analysis it follows that the rival wage effect

is positive for $s_l < 0.6156s_h$ and negative otherwise, and that the own wage effect is negative as well. Simple calculations show that the overall effect of s_l on p_h^* is negative.

To conclude, it is a matter of calculations to see that *at the optimal skill levels* of Lemmas 3 and 4, both ϵ_{r^*,s_l} and $\epsilon_{p_l^*,s_l}$ are positive as in the high-skill case. By contrast, the elasticity of the low-quality demand/low-skill workers employment level is negative, which is explained by the prevalence of the competitive effect over the quality appreciation one.

Algebraic expressions The elasticity of the wage with respect to the skill level amounts to

$$\epsilon_{r^*,s_l} = \frac{\partial r^*}{\partial s_l} \frac{s_l}{r^*} = \frac{320s_h^4 - 896s_h^3s_l + 759s_h^2s_l^2 - 260s_h s_l^3 + 32s_l^4}{(5s_h - 2s_l)(s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)},$$

where

$$\frac{\partial r^*}{\partial s_l} = \frac{2\bar{\theta}(320s_h^4 - 896s_h^3s_l + 759s_h^2s_l^2 - 260s_h s_l^3 + 32s_l^4)}{(64s_h^2 - 65s_h s_l + 16s_l^2)^2} \geq 0 \Leftrightarrow s_l \leq 0.6156s_h.$$

The elasticity of the price is equal to

$$\epsilon_{p_l^*,s_l} = \frac{\partial p_l^*}{\partial s_l} \frac{s_l}{p_l^*} = \frac{1280s_h^5 - 4096s_h^4s_l + 4319s_h^3s_l^2 - 2042s_h^2s_l^3 + 436s_h s_l^4 - 32s_l^5}{(5s_h - 2s_l)(s_h - s_l)(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)} \geq 0 \Leftrightarrow s_l \leq 0.5874s_h,$$

where

$$\frac{\partial p_l^*}{\partial s_l} = \frac{4\bar{\theta}(5s_h - 2s_l)(1280s_h^5 - 4096s_h^4s_l + 4319s_h^3s_l^2 - 2042s_h^2s_l^3 + 436s_h s_l^4 - 32s_l^5)}{(4s_h - s_l)^2(64s_h^2 - 65s_h s_l + 16s_l^2)^2} \geq 0 \Leftrightarrow s_l \leq 0.5874s_h.$$

The partial derivative $\frac{\partial p_l^*}{\partial s_l}$ could be decomposed into three effects (see C-4):

1. Direct effect

$$\left. \frac{\partial \hat{p}_l}{\partial s_l} \right|_{\substack{w=w^* \\ r=r^*}} = \frac{\bar{\theta}(2s_h - s_l)(160s_h^3 - 342s_h^2s_l + 153s_h s_l^2 - 16s_l^3)}{(s_l - 4s_h)^2(64s_h^2 - 65s_h s_l + 16s_l^2)} \geq 0 \Leftrightarrow s_l \leq 0.6375s_h.$$

2. Own wage effect

$$\frac{\partial \hat{p}_l}{\partial r} \frac{\partial r^*}{\partial s_l} = \frac{4\bar{\theta}s_h(320s_h^4 - 896s_h^3s_l + 759s_h^2s_l^2 - 260s_h s_l^3 + 32s_l^4)}{(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)^2} \geq 0 \Leftrightarrow s_l \leq 0.6156s_h,$$

where $\frac{\partial \hat{p}_l}{\partial r} = \frac{2s_h}{4s_h - s_l} > 0$ and $\frac{\partial r^*}{\partial s_l}$ is as above.

3. Rival wage effect

$$\frac{\partial \hat{p}_l}{\partial w} \frac{\partial w^*}{\partial s_l} = -\frac{3\bar{\theta}s_h^2s_l(144s_h^2 - 128s_h s_l + 29s_l^2)}{(4s_h - s_l)(64s_h^2 - 65s_h s_l + 16s_l^2)^2} < 0,$$

where

$$\frac{\partial \hat{p}_l}{\partial w} = \frac{s_l}{4s_h - s_l} > 0, \quad \frac{\partial w^*}{\partial s_l} = -\frac{3\bar{\theta}s_h^2s_l(144s_h^2 - 128s_h s_l + 29s_l^2)}{(64s_h^2 - 65s_h s_l + 16s_l^2)^2} < 0.$$

Finally, the elasticity of the demand amounts to

$$\epsilon_{D_l^*,s_l} = \frac{\partial D_l^*}{\partial s_l} \frac{s_l}{D_l^*} = \frac{6s_h(936s_h^4 - 1556s_h^3s_l + 993s_h^2s_l^2 - 288s_h s_l^3 + 32s_l^4)}{(4s_h - s_l)^2(64s_h^2 - 65s_h s_l + 16s_l^2)^2} > 0.$$

The partial derivative $\frac{\partial D_l^*}{\partial s_l}$ could be decomposed into three effects (see C-5):

1. Direct effect

$$\frac{\partial D_l}{\partial s_l} \Big|_{\substack{p_h=p_h^* \\ p_l=p_l^*}} = \frac{2s_h(5s_h-2s_l)(10s_h^2-8s_h s_l+s_l^2)}{s_l(s_h-s_l)(4s_h-s_l)(64s_h^2-65s_h s_l+16s_l^2)} > 0.$$

2. Own price effect

$$\frac{\partial D_l}{\partial p_l} \frac{\partial p_l^*}{\partial s_l} = -\frac{4s_h(5s_h-2s_l)(1280s_h^5-4096s_h^4 s_l+4319s_h^3 s_l^2-2042s_h^2 s_l^3+436s_h s_l^4-32s_l^5)}{s_l(s_h-s_l)(4s_h-s_l)^2(64s_h^2-65s_h s_l+16s_l^2)^2} \geq 0 \Leftrightarrow s_l \geq 0.5874s_h,$$

where $\frac{\partial D_l}{\partial p_l} = -\frac{s_h}{\theta s_l(s_h-s_l)} < 0$ and $\frac{\partial p_l^*}{\partial s_l}$ is as above.

3. Rival price effect

$$\frac{\partial D_l}{\partial p_h} \frac{\partial p_h^*}{\partial s_l} = -\frac{6s_h^2(3904s_h^4-6944s_h^3 s_l+4731s_h^2 s_l^2-1456s_h s_l^3+170s_l^4)}{(s_h-s_l)(s_l-4s_h)^2(64s_h^2-65s_h s_l+16s_l^2)^2} < 0,$$

where $\frac{\partial D_l}{\partial p_h} = \frac{1}{\theta(s_h-s_l)} > 0$ and $\frac{\partial p_h^*}{\partial s_l}$ is decomposed as follows (see C-6):

(a) Direct effect

$$\frac{\partial \hat{p}_h}{\partial s_l} \Big|_{\substack{w=w^* \\ r=r^*}} = \frac{2\bar{\theta}s_h(-176s_h^3+177s_h^2 s_l-48s_h s_l^2+2s_l^3)}{(4s_h-s_l)^2(64s_h^2-65s_h s_l+16s_l^2)} < 0.$$

(b) Rival wage effect

$$\frac{\partial \hat{p}_h}{\partial r} \frac{\partial r^*}{\partial s_l} = \frac{2\bar{\theta}s_h(320s_h^4-896s_h^3 s_l+759s_h^2 s_l^2-260s_h s_l^3+32s_l^4)}{(4s_h-s_l)(64s_h^2-65s_h s_l+16s_l^2)^2} \geq 0 \Leftrightarrow s_l \leq 0.6156s_h,$$

where $\frac{\partial \hat{p}_h}{\partial r}$ is as in (3) and $\frac{\partial r^*}{\partial s_l}$ is as above.

(c) Own wage effect

$$\frac{\partial \hat{p}_h}{\partial w} \frac{\partial w^*}{\partial s_l} = -\frac{6\bar{\theta}s_h^3(144s_h^2-128s_h s_l+29s_l^2)}{(4s_h-s_l)(64s_h^2-65s_h s_l+16s_l^2)^2} < 0,$$

where

$$\frac{\partial \hat{p}_h}{\partial w} = \frac{2s_h}{4s_h-s_l} > 0, \quad \frac{\partial w^*}{\partial s_l} = -\frac{3\bar{\theta}s_h^2(144s_h^2-128s_h s_l+29s_l^2)}{(64s_h^2-65s_h s_l+16s_l^2)^2} < 0.$$

APPENDIX D PROOF OF LEMMA 5

- (i) The objective functions of the agents are $E_h^* - C(s_h)$ and $\pi_l^* - C(s_l)$. The relevant first-order conditions are (B-1) and (A-2). We proceed as in the previous Lemmata, namely we set $s_h = \alpha s_l$, divide (B-1) by (A-2) and solve for α . The unique solution larger than one is $\alpha = 2.7787$, which yields $s_l^{uf} = 0.0359\bar{\theta}$ and $s_h^{uf} = 0.0997\bar{\theta}$. The candidate equilibrium total wage for the union is $E_h^{uf} = 0.0036\bar{\theta}^2$ and $\pi_l^{uf} = 0.0001\bar{\theta}^2$. It may be demonstrated that there are no profitable deviations for either agent. Second order conditions are fulfilled for all $s_h > s_l$. Direct substitution yields the remaining equilibrium values.
- (ii) The objective functions of the agents are $\pi_h^* - C(s_h)$ and $E_l^* - C(s_l)$. By setting $s_h = \alpha s_l$, dividing (A-1) by (B-2) and solving for α we obtain as unique solution larger than one $\alpha = 4.1847$, which yields $s_l = 0.0343\bar{\theta}$ and $s_h = 0.1437\bar{\theta}$. The associated profits for the firm and union are, respectively,

$0.0078\bar{\theta}^2$ and $0.0006\bar{\theta}$. As before, no profitable deviation exists for the firm or the union. The remaining equilibrium values obtain by direct substitution.

APPENDIX E PROOF OF PROPOSITION 3

The first-order conditions of this maximization problem generate the system

$$\begin{cases} \frac{3\bar{\theta}\Omega}{2(4s_h - s_l)^3(-65s_h s_l + 64s_h^2 + 16s_l^2)^3} = s_h, \\ \frac{6\bar{\theta}s_h^2(5s_h - 2s_l)\Xi}{(4s_h - s_l)^3(-65s_h s_l + 64s_h^2 + 16s_l^2)^3} = s_l, \end{cases} \quad (\text{E-1})$$

where $\Omega \equiv (-12939264s_h^8 s_l + 21219712s_h^7 s_l^2 - 19358928s_h^6 s_l^3 + 10509900s_h^5 s_l^4 - 3285932s_h^4 s_l^5 + 454635s_h^3 s_l^6 + 36336s_h^2 s_l^7 - 20864s_h s_l^8 + 3407872s_h^9 + 2048s_l^9)$ and $\Xi \equiv (-341360s_h^5 s_l + 434164s_h^4 s_l^2 - 291126s_h^3 s_l^3 + 109061s_h^2 s_l^4 - 21694s_h s_l^5 + 109568s_h^6 + 1792s_l^6)$.

By setting $s_h = \alpha s_l$ and dividing the first equation by the second we obtain

$$\frac{-2191360\alpha^{10} + 11111616\alpha^9 - 24353424\alpha^8 + 30515544\alpha^7 - 23869156\alpha^6 + 11816268\alpha^5 - 3495324\alpha^4 + 468971\alpha^3 + 36336\alpha^2 - 20864\alpha + 2048}{2191360\alpha^9 - 7703744\alpha^8 + 11414160\alpha^7 - 9295832\alpha^6 + 4510228\alpha^5 - 1306368\alpha^4 + 209392\alpha^3 - 14336\alpha^2} = 0. \quad (\text{E-2})$$

The only solution to (E-2) that is both real and greater than one is $\alpha = 1.471$. By following the same procedure as above, we obtain the welfare-maximizing training levels $s_h^W = 0.3056\bar{\theta}$ and $s_l^W = 0.2077\bar{\theta}$. It remains to confirm the concavity of the objective function $W(\cdot)$ with respect to s_h and s_l . The Hessian matrix of the problem is

$$H = \begin{pmatrix} \eta s_l^2 - 1 & -\eta s_h s_l \\ -\eta s_h s_l & \eta s_h^2 - 1 \end{pmatrix}, \quad (\text{E-3})$$

where $\eta \equiv \frac{12\bar{\theta}}{(4s_h - s_l)^4(64s_h^2 - 65s_h s_l + 16s_l^2)^4} (-105511104s_h^8 s_l + 230432448s_h^7 s_l^2 - 282164040s_h^6 s_l^3 + 217800999s_h^5 s_l^4 - 110985765s_h^4 s_l^5 + 37515072s_h^3 s_l^6 - 8128512s_h^2 s_l^7 + 1025184s_h s_l^8 + 19730432s_h^9 - 57344s_l^9)$.

It is a matter of calculations to demonstrate that (E-3) is negative definite at (s_h^W, s_l^W) , and its first order leading principal minor at (s_h^W, s_l^W) is -1.49 and the second order leading principal minor is 1.4712.

The values for prices, wages and quantity/employment follow from direct substitution.

APPENDIX F PROOF OF LEMMA 8

The problems in (28) are:

$$\max_{w, D_h} \{ \bar{\theta}[s_h(1 - D_h) - s_l D_l] - w, \} D_h^2 w, \quad (\text{F-1})$$

for the high-quality producer and

$$\max_{r, D_l} [(1 - s_h - s_l)s_l \bar{\theta} - r] D_l^2 r, \quad (\text{F-2})$$

for the low-quality one. By taking the first-order conditions of problem (F-1) and performing easy computations one obtains (29). Following the same steps, (30) is obtained starting from (F-2). The Hessian matrices of the two problems are negative semi-definite at the solution of each problem. Finally, by com-

binning (29) and (30) one obtains (31) and (32). Direct substitution yields the prices, and the firms profits and unions surplus, which we report hereafter.

$$\pi_h^* = E_h^* = \frac{\bar{\theta}^2 s_h (2s_h - s_l)^2}{2(4s_h - s_l)^2}, \quad \pi_l^* = E_l^* = \frac{\bar{\theta}^2 s_h^2 s_l}{2(4s_h - s_l)^2}. \quad (\text{F-3})$$

We derive the optimal training level for the case of firm-sponsored training, the case of union-sponsored training coinciding with this one. Each firm maximizes $\pi_i^* - C(s_i)$, $i = h, l$. The first-order conditions are

$$\frac{\partial[\pi_h^* - C(s_h)]}{\partial s_h} = 0 \Leftrightarrow \frac{\bar{\theta}(2s_h - s_l)(8s_h^2 - 2s_h s_l + s_l^2)}{2(4s_h - s_l)^3} = s_h, \quad (\text{F-4})$$

for the high-quality firm, and

$$\frac{\partial[\pi_l^* - C(s_l)]}{\partial s_l} = 0 \Leftrightarrow \frac{\bar{\theta}s_h^2(4s_h + s_l)}{2(4s_h - s_l)^3} = s_l, \quad (\text{F-5})$$

for the low-quality one.

By dividing (F-4) by (F-5) and setting $s_h = \alpha s_l$, with $\alpha > 1$, we obtain

$$\frac{(2\alpha - 1)(8\alpha^2 - 2\alpha + 1)}{\alpha^2(4\alpha + 1)} = \alpha.$$

The only solution larger than one of this equation is $\alpha = 2.7924$, which gives, by substituting back into the FOCs, $s_l^f = 0.0451\bar{\theta}$ and $s_h = 0.126\bar{\theta}$. Second-order conditions for profit maximization are met at these values. By plugging these values back into prices, wages, employment levels, one obtains the values reported in Lemma 8. To prove that a subgame-perfect Nash equilibrium actually exists, there remains to check the absence of profitable deviations. Consider the low-quality firm first. If it leapfrogs upwards the rival by setting $s_l^D > s_h^f$ it earns a profit equal to

$$\frac{0.25s_l^D \bar{\theta}(s_l^D - 0.063\bar{\theta})^2}{2(s_l^D - 0.0315\bar{\theta})^2} - \frac{(s_l^D)^2}{2}.$$

The unique maximizer of this expression is $s_l^D = 0.123\bar{\theta}$, yet, at this value, the deviation profit is negative. Let us now consider the high-quality firm. It is a matter of simple calculations to ascertain that, at the candidate equilibrium, the profit of this firm is $\pi_h^f = 0.0049\bar{\theta}$. If it leapfrogs downwards the rival by setting $0 < s_h^D < s_l^f$ it earns a profit equal to

$$\frac{0.0010\bar{\theta}^3 s_h^D}{(s_h^D - 0.1804\bar{\theta})^2} - \frac{(s_h^D)^2}{2},$$

which is always increasing in s_h^D (remember that, in the case of efficient bargaining, quantities are set, thus even if the goods are homogeneous, competition does not drive prices down to the zero). At $s_h^D = s_l^f$ the value of the deviation profit is 0.0014 which is lower than the π_h^f .

APPENDIX G CONTRACTIBLE INVESTMENT

We solve the training stage when the firm and the union share investment costs and jointly choose the training levels of the workforce. The high-quality firm and the high-skilled union aim to maximize

$\pi_h^* + E_h^* - C(s_h)$, whereas the low-quality firm and the low-skilled union $\pi_l^* + E_l^* - C(s_l)$. The first-order conditions with respect to the quality levels are

$$\frac{\partial [\pi_h^* + E_h^* - C(s_h)]}{\partial s_h} = 0 \Leftrightarrow (16s_h - 7s_l)\Theta\Upsilon = s_h, \quad (\text{G-1})$$

$$\frac{\partial [\pi_l^* + E_l^* - C(s_l)]}{\partial s_l} = 0 \Leftrightarrow 4s_h(5s_h - 2s_l)^2\Theta\Sigma = s_l, \quad (\text{G-2})$$

where $\Upsilon \equiv (40960s_h^7 - 137600s_h^6s_l + 209720s_h^5s_l^2 - 183378s_h^4s_l^3 + 97926s_h^3s_l^4 - 31709s_h^2s_l^5 + 5744s_hs_l^6 - 448s_l^7)$,

$\Theta \equiv \frac{6\bar{\theta}s_h}{(4s_h - s_l)^3(64s_h^2 - 65s_hs_l + 16s_l^2)^3}$ and $\Sigma \equiv (2560s_h^5 - 8536s_h^4s_l + 10242s_h^3s_l^2 - 5854s_h^2s_l^3 + 1633s_hs_l^4 - 180s_l^5)$.

By allowing $s_h = \alpha s_l$ with $\alpha > 1$, dividing (G-1) by (G-2) and rearranging terms we obtain:

$$\frac{256000\alpha^9 - 1713760\alpha^8 + 4236360\alpha^7 - 5860056\alpha^6 + 5197580\alpha^5 - 3092766\alpha^4 + 1233354\alpha^3 - 316747\alpha^2 + 47376\alpha - 3136}{4\alpha(5\alpha - 2)^2(2560\alpha^5 - 8536\alpha^4 + 10242\alpha^3 - 5854\alpha^2 + 1633\alpha - 180)} = 0. \quad (\text{G-3})$$

The only solution of (G-3) larger than one is $\alpha = 3.496$. Following the same procedure used in Appendix A, it can be easily shown that no agent has an incentive of leapfrogging the rival. By plugging the value of α back into the first-order conditions, we obtain $s_h^c = 0.2433\bar{\theta}$ and $s_l^c = 0.0732\bar{\theta}$. By direct substitution, we easily get that the sum of the payoffs of the high-quality firm and high-skilled union is equal to $0.0214\bar{\theta}^2$, whereas the sum of the payoffs of the low-quality firm and low-skilled union amounts to $0.0034\bar{\theta}^2$.

APPENDIX H ASYMMETRIC BARGAINING

The following tables report the equilibrium values of qualities, prices, demands, profit, total and individual wage and welfare under asymmetric bargaining Figure 2 summarizes them as a function of the bargaining powers.

TABLE 1
ALL BARGAINING POWER TO WORKERS (MONOPOLY UNION).

| | f | u | fu | uf | Soc. Op. |
|----------------|---|---|--------|---|---|
| s_h, s_l | 0.0671 $\bar{\theta}$, 0.028 $\bar{\theta}$ | 0.1404 $\bar{\theta}$, 0.0698 $\bar{\theta}$ | No Eq. | 0.1268 $\bar{\theta}$, 0.0327 $\bar{\theta}$ | 0.2583 $\bar{\theta}$, 0.245 $\bar{\theta}$ |
| p_h, p_l | 0.0396 $\bar{\theta}^2$, 0.0127 $\bar{\theta}^2$ | 0.0769 $\bar{\theta}^2$, 0.0294 $\bar{\theta}^2$ | – | 0.0839 $\bar{\theta}^2$, 0.0164 $\bar{\theta}^2$ | 0, 0.264 $\bar{\theta}^2$, 0.0199 $\bar{\theta}^2$ |
| D_h, D_l | 0.3106, 0.2378 | 0.3272, 0.2516 | – | 0.2831, 0.2149 | 0.5114, 0.4071 |
| π_h, π_l | 0.0015 $\bar{\theta}^2$, 0.0005 $\bar{\theta}^2$ | 0.0075 $\bar{\theta}^2$, 0.0022 $\bar{\theta}^2$ | – | 0.0075 $\bar{\theta}^2$, 0.0006 $\bar{\theta}^2$ | 0.0035 $\bar{\theta}^2$, 0.002 $\bar{\theta}^2$ |
| E_h, E_l | 0.0085 $\bar{\theta}^2$, 0.0021 $\bar{\theta}^2$ | 0.0077 $\bar{\theta}^2$, 0.0027 $\bar{\theta}^2$ | – | 0.0081 $\bar{\theta}^2$, 0.0024 $\bar{\theta}^2$ | 0.01 $\bar{\theta}^2$, 0.006 $\bar{\theta}^2$ |
| w, r | 0.0274 $\bar{\theta}^2$, 0.0088 $\bar{\theta}^2$ | 0.0538 $\bar{\theta}^2$, 0.0206 $\bar{\theta}^2$ | – | 0.0572 $\bar{\theta}^2$, 0.0112 $\bar{\theta}^2$ | 0.0196 $\bar{\theta}^2$, 0.0148 $\bar{\theta}^2$ |
| W | 0.0188 $\bar{\theta}^2$ | 0.0357 $\bar{\theta}^2$ | – | 0.0265 $\bar{\theta}^2$ | 0.0634 $\bar{\theta}^2$ |

TABLE 2
75% OF BARGAINING POWER TO WORKERS, 25% TO FIRMS.

| | f | u | fu | uf | Soc. Op. |
|----------------|---|---|--------|---|--|
| s_h, s_l | 0.1025 $\bar{\theta}$, 0.0353 $\bar{\theta}$ | 0.1261 $\bar{\theta}$, 0.0506 $\bar{\theta}$ | No Eq. | 0.1213 $\bar{\theta}$, 0.0373 $\bar{\theta}$ | 0.3915 $\bar{\theta}$, 0.255 $\bar{\theta}$ |
| p_h, p_l | 0.0565 $\bar{\theta}^2$, 0.0136 $\bar{\theta}^2$ | 0.066 $\bar{\theta}^2$, 0.0185 $\bar{\theta}^2$ | – | 0.0689 $\bar{\theta}^2$, 0.0148 $\bar{\theta}^2$ | 0, 1457 $\bar{\theta}^2$, 0.0677 $\bar{\theta}^2$ |
| D_h, D_l | 0.3617, 0.2529 | 0.3719, 0.2609 | – | 0.3554, 0.248 | 0.4287, 0.3058 |
| π_h, π_l | 0.0035 $\bar{\theta}^2$, 0.0008 $\bar{\theta}^2$ | 0.0104 $\bar{\theta}^2$, 0.002 $\bar{\theta}^2$ | – | 0.0106 $\bar{\theta}^2$, 0.0008 $\bar{\theta}^2$ | 0.0.251 $\bar{\theta}^2$, 0.0083 $\bar{\theta}^2$ |
| E_h, E_l | 0.0116 $\bar{\theta}^2$, 0.002 $\bar{\theta}^2$ | 0.0061 $\bar{\theta}^2$, 0.0015 $\bar{\theta}^2$ | – | 0.0065 $\bar{\theta}^2$, 0.0021 $\bar{\theta}^2$ | 0.0374 $\bar{\theta}^2$, 0.0124 $\bar{\theta}^2$ |
| w, r | 0.0322 $\bar{\theta}^2$, 0.0078 | 0.0379 $\bar{\theta}^2$, 0.0107 | – | 0.0391 $\bar{\theta}^2$, 0.0084 $\bar{\theta}^2$ | 0.0872 $\bar{\theta}^2$, 0.0405 $\bar{\theta}^2$ |
| W | 0.029 $\bar{\theta}^2$ | 0.0355 $\bar{\theta}^2$ | – | 0.0322 $\bar{\theta}^2$ | 0.0553 $\bar{\theta}^2$ |

TABLE 3
25% OF BARGAINING POWER TO WORKERS, 75% TO FIRMS.

| | f | u | fu | uf | Soc. Op. |
|----------------|---|---|---|--------|---|
| s_h, s_l | 0.1955 $\bar{\theta}$, 0.046 $\bar{\theta}$ | 0.0563 $\bar{\theta}$, 0.0148 $\bar{\theta}$ | 0.1919 $\bar{\theta}$, 0.0168 $\bar{\theta}$ | No Eq. | 0.3447 $\bar{\theta}$, 0.1726 $\bar{\theta}$ |
| p_h, p_l | 0.0915 $\bar{\theta}^2$, 0.0122 $\bar{\theta}^2$ | 0.0257 $\bar{\theta}^2$, 0.0038 $\bar{\theta}^2$ | 0.1015 $\bar{\theta}^2$, 0.005 $\bar{\theta}^2$ | – | 0.1175 $\bar{\theta}^2$, 0.0336 $\bar{\theta}^2$ |
| D_h, D_l | 0.4693, 0.2657 | 0.4733, 0.2681 | 0.4487, 0.253 | – | 0.5123, 0.2928 |
| π_h, π_l | 0.0138 $\bar{\theta}^2$, 0.0014 $\bar{\theta}^2$ | 0.0093 $\bar{\theta}^2$, 0.008 $\bar{\theta}^2$ | 0.0168 $\bar{\theta}^2$, 0.001 $\bar{\theta}^2$ | – | 0.0452 $\bar{\theta}^2$, 0.0074 $\bar{\theta}^2$ |
| E_h, E_l | 0.01 $\bar{\theta}^2$, 0.0007 $\bar{\theta}^2$ | 0.0013 $\bar{\theta}^2$, 0.0001 $\bar{\theta}^2$ | 0.0103 $\bar{\theta}^2$, 0.0001 $\bar{\theta}^2$ | – | 0.015 $\bar{\theta}^2$, 0.0025 $\bar{\theta}^2$ |
| w, r | 0.0214 $\bar{\theta}^2$, 0.003 $\bar{\theta}^2$ | 0.006 $\bar{\theta}^2$, 0.0009 $\bar{\theta}^2$ | 0.023 $\bar{\theta}^2$, 0.0011 $\bar{\theta}^2$ | – | 0.0294 $\bar{\theta}^2$, 0.0084 $\bar{\theta}^2$ |
| W | 0.0549 $\bar{\theta}^2$ | 0.0202 $\bar{\theta}^2$ | 0.05 $\bar{\theta}^2$ | – | 0.0743 $\bar{\theta}^2$ |

TABLE 4
ALL BARGAINING POWER TO FIRMS (SEE [MOTTA, 1993](#)).

| | f | u | fu | uf | Soc. Op. |
|----------------|--|--------|--------|--------|---|
| s_h, s_l | 0.2533 $\bar{\theta}$, 0.04824 $\bar{\theta}$ | No Eq. | No Eq. | No Eq. | 0.378 $\bar{\theta}$, 0.1428 $\bar{\theta}$ |
| p_h, p_l | 0.3435 $\bar{\theta}^2$, 0.1718 $\bar{\theta}^2$ | – | – | – | 0.1298 $\bar{\theta}^2$, 0.0245 $\bar{\theta}^2$ |
| D_h, D_l | 0.525, 0.0625 | – | – | – | 0.5521, 0.2761 |
| π_h, π_l | 0.02444 $\bar{\theta}^2$, 0.0015 $\bar{\theta}^2$ | – | – | – | 0.0717 $\bar{\theta}^2$, 0.0068 $\bar{\theta}^2$ |
| E_h, E_l | 0, 0 | – | – | – | 0, 0 |
| w, r | 0, 0 | – | – | – | 0, 0 |
| W | 0.0692 $\bar{\theta}^2$ | – | – | – | 0.0817 $\bar{\theta}^2$ |

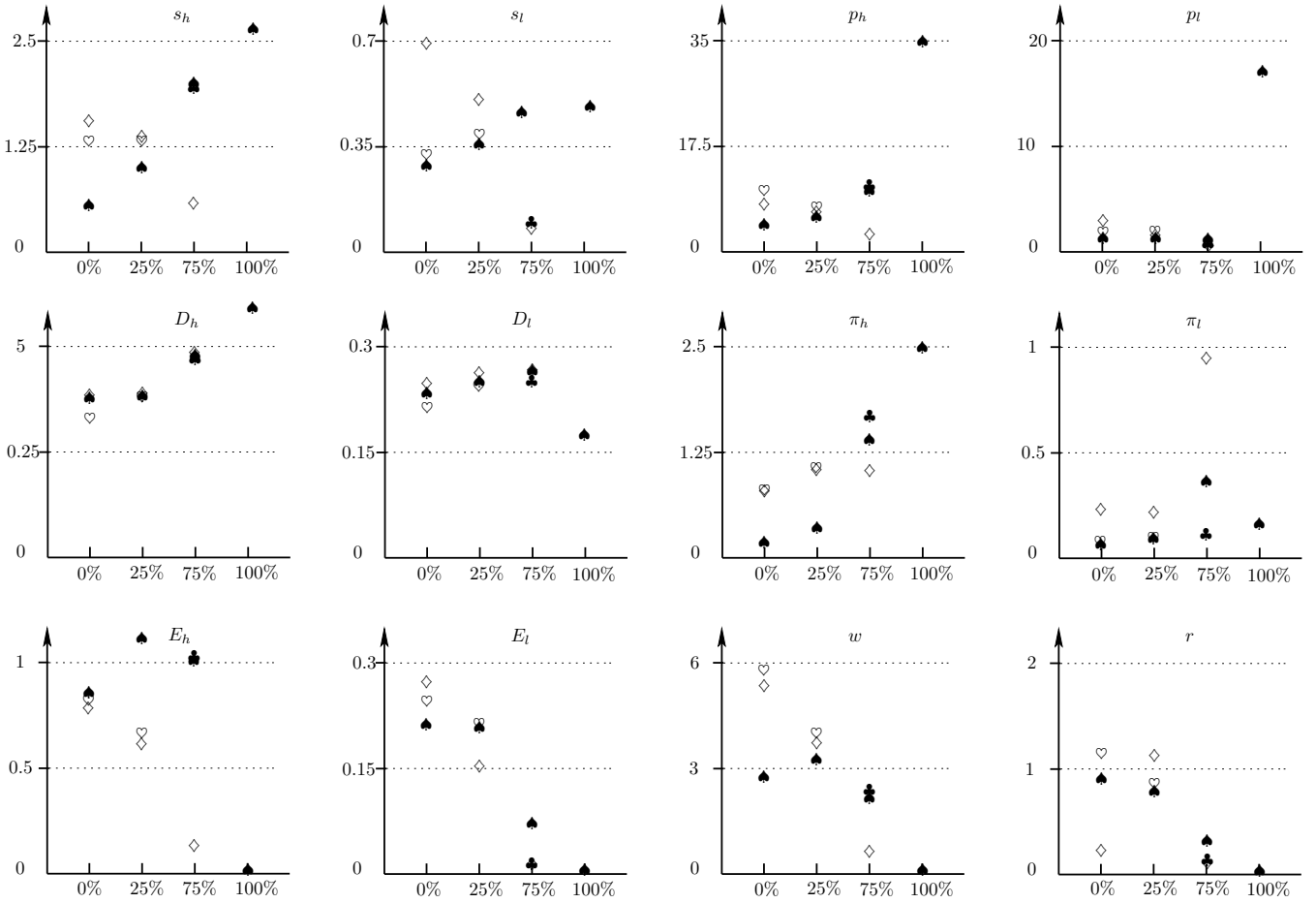


FIGURE 2. Equilibrium values under asymmetric bargaining powers. On the horizontal axis is represented the percentage of bargaining power attributed to the union. ♠ represents case f , \diamond represents case u , \heartsuit represents case uf and \clubsuit represents case fu .

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NOTES

¹Gabszewicz and Turrini (2000) consider a competitive labor market; thus, the wage rate for skilled workers is determined by the equality of supply and demand of skilled labour. The authors endogenize the decision *whether to acquire skills*, but the *skill level* remains exogenous. In a related paper, Bacchiega (2007) analyzes a non-competitive labor market and applies the Nash bargaining solution to determine the skilled workers' wage. In this paper, however, the process of skill acquisition is exogenous.

²Our main analysis is carried out for the case of right-to-manage bargaining. In section IV.5 we tackle the case of efficient bargaining.

³We assume that the quantity of labor that workers potentially supply is large enough to avoid rationing of consumers. A sufficient condition for this is that each union includes *at least* a unit mass of workers, because the total mass of consumers in the market is unit (see later in the paper). In this case, in fact, even if *all the consumers* only patronize a single firm, labor would be sufficient to satisfy the demand. Rationing in models of vertical differentiation raises several issues, see Boccard and Wauthy (2010).

⁴We use a model of vertical product differentiation, rather than of horizontal product differentiation, because the former setup allows us to link “product quality” with “workers' skills”. The vertical differentiation model, in fact, best suits the idea that different skill levels may be interpreted as different “qualities” in labor, which then are “transferred” into the characteristics of the final goods. This is in accordance with the vast empirical literature on the “quality transfer” from labor to products showing that countries with higher endowments in skills trade higher qualities in intra-industry trade (see the literature cited above). Moreover, recent empirical research shows that product quality increases with a firm's skilled-labor intensity, suggesting that firms that use their skilled workers relatively more intensively produce higher-quality goods (see Kugler and Verhoogen, 2012). By contrast, horizontal differentiation involves product features which are evaluated differently by consumers depending on their idiosyncratic preferences, and, according to our interpretation, the production of horizontally differentiated goods does not require the use of labor of different skill levels. A simple example may help clarifying this point. The presence of more sophisticated equipment in a car increases its quality (in the vertical differentiation sense) relative to the basic car model. Our idea is that to design and install sophisticated equipment, and in general, to produce a car equipped with it, requires highly skilled workers, while the production of the basic model requires less skilled workers. By contrast, the color of the car body does not influence the quality of the car, but only its attractiveness to specific groups of consumers, therefore cars differing only because of their color are horizontally differentiated. In this case the same worker may be tasked to paint the car any color.

⁵The present analysis, therefore, belongs to the class of Right-to-Manage oligopolized union models; see, e.g., Nickell and Andrews (1983). In Section IV.5 we delve into the case of Efficient Bargaining (McDonald and Solow, 1981). For a recent survey on unionized oligopolies, see Goeddeke (2010).

⁶In Section IV. we relax the assumption on the normalization of the mass of consumers.

⁷The assumption of a single-product monopolist, in our framework, is without loss of generality. See footnote 11 for a discussion.

⁸Alternatively, one could imagine that the training level defines the *maximal* attainable product quality, and that each firm then selects an *actual* quality no greater than that. In this case as well, each firm-union pair selects

different quality levels to avoid the Bertrand paradox. It is clear that, at equilibrium, any skill level “in excess” of that strictly necessary to produce the desired quality is a waste of resources.

⁹It should be noted that, although not directly, the variable costs of the firms *do depend* on the training levels. In fact, the skill of the workforce influences the wage rate, which, in turn, is the marginal production cost of the firm.

¹⁰Letting the firm and the union share the training cost is possible only when the investment is contractible. However, as Malcomson (1999, p. 2333) points out, this is a problematic assumption in labor markets. In fact, since training incentives differ between the firm and its union, the contracts are, in general, subject to opportunism. In such situations, Acemoglu and Pischke (1999b) show that, typically, one party only invests, namely that with the highest marginal return from training. These observations create a strong case in favor of the assumption that only one of the parties engages in training (see, e.g., Oosterbeek *et al.*, 2007). Nevertheless, as suggested by a Referee, in Section IV.6 we allow for the possibility that at the training stage the firm and the union share investment costs and jointly choose the training levels of the workforce.

¹¹Considering the alternative model where a multi-product monopolist serves the market with two variants of the good, high- and low-quality, by hiring skilled and unskilled workers from one union, does not affect the results of this section. Indeed, it is possible to show that the low-quality good would not be sold at equilibrium, which implies that no low-skill workers would be hired either, for *any skill/quality levels of the workers/goods* (see Acharyya, 1998 for the choice between pooling and separating menu in models of monopoly with vertical product differentiation). In this case the equilibrium profits of the monopolist and surplus of the union, gross of training costs, only depend on the quality/skill level of the high-quality good/high-skilled workers and have the same expressions as in the case of single-product monopoly. This entails that the optimal training levels coincide too. The proof is available upon request.

¹²The second order conditions of each problem are always satisfied.

¹³See Appendix H for some numerical examples of asymmetric bargaining. The monopoly analysis can be carried out by adopting the generalized Nash Bargaining solution because of its analytical easiness relative to the duopoly case. Yet, to provide comparable results, we have confined the monopoly analysis to the symmetric Nash bargaining.

¹⁴Each Nash product has only one point where the first-order derivative vanishes. At this point the (local) second order conditions are met, which guarantees the existence and uniqueness of a maximum.

¹⁵It can be demonstrated that, even if bargaining powers differ within each firm-union pair, the result that high-skilled workers earn a higher wage than low-skilled workers may be robust to the situation where the low-skilled union has greater relative bargaining power than the high-skilled union. The proof is available upon request.

¹⁶In Section IV.6 we explore some implications of contractible training investments.

¹⁷Appendix C thoroughly decomposes and describes the forces that enter condition (21).

¹⁸Appendix C contains a detailed analysis of the forces at stake.

¹⁹Freeman (1984), analyzing longitudinal data, confirms the finding of lower wage inequality in the union sector. In particular, Freeman documents that wage dispersion tends to fall when workers leave non-union for union jobs and to rise when they move in the opposite direction. Freeman (1991), using more recent longitudinal data, confirm that unionization reduces wage inequality. The author finds that declining unionization accounts for about 20 percent of the increase in the standard deviation of male wages in the U.S. between 1978 and 1988. More recently, a similar conclusion is also reached by Gosling and Machin (1994) and Card (1996).

²⁰As an example, s_l^{uf} is the training level of the low-skilled workers when the union (u) sets the high training level in one pair and the firm (f) sets the low training level in the other.

²¹Our modeling of the public training process implicitly assumes that the skill levels of workers still completely determines the quality levels of the products. Stated differently, firms can neither ask their workers to only partially use their skills to reduce the quality level of the final commodity, nor can they provide additional training to increase the quality level of the good with the aim of reducing price competition. One possible justification for this assumption is that, as we do not consider asymmetric information issues, the planner is able to verify the quality level of the final product and sanction the firms that under- or over-provide quality with respect to the socially optimal level.

²²It is instructive to this regard that, both under firm- and union-sponsored training, because of the competitive mechanisms presented above, the high-quality training level under duopoly is *larger* than the quality chosen in the corresponding training configuration under monopoly ($s_h^f > s_m^f$ and $s_h^u > s_m^u$) *but* the prices for the high-quality good are *lower* than the monopoly prices ($p_h^f < p_m^f$ and $p_h^u < p_m^u$).

²³All the calculations of the section are in Appendix F.

²⁴It may be proven that, in the configurations of pure firm- and union-sponsored training, letting the weights in the bargaining be asymmetric results in an increase of the training incentives of the agents with higher bargaining power and, conversely, in a proportional reduction of the incentives for the other agents. Numerical computations confirm this intuition for the mixed training configurations too. The proof is available upon request.

²⁵See Appendix G for calculation details.

²⁶Global concavity for firm h is satisfied for all $s_h > s_l$, while for firm l it can be demonstrated that local concavity at the optimal training levels is fulfilled.

²⁷In this case as well, the second-order conditions are satisfied for all $s_h > s_l$ for firm h , whereas local concavity is ensured for union l .

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