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ORIGINAL ARTICLE

# Revised KAD tool to optimize F1 cars through a combined-elitarian genetic-fuzzy algorithm

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**Abstract** The KAD (Knowledge Aided Design) tool is developed to predict the performance of an F1 car in different driving conditions and with different configurations. The regulations to put in trimming a car, also in the exasperated technology of the competitions, still demand a remarkable dose of luck and an elevated number of tests. It is then important to know a set of regulations close to the optimal trim before testing the car on the track. The difficult phase of this process is to evaluate the lap time. As a matter of fact driving style, track conditions and car behavior should be simulated. The optimisation of the fuzzy controller that simulates the pilot for an F1 racing car is difficult due to handling problems and velocity of response. For this purpose a specific Genetic Algorithm (GA) was conceived and tuned to work with a lumped mass model of an F1 racing car for the optimization of the fuzzy controller that simulates the pilot. A new mutation and a new crossover operator were defined to complement the standard crossover and mutation operators of the basic Holland's GA. This was necessary in order to increase the overall performance of the fuzzy pilot. This approach was tested on an F1 car due to the huge amount of data available (Donnarumma, 1998; Moelenbein, 1989; Lee and Takagi, 1993).


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## 1. Introduction

Some years ago, the project named "Nuvolari-fuzzy" began. Its aim was to provide a tool that would have provided designers with a prediction of the performances of cars in the phase of the conceptual design phase. This tool was thought to be used in a conventional automobile and it should also have been able to set the car in order to obtain possibly the best performances.

However the choice of the car is not easy, since the data are difficult to be obtained.

For this reason, it was decided to chose a F1 car, because a lot of data are known, just like telemetry data, power, masses, aerodynamic forces, tire performances. These data, previously kept secret, are now well known and measured at the best possible level of accuracy thanks to our collaboration with Ferrari F1 Team (Nig and Li, 1994; Rajendra, 1995). So, an old F1 racing car (Ferrari 126C2 of Gilles Villeneuve, 1982, see Figure 1) was chosen in order to study several issues and find relative solutions.

However, it is also important to underline that a model, which takes into consideration easiness of driving, is still lacking. For this reason, in our case a fuzzy model (i.e., a fuzzy model is a model based on fuzzy logic, which is a logic that differs from conventional logical systems because it aims to provide a model for approximate rather than precise reasoning, (Zadeh, 1996; Mamdani, 1976; Dubois and Prade, 1979) of the human pilot and a lumped mass model of a F1 racing car were used. This optimisation operates with an elitarian combined genetic algorithm (Hoffman and Pfister, 1994; Hanagandi et al., 1996; Furuhashi et al., 1994) that optimizes the fuzzy pilot in a specific car (i.e., F1 car). The fuzzy controller that simulates the pilot has several parameters that resemble pilot's driving style (Kargupta, 1996; Hoffman and Pfister, 1996; Ghini, 1998). These parameters are optimised by a specialized genetic algorithm that finds the best pilot for a certain part of the track. The process is repeated from the starting position to the end of the track until a record lap time, or a set of best lap times, can be found. The main advantages of this approach are the feasibility of the best lap by a human pilot and the sensitivity to modifications on the car. As confirmed by several F1 pilots, it is important to take into consideration the four most important parameters which control a racing car, in particular: the power of the engine; the aerodynamical download; the grip of the tyres; the weight. In other words it is important that an F1 car is settled to be easy driven. When the GA finds the record time, it is possible to vary the setting to find whether these variations improve the performances. If another GA is used it is possible to find the best setting possible. That is exactly what has been done and it is summarized in this paper. The paper is organized as follows. Next paragraph (the second one) introduces the overall structure of the optimisation procedure and defines the steps that conduct to the "best" solution. The model of the car is very briefly introduced in the third paragraph. For further information on lumped mass models of cars, with true suspension geometry and a model of the tires see the references nr.(Moelenbein, 1989; A. Lee and Takagi, 1993). The fourth paragraph describes the fuzzy controller that simulates pilot behaviour. A general introduction to fuzzy controller can be found in Zadeh (1996), Mamdani (1976), Dubois and Prade (1979). The combined elitarian GA that optimises the fuzzy pilots is introduced in the fifth paragraph. For this purpose several algorithms are described in Hoffman and Pfister (1994), Hanagandi et al. (1996), Furuhashi et al.(1994).


**Figure 1** The Ferrari 126C2.

## 2. The optimization procedure

The optimization process consists at first in choosing the more suitable racing circuit (which will be Imola circuit), considering that it should be divided into parts that can be covered in less than 10 s each one, by the worst fuzzy driver, so that a set of adjustments could be defined in terms of aerodynamic, damping, elastic and ground-tire friction coefficient. Then, at second, certain track conditions should be defined (perfect, dry but poor asphalt condition, dry but dirty, wet, very wet).

After this operation a test run is performed to check whether the input data are correct and everything works well. In this run the GA optimises the fuzzy control to obtain the record lap with that car, referred to the selected circuit and the indicated asphalt condition. The best fuzzy control is found for each sector of the track. The record lap time is calculated as the sum of the times simulated for each sector.

After this step of work, the general GA, which operates on car adjustments, is running. GA (Genetic Algorithm) evaluates the fitness of every setting (or set of adjustments) according to the record lap time of that car with those adjustments. At the end, the best setting can be found. A racing car has several adjustments that can be made directly on the track, starting from rear and anterior wings, to springs, shock absorbers, anterior height, posterior height, the bottom surface and many others. It should be taken into account that the computation of a single record lap time with a certain car of defined setting on a circuit like Imola requires more than two hours on a standard Pentium Personal Computer (PCs). In order to obtain the optimum trim with the normal regulations that can be made on the Ferrari 126C2 racing car with the aid of a simple screw-driver (about 20 regulations or 20 DOF to be optimised) the Holland type GA algorithm (Moelenbein, 1989) requires about 150 runs of the combined-elitarian GA that finds the record time for a total amount of 300 h (about 12 days). The optimisation should then be performed on a net of PCs in parallel (the ideal number is 10, one for each individual of the population) and it is still very time-consuming. The car model, the fuzzy controller optimizer and the car set optimiser were then kept as slim and efficient as possible in order to reduce computation time. A 1000 times faster PC is only adequate for this application.

## 3. The car model

The lumped mass model of the F1 racing car of Gilles Villeneuve's Ferrari 126C2 was implemented in 1984 and was refined to its final version of 1998. The model takes into account suspension geometry, non-linear spring-damper assembly, frame stiffness, concentrated aerodynamic loads, engine performances, and tire-soil contact. The model DOF (Degrees Of Freedom) are 14. These are the 3 translations and 3 rotations (pitch, roll and yaw) of the orthonormal reference system X, Y, Z (with X along the car axes and Z vertical that comes out of the ground) and the 8 flexural movements (4 vertical and 4 lateral), due to the excursion of suspensions. In other terms it can be asserted that the translations along the axes X and Y, together with the rotation movement (spin) around to the Z axis, interest and describe the motion of the car in its complex, while the translation along the Z axis and the rotation around X axis (roll) and Y (pitch) interest the elastic motion of the suspended part. It should be also pointed out that all the forces are applied in the contact point wheel-ground: the calculation of the equilibrium of the car-body, regarding to the ground, makes it possible to calculate the accelerations, the speeds (by integration vs. time) and the coordinates of the points of the suspensions, and this for every step of prefixed time. Without going too much in the details of these calculations, only some fundamental calculation steps can be summarized. Given the position of the suspended mass relatively to the step i-1, the forces and the aerodynamic moments applied on the car are reiterated and the coordinates of the points of the suspensions attached to the chassis are found (in this phase the programs considers also the yielding of the tire, that is responsible of a further displacement of the tire-ground contact point and a variation of its distance from the wheel holder). Given the positions of frame-to-suspension connection points, the suspension configuration is iterated until the correct (tire-ground) contact points are found. The true spring lengths, the roll bars angular displacements and the force on shock absorbers are then calculated. At the end equilibrium equation are solved and inertia forces on the unsuspended mass (wheels and wheel holders) are evaluated. The engine-transmission group is simulated by using the characteristic curves of the engine coupled with an automatic transmission that changes the gears automatically just after the point of maximum torque. The new gear has then the maximum available torque. A consistent time-lag in engine response was also introduced as it happened with the 126C2, this time lag depends on an estimation of the speed of the turbo and then of the available intake pressure. This point is very important since in Hoffman and Pfister (1996) Harvey Postlethwhite says: ‘‘That car, the original [Ferrari] 126C turbo had literally one quarter of the down force that, say Williams or Brabham had. It had a power advantage over the Cosworths for sure, but it also had massive throttle lag at that time. In terms of sheer ability I think Gilles was on a different plane to the other drivers. To win those races [the 1981 GPs at Monaco and Jarama] – on tight circuits – was quite out of this world. I know how bad that car was.’’ Then, this car should be a good chance to test a fuzzy pilot and its GA optimizer.

## 4. The fuzzy pilot

Before realizing Fuzzy pilot simulator, it is opportune to introduce the way in which the racing circuit is simulated. The simulator takes into consideration the lumping of every single section of the circuit centerline and considering the track width as a parameter associated to this line. The characteristic sections of track recognized by the simulator are three: the straight track, the right curve and the left curve. The straight track that starts from the point ($x_i$, $y_i$) and ends in the point ($x_f$, $y_f$) is represented as a line of Eq. (1):

$$y = mx + c \tag{1}$$

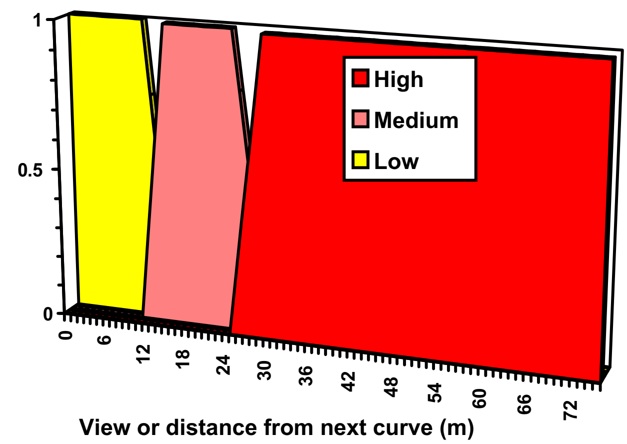


**Figure 2** The ‘‘view’’ membership functions.

where

$$m = \frac{(y_f - y_i)}{(x_f - x_i)} \tag{2}$$

and

$$c = y_i - mx_i \tag{3}$$

$$y = -\frac{1}{m}x + f \tag{4}$$

It is then possible to evaluate the distance from track centerline. This can be obtained from the $f$ coefficient from the Eq. (4) of the line normal to (1). The simulator is also able to evaluate whether the car is on the left or on the right of the centerline by evaluating the direction cosine of the position vector. The curve is individuated by crf center position ($x_c$, $y_c$), the curvature radius $R$ and the curve amplitude in radians. Curve equation is then

$$x^2 + y^2 + \alpha x + \beta y + \gamma = 0 \tag{5}$$

where

$$\alpha = -2x_c, \beta = -2y_c e \gamma - (x_c)^2 + (y_c)^2 + R^2 \tag{6}$$

The distance of the car from the curve center can be evaluated by a system of 2 equations composed by (5) and the equation of the line that passes through ($x_c$, $y_c$) and the car position. This distance d is particularly important for the evaluation of the car condition. In fact if the predicted car future position tends to increase the value of d it is necessary to close the curve. Another information, that the pilot should have, is the presence of very thigh turns. For this simulator thigh turns are curves with a radius inferior to 36 m.

### 4.1. The knowledge base of the fuzzy pilot

As mentioned by important pilots of the past, three input variables are necessary for realizing a car model (or car simulator): throttle position, steer velocity and braking torque.

These three variables are controlled separately. The view is given by the distance to the next curve (see Figure 2). The ‘‘view’’ and the car velocity control the throttle position. By the way the view controls also the integration step of the car simulator. The car velocity membership functions are depicted in Figure 3. The first rule of the controller is the following: *the*

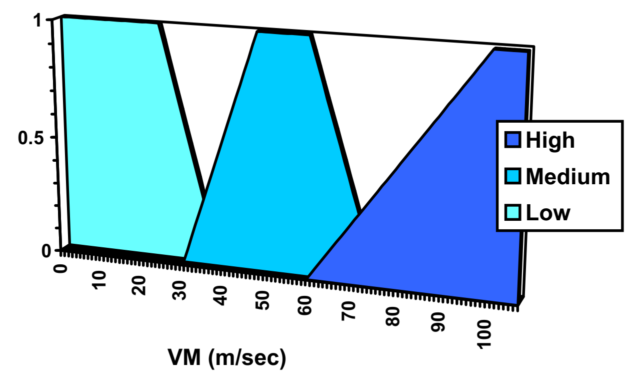


**Figure 3** Car velocity input membership functions.

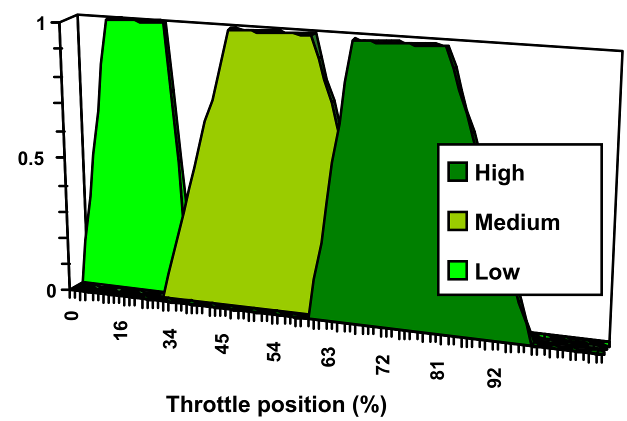


**Figure 4** Output throttle position membership functions.

*throttle should be increased if the view is high and the velocity is low*. An example of throttle position is given Figure 4. The second output membership function is the steering speed. It depends from two variables: the distance from the center line (see Figure 5) and the curvature radius (see Figure 6). The distance membership functions have been thickened in order to improve the correction ratio, as the car is closer to track margins. The second fuzzy rule of the fuzzy pilot is the following: *it is necessary to increment steer velocity as the distance from centerline increases and the curvature radius diminishes*. Steering velocity output membership functions are depicted in Figure 7. The third and last variable is the braking torque. The braking torque is applied only when the distance from the centerline is more than 35% of the available track width (and it is a curve) or when the danger signal is encountered. Input variables of the brake controllers are the distance from the centerline and car velocity.

## 5. The fuzzy pilot optimizer for record lap time determination

The racing circuit is subdivided into sectors than can be covered in less than 10 s each (see § 5.2). For each sector a best fuzzy pilot is defined. The record lap time is given by the sum of the record time for each sector. The car is started in the starting position of the track with the car still with motor running. The first three sectors are then used to define the initial condition of the fourth one. The first three partial times are discarded. Then the finishing condition of the sector i-1 is given as initial condition of the i sector. The choice of the sectors influences final results, but an accurate selection is enough to reduce this influence to a minimum. The time is calculated only in the current sector but the controller should negotiate with success also the following. As the car returns on the starting position the three initial sectors are re-negotiated in order to obtain the record lap time.

### *5.1. The initial population*

Every fuzzy controller (pilot) is defined by three controllers (throttle position, steering speed and braking torque). The different individuals are differentiated by the input and output membership functions. Each pilot is then coded by the genes relative to the input/output membership functions and by the gain constants. These constants amplify the response of the pilot and correspond to the aggressiveness of the pilot. Each pilot is then defined by 169 real numbers. Ten reasonable pilots are defined as an initial population. The number of ten is given by a compromise between convergence speed and quality of results. The reasonable pilots are defined by a set of pilots that are able to keep the car in the track in most conditions. A stochastic choice of the initial genes would have put on the car a set of poor pilots very far away from an F1 driver.

### *5.2. The fitness function*

The fitness function is defined as follows for each sector of the racing circuit.

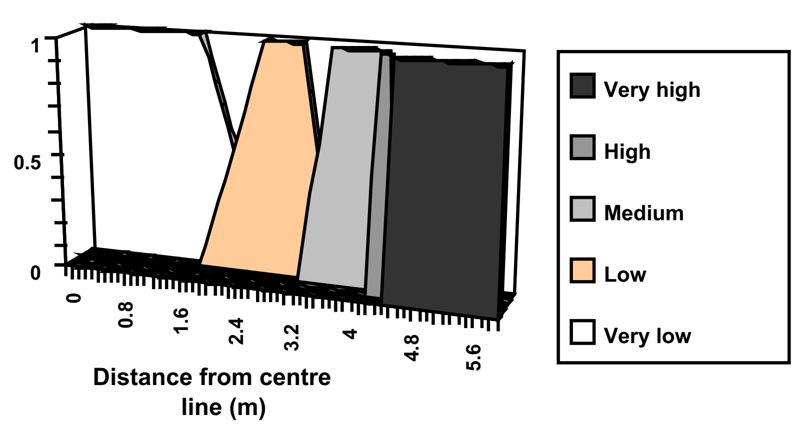


**Figure 5** Input membership functions for the input parameter distance from centre line. In this case the track width was fixed to 10 m.

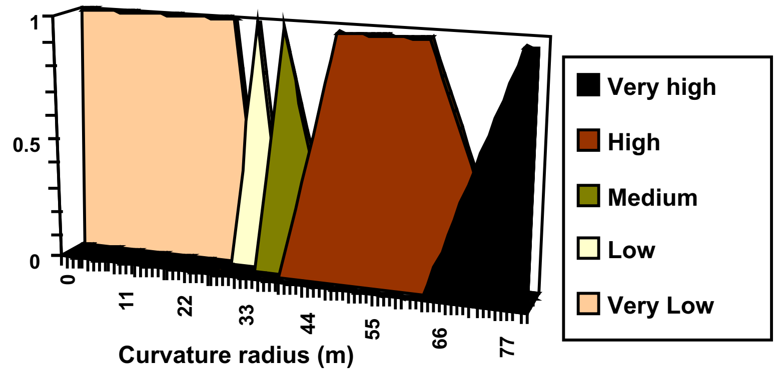


**Figure 6** Radius input membership functions.

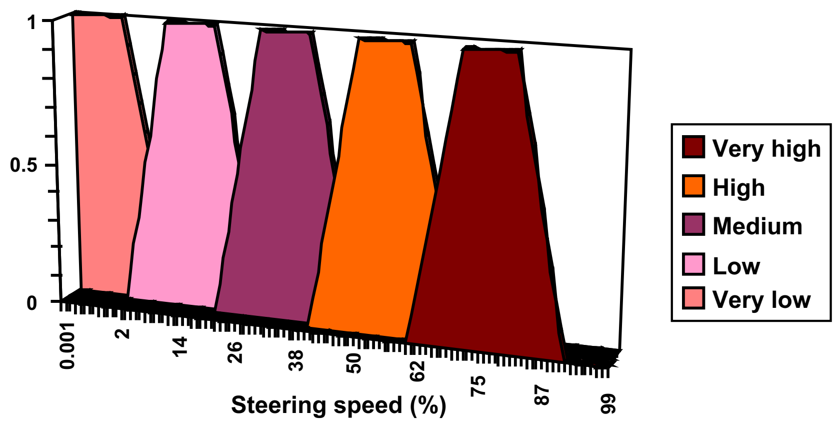


**Figure 7** Steering velocity output membership functions

$$fitness = \frac{100}{time} e^{\frac{10}{time}} \tag{6}$$

This fitness is particularly efficient in the selection of the best pilots and works as a penalty functions for the slowest controllers. The factor 10 is the maximum time allowed to cover a certain sector. The gain factor 100 is given to assign values close to 0 for the worst pilots and close to 1000 for the best.

### *5.3. The genetic operators*

In this optimizer two modified operators, that will operate only on some genes with a selective crossover procedure, assist the ‘‘standard’’ GA operators. The standard crossover operator makes the media of all the real genes of the parents and obtains a son with the multiple crossover approach. The additional factors that control the crossover and the generation of a new population are the probability (0–100) and the points (1–4). These number can be given as input to the GA and control the convergence ratio and quality of the solution as it will be see in § 6.. In fact when the probability is 100 all the individuals can be coupled. When its value is 80 only the first 8 individuals can be coupled, when it is 20 only two of the individuals can be coupled. The coupling is stochastic and the best ranking individuals have more probability to be coupled than the others. The influence of the parameter point is different from the standard crossover and for the modified crossover. The value of points indicates the number of points where the crossover takes place(1 $\Rightarrow$ 1 point, 2 $\Rightarrow$ 2 points, 3 $\Rightarrow$ 3 points). If point is equal to 4 it means that also the gain constants are interested by the crossover. The standard crossover does not influence the ranges of the input variables. The best individual takes part to the crossover but is inherited by the following generation without being subject to crossover. The modified crossover works in the same way of the ‘‘standard’’ crossover but operates on the output membership functions. The standard mutation and the modified mutation operate with the same philosophy of the crossover operators. The standard mutation works on the input functions while the modified works on the output ones. The sequence is similar to the crossover (1 $\Rightarrow$ 1 point, 2 $\Rightarrow$ 2 points, 3 $\Rightarrow$ 3 points, 4 also gain constants). In our case the mutation cannot be purely stochastic, otherwise the result would have been unsignificant. In fact the coordinates of the membership functions are incremented or decremented if the pilot ranking is odd or even. The amount of these changes is regulated by an ad-hoc step.

### *5.4. The lap time optimisation*

The combined-elitarian GA algorithm calculate the ranking of the population, the maximum ($f_{max}$), the minimum ($f_{min}$) and the media ($f_{medium}$) of the fitnesses. These three values are fundamental for the selection of the operators. Given the $f_{\max}, f_{\min}$ and $f_{\text{medium}}$ of the preceding (i-1) generation ($f_{(i-1)\max}, f_{(i-1)\min}$ and $f_{(i-1)\text{medium}}$ and the same values of the current (i) generation, only the traditional operators are used when:

$$f^{i-1}_{\max}\,1.05 \leqslant f^{i}_{\max}\ \ \boldsymbol{OR}\ \ f^{i-1}_{medium}\,1.05 \leqslant f^{i}_{medium}\ \ \boldsymbol{OR}\ \ f^{i-1}_{\min}\,1.05 \leqslant f^{i}_{\min}$$

Only the combined operators are used when

$$f^{i-1}_{\max}\,1.01 \geqslant f^{i}_{\max}\ \boldsymbol{AND}\ f^{i-1}_{medium}\,1.01 \geqslant f^{i}_{medium}\ \boldsymbol{AND}\ f^{i-1}_{\min}\,1.01 \geqslant f^{i}_{\min}$$

In the case of uncertainty the choice between traditional operators and the combined ones is stochastic.

## 6. Tests

These tests were performed on the Imola racing circuit and were aimed to optimize the GA algorithm for record lap determination. The parameters to be tuned are the probability and the number of points for crossover and mutation. The first tests of the combined elitarian GA were made using the following parameters: crossover, mutation, modified crossover and modified mutation probability = 100%. Points of crossover, mutation, modified crossover and modified mutation = 4. These parameters induce the maximum possible evolution spread. The results are depicted in Fig. 8. Convergence was reached after 34 generations. Other tests with mutation and modified mutation probability reduced to 60% were faster in convergence (25 generations instead of 34) but the maximum fitness value got down to 400. The original value of 100% was then restored. Afterwards the crossover and modified crossover points were reduced down to 3. Again the maximum fitness value was around 400 and the convergence was even faster (19 generations). Also this operation was discarded. The crossover and modified crossover probability were decreased to 60%. This modification proved to be successful. In fact to a slight reduction of maximum fitness (500 instead of 504) has corresponded a significant increase in convergence ratio (28 generations instead of 34). To further increase convergence speed the mutation probability was decreased to 60%. The solution was reached after only 11 generations but the best value was the worst ever had (350 instead of 504). The best solution is then the following:

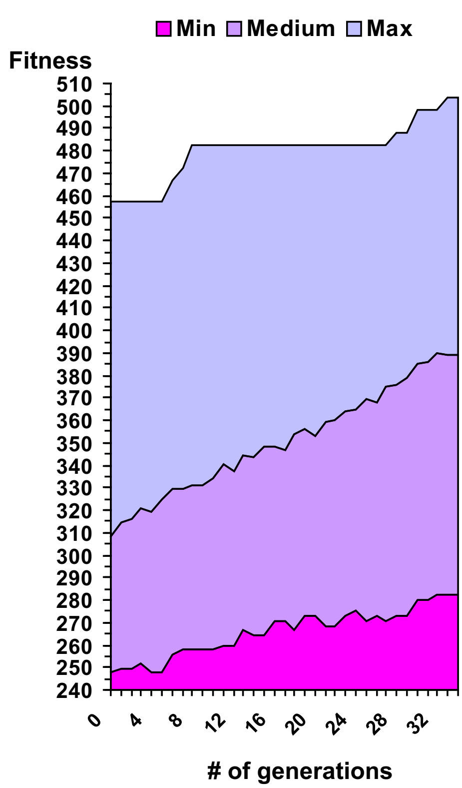


**Figure 8** GA with maximum spread.

- Crossover and modified crossover probability = 60%.
- Mutation and modified mutation probability = 100% (maximum).
- Points of crossover, mutation, modified crossover and modified mutation = 4 (multiple crossover and mutation on all the variables).

At the end our pilot would have qualified for the 1982 Imola Grand Prix but at the 9th row. A single test was also performed on the global optimization algorithm with the following DOF: front and rear spring stiffness, front and rear wing incidence. The result was feasible and close to the setting chosen by the Ferrari team in 1982. However, the direct experimental validation is obviously impossible.

Test results can be consulted in the file attached named "Test_Results_KADToolToOptimizaF1Cars_def".

## 7. Conclusions

As a result of the tests made it is possible to state that the two step optimisation of the F1 car trims and the fuzzy pilot, done using respectively the standard GA and the combined-elitarian GA algorithm, has been successfully implemented for the Ferrari 126C2 of Gilles Villeneuve (1982). As a matter of fact the GA algorithms have been capable of a relevant optimization of the fuzzy pilot, due to this fact it has been possible, at the end of the tests, to calculate a best lap time near to the real F1 worst lap times. As stated in the previous paragraph, the best lap time obtained by the fuzzy pilot has been good enough to be virtually qualified for the 1982 Imola Grand Prix. This demonstrate that even if human pilots are, obviously, better than fuzzy pilots, it is possible to approximate human driving behaviour using a fuzzy logic algorithm. Even if the lap time recorded is higher than 1982 Imola Grand Prix real lap times, it is important to notice that has been possible to obtain reasonable results, so it would be possible in the future to further refine this fuzzy pilot optimizer algorithm, along with the F1 car trim Optimizer algorithm, in order to simulate a better pilot.

The tests made have demonstrated also that the general optimization algorithm for the F1 car trims can give reasonable output results. In particular, as stated in the previous paragraph, the algorithm has indicated spring stiffness and wings incidence settings near to those that have been really implemented on the Ferrari 126C2 during Imola Grand Prix.

To further investigate the optimization possibility of the car trims, and how much they are close to those really implemented, it would be appropriate to make a comparison among tests carried on with different cars on different circuit. It is possible to carry on these additional tests, even if they require a detailed lumped mass model of the different cars, and appropriate models of the tracks. However if these additional tests, as we expect, will give results close to the real ones, the approach described in this paper would be suitable to adapt a certain car on a certain circuit. In this way it would be possible to reach a true optimisation, with the result that several real test sessions may be spared, permising to the racing teams to reach faster the optimal setting for their car.

Regarding to the technical issues related to the system required to run these algorithms, accurate tuning of the two GAs has been performed in order to reduce the computation time. However a very fast PC is required: at least 3 orders of magnitude faster than a Pentium II 300 MHz. Fortunately the GA can be easily parallelized and may run on a net of several computers (Piancastelli et al., 1999).

## Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jksues.2011.06.006.

## References


Donnarumma, A., Disegno di macchine, Utet, 1998.

Dubois, D., Prade, H., 1979. Decision Making under fuzziness. In: Gupta, M.M., Ragade, R.K., Yager, R.R. (Eds.), Advances in fuzzy set theory and applications. North Holland, Amsterdam.

Furuhashi, T., K. Nakaoka e Y. Uchikawa, A New Approach to Genetic Based Machine Learning and an Efficient Finding of Fuzzy Rules. In: Proc. Of the 1994 IEEE World Wisemen/women Workshop of Fuzzy Logic and Neural Networks/Genetic Algorithms, 1994, pp. 114–122.

Ghini, A., Ferrari 1947–1997, Rizzoli International Publications, 1998, ISBN: 0847821528.

Hanagandi, V., Buescher, K., Nikolaou, M., Design of Automatic Controllers Via Optimization Using Genetic Algorithms, Los Alamos National Laboratory, 1996.

Hoffman, F., Pfister, F.e G., Automatic Design of Hierarchical Fuzzy Controllers Using Genetic Algorithms, University of Kiel – Institute of Applied Physics, 1994.

Hoffman, F., Pfister, F.eG., A New Learning Method for the Design of Hierarchical Fuzzy Controllers Using Messy Genetic Algorithms, University of Kiel - Institute of Applied Physics, 1996.

Kargupta, H., Computational Processes In Evolution And The Gene Expression Messy Genetic Algorithm, Los Alamos National Laboratory, 1996.

Lee, M.A., Takagi, H., Embedding Apriori Knowledge into an Integrated Fuzzy System Design Method Based on Genetic Algorithms. In: Proc. of the 5th IFSA World Congress, 1993, pp. 1293–1296.

Mamdani, E.H., 1976. Application of Fuzzy Algorithms for the Control of a Dynamic Plant. Department of Electrical and Electronic Engineering, Queen Mary College, University of London, London.

Moelenbein, H., Parallel Genetic Algorithms, Population Genetics and Combinatorial Optimization. In: Proc. of ICGA '89, 1989, pp. 416–421.

Nig, K.C., Li, eY., Design of Sophisticated Fuzzy Logic Controllers using Genetic Algorithms. In: Proc. 3rd IEEE Int. Conf. Fuzzy Systems, Networks/Genetic Algorithms, 1994, pp. 114–122.

Piancastelli, L., Veroni, C., Mirri, M., A KAD Tool to Optimize F1 Cars Through a Combined Elitarian Genetic-Fuzzy Algorithm'', 11th Int. Conf. on Design Tools and Methods in Industrial Eng. (Palermo), 1999.

Rajendra, B.P., Intervals in Evolutionary Algorithms for Global Optimization, in APIC '95, 1995.

Zadeh, L.A., Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers of Lotfi A. Zadeh. in: Klir, G.J., Yuan, B. (Eds.), Advances in Fuzzy Systems – Applications and Theory, vol. 6, World Scientific, River Edge, NJ, 1996.