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Monetary Policy Indeterminacy and Identification Failures in the U.S.: Results from a Robust Test

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Abstract

We propose a novel identification-robust test for the null hypothesis that an estimated new-Keynesian model has a reduced form consistent with the unique stable solution against the alternative of sunspot-driven multiple equilibria. Our strategy is designed to handle identification failures as well as the misspecification of the relevant propagation mechanisms. We invert a likelihood ratio test for the cross-equation restrictions (CER) that the new-Keynesian system places on its reduced form solution under determinacy. If the CER are not rejected, sunspot-driven expectations can be ruled out from the model equilibrium and we accept the structural model. Otherwise, we move to a second-step and invert an Anderson and Rubin-type test for the orthogonality restrictions (OR) implied by the system of structural Euler equations. The hypothesis of indeterminacy and the structural model are accepted if the OR are not rejected. We investigate the finite sample performance of the suggested identification-robuts two-steps testing strategy by some Monte Carlo experiments and then apply it to a new-Keynesian AD/AS model estimated with actual U.S. data. In spite of some evidence of weak identification as for the 'Great Moderation' period, our results offer formal support to the hypothesis of a switch from indeterminacy to a scenario consistent with uniqueness occurred in the late 1970s. Our identification-robust full-information confidence set for the structural parameters computed on the 'Great Moderation' regime turn out to be more precise than the intervals previously reported in the literature through 'limited-information' methods.

Keywords: Confidence set, Determinacy, Identification failures, Indeterminacy, Misspecification, new-Keynesian business cycle model, VAR system.

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1 Introduction

The U.S. inflation and output growth processes have experienced dramatic breaks in the post-WWII. In particular, a marked reduction of the U.S. macroeconomic volatilities has been documented by Stock and Watson (2002), who coined the popular term 'Great Moderation' to indicate this stylized fact. A possible explanation for such phenomenon hinges upon the hypothesis of the switch to an aggressive monetary policy conduct occurred with the appointment of Paul Volcker as Chairman of the Federal Reserve at the end of the 1970s. With his appointment, the argument goes, the Fed moved from a weakly aggressive reaction to inflation to a much stronger one. Such a switch anchored private sector's inflation expectations, therefore leading the U.S. economy to move from an indeterminate equilibrium to determinacy. This story, popularized by Clarida *et al.* (2000), has subsequently been supported by Lubik and Schorfheide (2004), Boivin and Giannoni (2006), Benati and Surico (2009), Mavroeidis (2010), and Inoue and Rossi (2011*a*).

The above mentioned contributions implicitly assume the new-Keynesian model one works with to be correctly specified and, with the remarkable exception of Mavroeidis (2010), to feature identifiable parameters. As concerns the first issue, albeit new-Keynesian models can display several types of misspecification (An and Schorfheide, 2007), the omission of propagation mechanisms from the structural equations is a major concern in the empirical assessment of determinacy/indeterminacy. As discussed by Lubik and Schorfheide (2004) and Fanelli (2012), indeterminacy generally entails a richer correlation structure of the data. Therefore, the risk run by an econometrician is to confound a determinate case in which relevant propagation mechanisms are not embedded by the structural model at hand with the indeterminate scenario. In conducting their Bayesian analysis, Lubik and Schorfheide (2004) tackle this issue by analyzing versions of a small-scale new-Keynesian model featuring different dynamic structures, while Fanelli (2012) proposes a frequentist test of determinacy/indeterminacy that explicitly controls for the omission of propagation mechanisms from the specified system of structural Euler equations.

As concerns the identifiability of the structural parameters, aside from Mavroeidis (2010), who adopts a single-equation 'limited-information' approach, all existing empirical contributions in which the determinacy/indeterminacy issue of U.S. monetary policy is investigated assume that the structural parameters are identifiable. In general, both finite sample and asymptotic distributions for estimators and tests can be strongly affected if identification conditions are not satisfied, see e.g. Sargan (1983), Phillips (1989), Staiger and Stock (1997) and Stock and Wright (2000). Many authors have recently argued that estimated new-Keynesian systems like or similar to the one considered in this paper can be affected by 'weak identification' issues. Identification problems in a system of variables featuring highly nonlinear restrictions may involve the rank condition of the information matrix or suitable transformation of moments (Iskrev, 2008, 2010; Komunjer and Ng, 2011), or the relationship between the structural parameters and the sample objective function, which may display 'small' curvature in certain regions of the parameter space, see e.g. Canova and Sala (2009). The former concept of identification is also referred to as 'population identification', as opposed to the latter, often termed 'sample identification', because it is specific to a particular dataset and sample size (for proponents of this terminology, see Canova and Sala, 2009). Our paper is concerned with this second phenomenon, which we characterize as the situation in which the criterion used to estimate the structural parameters and test hypotheses on these parameters exhibit 'little curvature' in all or some directions of the parameter space, with the consequence of being nearly uninformative about these parameters. Weak identification of all or part of the estimated parameters can affect negatively the finite sample performances of the testing procedures commonly used by 'frequentist' practitioners. Robust inference under possible identification failure has received increasing attention by the literature on dynamic stochastic general equilibrium (DSGE) models, see e.g. Canova and Sala (2009), Dufour *et al.* (2009, 2013), Kleibergen and Mavroeidis (2009), Mavroeidis (2005, 2010), Guerron-Quintana *et al.* (2013) and Andrews and Mikusheva (2014), among others.[1]

This paper's contribution is twofold. On the methodological side, we propose a novel identification-robust test for the null hypothesis that an estimated new-Keynesian model has a reduced form consistent with the unique stable solution, against the alternative of sunspot-driven multiple equilibria. The test (i) can be applied regardless of the strength of identification of the model's structural parameters, and (ii) controls for the case of 'dynamic misspecification', where by this term we mean the omission of relevant propagation mechanisms from the specified system of structural Euler equations. On the empirical side, we use the small scale new-Keynesian model discussed in Benati and Surico (2009) and apply the proposed method to post-WWII U.S. data to investigate indeterminacy issues in the conduct of monetary policy on our selected 'pre-Volcker' and 'Great Moderation' samples.

As regards the methodological contribution, the proposed testing strategy is based on two steps. In the first-step, we use an identification-robust 'full-information' method to test the cross-equation restrictions (CER) that the new-Keynesian model places on its unique stable reduced

[1] Inoue and Rossi (2011*b*) and Andrews and Cheng (2012) tackle the issue from a more general perspective but their analysis can be adapted to the context of DSGE models.

form solution under determinacy. This requires the (numerical) inversion of a likelihood-ratio test for the CER implied by the new-Keynesian model along the lines recently suggested by Guerron-Quintana *et al.* (2013) and Dufour *et al.* (2013). If the CER are not rejected, we can rule out the occurrence of sunspot-driven expectations and arbitrary nuisance parameters from the model's equilibrium. Importantly, in this case we cannot rule out the possibility of a Minimum State Variable (MSV) equilibrium (McCallum, 1983), i.e. a solution nested within the class of indeterminate equilibria observationally equivalent to the determinate reduced form, see Evans and Honkapohja (1986), Lubik and Schorfheide (2004) and Fanelli (2012). Notably, the non-rejection of the CER amounts to an implicit acceptance of the hypothesis of correct specification of the new-Keynesian system. If instead the CER are rejected, we move to a second-step to determine whether the outcome obtained in the first-step depends on the multiple equilibria hypothesis, or to the omission of relevant propagation mechanisms from the specified structural equations. We apply an identification-robust 'limited-information' method and invert a test for the orthogonality restrictions (OR) implied by the system of Euler equations under the rational expectations hypothesis (and the assumption of correct specification). In principle, if the new-Keynesian system is correctly specified, the OR are valid irrespective of whether the implied equilibrium is determinate or indeterminate. However, conditional on the result in first-step, the non-rejection of the OR is in our framework evidence of indeterminacy, while their rejection suggests that the specified structural equations do not capture the dynamic of the data sufficiently well. The test inverted in this second-step is an Anderson Rubin-type test (Anderson and Rubin, 1949) that can be implemented in the multivariate framework along the lines suggested by Dufour *et al.* (2009, 2013).[2]

The tests computed in both steps are based on asymptotically pivotal test statistics regardless of the strength of identification of the model's structural parameters. The overall testing strategy is asymptotically correctly sized. We investigate its finite sample performance by some Monte Carlo experiments, using the new-Keynesian model by Benati and Surico (2009) as data generating process.

As regards the empirical contribution, the application of our testing strategy on U.S. quarterly data using Benati and Surico's (2009) model as benchmark, leads us to the following findings. The identification-robust test for the CER computed in the first-step implies the rejection of the hypothesis of determinacy on the 'pre-Volcker' sample. Conditional on this first-step, our identification-robust test for the OR computed in the second-step does not reject the new-Keynesian framework at hand. Therefore, our results support the multiple equilibria scenario,

[2] Alternatively, one can apply the 'S-test' approach by Stock and Wright (2000) or the 'K-LM test' approach by Kleibergen (2005), which require the evaluation of the criterion function associated with the continuos-updating version of the generalized method of moments.

which acknowledges a role for self-fulfilling expectations as a driver of the U.S. macroeconomic dynamics during the 1970s. Instead, when considering our 'Great moderation' sample, the identification-robust test for the CER computed in the first-step clearly supports the CER implied by the hypothesis of determinacy. While being unable to interpret this result as conclusive evidence of determinacy (recall the observational equivalence between the determinate and the indeterminate MSV solution), the case of sunspot shocks-driven expectations is clearly ruled out by the data. In line with Mavroeidis (2010), the 'limited-information' approach we implement in the second-step delivers wider projected confidence intervals for the policy parameters during the 'Great Moderation', as opposed to those computed for the 'Great Inflation' period. If taken in isolation, the projected confidence intervals of the policy parameters would be considered as uninformative as for the issue of determinacy. Differently, our 'full-information' inferential approach enables us to interpret such evidence as consistent with an economic system under determinacy, hence not affected by sunspot shocks. Therefore, our testing procedure is inherently more informative than a single-equation approach (even when the latter is designed to deal with weak identification), in that it allows the econometrician to go a step further in assessing (and, in this case, ruling out) the role of sunspot fluctuations as possible drivers of the U.S. economic dynamics.

The remained of this paper is organized as follows. Section 2 introduces the reference small scale new-Keynesian structural model, reports the time series representations of its reduced form solutions under determinacy (Sub-section 2.1) and indeterminacy (Sub-section 2.2), and analyzes the conditions under which observational equivalence occurs (Sub-section 2.3). Section 3 discusses how inference can be conducted under identification failure in a 'full-information' framework (Sub-section 3.1) and in a 'limited-information' (Sub-section 3.2) framework, and then combines these two approaches in a coherent testing strategy (Sub-section 3.3). Section 4 investigates the finite sample performance of the suggested testing strategy by some simulation experiments. Section 5 presents our empirical results obtained on U.S. quarterly data. Section 6 relates our work to the existing literature, and Section 7 contains some concluding remarks. Our Supplementary Material derives (i) the time series representations of the reduced form solutions associated with the new-Keynesian model, (ii) some asymptotic properties of the testing strategy and (iii) provides further Monte Carlo results on the finite sample properties of the testing strategy.

2 Structural model and reduced form solutions

This section presents the reference small-scale new-Keynesian business cycle model, summarize its time series representations under determinacy and indeterminacy, and discusses the conditions which give rise to observational equivalence. This is useful to understand the mechanics of the testing approach presented next.

Our reference new-Keynesian model is taken from Benati and Surico (2009). It features the following three equations:

$$\tilde{y}_t = \gamma E_t \tilde{y}_{t+1} + (1-\gamma)\tilde{y}_{t-1} - \delta(R_t - E_t \pi_{t+1}) + \omega_{\tilde{y},t} \tag{1}$$

$$\pi_t = \frac{\beta}{1+\beta\alpha} E_t \pi_{t+1} + \frac{\alpha}{1+\beta\alpha}\pi_{t-1} + \kappa \tilde{y}_t + \omega_{\pi,t} \tag{2}$$

$$R_t = \rho R_{t-1} + (1-\rho)(\varphi_\pi \pi_t + \varphi_{\tilde{y}} \tilde{y}_t) + \omega_{R,t} \tag{3}$$

where

$$\omega_{x,t} = \rho_x \omega_{x,t-1} + \varepsilon_{x,t} \quad , \text{-}1<\rho_x<1 \quad , \varepsilon_{x,t} \sim \mathrm{WN}(0, \sigma_x^2) \quad , \quad x = \tilde{y}, \pi, R \tag{4}$$

and expectations are conditional on the information set $\mathcal{F}_t$, i.e. $E_t\cdot := E(\cdot \mid \mathcal{F}_t)$. The variables $\tilde{y}_t$, π_t, and R_t stand for the output gap, inflation, and the nominal interest rate, respectively; γ is the weight of the forward-looking component in the intertemporal IS curve; α is price setters' extent of indexation to past inflation; δ is households' intertemporal elasticity of substitution; κ is the slope of the Phillips curve; ρ, φ_π, and $\varphi_{\tilde{y}}$ are the interest rate smoothing coefficient, the long-run coefficient on inflation, and that on the output gap in the monetary policy rule, respectively; finally, $\omega_{\tilde{y},t}$, $\omega_{\pi,t}$ and $\omega_{R,t}$ in eq. (4) are the mutually independent, autoregressive of order one disturbances and $\varepsilon_{\tilde{y},t}$, $\varepsilon_{\pi,t}$ and $\varepsilon_{R,t}$ are the structural (fundamental) shocks. This or similar small-scale models have successfully been employed to conduct empirical analysis concerning the U.S. economy. Clarida *et al.* (2000) and Lubik and Schorfheide (2004) have investigated the influence of systematic monetary policy over the U.S. macroeconomic dynamics; Boivin and Giannoni (2006), Benati and Surico (2009), and Lubik and Surico (2010) have replicated the U.S. Great Moderation, Benati (2008) and Benati and Surico (2008) have investigated the drivers of the U.S. inflation persistence; Castelnuovo and Surico (2010) have replicated the VAR dynamics conditional on a monetary policy shock in different sub-samples; Inoue and Rossi (2011*a*) have analyzed the role of parameter instabilities as drivers of the Great Moderation.

We compact the system composed by eq.s (1)-(4) in the representation

$$\Gamma_0 X_t = \Gamma_f E_t X_{t+1} + \Gamma_b X_{t-1} + \omega_t \tag{5}$$

$$\omega_t = \Xi \omega_{t-1} + \varepsilon_t \ , \ \ \varepsilon_t \sim \mathrm{WN}(0, \Sigma_\varepsilon) \tag{6}$$

$$\Xi{:=}dg(\rho_{\tilde{y}},\rho_\pi,\rho_R) \quad , \quad \Sigma_\varepsilon{:=}dg(\sigma^2_{\tilde{y}},\sigma^2_\pi,\sigma^2_R)$$

where $X_t{:=}(\tilde{y}_t,\pi_t,R_t)'$, $\omega_t{:=}(\omega_{\tilde{y},t},\omega_{\pi,t},\omega_{R,t})'$, $\varepsilon_t{:=}(\varepsilon_{\tilde{y},t},\varepsilon_{\pi,t},\varepsilon_{R,t})'$ and

$$\Gamma_0{:=}\begin{pmatrix} 1 & 0 & \delta \\ -\kappa & 1 & 0 \\ -(1-\rho)\varphi_{\tilde{y}} & -(1-\rho)\varphi_\pi & 1 \end{pmatrix}, \Gamma_f{:=}\begin{pmatrix} \gamma & \delta & 0 \\ 0 & \frac{\beta}{1+\beta\alpha} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_b{:=}\begin{pmatrix} 1-\gamma & 0 & 0 \\ 0 & \frac{\alpha}{1+\beta\alpha} & 0 \\ 0 & 0 & \rho \end{pmatrix}.$$

Let $\theta{:=}(\gamma,\delta,\beta,\alpha,\kappa,\rho,\varphi_{\tilde{y}},\varphi_\pi,\rho_{\tilde{y}},\rho_\pi,\rho_R,\sigma^2_{\tilde{y}},\sigma^2_\pi,\sigma^2_R)'$ be the $m\times 1$ vector of structural parameters ($m{:=}\dim(\theta)$). The elements of the matrices Γ_0, Γ_f, Γ_b and Ξ depend nonlinearly on θ and, without loss of generality, the matrix $\Gamma_0^{\Xi}{:=}(\Gamma_0+\Xi\Gamma_f)$ is assumed to be non-singular. The space of all theoretically admissible values of θ is denoted by $\mathcal{P}$.

For future uses, we consider the partition $\theta{:=}(\theta_s',\ \theta_\varepsilon')'$, where θ_ε contains the non-zero elements of $vech(\Sigma_\varepsilon)$ and θ_s all remaining elements. The 'true' value of θ, $\theta_0{:=}(\theta_{0,s}',\ \theta_{0,\varepsilon}')'$, is assumed to be an interior point of $\mathcal{P}$. Given the partition $\theta{:=}(\theta_s',\ \theta_\varepsilon')'$, we also consider the corresponding partition of the parameter space $\mathcal{P}{:=}\mathcal{P}_{\theta_s}\times\mathcal{P}_{\theta_\varepsilon}$. This distinction is important for two related reasons. First, the determinacy/indeterminacy of the system depends only on the values taken by θ_s, and not by θ_ε. Second, the sub-vector θ_ε is not directly recoverable (identifiable) from the estimation of the system of Euler equations (5)-(6) through 'limited-information' methods, and our procedure for testing determinacy/indeterminacy also relies on the direct estimation of θ_s from system (5)-(6).

Throughout the paper, we use the notations '$M(\theta)$' and '$M{:=}M(\theta)$' to indicate that the elements of the matrix M depend nonlinearly on the structural parameters θ, hence in our setup $\Gamma_0{:=}\Gamma_0(\theta)$, $\Gamma_f{:=}\Gamma_f(\theta)$, $\Gamma_b{:=}\Gamma_b(\theta)$, $\Xi{:=}\Xi(\theta)$ and $\Gamma_0^{\Xi}{:=}\Gamma_0^{\Xi}(\theta)$. Moreover, we call 'stable' a matrix that has all eigenvalues inside the unit disk and 'unstable' a matrix that has at least one eigenvalue outside the unit disk. Thus, denoted with $\lambda_{\max}(\cdot)$ the absolute value of the largest eigenvalue of the matrix in the argument, we have $\lambda_{\max}(M(\theta))<1$ for stable matrices and $\lambda_{\max}(M(\theta))>1$ for unstable ones.

The solution properties of the system of Euler equations (5)-(6) depend on whether θ_s lies in the determinacy or indeterminacy region of the parameter space. Thus, the theoretically admissible parameter space $\mathcal{P}_{\theta_s}$ is decomposed into two disjoint subspaces, the determinacy region, $\mathcal{P}^D_{\theta_s}$, and its complement $\mathcal{P}^I_{\theta_s}{:=}\mathcal{P}_{\theta_s}\backslash\mathcal{P}^D_{\theta_s}$. We assume that $\forall\theta_s\in\mathcal{P}_{\theta_s}$, an asymptotically stationary (stable) reduced form solution to system (5)-(6) exists, hence the case of non stationary and 'explosive' (unstable) indeterminacy is automatically ruled out. Since we consider only stationary solutions, $\mathcal{P}^I_{\theta_s}$ contains only values of θ_s that lead to multiple stable solutions. The whole set of regularity conditions assumed to hold in the specified structural system are reported in our Supplementary Material, where we show that the stability/instability of the matrix $G(\theta_s){:=}(\Gamma_0^{\Xi}-\Gamma_f\Phi_1)^{-1}\Gamma_f$, where Φ_1 stems from the solution of a quadratic matrix equation, can be associated with the determinacy/indeterminacy of the system.

Determinacy/indeterminacy is a system property that depends on all elements in θ_s. There are cases in which the new-Keynesian system is highly restricted and it becomes relatively simple to identify the region $\mathcal{P}^D_{\theta_s}$ ($\mathcal{P}^I_{\theta_s}$) of the parameter space. For instance, if system (1)-(4) is restricted such that $\gamma{:=}1$, $\alpha{:=}0$, and $\rho{:=}0$, $\rho_x{:=}0$, $x=\tilde{y},\pi,R$, the model collapses to a 'purely forward-looking' model. In this particular case, it can be shown that the inequality

$$\varphi_\pi+\frac{1-\beta}{\kappa}\varphi_{\tilde{y}}>1 \tag{7}$$

is sufficient and 'generically' necessary (Woodford, 2003, Proposition 4.3, p. 254) for determinacy. Consequently, the determinacy region of the parameter space is given by $\mathcal{P}^D_{\theta_s}{:=}\left\{\theta_s\in\mathcal{P}_{\theta_s},\,\varphi_\pi+\frac{1-\beta}{\kappa}\varphi_{\tilde{y}}>1\right\}$. However, it is in general not possible to work out a set of closed-form inequality constraints from system (5)-(6) that are both necessary and sufficient for determinacy (indeterminacy) and that can potentially be used to test whether $\theta_{0,s}$ lies in $\mathcal{P}^D_{\theta_s}$ or $\mathcal{P}^I_{\theta_s}$.[3]

2.1 Time series representation under determinacy

For values of θ_s such that the matrix $G(\theta_s){:=}(\Gamma_0^{\Xi}-\Gamma_f\Phi_1)^{-1}\Gamma_f$ is stable, i.e. $\lambda_{\max}(G(\theta_s)){<}1$, the system (5)-(6) has a unique stable reduced form solution that can be represented as the finite-order VAR

$$(I_3-\Phi_1(\theta_s)L-\Phi_2(\theta_s)L^2)X_t=u_t \quad , \quad u_t{:=}\Upsilon(\theta_s)^{-1}\varepsilon_t \tag{8}$$

where L is the lag/lead operator ($L^hX_t{:=}X_{t-h}$), X_0 and X_{-1} are fixed initial conditions, $\Phi_1(\theta_s)$, $\Phi_2(\theta_s)$ and $\Upsilon(\theta_s)$ are 3×3 matrices whose elements depend nonlinearly on θ_s and embody the cross-equation restrictions implied by the small new-Keynesian model (Hansen and Sargent, 1980, 1981). As shown in the Supplementary Material, the matrices $\Phi_1(\theta_s)$ and $\Phi_2(\theta_s)$ in eq. (8) are obtained as the unique solution to the second-order quadratic matrix equation

$$\mathring{\Phi}{=}(\mathring{\Gamma}_0-\mathring{\Gamma}_f\mathring{\Phi})^{-1}\mathring{\Gamma}_b \tag{9}$$

where $\mathring{\Gamma}_f$, $\mathring{\Gamma}_0$, $\mathring{\Gamma}_b$ and the stable matrix $\mathring{\Phi}$ are respectively given by

$$\mathring{\Gamma}_0{:=}\begin{pmatrix}\Gamma_0^{\Xi} & 0_{3\times3}\\ 0_{3\times3} & I_3\end{pmatrix} \;,\; \mathring{\Gamma}_f{:=}\begin{pmatrix}\Gamma_f & 0_{3\times3}\\ 0_{3\times3} & 0_{3\times3}\end{pmatrix},\; \mathring{\Gamma}_b{:=}\begin{pmatrix}\Gamma_{b,1}^{\Xi} & \Gamma_{b,2}^{\Xi}\\ I_3 & 0_{3\times3}\end{pmatrix},\; \mathring{\Phi}{:=}\begin{pmatrix}\Phi_1 & \Phi_2\\ I_3 & 0_{3\times3}\end{pmatrix},$$

[3] The following example shows that the condition in eq. (7) is not sufficient for determinacy, if the structural model in eq.s (1)-(4) involves lags of the variables, other than leads. Consider the system based on $\beta{:=}0.99$, $\kappa{:=}0.085$, $\delta{:=}0.40$, $\gamma{:=}0.25$, $\alpha{:=}0.05$, $\rho{:=}0.95$, $\varphi_{\tilde{y}}{:=}2$, $\varphi_\pi{:=}0.77$, $\rho_{\tilde{y}}{:=}\rho_\pi{:=}\rho_R{:=}0.9$. In this case, the condition $\varphi_\pi+\frac{1-\beta}{\kappa}\varphi_{\tilde{y}}>1$ is valid but the rational expectation-solution to system (1)-(4), while being stable, is not unique. Recall that we assume the existence of at least a solution under rational expectations.

and $\Gamma_{b,1}^{\Xi}{:=}(\Gamma_b+\Xi\Gamma_0)$, $\Gamma_{b,2}^{\Xi}{:=}-\Xi\Gamma_b$ and $\Upsilon(\theta){:=}(\Gamma_0-\Gamma_f\Phi_1(\theta))$. The matrix $\Phi_1{:=}\Phi_1(\theta_s)$ is the one entering the definition of $G(\theta_s)$. The constrained covariance matrix of the reduced form disturbances u_t, denoted with $\tilde{\Sigma}_u$, is given by

$$\tilde{\Sigma}_u(\theta){=}\Upsilon(\theta_s)^{-1}\Sigma_\varepsilon(\theta_\varepsilon)\Upsilon(\theta_s)'^{-1}. \tag{10}$$

Equations (9) and (10) define the CER implied by our new-Keynesian structural model on its reduced form solution under determinacy.

2.2 Time series representation under indeterminacy

For values of θ_s such that the matrix $G(\theta_s){:=}(\Gamma_0^{\Xi}-\Gamma_f\Phi_1)^{-1}\Gamma_f$ is unstable, i.e. $\lambda_{\max}(G(\theta_s)){>}1$,[4] the class of reduced form solutions associated with the new-Keynesian system (5)-(6) becomes more involved from a dynamic standpoint, see Lubik and Schorfheide (2003, 2004) and Fanelli (2012).

In particular, when $\lambda_{\max}(G(\theta_s)){>}1$, the matrix $G(\theta_s)$ can be decomposed in the form

$$G(\theta_s){=}P(\theta_s)\begin{pmatrix}\Lambda_1 & 0_{n_1\times n_2}\\ 0_{n_2\times n_1} & \Lambda_2\end{pmatrix}P^{-1}(\theta_s)$$

where $P(\theta_s)$ is a 3×3 non-singular matrix, Λ_1 is the $n_1\times n_1$ $(n_1<3)$ Jordan normal block that collects the eigenvalues of $G(\theta_s)$ that lie inside the unit disk and Λ_2 is the $n_2\times n_2$ $(n_2\leq 3)$ Jordan normal block that collects the eigenvalues of $G(\theta_s)$ that lie outside the unit disk. Notice that $n_1+n_2{=}3$, where $n_2{:=}\dim(\Lambda_2)$ determines the 'degree of multiplicity' of solutions, see below. In the Supplementary Material we prove that in this case the reduced form solutions can be given the VARMA-type representation:

$$(I_3-\Pi(\theta_s)L)(I_3-\Phi_1(\theta_s)L-\Phi_2(\theta_s)L^2)X_t=(M(\theta_s,\psi)-\Pi(\theta_s)L)V(\theta_s,\psi)^{-1}\varepsilon_t+\tau_t \tag{11}$$

$$\tau_t{:=}(M(\theta_s,\psi)-\Pi(\theta_s)L)V(\theta_s,\psi)^{-1}P(\theta_s)\zeta_t+P(\theta_s)\zeta_t. \tag{12}$$

In this system, the matrices $\Phi_1(\theta_s)$ and $\Phi_2(\theta_s)$ are defined as in the case of determinacy, see eq. (9), while the matrices $\Pi(\theta_s)$, $M(\theta_s,\psi)$ and $V(\theta_s,\psi)$ are given by

$$\Pi(\theta_s){:=}P(\theta_s)\begin{pmatrix}0_{n_1\times n_1} & 0_{n_1\times n_2}\\ 0_{n_2\times n_1} & \Lambda_2^{-1}\end{pmatrix}P^{-1}(\theta_s)\quad,\quad M(\theta_s,\psi){:=}P(\theta_s)\begin{pmatrix}I_{n_1} & 0_{n_1\times n_2}\\ 0_{n_2\times n_1} & \Psi\end{pmatrix}P^{-1}(\theta_s)$$

$$V(\theta_s,\psi){:=}(\Gamma_0(\theta_s)-\Gamma_f(\theta_s)\Phi_1(\theta_s))-\Xi(\theta_s)\Gamma_f(\theta_s)(I_3-M(\theta_s,\psi)).$$

[4] The case in which the matrix $G(\theta_s)$ has eigenvalues equal to one is deliberately ignored because it can be associated with the case of non-stationary processes.

In this framework, Ψ is a $n_2 \times n_2$ matrix ($n_2 \leq 3$) containing a set of arbitrary auxiliary parameters unrelated to θ_s. We collect these parameters in the vector $\psi{:=}vec(\Psi)$. The 'additional' moving average term τ_t which enters system (11)-(12) depends on a 3×1 martingale difference sequence (MDS) vector ζ_t which collects the 'sunspot shocks', and may be unrelated to the fundamental shocks. We assume ζ_t has a time-invariant covariance matrix Σ_ζ. The specific features of the ζ_t component are discussed in detail in the Supplementary Material.

While the determinate equilibrium in eq. (8) depends only on the state variables of the structural system (5)-(6), there are two sources of indeterminacy featured by the equilibria in eq.s (11)-(12). First, there is the 'parametric indeterminacy' that springs from the auxiliary parameters in the vector ψ. Such parameters index solution multiplicity and may amplify or dampen the fluctuations of X_t governed by the VMA part of the reduced form solution. Second, there is the 'stochastic indeterminacy' that stems from the term τ_t, which in turn depends on the sunspot shocks ζ_t (when $\Sigma_\zeta \neq 0_{3\times 3}$). These shocks may arbitrarily alter the dynamics and volatility of the new-Keynesian system, see Lubik and Schorfheide (2003, 2004) and Lubik and Surico (2009) for discussions.

2.3 Observational equivalence

The structure of the two reduced form solutions reported above reveals that, under indeterminacy, the parameter space associated with the new-Keynesian model is wider compared to the case of determinacy. Indeed, in addition to the structural parameters θ, the dynamics of the system is also governed by ψ and σ_ζ^+, where σ_ζ^+ collects the free elements of the covariance matrix Σ_ζ. Both ψ and σ_ζ^+ are unrelated to θ and are not identified under determinacy.

Let $\mathcal{N}$ be the open sub-space of $\mathbb{R}^{(n_2)^2}$ of all possible values taken by ψ, and let $\mathcal{Z}$ be the open sub-space of $\mathbb{R}^6$ of all possible values taken by the elements in σ_ζ^+; the 'complete' parameter space associated with indeterminacy is[5]

$$\mathcal{I}{:=}\left\{\theta^*{:=}(\theta', \psi', \sigma_\zeta^{+\prime})', \, \theta_s \in \mathcal{P}_{\theta_s}^I, \, \psi \in \mathcal{N}, \, \sigma_\zeta^+ \in \mathcal{Z}\right\}. \tag{13}$$

In the special case in which ψ and σ_ζ^+ fulfil the conditions

$$\psi{=}vec(I_{(n_2)^2}) \quad (\Rightarrow M(\theta_s, \psi){=}I_3) \quad , \quad \sigma_\zeta^+{=}0_{6\times 1} \quad (\Rightarrow \tau_t{=}0_{3\times 1} \text{ a.s. } \forall\, t), \tag{14}$$

system (11)-(12) collapses to a MSV solution (McCallum, 1983), i.e., a reduced form solution which has the same time series representation as the determinate VAR solution in eq. (8), and

[5] For given a $\theta_s \in \mathcal{P}_{\theta_s}^I$, the auxiliary parameters ψ might in principle lie in a region of $\mathcal{N}$ such that the VMA components of system (11)-(12) are non-invertible. Under this scenario, the possibility of recovering the structural shocks from the history of X_t is compromised even when the econometrician can observe all components of X_t. Thus, indeterminacy can be a further source of 'non-fundamentalness' in business cycle analysis.

it is subject to the same set of cross-equation restrictions, see Evans and Honkapohja (1986), Lubik and Schorfheide (2003, 2004), and Fanelli (2012).[6] This observational equivalence reflects on the properties of the testing strategy we present below.

3 Inferential issues

Let X_1, ..., X_T be a sample of T observations that are supposed to be generated by a solution of the new-Keynesian system (5)-(6). Our task is to decide whether the observations X_1, ..., X_T are consistent with the case of a unique stable equilibrium, or the case of multiple stable equilibria, controlling for two factors: (i) the possible identification failures, where by this term we denote the case in which the objective functions used to estimate the structural parameters and derive the test statistics may be poorly informative about θ or some of its components; (ii) the possible 'dynamic misspecification', where by this term we denote the situation in which the system (5)-(6) omits relevant propagation mechanisms.

An ideal test for the null $H_0 : \theta_{0,s} \in \mathcal{P}^D_{\theta_s}$ against the alternative $H_1 : \theta_{0,s} \in \mathcal{P}^I_{\theta_s}$ should be based on testing the set of inequality restrictions that identify the region $\mathcal{P}^D_{\theta_s}$ ($\mathcal{P}^I_{\theta_s}$) of the parameter space. For instance, Mavroeidis (2010) uses the standard 'Taylor principle' condition in eq. (7) to address the determinacy/indeterminacy issue in U.S. monetary policy by estimating a Taylor-type monetary policy rule in isolation from other structural equations. The typical risk with this 'single-equation' approach is that the 'Taylor principle' holds with certainty in the form of eq. (7) only if the structural system (5)-(6) fulfills e.g. the restrictions γ:=1, α:=0, and ρ:=0, ρ_x:=0, $x = \tilde{y}, \pi, R$. Our estimates reported in Section 5 show that these restrictions are invalid. In our framework, a 'generic' characterization of the indeterminacy region of the parameter space $\mathcal{P}^I_{\theta_s}$ is given by $\mathcal{P}^I_{\theta_s}$:=$\{\theta_s \in \mathcal{P}_{\theta_s},\ \lambda_{\max}(G(\theta_s))>1\}$, see Section 2 and the Supplementary Material. Unfortunately, even under strong identification, the condition $\lambda_{\max}(G(\theta_s))>1$ can hardly be used for testing purposes. Indeed, aside from very special cases, it is not easy to map the inequalities restrictions that characterize the unstable eigenvalues of the $G(\theta_s)$ matrix onto a set

[6] Observational equivalence between determinate and indeterminate reduced form solutions may be also obtained from system (5) when the vector of fundamental shocks is absent, i.e. when Σ_ε=$0_{3\times 3}$ (ε_t=$0_{3\times 1}$ a.s. $\forall\ t$). In this case, under a set of restrictions, including Ξ=$0_{n\times n}$, the structural model can be solved and represented as in eq. (8). Thus, there exists an intrinsic identification problem once we consider also 'exact' DSGE models: an indeterminate equilibrium of an 'exact' model (i.e. featuring ε_t=$0_{3\times 1}$ and Ξ=$0_{n\times n}$), can be observationally equivalent to the determinate equilibrium of an DSGE model with $\varepsilon_t \neq 0_{3\times 1}$ but richer dynamic structure, see Beyer and Farmer (2007) and Fanelli (2012) for a comprehensive discussion. While being interesting from a theoretical standpoint, the case of absence of fundamental shocks in the structural equations is empirically unpalatable, and it will not be considered in our analysis.

of 'manageable' restrictions on the elements of θ_s.[7] Even working out the inequalities associated with the condition $\lambda_{\max}(G(\theta_s)){>}1$ on a case-by-case basis, the resulting testing problem would involve nonstandard inference, see e.g. Silvapulle and Sen (2005) and references therein.

To circumvent the above mentioned difficulties, we address the testing problem from another viewpoint. We consider the following hypotheses:

$$H_0' : X_t \text{ is generated by the VAR system (8) under the CER in eq.s (9)-(10)} \tag{15}$$

$$H_1' : X_t \text{ is generated by the VARMA-type system (11)-(12), for } \theta^* {\in} \mathcal{I}^0 \tag{16}$$

where $\mathcal{I}^0$ is a subset of $\mathcal{I}$ in eq. (13) defined by

$$\mathcal{I}^0 {:=} \left\{ \theta^* {:=} (\theta', \psi', \sigma_\zeta^{+\prime})',\, \theta_s \in \mathcal{P}^I_{\theta_s},\, \psi \in \mathcal{N} \setminus \left\{ vec(I_{(n_2)^2}) \right\},\, \sigma_\zeta^+ \in \mathcal{Z} \setminus \left\{ 0_{6\times1} \right\} \right\} \subset \mathcal{I}. \tag{17}$$

Under H_0', the new-Keynesian system admits the same time series representation as the unique stable solution but is observationally indistinguishable from the indeterminate MSV equilibrium obtained from the system (11)-(12) when ψ and σ_ζ^+ satisfy the conditions in eq. (14). Under H_1', instead, the new-Keynesian system generates indeterminate non-MSV equilibria. A key observation is that the null of determinacy, H_0: $\theta_{0,s} \in \mathcal{P}^D_{\theta_s}$, implies the hypothesis H_0', while the converse is not true. Hence, the rejection of H_0' is evidence against determinacy, while the non-rejection of H_0' can not be considered conclusive evidence of determinacy. Indeed, the non-rejection of H_0' is only sufficient to rule out the case of 'parametric indeterminacy' generated by the presence of the auxiliary parameters ψ, and the 'stochastic indeterminacy' generated by the sunspot shocks ($\sigma_\zeta^+ \neq 0_{6\times1}$), but is not sufficient to rule out the case of a MSV solution nested in the class of models in eq.s (11)-(12).

To build our identification-robust test for H_0' against H_1', we exploit the well known fact that the construction of confidence sets is a dual problem to hypothesis testing, i.e. confidence sets are obtained by inverting tests, see e.g. Aitchison (1964).[8] In turn, inverting a test means considering all parameter values that are not rejected by the test at a pre-fixed significance level. Our robust testing strategy combines the information deriving from two types of identification-robust inferential approaches. The former, presented in Sub-section 3.1, is a 'full-information' identification-robust approach which allow us to build a confidence set for θ_s exploiting the

[7] Farmer and Guo (1995) use the inequality restriction that identify the indeterminacy region of the parameter space in their stylized business cycle model, and show that their point estimates of the structural parameters fulfil the restriction. However, no inference is provided is such paper.

[8] This approach has been used in the recent literature on inference in weakly identified DSGE models, see Dufour *et al.* (2009, 2013), Kleibergen and Mavroeidis (2009), Mavroeidis (2010), Qu (2011), Andrews and Mikusheva (2012) and Guerron-Quintana *et al.* (2013).

CER implied by the new-Keynesian system under determinacy. The latter, summarized in Sub-section 3.2, is a 'limited-information' identification-robust approach which allow us to build a confidence set for θ_s exploiting the OR implied by the new-Keynesian system under the rational expectations hypothesis. These two methods are condensed in Sub-section 3.3 in a coherent testing strategy for H_0' against H_1'.

3.1 Full-information approach for the CER

We consider the reduced form finite-order VAR solution of the new-Keynesian model in eq. (8), and the vector of reduced form coefficients $\phi{:=}(\phi^{*\prime}, vech(\Sigma_u)')'$, where $\phi^{*}{:=}vec(\Phi_u)$, and the matrix $\Phi_u{:=}[\Phi_1, \Phi_2]$ collects the VAR coefficients. In our setup, ϕ is assumed to be strongly identified. This assumption valid when all components of X_t are observed. For cases in which X_t features unobserved components, it is necessary to refer to the minimal state-space representation associated with the new-Keynesian system under determinacy on a model-by-model basis, see Komunjer and Ng (2011) and Guerron-Quintana *et al.* (2013) for examples and discussion. We denote with $\log L_T(\phi)$ the log-likelihood function associated with the finite-order VAR in eq. (8) before imposing the CER.

The CER that the new-Keynesian model places on its determinate reduced form solution in eq.s (9)-(10) can conveniently be compacted in the expression

$$f(\phi, \theta) = 0_{\dim(\phi)\times 1} \tag{18}$$

where $f(\cdot,\cdot)$ is a continuous, twice differentiable, vector function. By the implicit function theorem, the restrictions in eq. (18) can also be written in explicit form as follows (see Iskrev, 2008):

$$\phi{=}g(\theta) \tag{19}$$

where $g(\cdot)$ is a nonlinear twice differentiable function and the mapping from θ to ϕ is valid in a neighborhood of the true parameter values. Under the CER in eq. (19), the VAR log-likelihood depends on θ and is denoted with $\log L_T(g(\theta))$. In our setup, the shape of $\log L_T(g(\theta))$ may be poorly informative (or uninformative) about θ or some of its components, violating one of the standard regularity conditions behind ML estimation, see, *inter alia*, Andrews and Mikusheva (2012). Throughout the paper we maintain that θ_ε in $\theta{:=}(\theta_s', \theta_\varepsilon')'$ is strongly identified, and that identification failure may solely affect θ_s or some of its components. This assumption reflects the situation we typically observe in practice, where weak identification or unidentification typically involve the elements in θ_s and not the elements in θ_ε.[9] Under this assumption, for any given

[9] This assumption might be relaxed without changing the logic of the proposed testing strategy.

value of $\theta_s{=}\breve{\theta}_s \in \mathcal{P}_{\theta_s}$, the log-likelihood function $\log L_T(g(\breve{\theta}_s,\theta_\varepsilon))$ depends on θ_ε alone, and fulfills the regularity conditions discussed in e.g. Guerron-Quintana *et al.* (2013).

Keeping these observations in mind, we face the problem of computing a LR test for the null hypothesis that there exists a θ_ε such that

$$H_{0,cer}\colon\ \phi_{\breve{\theta}_s}{=}g(\breve{\theta}_s,\theta_\varepsilon)\quad,\quad \theta_s{=}\breve{\theta}_s\in\mathcal{P}_{\theta_s} \tag{20}$$

(against the alternative $H_{1,cer}:\phi_{\breve{\theta}_s}\neq g(\breve{\theta}_s,\theta_\varepsilon)$). The hypothesis $H_{0,cer}$ in eq. (20) is composite: it specializes the CER in eq. (19) to the 'guess' $\theta_s{=}\breve{\theta}_s$ about the parameters value. The notation '$\phi_{\breve{\theta}_s}$' used in eq. (19) remarks that under the CER, the VAR reduced form coefficients depend on the choice $\theta_s{=}\breve{\theta}_s$. When $H_{0,cer}$ is valid, also the hypothesis H_0' in eq. (15) is valid for $\theta_s{=}\breve{\theta}_s$. Likewise, when H_0' in eq. (15) is valid for some $\theta_s{=}\breve{\theta}_s$, the hypothesis $H_{0,cer}$ in eq. (20) will be automatically valid. However, while H_0' is accepted if there exists at least one $\theta_s{=}\breve{\theta}_s$ such that $H_{0,cer}$ holds, it is rejected if and only if $H_{0,cer}$ is rejected for all possible parameter values.

Let $LR_T(\hat{\phi}_{\breve{\theta}_s}){:=}{-}2(\log L_T(\hat{\phi}_{\breve{\theta}_s})-\log L_T(\hat{\phi}))$ be the likelihood-ratio test statistic for the hypothesis $H_{0,cer}$, where the vector $\hat{\phi}_{\breve{\theta}_s}$ is defined by $\hat{\phi}_{\breve{\theta}_s}{:=}g(\breve{\theta}_s,\hat{\theta}_\varepsilon^{\breve{\theta}_s})$, and $\hat{\theta}_\varepsilon^{\breve{\theta}_s}$ is the ML estimate of θ_ε obtained for $\theta_s{=}\breve{\theta}_s$. Under $H_{0,cer}$, the asymptotic null distribution of $LR_T(\hat{\phi}_{\breve{\theta}_s})$ is pivotal and $\chi^2_{d_1}$, where $d_1{:=}\dim(\phi)-\dim(\theta_\varepsilon)$, regardless of whether θ_s is identified or not, see e.g. Guerron-Quintana *et al.* (2013). In practice, there might be many possible choices $\theta_s{=}\breve{\theta}_s$ not rejected by the test $LR_T(\hat{\phi}_{\breve{\theta}_s})$. Since the components of θ_s typically lie within bounded (theoretically admissible) intervals, one can test $H_{0,cer}$ for many possible choices of $\breve{\theta}_s$ within a fine grid $\mathcal{G}_{\theta_s}$ in $\mathcal{P}_{\theta_s}$, giving rise to a 'grid testing' procedure. The numerical inversion of the test $LR_T(\hat{\phi}_{\breve{\theta}_s})$ for $H_{0,cer}$ gives rise to the identification-robust confidence set (or acceptance region of the test):

$$\mathcal{C}^{LR}_{1-\eta_1}{:=}\left\{\breve{\theta}_s\in\mathcal{G}_{\theta_s},\ LR_T(\hat{\phi}_{\breve{\theta}_s})<c^{\eta_1}_{\chi^2_{d_1}}\right\} \tag{21}$$

where $c^{\eta_1}_{\chi^2_{d_1}}$ is the η_1-level cut-off point associated with the $\chi^2_{d_1}$ distribution, and $0{<}\eta_1{<}1$ is the pre-fixed nominal level of significance (or type-I error) of the test.[10] The identification-robust confidence set $\mathcal{C}^{LR}_{1-\eta_1}$ has asymptotic coverage $100(1-\eta_1)$ (see Supplementary Material). A point estimate of θ_s can be obtained from the (nonempty) confidence set $\mathcal{C}^{LR}_{1-\eta_1}$ by

$$\hat{\theta}_{s,ML}:=\underset{\breve{\theta}_s\in\mathcal{C}^{LR}_{1-\eta_1}}{\arg\min}\ LR_T(\hat{\phi}_{\breve{\theta}_s}), \tag{22}$$

[10] Dufour *et al.* (2013) have proposed another identification-robust 'full-information' approach for the structural parameters of DSGE models based on the (numerical) inversion of a test for zero coefficients in the multivariate regression of the quantity $u_t(\breve{\theta}_s){:=}(I_3-\Phi_1(\breve{\theta}_s)L-\Phi_2(\breve{\theta}_s)L^2)X_t$ (which correspond to the disturbance of the VAR system (8) under the CER) on the regressors $Z_t{:=}(X_{t-1}',X_{t-2}')'$.

i.e. considering the parameter point within $\mathcal{C}^{LR}_{1-\eta_1}$ with associated largest p-value (or the 'least rejected' models at the pre-fixed level η_1).[11]

The identification-robust confidence set $\mathcal{C}^{LR}_{1-\eta_1}$ in eq. (21) is built in a 'full-information' framework, in the sense that inverting the test for the null in eq. (20) requires computing the determinate rational expectations solution associated with the new-Keynesian system.

3.2 Limited-information approach for the system of structural Euler equations

We now focus on the system of Euler equations (5)-(6), and consider the problem of testing the simple hypothesis

$$H_{0,spec}\colon\ \theta_s{=}\breve{\theta}_s \quad , \quad \breve{\theta}_s \in \mathcal{P}_{\theta_s} \tag{23}$$

against the alternative $H_{1,spec}$: $\theta_s \neq \breve{\theta}_s$, abstracting from the knowledge of the reduced form solution of the model. $H_{0,spec}$ is the hypothesis that the system of Euler equations (5)-(6) is valid in correspondence of the 'guess' $\theta_s{=}\breve{\theta}_s$ about the parameters value.

Following Dufour *et al.* (2013), a test for $H_{0,spec}$ can be computed as follows. By simple algebraic manipulations, we re-write system (5)-(6) in the form

$$\Gamma_0^{\Xi} X_t = \Gamma_f X_{t+1} + \Gamma_{b,1}^{\Xi} X_{t-1} + \Gamma_{b,2}^{\Xi} X_{t-2} + \Xi \Gamma_f \xi_t + \varepsilon_t - \Gamma_f \xi_{t+1},$$

where $\xi_t{:=}X_t - E_{t-1}X_t$ is a vector MDS, and then define the 3×1 vector function

$$v(X_t,\theta_s){:=}\Gamma_0^{\Xi} X_t - \Gamma_f X_{t+1} - \Gamma_{b,1}^{\Xi} X_{t-1} - \Gamma_{b,2}^{\Xi} X_{t-2} = \Xi \Gamma_f \xi_t + \varepsilon_t - \Gamma_f \xi_{t+1}. \tag{24}$$

Under $H_{0,spec}$, the term $v(X_t,\breve{\theta}_s)$ follows a VMA(1)-type process and fulfills the OR:

$$E\left(v(X_t,\breve{\theta}_s) \mid \mathcal{F}_{t-1}\right) = 0_{3\times 1}. \tag{25}$$

Therefore, we can associate the multivariate linear regression model:

$$v(X_t,\breve{\theta}_s) = \Pi_{\breve{\theta}_s} Z_t + \epsilon_t \quad , \quad Z_t \in \mathcal{F}_{t-1} \quad , t = 1,...,T \tag{26}$$

to the hypothesis $H_{0,spec}$. In eq. (26), $\Pi_{\breve{\theta}_s}$ is a $3{\times}r$ matrix of coefficients, Z_t is a $r\times 1$ vector of regressors selected from the information set $\mathcal{F}_{t-1}$, and ϵ_t is a disturbance term whose covariance matrix, Σ_ϵ, can possibly be non-diagonal. The notation '$\Pi_{\breve{\theta}_s}$' for the regression coefficients remarks that we have a multivariate regression system like that in eq. (26) for each choice

[11] The point estimates in eq. (22) can be interpreted as 'Hodges-Lehmann' estimates of θ_s, see e.g. Dufour *et al.* (2006, 2009, 2010).

$\theta_s{=}\breve{\theta}_s$. Under $H_{0,spec}$, additional information from predetermined variables should be irrelevant, hence the associated problem

$$H^*_{0,spec} : \Pi_{\breve{\theta}_s}{=}0_{3\times r} \text{ vs } \; H^*_{1,spec} : \Pi_{\breve{\theta}_s} \neq 0_{3\times r} \tag{27}$$

should lead us to accept $H^*_{0,spec}$. We have thus transformed the problem of testing the hypothesis $H_{0,spec}$ (against $H_{1,spec}$) into the problem of testing the hypothesis $H^*_{0,spec}$ (against $H^*_{1,spec}$) in the multivariate linear regression system (26). Standard asymptotic theory applies for the testing problem in eq. (27) irrespective of whether θ_s is identifiable or not.

Let $AR_T(\breve{\theta}_s)$ be any asymptotically pivotal test statistic for the problem in eq. (27). In practice, $AR_T(\breve{\theta}_s)$ can be a Wald-type, a Lagrange Multiplier or (quasi-)LR test, and can be interpreted as an Anderson Rubin-type test (Anderson and Rubin, 1949).[12] Under $H_{0,spec}$, the asymptotic null distribution of $AR_T(\breve{\theta}_s)$ is $\chi^2_{d_2}$, $d_2{:=}3r$ and also in this case there might be many choices $\theta_s{=}\breve{\theta}_s$ not rejected by the test $AR_T(\breve{\theta}_s)$. The numerical inversion of the test $AR_T(\breve{\theta}_s)$ for $H_{0,spec}$ leads to the identification-robust confidence set (or acceptance region):

$$\mathcal{C}^{AR}_{1-\eta_2}{:=}\left\{\breve{\theta}_s \in \mathcal{D}_{\theta_s},\ AR_T(\breve{\theta}_s) < c^{\eta_2}_{\chi^2_{d_2}}\right\} \tag{28}$$

where $\mathcal{D}_{\theta_s}$ is a fine grid in $\mathcal{P}_{\theta_s}$, $c^{\eta_2}_{\chi^2_{d_2}}$ is the η_2-level cut-off point associated with the $\chi^2_{d_2}$ distribution, and $0{<}\eta_2{<}1$ is the pre-fixed nominal level of significance (or type-I error) of the test. The identification-robust confidence set $\mathcal{C}^{AR}_{1-\eta_2}$ has asymptotic coverage $100(1-\eta_2)$ (see Supplementary Material) and defines the set of parameter points in $\mathcal{P}_{\theta_s}$ which are consistent with the new-Keynesian model at the significance level η_2 regardless of the multiplicity/uniqueness of its solutions. A point estimate of θ_s can be obtained from the (nonempty) confidence set $\mathcal{C}^{AR}_{1-\eta_2}$ by

$$\hat{\theta}_{s,LI} := \underset{\breve{\theta}_s\in\mathcal{C}^{AR}_{1-\eta_2}}{\arg\min}\ AR_T(\breve{\theta}_s). \tag{29}$$

It is worth observing that both methods discussed in this and in the previous sub-section refer to estimation of the full system of equations. However, while the 'full-information' method presented in Sub-section 3.1 imposes the additional restriction that the reduced form is a finite-order VAR and exploits the CER implied by the structural model, the 'limited-information'

[12] Since the ϵ_t term follows a VMA-type process in system (26), HAC-type versions of the tests can be applied as suggested by Dufour *et al.* (2013). Alternatively, one can use the 'S-test' method by Stock and Wright (2000), or the 'K-LM test' by Kleibergen (2005), both based on the evaluation of the criterion function corresponding to the continuos-updating version of generalized method of moments. Some computational issues make us prefer the approach in Dufour *et al.* (2009, 2013). Kleibergen and Mavroeidis (2009) discuss weak instrument robust statistics for testing hypotheses on θ_s or its subset in the GMM framework, and then apply these methods to the new-Keynesian Phillips curve.

approach summarized here ignores, by construction, any information stemming from the reduced form solutions. Mavroeidis *et al.* (2014), Section 3, discuss the difference between the two approaches in the context of a single structural equation.

3.3 Testing strategy

The two estimation/testing methods discussed in the previous sub-sections form the basis of our identification-robust testing strategy for H_0' in eq. (15) against H_1' in eq. (16).

Our approach is based on the following two steps:

Step 1: LR test for the CER. Invert the test $LR_T(\hat{\phi}_{\breve{\theta}_s})$ for $H_{0,cer}$ discussed in Sub-section 3.1 at the level η_1, considering points $\theta_s{=}\breve{\theta}_s$ taken from a fine grid $\mathcal{G}^*_{\theta_s}$ in $\mathcal{P}_{\theta_s}$. This yields the identification-robust confidence set

$$\mathcal{C}^{*LR}_{1-\eta_1}{:=}\left\{\breve{\theta}_s \in \mathcal{G}^*_{\theta_s},\ LR_T(\hat{\phi}_{\breve{\theta}_s}) < c^{\eta_1}_{\chi^2_{d_1}}\right\} \tag{30}$$

whose asymptotic coverage is at least $1-\eta_1$ (see Supplementary Material). If $\mathcal{C}^{*LR}_{1-\eta_1}$ is nonempty, the null H_0' is accepted and the analysis is stopped. If instead $\mathcal{C}^{*LR}_{1-\eta_1}$ is empty, i.e. the hypothesis $H_{0,cer}$ is rejected for all possible parameter values in the grid implying the rejection of H_0', we move to the next step.

Step 2: Anderson-Rubin test for the OR. Conditional on the confidence set $\mathcal{C}^{LR}_{1-\eta_1}$ being empty, we invert the test $AR_T(\breve{\theta}_s)$ for $H_{0,spec}$ discussed in Sub-section 3.2 at the level η_2, considering points $\theta_s{=}\breve{\theta}_s$ taken from a fine grid $\mathcal{D}^*_{\theta_s}$ such that $\mathcal{D}^*_{\theta_s}{:=}\left\{\breve{\theta}_s \in \mathcal{P}_{\theta_s},\ \lambda_{\max}(G(\breve{\theta}_s)){>}1\right\}$. This yields the identification-robust confidence set

$$\mathcal{C}^{*AR}_{1-\eta_2}{:=}\left\{\breve{\theta}_s \in \mathcal{D}^*_{\theta_s},\ AR_T(\breve{\theta}_s) < c^{\eta_2}_{\chi^2_{d_2}}\right\} \tag{31}$$

whose asymptotic coverage is at least $1-\eta_2$ (see Supplementary Material). If $\mathcal{C}^{*AR}_{1-\eta_2}$ is nonempty, we accept the hypothesis H_1' in eq. (16). If instead $\mathcal{C}^{*AR}_{1-\eta_2}$ is empty, i.e. $H_{0,spec}$ is rejected for all possible parameter values in the grid, we reject H_1' and conclude that the new-Keynesian system (5)-(6) omits relevant propagation mechanisms.

Hereafter, we conventionally denote the testing strategy obtained by combining the two steps described above with the symbol '$LR_T \rightarrow AR_T$'. Several remarks are in order.

Remark 1. The idea underlying the '$LR_T \rightarrow AR_T$' approach is that if the identification-robust confidence set $\mathcal{C}^{*LR}_{1-\eta_1}$ computed in the first-step is nonempty, there exists at least one point in the parameter space consistent with H_0'. This means that the time series representation of the new-Keynesian model summarized in eq.s (8)-(10) is supported by the data for some θ. If instead the identification-robust confidence set $\mathcal{C}^{*LR}_{1-\eta_1}$ is empty, H_0' is rejected and a second-step is run to decide between H_1' and the dynamic misspecification of the structural new-Keynesian system (5)-(6). The second-step is therefore run conditionally on the rejection of the CER in the first-step. If the identification-robust confidence set $\mathcal{C}^{*AR}_{1-\eta_2}$ computed in the second-step is nonempty, there exists at least one θ in the parameter space consistent with H_1'. Finally, when both $\mathcal{C}^{*LR}_{1-\eta_1}$ and $\mathcal{C}^{*AR}_{1-\eta_2}$ are empty, the new-Keynesian system omits relevant propagation mechanisms and is rejected.

Remark 2. The procedure is asymptotically valid irrespective of the strength of identification, hence it can be applied also when θ is strongly identified. Notably, it does not require the identification of the set of parametric inequality restrictions that define the sub-regions $\mathcal{P}^D_{\theta_s}$ ($\mathcal{P}^I_{\theta_s}$) of the parameter space. The practitioner is therefore not committed to the use of non-standard asymptotic inference. Moreover, it is not necessary to specify prior distributions for θ and the auxiliary parameters ψ (and σ^+_ζ) that govern solution multiplicity in eq.s (11)-(12). In this respect, the suggested approach can be regarded as an identification-robust alternative to the test proposed by Fanelli (2012) for strongly identified models.

Remark 3. Many NK-DSGE models feature unobserved states and reliable proxies for these states are not always available. In these situations, we can still compute the LR test in the first-step along the lines suggested by Guerron-Quintana *et al.* (2013), but the implementation of the Anderson Rubin-type test in the second-step may become problematic. Thus, if LR test for the CER rejects H_0' in the first-step, it is not possible to decide whether the rejection is due to the occurrence of multiple equilibria (H_1'), or to the omission of relevant propagation mechanisms. The extension of the '$LR_T \rightarrow AR_T$' testing strategy towards this direction is the subject for future research.

Remark 4. In our setup, the hypothesis of no dynamic specification of the NK-DSGE model is given by the composite hypothesis $H^* = H_0' \vee H_1'$. In the Supplementary Material we prove that as a test for H^*, the '$LR_T \rightarrow AR_T$' sequential procedure has significance level which is bounded above by the maximum of the nominal type-I errors used for the $LR_T(\hat{\phi}_{\breve{\theta}_s})$ test in the first-step and the $AR_T(\breve{\theta}_s)$ test in the second-step. Thus, if e.g. $\eta_1 = \eta_2$:=0.10, the significance level of the procedure as a test for H^* is asymptotically no larger than 10%.

4 Monte Carlo simulations

In this section, we use Benati and Surico's (2009) new-Keynesian system in eq.s (1)-(4) to investigate the finite sample size performance of the '$LR_T \rightarrow AR_T$' testing strategy through some Monte Carlo experiments. Further Monte Carlo results about the rejection frequency of the testing strategy under indeterminate equilibria that belong to the class of models defined by H_1', and the case of 'dynamic misspecification' are confined in the Supplementary Material.

It is worth noting that we work with a 'semi-structural' expression for the NKPC in eq. (2). Such expression features a slope parameter, κ. According to the new-Keynesian theory of the business cycle, κ is a composite parameter influenced by the Calvo-price stickiness parameter, the discount factor, households' risk aversion, and the elasticity of labor. Identification issues are likely to be (even) more severe when referring to such a 'fully-microfounded' version of the NKPC, see Fukaĉ and Pagan (2006, p.17). Our focus on eq. (2) is justified by our willingness to work with a representative version of the NKPC. This is intended to maximize the comparability of our results to the vast literature dealing with specifications similar to ours.[13]

Artificial data sets are generated from the reduced form solutions discussed in Section 2. In all experiments, we consider $M = 1,000$ replications and samples of length $T = 100$ (not including initial lags). The chosen sample size corresponds roughly to the number of quarterly observations we consider for the 'pre-Volcker' (1954q1-1979q2) and 'Great Moderation' (1985q1-2008q2) samples in the empirical section (see Section 5). For each generated data set, we treat the output gap as observable, reproducing the situation we face in Section 5.

To evaluate the empirical size of the '$LR_T \to AR_T$' test for the hypothesis H_0', the Monte Carlo design is calibrated to match the model estimated by Benati and Surico (2009) using U.S. data with Bayesian methods. The discount factor β:=0.99 is treated as known and estimation involves 13 free parameters, 10 of which are collected in the sub-vector θ_s, and 3 in the sub-vector θ_ε. The true vector of parameters $\theta_0:=(\theta_{0,s}', \theta_{0,\varepsilon}')'$ is calibrated to the medians of the 90% coverage percentiles of the posterior distribution reported in Table 1 of Benati and Surico (2009) (see the 'After the Volcker stabilization' column). The data are generated from the reduced form VAR solution in eq. (8) subject to the CER in eq.s (9)-(10), using a Gaussian distribution for the structural shocks ε_t and a diagonal covariance matrix Σ_ε (hence the elements of the sub-vector $\theta_{0,\varepsilon}$ correspond to the diagonal components of Σ_ε). With this calibration, $\lambda_{\max}(G(\theta_{0,s}))$=0.964.

The numerical inversion of the $LR_T(\hat{\phi}_{\breve{\theta}_s})$ test (first-step) is obtained on each simulated dataset by using a grid of points described in detail in Table 1. We refer to Andrews and Mikusheva (2014) for practical details about the implementation of grid-testing methods. The

[13] The same choice is adopted by e.g. Mavroeidis *et al.* (2014) in their recent review of the NKPC empirical literature. Moreover, severe identification issues affect even the 'semi-structural' version of the NKPC we focus on (at least in the widely adopted uni-equational context), as documented and discussed by, among others, Kleibergen and Mavroeidis (2009) and Mavroeidis *et al.* (2014). Hence, while not fully exploiting the restrictions coming from the theory, our version of the NKPC and the chosen new-Keynesian system in general, represents an interesting data generating process to investigate the properties of the proposed identification-robust testing strategy.

log-likelihood maximization algorithm under the CER is adapted from the grid-search numerical method discussed in Bårdsen and Fanelli (2014). The empirical size of the test for H_0' is evaluated by fixing the type-I error of the test at the level η_1=0.10. The results are reported in Table 1, where we summarize the rejection frequency of the $LR_T(\hat{\phi}_{\hat{\theta}_{s,ML}})$ test and the average point estimates of the structural parameters (along with the corresponding Monte Carlo standard errors) obtained from the problem in eq. (22) by replacing $\mathcal{C}_{1-\eta_1}^{LR}$ with $\mathcal{C}_{0.90}^{*LR}$.[14] For completeness, Table 1 also reports the empirical size of the $LR_T(\hat{\phi}_{\theta_{0,s}})$ test for the hypothesis $H_{0,cer}$: $\phi_{\breve{\theta}_{0,s}}=g(\theta_{0,s},\theta_\varepsilon)$ ($\theta_s=\theta_{0,s}$), see eq. (20) and Section 4 in the Supplementary Material for details.

The inverted test for H_0' tends to be slightly conservative, as the empirical size is 7.9% as opposed to the nominal size of 10% (instead the empirical size of the test $LR_T(\hat{\phi}_{\theta_{0,s}})$ for the specific hypothesis $H_{0,cer}$: $\phi_{\breve{\theta}_{0,s}}=g(\theta_{0,s},\theta_\varepsilon)$ is 12.1%). Moreover, the grid-testing procedure delivers point estimates of the structural parameters relatively close to the true values. Table 1 also reports the average projected 90% confidence intervals for the individual structural parameters (fourth column), and these intervals are contrasted with the actual intervals used to define the parametric grid (fifth column).

5 Empirical evidence

In this section, we apply the '$LR_T \to AR_T$' testing strategy to post-WWII U.S. monetary policy. We employ quarterly data, sample 1954q3-2008q3, and three observable variables, X_t:=$(\tilde{y}_t, \pi_t, R_t)'$. The output gap $\tilde{y}_t$ is computed as percent log-deviation of the real GDP with respect to the potential output estimated by the Congressional Budget Office. The inflation rate π_t is the quarterly growth rate of the GDP deflator. For the short-term nominal interest rate R_t we consider the effective Federal funds rate expressed in quarterly terms (averages of monthly values). The source of the data is the Federal Reserve Bank of St. Louis' web site. The beginning of the sample is due to data availability (in particular, of the effective Federal Funds rate. The end of the sample is justified by our intention to avoid dealing with the 'zero-lower bound'

[14] To invert the $LR_T(\hat{\phi}_{\breve{\theta}_s})$ test numerically, we should consider a multi-dimensional grid search for the log-likelihood $\log L_T(g(\breve{\theta}_s, \theta_\varepsilon))$ on a large number of evenly spaced parameter points. Since in our setup dim(θ_s)=10, this approach is computationally cumbersome. For instance, if one considers only 10 evenly spaced points within each of the 10 intervals that define the admissibe parameter space (see the last column of Table 1), then it is necessary to evaluate the log-likelihood 10^{10} times for each simulated dataset. To speed up computation time and line with what suggested by Andrews and Mikusheva (2014), we decided to select only 300 points randomly (using the uniform distribution) from the rectangle formed by the Cartesian products of the 10 intervals. Of course, the employment of more sophisticated and efficient algorithms could lead to an even more satisfactorily empirical size-control of the test.

phase began in December 2008, which triggered a series of non-standard policy moves by the Federal Reserve whose effects are hardly captured by our standard new-Keynesian framework.

Our reference structural model is given by the new-Keynesian system (1)-(4). Following most of the literature on the 'Great Moderation', we divide the post-WWII U.S. era in two periods, roughly corresponding to the 'Great Inflation' and the 'Great Moderation' samples. We take the advent of Paul Volcker as Chairman of the Federal Reserve to identify our first sub-sample, i.e. 1954q3-1979q2, which we call 'pre-Volcker' sample. As for the 'Great Moderation' sample, we consider the period 1985q1-2008q3. McConnell and Pérez-Quirós (2000) find a break in the variance of the U.S. output growth in 1984q1. Our empirical investigation deals with a measure of the output gap, inflation, and the federal funds rate. Signs of the 'Volcker disinflation' are still evident in 1984. This is possibly due to the 'credibility build-up' undertaken by the Federal Reserve in the early 1980s, a period during which private agents gradually changed their view on the Federal Reserve's ability to deliver low inflation (Goodfriend and King, 2005). Moreover, the first years of Volcker's tenure (until October 1982) were characterized by non-borrowed reserves targeting. Hence, the fit of our policy rule would substantially worsen if we included the Volcker disinflation (Estrella and Fuhrer, 2003; Mavroeidis, 2010), a fact that would carry consequences on the estimates of all parameters of the system. To circumvent this problem, we postpone the beginning of our second sub-sample to 1985q1. A similar choice is undertaken by Christiano *et al.* (2013). Thus, our 'Great Moderation' sample is given by the period 1985q1-2008q3 and will be denoted as 'post-1985' sample throughout this section.

The first-step of the '$LR_T \to AR_T$' testing strategy requires computing the 'full-information' $LR_T(\hat{\phi}_{\breve{\theta}_s})$ test discussed in Sub-section 3.1. As is common in the literature, we pre-fix the nominal level of significance at the 10% level (η_1=0.10). The log-likelihood maximization algorithm is inspired to the grid-search approach discussed in Bårdsen and Fanelli (2014). Table 2 summarizes the results of the $LR_T(\hat{\phi}_{\breve{\theta}_s})$ test on the 'pre-Volcker' and 'post-1985' samples, respectively. In the upper panel of Table 2, we summarize the projected 90% confidence intervals for the individual elements of θ_s derived from the identification-robust confidence set $\mathcal{C}^{*LR}_{0.90}$ (see eq. (21)) and the point estimate of θ_s. The projected confidence intervals are computed using Dufour's (1997) method. In the lower panel, we indicate whether the grid-testing procedure leads to an empty or nonempty identification-robust confidence set, and report the value of $LR_T(\hat{\phi}_{\breve{\theta}_s})$ associated with $\breve{\theta}_{s,ML}$ and corresponding p-value.

Table 2 suggests two important facts. First, the CER that the new-Keynesian system implies under determinacy are firmly rejected on the 'pre-Volcker' sample (the set $\mathcal{C}^{*LR}_{0.90}$ is empty), and are firmly accepted on the 'post-1985' sample by the data (the set $\mathcal{C}^{*LR}_{0.90}$ is nonempty and the p-value associated with the 'least rejected' model is 0.36). We reject the hypothesis of determinacy on the 'pre-Volcker' sample and do not reject the hypothesis H_0' in eq. (15) on the 'post-1985' sample. Despite we can not interpret the result relative to the chosen 'Great Moderation' regime as conclusive evidence of determinacy (see the discussions in Sub-section 2.3 and Sub-section 3.3), our inference is sufficient to rule out the scenario according to which the U.S. business cycle was driven by sunspot expectations extraneous to fundamental shocks. Interestingly, the fact that the CER entailed by the hypothesis of determinacy are not rejected on the period 1985q1-2008q3, suggests an implicit non-rejection of the new-Keynesian system (1)-(4) on that sample. Second, the 90% projected identification-robust confidence intervals for the policy (feedback) parameters $\varphi_{\tilde{y}}$ and φ_π are surprisingly tighter than the confidence sets documented by e.g. Mavroeidis (2010). In particular, the estimation of the value of the parameter φ_π, which captures the systematic reaction of the Federal Reserve to inflation, has attracted a lot of attention. The debate has been intense also because of the lack of precision surrounding the estimates of such parameter. A prominent example in the literature is represented by Mavroeidis (2010). He convincingly shows that, in a single-equation context, the estimation of φ_π tends to be imprecise, and the formal evidence in favor of an aggressive systematic policy response to inflation is scant. Possible reasons include (a) the absence of sunspot shocks under determinacy, which implies a lower volatility of inflation and output and, therefore, a harder identification of the systematic relationship between the policy rate and the policy relevant-macroeconomic variables, and (b) a higher degree of interest rate smoothing, which limits the reaction of the policy rate in presence of shocks hitting inflation and output. Interestingly, our empirical analysis allows us to formally rule out any role for sunspot fluctuations in the 'post-1985' period on the one hand, and a fair amount of interest rate smoothing (ranging from 0.569 to 0.697, according to our 90% confidence interval) by the Federal Reserve, on the other hand. Importantly, our identification-robust approach does not lead us to reject the correct specification of the specified new-Keynesian model during the 'Great Moderation'. Our findings are particularly important in light of a recent paper by Cochrane (2011), who argues that the parameters of Taylor-type rules like that in eq. (3) are not identifiable in prototypical new-Keynesian models. Cochrane (2011), however, considers formulations of the new-Keynesian system which are 'less involved', from a dynamic standpoint, than our 'hybrid' model in eq.s (1)-(4). Table 2 shows that the 'full-information' approach delivers relatively tight confidence sets not only for $\varphi_{\tilde{y}}$ and φ_π, but also for δ (intertemporal elasticity of substitution), α (indexation to past inflation), and κ (slope of the NKPC), which are notoriously difficult to estimate precisely from the data.[15]

[15] It can be noticed that some of the elements of $\hat{\theta}_{s,ML}$ (fifth column of Table 2) lie exactly on the boundaries of the corresponding intervals used to define the grid (e.g. the point estimate of α). This is perfectly consistent with the identification-robust inference approach, see, e.g., Dufour *et al.* (2006, 2009, 2010, 2013).

We then proceed with the 'limited-information' second-step of the '$LR_T \to tR_T$' testing strategy, which requires the inversion of the Anderson and Rubin-type $AR_T(\breve{\theta}_s)$ test for the OR implied by the system of Euler equations (1)-(4) on the 'pre-Volcker' sample. Recall, indeed, that the CER implied by the new-Keynesian model under the hypothesis of determinacy have been rejected by the data on the 'pre-Volcker' sample. The second-step is conducted to establish whether the rejection of the hypothesis of determinacy can be ascribed to the multiple equilibria hypothesis, or to the inability of the estimated system to capture the propagation mechanisms at work in the data. For completeness, we invert the $AR_T(\breve{\theta}_s)$ test not only on the 'pre-Volcker' sample, but also on the 'post-1985' sample, albeit this calculation would not be required by our testing strategy (recall that we have accepted the new-Keynesian system on the 'post-1985' sample in the previous step). We pre-fix the nominal type-I error η_2 at the level η_2=0.10.

The results of this second-step are summarized in Table 3. In the upper panel, we report the projected confidence intervals for the individual elements of θ_s derived with Dufour's (1997) method from the identification-robust confidence set $\mathcal{C}^{*AR}_{0.90}$, along with the point estimate obtained from the problem in eq. (29) replacing $\mathcal{C}^{AR}_{1-\eta_2}$ with $\mathcal{C}^{*AR}_{0,90}$. In the lower panel, we indicate whether the grid-testing procedure leads to an empty or nonempty identification-robust confidence set and, in the second case, we report the value of the test statistic associated with the point estimate $\hat{\theta}_{s,LI}$ and corresponding p-value.

Table 3 shows that the new-Keynesian model is not rejected by the $AR_T(\breve{\theta}_s)$ test on the 'pre-Volcker' sample (the set $\mathcal{C}^{*AR}_{0.90}$ is nonempty and the p-value associated with the 'least rejected' model is 0.14). As expected, we also find that the new-Keynesian model is not rejected by the $AR_T(\breve{\theta}_s)$ test on the 'post-1985' sample (the set $\mathcal{C}^{*AR}_{0.90}$ is nonempty and the p-value associated with the 'least rejected' model is 0.37). This is a 'reassuring' result, as it corroborates the outcome obtained with the $LR_T(\hat{\phi}_{\breve{\theta}_s})$ test in the first-step. Moreover, if we compare the projected identification-robust confidence intervals built with the 'full-information' method (sixth column of Table 2) with the corresponding intervals built with the 'limited-information' method (sixth column of Table 3), we find that the former are remarkably more informative than the latter. This result confirms that 'full-information' methods designed to deal with identification failure provide more precise information than 'limited-information' approaches.

By combining the evidence in Table 3 with that in Table 2, we argue that if one interprets the U.S. business cycle through the lens of the estimated (and not rejected) new-Keynesian system (1)-(4), any inference based on finite-order structural VARs on the 'pre-Volcker' sample is inherently misspecified. Indeed, our test suggests that the 'right' time series model for X_t:=$(\tilde{y}_t, \pi_t, R_t)'$ on the 'pre-Volcker' period belongs to the class of VARMA-type systems in eq.s (11)-(12). Accordingly, any finite-order VAR for X_t would represent a truncated approximation to the actual equilibrium and might in principle return largely incorrect estimates of the impulse response function and the parameters of interest; see e.g. Ravenna (2007) for a similar point.

Overall, we can conclude that the '$LR_T \to AR_T$' testing strategy leads us to accept the hypothesis of indeterminacy (H_1' in eq. (16)) on the 'pre-Volcker' sample, for which the set $\mathcal{C}_{0.90}^{*LR}$ is empty and the set $\mathcal{C}_{0.90}^{*AR}$ is nonempty, and not to reject the hypothesis H_0' in eq. (15) on the 'Great Moderation' sample, for which the set $\mathcal{C}_{0.90}^{*LR}$ is nonempty. Our conclusions are consistent with the occurrence of a policy switch in the late 1970s. Our prior-free approach maximizes the role attached to the data in determining these results.[16]

6 Relation to the literature

Our paper has several connections with the literature. On the methodological side, our analysis is related to the recent works of Guerron-Quintana *et al.* (2013) and Dufour *et al.* (2013) on identification-robust frequentist inference in DSGE models. The first-step of our testing procedure is essentially based on the pointwise inversion of the likelihood ratio test proposed by Guerron-Quintana *et al.* (2013) as a tool to build identification-robust confidence sets for the structural parameters. Our methodology is also connected to the contributions by Stock and Wright (2000), Kleiberger and Mavroeidis (2009) and Dufour *et al.* (2006, 2009, 2010, 2013), among others. Indeed, conditional on the first-step, the second-step of the suggested testing strategy requires the pointwise inversion of an Anderson Rubin-type test for the OR implied by the system of Euler equations. Compared to Fanelli (2012), who proposes a test for determinacy/indeterminacy in new-Keynesian models controlling for the omission of propagation mechanisms, our procedure is robust to identification failures and can be applied regardless of the strength of identification. Moreover, the logic of the test and its properties are completely different: while we test the OR in the system of Euler equations only if the CER obtained under determinacy are rejected in the first-step, in Fanelli (2012) the CER obtained under determinacy are tested in a second-step, conditionally on the OR implied by the system of Euler equations

[16] An approximate and purely indicative measure of the extent of the change characterizing the parameters of the model across the two regimes can be broadly obtained by comparing the identification-robust confidence intervals and the point estimates reported in Table 2 and Table 3. For instance, we find that as for the parameters δ (intertemporal elasticity of substitution) α (indexation to past inflation), φ_π (long run reaction to inflation) and ρ_π (inflation shock persistence), the 'full-information' point estimates computed on the 'post-1985' sample (see the fifth column of Table 2) do not lie within (or lie on the border of) the corresponding 'limited-information' identification-robust confidence intervals computed on the 'pre-Volcker' sample (see the fourth column of Table 3). Evidence of instability in the parameters of the private sector, other than the policy parameters, has also been found, among others, by Canova (2009), Inoue and Rossi (2011*a*), Canova and Menz (2011), Canova and Ferroni (2012), Castelnuovo (2012a), and Cantore *et al.* (2013).

not being rejected in the first-step.

Finally, it worth stressing that our testing approach is not related to situations in which the agents know that an economy fluctuates between determinate and indeterminate states driven by a Markov-switching process as in e.g. Farmer *et al.* (2009).

On the empirical side, Lubik and Schorfheide (2004) test for determinacy in the U.S. economy with a model similar to ours, by undertaking a Bayesian investigation in which posterior weights for the determinacy and indeterminacy regions of the parameter space are constructed and compared. Our paper implements a frequentist approach, which neither requires the use of a-prior distributional assumptions, nor the commitment to non-standard inference. In particular, we are not forced to choose a prior distribution for some arbitrary auxiliary parameters that index the multiplicity of solutions under rational expectations as in Lubik and Schorfheide (2004). With respect to Boivin and Giannoni (2006), our method is based on the direct estimation of the structural new-Keynesian model and provides a direct control for the cases of identification failure and dynamic misspecification. Hence, we need not minimize the distance between some selected impulse responses taken from a VAR modeling the macroeconomic variables of interest and the structural model-based responses, a methodology which is unfortunately bias-prone as for expectations-based models like ours (Canova and Sala, 2009). More importantly, we need not make restrictive assumptions on the solution under indeterminacy, as opposed to the MSV solution assumed by Boivin and Giannoni (2006). While being plausible, such solution is anyhow arbitrary, and it may importantly affect the simulated moments of interest (Castelnuovo, 2012*b*).

Mavroeidis (2010) applies identification-robust 'limited-information' methods to investigate the determinacy/indeterminacy of U.S. monetary policy conditional on the estimation of the policy rule in isolation. Compared to Mavroeidis (2010), we investigate the issue of macroeconomic stability of U.S. monetary policy by using a fully specified 'hybrid new-Keynesian model' à la Benati and Surico (2009), and apply a testing strategy which combines 'limited-' and 'full-information' methods and is robust to identification failure. Mavroeidis (2010) conjectures that the difference between the (precise) confidence intervals in the 'pre-Volcker' period and the (imprecise) ones in the 'post-Volcker' phase may be interpreted as (a) absence of sunspot fluctuations during the 'Great Moderation'; (b) increase in the policy inertia; (c) larger variability of the policy shocks during the first years of the Volcker era. Our methodology formally shows that sunspot fluctuations are unlikely to have played a role during the 'Great Moderation'. We therefore offer statistical support to Mavroeidis' conjecture (a). Differently, we do not find clear evidence in favor of an increase in the policy inertia when moving from our first to our second sub-sample. However, the confidence interval surrounding the point estimate of the degree of interest rate smoothing during the 'Great Moderation' does not exclude Mavroeidis' second conjecture (b) either. Finally, our 'Great Moderation' sub-sample begins in 1985, i.e., after the end of the 'Volcker experiment' related to the targeting of non-borrowed reserves by the Federal Reserve. Hence, our results are not necessarily driven by a large volatility of the policy shocks, whose variance has drastically reduced since 1985 (see Mavroeidis (2010), Figure 3 - left panel). More importantly, however, we show that, when applying a system based 'full-information' approach designed to handle weak identification, the precision of the estimates obtained for the 'Great Moderation' sample is higher than the one achieved via a single-equation approach.

7 Concluding remarks

This paper has proposed and implemented a novel identification-robust approach to test the null hypothesis that a fully specified small-scale new-Keynesian monetary policy model has a reduced form consistent with the unique stable solution, versus the alternative of indeterminacy. The testing strategy is designed such that when the null hypothesis is rejected, a second-step is run to establish whether the rejection is due to the occurrence of multiple equilibria or to the omission of relevant propagation mechanisms from the specified system of structural Euler equations. Our methodology can be applied regardless of the strength of identification of the structural parameters, and it requires neither the use of prior distributions nor that of nonstandard inference. Hence, our procedure works in favor of reducing the degree of arbitrariness of our empirical results.

We have applied our novel methodology to a standard dataset of U.S. macroeconomic data by using the new-Keynesian framework recently employed by Benati and Surico (2009) as our reference structural model. The results of our testing strategy conform to the case of a switch from indeterminacy to a framework consistent with determinacy, in correspondence to the advent of Paul Volcker as Chairman of the Federal Reserve. Nevertheless, it is not possible to claim that our analysis supports the hypothesis of a unique equilibrium after Volcker. With respect to Mavroeidis (2010), who works with a single-equation 'limited-information' approach, we find tighter confidence bands for our estimated parameters. We attribute this difference to the 'full-information' nature of the first-step of our robust test and to the fact that the estimated new-Keynesian model is not rejected by the data on the 'Great Moderation' period.

To be clear, our findings, which line up with a number of previous contributions in the literature, are consistent with, but do not necessarily point to, the 'good policy' explanation of the U.S. Great Moderation. In light of the recent financial crisis, our analysis as for the period mid-1980s-onwards may very well be over. When enough data become available, our methodology will help to shed further light on this issue.

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References

[1] Aitchison J. 1964. Confidence-region tests. *Journal of the Royal Statistical Society, Series B* **26**: 462-476.

[2] An S, Schorfheide F. 2007. Bayesian analysis of DSGE models. *Econometric Reviews* **26**: 113-172.

[3] Anderson TW, Rubin H. 1949. Estimation of the parameters of a single equation in a complete system of stochastic equations. *Annals of Mathematical Statistics* **20**: 46-63.

[4] Andrews DWK, Cheng X. 2012. Estimation and inference with weak, semi-strong, and strong identification. *Econometrica* **80**: 2153-2211.

[5] Andrews I, Mikusheva A. 2014. Maximum likelihood inference in weakly identified DSGE models. *Quantitative Economics*. Forthcoming.

[6] Bårdsen G, Fanelli L. 2014. Frequentist evaluation of small DSGE models. *Journal of Business and Economic Statistics*. Forthcoming.

[7] Benati, L. 2008. Investigating inflation persistence across monetary regimes. *Quarterly Journal of Economics* **123**: 1005-1060.

[8] Benati L, Surico P. 2009. VAR analysis and the Great Moderation. *American Economic Review* **99**: 1636-1652.

[9] Beyer A, Farmer REA. 2007. Testing for indeterminacy: an application to U.S. monetary policy: Comment. *American Economic Review* **97**: 524-529.

[10] Binder M, and Pesaran MH. 1995. Multivariate rational expectations models and macroeconomic modelling: a review and some new results. In *Handbook of Applied Econometrics*, Pesaran MH, Wickens M (eds.). Blackwell: Oxford; 139-187.

[11] Boivin J, Giannoni MP. 2006. Has monetary policy become more effective? *Review of Economics and Statistics* **88**: 445-462.

[12] Cantore C, Ferroni F, Leon-Ledesma M. 2013. The dynamics of hours worked and technology, University of Surrey, Bank de France, and University of Kent, mimeo.

[13] Canova F, Menz T. 2011. Does money matter in shaping domestic business cycles? An international investigation. *Journal of Money, Credit and Banking* **43**: 577-609.

[14] Canova F, Ferroni F. 2011. Multiple filtering devices for the estimation of cyclical DSGE models. *Quantitative Economics* **2**: 73-98.

[15] Canova F, Ferroni F. 2012. The dynamics of US inflation: Can monetary policy explain the changes? *Journal of Econometrics* **167**: 47-60.

[16] Canova F, Gambetti L, Pappa E. 2008. The structural dynamics of U.S. output and inflation: What explains the changes? *Journal of Money, Credit and Banking* **40**: 369-388.

[17] Canova F, Sala L. 2009. Back to square one: identification issues in DSGE models. *Journal of Monetary Economics* **56**: 431-449.

[18] Castelnuovo E, Surico P. 2010. Monetary policy, inflation expectations and the Price Puzzle. *Economic Journal* **120**: 1262-1283.

[19] Castelnuovo E. 2012*a*. Estimating the evolution of money's role in the U.S. monetary business cycle. *Journal of Money, Credit and Banking* **44**: 23-52.

[20] Castelnuovo E. 2012*b*. Policy switch and the Great Moderation: the role of equilibrium selection. *Macroeconomic Dynamics* **16**: 449-471.

[21] Christiano L, Motto R, Rostagno M. 2013. Risk shocks. *American Economic Review*. Forthcoming.

[22] Clarida RJ, Galì J, Gertler M. 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. *Quarterly Journal of Economics* **115**: 147-180.

[23] Cochrane J. 2011. Determinacy and identification with Taylor rules, *Journal of Political Economy* **119**: 565-615.

[24] Del Negro M, Schorfheide F. 2009. Monetary policy analysis with potentially misspecified models. *American Economic Review* **99**:1415-50.

[25] Del Negro M, Schorfheide F, Smets F, Wouters R. 2007. On the fit of New Keynesian models. *Journal of Business and Economic Statistics* **25**: 123-143.

[26] Dufour J-M. 1997. Some impossibility theorems in econometrics, with applications to structural and dynamic models. *Econometrica* **65**: 1365-1389.

[27] Dufour J-M., Khalaf L, Kichian M. 2006. Inflation dynamics and the New Keynesian Phillips Curve: an identification robust econometric analysis. *Journal of Economic Dynamics and Control* **30**: 1707-1728.

[28] Dufour J-M, Khalaf L, Kichian M. 2009. Structural multi-equation macroeconomic models: identification-robust estimation and fit. *Bank of Canada, Working Paper* 2009-10.

[29] Dufour J-M, Khalaf L, Kichian M. 2010. On the precision of Calvo parameter estimates in structural NKPC models, *Journal of Economic Dynamics and Control* **34**: 1582-1595.

[30] Dufour J-M, Khalaf L, Kichian M. 2013. Identification-robust analysis of DSGE and structural macroeconomic models. *Journal of Monetary Economics* **60**: 340-350.

[31] Estrella A, Fuhrer JC. 2003. Monetary policy shifts and the stability of monetary policy models. *Review of Economics and Statistics* **85**: 94-104.

[32] Evans G, Honkapohja S. 1986. A complete characterization of ARMA solutions to linear rational expectations models. *Review of Economic Studies* **53**: 227-239.

[33] Fanelli L. 2012. Determinacy, indeterminacy and dynamic misspecification in Linear Rational Expectations models. *Journal of Econometrics* **170**: 153-163.

[34] Farmer R, Guo J. 1995. The econometrics of indeterminacy: an applied study. In: *Carnegie–Rochester Conferences on Public Policy*, Vol. 43; 225-271.

[35] Farmer R, Waggoner DF, Zha T. 2009. Understanding markov switching Rational Expectations models. *Journal of Economic Theory* **144**: 1849-1867.

[36] Fuhrer JC, Rudebusch G. 2004. Estimating the Euler equation for output, *Journal of Monetary Economics* **51**: 1133-1153.

[37] Fukać M, Pagan A. 2006. Issues in adopting DSGE models for use in the policy process, CAMA *Working Paper* 10-2006.

[38] Goodfriend M, King R. 2005. The incredible Volcker disinflation. *Journal of Monetary Economics* **52**: 981-1015.

[39] Guerron-Quintana P, Inoue A, Kilian L. 2013. Frequentist inference in weakly identified DSGE models. *Quantitiative Economics* **4**: 197-229.

[40] Hansen LP, Sargent TJ. 1980. Formulating and estimating dynamic linear rational expectations models. *Journal of Economic Dynamics and Control* **2**: 7-46.

[41] Hansen LP, Sargent TJ. 1981. *Linear rational expectations models for dynamically interrelated variables*. In *Rational Expectations and Econometric Practice*, Lucas RE Jr, Sargent TJ (eds). Minneapolis: University of Minnesota Press; 127-156.

[42] Herbst E, Schorfheide F. 2012. Evaluating DSGE model forecasts of comovements. *Journal of Econometrics* **171**: 152-166.

[43] Inoue A, Rossi B. 2011*a*. Identifying the sources of instabilities in macroeconomic fluctuations. *Review of Economics and Statistics* **93**: 1186-1204.

[44] Inoue A, Rossi B. 2011*b*. Testing for identification in possibly nonlinear models. *Journal of Econometrics* **161**: 246-261.

[45] Iskrev N. 2008. Evaluating the information matrix in linearized DSGE models. *Economic Letters* **99** : 607-610.

[46] Iskrev N. 2010. Local identification in DSGE models. *Journal of Monetary Economics* **57**: 189-202.

[47] Kleibergen F. 2005. Testing parameters without assuming that they are identified. *Econometrica* **73**: 1103-1123.

[48] Kleibergen F, Mavroeidis S. 2009. Weak instrument robust tests in GMM and the New Keynesian Phillips Curve. *Journal of Business and Economic Statistics* **27**: 293-311.

[49] Komunjer S, Ng S. 2011. Dynamic identification in DSGE models. *Econometrica* **79**: 1995-2032.

[50] Lubik TA, Schorfheide F. 2003. Computing sunspot equilibria in Linear Rational Expectations models. *Journal of Economic Dynamics and Control* **28**: 273-285.

[51] Lubik TA, Schorfheide F. 2004. Testing for indeterminacy: an application to U.S. monetary policy. *American Economic Review* **94**: 190-217.

[52] Lubik TA, Surico P. 2009. The Lucas critique and the stability of empirical models. *Journal of Applied Econometrics* **25**: 177-194.

[53] Mavroeidis S. 2005. Identification issues in forward-looking models estimated by GMM, with an application to the Phillips curve. *Journal of Money Credit and Banking* **37**: 421-448.

[54] Mavroeidis S. 2010. Monetary policy rules and macroeconomic stability: some new evidence. *American Economic Review* **100**: 491-503.

[55] Mavroeidis S, Plagborg-Møller M, Stock JH. 2014. Empirical evidence on inflation expectations in the new Keynesian Phillips curve. *Journal of Economic Literature* **52**: 124-188.

[56] McCallum B. 1983. On non-uniqueness in rational expectations models: An attempt at perspective. *Journal of Monetary Economics* **11**: 139-168.

[57] McConnell MM, Pérez-Quirós G. 2000. Output fluctuations in the United States: what has changed since the early 1980's? *American Economic Review* **90**: 1464-1476.

[58] Mikusheva A. 2010. Robust confidence sets in the presence of weak instruments. *Journal of Econometrics* **157**: 236-247.

[59] Pesaran HM. 1987. *The limits to rational expectations*, Basil Blackwell, Oxford.

[60] Phillips PCB. 1989. Partially identified econometric models, *Econometric Theory* **5**: 181-240.

[61] Ravenna F. 2007. Vector autoregressions and reduced form representations of DSGE models. *Journal of Monetary Economics* **54**: 2048-2064.

[62] Sargan JD. 1983. Identification and lack of identification, *Econometrica* **51**: 1605-1633.

[63] Schorfheide F. 2011. Estimation and evaluation of DSGE models: progress and challenges, *NBER Working Paper* n.16781.

[64] Silvapulle MJ, Sen PK. 2005. *Constrained statistical inference*. Wiley.

[65] Staiger D, Stock JH. 1997. Instrumental variables regressions with weak instruments. *Econometrica* **65**: 557-586.

[66] Stock JH, Watson M. 2002. Has the business cycle changed and why? *NBER Macroeconomics Annual* **17**: 159-218.

[67] Stock JH, Wright JH. 2000. GMM with weak identification. *Econometrica* **68**: 1055-1096.

[68] Woodford M. 2003. *Interest and prices*. Princeton University Press, Princeton.

TABLES

Table 1. Empirical size of the '$LR_T \rightarrow AR_T$' testing strategy when the data are generated from the new-Keynesian system (5)-(6) under the hypothesis H_0' in eq. (15).

'true' $\theta_{0,s}$		T=100	η_1=0.10	
$\lambda_{\max}(G(\theta_{0,s}))$:=0.964	Interpret.	$\hat{\theta}_{s,ML}$	Average proj. 90% c.i. & grid intervals	
γ_0:=0.744	IS, forward looking term	0.694 (0.206)	[0.728-0.784]	[0.688-0.822]
δ_0:=0.124	IS, inter. elast. of substitution	0.117 (0.038)	[0.113-0.141]	[0.090-0.160]
α_0:=0.059	NKPC, indexation past inflation	0.058 (0.026)	[0.047-0.081]	[0.030-0.099]
κ_0:=0.044	NKPC, slope	0.041 (0.013)	[0.039-0.051]	[0.035-0.056]
ρ_0:=0.834	Rule, smoothing term	0.747 (0.224)	[0.772-0.841]	[0.515-0.877]
$\varphi_{\widetilde{y},0}$:=1.146	Rule, reaction to output gap	0.925 (0.434)	[0.705-1.237]	[0.383-1.610]
$\varphi_{\pi,0}$:=1.749	Rule, reaction to inflation	1.463 (0.637)	[1.228-1.917]	[0.700-2.570]
$\rho_{\widetilde{y},0}$:=0.796	Output gap shock, persistence	0.729 (0.215)	[0.765-0.818]	[0.738-0.834]
$\rho_{\pi,0}$:=0.418	Inflation shock, persistence	0.378 (0.126)	[0.356-0.462]	[0.300-0.520]
$\rho_{R,0}$:=0.404	Policy rate shock, persistence	0.371 (0.125)	[0.354-0.453]	[0.289-0.518]
		Rej($LR_T(\hat{\phi}_{\hat{\theta}_{s,ML}})$)=0.079	Rej($LR_T(\hat{\phi}_{\theta_{0,s}})$)=0.121	

NOTES. Results are obtained using M=1,000 replications. Each simulated sample is initiated with 200 additional observations to get a stochastic initial state and then are discarded. The structural parameters are calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'After the Volcker stabilization'. The numerical inversion of the $LR_T(\hat{\phi}_{\breve{\theta}_s})$ test for the CER (first-step) is obtained on each generated dataset by considering 300 points $\breve{\theta}_s$ randomly chosen (using the uniform distribution) from the grid delimited by the rectangle formed by the Cartesian product of the intervals reported in the last column. '$\hat{\theta}_{s,ML}$' is the point estimates of θ_s obtained from the problem in eq. (22) replacing $\mathcal{C}^{LR}_{1-\eta_1}$ with $\mathcal{C}^{*LR}_{0.90}$, and the associated values in parentheses are the corresponding Monte Carlo standard errors. 'Average proj. 90% c.i. & grid intervals' reports the average projected 90% confidence interval computed as in Dufour (1997) and the actual intervals used for the individual parameters in the grid-testing procedure. 'Rej(·)' stands for 'rejection frequency'. $LR_T(\hat{\phi}_{\theta_{0,s}})$ is the test statistic for the hypothesis $H_{0,cer}$ in eq. (20) evaluated at the specific point $\breve{\theta}_s = \theta_{0,s}$, see Section 4 in the Supplementary Material.

Table 2. Projected 90% identification-robust confidence intervals, point estimates of the structural parameters θ_s:=$(\gamma, \delta, \alpha, \kappa, \rho, \varphi_{\tilde{y}}, \varphi_\pi, \rho_{\tilde{y}}, \rho_\pi, \rho_R)'$ and results of the first-step of the '$LR_T \rightarrow AR_T$' testing strategy on U.S. quarterly data.

		1954q3-1979q2 'pre-Volcker'		1985q1-2008q3 'Great Moderation'	
Parameter	Interpretation	$\hat{\theta}_{s,ML}$	proj. 90% c.i.	$\hat{\theta}_{s,ML}$	proj. 90% c.i.
γ	IS, forward looking term	-	-	0.729	0.652-0.772
δ	IS, inter. elast. of substitution	-	-	0.082	0.082-0.154
α	NKPC: indexation past inflation	-	-	0.020	0.020-0.059
κ	NKPC: slope	-	-	0.048	0.042-0.098
ρ	Rule, smoothing term	-	-	0.666	0.569-0.697
$\varphi_{\tilde{y}}$	Rule, reaction to output gap	-	-	0.339	0.127-0.479
φ_π	Rule, reaction to inflation	-	-	5.439	2.318-5.445
$\rho_{\tilde{y}}$	Output gap shock, persistence	-	-	0.920	0.720-0.978
ρ_π	Inflation shock, persistence	-	-	0.925	0.748-0.970
ρ_R	Policy rate shock, persistence	-	-	0.794	0.730-0.806
identification-robust c.s. $\mathcal{C}^{*LR}_{0.90}$		empty		nonempty (card($\mathcal{C}^{*LR}_{0.90}$)=15)	
$\lambda_{\max}(G(\hat{\theta}_{s,ML}))$		–		0.946	
$LR_T(\hat{\phi}_{\hat{\theta}_{s,ML}})$ test (first-step)		–		19.54 [0.36]	

NOTES. The projected 90% identification-robust confidence intervals (proj. 90% c.i.) have been obtained from the 90% identification-robust confidence set $\mathcal{C}^{*LR}_{0.90}$ (see eq. (30)) as in Dufour (1997). The set $\mathcal{C}^{*LR}_{0.90}$ has been obtained by inverting numerically the $LR_T(\hat{\phi}_{\breve{\theta}_s})$ test considering 5,000,000 points $\breve{\theta}_s$ chosen randomly (using the uniform distribution) from the rectangle formed by the Cartesian product of the following intervals: [0.65, 0.85] for γ, [0.08, 0.16] for δ, [0.02, 0.10] for α, [0.04, 0.10] for κ, [0.50, 0.70] for ρ, [0.05, 1.5] for $\varphi_{\tilde{y}}$, [0.5, 5.5] for φ_π, [0.40, 0.98] for $\rho_{\tilde{y}}, \rho_\pi$ and ρ_R. '$\hat{\theta}_{s,ML}$' is the point estimate derived from the problem in eq. (22) replacing $\mathcal{C}^{LR}_{1-\eta_1}$ with $\mathcal{C}^{*LR}_{0.90}$. $LR_T(\hat{\phi}_{\hat{\theta}_{s,ML}})$ correspondes to the value of the test statistics obtained in correspondence of the 'least rejected' model within $\mathcal{C}^{*LR}_{0.90}$. P-values in brackets. Estimation on each sub-period is carried out by considering within-periods initial values and variables are demeaned within each sub-period.

Table 3. Projected 90% identification-robust confidence intervals, point estimates of the structural parameters $\theta_s{:=}(\gamma, \delta, \alpha, \kappa, \rho, \varphi_{\tilde{y}}, \varphi_\pi, \rho_{\tilde{y}}, \rho_\pi, \rho_R)'$ and results of the second-step of the '$LR_T \to AR_T$' procedure on U.S. quarterly data.

		1954q3-1979q2 'pre-Volcker'		1985q1-2008q3 'Great Moderation'	
Parameter	Interpretation	$\hat{\theta}_{s,LI}$	proj. 90% c.i.	$\hat{\theta}_{s,LI}$	proj. 90% c.i.
γ	IS: forward looking term	0.841	0.660-0.845	0.821	0.650-0.850
δ	IS: inter. elast. of substitution	0.088	0.084-0.160	0.132	0.080-0.160
α	NKPC: indexation past inflation	0.025	0.020-0.070	0.097	0.020-0.099
κ	NKPC: slope	0.042	0.040-0.058	0.087	0.040-0.100
ρ	Rule, smoothing term	0.520	0.500-0.698	0.699	0.500-0.700
$\varphi_{\tilde{y}}$	Rule, reaction to output gap	0.138	0.050-0.325	0.295	0.050-1.043
φ_π	Rule, reaction to inflation	0.687	0.500-0.906	2.123	0.500-5.499
$\rho_{\tilde{y}}$	Output gap shock, persistence.	0.900	0.620-0.964	0.911	0.400-0.980
ρ_π	Inflation shock, persistence.	0.578	0.414-0.793	0.907	0.400-0.980
ρ_R	Policy rate shock, persistence	0.798	0.565-0.916	0.795	0.674-0.980
identification-robust c.s. $\mathcal{C}^{*AR}_{0.90}$		nonempty (card($\mathcal{C}^{*AR}_{0.90}$)=26)		nonempty (card($\mathcal{C}^{*AR}_{0.90}$)=41891)	
$\lambda_{\max}(G(\hat{\theta}_{s,LI}))$		1.012		0.965	
$AR_T(\hat{\theta}_{s,LI})$ test (second-step)		24.44 [0.14]		19.27 [0.37]	

NOTES. The projected 90% identification-robust confidence intervals (proj. 90% c.i.) have been obtained from the 90% identification-robust confidence set $\mathcal{C}^{*AR}_{0.90}$ (see eq. (31)) as Dufour (1997). The confidence sets have been obtained by inverting the test $AR_T(\breve{\theta}_s)$ (second-step); in practice, $AR_T(\breve{\theta}_s)$ is computed as a quasi-LR test using $Z_t{:=}(X'_{t-1}, X'_{t-2})'$ in the auxiliary multivariate regression system (26), considering 5,000,000 points $\breve{\theta}_s$ randomly chosen (using the uniform distribution) from the rectangle formed by the Cartesian product of the same intervals as in Table 2 and imposing the condition $\lambda_{\max}(G(\breve{\theta}_s)){>}1$ on the 1954q3-1979q2 period. '$\hat{\theta}_{s,LI}$' is the point estimate derived the problem in eq. (29) by replacing $\mathcal{C}^{AR}_{1-\eta_2}$ with $\mathcal{C}^{*AR}_{0.90}$. $AR_T(\hat{\theta}_{s,LI})$ reports the value of the test statistics obtained in correspondence of the 'least rejected' model within $\mathcal{C}^{*AR}_{0.90}$. P-values in brackets. Estimation on each sub-period is carried out by considering within-periods initial values and variables are demeaned within each sub-period.