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Coupling Response Surface and Differential Evolution for Parameter Identification Problems

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Abstract: *In the present paper, a new surrogate-assisted evolutionary algorithm for dynamic identification problems with unknown parameters is presented. It is based on the combination of the Response Surface approach (the surrogate model) with Differential Evolution algorithm for global search. Differential evolution (DE) is a evolutionary algorithm where N different vectors collecting the parameters of the system are chosen randomly or by adding weighted differences between vectors obtained from two populations. In the proposed algorithm (called DE-Q), the Response Surface is introduced in the mutation operation. The new parameter vector is defined as the one minimizing the second-order polynomial function (the Response Surface, RS), approximating the objective function. The performances in term of speed rate are improved by introducing the second-order approximation; nevertheless, robustness of DE algorithm for global minimum search of objective function is preserved, since multiple search points are used simultaneously. Numerical examples are presented, concerning: search of the global minimum of analytical benchmark functions; parameter identification of a damaged beam; parameter identification of mechanical properties (masses and member stiffnesses) of a truss-girder steel bridge starting from frequencies and eigenvectors obtained from an experimental field test.*

1 INTRODUCTION

Model updating methods are widely employed to develop accurate models representing the real structure in the framework of optimization design (Sgambi et al., 2012), damage identification (Moaveni et al., 2008; Teughels and De Roeck, 2004) (Vincenzi et al, 2013), structural control (Bitaraf et al., 2012; Jiang and Adeli, 2008), and structural health monitoring (Doebling et al., 1996; Soyoz and Feng, 2009). The basic procedure of model updating is to adjust some parameters of the structure so that the model predictions agree as closely as possible with the measurements. In dynamic problems, the validation and updating of numerical models is mainly based on comparing modal parameters obtained numerically (from the model) and experimentally (from tests). Several model updating methods have been proposed in the past to reconcile FE models with modal data obtained from experimental modal analysis. Comprehensive reviews of classical structural methods can be found in (Maia and Silva, 1997; Mottershead and Friswell, 1993). Usually, they define an objective (cost) function, by comparing the numerical and experimental results in term of vibrations parameters (natural frequencies and mode shapes).

Due to the nonlinear relation between the vibration data and the physical parameters, an iterative optimization process must be defined. The success of the application of the updating method depends on the definition of the optimization problem and the mathematical capabilities of the optimization algorithm. Conventional gradient-based methods (as Newton and Quasi-Newton Algorithms) have an efficient convergence rate (at least in the case of few identification parameters), but they may reach local minima, depending on the starting vector adopted. Moreover, in gradient method, local curvatures of the objective function are used to define an approximate quadratic model function. Hence, gradient-based methods often fail (or low accuracy is achieved) due to ill conditioning of the optimisation problem when the objective function has low sensitivity to parameter variation close to the solution point (Friswell and Mottershead,

1995). For this reason, many authors focused the research on regularization techniques to reduce the ill-conditioning in model updating (D'Ambrogio and Fregolent, 1998; Natke, 1993; Tikhonov, 1995). Nevertheless, the conventional model updating procedures are usually computationally expensive for large-scale numerical models, especially for the calculations of the eigensensitivities.

Response Surface (RS) methodology might constitute a good alternative for solving model updating problems. By using RS, the relationships between the objective function to be minimized and the model parameters are explored by selecting a set of numerical simulations with the aid of the design of experiment technique (Khuri and Cornell, 1996). The major benefit is the significant reduction of the number required of numerical evaluations of the objective function (and, consequently, the computational effort). Originally introduced by Box and Wilson (1951) for chemical processes, RS method has gone through several stages of development that were aimed at making it more applicable to wider situations, covering many useful applications in a variety of fields (Myers and Montgomery, 1995). Some researches have been conducted into the direct application of RS to model updating (Fang and Perera, 2011). Uniform designs were employed by (Ren et al., 2011) to construct RS models for updating a box-girder bridge using the static deflection at the midspan as the object function to be minimized. The main disadvantage of using RS approximations is that, adopting a second-order approximation of the objective function, a local minimum instead of the global minimum can be reached when the objective function is not strictly convex. The use of the so-called general response surface method (GRSM) (Alotto et al., 1997) can partially solve this problem, but these approaches are applicable to low dimensional problems (few parameters to be identified) only.

Non-gradient methods, as evolutionary and genetic algorithms and artificial intelligence techniques (neural networks) (Titurus et al., 2003) have been widely utilized recently in optimization problems. Genetic and evolutionary algorithms are in general very robust and the starting point selection has a low influence on optimization results (Savoia and Vincenzi, 2008). Unfortunately, these methods have the disadvantage of requiring a large number of objective function computations, since they are based on probabilistic search without any information on the shape of objective function. The computational effort can then be very high if, for instance, for each computation a new FE model must be built and a dynamic solution obtained.

Several studies have been developed in the last decades to reduce the computational effort in Genetic and Evolutionary strategies (Khoo and Chen, 2001; Lin and Wang, 2012). Surrogate - assisted evolutionary strategies (Jin, 2011; Mueller et al., 2013) use efficient computational models, as response surface or high polynomial functions (Bach et al., 2012), to approximate the objective functions. They received considerably increasing interest in the recent years in reducing the computational effort in optimization problem, mainly when the computational simulation of the objective function evaluation is highly time consuming .

Several studies adopt a surrogate model approach combined with the objective function minimization (Jin, 2011); the strategy for properly managing the use of the surrogates (i.e. when and how to use the surrogate model) is a serious challenge and an efficient strategy still remains an interesting research topic.

In the present paper, the Response Surface (RS) methodology is combined with Differential Evolution (DE) algorithm to perform dynamic parameter identification. DE is a parallel direct search method where *NP* different vectors, collecting the parameters of the system, are used in the minimization process (Storn and Price, 1997). In the original version of DE algorithm, the vector population is defined randomly or by adding weighted differences between vectors obtained from two populations.

The DE algorithm has been used in (Savoia and Vincenzi, 2008) to perform dynamic structural identification and in (Savoia et al., 2009) for inverse analysis problems concerning derivation of parameters of material constitutive laws from experimental data. In (Hrstka et al., 2003), a comparison of DE with other types of evolutionary algorithms has been presented.

Surrogates approximations can be applied in almost all operations of evolution algorithms, such as the definition of the initial population, the local search or in objective function evaluations (Jin, 2011). In the proposed DE-Q method, the surrogate model is used in the mutation operation; the new parameter vector is defined as the minimizer of a second-order polynomial RS, approximating the objective function. The use of surrogates models in the mutation operation is almost uninvestigated; some preliminary result on the application of surrogates in mutation can be found in (Abboud and Schoenauer, 2001) where the Support Vector Machines with Gaussian kernels is used to approximate the objective function.

In several proposals, the surrogate function is used to approximate the objective function globally, adopting high order polynomials (Bach et al., 2012). Globally accurate approximations require several samples and generally have large computational complexity. Furthermore, high computational effort is needed to find the surrogate global minimum. Nevertheless, in dynamic parameter identification and model updating procedures, the interest about the shape of the objective function is only in the neighbourhood of the global minimum. Trying to obtain a good model over the whole search space can then be unnecessary (Lin and Wang, 2012). The use of a second order polynomial approximation function in the mutation operation can locally fit the objective function; close to the solution, the quadratic approximation gives high accuracy

and very high convergence speed. Moreover, when the objective function presents only one (global) minimum, the second-order approximation provides for the solution with a very low number of iterations. Nevertheless, introducing the RS second-order approximation in mutation of DE, robustness of DE algorithm for global minimum search is preserved, since multiple search points are used simultaneously. Adopting the proposed DE-Q algorithm, the performances in term of speed rate are improved and higher precision of results is obtained.

The paper is structured as follows. Section 2 describes the identification problem and the objective function to be minimized. Section 3 recalls some basic information concerning the Response Surface methodology. Section 4 presents the proposed optimization algorithm, and results concerning minimization of some analytical benchmark functions are illustrated in Section 5. Finally, in Sections 6 and 7 the results of model updating of two structures are described.

The first numerical example concerns a damaged beam, with two unknown parameters, the location and the stiffness (related with the crack depth) of the spring modelling the crack, to be identified. 100 minimization tests, starting from exact input data (in term of frequencies and eigenvectors), are performed and identification results are statistically presented, comparing the performances of classical DE and proposed DE-Q algorithms. Then, identification of mechanical parameters is performed starting from a set of pseudo-experimental data, simulating the statistical scattering of results due to measurement errors.

In the second example, parametric identification of masses and stiffnesses of an existing steel truss-girder bridge is presented. Experimental frequencies and mode shapes obtained by Operational Modal Analysis technique are used to identify mechanical properties of the bridge. The proposed method allows to identify the unknown parameters of a FE model matching the first six frequencies obtained from field tests.

2 PARAMETER IDENTIFICATION VIA OBJECTIVE FUNCTION MINIMIZATION

In general, FE model updating methods may be divided into two groups. The first group directly updates the individual elements of stiffness and mass matrices (Baruch and Bar-Itzhack, 1978; Berman and Nagy, 1983; Franco et al., 2006). Resulting updated matrices can reproduce very closely the structural modal properties, but often the matrix terms are not related with physically meaningful structural parameters. Due to these reasons, these methods are rarely used in model updating of real structures.

Methods included into the second group pre-select a set of unknown physical parameters and pose the model updating as an optimization problem. The updated parameters can be, for instance, structural or non-structural masses, elastic moduli and stiffnesses of external constraints. These models define an objective function to be minimized, based on the comparison between experimental results and predictions of the numerical model. Due to the nonlinear relationship between the structural response (for example, natural frequencies and mode shapes) and the physical parameters to be determined, an iterative optimization process is performed.

In dynamical problems, the objective function is often formulated as a weighted least-squares problem in which modal metrics, measuring the residuals between measured and model predicted modal properties, are build up into a scalar function. The possible objective functions can be grouped into three main categories, the frequency and mode shape residual (Teughels et al., 2002) and the modal flexibility residual (Jaishi and Ren, 2007). The modal flexibility residual was formulated in FE model updating for the purpose of damage detection. Moreover, other authors use advanced functions to improve the correlation in modal parameters. For instance, (Gentile and Cabrera, 2001) adopts the normalized modal difference (NMD) to define the mode shape correlation, whereas (Doebling et al., 1997) introduces a modal strain energy index as a residual for FE model updating.

In this paper, the aim is to find the optimal values of a set of physical parameters for dynamic identification purpose, and the numerical tests are performed by adopting, as input data, frequencies and/or mode shape vectors. First, the reference (or experimental) and the numerical modes are coupled by using the MAC (Modal Assurance Criterion) (Ewins, 2000). Then, the objective (cost) function H to be minimized during the identification procedure is defined as the relative error between modal frequencies and mode shapes obtained adopting a given set of identification parameters $(\omega_i, \boldsymbol{\varphi}_i)$ in the numerical model and the reference solution $(\overline{\omega}_i, \overline{\boldsymbol{\varphi}}_i)$ obtained experimentally, i.e.:

$$H(\mathbf{x}) = \sum_{i=1}^{N} \left[w_1 \left(\frac{\omega_i - \overline{\omega}_i}{\overline{\omega}_i} \right)^2 + w_2 \, NMD_i^{\,2} \right] \tag{1}$$

where $\mathbf{x}$ denotes the D-dimensional vector of unknown mechanical parameters and NMD is the so called "*Normalized Modal Difference*" defined as:

$$NMD_i = \sqrt{\frac{1 - MAC(\boldsymbol{\varphi}_i, \overline{\boldsymbol{\varphi}}_i)}{MAC(\boldsymbol{\varphi}_i, \overline{\boldsymbol{\varphi}}_i)}} \tag{2}$$

In Eq. (1), N is the number of mode shapes considered and w_1, w_2 are weight constants.

Other authors formulated the updating procedure in a multi-objective context (Paya et al., 2008) or defining a multicriteria optimization process (Rasma and Adeli, 2000).

Since the NMD is very sensitive to mode shapes changing, in all the examples presented in the paper w_1 is

always set equal to 1, while w_2 is assumed equal to 0.01.

3 THE USE OF RESPONSE SURFACE METHOD IN IDENTIFICATION PROBLEMS

The basic concept of the Response Surface (RS) method is to approximate the original complex or even implicit objective function H using simpler and explicit interpolation functions. The response surface method was originally proposed by (Box and Winson, 1951) as a statistical tool, to find the operating conditions of a chemical process at which some response was optimized. Subsequently, the use of RS methods has been extended to other fields, especially to engineering problems involving the execution of complex computer analysis codes. In this case, in fact, RS method can be used to alleviate the computational effort. Moreover, Khuri and Cornell (1996) provided modern perspectives of RS method applied to structural reliability analyses.

The basic idea of the RS method is that an objective function H can be approximated by an analytical estimation function $\hat{H}$:

$$\hat{H} = g(\mathbf{x}) \tag{3}$$

where $\mathbf{x}$ denotes the D-dimensional vector collecting the unknown parameters to be identified and $g(\mathbf{x})$ is the response function. Many methods are available to define $g(\mathbf{x})$. In classical RS methodology, the RS is obtained by combining first or second order polynomials fitting the objective function calculated in a set of sampling points. Second-order approximations are commonly used in structural problems due to the computational efficiency with acceptable accuracy. Higher order polynomials are rarely used because the number of coefficients to be determined strongly increases with the order. A proposal for multi dimensional functions with higher order polynomials can be found in Bach et al. (2012). Furthermore, some authors used quadratic polynomials without the cross terms, originating incomplete polynomials (Bucher and Bourgund, 1990).

Adopting a second-order approximation function, Eq. (4) can be written as follows:

$$\hat{H} = \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{L}^T\mathbf{x} + \beta_0 \tag{4}$$

where $\mathbf{Q}$ is a $D\times D$ coefficient matrix collecting the quadratic terms, $\mathbf{L}$ is a D-dimension vector and β_0 a constant.

Following the procedure proposed in (Khuri and Cornell, 1996), a limited number of selected numerical simulations (called experiments) is used in order to obtain an analytical relation between the values of identification parameters and the approximate objective function $\hat{H}$. Without loss of generality and for the sake of simplicity, in the following only 2 unknown parameters $\mathbf{x}=\{x_1, x_2\}$ are considered. Therefore, Eq. (5) can be written as follows:

$$\hat{H} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 \tag{5}$$

where coefficients β_i are unknown. If NS observations (i.e. NS evaluation of $\hat{H}$ starting from NS different vectors $\mathbf{x}$) are available, Eq. (4) gives:

$$\hat{\mathbf{H}} = \mathbf{Z}\cdot\boldsymbol{\beta} \tag{6}$$

in matrix notation, where vector $\boldsymbol{\beta}^T = \{\beta_0, \beta_1, ...\beta_5\}$ collects the unknown coefficients of the RS, $\mathbf{Z}$ is the matrix of components $\mathbf{Z}_i(\mathbf{x}_i)$ which contains the family of polynomials with constant, linear, quadratic and cross terms (Khuri and Cornell, 1996), and:

$$\hat{\mathbf{H}} = \begin{bmatrix} H_1(\mathbf{x}_1) \\ H_2(\mathbf{x}_2) \\ \vdots \\ H_{NS}(\mathbf{x}_{NS}) \end{bmatrix} \tag{7}$$

Typically, the number NS of observations is greater than the number of parameters collected in vector $\boldsymbol{\beta}$. Then, the latter is determined by applying the least square estimate method, so obtaining:

$$\boldsymbol{\beta} = (\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T\hat{\mathbf{H}}\,. \tag{8}$$

Eq. (10) is obtained by setting equal weight for all observation. However, according to the weighted regression method (Kaymax and McMahon, 2005; Myers and Montgomery, 1995), a RS with greater accuracy can be generated by increasing the weight of the points close to the solution point, so obtaining:

$$\boldsymbol{\beta} = (\mathbf{Z}^T\mathbf{W}\mathbf{Z})^{-1}\mathbf{Z}^T\mathbf{W}\hat{\mathbf{H}}\,. \tag{9}$$

where $\mathbf{W}$ is an $NS\times NS$ diagonal matrix of weight coefficients.

When the parameters $\boldsymbol{\beta}$ of the RS are determined, matrices $\mathbf{Q}$, $\mathbf{L}$ in Eq. (4) are assembled and the optimal vector $\mathbf{x}^*$ minimizing $\hat{H}$ is easily computed:

$$\mathbf{x}^* = -\mathbf{Q}^{-1}\mathbf{L} \tag{10}$$

Many algorithms have been proposed to select appropriate sets of sampling points $\mathbf{x}_k$, in order to obtain a response function better representing the objective function to be minimized. A detailed description of implementation and sampling strategies for RS method is given in (Khuri and Cornell, 1996).

A possible drawback of the use of a quadratic RS is that it predicts the presence of a single minimum also when the actual objective function presents more local minima. For this reason, in the proposed algorithm, described in the

following section, Response Surface methodology is used as surrogate model for Differential Evolution algorithm, in the mutation operation.

4 THE PROPOSED DE-Q METHOD

4.1 DE-Q Algorithm

The RS methodology is combined with a Differential Evolution algorithm to improve the performances in term of computational speed, but avoiding the convergence to local minima of the objective function.

Differential Evolution (*DE*) is a heuristic direct search approach where *NP* vectors (called *population*) are used simultaneously (Storn and Price, 1997).

The basic algorithm for DE is as follows. Each vector $\mathbf{x}_{i,G}$ contains a number *D* of optimization parameters, where subscript *G* indicates the *G*-th generation of parameter vectors. The number *NP* of vectors of the population is kept constant during the minimization process.

In order to minimize the objective function, a direct search method generates variations of parameter vectors. After the random selection of the initial population, for each vector DE generates a new parameter vector by adding to it the weighted difference vector between two other vectors of the population, so generating a third vector (the *mutant vector*). This operation is called *Mutation*. Then, in the *Crossover* operation, a new trial vector is generated by selecting some components of the mutant vector and some of the original vector. Once a variation is generated, a decision must be made whether or not to accept the new parameters. Only if the *trial vector* reduces the value of the objective function, the new generated vector replaces the old one (*Selection* operation). An exhaustive description of the basic DE algorithm and the rules of parameters involving in the procedure can be found in (Storn and Price, 1997). Applications to model updating and inverse analysis are described in (Savoia and Vincenzi, 2008; Savoia et al., 2009) (Vincenzi et al., 2013)

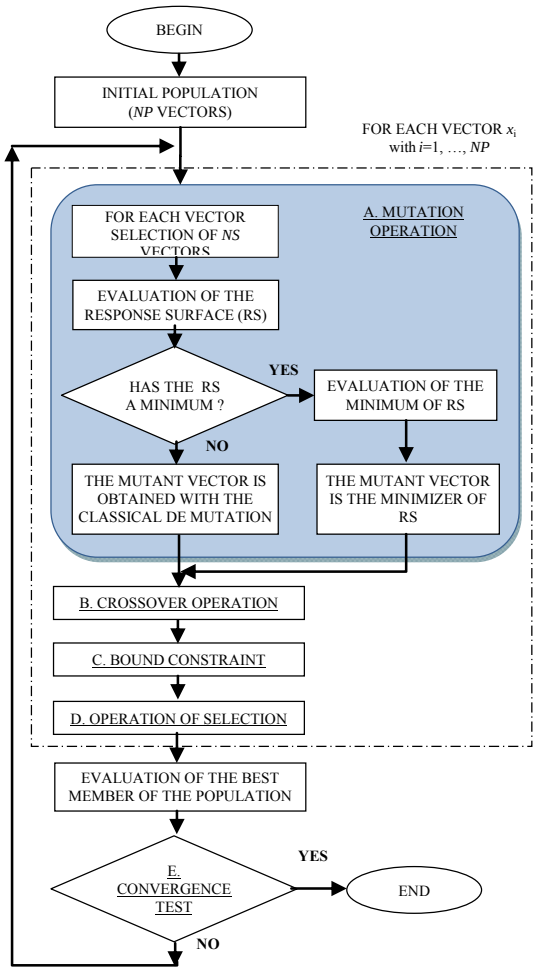


Figure 1 Flowchart of the proposed DE-Q algorithm.

Starting from the architecture of the basic DE algorithm, a modified DE-Q algorithm is proposed here. The *Mutation* operation is changed: when the new vector must be generated, a subset of vectors of the population is used to generate a quadratic RS as a local approximation of the objective function. If the RS is a convex surface, the new vector is selected as the minimum.

The algorithmic scheme to be followed for the generation of the new (*G*+1) population of search vectors is shown in Figure 1 and each phase is detailed in the following.

A . Mutation

For each vector of the *G*-th population:

$$\mathbf{x}_{i,G}, \quad i = 1,2,..., NP \tag{11}$$

a trial vector $\mathbf{v}_{i,G}$ must be generated. To do that, a subset of *NS* vectors (with *NS*<*NP*) is selected, containing the vector $\mathbf{x}_{i,G}$ and other *NS*-1 vectors randomly selected among the remaining vectors of the *G*-th population. The *NS* search vectors are used to calibrate an approximate RS, $\hat{H}$, fitting the cost function *H*, see Eq. (4). It is obtained by solving the linear system in Eq. (9) to obtain coefficients $\boldsymbol{\beta}$. For the weight coefficients, the following expression is used:

$$w_j = \exp\left(-\frac{g(\mathbf{x}_{j,G}) - H_{best}}{H_{best}}\right) \tag{12}$$

where $\mathbf{x}_{j,G}$ is the *j*-th vector of the subset of *NS* vectors and:

$$H_{best} = \min(g(\mathbf{x}_{j,G})) \tag{13}$$

The shape of the RS function is then checked, and two possibilities arise. If the approximation function is convex (Figure 2*a*,*b*), the new parameter vector is defined as the minimizer of the second-order polynomial approximation, i.e. (see Eq. (12)):

$$\mathbf{v}_{i,G} = \mathbf{x}^* = -\mathbf{Q}^{-1}\mathbf{L} \tag{14}$$

Otherwise (Figure 2*c*), classical Mutation operation based on linear combination of vectors of the same population is performed to obtain the trial vector $\mathbf{v}_{i,G}$, i.e.:

$$\mathbf{v}_{i,G} = \mathbf{x}_{r_1,G} + F \cdot (\mathbf{x}_{r_2,G} - \mathbf{x}_{r_3,G}) \tag{15}$$

where $r_1, r_2, r_3 \in \{1,2,...,NP\}$ are mutually different integer numbers. Moreover, F is a positive constant (scale parameter) controlling the amplitude of the mutation. The scale parameter F is taken smaller than 2. In all example presented in the following, F=0.6 is set.

NS must be large enough to assure that the least squares problem is determined but smaller than *NP*, in order to preserve the diversity of the RSs and the new members of the next population.

B . Crossover

In order to increase the diversity of the vectors, the crossover operation defined in the original DE algorithm is maintained. Accordingly, the *trial vector* $\mathbf{u}_{i,G}$ is obtained by randomly exchanging the values of optimization parameters between the original vectors of the population $\mathbf{x}_{i,G}$ and those of mutant population $\mathbf{v}_{i,G}$, i.e.:

$$\mathbf{u}_{i,G} = (u_{1i,G}, u_{2i,G}, ..., u_{Di,G}) \tag{16}$$

where:

$$u_{ji,G} = \begin{cases} v_{ji,G} & if\ rand(j) \le CR \\ x_{ji,G} & if\ rand(j) > CR \end{cases} \tag{17}$$

In Eq. (17), $j = 1,2,...,D$, where D is the number of optimization parameters, and u_{ji} is the j-th component of vector $\mathbf{u}_i$. Moreover, *rand*(j) is the j-th value of a vector of uniformly distributed random numbers in [0, 1], and *CR* is the crossover constant, with 0 < *CR* < 1. Constant *CR* indicates the percentage of mutations considered in the trial vector. In all applications presented in the following, CR is set equal to 0.5.

C. Bound constraints

Usually, in engineering applications, the optimization parameters are constrained to belong in given intervals, i.e.:

$$x_{ji,G} \in \left[x_j^{\min}, x_j^{\max}\right] \tag{18}$$

where $j = 1,2,...,D$. Introducing the bound constraints for the optimization parameters is useful in order to restrain the analysis to ranges of unknown parameters which are meaningful from the physical point of view. To this purpose, a projection algorithm is introduced. After the mutant operation, if a vector out of range is obtained, its projection on the prescribed interval of parameters is calculated and considered as the new vector (see (Savoia and Vincenzi, 2008) for details).

D. Selection

In order to decide if a vector $\mathbf{u}_i$ may be element of the new population of generation G+1, each vector $\mathbf{u}_{i,G}$ is compared with the previous vector $\mathbf{x}_{i,G}$. If vector $\mathbf{u}_{i,G}$ gives a smaller value of objective function H than $\mathbf{x}_{i,G}$, $\mathbf{u}_{i,G}$ is selected as the new vector of population G+1; otherwise, the old vector $\mathbf{x}_{i,G}$ is retained:

$$\mathbf{x}_{i,G+1} = \begin{cases} \mathbf{u}_{i,G} & \text{if } H(\mathbf{u}_{i,G}) < H(\mathbf{x}_{i,G}) \\ \mathbf{x}_{i,G} & \text{if } H(\mathbf{u}_{i,G}) \ge H(\mathbf{x}_{i,G}) \end{cases} \tag{19}$$

with $i = 1,2,...,NP$.

E. Convergence rule

In the convergence rule, the values of the objective function obtained from the population G+1 are compared. Vectors $\mathbf{x}_{i,G+1}$ are ordered depending on values of objective function as:

$$\tilde{\mathbf{x}}_{1,G+1} \prec \tilde{\mathbf{x}}_{2,G+1} \prec ... \prec \tilde{\mathbf{x}}_{NP,G+1} \tag{20}$$

such that:

$$H\left(\tilde{\mathbf{x}}_{1,G+1}\right) < H\left(\tilde{\mathbf{x}}_{2,G+1}\right) < ... < H\left(\tilde{\mathbf{x}}_{NP,G+1}\right) \tag{21}$$

Convergence rule is then based on the difference of values H of the objective function of the first *NC* vectors and the distances between the same vectors, *NC* being the number of controlled vectors. The first convergence test can be expressed as:

$$\Delta_i^H = \frac{\left|H(\tilde{\mathbf{x}}_{i,G+1}) - H(\tilde{\mathbf{x}}_{i+1,G+1})\right|}{\left|H(\tilde{\mathbf{x}}_{i,G+1})\right|} < VTR_1 \tag{22}$$

where $i = 1,...,NC$ and VTR_1 is the prescribed precision.

Control of values of objective function H only can be not sufficient when the objective function has low sensitivity close to the minimum solution. For this reason, convergence requires also that the relative distance between the individual components of the first *NC* vectors be small, i.e.:

$$\Delta_{ij}^x = \frac{\left|\tilde{x}_{ji,G+1} - \tilde{x}_{ji+1,G+1}\right|}{\left|\tilde{x}_{ji,G+1}\right|} < VTR_2 \tag{23}$$

Convergence requirement also on each component of the vector is very important when more local minima with similar values of the objective function are present (see Section 5).

4.2 Discussion

It must be recalled that, in civil engineering updating problems, the shape of objective function is usually unknown and may have more than one minimum,

especially when model updating requires the modification also of the geometry of the model, as in the case of damage detection problems. Then, the use of local search technique (e.g., classic gradient-based algorithms) is often inappropriate. On the other hand, in real problems, usually the objective function has only few minima, especially if the identification problem is properly defined, together with the variation intervals of the pre-selected physical parameters to be identified.

Another important issue concerns the number of unknown parameters to be identified. With more than 3-4 unknowns, gradient-based algorithms have a very low sensitivity to parameter variations, and convergence often fails. On the other hand, techniques like genetic and evolution algorithms are based on a calculation involving several vectors simultaneously. Hence, if some vectors reach local minima, they can be excluded because they are associated with higher values of the cost function. Nevertheless, basic genetic and evolution algorithms require a large number of objective function computations, since they are based on probabilistic search without any information on the shape of objective function. Moreover, after having identified the global minimum, evolutionary algorithms require several computations to obtain the prescribed precision, because no information about the shape of the cost function is introduced.

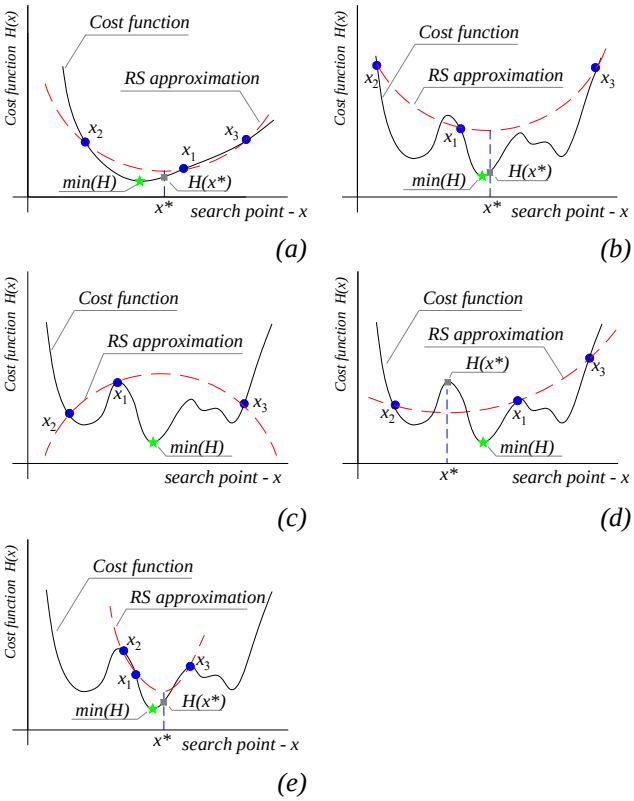


Figure 2 Approximation of cost function by quadratic response surface, with *NS*=3.

In this context, the proposed DE-Q algorithm combines the advantages and versatility of direct search algorithms and the shape approximation technique like RS. If the objective function presents only one (global) minimum, second-order approximation provides for the solution in a very low number of iterations. On the other hand, even if local minima are present, global minimum is expected to be reached, since multiple search points are used simultaneously. Moreover, if the quadratic approximation fails (i.e., it is not a convex function), classical Mutation operation is used (Figure 2*c*) whereas, if the minimum of second-order approximation gives a higher values of the objective function (Figure 2*d*), the new vector is rejected in the Selection operation (i.e., the old vector $\mathbf{x}_{i,G}$ is retained). Close to the solution (Figure 2*e*), the second-order function is expected to approximate quite well the objective function, so that the convergence can be reached in a very limited number of iterations.

For all these reasons, the performances in term of convergence speed rate are expected to be significantly improved by introducing the second-order approximation, but maintaining the capability of the original DE algorithm to reach the global minimum of the objective function.

Numerical performances of DE-Q algorithm will be shown and discussed in the following, with reference to some analytical benchmark functions first, and then to two dynamical identification problems.

5 ANALYTICAL BENCHMARK FUNCTION MINIMIZATION PROBLEMS

In order to assess the numerical performances of DE-Q algorithm, the problem of finding the global minima of two benchmark functions is studied first. These functions are often used to test the performance of optimization algorithms and their mathematical formulas and properties are described in the following.

5.1 Benchmark function no. 1: comparison between DE and DE-Q.

The benchmark function No. 1 is a two dimension non-convex function defined by the equation:

$$H(x_i) = 5.233 + 0.01\sum_{i=1,2}(x_i - 0.5)^4 - 30x_i^2 - 20x_i \qquad (24)$$

with $-6.0 \le x_i \le +6.0$ (i =1, 2). The 3D plot of the objective function is shown in Figure 3.

The function has three local minima and one global minimum, the latter placed at $\mathbf{x}^*$ = (-4.454, -4.454) (Alotto et al., 1997). The convergence to the global minimum is a difficult task also for global search methods, due to the similar values of the objective function at the four local minima.

The performances of the proposed DE-Q algorithm and the original DE algorithm are compared. For each

benchmark function, 100 optimization tests are performed, starting from different sets of 15 initial search vectors selected randomly in the intervals of definition of unknown variables. A number of sampling NS = 8 is chosen and constants F and CR are set equal to 0.6 and 0.5, respectively.

The statistical analyses of results are reported in Table 1. Being the convergence to the solution and the number of iterations required strictly correlated, the same convergence criteria are adopted for the two algorithms. The values adopted for the convergence constants are $VTR_1 = 10^{-3}$ and $VTR_2 = 10^{-2}$ (see Eqs. 24-25).

In Table 1, for the unknown parameters obtained at the end of the process, the mean values and the coefficients of variation (C.V.) of all vectors obtained at convergence from 100 tests performed are reported. For both algorithms, the mean values are very close to the reference values with small coefficients of variation. DE-Q algorithm exhibits better performances than original DE in term of speed rate and precision of results. The average number of iterations required for convergence is 40% smaller than adopting the original DE algorithm but, above all, the C.V. is one order of magnitude smaller: in fact, once the global minimum is detected, all the vectors rapidly converge close to the exact solution.

The better convergence properties can be clearly verified by comparing the values of the objective function during the optimization process and the position of search points at each iteration. A typical example is shown in Figure 4, 5, 6 For each iteration (from 1 to 20), the errors of the objective function of all search points are reported in Figure 4. During the first few iterations, both DE and DE-Q algorithms are involved to find the global minimum, and the objective function values are comparable. Then, after 4-5 iterations, the RS applied to global search strongly improves the speed performances of DE-Q, and the global minimum is reached with a lower number of iterations and a much higher precision.

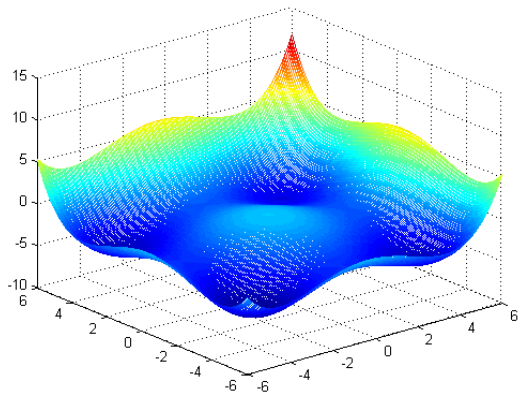
The contour plot of the objective function with indication of the distribution of search vectors at iterations n. 1,5,10, 15 are shown in Figure 5-6. Note that, at the beginning of the process, all points are randomly spread on the research space (Figure 5*a* and 6*a*).

After five iterations, the search points are grouped close to the global minimum and, sometimes, to some local minima. Then, the remaining iterations are needed to select the global minimum and to attain the prescribed convergence precisions. Due to the second-order approximation of the objective function, in this phase DE-Q gives much better results, bringing together the search vectors close to the global minimum much faster than DE.

Finally, a sensitivity analysis is performed to investigate the role of the parameter NS with respect to the total number of search vector NP in DE-Q.

15 and 30 search vectors are considered. To calibrate a complete quadratic approximation, a minimum of 6 parameters is needed; otherwise the least squares problem is underdetermined. However, the relation $NS < NP$ must be satisfied to preserve the diversity of the calibrated surfaces. Thus, the number of sampling to construct the quadratic approximation is selected from 6 to 24, i.e., the 80% of NP.

Results are shown in Figure 7. Adopting 15 search points, convergence is reached in 100-130 evaluation of the objective function; increasing the number of search points NP, the number of evaluations increases. Roughly, the number of evaluations increases approximately in proportion to the number of search vectors. However, no considerable differences arise by varying NS. Even if this is



(b)

Figure 3 Benchmark no. 1: 3D plot.

Table 1

Statistical results – Benchmark No. 1

	Ref. value	*DE*		*DE-Q*	
		Mean	*CV%*	*Mean*	*CV%*
x_1	-4.454	-4.452	0.77	-4.454	0.05
x_2	-4.454	-4.450	0.62	-4.453	0.06
Iterations	-	19.10	20.12	11.06	19.02
Failed test	-	4	-	3	-

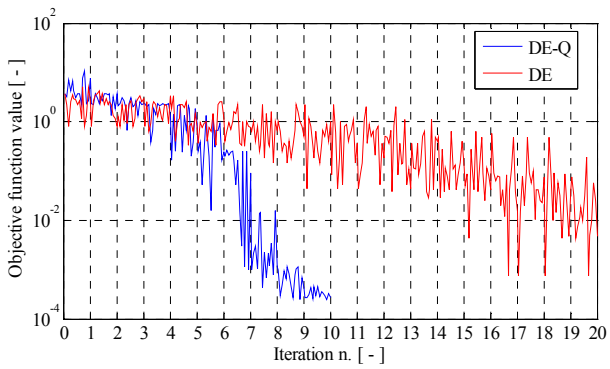


Figure 4 Benchmark No. 1: values of the objective function vs. Iteration number.

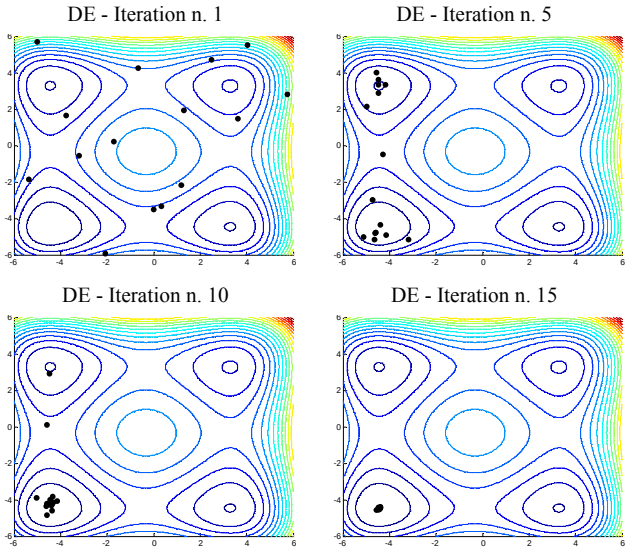


Figure 5 Benchmark No. 1 – Search vectors at iterations n. 1, 5, 10, 15 – DE algorithm.

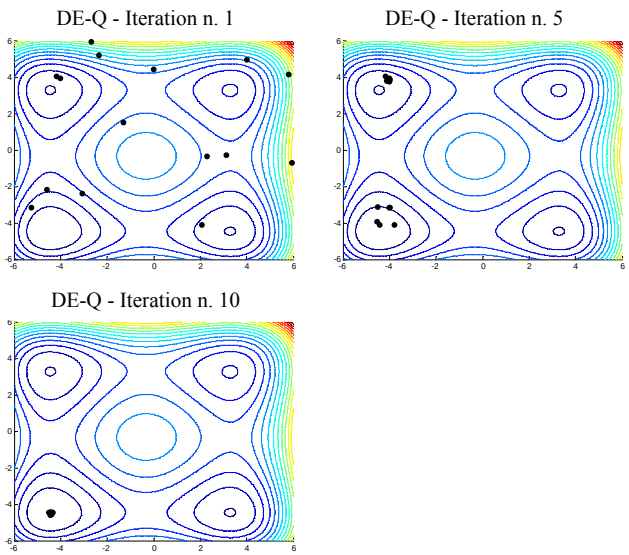


Figure 6 Benchmark No. 1 – Search vectors at iterations n. 1, 5 and 10 – DE-Q algorithm.

only an example, in many computational experiments performed with the proposed algorithm it is confirmed that results are almost insensitive to changes in *NS*, for a given *NP*.

5.2 Benchmark function no. 2: comparison of DE with other global search techniques.

The benchmark function No. 2 is known as Shifted Ackley's Function. Ackley's problem requires to find the parameter vector $\mathbf{x} = [x_1,\ldots, x_i, \ldots, x_D]$ minimizing the following equation (Ackley, 1987):

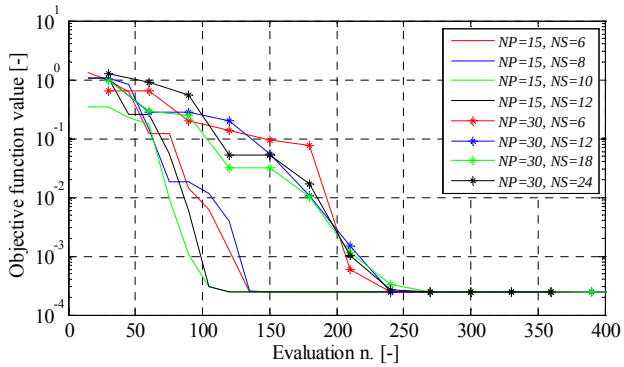


Figure 7 Benchmark No. 1 – Sensitivity of results with respect to *NP* and *NS*.

$$H(x_i) = 20 - 20 \cdot e^{\left[-0.2\sqrt{\frac{1}{D}\sum(x_i-1)^2}\right]} - e^{\left[\frac{1}{D}\sum\cos(2\pi(x_i-1))\right]} + e \quad (25)$$

where D indicates the problem dimension.

First, $D = 2$ is considered. The global minimum is given by $\mathbf{x}^* = (1.0, 1.0)$. The solution is searched in the range [-2.0, +2.0] for both variables x_1 and x_2. In the selected range, the function presents 24 local minima, 16 of them on the boundary (see Figure 8).

Benchmarck function No. 2 is used to compare the performance of the proposed DE-Q algorithm with those of other global optimization techniques, i.e.:

- the (original) DE algorithm;
- the Genetic Algorithm (GA) implemented in the Matlab "Genetic Algorithm and Direct Search Toolbox" (The Mathworks, 2011);
- an improved surrogate model algorithm developed by Mueller (2012), also implemented in the Matlab "Modularized Surrogate Model Toolbox" (SMT - (Mueller, 2012).

The Matlab GA toolbox implements a standard but robust algorithm where the selection operation is applied at first, the crossover is then used to combine two vector

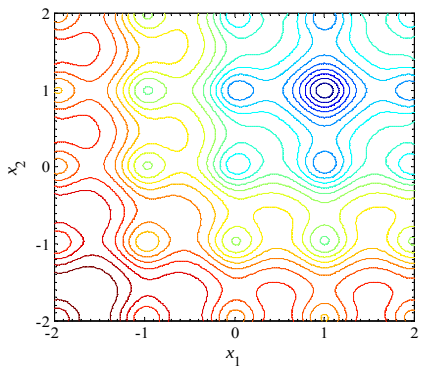


Figure 8 Contour map for the Ackley's Function (benchmark analytical function No. 2) for $D = 2$.

(parents) to form new ones (children) for the next generation and, finally, the mutation applies random changes to parents to form children.

The SMT collects several surrogate models such as Kriging models with Gaussian, linear or spline correlation function, polynomial regression models or cubic radial basis interpolation functions. The toolbox allows the user to choose also between different mixed models.

First, DE and DE-Q are compared considering the results obtained over 100 tests. The same parameters indicated for Benchmark No. 1 are used in DE-Q and DE algorithms. The values adopted for the convergence constants are $VTR_1 = 10^{-4}$ and $VTR_2 = 10^{-3}$.

Values of the objective functions vs. iteration number over 100 tests are shown in Figure 9. Each line collects the best objective function values for each iteration (i.e. the objective function value of the best member of each generation). Upper and lower envelopes (dashed lines inFigure 9) clearly show that DE-Q algorithm significantly improves the speed rate: for a given iteration, the objective function value is lower than the one obtained by DE, with exception only for the first iterations.

In Figure 10, the comparison with GA and SMT is shown. For each algorithm, a typical run is reported. For all algorithms, 15 search vectors are used, and default options are chosen for both GA and SMT algorithms. Due to the different approaches characterizing the optimization algorithms, the total evaluation of the objective function is considered instead of the iteration number.

The standard Genetic Algorithm provides for results with a very low convergence rate: for several evaluation, the objective function value associated to the best member of the generation remains almost unvaried. On the contrary, the improved SMT gives higher accuracy than other algorithms, with a speed rate (in terms of number of function evaluations) about 4 times greater than DE-Q. Nevertheless, the SMT adopts a very complex algorithm (based on higher polynomials) to manage the surrogate model, so requiring much more time at each iteration to decide the candidate to be used in the further generation. In this example, DE-Q requires an average time of 0.4 seconds to reach the global minimum, while SMT needs about 1 minute to obtain the same precision in the minimum search.

To validate the results for a large dimension problem, the Ackley's Function in Eq. (25) with $D = 15$ is finally considered. To calibrate a complete quadratic approximation in DE-Q, 136 parameters must be estimated (15 parameters for the linear terms, 15 for the quadratic terms, 105 for the cross term and one constant), and so, a minimum of 136 search vector is needed. To obtain an over-determined least square problem, a number of vectors $NP = 170$ is selected. The same number is also adopted for all other algorithms. Default options are chosen for both GA and SMT algorithms.

The values adopted for the convergence constants are

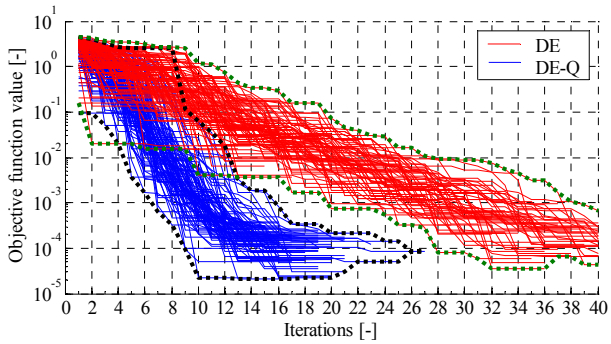


Figure 9 Benchmark No. 2: values of the objective function vs. iteration number over 100 tests.

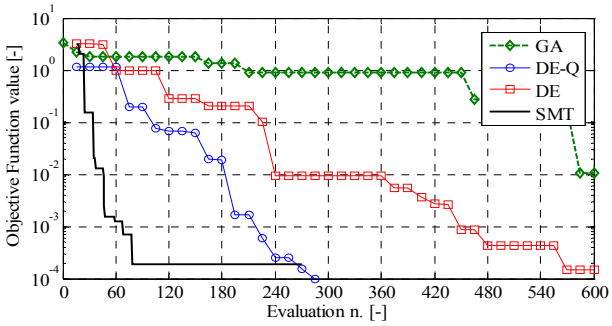


Figure 10 Benchmark No. 2: values of the objective function vs. number of function evaluations.

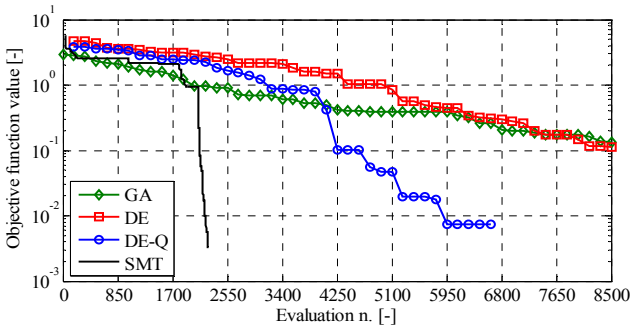


Figure 11 Ackley's Function for $D = 15$ and NP=170: values of the objective function vs. number of function evaluations.

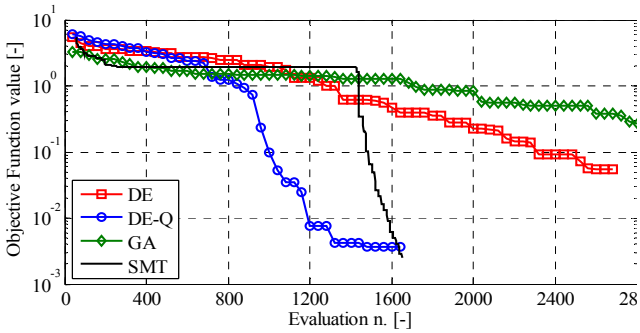


Figure 12 Ackley's Function for $D = 15$ and NP=40: values of the objective function vs. number of function evaluations.

VTR_1 = VTR_2 = 10^{-2}; moreover, a maximum of 8500 evaluations is prescribed for all algorithms (i.e. 50 iterations for DE and DE-Q). Results reported in Figure 11 show comparable results for all algorithms for the first 1700 evaluations (10 iterations); after that, the SMT algorithm strongly improves its performances matching the global minimum with very few evaluations. The worst performances of DE-Q in terms of speed rate are due to the very high number of vectors needed to guarantee the over-determined least square problem and to built a complete quadratic function. To reduce the number of vectors, an incomplete quadratic function is then considered. Eliminating the cross terms in RS, the number of unknown parameters goes drastically down to 31. Consequently, the number of search vectors is set equal to 40. The same convergence constant values of the previews analysis are used. The results, shown in Figure 12, indicate a strongly improvements of DE-Q, with a number of evaluations comparable with SMT and, consequently, a much smaller computational cost due to the reduced time required for each function evaluation.

The last comments concern the computational time required to find the global minimum with the accuracy prescribed. In the last example, only few seconds are required to DE-Q to reach the global minimum, and about two times for DE and GA. On the contrary, in spite of the reduced number of iterations, SMT requires more than 10 hours to find the solution, mainly due to the complexity of the managing algorithm. Thus, SMT is adequate only for very expensive computational optimization problems, while DE-Q can be efficiently used also in common updating procedures.

6 CASE STUDY No. 1 – CRACKED BEAM

In order to test the numerical performances of DE-Q algorithm, the damage assessment of a simply-supported, cracked aluminum beam is performed by FE model updating. The identification problem has two unknown parameters, the location and the stiffness (related with the crack depth) of the spring modeling the crack. Dynamic modal data are used to identify the damage, i.e., frequency and mode shape (measured in a number of point along the beam) of transverse modes. For details about the cracked beam model, see (Chondros et al., 1998; Vincenzi et al., 2013).

The choice of this simple problem is justified by the presence in the objective function of local minima even if only two unknown parameters are considered in the identification problem (see Figure 13).

The reference solution is analyzed at first. The beam has a rectangular cross-section 6×25.4 (b×h) mm and length l=235 mm. The material properties are: Young's modulus E = $7.2 \cdot 10^4$ MPa, density 2800 kg/m^3 and Poisson's ratio 0.35. The beam has one crack, with depth 9.7 mm, placed at 2/3 of the length l. The cost function reported in Eq. (1) is used, where the first 2 natural frequencies and mode shapes (measured in 12 sections along the beam) are used, and weight coefficients are set w_1=1 and w_2=0.01.

The search space of the two updating variables, the stiffness $X_1 = K$ and the position $X_2 = y$, is limited to the intervals]7000, 35000[Nm, and]0,235[mm, respectively. For numerical convenience, the normalized value x_j (with j=1,2) of the identification parameter X_j is defined as:

$$x_j = \frac{X_j - X_{jS}}{X_{jD}} \quad \Rightarrow \quad X_j = x_j X_{jD} + X_{jS} \tag{26}$$

where:

$$X_{jS} = \frac{X_{j2} + X_{j1}}{2}, \quad X_{jD} = \frac{X_{j2} - X_{j1}}{2} \tag{27}$$

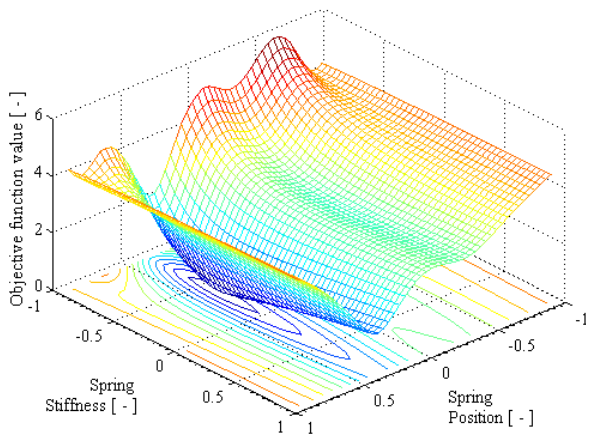


Figure 13 Case study No. 1: The objective function vs. spring stiffness and position.

Table 2
Case study No.1: results adopting exact input data

	Ref. value	*DE*		*DE-Q*	
		Mean	*CV%*	*Mean*	*CV%*
Position x_1	+0.333	+0.334	0.45	+0.334	0.11
Stiffness x_2	-0.385	-0.386	3.29	-0.385	2.57
Iterations	-	18.84	19.5	9.82	20.4
Failed test	-	0	-	0	-

Table 3
Case study No.1: results adopting pseudo-experimental input data

	Ref. value	*DE*		*DE-Q*	
		Mean	*CV%*	*Mean*	*CV%*
Position x_1	+0.333	+0.333	6.58	+0.334	6.60
Stiffness x_2	-0.385	-0.341	64.0	-0.347	60.7
Iterations	-	18.86	21.3	7.80	14.8
Failed test	-	0	-	0	-

are the mean value and width of the interval of variation of X_j, so that the normalized parameters vary in the range] -1, +1[. In Eq. 29 X_{j1} and X_{j2} are the upper and lower bound of the search intervals, respectively. The reference solution is so given by **x***= (+0.333, -0.385).

Number of search vector is set to 15 and the number of sampling to calibrate the RS is NS = 10. Values adopted for constants governing the DE mutation and crossover operations are F = 0.6 and CR =0.5, while the convergence constants are $VTR_1 = 10^{-3}$ and $VTR_2 = 10^{-2}$.

In a general identification problem, the estimate of the optimal model is sensitive to: a) uncertainties due to limitations of the adopted numerical models to represent the behavior of the real structure; b) the presence of uncertainty in modal parameters adopted as input in identification procedure, mainly due to noise in experimental measurement and the processing errors in estimating the modal data. The first type of errors is commonly called "model error", while the last is denoted in the following as "input error".

6.1 Exact input data

The numerical tests have been first performed by adopting, as input data, exact values of frequencies and mode shape vectors (no input error).

In order to verify the robustness of the DE-Q algorithm to obtain the unknown parameters starting from the modal data, 100 optimization tests have been performed. The number of vectors for each population is NP = 15, and the values adopted for the convergence targets in Eqs. (22-25) are $VTR_1 = 10^{-3}$ and $VTR_2 = 10^{-2}$. The results are then compared with those obtained by the DE algorithm.

Statistical analysis of identification results obtained from numerical tests is reported in Table 2. For both algorithms, the mean values and coefficients of variation (C.V.) of the updating parameters (all vectors obtained at the end of 100 tests) are reported. Having adopted the same convergence rules, similar results in terms of accuracy of the solution are obtained from the proposed DE-Q and the original DE algorithm: mean values of spring position and stiffness are very close to the expected values in both cases. As for the C.V.s, results obtained via DE-Q are more accurate, especially as far as the crack position is concerned (0.11% for DE-Q versus 0.45% for DE). As for the computational performances, the number of iterations required for convergence of DE-Q algorithm is about half of that required by the original DE.

The values of identification parameters (spring position and stiffness) and objective function H, for each search point during convergence to the solution, are reported in Figure 14-12. The better performances of DE-Q algorithm are evident: during the very first iterations, both DE-Q and DE populations explore the search domain (values of parameters in the range]-1,+1[ ) and encounter very high values of objective function. Then, after 3-4 iterations, the

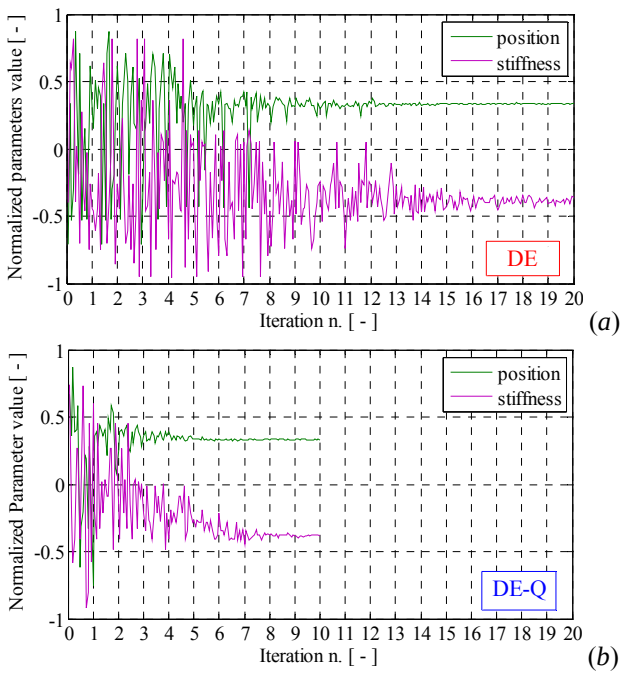


Figure 14 Case study No. 1: parameter values vs. Iteration number – (*a*) DE and (*b*) DE-Q algorithm.

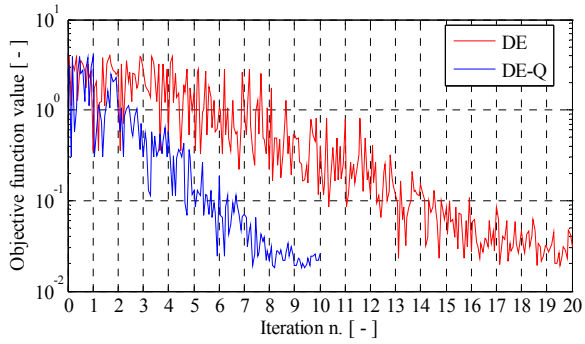


Figure 15 Case study No. 1: objective function of search points vs. Iteration number.

second-order approximation of DE-Q algorithm becomes very effective, and the convergence rate is strongly improved up to final convergence.

6.2 Pseudo-Experimental input data

The same problem is now faced by adopting pseudo-experimental data, in order to simulate noise in experimental measures (input error). Pseudo-experimental data have been obtained by multiplying exact values of frequencies and components of mode eigenvectors by uncorrelated coefficients, extracted from normal probability distributions with unit mean value and C.V. equal to 5 percent for frequencies and 10 percent for eigenvector components.

Statistical results from 100 identification tests are

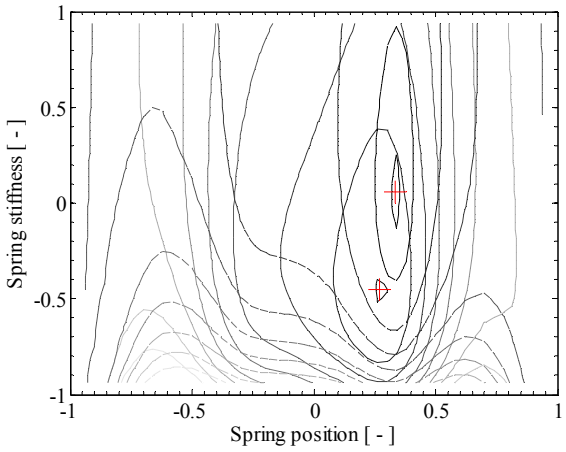


Figure 16 Case study No. 1: two objective functions starting from 2 different sets of pseudo-experimental input data (frequencies and mode shapes).

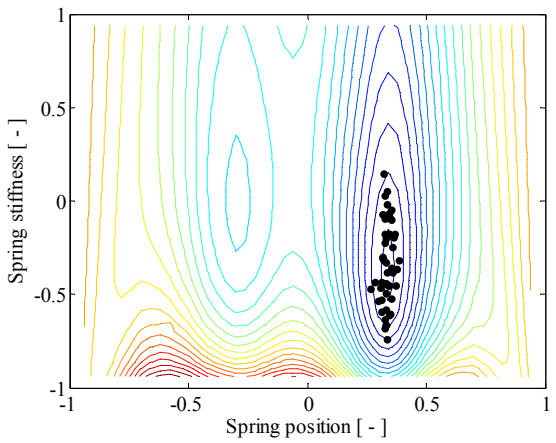


Figure 17 Case study No. 1: Identification results from different sets of input pseudo-experimental data, compared with contour lines of the reference objective function (exact input data).

reported in Table 3. In this case, the high dispersion of identification results (C.V. about 6.5% for spring position and 60% for spring stiffness, for both algorithms) is due to the random error of input results. In fact, changing the input modal parameters, the position of the global minimum of objective function will change (see, for instance, Figure 16 where 2 contour plots of the objective functions obtained starting from 2 different sets of input data are shown). The global minima of several identification tests are given in Figure 17, confirming the high sensitivity of the spring stiffness to uncertainties in the input data (frequencies and mode shapes). Note that the average value of the spring stiffness is off by 10% from the exact value even if the noise on frequencies and mode shapes is centered. This is due to the nonlinear correlation between modal parameters and stiffness. Finally, the number of iterations required for convergence and reported in Table 4 confirms the better

Figure 18 Case study No. 2: Old and new Pontelagoscuro truss-girder viaducts.

performances of DE-Q algorithm, the iterations required being reduced by a factor of 2.5 with respect to DE algorithm.

7 CASE STUDY No. 2 - STEEL-TRUSS GIRDER BRIDGE

Case study No. 2 refers to a real case study, the parametric identification of masses and stiffnesses of a steel truss-girder bridge (see Figure 18), starting from experimental frequencies and mode shapes. In this case, the identification problem is affected by both “input error” and “model error”, the latter due to the modelization of the structure by FE method.

The Pontelagoscuro viaduct is a steel viaduct on the Bologna –Venice railway line (Italy). It is composed by 2 distinct parallel steel railway viaducts crossing the Po river (Figure 18). The first viaduct was built in 1948 and is composed by 9 single span truss-girder bridges.The 5 inner spans are 75 meters long each, while the 4 end spans are

Table 4
Case study no. 2: Identified frequencies and mode shapes

Mode n.	*Experimental Frequency [Hz]*	*Mode shape*
1	2.143	1st lateral
2	3.857	1st vertical
3	4.307	1st torsional
4	4.700	2nd lateral
5	7.907	2nd torsional
6	8.139	3rd lateral

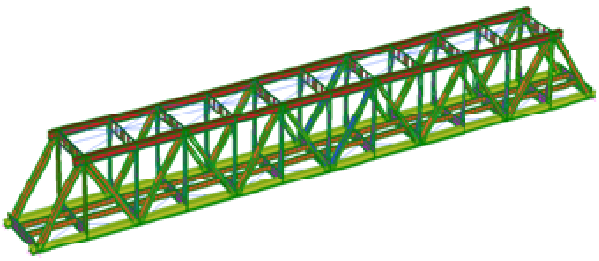


Figure 19 Case study No. 2: Finite Element Model of an end span of the old Pontelagoscuro bridge.

about 60 meters long. Two main truss-girders support the vertical load, while X-shaped upper and lower lateral bracings assure the transverse stability. The upper and the lower chord, the diagonals and the stunts are composed of 4 L-shaped steel elements, riveted together by means of plates. Stiffening plates are introduced in order to improve the transverse stiffness of the bridge box cross-section (in the transverse direction). For each bridge, 60 or 75 meter long, end supports are constituted by steel bearings directly in contact with lower main strings, so that a simply-supported scheme is realized. Stringers and additional elements supporting the railway lines are also composed of riveted steel elements.

In the following, results of the dynamic updating procedure of an end span of the old Pontelagoscuro riveted bridge, 60 meters long, will be described. First, operational modal analysis technique has been applied to find the modal frequencies and mode shapes of the truss girder bridge. For this purpose, the accelerations in the vertical and (transverse) horizontal directions were measured. A ambient vibrations have been used as excitation source. Starting from the recorded data, the global modes have been identified by Operative Modal Analysis technique (Guidorzi et al., 2013; Soyoz and Feng, 2009) Six global modes of vibration were clearly identified from the experimental tests, whose frequencies are reported in Table 4. The first mode shape (2.14 Hz) is characterized by the horizontal deformation of the bridge. In fact, the old Pontelagoscuro bridge was designed mainly to support vertical loads: due to the presence of the two vertical truss girders, the bridge is very stiff in the vertical plane but quite deformable in the transverse direction. The second mode shape (2.86 Hz) is a flexural mode with deflection in the vertical plane. The third mode is a torsional mode with a significant distortion of the transverse bridge cross-sections and the fourth is the second lateral mode (4.3 and 4.7 Hz, respectively).

The dynamic behavior of the structure has been modeled with FEM with linear elastic elements for the two main steel truss girders, stunts and floor-beams, and beam elements for the upper and lower bracings (Figure 19). As far as the external supports are concerned, a simply-supported scheme has been adopted.

A preliminary sensitivity analysis has been performed to select the parameters to match numerical modal parameters with experimental results. The selected identification parameters are:

1. the equivalent density m_{eq} of steel members (due to the large amount of plates and rivets);
2. the equivalent moment of inertia J_d of the diagonals (due to the presence of stiffening brackets);
3. the stiffness of axial bracings at the top and bottom level, through the equivalent area of each brace (A_b) (being their sections roughly estimated – the element are almost inaccessible – and being some bracings not always efficiently connected with the main bridge structural elements);
4. the equivalent density m of deck steel members (due to the presence of secondary masses at the deck level).

The parameters governing the DE-Q algorithm are

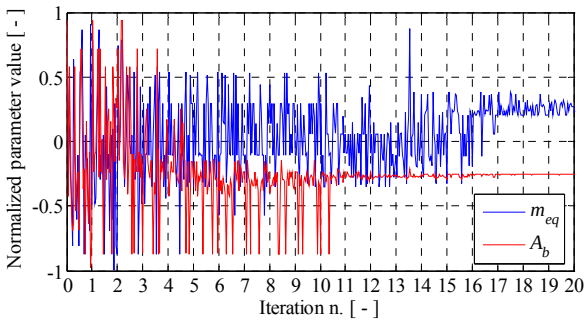
7.1 Results

The intervals for each parameter to be identified are set as follows:

m_{eq} = [7.85·10^3 kg/m^3, 13.85·10^3 kg/m^3],
J_d = [0.10·10^{-3} m^4, 0.20·10^{-3} m^4],
A_b =]0.0 m^2, 11·10^{-3} m^2],
m = [15.3·10^3 kg/m^3, 21.3·10^3 kg/m^3].

Also in this case, for numerical convenience, the normalized values of the identification parameter are defined according to Eqs. (26) and (27), so obtaining a search domain in the range]-1,+1[for all parameters. The number of vectors for each population is set equal to 25 and *NS* is set equal to 19. Values adopted for the convergence targets are set $VTR_1 = 10^{-2}$ and $VTR_2 = 10^{-1}$.

After the optimization process, the values of identified parameters are found, i.e., m_{eq} = 9.59·10^3 kg/m^3, J_d = 0.114·10^{-3} m^4, A_b = 3.5·10^{-3} m^2, m =19.18·10^3 kg/m^3. As for the equivalent density of steel members (m_{eq}), it is worth noting that the value is 22% greater than the nominal steel density (7.85·10^3 kg/m^3) due to the presence of connection plates and rivets, comparable with the value typically taken into account in design of new steel structures. Moreover, the additional masses on the deck (sleepers, railway lines



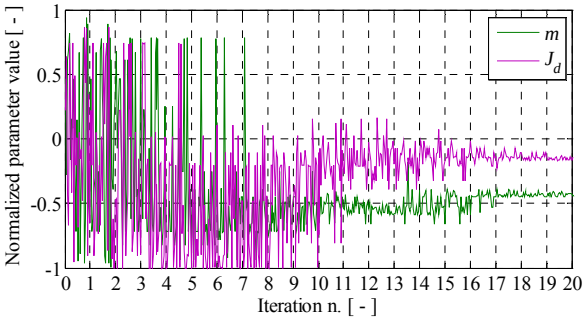


Figure 22 Case study No. 2: parameter values vs. Iteration number.

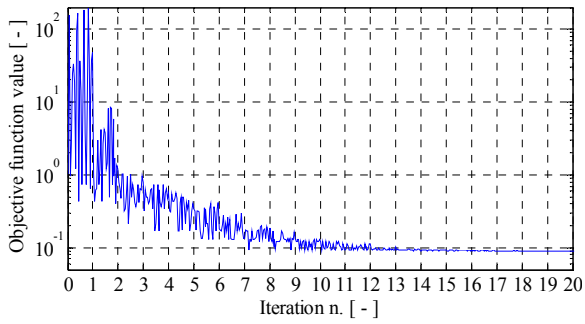


Figure 23 Case study No. 2: objective functions vs. Iteration number.

and iron grate for protection) are about 1.5 times greater than the masses introduced in the finite element model for the structural elements (lower chords, floorbeams and stringers), so obtaining a total mass 2.5 times the mass of the only structural members. Finally, the value of equivalent area of transverse bracings is about the 50% smaller than the initially assumed value. It is worth noting that

For each iteration during the optimization process, the values of the components of the search vectors and the objective function are depicted in Figure 22-19. Moreover, the convergence of the 4 first frequencies of the updated model to the optimal values are also given in Figure 20. It is shown that, after 8-9 iterations, results in term of frequencies and objective function (indicating the error between experimental and numerical results) are very stable. Iterations from n. 10 to n. 20 are required to reach

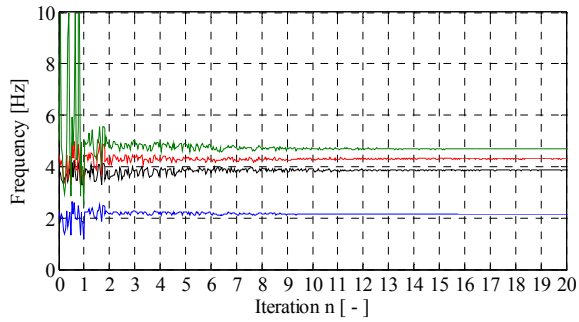


Figure 20 Case study No. 2: first 4 frequencies vs. Iteration number.

Table 5
Case study no. 2: First 6 experimental and numerical frequencies and mode shapes

Mode n.	*Experimental Frequency [Hz]*	*Numerical frequency [Hz]*	*Error [%]*	*MAC [%]*
1	2.143	2.153	+0.47	98.3
2	3.857	3.854	-0.08	96.4
3	4.307	4.304	-0.07	86.3
4	4.700	4.682	-0.38	98.1
5	7.907	8.161	+3.21	94.9
6	8.139	8.490	+4.31	79.2

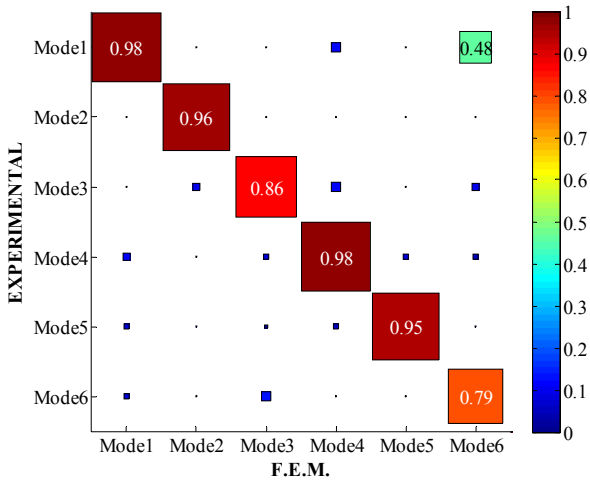


Figure 21 Case study No. 2: MAC values between numerical (F.E.M.) and experimental modes.

the imposed convergence value and to reduce the uncertainties on the optimization parameters, especially as far as the global mass m_{eq} is concerned. The other parameters are stable after iteration n. 11.

Finally, the value of the first 6 frequencies obtained after the optimization process are compared with the experimental values in Table 5. Frequencies of the numerical model are very close to the experimental values,

with errors never greater than 0.5% for the first 4 modes, and smaller than 5% for all modes. Values of MAC are also reported in Table 5 and Figure 21. For all modes, the identification attained almost perfect correlations between experimental and numerical mode shapes, with MAC>0.95 for almost all modes and smaller values (0.86 and 0.79) only for the third and sixth modes.

8 CONCLUSIONS

A new global search method for dynamic identification problems with unknown parameters is presented. It is based on the combination of the Response Surface approach with Differential Evolution algorithm for global search. Numerical examples are presented, concerning the optimization of analytical benchmark functions, the parameter identification of a damaged beam, the parameter identification of mechanical properties (masses and member stiffnesses) of a truss-girder steel bridge starting from frequencies and eigenvectors obtained from an experimental field test. Results show that the performances in term of speed rate are improved over classical global search methods by introducing the second-order approximation; nevertheless, numerical tests shows that robustness of DE algorithm for global minimum search of objective function is preserved, since multiple search points are used simultaneously.

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