International Communications in Heat and Mass Transfer 178 (2026) 111933

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Evolution of the constructal design application on fins and cavities as a passive element for heat transfer problems☆

Giovani Dambros Telli [a], Cesare Biserni [b], Claudia Naldi [b], Liércio André Isoldi [c], Elizaldo Domingues dos Santos [c,d], Luiz Alberto Oliveira Rocha [c,d,e,*]

[a] Área do Conhecimento de Ciências Exatas e Engenharias, University of Caxias do Sul, 1130 Francisco Getúlio Vargas Street, Caxias do Sul, RS 95070-560, Brazil
[b] Department of Industrial Engineering (DIN), Alma Mater Studiorum, University of Bologna, Bologna, Italy
[c] Graduate Program in Computational Modeling, Federal University of Rio Grande - FURG, Italia Avenue, km 8, 96201-900, Rio Grande, Brazil
[d] Institute of Earth Sciences, Complex Flow Systems Lab, 59 Romao Ramalho Street, 7000-671, Evora, Portugal
[e] Graduate Program in Mechanical Engineering, Federal University of Rio Grande do Sul – UFRGS, 425 Sarmento Leite Street, Porto Alegre, RS 90050-170, Brazil.

A R T I C L E I N F O

Keywords:
Constructal design
Constructal law
Cavity
Fin
Heat transfer
Energy storage

A B S T R A C T

The demand for efficient thermal management technologies is increasing as chip technology advances and large-scale data centers grow. High-conductivity fins and cavities are widely employed to enhance heat transfer. However, identifying the best fin or cavity configuration and understanding how it improves thermal performance remains a challenge. In this context, constructal design has been extensively applied to develop flow architectures that enhance access to internal flow and reduce thermodynamic imperfections. This work systematically reviews the evolution and application of constructal design in heat transfer problems involving fins and cavities. A total of 195 research articles from two major databases were analyzed. This study shows how fin and cavity designs have evolved and how these changes have influenced heat transfer performance. A bibliometric analysis also illustrates the growth of this research field and identifies recent research trends. Cavity geometries evolved from simple I-shaped cavities to complex branching configurations, including H-, X-, double-Y-, and tree-shaped designs. Under the same comparison conditions, the best-performing isothermal cavity was the tree-shaped configuration with four branches, which reduced the dimensionless maximum excess temperature by up to 93.6% compared with the baseline I-shaped cavity. Several fin configurations were also evaluated, including disk-shaped, circular, tree-shaped, Y-shaped, rectangular, and pin-fin arrays. The results indicate that the transition from simple fin geometries to hierarchical and branching designs improves thermal performance. The research field continues to grow, with recent studies focusing on mixed and forced convection and the application of constructal design to thermal energy storage systems with phase change materials.

1. Introduction

Nowadays, the development of new technologies is associated with continuous increases in electronic and computational power. From battery energy storage systems to high-frequency chips and large-scale data centers for artificial intelligence, these technologies are steadily advancing toward higher power density and energy consumption, posing real challenges for thermal management [1,2]. Moreover, in some technologies, particularly electronic chips, the power levels are increasing while physical dimensions continue to shrink, making heat dissipation during operation increasingly difficult [3].

In this regard, a wide range of heat dissipation strategies has been developed to improve the thermal performance of engineering systems. One widely employed example is the use of fins or cavities to enhance heat transfer by increasing fluid–thermal interaction and reducing thermal resistance in compact, lightweight systems [4–6]. Fins and cavities can assume several geometric configurations, such as triangular, convex, and wavy shapes, which influence heat transfer and system performance [7–9]. Nevertheless, in most heat transfer problems, a major question is how to identify the configuration that most effectively promotes heat removal and how this flow architecture influences the

☆ This article is part of a Special issue entitled: 'Design in Nature and Society' published in International Communications in Heat and Mass Transfer.

* Corresponding author at: Graduate Program in Computational Modeling, Federal University of Rio Grande - FURG, Italia Avenue, km 8, 96201-900, Rio Grande, Brazil.

E-mail address: luizrocha@furg.br (L.A.O. Rocha).

https://doi.org/10.1016/j.icheatmasstransfer.2026.111933
Received 8 March 2026; Received in revised form 17 June 2026; Accepted 29 June 2026
Available online 3 July 2026

Nomenclature

A	Domain area (channel) m^2
A_c	Area filled by the fin or cavity m^2
Gr	Grashof number ($Gr = g\beta(T_s - T_\infty)L^3/\nu^2$)
h	Convection heat transfer coefficient $W/(m^2 \cdot K)$
H	Domain height m
H_1	Height of the fin/cavity m
k	Thermal conductivity $W/(m \cdot K)$
L	Domain length m
L_1	Length of the fin/cavity m
Nu_H	Nusselt number for characteristic height ($Nu_H = h \cdot H/k$)
Pr	Prandtl number ($Pr = \mu c_p/k$)
q'''	Heat generation per unit volume W/m^3
q^*	Dimensionless heat transfer rate
$\tilde{q}$	Dimensionless thermal conductance
Ra	Rayleigh number ($Ra = Gr \times Pr$)
Re_H	Reynolds number ($Re_D = \rho V_\infty H/\mu$)
Ri	Richardson number ($Ri = Gr_H / Re_H^2$)
T_{max}	Maximum temperature of the cavity K
T_{min}	Minimum temperature of the cavity K
T_∞	Bulk temperature K
Greek symbols	
ϕ	Fraction of the area filled by the fin or cavity
θ_{max}	Dimensionless maximum excess of temperature
Subscripts	
o	Once optimized
oo	Twice optimized

system's overall thermal behavior. Then, fundamental heat transfer problems and the development of new heat-dissipation technologies could be addressed based on the constructal law.

The constructal law was postulated by Bejan in 1996 [10,11] and considers the time evolution of the configuration (design) of all flow systems that are free to morph, including animate and inanimate, human-made and natural, and social and individual systems [12]. According to the constructal law, any finite-size flow system must evolve its configuration in order to persist over time. This evolution occurs through a natural tendency to provide greater access to the currents that flow through the system [13–15]. In heat transfer problems, heat itself is the flowing entity, and the evolution of the system configuration enables more efficient pathways for heat dissipation or absorption.

The constructal law is considered a law of physics under the broad umbrella of thermodynamics. The first law of thermodynamics governs the conservation of energy in "any system", and the second law of thermodynamics identifies the presence of irreversibility for "any system" as if it were a black box [16]. However, the two laws of thermodynamics are global statements that consider only the flow of heat, work, and mass across a boundary and do not account for the system's design, organization, freedom, or evolution [17]. Then, the constructal law sheds light on the evolution of system designs over time. The occurrence of a design appears to be a dynamic phenomenon, with continuous, never-ending evolution [16,18].

The constructal research area has grown rapidly over the years and expanded beyond the realm of thermodynamics into the most diverse areas of engineering, physics, geophysics, biology, social sciences, economics, architecture, the arts, and health sciences [19–22]. The meaning of the constructal law has become broader and more powerful because, by describing flow systems, it enables the prediction, design, and fast development of technologies based on novel evolutionary flow architectures [12]. In engineering studies, especially in heat transfer problems, constructal design has been used as a method to help systems change their geometry to distribute their imperfections and improve the thermal performance [23]. Some examples of the constructal design in heat transfer problems include microchannel heat exchangers [24], latent heat exchangers [25], liquid cooling [26,27], and battery thermal management systems [28–30]. From these examples, it is clear that hierarchical flow architecture plays a key role in developing more efficient heat-transfer systems.

Since it was postulated, the constructal law has been extensively reviewed in journals and books because of its impact on engineering and its expansion into other fields of knowledge [31–36]. In the heat transfer domain, Chen et al. [37] reviewed the progress of constructal theory in China, addressing studies involving heat conduction, thermal insulation, convective heat transfer, and mass transfer in porous media. In another review, Chen et al. [38] summarize the progress in constructal design across eight heat sink configurations and multiple thermal performance indicators. Marzouk et al. [39] presented a comprehensive review of the applications of constructal theory to heat conduction, fluid flow, and convective heat transfer problems. Table 1 summarizes the main characteristics of prior review studies on constructal theory and its applications in the heat transfer domain. Although previous works have discussed constructal theory and thermal optimization problems more broadly, no review has specifically focused on the evolution of constructal design in heat transfer problems involving fins and cavities, using bibliometric and systematic literature review approaches.

In this regard, three decades after the constructal law was postulated, the present work systematically reviews the applications and evolution of constructal design for fins and cavities in heat transfer problems. This study maps the evolution of constructal designs over time in these applications to improve the thermal performance of the systems under investigation. It also analyzes the expansion of research on constructal design of fins and cavities by identifying trends in publications, citations, and authors' keywords. Therefore, this study clarifies the current state of development and recent research trends in constructal design for fins and cavities, thereby contributing to a comprehensive understanding of the field.

2. Methodological procedures

A systematic literature review (SLR) is a structured methodology for organizing and mapping a research area through methodological consistency. Its primary goal is to deliver a comprehensive and unbiased summary of existing evidence on a specific research topic, while also highlighting research gaps and opportunities for future studies [40]. Unlike other reviews, an SLR is guided by a well-defined research question and follows a predetermined review protocol [40]. The SLR method has been widely adopted in engineering research to support reliable and evidence-based conclusions [41,42]. In an SLR, transparency throughout the review process is important, and all methodological decisions and steps must be documented to ensure clarity and methodological rigor [43]. Employing structured procedures for the identification, screening, and synthesis of studies mitigates the risk of bias [44].

2.1. Scope

An SLR aims to identify and synthesize all available evidence that meets predefined inclusion criteria to address a research question or hypothesis [45]. The present study aims to map the state of the art and answer the main research question: How has the constructal design of fins and cavities in heat transfer architecture evolved over time?

In addition to this primary question, the SLR addresses the following secondary questions:

Table 1
Characteristics of previous review studies related to constructal theory and heat transfer applications

Ref.	Scope	Systematic Review	Database / Period covered	Bibliometric Analysis	Constructal Design Applied to Fins/ Cavities	Contribution
Chen et al. [37]	Developments of Constructal Theory in China over the past decade	No	Not explicitly reported	No	Broadly	Broad review of constructal applications in engineering focused on developments in China
Chen et al. [38]	Constructal design in eight types of heat sinks	No	Not explicitly reported	No	Yes (heat sinks)	Focused on optimization of eight heat sink configurations and performance indicators
Marzouk et al. [39]	Heat transfer enhancement through Constructal Theory	No	Scopus (1997–2025)	Yes	Broadly	Broad review of constructal applications in heat transfer and flow systems
Present study	Constructal Design applied to fins and cavities in heat transfer problems	Yes	Scopus + WoS / No restriction	Yes	Yes	Systematic and bibliometric review focused on the evolution of Constructal Design applied to fins and cavities heat transfer problems.

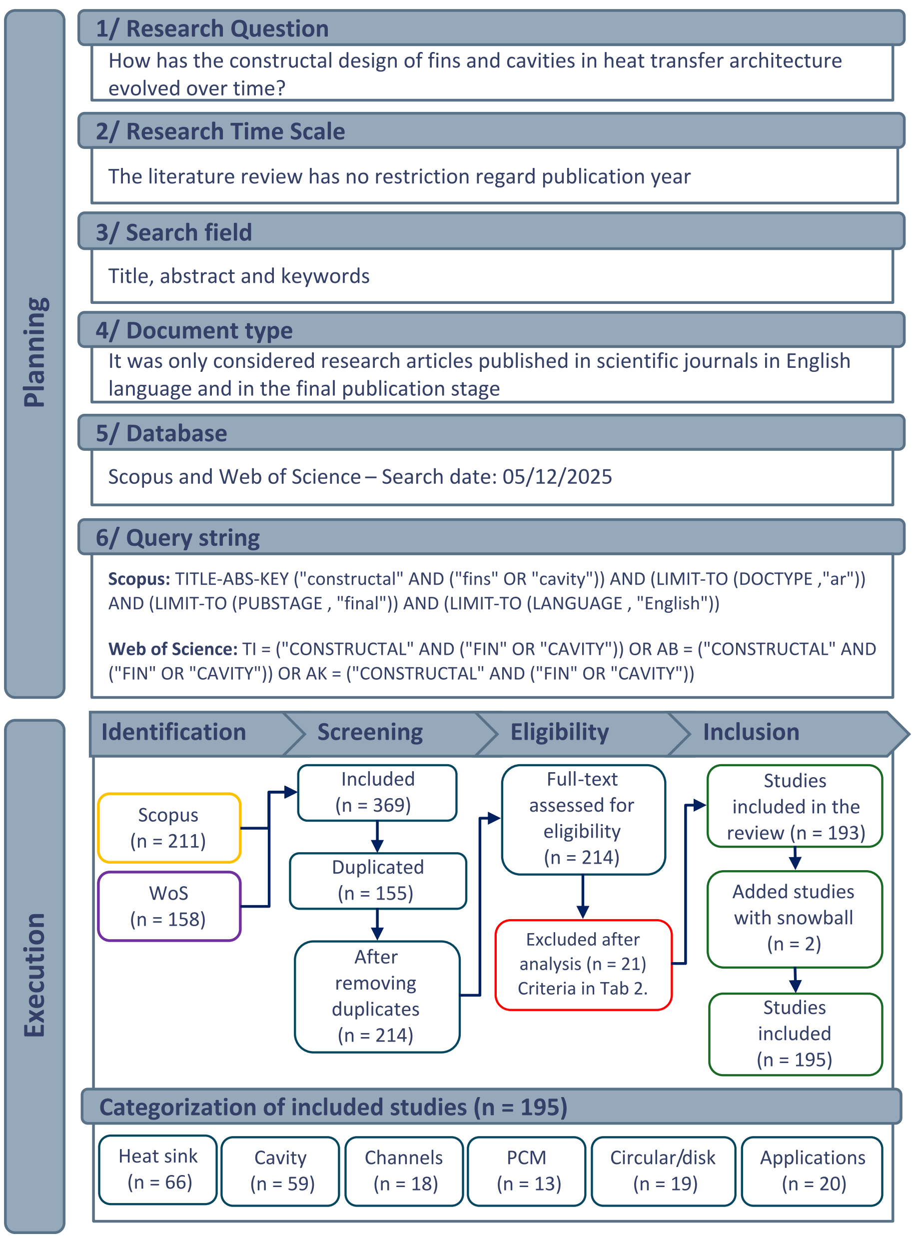


Fig. 1. Systematic review protocol.

- How has the constructal design of fins and cavities research field expanded over time?
- How did the geometry of the fins and cavities evolve in the different heat transfer problems (from I-shaped to double-Y-shaped)?
- What are the main performance indicators used?
- What are the most prominent topics in this specific research field?

2.2. Review structure

In this study, the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines were followed to report the literature search strategy and article selection process [42,46]. The review was conducted in three main stages: (i) planning, during which the objectives, protocols, and inclusion criteria were defined; (ii) execution, which involved screening and selecting relevant studies; and (iii) analysis, in which the selected articles were synthesized and interpreted.

Fig. 1 presents the review protocol used for this work. The SLR was conducted without a year restriction. Data were retrieved from the Scopus and Web of Science (WoS) databases. Scopus and WoS were selected because they are the largest databases of peer-reviewed literature and are comprehensive, reliable tools with various selection criteria. Scopus and WoS are widely used by academia, especially in the engineering field, and provide standardized export data. Data were collected in December 2025.

To identify all documents in the constructal design field related to fins and cavities in heat-transfer problems, the term “constructal” was combined with “fins” or “cavity” in the articles’ titles, abstracts, or author keywords. Only research articles in English were considered. Review articles, chapter books, books, conference papers, conference review papers, notes, editorials, and letters were excluded. Therefore, only peer-reviewed research articles were included in this study, as they typically provide more comprehensive content than other publication types, such as conference papers. Additionally, most conference proceedings are not indexed in major databases such as Scopus and WoS.

In total, we found 369 documents, comprising 211 from Scopus and 158 from the WoS databases. In the screening phase, we first removed 155 duplicate documents between the Scopus and WoS databases, as presented in Fig. 1. After duplicate removal, the remaining studies were evaluated according to the eligibility criteria presented in Table 2. Studies unrelated to constructal design applications involving fins and cavities in heat transfer problems, full-text manuscripts unavailable, and non-research article publications were excluded at this stage. A total of 21 documents were removed in this step, as shown in Fig. 1. Finally, two more works were added using the snowball technique, identified by examining the references of the already selected documents. A total of 195 documents were found following the current methodology.

The quality assessment of the primary studies was conducted manually to verify compliance with the eligibility criteria and their relevance to the research questions [47]. All studies were independently evaluated by the reviewers during the full-text screening stages. Disagreements regarding study inclusion were resolved through discussion and consensus among the authors. The selected studies were evaluated based on methodological clarity, the definition of geometric degrees of freedom, boundary conditions, validation procedures, and the consistency of the reported thermal performance indicators. Studies lacking sufficient methodological detail were excluded from the final database.

After screening and selecting the relevant studies, the collected works were classified into several categories (heat sinks, cavities, channels, circular/disk, phase change materials (PCMs), and applications), depending on the type of heat transfer problem. The largest group focused on heat sinks with rectangular or pin fin arrays. The second-largest category comprised studies on cavities with various geometric designs, followed by studies on channel flows and PCMs. All screening, study selection, and categorization stages were conducted using the Rayyan platform to improve the organization, consistency, and management of the systematic review process.

The search results were exported and analyzed using VOSviewer and Bibliometrix software to screen, quantify, and map data from the selected studies. The number of documents and citations per year, and the main countries and sources researching the constructal field applied to fins and cavities, were quantified using the Bibliometrix tool [48]. The VOSviewer software [49] was also used in this work to assist in scientific mapping analyses and to identify relationships among authors’ keywords. In the keyword co-occurrence network, keyword clusters closely reflect the themes or topics of the research field under study. It is possible to identify the main researched topics and the most prominent ones in the literature.

It is important to emphasize the scope and limitations of the present work. The adopted search strategy focused on studies explicitly related to the constructal law and the constructal design field by using the term “constructal” in the searchable fields. As a result, studies involving fins, cavities, heat sinks, PCM systems, and related thermal problems that did not explicitly identify themselves within the constructal domain may not have been included in the final database. Therefore, the present review should be interpreted as a systematic review of the constructal field literature applied to fins and cavities in heat transfer problems rather than as a comprehensive review of fins and cavities in the broader thermal sciences literature.

Table 2
Inclusion and exclusion criteria adopted during the selection of the studies included in the systematic literature review.

Type	Criteria
Included	• Studies whose main focus was the constructal design involving fins and cavities in heat transfer problems • Research articles in the English language
Excluded	• Unavailable online • Review paper; Conference paper, Notes, Editorial, Letters • Articles where the main focus was not the constructal design on fins and cavities heat transfer problems

3. Bibliometric analysis

Bibliometric analysis is a technique for assessing scientific research output using specific indicators and identifying research trends across fields and their evolution over time [50]. It is particularly useful when the objective is to contextualize a specific line of research within a broader scientific landscape, especially in areas with a large volume of publications [51]. In this context, this type of analysis helps identify how the constructal field applied to fins and cavities has grown over the years, as well as the main research topics and the leading countries and journals publishing on this topic. The bibliometric analyses were conducted using 211 documents from the Scopus database, as it contained more indexed documents than the Web of Science database for the adopted search strategy. It is important to emphasize that bibliometric indicators such as country rankings, journal productivity, citation counts, and keyword occurrence trends may vary across databases due to differences in indexing coverage and metadata structures.

The number of documents published and the number of citations per year are presented in Fig. 2. The first two documents identified date from 1999, a few years after Bejan postulated the Constructal Law in 1996 [10,11]. From 1999 onward, the number of published documents increased, particularly after 2008, peaking at 17 in 2017. The number of citations also increased over the years, exceeding 400 citations per year almost continuously since 2015. Over the period analyzed, a total of 5,907 citations were recorded. The 211 analyzed documents indexed in Scopus yield an h-index of 48. This growth trend in both publications and citations highlights the increasing relevance of constructal design in heat transfer research. It is worth noting that the number of documents and citations from 2025 continues to increase, as it takes time for all the articles published in 2025 to be indexed in Scopus.

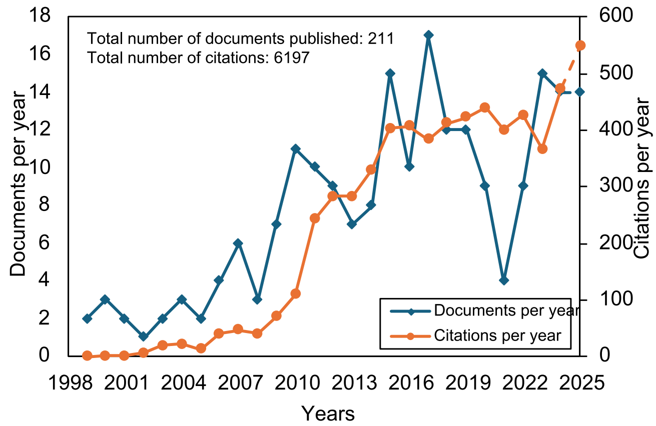


Fig. 2. Number of documents and citations per year related to the constructal design method applied to fins and cavities problems from 1999 to 2025.

Fig. 3 shows the co-occurrence of authors' keywords, which helps identify the main research topics and their relationships across different areas within the constructal field applied to heat transfer problems involving fins and cavities. The figure was generated using VOSviewer software, with a minimum occurrence threshold of 5 occurrences per keyword and the full counting method without merging synonyms [49]. Larger nodes represent keywords with higher frequency of occurrence, while the thickness of the connecting lines indicates stronger relationships between keywords based on their simultaneous occurrence in the selected documents. Keywords positioned closer to each other tend to appear more frequently within the same studies. The color scale represents the average year of keyword occurrence in literature, with lighter colors (yellow or light green) indicating more recent keywords and darker colors (purple or dark blue) representing earlier occurrences. Although the first studies identified in the review were published in 1999, the average year of occurrence for the most frequent keywords ranged from approximately 2012 to 2020, reflecting their continued use throughout the analyzed period.

The network reveals that "constructal theory" and "constructal design" occupy central positions and present strong connections with terms such as "optimization", "heat transfer", "cavity", and "enhanced heat transfer", indicating that these topics form the core structure of the constructal field literature applied to fins and cavities. It is noteworthy that, on average, the expression "constructal design" has been used more recently in the literature (on average since 2018) than "constructal theory" or "constructal". A possible explanation for this trend is the increasing differentiation in the use of the term "constructal", with authors adopting more specific expressions, such as "constructal theory", "constructal design", or "constructal law", rather than using "constructal" as a generic keyword. In addition, the terms "optimization" and "generalized thermodynamic optimization" have appeared less frequently in recent years. This may reflect a clearer distinction between optimization techniques and the constructal law itself, which is a law of physics. In this regard, the term "optimization" is often linked to specific methodologies that can be coupled with the constructal design approach.

Besides the core structure, more specific keywords such as "forced convection", "mixed convection", "PCM", and "multi-objective optimization" appear more peripherally in the network and with more recent average occurrence years, suggesting the diversification of research applications and topics with greater recent prominence in heat transfer research. The link between constructal research and PCM-related topics aligns with global trends toward developing thermal energy storage technologies to support the integration of renewable energy.

Fig. 4(a) presents the number of publications and citations by country, while Fig. 4(b) shows the number of publications by scientific journal. China ranks first with 56 documents, followed by Brazil (51), Italy (46), Iran (26), and the USA (24). In terms of citations, the ranking changes to China, Italy, Brazil, and the USA. These results indicate that the constructal field applied to fins and cavities in heat transfer problems is widely explored worldwide, with contributions from 26 countries. The bibliometric analysis also indicates that most studies were published in the International Journal of Heat and Mass Transfer (38 papers), International Communications in Heat and Mass Transfer (20 papers), and the International Journal of Thermal Sciences (18 papers), as shown in Fig. 4(b).

4. Synthesis and interpretation

This section presents the synthesis and interpretation of the selected

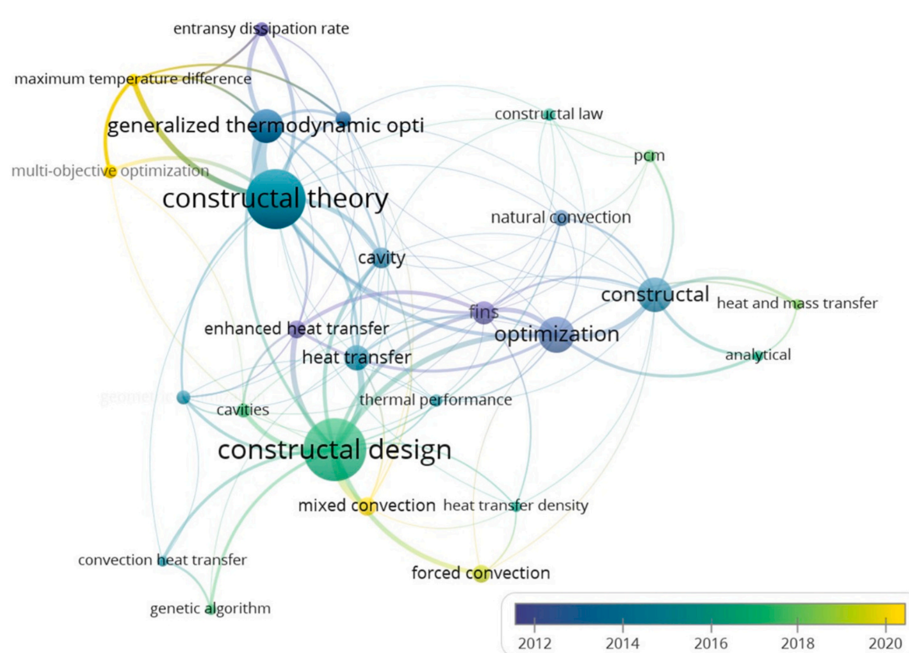


Fig. 3. Keywords occurrence map over time. Circle sizes denote how often the keyword was adopted.

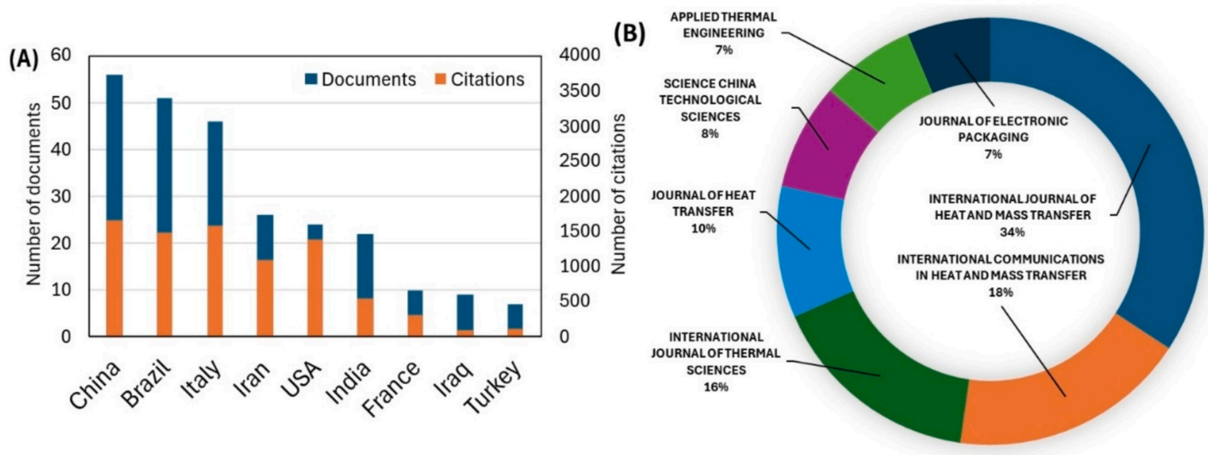


Fig. 4. Results of the bibliometric analysis regarding the number of documents and citations per country (a) and the percentage of documents by source (b).

documents to answer the questions proposed in the methodological procedures. To organize and clarify the analysis, the documents were grouped into categories based on the type of heat transfer problem, as shown in Fig. 5.

4.1. Cavities

4.1.1. Isothermal and convective cavities

Among heat transfer problems involving cavities, the study of cavities embedded in a solid wall with internal heat generation has been widely investigated from the lens of constructal design. In this kind of problem, the main goal is to determine the cavity geometry that increases the heat transfer performance from the solid wall and the intruded cavity. From this heat transfer problem, it is possible to extrapolate to several other problems composed of low-resistance media (cavities) and a high-resistance domain (a solid wall).

In most of these studies, a square numerical domain is used, and several different intruded cavity shapes are explored. In general, the domain is considered as a two-dimensional solid body subjected to a constant heat generation per unit volume (q''') and with a constant thermal conductivity (k). The domain is cooled by inserting a cavity. The solid domain external surfaces are considered adiabatic. The thermal field is assumed to be in a steady state. In this regard, the heat generated by the solid domain is removed only by the cavity surface at a minimum temperature (T_{min}), which is considered an isothermal cavity (simulating fluid flow with a constant high heat transfer coefficient, h). However, some works considered the heat transfer coefficient and a specific bulk temperature (T_∞) in the cavity as boundary conditions rather than a T_{min} on the cavity surface. In these cases, the intensity of convective heat transfer is characterized by the parameter a, introduced by [52] and defined as:

$$a = \left(\frac{2hA^{\frac{1}{2}}}{k}\right)^{1/2} \tag{1}$$

where h is the convective heat transfer coefficient, A is the cavity area, and k is the thermal conductivity of the solid material. The parameter a describes the ratio of convective and conductive heat transfer within the solid domain and is proportional to the square root of the Biot number based on the characteristic length scale. Low values of a are associated with practical industrial forced-convection applications, whereas $a \rightarrow \infty$ corresponds to the case of isothermal cavity surfaces.

For square or rectangular solid domains, the total domain area is expressed by $A = H \times L$, where H is the height, and L is the length of the domain. From this equation, it is possible to calculate the relation between the total domain area and area occupied by the cavity (A_c), which

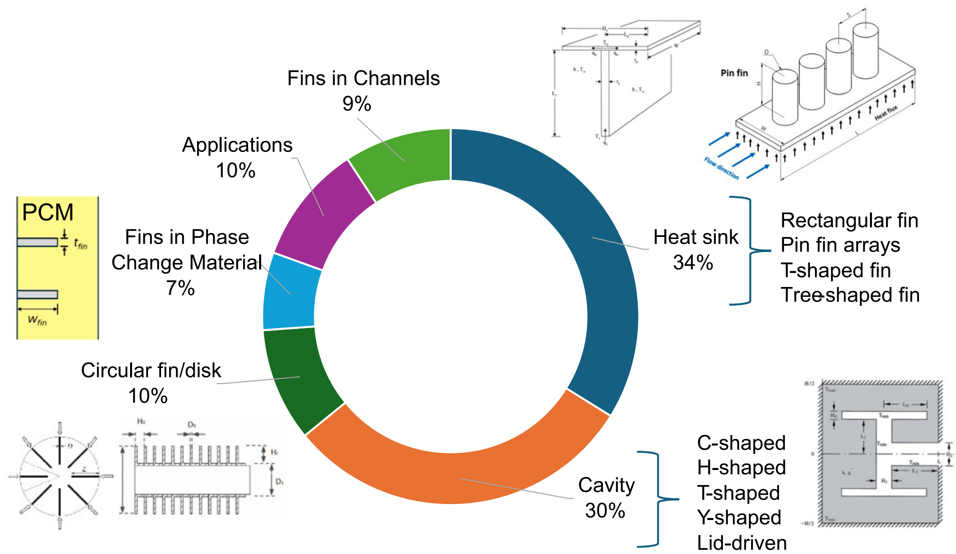


Fig. 5. Types of heat transfer problems involving constructal design applied to fins and cavities.

Table 3
Studies about the cavity geometrical investigation regarding cavity shape, boundary condition, and degrees of freedom for a square or rectangular solid domain.

Ref.	Cavity	Boundary condition	Geometric degrees of freedom (DoF)
Biserni et al. [54]	I and T-shaped	Isothermal	C-shaped: 1 DoF; H_0/L_0 T-shaped: 3 DoF; H/L, L_0/L_1, D_0/D_1
Biserni et al. [55]	H-shaped	Isothermal	5 DoF: L_0/L_2, L_1/L_2, H_0/H_2, H_1/H_2, H_2/L_2
Lorenzini e Rocha [56]	T-Y-shaped	Convective ($a = 0.1$)	3 DoF: H_0/L_0, H_1/L_1 and H_2/L_2
Rocha et al. [57]	I-shaped	Convective ($a = 0.005$ - 1.0)	2 DoF: H_0/L_0, H/L
Lorenzini et al. [58]	Y- and T-shaped	Isothermal	T-shaped: 2 DoF: D_0/D_1, L_0/L_1 Y-shaped: 2 DoF: α and volume ratio
Lorenzini et al. [59]	Y-shaped	Isothermal	4 DoF: H/L, L_1/L_0, t_1/t_0, α
Lorenzini et al. [60]	T-shaped	Convective ($a = 0.1$)	4 DoF: H_1/L_1, H_0/L_0, H/L, L_0/L_1
Lorenzini et al. [61]	T-Y-I -shaped	Convective ($a = 0.1$)	4 DoF: H_3/L_3, H_2/L_2, H_1/L_1, H_0/L_0
Hajmohammadi et al. [62]	Various I-shaped	Convective ($a = 0.1$) and Isothermal	2 DoF: H_0/L_0, N branches
Lorenzini et al. [63]	I-shaped	Isothermal and Convective	2 DoF: H_0/L_0, H/L
Lorenzini et al. [64]	T-shaped	Convective ($a = 0.1 - 2.0$)	4 DoF: L_0/L_1, H_1/L_1, H_0/L_0, H/L
Lorenzini et al. [65,66]	X-shaped	Isothermal	2 DoF: L_1/L_0, D_1/D_0
Lorenzini et al. [67]	Y-shaped	Convective ($a = 0.1 - 100$)	4 DoF: H/L, t_1/t_0, L_1/L_0, α
Lorenzini et al. [68] Gonzales et al. [69]	Y-shaped	Isothermal	4 DoF: H/L, t_1/t_0, L_1/L_0, α
Gonzales et al. [70]	Double T-shaped	Isothermal	4 DoF: H_0/L_0, H_2/L_2, H_1/L_1, S_1/H_0
Hajmohammadi [71]	Omega-shaped	Convective ($a = 0.1$) and Isothermal	5 DoF: H_R, R_0/R_1, ϕ_s, ϕ_b, D_b/H_b
Hajmohammadi [72]	Tree-shaped	Convective ($a = 0.1$) and Isothermal	2 DoF: $\widehat{H_1}$, $\widehat{L_1}$ for N branches
Gonzales et al. [73,74]	Double-T	Isothermal	5 DoF: H/L, H_0/L_0, H_2/L_2, H_1/L_1, S_1/H_0
Gonzales et al. [75,76]	Double-Y	Isothermal	5 - 7 DoF: H/L, H_0/L_0, H_1/L_1, H_2/L_2, S_1/H_0, α, β
Zhu et al. [53]	Fork-shaped (Various I-shaped)	Isothermal	4 DoF: H_0/L_0, H/L, porosity, N
Xie et al. [77]	Fork-shaped (Various I-shaped	Convective (a =0.1-0.5)	2 DoF: H_0/L_0, H/L

is defined as (ϕ):

$$\phi = \frac{A_c}{A} = \frac{Cavity\ area}{Total\ domain\ area} = 0.1; 0.2; 0.3\ldots \tag{2}$$

Usually, the relationship between the cavity area and the domain area is treated as a constraint in the analyzed works. Additionally, different aspect ratios of the specific cavity (I-shaped, T-shaped, etc.) are explored to improve the heat transfer performance. In these studies, the main performance indicator was the dimensionless maximum excess of temperature (θ_{max}), which is defined as:

$$\theta_{max} = \frac{T_{max} - T_{min}}{q''' \frac{A}{k}} \tag{3}$$

where T_{max} is the maximum temperature of the numerical domain, T_{min} is the minimum isothermal cavity temperature (or the bulk temperature, T_∞, when considering the heat transfer coefficient as a boundary condition). In some works, Equation (3) is also considered as the definition of the dimensionless global thermal resistance, and the main goal is to minimize it. Besides the dimensionless maximum excess of temperature as a performance indicator, some works also employed the entropy generation as a performance metric [53].

Table 3 presents studies on the geometric investigation of cavities inserted into conducting solid walls with internal heat generation based on the constructal design, focusing on cavity shape, the boundary conditions imposed, and geometrical degrees of freedom. It is important to note that most of the cavity studies reviewed in this section adopted the dimensionless maximum excess temperature or the dimensionless global thermal resistance as the performance indicator. These metrics facilitate the comparison of thermal performance reported in the literature. However, direct comparisons among studies should be interpreted with caution, as the works considered different boundary conditions, convection parameter (a) values, geometric constraints, and degrees of freedom.

Fig. 6 presents the evolution of the geometric configuration obtained through the application of constructal design in the cavity heat transfer problems over the years. In this figure, only square or rectangular domains were considered, presented in chronological order as they appeared in the first paper to use cavity geometry in the literature. It is possible to see the complex evolution of the cavity shape over time. The simplest designs were the I-shaped (C-shaped) and T-shaped cavities, which were the first ones studied. Subsequently, more complex designs, including H-, T-, and X-shaped cavities and, most recently, the double-Y-shaped cavity in 2023, were evaluated. All studies analyzed different cavity aspect ratios to determine the optimal cavity geometry for improving heat transfer performance.

To enable a consistent comparison among the main isothermal cavity studies, Fig. 7 presents the best dimensionless maximum excess temperature (θ_{max}) reported for each cavity shape, considering a square domain $H/L = 1$ and the ratio of the cavity area and domain area (ϕ) equal to 0.1. Since all studies employ the same performance indicator and geometric constraints, the comparison provides a consistent overview of the thermal improvements achieved through cavity evolution over time. All configurations are compared relative to the isothermal I-shaped cavity, which represents the simplest design and serves as the baseline case.

The first work found in the literature was developed by Biserni et al. [54], which explored the design of elemental isothermal I-shaped and T-shaped cavities, evaluating the impact of the first construct degree of freedom (bifurcation of T) on the configuration of the elemental trunk. The results showed that for the I-shaped cavity, the best value was $\theta_{max} = 0.1008$ for $H/L = 1$ and $\phi = 0.1$ (baseline case). However, the best T-shaped cavity performed about 29% better than the I-shaped configuration under equivalent boundary conditions, achieving $\theta_{max} = 0.0710$. It is noteworthy that increased cavity intrusion into the solid wall improved the system's heat transfer performance. A few years later, Biserni et al. [55] studied an isothermal H-shaped cavity, using the same boundary conditions as the previous work. The results indicated that optimal architecture is achieved when the first branch is as minimal as possible, making the second branch thicker and extending almost to the end of the domain, as seen in Fig. 7. For the $H/L = 1$ and $\phi = 0.1$, it was found that $\theta_{max} = 0.00245$, which exhibits a thermal performance approximately four times higher than that of the elemental I-shaped cavity and nearly three times higher than that of the T-shaped cavity.

Lorenzini et al. [67,68] investigated the isothermal and convective Y-shaped cavity using exhaustive search and a genetic algorithm to assess the effects of the Y-shaped aspect ratio and the heat transfer coefficient (h) on thermal performance, and to determine the optimal configuration considering four degrees of freedom. Regarding the isothermal Y-shaped cavity, the best geometry had a short first branch and longer second branches, with the Y branches extending almost to the domain boundary. This geometry led to $\theta_{max} = 0.0611$ for $H/L = 1$ and $\phi = 0.1$, representing a reduction of about 39.4% relative to the optimal I-

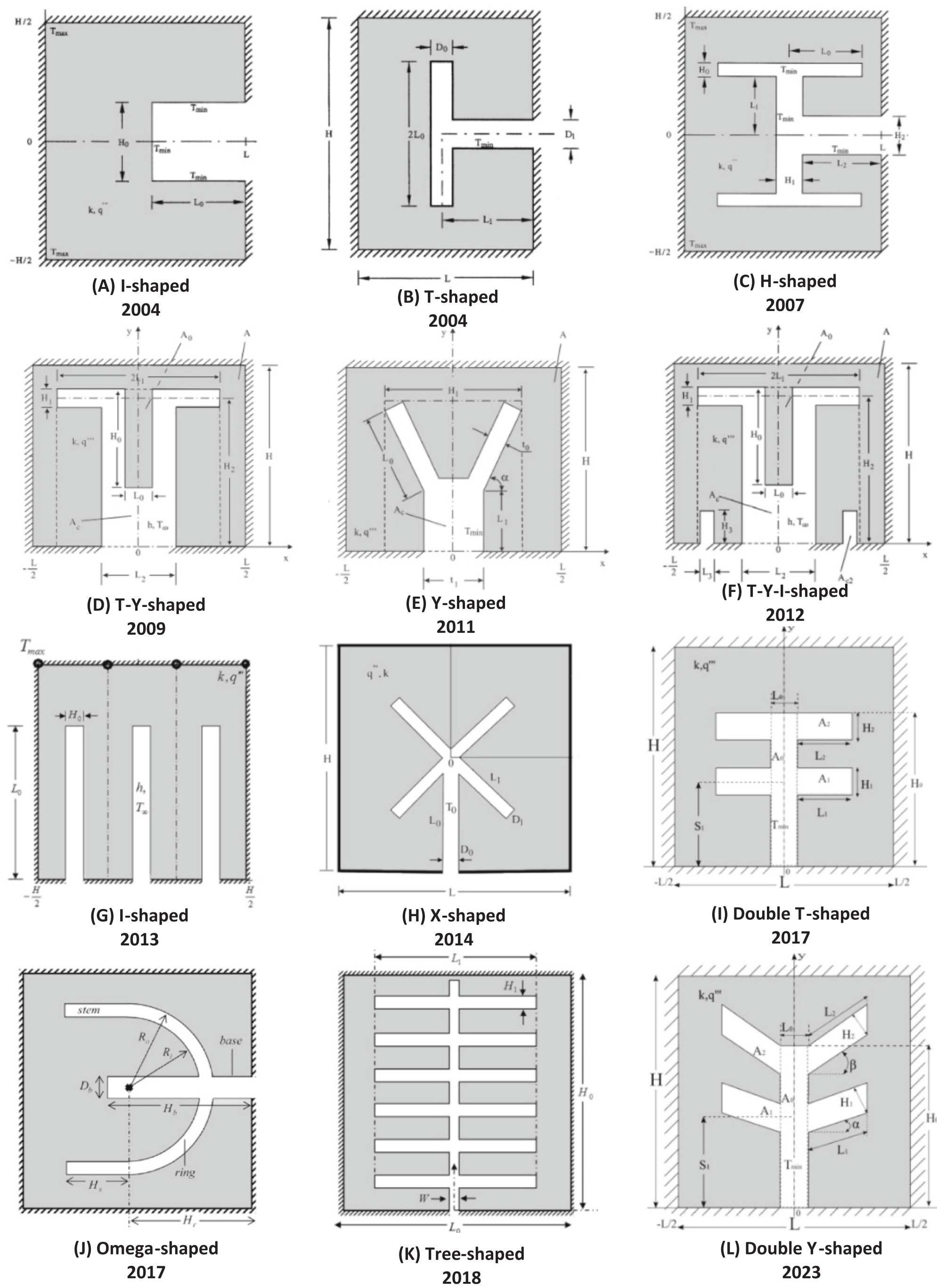


Fig. 6. Evolution of geometric configuration through the constructal design method applied to cavity studies. Adapted from [54–56,59,61,62,65,66,70–72,75].

shaped baseline. In another work from Lorenzini et al. [59], they also found that the optimal Y-shaped cavity performed 39.9% and 45.8% better than the optimal T-shaped and I-shaped cavities under $H/L = 1$ and $\phi = 0.05$ conditions, respectively.

Hajmohammadi et al. [62] proposed reducing the complexity of cavity geometries and studied N-branched I-shaped cavities intruded into square and trapezoidal solid domains. In this work, the cavities do not enter at a single point (not a single cavity), as in previous works, but at several points. The N branches were separated nearly symmetrically into equal divisions. In this work, heat was removed from the cavity surface through the free stream with a heat transfer coefficient, and also as an isothermal cavity. For the sake of comparison, for a solid domain of $H/L = 1$ and $\phi = 0.1$, the best isothermal cavity results were the ones where the cavity was intruded from the bottom to the upper part of the solid domain, cutting the entire solid and improving the heat transfer performance, as presented in Fig. 7. In particular, the θ_{max} was 0.0253, 0.0112, and 0.0064 for $N = 2$, 3, and 4 branches, respectively, representing an improvement over the baseline I-shaped cavity. Xie et al. [77] and Zhu et al. [53] also studied the geometry of 3-branched I-shaped cavities (fork-shaped) to find the best heat transfer performance.

Hajmohammadi [71] explored ω-shaped cavities with the same boundary conditions (convective and isothermal). The optimal isothermal ω-shaped cavity geometry achieved 26% better results than the optimal H-shaped cavity for $\phi = 0.1$. Later, Hajmohammadi [72] also presented a tree-shaped cavity. Different than the various I-shaped cavities (N branches), the tree-shaped cavity is a single cavity, since there is only one root, and from this one root derive N-branches. The author evaluated the thickness of the primary root and branches, as well

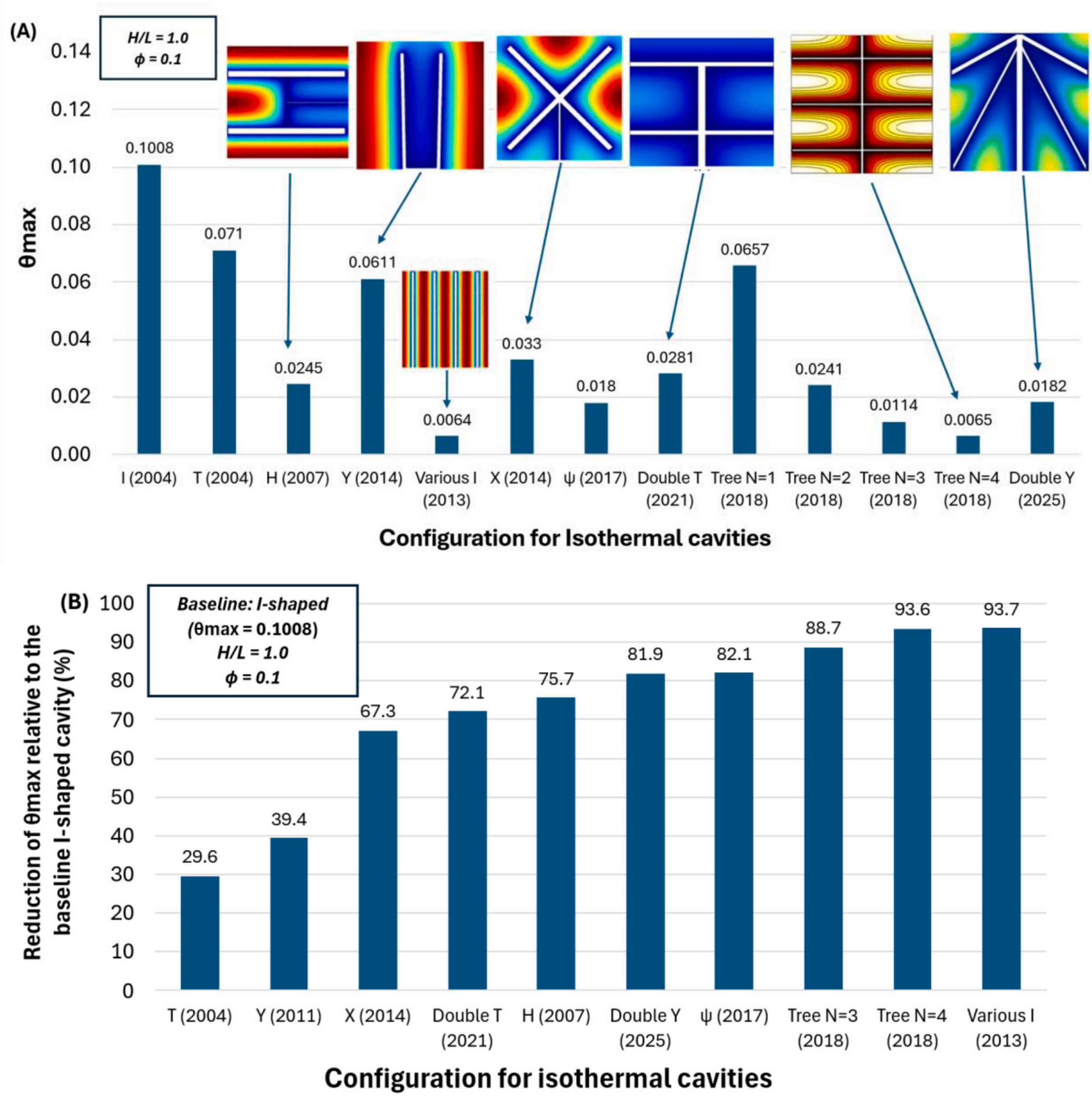


Fig. 7. Design evolution over time in isothermal cavity problems (a) and their reduction of θ_{max} in relation to the baseline I-shaped cavity [54] (b) for: Various I-shaped [62]; Tree-shaped [72]; Omega-shaped [71]; Double Y-shaped [76]; H-shaped [55]; Double T-shaped [74]; X-shaped [65,66]; Y-shaped [67,68]; T-shaped [54].

as the number of branches in the tree configuration. The optimal tree design appears to be the one with $N = 4$, featuring a thick primary root extending from the bottom to the top of the cavity and a thicker secondary root almost reaching from the left to the right side of the solid domain. The θ_{max} results for the $N = 1$ were higher for almost all the cavities in the literature. Nevertheless, the results for $N = 3$ and 4 were lower than almost all configurations in the literature, except for the various I-shaped configurations with $N = 4$. The best isothermal cavity result was for $N = 4$, yielding $\theta_{max} = 0.0065$, corresponding to a reduction of about 93.6% relative to the isothermal baseline cavity (I-shaped) for $H/L = 1$ and $\phi = 0.1$.

The isothermal X-shaped cavity was investigated by [65,66]. In these works, the authors evaluated two degrees of freedom: the ratios of the cavity lengths and the cavity thicknesses. They also explored different volume fractions occupied by the cavity. The results indicate that the optimal X-shaped cavity reduced about 67.3% the dimensionless maximum excess of temperature in relation to the optimal isothermal I-shaped cavity for $H/L = 1$ and $\phi = 0.1$. However, it performs worse than the H-shaped cavity, suggesting that additional degrees of freedom could improve the X-shaped cavity's performance. On the other hand, the isothermal X-shaped cavity improves heat transfer performance by about 53% over the Y-shaped cavity and by about 68% over the I-shaped cavity at $\phi = 0.05$.

Gonzales et al. [70,73,74] evaluated a isothermal double-T-shaped cavity embedded in a wall with internal heat generation. In their first study [70], the authors compared two stochastic optimization methods (Simulated Annealing and the Luus–Jaakola algorithm) combined with the constructal design method to optimize the double-T cavity geometry with four degrees of freedom. The Luus–Jaakola algorithm achieved the best performance. In a subsequent study [73], they investigated the application of the Differential Evolution (DE) algorithm and its parameter effects for predicting cavity geometry, coupled with the constructal design method. Finally, in [74], the authors studied the application of exhaustive search and Simulated Annealing optimization methods combined with the constructal design approach to optimize the degrees of freedom of a isothermal double-T cavity. The optimal isothermal double-T cavity geometry for $H/L = 1$ and $\phi = 0.1$ yielded a maximum dimensionless temperature of $\theta_{max} = 0.0281$, lower than those of the optimal I-, T-, Y-, and X-shaped cavities. The optimal double-T configuration was geometrically similar to a tree-shaped cavity with two branches, in which the first construct extended almost to the top of the domain, while the second T-branch spanned nearly from the left to the right side of the domain (Fig. 7). Instead of using optimization algorithms to determine the cavity geometry for a prescribed set of degrees of freedom, Vianna et al. [78,79] proposed a general methodology to understand how the cavity shape evolves in this type of problem. In their approach, new algorithms were employed in which the cavity geometry was progressively built according to the maximum temperature at each evolutionary step.

More recently, Gonzales et al. [75,76] proposed the isothermal double-Y cavity. In [76], optimization was performed via exhaustive search and differential evolution, using four degrees of freedom. The best cavity geometry was obtained by considering the first construct up to the top of the domain and extending it downward to the bottom of the cavity, as shown in Fig. 7. This configuration produced a more homogeneous temperature field and yielded $\theta_{max} = 0.0182$, which was 25-82% better than I-, T-, Y-, X-, double-T, and H-shaped cavities. In another work, the canonical differential evolution algorithm and its parameters were explored to optimize the geometry of an isothermal double Y-shaped [75].

Over the years, cavity designs have evolved from simple I-shaped geometries to more complex configurations, such as H-, Y-, X-, tree-shaped, double-T, and double-Y cavities, thereby improving heat transfer performance. The first I-shaped cavity was a non-hierarchical architecture and had a limited number of geometric degrees of freedom. For both isothermal and convective boundary conditions, introducing trunks and branches generally gave the cavity more freedom to morph, resulting in cavity shapes with better thermal performance. Under the same geometric constraints and using the same performance indicator, reductions of up to 93.7% in the dimensionless maximum excess temperature were achieved in relation to the baseline I-shaped cavity. The cavity geometries with lower thermal resistance are hierarchical architectures that allow the cavity to penetrate deeper into the solid domain and distribute the cooling branches more uniformly throughout the heat-generating region. These flow architectures reduce the conduction path between the heat source and the cooling cavity, resulting in a more homogeneous temperature field and lower thermal resistance. In addition, the evolution of cavity designs was followed by an increase in the number of geometric degrees of freedom, providing greater flexibility during analysis. However, comparisons among different cavity geometries should be interpreted with caution, as the studies employed different geometric constraints and degrees of freedom.

4.1.2. Cavities intruded in a trapezoidal solid domain

Unlike previous works that analyzed an intruded cavity in a square or rectangular domain, some works evaluated it in a trapezoidal solid domain [62,80–83], as shown in Fig. 8. Rocha et al. [80] investigated an I-shaped cavity intruded into the lateral of a trapezoidal solid domain having an internal heat generation and adiabatic surfaces. They found that to improve thermal performance, the cavity should penetrate into the solid domain and be slender. Xie et al. [81] evaluated a T-shaped cavity in the lateral surface of a trapezoidal solid domain that generates heat, and the external surfaces are insulated. The best results were achieved when the solid domain is narrower in the region where the cavity is inserted and when the T-shaped cavity has a thicker trunk that penetrates deeply into the solid domain. In addition, the branches tend to be considerably thinner. Hajmohammadi et al. [62] also evaluated various I-shaped (N-branch) cavities embedded in a trapezoidal solid. Compared with the T-shaped cavity reported in Ref. [81], the single-branch case ($N = 1$) yields higher values than the reference T-shaped cavity. The introduction of additional branches ($N = 2$, 3, and 4) improves thermal performance across all solid aspects of the ratios investigated. This behavior can be explained by a more uniform distribution of cooling paths within the solid as the domain is progressively divided into nearly symmetric regions, each intruded by equal branches. As a result, the temperature field becomes more homogeneous, and the peak temperature is considerably reduced, corroborating the constructal theory that uniform distribution of imperfections leads to optimal thermal designs. Pouzesh et al. [83] studied four different cavity shapes (triangular, elliptical, trapezoidal, and Y-shaped) inserted in a trapezoidal solid domain with uniform heat generation. The optimal cavity shape depends on the cavity aspect ratio: for ($H_0/L_0 < 0.7$), trapezoidal, triangular, and Y-shaped cavities perform similarly, whereas for ($H_0/L_0 > 0.7$), the Y-shaped cavity provides the lowest peak temperature, making triangular and Y-shaped cavities the best designs.

These studies indicate that optimal cavity design is influenced by the external geometry of the solid. Unlike rectangular domains, where cavity evolution was primarily associated with increased branching complexity, the trapezoidal geometry imposes additional geometric constraints that influence the optimal cavity shape and penetration depth. Nevertheless, the same principles of the constructal law remain valid: cavity configurations with better thermal performance improve the distribution of cooling paths within the solid domain and promote a more uniform temperature field. Therefore, the interaction between the cavity and domain geometries is also an important factor in determining the optimal cavity design.

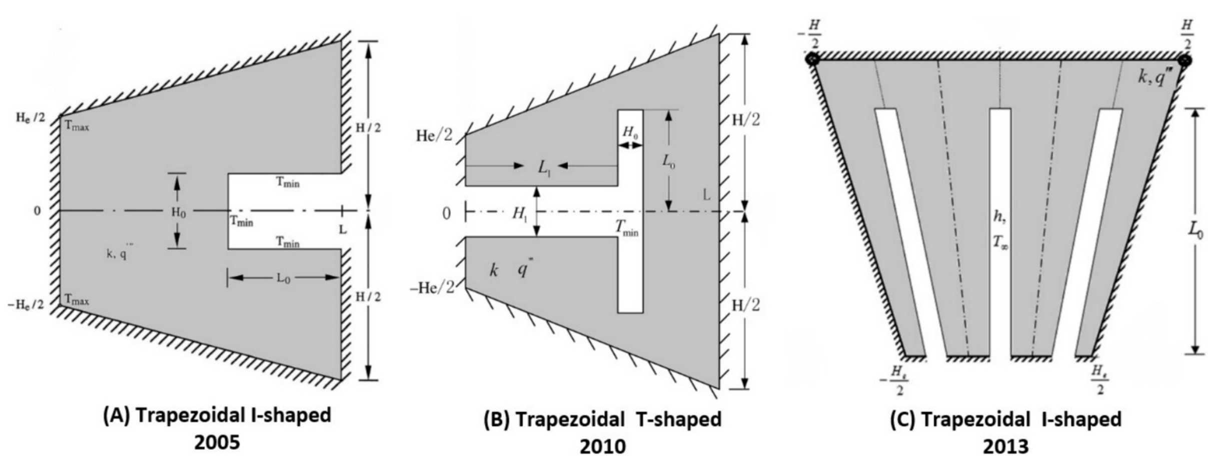


Fig. 8. Trapezoidal cavities. Adapted from [62,80,81].

4.1.3. Cavities intruded in a cylindrical solid domain

Besides the square, rectangular, and trapezoidal solid domains, Lorenzini et al. [84] studied the geometry of rectangular cavities intruding into a cylindrical solid with uniform internal heat generation, aiming to minimize the global thermal resistance for an isothermal cavity with adiabatic outer surface conditions. The effects of the number of cavities (N = 3-12) and the cavity volume fraction (ϕ = 0.02-0.15) were investigated while keeping the total volume fixed. Results show that the minimized global thermal resistance decreases with increasing N and ϕ, although it tends to stabilize for large N. This design evolution promotes a more uniform temperature distribution, improving the overall thermal performance. In another work, Lorenzini et al. [85] consider the same I-shaped cavities intruded in a cylindrical body with internal heat generation, but with the cavity bathed by a fluid with a heat transfer coefficient, h, at T_∞. In this work, they evaluated the impact of the heat transfer coefficient using the parameter $\lambda = hA^{1/2}/k$, varying the number of cavities from N = 3-32 and the cavity area fraction from 0.02 to 0.15. In general, the intruded cavities penetrate almost to the middle of the solid domain to improve temperature uniformity. The results show that for fixed λ and ϕ, there is an optimal number of cavities that minimizes the θ_{max}, and this number, in general, increases as λ and ϕ increase.

Estrada et al. [86] studied the intrusion of a tree-shaped cavity with two branches added to a single branch in a cylindrical solid under internal uniform heat generation. The heat was dissipated by the cavity with a heat transfer coefficient, h. The authors combined the constructal design with the exhaustive search optimization method, imposing two restrictions (the area occupied by the trunk and the area occupied by the branches of the cavity) and four degrees of freedom. The authors compared the tree-shaped results with the I-shaped cavity from [85] as shown in Fig. 9a. The results show that optimizing each degree of freedom (DOF) improves the system's thermal performance. However, the θ_{max} results for a more complex cavity (tree-shaped) were similar to those obtained for the I-shaped cavity. Fig. 9b presents the results for the best designs found in [85,86], considering N = 4, 8, and 16 cavities. A comparison with previous exhaustive optimizations reveals that the choice of optimized parameters and imposed constraints influences the optimal configuration, with the best designs being those that most effectively distribute high-temperature regions rather than simply increasing the number of cavities. Finally, an example of a study of an H-shaped fin inserted in a cylindrical domain can be found in [87].

In the end, these studies of cavities in cylindrical domains indicate that increasing the number of cavities improves heat transfer performance by promoting a more uniform temperature distribution within the solid domain. However, thermal improvement tends to plateau as the number of cavities increases, indicating an optimal number of cavities for a given set of conditions. Besides that, the comparison between I-shaped and tree-shaped cavities suggests that increasing geometric complexity does not necessarily result in better thermal performance. On the other hand, the optimal cavity design depends not only on the design itself but also on the imposed constraints and the number of geometric degrees of freedom considered in each study.

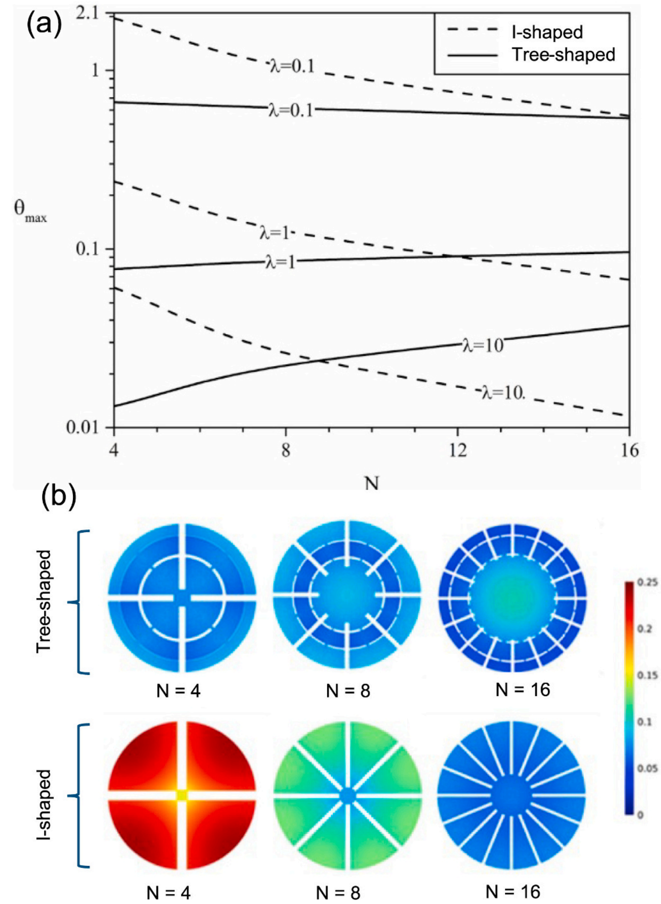


Fig. 9. (a) Comparison of θ_{max} for I and tree-shaped cavities in relation to the number of branches and λ; (b) Best designs found in tree-shaped [86] and the best design found in the I-shaped [85], considering N = 4, 8, and 16 (Adapted from [86]).

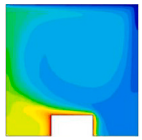
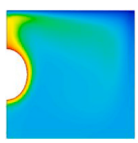
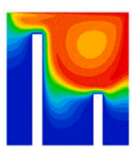
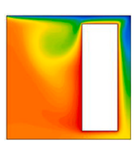
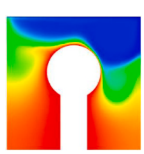
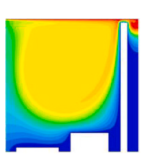
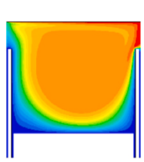
4.1.4. Lid-driven cavity flows and natural convection in cavities

Another type of problem explored in the literature is the intrusion of fins into lid-driven cavity flows. In this problem, an object (a fin) is typically inserted into a square cavity. Fluid motion is generated by the movement of an infinite plate at the upper surface. Heat transfer occurs due to the temperature difference between the fin surface and the surrounding flow. This temperature difference generates buoyancy forces and drives the flow. This type of problem is considered a mixed convection problem. Forced convection is driven by the motion of the plate, while natural convection is driven by the temperature difference between the fin surface ($T^* = 1$) and the bottom and lateral walls ($T^* = 0$). The lateral and lower surfaces are considered adiabatic. Performance indicators found in the literature include the (i) spatial-averaged Nusselt number (Nu_H) and also the (ii) dimensionless heat transfer rate (q^*). Table 4 presents some of the best-performing results for fins intruded into lid-driven cavities under various conditions.

Lorenzini et al. [88] investigated the intrusion of a rectangular fin in the center of the bottom surface of the cavity. The fin-to-cavity area ratio, ϕ, was fixed at 0.05, and the fin aspect ratio varied while the cavity aspect ratio was held constant. They also varied the Rayleigh (Ra) and Reynolds (Re) numbers, keeping the Prandtl number (Pr) fixed at 0.71. The results demonstrate that the rectangular fin aspect ratio affects the Nusselt number across flow regimes, and that the application of constructal design improves thermal performance, with differences of up to ~200% between optimal and non-optimal shapes. Aldrighi et al. [89] considered the same problem and the same boundary conditions as [88]. However, they studied the fin intruded into the middle of the cavity's bottom surface, as well as the downstream (right) and upstream (left) surfaces of the cavity. They found that the best Nu_H results were higher for fins intruded into the downstream surface for Re_H = 50-500, whereas for Re_H = 1000, the best results were obtained for the fin placed at the upstream surface. Aldrighi et al. [90] also investigated the variation in the position of the inserted fin across the cavity surfaces, rather than fixing it at the midpoint of the bottom, downstream, and upstream. The results show that, for the bottom surface, the best results are still in the middle. However, for the downstream and upstream surfaces, the best results were obtained when the fin was inserted at the top of the surfaces, yielding Nu_H = 28.55 and 26.88, respectively (Table 4).

Robinson and Das [91], based on our respiratory system, investigated an alveolus-shaped fin intruded into a mixed-convection lid-driven cavity. The authors evaluated the fin with one, two, and four

Table 4
Summary of some conditions tested, and the best performance achieved for fins intruded in lid-driven cavities

Ref.	Conditions tested	Best Performance	Optimal design in temperature maps	Main finding
Lorenzini et al. [88]	$\phi = 0.05$ $Ra_H = 10^3$ - 10^6 $Re_H = 10$ - 1000 $Pr = 0.71$	$Nu_H = \sim 14$ $\phi = 0.05$ $Ra_H = 10^6$ $Re_H = 1000$		Optimal fin shape depends on the flow regime and strongly affects heat transfer
Aldrighi et al. [89]	$\phi = 0.05$ $Re_H = 10$ -1000 $Pr = 0.71$	$Nu_H = 11.32$ $(H_1/L_1)o =$ 0.3 $\phi = 0.05$ $Re_H = 1000$		Fin placement on different cavity surfaces affects thermal performance.
Aldrighi et al. [90]	$\phi = 0.05$ $Re_H = 10$ - 1000 $Pr = 0.71$	$Nu_H = 28.55$ $(H_1/L_1)_o =$ 2.0 $\phi = 0.05$ $Re_H = 1000$		Fin surface location and position influence thermal performance.
Razera et al. [103]	$\phi = 0.05$ - 0.30 $Ra_H = 10^3$ - 10^6 $Re_H = 10$ - 1000 $Pr = 0.71$	$Nu_H = 14.21$ $(H_1/L_1)_o =$ 0.4 $\phi = 0.05$ $Ra_H = 10^6$ $Re_H = 100$		Semi-elliptical fins outperformed rectangular fin
Rodrigues et al. [98]	$\phi_{total} = 0.1$ $Ri = 0.1$ - 1.0 $Re_H = 400$ $Pr = 6.0$	$q^* = 7.84$ $Nu_H = 2.55$ $\phi_1 = 0.07$; ϕ_2 $= 0.03$ $Ri = 1.0$ $(H_1/L_1)_{oo} =$ 0.5 $(H_2/L_2)_o =$ 0.5		Asymmetric fin distributions improve heat transfer
Borahel et al. [108]	$\phi = 1/4, 1/8, 1/16$, and 1/32 $Ri = 0.1$ - 10 $Re_H = 500$ $Pr = 0.71$	$q^* = 19.83$ $\phi = ¼$ $Ri = 0.1$ Cavity aspect ratio = 3.0 Position = 3/4		Obstacle position and aspect ratio govern heat transfer.
Robinson and Das [91]	$\phi = 0.20$ $Ra_H = 10^3$ - 10^6 $Re_H = 10$ - 1000	$Nu_H = \sim 16$ - 21 $Re_H = 500$		Biomimetic fins perform better than conventional designs
Rodrigues et al. [101]	$\phi_{total} = 0.1$ $Ri = 0.1$ $Re_H = 400$ $Pr = 6.0$	$q^* = 22.84$ $\phi_1 = 0.01$, $\phi_2 = 0.04$, $\phi_3 = 0.05$		Larger third-fin increase thermal performance
Rodrigues et al. [100]	$\phi_{total} =$ 0.05 – 0.2 $Ri = 0.1$ $Re_H = 400$ $Pr = 6.0$	$q^* = 19.59$ $\phi_T = 0.2$ $\phi_1 = 0.02$, $\phi_2 = 0.16$ and $\phi_3 =$ 0.02		Multi-fin asymmetry improves flow access and heat transfer.

facsimile alveoli and compared it with a rectangular fin. Results indicated that the fin with one alveolus achieved the highest thermal performance, followed by the four and two-alveoli fins, while the rectangular fin showed the lowest performance due to its smaller surface area. Nevertheless, despite having nearly 25% more surface area than the rectangular fin, the two-alveoli fin provided a slight improvement in heat transfer performance, demonstrating that geometry is the key factor for maximizing heat transfer. Unrestricted fluid motion within the cavity prevents heat accumulation and enhances heat dissipation. Cong et al. [92] and Mountrichas et al. [93] studied rectangular fins intruded in a lid-driven cavity using Al_2O_3–water and CuO-water nanofluids in different concentrations, respectively. They found that adding nanoparticles enhances heat transfer compared with the base fluid, improving heat-transfer rates and reducing entropy generation. Kan et al. [94] also studied the application of Al_2O_3–Cu–water hybrid nanofluids in a Y-shaped fin intruded in a split lid-driven square cavity (the moving plate that generates the flow begins from both sides of the upper surface of the cavity). The results indicate symmetrical streamlines and isotherms, as the fluid motion is generated from both sides, while the local Nusselt number increases with lid motion and reaches a maximum when both lids meet. Other works considering non-Newtonian fluid and nanofluids in cavities can be found in refs. [95–97].

Rodrigues et al. [98,99] evaluated the insertion of two rectangular fins in a mixed convection lid-driven cavity. They varied the height-to-length ratios of the left and right rectangular fins, considering three different fin area fractions and two Richardson numbers (Ri) to maximize the dimensionless heat transfer rate and the Nusselt number. The best results were obtained with asymmetric fins, each with a different area fraction, resulting in the best thermal performance. The height/length ratios depend on the Ri and the fraction areas. Notably, designs that maximize the Nu number do not yield the highest heat-transfer rate, since thermal performance depends on the trade-off between Nu number and fin heat-exchange area [98]. Building on this, Rodrigues et al. [100,101] later investigated three rectangular fins intruded into the bottom of a lid-driven square cavity. In this study, the authors combined the constructal design method with response surface methodology (RSM) to improve dimensionless heat transfer rate [100]. Again, they varied the fin area fraction (0.05 to 0.2) and the height-to-length of the tree fins. The highest thermal performance ($q^* = 19.59$) was achieved for an asymmetric design with a total fin area fraction of 0.2, distributed as 0.02, 0.16, and 0.02 for the first, second, and third fins, respectively. The first and third fins had tall, slender geometries. The second fin was shorter and wider. This configuration allowed free fluid flow between the fins and restricted edge flow, enhancing convection, as shown in Table 4. Subsequent works explored different fin shapes and cavity geometries. These included triangular [102], semi-elliptical fins [103] in a mixed convection square lid-driven cavity, and a U-shaped fin intruded into a trapezoidal lid-driven cavity [104]. Beyond the constructal design domain, recent studies have also investigated cavity heat transfer problems involving flexible fins and deformable cavity walls, as well as the effects of magnetic fields and nanofluids on thermal performance [105,106]. Although these studies do not explicitly employ the constructal design idea, they illustrate current research trends in cavity problems using alternative design approaches and physical mechanisms.

Unlike previous works, which studied intruded fins in mixed-convection lid-driven cavities, Borahel et al. [107,108] investigated the constructal design method to optimize heat transfer from a rectangular isothermal block placed inside an adiabatic lid-driven cavity under mixed convection. The cavity area and the block–to–cavity area fraction (ϕ = 1/4, 1/8, 1/16, and 1/32) are fixed, while the block aspect ratio and horizontal position are treated as DOFs [108]. The analysis considered Richardson numbers of 0.1, 1.0, and 10 at fixed Grashof ($Gr = 10^5$) and Prandtl ($Pr = 0.71$) numbers. Results indicated that heat transfer performance is strongly governed by isothermal block geometry, with higher dimensionless heat transfer rates associated with larger block aspect ratios. For $\phi = 1/16$ and $Ri = 0.1$, a tall block improved thermal performance by up to 143.2% compared to the square configuration. In another work [107], the impact of the cavity geometry on heat transfer performance was investigated.

The reviewed studies of lid-driven cavities show that the system's thermal performance is affected by the fin geometry, position, and fin area distribution. These parameters influence how the fluid motion interacts with the intruded fins and how heat is removed from the system. Asymmetric fin configurations tend to provide better thermal performance than symmetric designs because they improve fluid access and heat transfer within the cavity. The results also show that maximizing the Nusselt number does not necessarily maximize the heat transfer rate, since thermal performance depends on the trade-off between heat transfer coefficient and heat transfer area. Therefore, for this type of heat transfer problem, the optimal fin architecture and position depend on the adopted performance indicator and the flow conditions considered in each study. As a result, there is no universal design answer for the fin design and position, since they depend on the constraints and conditions imposed on each system.

Different from the specific mixed convection lid-driven cavity, Da Silva and Gosselin [109] numerically maximize heat transfer in a rectangular thermal loop composed of two superimposed rectangles of different sizes, with coolant between them. Buoyancy-driven flow is induced by maintaining a constant temperature difference across the outer vertical walls, which is characterized as a free convection problem. Results show that the relative dimensions of the inner and outer rectangles strongly influence thermal performance, with the optimal gap size decreasing as the Rayleigh number increases. Increasing the circuit aspect ratio enhances heat transfer, whereas increasing the eccentricity between the rectangles decreases thermal performance. Other natural convection and ventilated cavity studies can be found in [110–112].

4.2. Disc and circular fins

In studies of disc and circular fin heat transfer, the problems typically involve convective heat transfer in a main stem or tube at high temperature, or in a tube carrying a hot fluid internally. To enhance heat transfer, disc-shaped or radial fins with different geometries, numbers, and spatial arrangements are attached to the main stem. Other studies consider cylindrical bodies with internal heat sources or heat generation at the tube walls, with fins placed inside the tube to promote heat transfer to the flowing fluid. The main goal of these studies is to use the constructal design to identify the optimal design of disc or radial fins attached to a main stem, as well as the optimal positioning of heat sources, considering parameters such as the number, spacing, diameter, thickness, and arrangement of the flow architectures. The performance indicators used for this type of problem are diverse and include: (i) overall thermal resistance; (ii) global thermal conductance; (iii) heat transfer rate; (iv) maximum dimensionless temperature; and (v) heat exchanger effectiveness.

Alebrahim and Bejan [113] applied the constructal design method to minimize the thermal resistance of a conduction heat transfer problem in which a heat-generating volume of low-conductivity material contains high-conductivity inserts in the form of constant-thickness disks assembled on a common stem. The elemental and assembly geometries were numerically optimized under a constant total volume constraint. Subsequently, the interstitial material was replaced by forced convection, and the inserts acted as fins, forming a cylindrically symmetric finned assembly. The results showed that the overall optimization sequence can be simplified by using optimal geometric ratios obtained at the elemental level.

Almogbel and Bejan [114] performed a constructal design geometric optimization of a cylindrical stem with a large number of radially mounted pin fins. The heat is generated inside the stem and transferred to a free stream (forced convection) via the fins. The geometry is optimized to maximize global conductance under constant total volume and fin material constraints, exploring the effects of the external shape's height and diameter, as well as the internal ratio of the stem diameter to the fin diameters. The results found that the optimized design slightly improves global conductance. Similarly, Shokouhmand et al. [115] studied the constructal design and geometric optimization of a finned-tube heat exchanger to minimize the overall thermal resistance. It was found to have optimal fin numbers and heights. Hajmohammadi [116] also evaluated the geometry of annular fins attached to a pin fin based on the constructal design.

Mustafa et al. [117] investigated the maximization of heat transfer performance in a single-row crossflow heat exchanger with concentric and eccentric circular fins using the constructal design approach. The authors varied the fin tip-to-fin spacing, tube diameter, fin number, fin thickness, and the angle between the fins. The maximum volumetric heat transfer density occurs at $N = 2$ and decreases with increasing fin number, with minimum values observed for vertical fin configurations at $N = 4$, 8, and 12. Matos et al. [118] investigated the geometry of finned tube bundles to maximize heat transfer within a fixed volume under external flow by the lens of the constructal theory. The authors identified optimal tube spacing, eccentricity, and fin spacing for staggered circular and elliptical arrangements, with elliptical tubes achieving up to 19% higher heat transfer and 32% lower material mass than circular tubes.

Fig. 10 presents some of the cases studied in the literature. Da Silva et al. [119] investigated the optimal shape of a rectangular fin positioned on the rim of a heat-generating disc to minimize global thermal resistance. Fig. 10a shows the studied domain. The heat is generated in the disk and removed by N radial fins with higher thermal conductivity. The fins are uniformly distributed along the disc perimeter and project toward the interior. Geometric optimization was performed considering the distribution of the fin material, the ratio of high- to low-conductivity regions, the fraction of fin material, and the number of sectors in the disc domain. The analytical and numerical results show good agreement and identify optimal fin aspect ratios. Salimpour et al. [120] studied a similar tube with radial fins based on the constructal theory. However, in this study, the round tube has constant heat flux and temperature. Then, the heat is transferred from the tube wall to the fluid inside the tube by the radial fins (Fig. 10b). The authors evaluated the conductivity ratio, fin volume fraction, and number of fins. The maximum heat transfer rate and the minimum temperature were achieved for $N = 16$ with a fin fraction ratio of 0.1.

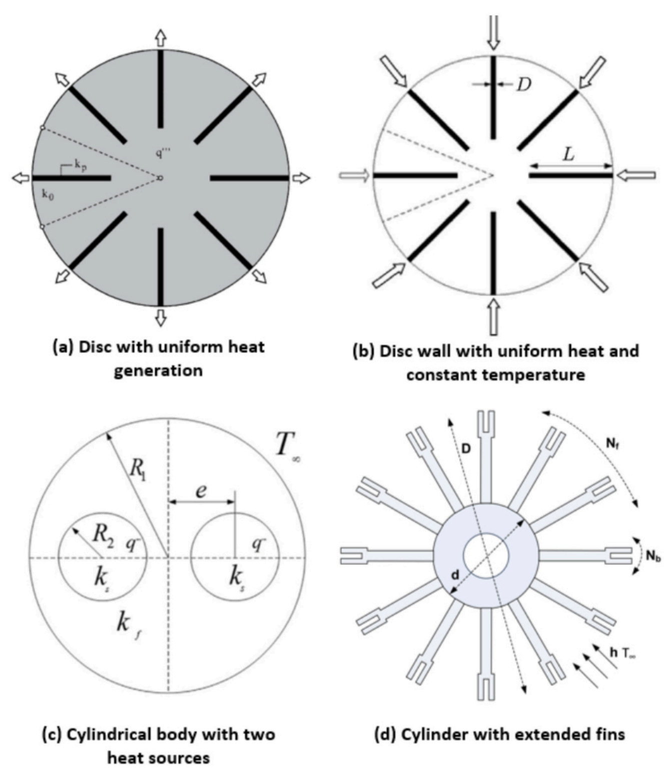


Fig. 10. Disc and cylindrical domains studied in the literature (Adapted from [119,120,122,124]).

Hajmohammadi et al. [121] applied the constructal design to minimize the maximum dimensionless temperature (θ_{max}) in an architecture of multiple cylindrical heat sources cooled by a circular fin. The heat is uniformly generated in the inner circles and dissipated to a cold free stream as shown in Fig. 10c. The authors investigated the optimal locations of the heat sources (from 2 to 4 heat sources) for several configurations, and then the optimized configurations were compared to identify the best thermal performance. The authors found that the triangular configuration had better thermal performance than the in-row distribution of three sources, resulting in lower thermal resistance under identical conditions. In general, the optimal heat-source locations lie near the centroids of equal fin sectors.

In the study of Gong et al. [122], they similarly applied constructal theory to identify the geometry of cylindrical heat sources with convective cooling and cylindrical or conical fins. For conical fins, the hot-spot temperature increases with the radius ratio of the top to the bottom circles and reaches a minimum when the heat sources are closely spaced. The minimum dimensionless hot-spot temperature with conical fins is about 7.26% lower than with cylindrical fins. Feng et al. [123] apply the constructal law to determine the optimal geometry for removing heat from a cylindrical internal heat source with an external circular helm-shaped fin. The helm-shaped fin reduced the dimensionless thermal resistance by 6.57% compared with the circular-shaped fin configuration.

Hajabdollahi et al. [124] studied the heat transfer performance of a cylinder with extended fins to dissipate heat via constructal design, as shown in Fig. 10d. The optimization was performed to improve the total heat transfer rate and reduce the fin material. The authors investigated simple fins and two- and three-branch (fork-shaped) fins, considering five design parameters. The results indicate that increasing the number of fin branches improves the heat transfer rate for a fixed fin length, with the three-branch configuration achieving the required heat transfer with the shortest fin length, followed by the two-branch and simple fins. Later, Hajabdollahi et al. [125] explored the same problem using a multi-objective optimization method, with the total heat transfer area and the heat exchanger effectiveness as objective functions. With simple fins, the heat exchanger effectiveness increased to 5.3% compared to the case without fins. More works related to circular fork-shaped fins, especially under dry, partial, and fully wet conditions, can be found in [126–128].

Similarly, Mustafa et al. [129] also studied the design of a row of radially finned tubes with simple fins. Using the constructal design method, the configuration is optimized by maximizing the volumetric heat transfer density while keeping the domain dimensions constant. Five geometric degrees of freedom are considered: fin spacing, number of radial fins, tube diameter, fin thickness, and the angular spacing between fins. In the analyzed cases, the maximum volumetric heat transfer density is achieved with two fins and decreases as the number of fins increases. Unlike the studies presented, which usually consider air or conventional liquids, only one study analyzed the magnetohydrodynamic natural convection cooling of liquid metal in a horizontal concentric annulus with tree-shaped fins [130].

In summary, the studies reviewed in this section indicate that increasing the number or complexity of fins does not necessarily improve thermal performance, since an optimal fin configuration generally exists for a given set of constraints. In many cases, the best results were achieved with a limited number of fins and an appropriate distribution of fin material. In addition, the location of heat sources and the arrangement of fins are important factors in reducing thermal resistance and improving heat transfer performance. These results show that the distribution of the flow architecture is often more important than simply increasing fin area.

4.3. Constructal design applied to heat sinks with different types of fins

4.3.1. Constructal fins

One of the most researched heat transfer problems is the convective fins or trees used in several cooling applications. This is also a fundamental topic, as it investigates the influence of fin geometry on thermal performance. In general, the heat current enters the base (root) of the fin and dissipates through it into a highly conductive material. The fin is usually under natural convection (T_∞ and h) or forced convection. In this type of problem, the main performance indicators are: (i) dimensionless global thermal conductance, (ii) fin efficiency, and (iii) fin effectiveness.

The fin efficiency is the ratio of the actual heat transfer to the heat transfer that would occur if the entire fin were at the base temperature. Fin effectiveness is the ratio of the heat-transfer rate with the fin to that without the fin. The dimensionless global thermal conductance is defined as:

$$\widetilde{q} = \frac{q}{kW(T_1 - T_\infty)} \tag{4}$$

where q is the heat current, T_1 is the temperature of the root, T_∞ is the ambient temperature, k is the thermal conductivity, and W is the width of the fin. Besides these performance indicators, some studies also use the entropy-generation perspective to study constructal fins [131].

Fig. 11 illustrates the evolution of constructal design applications for various fin shapes over time. The fin geometries are presented in chronological order of their first appearance in the literature. A clear increase in geometric complexity is observed, with designs evolving from simple T-shaped fins to more complex configurations, including Y-shaped, T-Y-shaped, double T-shaped, and two-stage T-Y-shaped fins, to enhance the thermal performance of the system.

This type of problem was first analyzed through the lens of the constructal design by Bejan and Almogbel [52]. The authors investigated the best design to maximize the dimensionless global thermal conductance of a T-shaped fin, as presented in Fig. 11a. The T-shaped fin has two elemental fins with thicknesses t_1 and t_0 and lengths L_1 and L_0, respectively. The fin receives a heat current from its root (q_1) with temperature (T_1), and the heat transfer occurs to a fluid with temperature T_∞ and constant heat transfer coefficient. The main goal was to find the best design in relation to L_1/L_0 and t_1/t_0 that maximizes global thermal conductance. The results indicate that $\widetilde{q}$ increases as the fin material volume fraction and T-shaped construct increase. The constructal design fin improved the global conductance by 29% compared to optimized fins reported in the literature. The authors also investigated Tau-shaped and umbrella cylindrical fins. The impact of the heat transfer coefficient on the T-shaped fin problem is discussed in [138]. The constructal approach was validated using a CFD code in [139].

Lorenzini and Rocha [132] applied the constructal design to determine the geometry of the Y-shaped fin, similar to the previous work using the T-shaped fin [52], but also considering the angle between the branches as a degree of freedom (Fig. 11b). The main goal was to minimize global thermal resistance by keeping the total volume and fin material constant. The minimized thermal resistance of Y-shaped fins is lower than that of T-shaped fins (~11% in one case), and the optimal thickness ratio for Y-shaped fins is about 12% higher than that for T-shaped fins.

Later, Lorenzini and Moretti [140,141] proposed Y-shaped fin profiles varying the angle of the arms of the original T-shaped fin. For a single fin, the highest global conductance was for an angle of 180° in the vicinity, which is similar to a T-shaped fin. However, they emphasize the importance of using the new performance indicators, such as fin efficiency and fin effectiveness. The authors also analyzed multiple fin systems and found that for five Y-shaped fins, the best angle was 78°, which increased performance by 4 times compared to the baseline case [140]. The geometry of the T-shaped fin was also optimized to minimize the entropy generation [142]. Hui et al. [134] studied the constructal

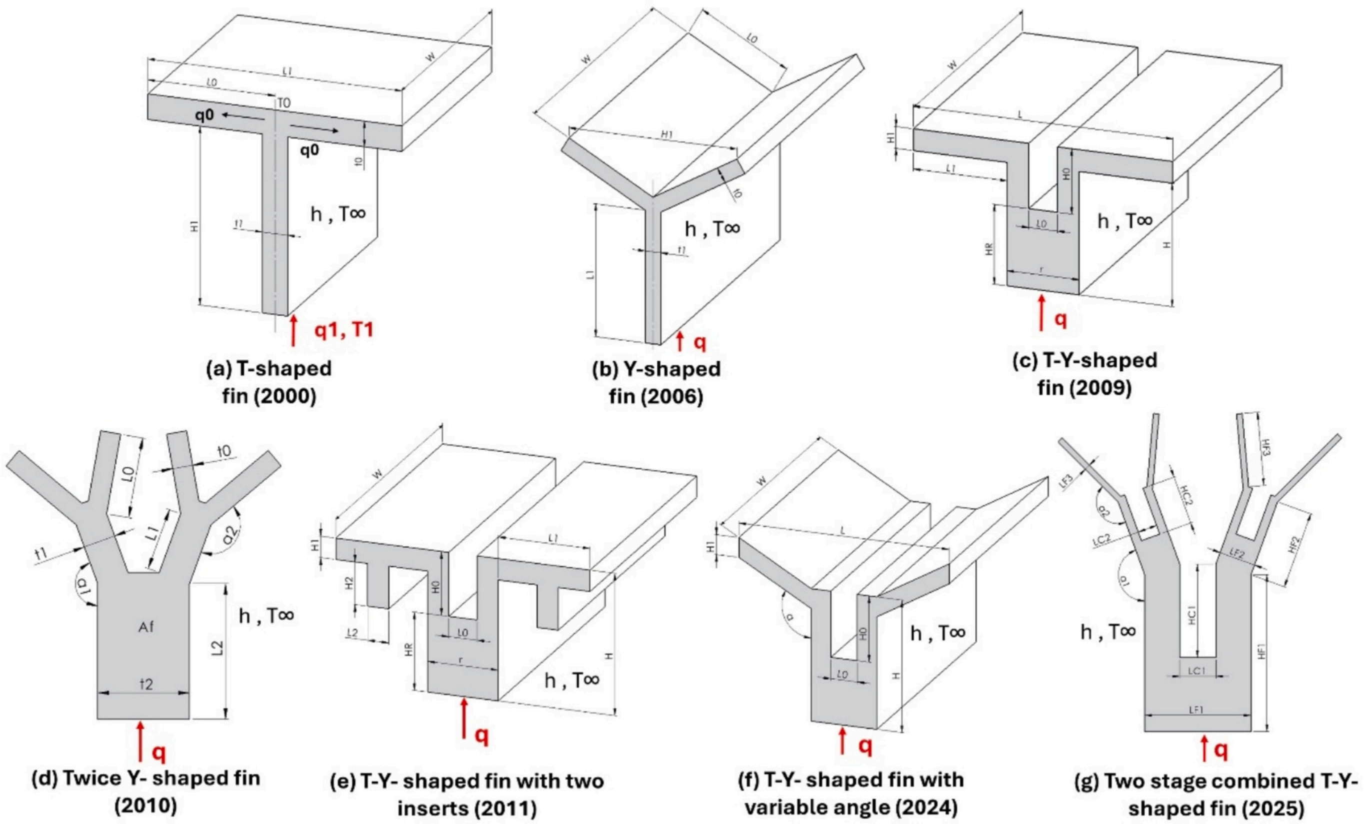


Fig. 11. Evolution of the constructal design method applied to different fins shapes (Adapted from [52,132–137]).

design of twice Y-shaped fin assemblies with six geometric degrees of freedom to minimize the dimensionless maximum thermal resistance, as shown in Fig. 11d. The results show that the twice Y-shaped configuration improves heat transfer performance, reducing the maximum thermal resistance by 36.4% compared to a single Y-shaped assembly.

The performance of the T-shaped fin under variable thermal conductivity and convective heat transfer coefficient is analytically investigated in [143]. The results show that the performance of constructal fins is influenced by both base and ambient temperatures: increasing base temperature reduces performance, whereas higher ambient temperature enhances it. Bhanja and Kundu [144,145] increased the complexity of their analytical models by incorporating radiation into the heat transfer problem and treating the T-shaped fin as a porous fin. Increasing the radiation parameter enhances the dimensionless heat transfer and slightly increases the optimal thickness-to-length ratio of the stem [145]. The performance also increases with higher porosity and decreases with increasing thermogeometric parameter [144]. Hajmohammadi et al. [146] also considered the radiation effect on a T-Y-shaped fin.

Hazarika et al. [147] developed an analytical method to predict the performance of a fully wet constructal T-shaped fin assembly under dehumidifying conditions. The maximum heat transfer rate and the optimal thickness-to-length ratio of the stem aspect ratio increase with relative humidity and ambient temperature, whereas a higher base temperature reduces the heat transfer rate and shifts the optimum toward a larger thickness-to-length ratio of the stem. Hazarika et al. [148] also considered the temperature distribution, fin efficiency, and optimal design parameters of a constructal T-shaped porous fin operating under fully wet conditions.

In other studies, the authors also analytically predict the thermal performance and optimal design parameters of a wet T-shaped fin, accounting for temperature-dependent thermal conductivity of the fin material and a variable convective heat transfer coefficient [149,150]. They also evaluated the relationship between the humidity ratio of saturated air adjacent to the fin surface and the fin surface temperature, and the effect on the T-shaped fin performance under fully wet conditions [151]. Recently, they also determined the dry-region length, temperature distribution, performance, and optimal design parameters of a T-shaped fin under partially wet conditions [152].

Das and Kandu [153] performed direct and inverse analyses of dry and wet T-shaped fins to evaluate heat transfer performance and determine optimal geometric parameters under a prescribed fin volume. Diao et al. [154] studied the constructal design of a T-shaped porous fin by minimizing a composite objective function that combined the maximum temperature difference and the pumping power. Compared with the original design, the optimized configuration reduces the composite function by 12.2% and the maximum temperature difference by 42.7%.

Lorenzini and Rocha [133] evaluated the geometry of a T-Y fin assembly with an internal cavity by minimizing the global thermal resistance subject to constraints on the total, fin-material, and cavity volumes, as shown in Fig. 11c. The results show that smaller cavity volumes and larger fin volumes enhance thermal performance and the twice-minimized global thermal resistance. The T-Y-shaped fin showed slightly higher performance than the Y-shaped fin. Zhang et el. [136] applied the constructal design method to a variable-angle T-Y-shaped fin, where the branched fin angle is an additional variable (Fig. 11f). The optimization results indicated that a branched angle of 17.2° reduces the maximum dimensionless excess temperature by 19.3% compared to a 90° configuration, while 3-DOF optimization further lowers it by 28.7%.

Based on the T-Y-shaped fin, a new fin geometry was proposed with a cavity between the branches of a T-Y configuration and two additional extended surfaces, as presented in Fig. 11e [135]. Global thermal resistance is minimized through geometric optimization under fixed-volume constraints, demonstrating that increased fin material enhances performance. The optimized complex assembly achieves about

32% lower thermal resistance than the best T-Y configuration under identical conditions. Fig. 12 compares the two designs and their respective thermal performance.

Zhang et al. [137] apply constructal theory in a two-stage combined T-Y-shaped fin by minimizing the maximum dimensionless thermal resistance, which decreased by 20.4% from 1-DOF to 6-DOF optimization, with the branched fin length-width ratio identified as the most influential parameter (Fig. 11g). Compared with other configurations, the optimized two-stage T-Y fin reduces by 33.7% relative to a single-stage T-Y fin and by 1.5% relative to a variable-angle T-Y fin. A geometric optimization of a two-stage T-shaped porous fin is also reported in Ref. [155]. Different fin shapes, such as cascade fins composed of two different materials, and I-shaped fins cooling a heat-generating plate, have also been studied in [156,157].

Other studies have evaluated T-shaped fins attached to a uniformly heated, insulated basement under steady convective cooling [158–161]. The heated solid base may have semicircular, cylindrical, rectangular, or trapezoidal geometries, as shown in Fig. 13. The results show that global thermal resistance can be minimized through constrained geometric optimization guided by constructal design.

From the presented constructal fins literature, it is clear that evolution has progressed from simple T-shaped fins to more complex Y-, T-Y-, and hierarchical branched configurations. It is evident that increasing the number of branches and the geometric degrees of freedom improves thermal performance by distributing the fin material more effectively and increasing the heat-transfer area available for heat dissipation. Then, the best constructal fin architectures are those that distribute heat more uniformly from the heat source (fin base) toward the fin extremities, reducing thermal resistance and improving overall conductance. However, the thermal improvement tends to diminish as the fin's complexity increases with the addition of more branches. Since different performance indicators are used for this type of problem, the one chosen to maximize may lead to different optimal geometries. Therefore, the optimal constructal fin design depends not only on the fin design but also on the performance indicator and optimization objective adopted.

4.3.2. Arrangement of different types of fins, rectangular and pin fins

The constructal design of rectangular and pin-fin heat transfer problems typically concerns finding the optimal geometry and arrangement of a fin array to improve the system's thermal performance. Fig. 14 presents generic physical models of rectangular (a) and cylindrical pin fins (b). Most of the literature relies on numerical studies in which a heat flux is imposed at the base, while the fluid flow is parallel or perpendicular to the fins. An alternative approach is to prescribe a constant temperature on the fin surfaces, rather than using a heat-generation parameter, while maintaining the surrounding fluid at a lower temperature. In general, the literature on this problem has

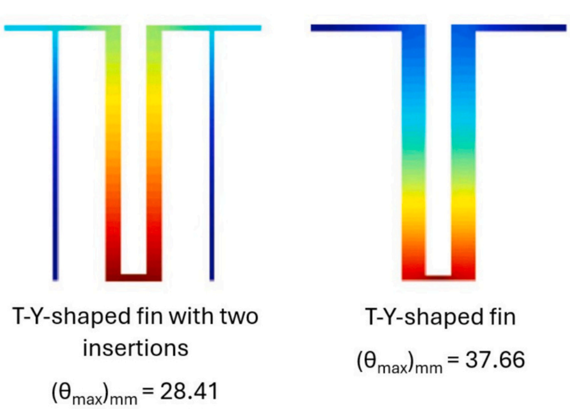


Fig. 12. Comparison of the best results found for T-Y-shaped and T-Y-shaped with insertions for $H/L = 1$, $\phi = 0.1$ and $\phi = 0.05$ (Adapted from [133,135]).

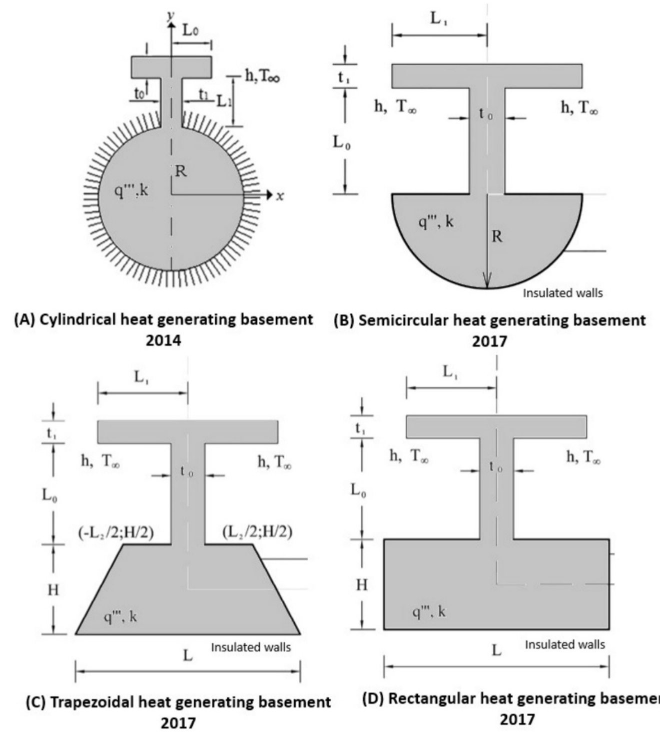


Fig. 13. Different basement geometries generating heat with T-shaped fins (Adapted from [158–161]).

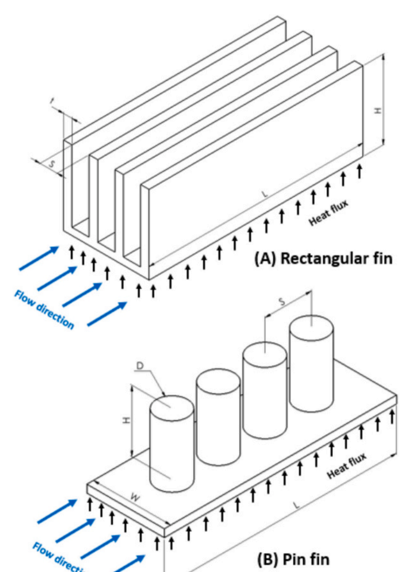


Fig. 14. Rectangular fins (a) and cylindrical pin fins (b).

investigated the influence of position, height, spacing, fin diameter (or thickness), and the number of fins on the system's performance. The main performance indicators for these problems are the overall heat transfer rate, the pumping power (or pressure drop), the maximum temperature difference, and the thermal resistance or conductance.

Bejan et al. [162] investigated a parallel rectangular fin with heat transfer from the base and a flow direction parallel to the base, as shown in Fig. 14a. In this work, the authors simultaneously optimize fin and flow-channel aspect ratios to achieve a high overall heat transfer rate. For a specified total volume and known inlet flow conditions, the constructal theory predicts the optimal configuration, the overall heat transfer rate, and the required pumping power as functions of multiple DOFs. With similar rectangular fins and boundary conditions, Liu et al. [163] applied constructal design, combined with optimization techniques, to optimize the geometry of rectangular porous fins with respect to the maximum temperature difference and pumping power. Optimizing the fin number and length reduced the maximum temperature difference by 66.3% compared with the original design.

Previous works considered rectangular fins under forced convection. On the other hand, Zhang et al. [164] investigated the space between the rectangular fins under natural convection to maximize the heat transfer. The results indicate that optimal spacing differs from conventional designs, with enhanced heat transfer driven by outlet velocity effects rather than early boundary-layer merging. By an analytical expression, the optimal plate-to-plate spacing is $\frac{4}{3\delta}$ (being δ the velocity boundary-layer thickness). Zadhoush et al. [165] also studied the geometry of rectangular fins under natural convection using the constructal theory. Longitudinal and transverse fin arrangements are analyzed to assess their cooling performance, showing that fin arrangement strongly influences thermal efficiency. The optimal configuration is a longitudinal arrangement with six fins, achieving a maximum heat transfer rate of 34.47 W. Bejan [166], through the constructal design, identified the optimal internal structures geometry (spacing, aspect ratio, and sizes) that improve the thermal performance of parallel plates, rectangular pin fins, and an array of horizontal cylinders under forced and natural convection. The optimal aspect ratio for ducts relates to the geometry and flow conditions in which thermal boundary layers merge near the channel exit.

Bello-Ochende and Bejan [167] applied the constructal theory to find the configuration of two rows of cylindrical pin fins under laminar forced convection that maximizes the total heat transfer rate for a fixed fin material volume (similar to Fig. 14b). The optimal design results from balancing conduction along the fins and transverse convection. Numerical results show good agreement with scaling analysis predictions and indicate that non-uniform fin diameters and heights resulted in the best thermal performance. Later, Bello-Ochende [168] investigated the thermal performance of cylindrical pin-fin heat sinks with multiple array configurations under forced convection. The results show that varying fin height has a limited impact, while optimized fin diameter plays a dominant role in improving thermal performance. It was also found that the three-row configuration improves thermal performance by 10% for $Re > 100$ compared to the two-rows configuration.

Similarly, Yang et al. [169] evaluated the inline arrangement of cylindrical pin fins under forced convection to maximize heat transfer rate and minimize pressure drop. They identified the optimal pin diameter, fin number, and heat sink aspect ratio that maximizes thermal performance. They also studied the same problem, considering the entropy generation rate [170]. Other works also use the minimum entropy generation rate as an objective function to optimize the cylindrical pin-fin heat sink geometry, as reported in [171,172]. Another study examined a multi-scale cylindrical porous fin, minimizing a composite function of the maximum temperature difference and pumping power. The optimized design reduces the composite function by 51% and, via ANN-assisted NSGA-II optimization, decreases the maximum temperature difference by 81.9% and the pumping power by 18.7%, demonstrating the superior performance of multi-scale porous fins [173].

Besides the pin-fin arrangement under forced convection, some works have considered different fin arrangements under natural convection conditions [174–176]. Joo et al. [175] conducted a topology optimization of a pin fin array to thermally optimize natural-convection heat sinks. The resulting multiscale fin geometries align with constructal theory predictions and result in 13% lower thermal resistance and 48% lower mass than analytically optimized radial plate-fin heat sinks.

Most of the literature addresses cylindrical pin fins, since they are the most suitable geometry in industry. However, other works also investigated different pin fin shapes such as: diamond-shaped pin fins cooled by mixed convection [177]; concave/convex and tapered fins under forced convection [178,179]; T-shaped under forced convection by parallel and cross-flow [180]; snow-flake fin under conduction [181]; longitudinally finned tubes cooled by forced convection [182,183]; and the position of various heat sources connected and cooled by square-shaped fin [184].

More recently, Godi [185–187] studied micro-channel heat sinks with cylindrical or rectangular pin fins, as shown in Fig. 15. Godi [185] inserted solid, half-hollow, and fully hollow fin configurations on a microchannel to maximize the dimensionless thermal conductance with fixed axial length and total volume. The results show that hollow fins performed better than solid fins at moderate Reynolds numbers ($Re <$ 475) by promoting internal fluid circulation and enhanced heat removal, while solid fins perform better at higher Reynolds numbers due to reduced flow complexity. Compared with conventional micro-channels (without fins), solid fins improve thermal conductance by 17.1%, half-drilled fins reduce peak temperature and increase conductance by 13.4%, and fully hollow fins provide the best performance, increasing conductance by up to 17.5%.

In general, from the studies reviewed in this section, it is possible to observe that the thermal performance of rectangular fins, pin fins, and the heat exchanger with cylindrical pin fins is determined by the distribution of the fins within the available volume. More specifically, for parallel rectangular fins, the best design depends on the fin spacing and aspect ratio, which allow thermal boundary layers to develop efficiently. For pin-fin heat sinks, the results indicate that non-uniform fin design and arrangements often provide better thermal performance than uniform configurations, since they improve the balance between heat transfer and pressure losses. More recently, studies on heat exchangers with cylindrical pin fins and microchannels have shown that both the fin arrangement and the internal fin geometry can affect thermal performance. Finally, the reviewed studies indicate that thermal performance improves when the available fin material and flow passages are properly distributed within the domain, rather than simply increasing the number or size of the fins.

4.3.3. Trees of convective fins

The first work related to convective tree fins was developed by Bejan and Dan [188]. In this work, the main goal was to identify the geometry that maximizes the overall thermal conductance. In other words, the geometry that best distributes a limited amount of high-conductivity material within this domain to collect heat from a localized boundary source and spread it efficiently throughout the volume. The problem was treated as two-dimensional with fixed volume (frontal area and length), consistent with forced convection. The authors investigated several configurations, changing the number of branches from the main root and also their aspect ratio. Using the constructal approach, it was possible to identify that the optimized structures in the first construct

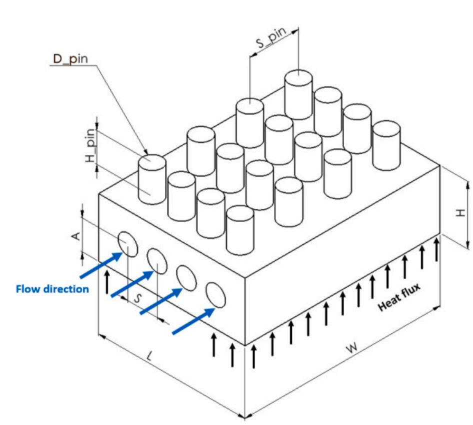


Fig. 15. Heat exchanger with cylindrical pin fins.

consist of a single thicker main root with multiple thin branches, whereas in the second optimized construct, the geometry evolves into a T-shaped thicker main root with several thicker secondary branches. Bejan [189] reviewed developments in minimizing flow resistance between a point source (or sink) and a finite volume via geometric optimization under volume constraints. Using steady volume-to-point heat conduction as a reference problem, the method begins by optimizing an elemental volume and progressively extends to larger assemblies. The resulting optimal configurations naturally evolve into tree-shaped structures. It is also highlighted that constructal theory is relevant for explaining similar tree-like patterns observed in nature.

Almogbel [190] investigated the optimization of a convective tree-shaped fin from the constructal perspective. The total volume and fin material were considered as constraints. At each optimization stage, additional branches are introduced to increase the fin material's access to the heat transfer surface. It was found that a uniform tree-shaped fin with equal branch lengths and thicknesses exhibits better thermal performance than conventional longitudinal and optimized T-shaped fins, while maintaining compact geometry. Kephart and Jones [191] explored the optimization of metal-matrix heat sinks in a convective medium. The results revealed that optimal fin architectures naturally evolve into tree-like structures with functionally graded porosity. When the fin shape, aspect ratio, and base-to-fin thickness are simultaneously optimized, total heat transfer can increase by up to 55% compared with conventional fin configurations. Çetkin [192] reported that the peak temperature of a non-uniformly heated 3D solid can be reduced by adding high-conductivity tree-shaped inserts connected to a heat sink through their stem. For a fixed solid volume and a high-conductivity material fraction, the influence of hot-spot location and insert geometry is examined. Increasing the number of bifurcation levels from zero to two lowered the peak temperature. Optimal diameter ratios and bifurcation angles at each branching level were identified, and Y-shaped trees performed better than T-shaped trees.

4.3.4. Composite heat transfer

More recently, composite heat-dissipating architectures have been proposed in the literature [193–195]. In this type of work, high-conductivity channels of various shapes are inserted into a heat-generating body, which is externally connected to a T-shaped fin. The heat generated is collected from the high-conductivity channel (conduction), transferred to the external T-shaped channel, and dissipated to ambient air by convection.

Fig. 16 presents an arrow-shaped high-conductivity channel connected to the T-shaped fin proposed by Chen et al. [193]. In this case, an arrow-shaped high-conductivity channel (k_p) is embedded in a square heat-generating body composed of low-conductivity material (k_0) and externally connected to a T-shaped fin (k_l). A thin interfacial layer with conductivity (k_r) is introduced at the junction between the channel and the fin to model contact thermal resistance. Uniform volumetric heat generation occurs within the low-conductivity material, with the generated heat first converging toward the arrow-shaped channel and then flowing into the T-shaped fin. Heat is dissipated to the surrounding air by convection with a heat transfer coefficient, while the remaining boundaries of the heat-generating body are considered adiabatic. Using the constructal theory and optimization techniques, they found that the optimal geometry with four degrees of freedom reduces the minimum dimensionless maximum temperature difference by 43.9% and the entropy production rate by 19.6% compared with single-degree-of-freedom optimization.

In another work [194], an X-shaped high-conductivity channel connected to a T-shaped fin under convective conditions was proposed and optimized. Compared with the arrow-shaped channels [193], the X-shaped channels reduced the minimum dimensionless maximum temperature difference by 28.1% and lowered the hotspot temperatures. Finally, Chen et al. [195] studied the incorporation of fan- and clover-shaped conductive channels into heat-generation bodies coupled to an externally T-shaped fin. The performance indicators are the minimization of the maximum temperature difference and the entropy generation rate in a multi-objective optimization. In general, slender channels and fin geometries improved heat dissipation by enabling more effective heat conduction and increasing the available convective surface area. Overall, the clover-shaped configuration performs better, resulting in about 23% lower maximum temperature difference and 1.4% lower entropy generation rate than the fan-shaped design.

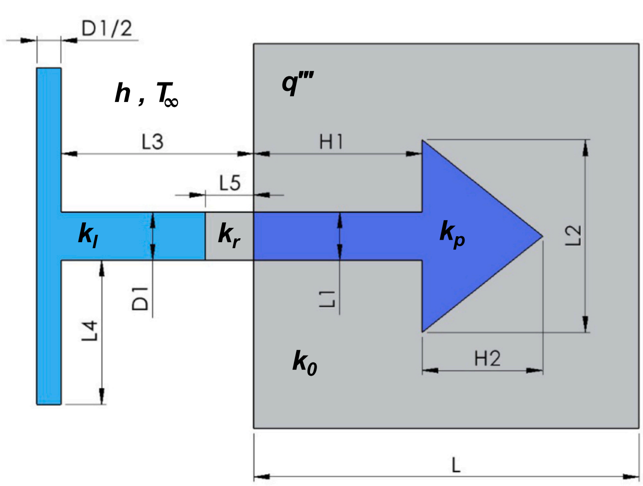


Fig. 16. A composite heat-dissipating structure composed of an arrow-shaped high-conductivity channel externally connected to a T-shaped fin (Adapted from [193]).

4.4. Constructal design applied to fins in latent heat thermal energy storage

Latent heat thermal energy storage (LHTES) has attracted attention in the literature due to its strong connection to energy storage research and its importance for renewable energy systems. The use of phase change materials (PCMs) for cooling applications is also important, as it is defined as a passive cooling approach that does not require external power sources, such as fans or pumps, to drive fluid flow. This characteristic makes PCM-based cooling an attractive energy-efficient solution. However, the main challenge with LHTES is the inherently low thermal conductivity of most PCMs, which results in relatively long charging and discharging times. In this context, several researchers have investigated the use of finned structures made of highly conductive materials to address these limitations. Nonetheless, most studies on fin insertion in PCM heat sinks do not explicitly use the term “constructal” and were therefore excluded from this systematic literature review. Using the adopted methodology, only thirteen documents were identified.

In these works, the main goal was to study different flow architectures (fins) in PCM reservoirs to improve the system's heat-transfer performance. It is important to note that, during the early stages of the PCM melting process, heat transfer is dominated by conduction because the PCM remains in the solid phase. As melting progresses, natural convection becomes more important and must be accounted for to achieve an optimal design for a latent thermal energy storage system. During the solidification process, the heat transfer mechanisms act in the opposite sequence. Then, the optimal flow architecture for these problems should perform well in both conduction and convection heat transfer mechanisms. The main performance indicators found in the literature are as follows: (i) melting or solidification time, (ii) average temperature or maximum temperature evolution by time, and (iii) energy absorbed or released.

The first document to use the term “constructal” and fins in LHTES was published by Wang et al. [196] in 2006. In this work, the energy is stored in a PCM cavity at −1 °C, released from a hatch on the left side at

60 °C, and the other surfaces are insulated. The main goal was to design the distribution of highly conductive material to improve the melting process. The best distribution of the highly conductive material exhibits a dendritic shape and performs better than arbitrary shapes. It is clear that the developed hierarchy in the best designs features a few thicker trunks and several small ramifications. The optimized constructal design fin presented a melting time of 923 s, which was around 29.5% of the melting time of the first structure (rectangular fin).

Fig. 17 presents some of the fins' configurations studied in the literature. Kalbasi and Salimpour [197] investigated the melting process in a PCM heat sink with the intrusion of horizontal fins. Fig. 17a illustrates the computational domain considered in this study. The authors varied the enclosure aspect ratio (AR), the number of fins, the fin length, and the number of enclosures. With a low number of fins (meaning longer fins), the space between the enclosures is almost blocked. As a result, the heat from the bottom is trapped in the lower levels, increasing the base temperature and decreasing the maximum time required to reach the system's critical temperature. On the contrary, increasing the number of fins shortens their length, promoting a less uniform temperature distribution within the enclosure and reducing the maximum time. Thus, the optimal maximum time for safe operation corresponds to the optimal number of fins. In another work, Kalbasi and Salimpour [198] evaluated the use of vertical fins to maximize the safe operating time while avoiding critical temperatures. Results show that PCM melting in finned rectangular enclosures is governed by fin-PCM contact area and natural convection, with wider enclosures outperforming square or slender ones. The fin thickness ratio has little effect and increasing the number of enclosures does not necessarily improve the system's thermal performance. Salimpour et al. [199] also evaluated boundary-layer development for vertical fins in a PCM-based heat sink and concluded that, for a given enclosure length, there exist optimal values of volume fraction and fin spacing that maximize heat sink performance.

Joshi and Rathod [200] investigated the PCM melting time using three horizontal rectangular fins positioned on the right side of the PCM reservoir, based on the constructal theory. The right wall has a constant temperature of 70 °C, and the others are insulated. In the first stage, the authors evaluated the impact of changing the length of the tree fins. The first, second, and third fins were 50, 75, and 100% of the length of the baseline (Config. 1 - Fig. 17b). After 100 min, the Config. 1 attains a melt fraction of 0.93 and absorbs 270.19 kJ, while the baseline reaches 0.95 and 273.38 kJ, respectively. The small differences are attributed to the larger PCM volume resulting from reduced fin volume. In the second stage, the authors identified reservoir hotspots and relocated the Config. 1 fin. The constructal design results in a 6.3% increase in melt fraction despite a 50% reduction in the fin-to-PCM volume ratio, and it also improves the total energy absorbed by the PCM by 4.4%.

Barik and Swain [201] investigated the melting time in tree-shaped fins intruded into a cylindrical PCM reservoir as shown in Fig. 17c. They studied seven configurations: the 2^{nd} construct is a Y-shaped fin, the 3^{rd} is a tree-shaped fin with two ramifications, and the 4^{th} is a tree-shaped fin with four ramifications from the principal trunk (Fig. 17c). The results show that melting time decreases with increasing fin complexity. The 4^{th} construct fins reduced melting time by 42.5% compared with the 2^{nd} construct fins, while configurations with fins on both inner and outer surfaces performed better than those with fins on a single surface. In particular, the 3^{rd} with inner and outer fins achieves a 50.3% reduction in melting time relative to the 2^{nd} configuration. Shukla et al. [202] examined the melting and solidification behavior of a metallic heat exchanger filled with PCM and equipped with tree-shaped fins as shown in Fig. 17d. The number of fins generated ranges from no fins to one branch (rectangular), two branches (Y-shaped), to three branches (tree-shaped). The branch angles considered are 30°, 45°, and 60°, while the fin branch length ratios were 0.75, 1.0, 1.25, and 1.50, and the fin volume fractions were 3%, 5%, 7%, and 9%. The results show that, at a constant fin volume, increasing the number of fin branches improves melting and solidification rates by increasing the heat-transfer area, with tree-shaped fins performing better than rectangular ones. With higher fin volumes, the total stored energy is reduced, while fin length ratio and branch angle have only minor effects.

More recently, Arshad [203] studied the constructal design applied to PCM-based heat sinks for thermal management. A novel tree-shaped constructal fin heat sink is analyzed by varying fin height (0-20 mm) and

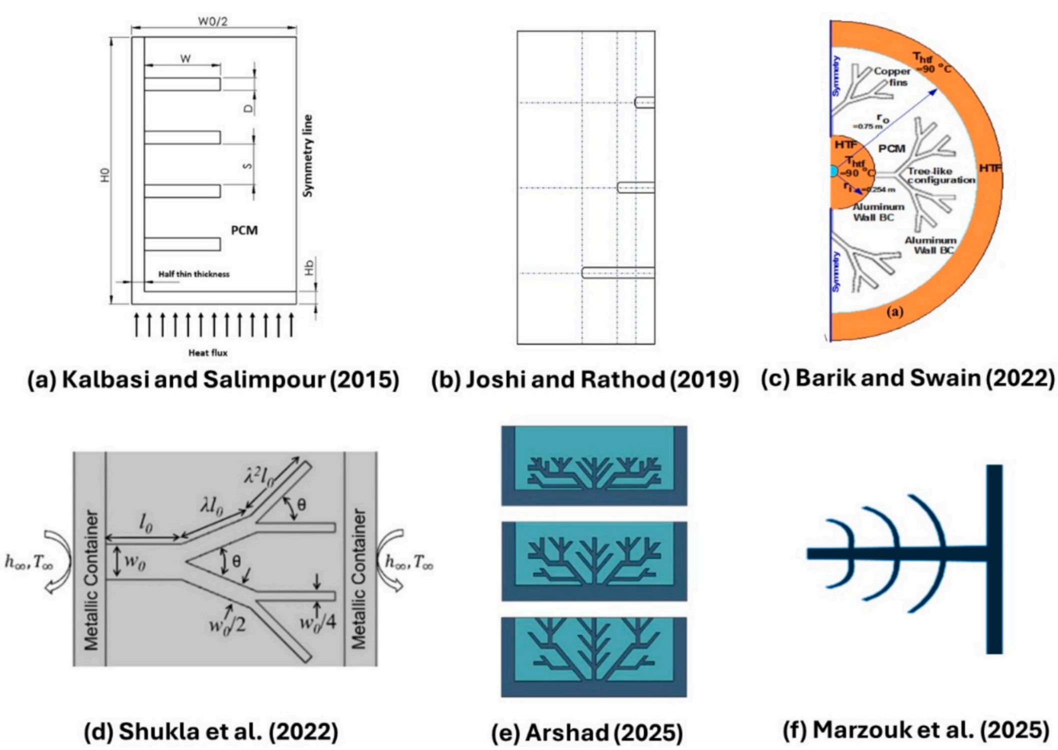


Fig. 17. PCM-based heat sinks with different fin designs. Adapted from [197,200–204].

volume fraction (0-20%) at a constant heat flux (Fig. 17e). The results show that heat-transfer performance is governed by fin height and branching, with tree-shaped fins reducing the average heat-sink temperature by about 8% compared to unfinned configurations. Higher fin heights increase the time required to reach a target temperature, as natural convection dominates the PCM melting process. An optimal configuration with a 20 mm fin height and a 10% volume fraction is identified as the most effective for passive cooling. Marzouk et al. [204] investigated the PCM melting in a rectangular enclosure with the intrusion of rectangular, constructal fin (Y-shaped), and tree-shaped fins, evaluating liquid fraction, melting time, and heat transfer rate. The constructal fin design is shown in Fig. 17f. The results show that fins enhance melting performance, with tree-shaped fins providing the best results, achieving complete melting within 130 min and reducing the melting time by 67% relative to the baseline case (without fins). Rectangular and constructal fins reduce melting time by 41% and 53%, respectively, while tree-shaped fins also achieve the highest heat transfer rates and energy storage capacity.

Eslami and Bahrami [205] studied the transient behavior of a sensible and latent heat storage tank with fin configurations optimized based on constructal theory. The fins were responsible for the heat conduction to the PCM tank. The fin-to-PCM volume ratio was kept constant at 10%. The authors investigated I-, V-, and pencil-shaped fins. In sensible heat storage, the pencil-shaped fin improved thermal performance by up to 42% compared to the I-shaped fin. In PCM latent heat storage systems, conduction dominates during the early charging stage. Although V- and pencil-shaped designs enhance early melting, the total charging time remains similar across configurations. Solé et al. [206] performed a geometrical optimization of a PCM fin heat exchanger used in a solar receiver.

Khosravi et al. [207] investigated seven fin designs to reduce the melting and solidification times of a tank filled with PCM, based on the constructal theory. The authors studied horizontal and vertical parallel and snowflake-shaped fins in a rectangular PCM tank, as well as radial, V-shaped, Y-shaped, and triple fins in a cylindrical PCM tank. Fig. 18 presents the temperature and mass fraction for the configuration tested during the melting process at 30,000 s. The triple-fin with three branches presented the fastest melting time due to strong vortex formation and enhanced recirculation in the upper region. To minimize solidification time, the parallel-horizontal fin configuration was most effective because its uniform fin distribution enhances conductive heat transfer and maintains a more symmetric temperature field. When both charging and discharging are considered, the parallel-horizontal configuration provides the best overall performance based on the 100% melting/solidification criterion. At full charge and discharge, Y-shaped fins achieve the highest energy storage and release.

Finally, the literature on constructal design applied to fin-PCM systems indicates that fin design can overcome the low thermal conductivity of PCMs and reduce melting and solidification times. The design evolution in this field began with simple I-shaped fins and progressed to more complex Y- and tree-shaped configurations with multiple branches. In general, the best fin architectures are those that distribute heat more uniformly within the PCM reservoir and increase the heat transfer area available for conduction during the early stages of melting. However, most studies have focused on the melting process while neglecting the solidification. This may lead to designs that are optimal for melting but not necessarily for both thermal regimes. Additionally, increasing the fin volume fraction does not always improve overall thermal performance, as part of the PCM volume is replaced by highly conductive material. Therefore, the optimal fin design should balance melting time, solidification time, and thermal energy storage capacity of the system.

4.5. *Constructal fins applied to channel heat transfer problems*

The category of constructal fins in channels or ducts refers to heat-transfer problems in which fins of different shapes, aspect ratios, and numbers are inserted into channels or ducts. These problems typically involve forced convection as the dominant heat-transfer mechanism, since the fluid flows through the channels and removes heat from the fins, which are usually maintained at a constant temperature or subjected to heat generation at the channel base. In this type of problem, it is important to study heat transfer combined with flow separation and its subsequent reattachment, which is important in several engineering problems [208]. The performance indicators for this type of problem are usually the heat transfer rate, peak temperature, thermal conductance, and pressure losses.

Lorenzini and Moretti [209,210] investigated the thermal conductance and pressure losses of I- and Y-shaped fins inserted inside a channel under laminar forced convection. The authors evaluated these fins positioned only at the bottom, as well as at the bottom and top of the channel. The air was flowing at $Re = 1500$ and 297.15 K while the fin root temperature was 373.15 K. The maximum thermal conductance was achieved for Y-shaped fins at the top and bottom of the channel's surface. However, it resulted in the second-highest pressure drop, while the Y-shaped fin, only on the bottom, presented the lowest values of pressure losses. Thus, Y-shaped fins presented better thermo-hydraulic performance than I-shaped fins. Lorenzini and Moretti [211] also

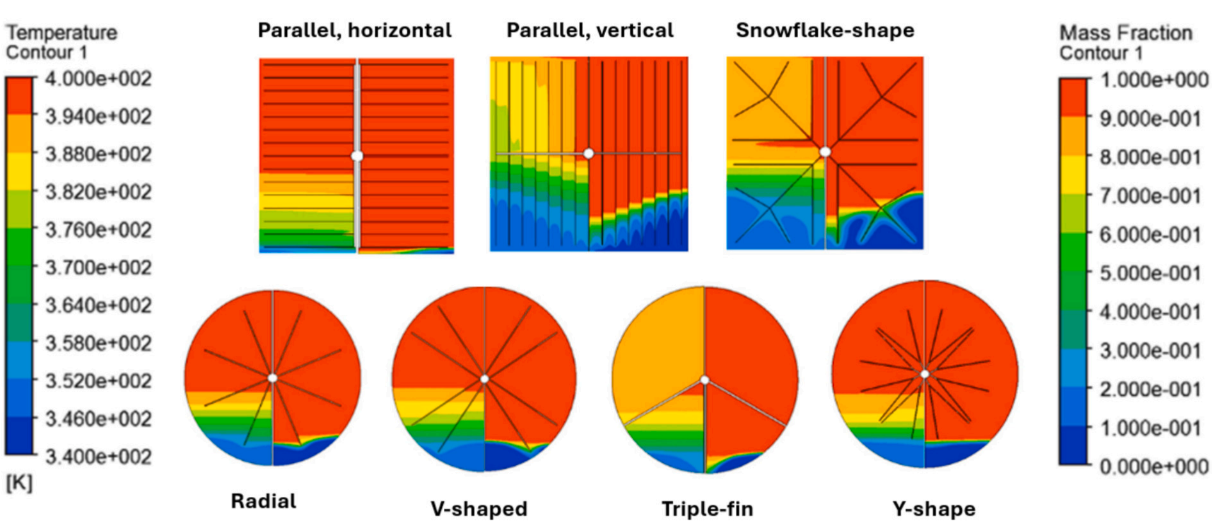


Fig. 18. Temperature (left halves) and mass fraction (right halves) of seven different fin designs during the melting process at 30,000 s (Adapted from [207]).

compared the same system with Y-shaped fins at the bottom only, using water and air as the fluids. Cetkin [212,213] investigated a heat generation domain that is cooled by a channel with I- and T-shaped fins. The author found the optimal design, based on constructal theory, for a given condition. The application of triangular fins based on the constructal design was also investigated, and the optimal case improved the thermal performance by 65.7% compared with the worst case [214].

An interesting design investigated is the wavy-fin channel in compact heat exchangers, which resembles a corrugated channel [215,216]. Song et al. [216] investigated the optimal wavy-fin channel geometry that maximizes the heat transfer rate and minimizes the pressure drop using constructal theory. The geometric parameters analyzed were the channel spacing, wavelength ratio, and amplitude ratio of the two wavy walls. The results show that the constructal wavy-fin channels can reduce the pressure drop by about 54% while improving the heat transfer rate by 26% compared with conventional designs. Another interesting architecture investigated in the literature is the louvered fin channel configuration reported in Refs. [217, 218].

Adewumi et al. [219] explored the geometry of a microchannel with the addition of a cylindrical pin fin using constructal theory to minimize the peak temperature of the system. The results indicate that the minimized peak temperature decreases with increasing Bejan number, while the maximum thermal conductance increases for optimized configurations containing three to six rows of micro pin fins. Wang et al. [220] did a similar study, adding rectangular fins to a microchannel and using the dimensionless entropy generation rate as the objective. They found that a hybrid staggered finned configuration decreases the entropy generation rate by 60.36% compared with the finless configuration. De Moura et al. [221] is another example of work that investigated the fin design in a microchannel heat sink using the constructal law and found the optimal design for its boundary conditions. Recently, Zhang et al. [222] investigated a microchannel with different pin-fin arrays to cool a chip with non-uniform heat generation using a genetic algorithm. Compared with a dense pin-fin array, the optimized fin layout reduced the maximum temperature by 6.75 K and the pressure dropped by 54.1% at $Re = 750$, achieving a better balance between heat-transfer performance and flow resistance under non-uniform heating.

Lu et al. [223] applied the constructal theory to an embedded finned microchannel heat sink for circuit liquid cooling under fixed total volume and fin-volume constraints. A conditional generative adversarial network was employed to predict performance, enabling a 5-DOF evolutionary optimization of fin cross-section geometry via a genetic algorithm. Fig. 19 presents the performance evaluation criteria (PEC) used in the study, which consider the average Nusselt number and the apparent friction factor of the microchannel, along with their benchmarks. It compares different fin cross-section shapes evaluated in the study. The circular figure inside the graph shows the degree of freedom of the fin cross-section. The optimized "butterfly-wing"-like fin shape reduces maximum and average temperatures while improving overall thermal performance. Compared with heat sink designs using near-elliptical, rectangular, square, and rhombus fin cross-sections, the optimized design improved PEC by 85.9%, 65.7%, 23.7%, and 21.9%, respectively. Other works also consider nanofluids and porous fins in channels, as reported in Refs. [224, 225].

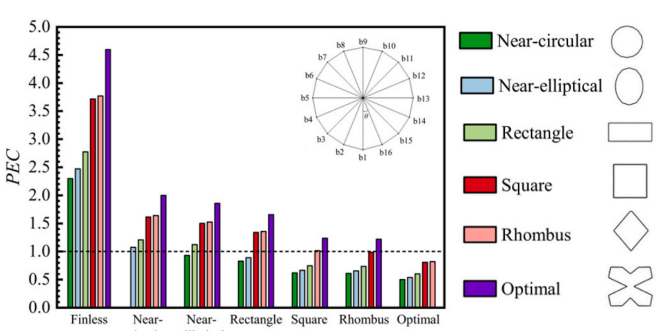


Fig. 19. Comparison of different fins cross section shapes in relation to the performance evaluation criteria (PEC) for $Re = 200$ (Adapted from [223]).

For the works involving fins applied to channels reviewed in this section, the system performance does not depend exclusively on heat transfer enhancement, but rather on the balance between heat transfer and pressure losses, which are directly related to the required pumping power. In this regard, increasing the complexity of the flow architecture usually improves heat transfer by promoting fluid mixing and disrupting the thermal boundary layer. However, these modifications also tend to increase flow resistance and pressure losses. Therefore, for this type of problem, the best configurations are those that achieve the highest thermo-hydraulic performance by balancing heat transfer rate and pressure drop. Studies on microchannels also indicate that the fin arrangement and cross-sectional geometry are important geometric parameters for improving thermal performance under volume and flow constraints.

4.6. Applications

The applications category includes documents identified in the systematic literature review that are not directly related to the fundamental heat transfer problems discussed previously. In this context, a variety of studies are presented that apply constructal law to diverse types of problems and applications, using various performance indicators.

Most applications use the constructal law to improve the geometry of heat exchangers. Hajabdollahi [226] performed multi-objective optimization of a constructal plate-fin heat exchanger, using effectiveness and total annual cost as objectives, and compared the results with a conventional design. The constructal configuration is preferable for effectiveness above 0.747, while the conventional design is optimal below this value. Sotoodeh and Amidpour [227] also modelled and optimized a multiple-stage plate-fin exchanger by the constructal theory. They found that constructal heat exchangers improve heat recovery by 7% and 11.5% for two-section and three-section heat exchangers, respectively, compared with the conventional heat exchanger. In turn, Dehaj and Hajabdollahi [228] studied a constructal fin-and-tube heat exchanger using effectiveness and total annual cost as objective functions. The constructal branch-type design improves the maximum effectiveness by 6.65% compared with the conventional heat exchanger. The constructal design was also applied to improve the thermal performance of coaxial and finned crossflow heat exchangers, and microjet liquid-cooled heat sinks [229–231].

The constructal design method was also used to identify the best geometries of insulating walls, accounting for thermal insulation and mechanical strength constraints [232–235]. The optimal number of air gaps (cavities) increases with stronger natural convection effects and lower prescribed wall stiffness, while the maximum wall thermal resistance is higher when natural convection within the air gaps is weaker, and the wall stiffness is reduced [232]. This type of problem extends constructal design, typically applied to fluid flow or heat transfer problems, showing that it can also be effectively integrated with mechanical strength requirements [232]. The study of the constructal theory on flow architectures in mechanical engineering and civil engineering is reviewed in Ref. [236]. For example, within mechanical engineering, constructal design has been employed to investigate gas turbine blades and to optimize the design of low-speed turbines [237,238].

Another application is the use of the constructal design in solar thermochemical reactors to identify the geometry that maximizes reactor efficiency [239,240]. Different reactor designs were evaluated, such as a filled cylinder, a cavity, and a honeycomb. The optimized honeycomb reactor allows an efficiency increase (~10%) with respect to the single cavity reactor [240]. Neveu et al. [241] also combined the constructal theory and exergy optimization to study the design of thermochemical reactors. The application of the constructal design was also found in problems involving solar collectors and thermophotovoltaic systems [242,243]. Kant and Pitchumani [244] investigated the influence of incorporating constructal tree-shaped fins on the thermal performance and cost of a thermochemical energy storage system for storing solar thermal energy in building applications. The constructal tree-shaped fins improve heat transport during the discharge phase. Systems without fins fail to meet U.S. Department of Energy targets, exhibiting low energy density and high storage cost, whereas embedding constructal fin trees enhances performance, achieving energy storage densities above 140 kWh/m^3 and levelized storage costs below $4/kWh, thereby meeting the energy storage requirements.

The applications shown here clearly indicate that constructal design for fins and cavities extends beyond just heat transfer. It is increasingly being used in thermal energy systems, civil engineering, and thermochemical materials. These studies demonstrate the predictive power of constructal law in guiding the development of new flow architectures across different technologies.

5. Conclusion and future directions

This work presents the constructal design evolution of fins and cavities in heat transfer problems, showing how this research field has evolved over time, the main performance indicators, and the most prominent topics. To address these aspects, this research was conducted through a systematic literature review following a review protocol. The searched terms were "constructal" combined with "fin" or "cavity". The documents selected were carefully analyzed.

From the bibliometric analysis, it can be concluded that the constructal design applied to fins and cavities heat transfer problems is expanding and growing steadily, as evidenced by the increasing number of documents and citations per year. It was also evident that this research topic is being pursued worldwide, with several countries and research groups studying how the evolutionary flow architectures of fins and cavities impact the heat transfer performance of different systems. The authors' keywords also showed some promising research topics in this field, some of which are more recent in the literature. Topics related to forced and mixed convection, and to heat transfer problems involving phase change materials, appear to be more recent in the literature and more prominent for future research. Besides that, the combination of the constructal design method with multi-objective optimization techniques has also been shown to be a recent and prominent topic for future research.

The evolution of fin and cavity designs is evident over time in the evaluated heat transfer problems. Some common design principles emerged across the literature. In general, thermal performance improves when the flow architecture provides better access for heat flow, distributes cooling paths more uniformly throughout the domain, and promotes a more effective distribution of the available conductive material. The reviewed studies also showed that increasing geometric complexity and the degrees of freedom often improve thermal performance.

In cavity problems, it was possible to see the evolution of the cavity's geometries studied over the years. The results show that when a geometry is free to morph, it facilitates access to its currents and improves its performance. In the case of cavities in square domains, the elemental cavity (C- or I-shaped cavity) has a dimensionless global thermal resistance of 0.113. With the evolution of more complex cavity designs, the global thermal resistance was reduced by up to 93.7%.

This trend is also evident in constructal fins in heat sink problems. The most fundamental problem was the T-shaped fin, which evolved into a twice-Y-shaped fin, or into several pin-fin arrays, whose size, spacing, and distribution completely affect heat transfer and could determine the best design by the lens of constructal design. The literature clearly indicates that design modifications to fins and cavities based on constructal design can improve heat-transfer performance in a wide range of heat-transfer problems. However, it is worth mentioning that, since we are dealing with heat transfer problems involving a large number of variables, it is difficult to perform a fair comparison among different research articles. This is because the boundary conditions, performance indicators, and geometric degrees of freedom may differ from one study to another, making direct comparisons among different designs reported in the literature difficult. Nevertheless, in all heat transfer problems analyzed, it was possible to observe the evolution of the flow architectures and their ability to improve system performance. In phase change material problems, the first design studied was the I-shaped fin, which evolved into Y-shaped and complex tree-shaped fins that improve melting or solidification through conduction and convection heat transfer mechanisms.

It is also interesting to note that, even though most of the works presented here are related to more fundamental heat transfer problems, some works are also related to more applied problems, as seen in the application category, where constructal design was used to identify the best design of fins or cavities in insulation walls, heat exchangers, thermochemical reactors, solar collectors, and thermophotovoltaic systems. This shows how broad the constructal law is becoming and how the heat transfer problems presented here can be coupled with several applications, including thermal energy storage systems, other engineering fields, and even beyond the field of thermodynamics.

Then, from this research, it is possible to point out some future research directions:

(i) For cavity and fin heat transfer problems, most existing studies rely on analytical or numerical models, leaving opportunities for future experimental investigations and three-dimensional studies to validate the thermal performance of complex cavity architectures under realistic operating conditions.
(ii) For finned heat sinks and channel-flow applications, future research should further explore multi-objective optimization approaches that simultaneously consider heat transfer enhancement and pressure losses (pumping power) requirements. The incorporation of manufacturability constraints into the development of new flow architecture is also an important research opportunity.
(iii) For constructal fins and heat exchangers, investigations involving new hierarchical branched architectures, new objective functions, and experimental validation are required. The integration of constructal design with artificial intelligence, machine learning, artificial neural networks, and genetic algorithms may also reduce computational costs and accelerate the development of new flow architectures.
(iv) For phase-change material systems, future studies should focus on fin architectures that simultaneously improve melting and solidification performance. Most existing studies evaluate only the melting process, leaving opportunities to develop designs optimized for the complete charging/discharging cycle. Furthermore, future research should seek evolutionary architectures that perform well under both conduction and convection heat transfer mechanisms, which are inherent in PCM thermal energy storage systems.
(v) For application problems, more studies are needed to transfer constructal design concepts from fundamental heat transfer problems to industrial applications, including thermal energy storage systems, heat exchangers, electronic cooling, renewable energy technologies, and other engineering systems.

Finally, this research demonstrates the power of the constructal law to predict and design new flow architectures across a wide range of heat transfer problems. The examples presented show that when flow architectures are designed with purpose and allowed to evolve within appropriate degrees of freedom, improvements in thermal performance can be achieved. Finally, as the demand for high-performance thermal management continues to grow in emerging technologies, the application of the constructal law is expected to expand across heat transfer research and industrial practice.

CRediT authorship contribution statement

Giovani Dambros Telli: Writing – review & editing, Writing – original draft, Software, Methodology, Funding acquisition, Conceptualization. **Cesare Biserni:** Writing – review & editing, Methodology, Investigation, Formal analysis, Data curation. **Claudia Naldi:** Writing – review & editing, Visualization, Methodology, Investigation, Formal analysis. **Liércio André Isoldi:** Writing – review & editing, Visualization, Methodology, Investigation, Funding acquisition, Formal analysis. **Elizaldo Domingues dos Santos:** Writing – review & editing, Visualization, Methodology, Investigation, Funding acquisition, Formal analysis. **Luiz Alberto Oliveira Rocha:** Writing – review & editing, Writing – original draft, Visualization, Supervision, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Luiz Alberto Oliveira Rocha reports financial support was provided by CNPq (Brasília, DF, Brazil). Liercio Andre Isoldi reports financial support was provided by CNPq (Brasília, DF, Brazil). Elizaldo Domingues dos Santos reports financial support was provided by CNPq (Brasília, DF, Brazil). Giovani Dambros Telli reports financial support was provided by Foundation for Research Support of Rio Grande do Sul State. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The author G.D. Telli thanks FAPERGS for financial support (Process: 25/2551-0002899-0). The authors E. D. dos Santos, L. A. Isoldi, and L. A. O. Rocha thank CNPq (Brasília, DF, Brazil) for research Grant (Processes: 308396/2021-9, 309648/2021-1, 307038/2025-4) and for financial support in the Universal Project (Process: 445095/2014-8).

Data availability

No data was used for the research described in the article.

References

[1] H.B. Ghadim, A. Godin, A. Veillere, M. Duquesne, D. Haillot, Review of thermal management of electronics and phase change materials, Renew. Sust. Energ. Rev. 208 (2025) 115039, https://doi.org/10.1016/j.rser.2024.115039.

[2] A.A. Alkrush, M.S. Salem, O. Abdelrehim, A.A. Hegazi, Data centers cooling: a critical review of techniques, challenges, and energy saving solutions, Int. J. Refrig. 160 (2024) 246–262, https://doi.org/10.1016/j.ijrefrig.2024.02.007.

[3] Z. Li, H. Luo, Y. Jiang, H. Liu, L. Xu, K. Cao, H. Wu, P. Gao, H. Liu, Comprehensive review and future prospects on chip-scale thermal management: core of data center's thermal management, Appl. Therm. Eng. 251 (2024) 123612, https://doi.org/10.1016/j.applthermaleng.2024.123612.

[4] F.L. Rashid, N.S. Dhaidan, A.J. Mahdi, S.A. Kadhim, K.A. Hammoodi, M.A. Al-Obaidi, H.I. Mohammed, S. Ahmad, S. Salahshour, E.B. Agyekum, Heat transfer enhancement of phase change materials using tree shaped fins: a comprehensive review, Int. Commun. Heat Mass Transf. 162 (2025) 108573, https://doi.org/10.1016/j.icheatmasstransfer.2024.108573.

[5] F.L. Rashid, Z.A.K. Alkhekany, H.J. Kurji, S. Ahmad, A. Chibani, M. Kezzar, K. A. Hammoodi, S.A. Kadhim, M.A.N. Haddad, E.B. Agyekum, Enhancing the thermal performance of different types of heat exchangers with fins: a comprehensive review, Int. Commun. Heat Mass Transf. 166 (2025) 109177, https://doi.org/10.1016/j.icheatmasstransfer.2025.109177.

[6] P.V. Ananth Subray, B.N. Hanumagowda, S.V.K. Varma, A.M. Zidan, M. Kbiri Alaoui, C.S.K. Raju, N.A. Shah, P. Junsawang, Dynamics of heat transfer analysis of convective-radiative fins with variable thermal conductivity and heat generation: differential transformation method, Mathematics 10 (2022), https://doi.org/10.3390/math10203814.

[7] M.D. Sagar, M.R. Puttaraja, G.K. Gowtham, R. Naveen Kumar, R.S. Varun Kumar, B.C. Prasannakumara, N.A. Shah, A comprehensive review on geometric variations in fin configurations: experimental and mathematical studies, in: Archives of Computational Methods in Engineering, 2025, https://doi.org/10.1007/s11831-025-10470-x.

[8] C. Kumar, P. Nimmy, K.V. Nagaraja, R.S.V. Kumar, A. Verma, S. Alkarni, N. A. Shah, Analysis of heat transfer behavior of porous wavy fin with radiation and convection by using a machine learning technique, Symmetry (Basel) 15 (2023), https://doi.org/10.3390/sym15081601.

[9] C. Kumar, P. Srilatha, K. Karthik, C. Somashekar, K.V. Nagaraja, R.S. Varun Kumar, N.A. Shah, A physics-informed machine learning prediction for thermal analysis in a convective-radiative concave fin with periodic boundary conditions, ZAMM Zeitschrift Fur Angewandte Mathematik Und Mechanik 104 (2024) 1–22, https://doi.org/10.1002/zamm.202300712.

[10] A. Bejan, Constructal-theory network of conducting paths for cooling a heat generating volume, Int. J. Heat Mass Transf. 40 (1997) 799–811, https://doi.org/10.1016/0017-9310(96)00175-5.

[11] A. Bejan, Street network theory of organization in nature, J. Adv. Transp. 30 (1996) 85–107, https://doi.org/10.1002/atr.5670300207.

[12] S. Lorente, A. Bejan, Current trends in constructal law and evolutionary design, Heat Transf. Asian Res. 48 (2019) 3574–3589, https://doi.org/10.1002/htj.21556.

[13] A. Bejan, Shape and Structure, from Engineering to Nature, Cambridge University Press, 2000.

[14] A. Bejan, J.P. Zane, Design in nature: how the constructal law governs evolution in biology, physics, technology, and social organizations, Anchor 1 (2013) 1–296.

[15] A. Bejan, The Physics of Life: The Evolution of Everything, St. Martin's Press, New York, 2016.

[16] A. Bejan, S. Lorente, Constructal law of design and evolution: physics, biology, technology, and society, J. Appl. Phys. 113 (2013), https://doi.org/10.1063/1.4798429.

[17] A. Bejan, Freedom and Evolution, Springer International Publishing, Cham, 2020, https://doi.org/10.1007/978-3-030-34009-4.

[18] A. Bejan, S. Lorente, The constructal law of design and evolution in nature, Philosoph. Trans. Royal Soc. B: Biol. Sci. 365 (2010) 1335–1347, https://doi.org/10.1098/rstb.2009.0302.

[19] A. Bejan, J.H. Marden, The constructal unification of biological and geophysical design, Phys Life Rev 6 (2009) 85–102, https://doi.org/10.1016/j.plrev.2008.12.002.

[20] U. Lucia, D. Fino, T.S. Deisboeck, G. Grisolia, A thermodynamic perspective of cancer cells' volume/area expansion ratio, Membranes (Basel) 13 (2023) 895, https://doi.org/10.3390/membranes13120895.

[21] A. Bejan, J.D. Charles, Animal design advantage from the analogy between friction and body heat loss, Biosystems 235 (2024) 105096, https://doi.org/10.1016/j.biosystems.2023.105096.

[22] M.E.F. Capponero, G.D. Telli, E.D. dos Dos Santos, L.A. Isoldi, M.N. das Gomes, L. A.O. Rocha Cesare, Constructal design method applied to wave energy converters: a systematic literature review, Dynamics 5 (2025) 36, https://doi.org/10.3390/dynamics5030036.

[23] A. Bejan, J.A. Jones, Max Jakob, Memorial Award Lecture From Heat Transfer Principles to Shape and Structure in Nature: Constructal Theory 122, 2000.

[24] Y.B. El Halal, G. Lorenzini, G.D. Telli, R.A.A.C. Gonçalves, L.A. Isoldi, L.A. O. Rocha, E.D. dos Santos, Geometrical analysis of heat transfer in a corrugated channels heat exchanger under forced convection and turbulent flow, Int. J. Comp. Methods Exp. Measur. 13 (2025) 1–14, https://doi.org/10.18280/ijcmem.130101.

[25] S. Lorente, A. Bejan, J.L. Niu, Constructal design of latent thermal energy storage with vertical spiral heaters, Int. J. Heat Mass Transf. 81 (2015) 283–288, https://doi.org/10.1016/j.ijheatmasstransfer.2014.09.077.

[26] J.F. Guil-Pedrosa, L.M. García-Gutiérrez, A. Soria-Verdugo, S. Lorente, Experimental and numerical analysis of a novel constructal design for canopy-to-canopy liquid cooling systems, Int. J. Therm. Sci. 214 (2025) 109918, https://doi.org/10.1016/j.ijthermalsci.2025.109918.

[27] M. Mosa, M. Labat, S. Lorente, Constructal design of flow channels for radiant cooling panels, Int. J. Therm. Sci. 145 (2019) 106052, https://doi.org/10.1016/j.ijthermalsci.2019.106052.

[28] G. Dambros Telli, S. Gungor, S. Lorente, Counterflow canopy-to-canopy and U-turn liquid cooling solutions for battery modules in stationary battery energy storage systems, Appl. Therm. Eng. 238 (2024) 121997, https://doi.org/10.1016/j.applthermaleng.2023.121997.

[29] S. Gungor, E. Cetkin, S. Lorente, Canopy-to-canopy liquid cooling for the thermal management of lithium-ion batteries, a constructal approach, Int. J. Heat Mass Transf. 182 (2022) 121918, https://doi.org/10.1016/j.ijheatmasstransfer.2021.121918.

[30] R.S. da Borahel, G.D. Telli, A.A. Bolzoni, L.A. Isoldi, E.D. dos Santos, L.A. O. Rocha, Numerical analysis of the impact of PCM reservoir width on lithium-ion battery cooling: a constructal design study, Res. Eng. Struct. Mater. 11 (2025) 2421–2435, https://doi.org/10.17515/resm2025-557en1202rs.

[31] A.H. Reis, Constructal theory: from engineering to physics, and how flow systems develop shape and structure, Appl. Mech. Rev. 59 (2006) 269–282, https://doi.org/10.1115/1.2204075.

[32] A. Bejan, G. Tsatsaronis, Purpose in thermodynamics, Energies (Basel) 14 (2021), https://doi.org/10.3390/en14020408.

[33] A.H. Reis, C. Gama, Sand size versus beachface slope — an explanation based on the constructal law, Geomorphology 114 (2010) 276–283.

[34] Y. Ventikos, The importance of the constructal framework in understanding and eventually replicating structure in tissue, Phys Life Rev 8 (2011) 241–263.
[35] A. Bejan, S. Lorente, Design with Constructal Theory, John Wiley & Sons, Inc., Hoboken, NJ, USA, 2008, https://doi.org/10.1002/9780470432709.
[36] A. Bejan, Advanced Engineering Thermodynamics, John Wiley & Sons, 2016, https://doi.org/10.1002/9781119245964.
[37] L. Chen, H. Feng, Z. Xie, F. Sun, Progress of constructal theory in China over the past decade, Int. J. Heat Mass Transf. 130 (2019) 393–419, https://doi.org/10.1016/j.ijheatmasstransfer.2018.10.064.
[38] L.G. Chen, A.B. Yang, H.J. Feng, Y.L. Ge, S.J. Xia, Constructal design progress for eight types of heat sinks, Sci. China Technol. Sci. 63 (2020) 879–911, https://doi.org/10.1007/s11431-019-1469-1.
[39] S.A. Marzouk, M.M. Abou Al-Sood, E.M.S. El-Said, M.M. Younes, M.K. El-Fakharany, Comprehensive review of heat transfer enhancement through constructal theory, J. Therm. Anal. Calorim. 150 (2025) 18089–18127, https://doi.org/10.1007/s10973-025-14630-w.
[40] B. Kitchenham, Procedures for Performing Systematic Reviews, Keele University, UK, 2004.
[41] F.L. Rashid, M.A. Al-Obaidi, Y.A. Jebbar, N.M.L. Al Maimuri, A. Bouabidi, A. M. Ashour, A. Chibani, E.B. Agyekum, Impact of vibration on thermal energy storage performance: a comprehensive review, Int. Commun. Heat Mass Transf. 169 (2025) 109723, https://doi.org/10.1016/j.icheatmasstransfer.2025.109723.
[42] Y. Wirani, I. Eitiveni, Y.G. Sucahyo, Framework of smart and integrated household waste management system: a systematic literature review using PRISMA, Sustainability 16 (2024) 4898, https://doi.org/10.3390/su16124898.
[43] A. Dresch, D.P. Lacerda, J.A.V. Antunes Jr., Design Science Research, Springer International Publishing, Cham, 2015, https://doi.org/10.1007/978-3-319-07374-3.
[44] V. Pillai, Julia H. Littell, Jacqueline Corcoran, Systematic Reviews and Meta-Analysis, Oxford University Press, 2008.
[45] H. Snyder, Literature review as a research methodology: an overview and guidelines, J. Bus. Res. 104 (2019) 333–339, https://doi.org/10.1016/j.jbusres.2019.07.039.
[46] D. Moher, A. Liberati, J. Tetzlaff, D.G. Altman, D. Altman, G. Antes, D. Atkins, V. Barbour, N. Barrowman, J.A. Berlin, J. Clark, M. Clarke, D. Cook, R. D'Amico, J.J. Deeks, P.J. Devereaux, K. Dickersin, M. Egger, E. Ernst, P.C. Gøtzsche, J. Grimshaw, G. Guyatt, J. Higgins, J.P.A. Ioannidis, J. Kleijnen, T. Lang, N. Magrini, D. McNamee, L. Moja, C. Mulrow, M. Napoli, A. Oxman, B. Pham, D. Rennie, M. Sampson, K.F. Schulz, P.G. Shekelle, D. Tovey, P. Tugwell, Preferred reporting items for systematic reviews and meta-analyses: the PRISMA statement, PLoS Med. 6 (2009), https://doi.org/10.1371/journal.pmed.1000097.
[47] A.P. Cardoso Ermel, D.P. Lacerda, M.I.W.M. Morandi, L. Gauss, Literature Reviews: Modern Methods for Investigating Scientific and Technological Knowledge, 1st ed, Springer International Publishing, Cham, 2021, https://doi.org/10.1007/978-3-030-75722-9.
[48] M. Aria, C. Cuccurullo, bibliometrix : an R-tool for comprehensive science mapping analysis, J. Inf. Secur. 11 (2017) 959–975, https://doi.org/10.1016/j.joi.2017.08.007.
[49] N.J. van Eck, L. Waltman, Software survey: VOSviewer, a computer program for bibliometric mapping, Scientometrics 84 (2010) 523–538, https://doi.org/10.1007/s11192-009-0146-3.
[50] M. Gutiérrez-Salcedo, M.Á. Martínez, J.A. Moral-Munoz, E. Herrera-Viedma, M. J. Cobo, Some bibliometric procedures for analyzing and evaluating research fields, Appl. Intell. (2017), https://doi.org/10.1007/s10489-017-1105-y.
[51] S. Rousseau, R. Rousseau, Bibliometric techniques and their use in business and economics research, J. Econ. Surv. 35 (2021) 1428–1451, https://doi.org/10.1111/joes.12415.
[52] A. Bejan, M. Almogbel, Constructal T-shaped fins, Int. J. Heat Mass Transf. 43 (2000) 2101–2115, https://doi.org/10.1016/S0017-9310(99)00283-5.
[53] H. Zhu, L. Chen, H. Feng, Y. Ge, Multi-objective constructal designs for fork-shaped cavities with minimizing maximum-temperature-difference and entropy-generation-rate, Therm. Sci. Eng. Progr. 67 (2025) 104161, https://doi.org/10.1016/j.tsep.2025.104161.
[54] C. Biserni, L.A.O. Rocha, A. Bejan, Inverted fins: geometric optimization of the intrusion into a conducting wall, Int. J. Heat Mass Transf. 47 (2004) 2577–2586, https://doi.org/10.1016/j.ijheatmasstransfer.2003.12.018.
[55] C. Biserni, L.A.O. Rocha, G. Stanescu, E. Lorenzini, Constructal H-shaped cavities according to Bejan's theory, Int. J. Heat Mass Transf. 50 (2007) 2132–2138, https://doi.org/10.1016/j.ijheatmasstransfer.2006.11.006.
[56] G. Lorenzini, L.A.O. Rocha, Geometric optimization of T-Y-shaped cavity according to Constructal design, Int. J. Heat Mass Transf. 52 (2009) 4683–4688, https://doi.org/10.1016/j.ijheatmasstransfer.2009.06.020.
[57] L.A.O. Rocha, G. Lorenzini, C. Biserni, Y. Cho, Constructal design of a cavity cooled by convection, Int. J. Design Nat. Ecodyn. 5 (2010) 212–220, https://doi.org/10.2495/DNE-V5-N3-212-220.
[58] G. Lorenzini, C. Biserni, L.A.O. Rocha, Geometric optimization of isothermal cavities according to Bejan's theory, Int. J. Heat Mass Transf. 54 (2011) 3868–3873, https://doi.org/10.1016/j.ijheatmasstransfer.2011.04.042.
[59] G. Lorenzini, C. Biserni, L.A. Isoldi, E.D. dos Santos, L.A.O. Rocha, Constructal design applied to the geometric optimization of Y-shaped cavities embedded in a conducting medium, J. Electron. Packag. 133 (2011) 1–8, https://doi.org/10.1115/1.4005296.
[60] G. Lorenzini, C. Biserni, F.L. Garcia, L.A.O. Rocha, Geometric optimization of a convective T-shaped cavity on the basis of constructal theory, Int. J. Heat Mass Transf. 55 (2012) 6951–6958, https://doi.org/10.1016/j.ijheatmasstransfer.2012.07.009.
[61] G. Lorenzini, F.L. Garcia, E.D. Dos Santos, C. Biserni, L.A.O. Rocha, Constructal design applied to the optimization of complex geometries: T-Y-shaped cavities with two additional lateral intrusions cooled by convection, Int. J. Heat Mass Transf. 55 (2012) 1505–1512, https://doi.org/10.1016/j.ijheatmasstransfer.2011.10.057.
[62] M.R. Hajmohammadi, S. Poozesh, A. Campo, S.S. Nourazar, Valuable reconsideration in the constructal design of cavities, Energy Convers. Manag. 66 (2013) 33–40, https://doi.org/10.1016/j.enconman.2012.09.031.
[63] G. Lorenzini, C. Biserni, L.A.O. Rocha, Geometric optimization of c-shaped cavities according to bejan's theory: general review and comparative study, J. Electron. Packag. 135 (2013) 1–7, https://doi.org/10.1115/1.4024113.
[64] G. Lorenzini, C. Biserni, F.B. Link, L.A. Isoldi, E.D. dos Santos, L.A.O. Rocha, Constructal design of T-shaped cavity for several convective fluxes imposed at the cavity surfaces, J. Eng. Thermophys. 22 (2013) 309–321, https://doi.org/10.1134/S1810232813040048.
[65] G. Lorenzini, C. Biserni, L.A.O. Rocha, Geometric optimization of X-shaped cavities and pathways according to Bejan's theory: comparative analysis, Int. J. Heat Mass Transf. 73 (2014) 1–8, https://doi.org/10.1016/j.ijheatmasstransfer.2014.01.055.
[66] G. Lorenzini, C. Biserni, F.B. Link, E.D. Dos Santos, L.A. Isoldi, L.A.O. Rocha, Constructal design of isothermal X-shaped cavities, Therm. Sci. 18 (2014) 349–356, https://doi.org/10.2298/TSCI120804005L.
[67] G. Lorenzini, C. Biserni, E.D. Estrada, L.A. Isoldi, E.D.D. Santos, L.A.O. Rocha, Constructal design of convective Y-shaped cavities by means of genetic algorithm, J. Heat Transf. 136 (2014) 1–11, https://doi.org/10.1115/1.4027195.
[68] G. Lorenzini, C. Biserni, E. Da Silva Diaz, E.D. Dos Estrada, L.A. Santos, L.A.O. Rocha Isoldi, Genetic algorithm applied to geometric optimization of isothermal Y-shaped cavities, J. Electron. Packag. 136 (2014) 1–9, https://doi.org/10.1115/1.4027421.
[69] G.V. Gonzales, E.S.D. da Estrada, L.R. Emmendorfer, L.A. Isoldi, G. Xie, L.A. O. Rocha, E.D. dos Santos, A comparison of simulated annealing schedules for constructal design of complex cavities intruded into conductive walls with internal heat generation, Energy 93 (2015) 372–382, https://doi.org/10.1016/j.energy.2015.09.058.
[70] G.V. Gonzales, E.D. Dos Santos, L.A. Isoldi, L.A.O. Rocha, A.J. Da Silva Neto, W. R. Telles, Constructal design of double-T shaped cavity with stochastic methods Luus-Jaakola and simulated annealing, Defect Diffusion Forum 370 (2017) 152–161, https://doi.org/10.4028/www.scientific.net/DDF.370.152.
[71] M.R. Hajmohammadi, Introducing a ψ-shaped cavity for cooling a heat generating medium, Int. J. Therm. Sci. 121 (2017) 204–212, https://doi.org/10.1016/j.ijthermalsci.2017.07.010.
[72] M.R. Hajmohammadi, Optimal design of tree-shaped inverted fins, Int. J. Heat Mass Transf. 116 (2018) 1352–1360, https://doi.org/10.1016/j.ijheatmasstransfer.2017.09.042.
[73] G.V. Gonzales, L.A. Isoldi, L.A.O. Rocha, A.J. Da Silva Neto, E.D., Dos Santos, Evaluation of a differential evolution/constructal design algorithm for geometrical optimization of double t-shaped cavity intruded into a heat generating wall, Defect Diffusion Forum 396 (2019) 145–154, https://doi.org/10.4028/www.scientific.net/DDF.396.145.
[74] G.V. Gonzales, G. Lorenzini, L.A. Isoldi, L.A.O. Rocha, E.D. dos Santos, A.J. S. Neto, Constructal design and simulated annealing applied to the geometric optimization of an isothermal Double T-shaped cavity, Int. J. Heat Mass Transf. 174 (2021) 121268, https://doi.org/10.1016/j.ijheatmasstransfer.2021.121268.
[75] G.V. Gonzales, C. Biserni, E. da Silva Diaz Estrada, G.M. Platt, L.A. Isoldi, L.A. O. Rocha, A.J. da Silva Neto, E.D. dos Santos, Investigation on the association of differential evolution and constructal design for geometric optimization of double Y-shaped cooling cavities inserted into walls with heat generation, Appl. Sci. (Switz.) 13 (2023), https://doi.org/10.3390/app13031998.
[76] G.V. Gonzales, C. Biserni, L.A.O. Rocha, E. da Silva Diaz, L.A. Isoldi Estrada, A. J. da Silva Neto, E.D. dos Santos, Geometrical optimization of an isothermal double Y-shaped cavity employing differential evolution algorithm with a constructal approach, Int. Commun. Heat Mass Transf. 161 (2025) 28–34, https://doi.org/10.1016/j.icheatmasstransfer.2024.108447.
[77] Z. Xie, Y. Lin, W. Guan, L. Gan, Determining the optimal structure of fork-shaped cavities for cooling electronic components generating heat, Int. J. Interact. Des. Manuf. (IJIDeM) (2025), https://doi.org/10.1007/s12008-024-02184-4.
[78] J.C.B. Vianna, E.D.S.D. Estrada, L.A. Isoldi, E.D. Dos Santos, J.A. Souza, Design of cooling cavities by the application of the constructal design, Defect Diffusion Forum 372 (2017) 163–169, https://doi.org/10.4028/www.scientific.net/DDF.372.163.
[79] J.C.B. Vianna, E.S.D. da Estrada, L.A. Isoldi, E.D. dos Santos, J.A. Souza, A new constructal theory based algorithm applied to thermal problems, Int. J. Therm. Sci. 126 (2018) 118–124, https://doi.org/10.1016/j.ijthermalsci.2017.12.023.
[80] L.A.O. Rocha, E. Lorenzini, C. Biserni, Geometric optimization of shapes on the basis of Bejan's Constructal theory, Int. Commun. Heat Mass Transf. 32 (2005) 1281–1288, https://doi.org/10.1016/j.icheatmasstransfer.2005.07.010.
[81] Z. Xie, L. Chen, F. Sun, Geometry optimization of T-shaped cavities according to constructal theory, Math. Comput. Model. 52 (2010) 1538–1546, https://doi.org/10.1016/j.mcm.2010.06.017.
[82] Z.H. Xie, L.G. Chen, F.R. Sun, Constructal design of conducting body cooled by open cavity, J. Energy Inst. 85 (2012) 220–226, https://doi.org/10.1179/1743967112Z.00000000031.
[83] A. Pouzesh, M.R. Hajmohammadi, S. Poozesh, Investigations on the internal shape of constructal cavities intruding a heat generating body, Therm. Sci. 9 (2015) 609–618, https://doi.org/10.2298/TSCI120427164P.

[84] G. Lorenzini, L. Alberto Oliveira Rocha, C. Biserni, E. Domingues Dos Santos, L. André Isoldi, Constructal design of cavities inserted into a cylindrical solid body, J. Heat Transf. 134 (2012) 1–6, https://doi.org/10.1115/1.4006103.

[85] G. Lorenzini, E. Da Silva Diaz, E.D. Dos Estrada, L.A. Santos, L.A.O. Rocha Isoldi, Constructal design of convective cavities inserted into a cylindrical solid body for cooling, Int. J. Heat Mass Transf. 83 (2015) 75–83, https://doi.org/10.1016/j.ijheatmasstransfer.2014.11.065.

[86] E.S.D. Estrada, E.X. Barreto, L.A. Isoldi, E.D. dos Santos, S. Lorente, L.A.O. Rocha, Constructal design of tree shaped cavities inserted into a cylindrical body with heat generation, Int. J. Therm. Sci. 152 (2020) 106342, https://doi.org/10.1016/j.ijthermalsci.2020.106342.

[87] F. Wang, Investigation on heat transfer in circular fins with H-shaped cavity using finite element method and constructal design, Comput. Therm. Sci. 15 (2023) 83–96, https://doi.org/10.1615/ComputThermalScien.2023045015.

[88] G. Lorenzini, B.S. Machado, L.A. Isoldi, E.D. dos Santos, L.A.O. Rocha, Constructal design of rectangular fin intruded into mixed convective lid-driven cavity flows, J. Heat Transf. 138 (2016) 1–12, https://doi.org/10.1115/1.4033378.

[89] E.S. Aldrighi, P.M. Rodrigues, B.D.D.A. Rodriguez, L.A. Isoldi, L.A.O. Rocha, E. D. Dos Santos, Constructal design of rectangular fin intruded into different surfaces of forced convective lid-driven cavity flow, Int. J. Fluid Mechan. Res. 43 (2016) 418–440, https://doi.org/10.1615/InterJFluidMechRes.v43.i5-6.50.

[90] E. Sias, G. Lorenzini, G.D. Telli, F. Schwarz, L.A. Isoldi, L.A.O. Rocha, E.D. Dos Santos, Geometrical assessment of rectangular fins at different surfaces and positions on Nusselt number of lid-driven cavities under laminar forced convection, J. Comput. Appl. Mech. 55 (2024) 744–770, https://doi.org/10.22059/jcamech.2024.380432.1183.

[91] A. Robinson, P.K. Das, Biomimetic and constructal design of alveolus-inspired extended surfaces for heat dispersion, Energies (Basel) 16 (2023), https://doi.org/10.3390/en16010066.

[92] R. Cong, Y. Ozaki, B.S. Machado, P.K. Das, Constructal design of a rectangular fin in a mixed convective confined environment, Inventions 3 (2018), https://doi.org/10.3390/inventions3020027.

[93] P. Mountrichas, W. Zhao, M.S. Randeva, P.K. Das, Entropy generation of CuO-water nanofluid in a cavity with an intruded rectangular fin, Energies (Basel) 16 (2023), https://doi.org/10.3390/en16020912.

[94] Z.H. Khan, W.A. Khan, M. Hamid, H. Liu, Finite element analysis of hybrid nanofluid flow and heat transfer in a split lid-driven square cavity with Y-shaped obstacle, Phys. Fluids 32 (2020), https://doi.org/10.1063/5.0021638.

[95] Z.H. Khan, W.A. Khan, M. Hamid, Non-Newtonian fluid flow around a Y-shaped fin embedded in a square cavity, J. Therm. Anal. Calorim. 143 (2021) 573–585, https://doi.org/10.1007/s10973-019-09201-9.

[96] N. Vishnu Ganesh, Q.M. Al-Mdallal, G. Hirankumar, R. Kalaivanan, Impact of a hot constructal tree-shaped fin on the convection flow of single wall carbon nanotube water nanofluid inside a sinusoidal enclosure, Int. Commun. Heat Mass Transf. 137 (2022) 106279, https://doi.org/10.1016/j.icheatmasstransfer.2022.106279.

[97] S.Z. Abbas, X. Wang, W.A. Khan, A. Hobiny, K. Iqbal, Finite element analysis of nanofluid flow and heat transfer in a square cavity with two circular obstacles at different positions in the presence of magnetic field, J. Energy Storage 51 (2022), https://doi.org/10.1016/j.est.2022.104462.

[98] P.M. Rodrigues, C. Biserni, C.C. de Escobar, L.A.O. Rocha, L.A. Isoldi, E.D. dos Santos, Geometric optimization of a lid-driven cavity with two rectangular intrusions under mixed convection heat transfer: a numerical investigation motivated by constructal design, Int. Commun. Heat Mass Transf. 117 (2020) 104759, https://doi.org/10.1016/j.icheatmasstransfer.2020.104759.

[99] P.M. Rodrigues, C.C. De Escobar, L.A.O. Rocha, L.A. Isoldi, E.D. Dos Santos, Geometry evaluation of heat transfer by mixed convention in driven cavities with two inserted fins, Defect Diffusion Forum 396 (2019) 164–173, https://doi.org/10.4028/www.scientific.net/DDF.396.164.

[100] P.M. Rodrigues, E.H.T. Cunegatto, C.C. de Escobar, F.S.F. Zinani, E.D. dos Santos, L.A. Isoldi, L.A.O. Rocha, Geometric evaluation and constructal design applied to a lid-driven cavity with three fins of distinct sizes, J. Braz. Soc. Mech. Sci. Eng. 47 (2025) 1–20, https://doi.org/10.1007/s40430-025-05430-2.

[101] P.M. Rodrigues, C.C. de Escobar, F.S.F. Zinani, E.D. dos Santos, L.A. Isoldi, L.A. O. Rocha, Constructal design of three fins inside a lid-driven cavity, J. Braz. Soc. Mech. Sci. Eng. 46 (2024) 1–15, https://doi.org/10.1007/s40430-024-04991-y.

[102] A.L. Razera, T.M. Fagundes, F.M. Seibt, R.J.C. Da Fonseca, D.J.C. Varela, P.R. B. Ortiz, F.R. Coelho, L.Z. Lessa, A. Schmidt, G.M. Furtado, E.D. Dos Santos, L. A. Isoldi, L.A.O. Rocha, Constructal design of a triangular fin inserted in a cavity with mixed convection lid-driven flow, Defect Diffusion Forum 372 (2017) 188–201, https://doi.org/10.4028/www.scientific.net/DDF.372.188.

[103] A.L. Razera, R.J.C. da Fonseca, L.A. Isoldi, E.D. dos Santos, L.A.O. Rocha, C. Biserni, Constructal design of a semi-elliptical fin inserted in a lid-driven square cavity with mixed convection, Int. J. Heat Mass Transf. 126 (2018) 81–94, https://doi.org/10.1016/j.ijheatmasstransfer.2018.05.157.

[104] S. Bilal, M. Rehman, S. Noeiaghdam, H. Ahmad, A. Akgül, Numerical analysis of natural convection driven flow of a non-Newtonian power-law fluid in a trapezoidal enclosure with a U-shaped constructal, Energies (Basel) 14 (2021), https://doi.org/10.3390/en14175355.

[105] F. Selimefendigil, H.F. Oztop, A.J. Chamkha, MHD mixed convection in a nanofluid filled vertical lid-driven cavity having a flexible fin attached to its upper wall, J. Therm. Anal. Calorim. 135 (2019) 325–340, https://doi.org/10.1007/s10973-018-7036-y.

[106] F. Selimefendigil, H.F. Öztop, Natural convection in a flexible sided triangular cavity with internal heat generation under the effect of inclined magnetic field, J. Magn. Magn. Mater. 417 (2016) 327–337, https://doi.org/10.1016/j.jmmm.2016.05.053.

[107] R.S. da Borahel, F.S.F. Zinani, L.A. Isoldi, E.D. dos Santos, L.A.O. Rocha, Analysis of the design of cavities with isothermal blocks under mixed convection, J. Appl. Comp. Mechan. 11 (2025) 541–556, https://doi.org/10.22055/jacm.2024.47208.4676.

[108] R. da Silveira Borahel, F.S.F. Zinani, L.A.O. Rocha, E.D. dos Santos, L.A. Isoldi, C. Biserni, Geometric optimization of a rectangular isothermal block inside a lid-driven cavity by means of constructal design, Int. Commun. Heat Mass Transf. 139 (2022), https://doi.org/10.1016/j.icheatmasstransfer.2022.106499.

[109] A.K. da Silva, L. Gosselin, Volumetric maximization of coolant usage in closed self-driven circuits, Int. J. Heat Mass Transf. 53 (2010) 3969–3976, https://doi.org/10.1016/j.ijheatmasstransfer.2010.05.016.

[110] A. Müftüoğlu, E. Bilgen, Natural convection in an open square cavity with discrete heaters at their optimized positions, Int. J. Therm. Sci. 47 (2008) 369–377, https://doi.org/10.1016/j.ijthermalsci.2007.03.015.

[111] E. Bilgen, A. Balkaya, Natural convection on discrete heaters in a square enclosure with ventilation ports, Int. J. Heat Fluid Flow 29 (2008) 1182–1189, https://doi.org/10.1016/j.ijheatfluidflow.2008.01.013.

[112] R.R. Madadi, C. Balaji, Optimization of the location of multiple discrete heat sources in a ventilated cavity using artificial neural networks and micro genetic algorithm, Int. J. Heat Mass Transf. 51 (2008) 2299–2312, https://doi.org/10.1016/j.ijheatmasstransfer.2007.08.033.

[113] A. Alebrahim, A. Bejan, Constructal trees of circular fins for conductive and convective heat transfer, Int. J. Heat Mass Transf. 42 (1999) 3585–3597, https://doi.org/10.1016/S0017-9310(99)00021-6.

[114] M. Almogbel, A. Bejan, Cylindrical trees of pin fins, Int. J. Heat Mass Transf. 43 (2000) 4285–4297, https://doi.org/10.1016/S0017-9310(00)00049-1.

[115] H. Shokouhmand, S. Mahjoub, M.R. Salimpour, Constructal design of finned tubes used in air-cooled heat exchangers, J. Mech. Sci. Technol. 28 (2014) 2385–2391, https://doi.org/10.1007/s12206-014-0145-z.

[116] M.R. Hajmohammadi, Design and analysis of multi-scale annular fins attached to a pin fin, Int. J. Refrig. 88 (2018) 16–23, https://doi.org/10.1016/j.ijrefrig.2017.11.032.

[117] A.W. Mustafa, U.A. Sulaiman, M.M. Awad, Constructal design of crossflow heat exchanger with concentric and eccentric circular fins, Heat Transf. 53 (2024) 1168–1189, https://doi.org/10.1002/htj.22989.

[118] R.S. Matos, T.A. Laursen, J.V.C. Vargas, A. Bejan, Three-dimensional optimization of staggered finned circular and elliptic tubes in forced convection, Int. J. Therm. Sci. 43 (2004) 477–487, https://doi.org/10.1016/j.ijthermalsci.2003.10.003.

[119] A.K. da Silva, C. Vasile, A. Bejan, Disc cooled with high-conductivity inserts that extend inward from the perimeter, Int. J. Heat Mass Transf. 47 (2004) 4257–4263, https://doi.org/10.1016/j.ijheatmasstransfer.2004.04.024.

[120] M.R. Salimpour, J. Darabi, Sh. Mahjoub, Design and optimization of internal longitudinal fins of a tube using constructal theory, J. Eng. Thermophys. 28 (2019) 239–254, https://doi.org/10.1134/S1810232819020061.

[121] M.R. Hajmohammadi, S. Poozesh, S. Salman Nourazar, A.H. Manesh, Optimal architecture of heat generating pieces in a fin, J. Mech. Sci. Technol. 27 (2013) 1143–1149, https://doi.org/10.1007/s12206-013-0217-5.

[122] S. Gong, L. Chen, H. Feng, Z. Xie, F. Sun, Constructal optimization of cylindrical heat sources surrounded with a fin based on minimization of hot spot temperature, Int. Commun. Heat Mass Transf. 68 (2015) 1–7, https://doi.org/10.1016/j.icheatmasstransfer.2015.08.004.

[123] H. Feng, L. Chen, Z. Xie, F. Sun, Constructal design for helm-shaped fin with internal heat sources, Int. J. Heat Mass Transf. 110 (2017) 1–6, https://doi.org/10.1016/j.ijheatmasstransfer.2017.02.074.

[124] Z. Hajabdollahi, H. Hajabdollahi, P.F. Fu, Improving the rate of heat transfer and material in the extended surface using multi-objective constructal optimization, Int. J. Heat Mass Transf. 115 (2017) 589–596, https://doi.org/10.1016/j.ijheatmasstransfer.2017.08.055.

[125] H. Hajabdollahi, M. Shafiey Dehaj, B. Masoumpour, M. Ataeizadeh, Optimal design analysis of a tubular heat exchanger network with extended surfaces using multi-objective constructal optimization, Front. Energy 16 (2022) 862–875, https://doi.org/10.1007/s11708-022-0839-3.

[126] S.A. Hazarika, D. Bhanja, S. Nath, A novel optimal constructal fork-shaped fin array design to ascertain thermo-heightened performance under dehumidifying conditions, Int. J. Therm. Sci. 144 (2019) 67–78, https://doi.org/10.1016/j.ijthermalsci.2019.06.003.

[127] S.A. Hazarika, D. Bhanja, S. Nath, Fork-shaped constructal fin array design a better alternative for heat and mass transfer augmentation under dry, partially wet and fully wet conditions, Int. J. Therm. Sci. 152 (2020) 106329, https://doi.org/10.1016/j.ijthermalsci.2020.106329.

[128] S.A. Hazarika, T. Deshmukhya, D. Bhanja, S. Nath, A novel optimum constructal fork-shaped fin array design for simultaneous heat and mass transfer application in a space-constrained situation, Int. J. Therm. Sci. 150 (2020) 106225, https://doi.org/10.1016/j.ijthermalsci.2019.106225.

[129] A.W. Mustafa, H.M. Salman, B.O. Hasan, Maximization of heat transfer density from radially finned tubes in cross-flow using the constructal design method, Heat Transf. 52 (2023) 354–377, https://doi.org/10.1002/htj.22698.

[130] A. Hickie-Bentzen, S.H. Tasnim, S. Mahmud, Magnetohydrodynamic natural convection of In-Ga-Sn alloy in a horizontal concentric annulus with tree-shaped internal fins, Int. Commun. Heat Mass Transf. 147 (2023), https://doi.org/10.1016/j.icheatmasstransfer.2023.106948.

[131] P. Ribeiro, D. Queiros-Condé, On the entropy production of the elemental construct of the constructal designed plate generating heat, Int. J. Therm. Sci. 145 (2019), https://doi.org/10.1016/j.ijthermalsci.2019.106043.

[132] G. Lorenzini, L.A.O. Rocha, Constructal design of Y-shaped assembly of fins, Int. J. Heat Mass Transf. 49 (2006) 4552–4557, https://doi.org/10.1016/j.ijheatmasstransfer.2006.05.019.
[133] G. Lorenzini, L.A.O. Rocha, Constructal design of T-Y assembly of fins for an optimized heat removal, Int. J. Heat Mass Transf. 52 (2009) 1458–1463, https://doi.org/10.1016/j.ijheatmasstransfer.2008.09.007.
[134] X.Z. Hui, C.L. Gen, S.F. Rui, Constructal optimization of twice Y-shaped assemblies of fins by taking maximum thermal resistance minimization as objective, Sci. China Technol. Sci. 53 (2010) 2756–2764, https://doi.org/10.1007/s11431-010-4037-x.
[135] G. Lorenzini, R.L. Corrȁ, E. Domingues Dos Santos, L.A.O. Rocha, Constructal design of complex assembly of fins, J. Heat Transf. 133 (2011) 1–7, https://doi.org/10.1115/1.4003710.
[136] W. Zhang, L. Chen, H. Feng, Y. Ge, Adding branched fin angle as degree-of-freedom improves performance of a T-Y-shaped fin, Case Stud. Therm. Eng. 61 (2024) 105031, https://doi.org/10.1016/j.csite.2024.105031.
[137] W. Zhang, L. Chen, H. Feng, Y. Ge, Constructal design of two-stage combined T-Y-shaped fin considering minimization of maximum thermal resistance, Therm. Sci. Eng. Progr. 60 (2025) 103394, https://doi.org/10.1016/j.tsep.2025.103394.
[138] G. Lorenzini, M. Medici, L.A.O. Rocha, Convective analysis of constructal T-shaped fins, J. Eng. Thermophys. 23 (2014) 98–104, https://doi.org/10.1134/S1810232814020027.
[139] G. Lorenzini, S. Moretti, A CFD application to optimize T-shaped fins: comparisons to the constructal theory's results, J. Electron. Packag. 129 (2007) 324–327, https://doi.org/10.1115/1.2756852.
[140] G. Lorenzini, S. Moretti, Numerical analysis of heat removal enhancement with extended surfaces, Int. J. Heat Mass Transf. 50 (2007) 746–755, https://doi.org/10.1016/j.ijheatmasstransfer.2006.07.012.
[141] G. Lorenzini, S. Moretti, Numerical analysis on heat removal from Y-shaped fins: Efficiency and volume occupied for a new approach to performance optimisation, Int. J. Therm. Sci. 46 (2007) 573–579, https://doi.org/10.1016/j.ijthermalsci.2006.08.004.
[142] S. Zehisaadat, R.K. Khalajzadeh, M.R. Hajmohammadi, G. Lorenzini, Geometric optimization of T-shaped fin and inverted fin based on minimum entropy generation objective, J. Eng. Thermophys. 31 (2022) 668–687, https://doi.org/10.1134/S1810232822040129.
[143] B. Kundu, D. Bhanja, Performance and optimization analysis of a constructal T-shaped fin subject to variable thermal conductivity and convective heat transfer coefficient, Int. J. Heat Mass Transf. 53 (2010) 254–267, https://doi.org/10.1016/j.ijheatmasstransfer.2009.09.034.
[144] D. Bhanja, B. Kundu, Thermal analysis of a constructal T-shaped porous fin with radiation effects, Int. J. Refrig. 34 (2011) 1483–1496, https://doi.org/10.1016/j.ijrefrig.2011.04.003.
[145] D. Bhanja, B. Kundu, Radiation effect on optimum design analysis of a constructal T-shaped fin with variable thermal conductivity, Heat Mass Transf./Waerme- Und Stoffuebertragung 48 (2012) 109–122, https://doi.org/10.1007/s00231-011-0845-1.
[146] M.R. Hajmohammadi, S. Poozesh, R. Hosseini, Radiation effect on constructal design analysis of a T-Y-shaped assembly of fins, J. Therm. Sci. Technol. 7 (2012) 677–692, https://doi.org/10.1299/jtst.7.677.
[147] S.A. Hazarika, D. Bhanja, S. Nath, B. Kundu, Analytical solution to predict performance and optimum design parameters of a constructal T-shaped fin with simultaneous heat and mass transfer, Energy 84 (2015) 303–316, https://doi.org/10.1016/j.energy.2015.02.102.
[148] S.A. Hazarika, T. Deshamukhya, D. Bhanja, S. Nath, Thermal analysis of a constructal T-shaped porous fin with simultaneous heat and mass transfer, Chin. J. Chem. Eng. 25 (2017) 1121–1136, https://doi.org/10.1016/j.cjche.2017.03.034.
[149] S.A. Hazarika, D. Bhanja, S. Nath, B. Kundu, Geometric optimization and performance study of a constructal T-shaped fin under simultaneous heat and mass transfer, Appl. Therm. Eng. 109 (2016) 162–174, https://doi.org/10.1016/j.applthermaleng.2016.08.007.
[150] S.A. Hazarika, D. Bhanja, S. Nath, B. Kundu, Prédictions exactes pour la conception optimale d'une ailette en forme de T sous condition de charge humide actuelle, Int. J. Refrig. 82 (2017) 141–162, https://doi.org/10.1016/j.ijrefrig.2017.06.032.
[151] S.A. Hazarika, D. Bhanja, S. Nath, B. Kundu, Thermal design parameters of a wet T-shaped fin for linear variation of humidity ratio with saturation temperature, J. Mech. Sci. Technol. 32 (2018) 2391–2397, https://doi.org/10.1007/s12206-018-0451-y.
[152] S.A. Hazarika, D. Bhanja, S. Nath, Optimal design of constructal T-shaped fin under partially wet condition for enhanced thermal performance: an analytical approach, Heat Transf. Eng. 41 (2020) 1156–1173, https://doi.org/10.1080/01457632.2019.1611137.
[153] R. Das, B. Kundu, Forward and inverse nonlinear heat transfer analysis for optimization of a constructal T-shape fin under dry and wet conditions, Int. J. Heat Mass Transf. 137 (2019) 461–475, https://doi.org/10.1016/j.ijheatmasstransfer.2019.03.097.
[154] X. Diao, H. Feng, L. Chen, Y. Ge, B. Wu, Constructal design of a T-shaped porous fin adopting minimum composite function, ANN and NSGA-II, Int. Commun. Heat Mass Transf. 162 (2025) 108581, https://doi.org/10.1016/j.icheatmasstransfer.2025.108581.
[155] X. Diao, H. Feng, L. Chen, Y. Ge, A proposed two-stage 3D T-shaped porous-fin model and multi-objective constructal designs for it via composite-function, NSGA-II and ANN, Appl. Therm. Eng. 280 (2025) 128339, https://doi.org/10.1016/j.applthermaleng.2025.128339.
[156] C. Zamfirescu, M. Feidt, Cascaded fins for heat transfer enhancement, Heat Transf. Eng. 28 (2007) 451–459, https://doi.org/10.1080/01457630601163835.
[157] P. Ribeiro, D. Queiros-Condé, The space and time behaviour of the constructal plate generating heat, Int. J. Therm. Sci. 135 (2019) 173–186, https://doi.org/10.1016/j.ijthermalsci.2018.08.045.
[158] C. Biserni, F.L. Dalpiaz, T.M. Fagundes, M. Garai, L.A.O. Rocha, Geometric optimization of morphing fins coupled with a semicircular heat generating body: a numerical investigation on the basis of Bejan's theory, Int. Commun. Heat Mass Transf. 86 (2017) 81–91, https://doi.org/10.1016/j.icheatmasstransfer.2017.05.006.
[159] G. Lorenzini, C. Biserni, R.L. Correa, E.D. Dos Santos, L.A. Isoldi, L.A.O. Rocha, Constructal design of T-shaped assemblies of fins cooling a cylindrical solid body, Int. J. Therm. Sci. 83 (2014) 96–103, https://doi.org/10.1016/j.ijthermalsci.2014.04.011.
[160] G. Lorenzini, C. Biserni, F.L. Dalpiaz, T.M. Fagundes, L.A.O. Rocha, Geometric optimization of T-shaped constructs coupled with a heat generating basement: a numerical approach motivated by Bejan's constructal theory, J. Eng. Thermophys. 26 (2017) 485–497, https://doi.org/10.1134/S1810232817040051.
[161] C. Biserni, F.L. Dalpiaz, T.M. Fagundes, L.A.O. Rocha, Constructal design of T-shaped morphing fins coupled with a trapezoidal basement: a numerical investigation by means of exhaustive search and genetic algorithm, Int. J. Heat Mass Transf. 109 (2017) 73–81, https://doi.org/10.1016/j.ijheatmasstransfer.2017.01.033.
[162] A. Bejan, H. Almahmoud, U. Gunes, Evolutionary design with freedom - Fin shapes and entrance length channels for heat sinks, Int. Commun. Heat Mass Transf. 147 (2023) 106946, https://doi.org/10.1016/j.icheatmasstransfer.2023.106946.
[163] X.Y. Liu, H.J. Feng, L.G. Chen, Y.L. Ge, Constructal design of a rectangular porous fin considering minimization of maximum temperature difference and pumping power consumption, Sci. China Technol. Sci. 67 (2024) 919–929, https://doi.org/10.1007/s11431-023-2495-y.
[164] X. Zhang, D. Liu, Optimum geometric arrangement of vertical rectangular fin arrays in natural convection, Energy Convers. Manag. 51 (2010) 2449–2456, https://doi.org/10.1016/j.enconman.2010.05.009.
[165] M. Zadhoush, A. Ahmadi Nadooshan, M. Afrand, H. Ghafori, Constructal optimization of longitudinal and latitudinal rectangular fins used for cooling a plate under free convection by the intersection of asymptotes method, Int. J. Heat Mass Transf. 112 (2017) 441–453, https://doi.org/10.1016/j.ijheatmasstransfer.2017.04.108.
[166] A. Bejan, Optimal internal structure of volumes cooled by single-phase forced and natural convection, J. Electron. Packag. 125 (2003) 200–207, https://doi.org/10.1115/1.1566970.
[167] T. Bello-Ochende, J.P. Meyer, A. Bejan, Constructal multi-scale pin-fins, Int. J. Heat Mass Transf. 53 (2010) 2773–2779, https://doi.org/10.1016/j.ijheatmasstransfer.2010.02.021.
[168] T. Bello-Ochende, Multi-scale pin fins: Scale analysis and mathematical optimization of micro-pin fins arranged in rows, Open Eng. 5 (2015) 238–247, https://doi.org/10.1515/eng-2015-0023.
[169] A. Yang, L. Chen, Z. Xie, H. Feng, F. Sun, Constructal heat transfer rate maximization for cylindrical pin-fin heat sinks, Appl. Therm. Eng. 108 (2016) 427–435, https://doi.org/10.1016/j.applthermaleng.2016.07.150.
[170] A. Yang, L. Chen, Z. Xie, H. Feng, F. Sun, Constructal operation cost minimization for in-line cylindrical pin-fin heat sinks, Int. J. Heat Mass Transf. 129 (2019) 562–568, https://doi.org/10.1016/j.ijheatmasstransfer.2018.09.129.
[171] L. Chen, A. Yang, Z. Xie, F. Sun, Constructal entropy generation rate minimization for cylindrical pin-fin heat sinks, Int. J. Therm. Sci. 111 (2017) 168–174, https://doi.org/10.1016/j.ijthermalsci.2016.08.017.
[172] G. Xie, Y. Song, M. Asadi, G. Lorenzini, Optimization of pin-fins for a heat exchanger by entropy generation minimization and constructal law, J. Heat Transf. 137 (2015), https://doi.org/10.1115/1.4029851.
[173] X. Liu, H. Feng, L. Chen, Y. Ge, Design of a multi-scale cylindrical porous fin based on constructal theory, Int. Commun. Heat Mass Transf. 153 (2024), https://doi.org/10.1016/j.icheatmasstransfer.2024.107352.
[174] C.J. Kobus, T. Oshio, An experimental and theoretical investigation into the thermal performance characteristics of a staggered vertical pin fin array heat sink with assisting mixed convection in external and in-duct flow configurations, Exp. Heat Transf. 19 (2006) 129–148, https://doi.org/10.1080/08916150500479364.
[175] Y. Joo, I. Lee, S.J. Kim, Efficient three-dimensional topology optimization of heat sinks in natural convection using the shape-dependent convection model, Int. J. Heat Mass Transf. 127 (2018) 32–40, https://doi.org/10.1016/j.ijheatmasstransfer.2018.08.009.
[176] A.W. Mustafa, H.S. Hasan, H.H. Khlaif, Constructal design of vertical multiscale triangular fins in natural convection, Heat Transf. 52 (2023) 5454–5474, https://doi.org/10.1002/htj.22935.
[177] A.W. Mustafa, Constructal design of multi-scale diamond-shaped pin fins cooled by mixed convection, Int. J. Therm. Sci. 145 (2019) 106018, https://doi.org/10.1016/j.ijthermalsci.2019.106018.
[178] A.W. Mustafa, Maximal heat transfer density from cross-flow heat exchanger with tapered fins using constructal design method, Heat Transf. (2024) 749–766, https://doi.org/10.1002/htj.23199.
[179] A.W. Mustafa, I.R. Jawad, A.A. Mohammed, Constructal design of cross-flow heat exchanger with concave/convex fins, Heat Transf. 54 (2025) 21–40, https://doi.org/10.1002/htj.23158.

[180] E. Cetin, E. Cetkin, The effect of cavities and T-shaped assembly of fins on overall thermal resistances, Int. J. Heat Technol. 35 (2017) 944–952, https://doi.org/10.18280/ijht.350430.
[181] H.C. Konan, E. Cetkin, Snowflake shaped high-conductivity inserts for heat transfer enhancement, Int. J. Heat Mass Transf. 127 (2018) 473–482, https://doi.org/10.1016/j.ijheatmasstransfer.2018.08.063.
[182] N.Y. Abbas, A.W. Mustafa, M.K.A. Abbas Asker, Constructal design of longitudinally finned tubes cooled by forced convection, Heat Transf. 49 (2020) 1613–1631, https://doi.org/10.1002/htj.21681.
[183] A.H. Ahmed, M.H. Zaidan, M.S.M. Al-Jethelah, Effect of upstream fin length on longitudinally finned flat tubes bank performance based on constructal design and fuzzy logic control, Int. J. Heat Technol. 42 (2024) 1455–1464, https://doi.org/10.18280/ijht.420435.
[184] M.R. Hajmohammadi, S. Poozesh, S.S. Nourazar, Constructal design of multiple heat sources in a square-shaped fin, Proc. Inst. Mechan. Eng. Part E: J. Process Mechan. Eng. 226 (2012) 324–336, https://doi.org/10.1177/0954408912447720.
[185] N.Y. Godi, Complex heat transfer analysis in heat exchanger with constructal cylindrical fins, Int. Commun. Heat Mass Transf. 159 (2024), https://doi.org/10.1016/j.icheatmasstransfer.2024.108116.
[186] N.Y. Godi, M.O. Petinrin, Heat transfer analysis in constructal designed microchannels with perforated micro fins, J. Inst. Eng. (India): Ser. C 104 (2023) 529–540, https://doi.org/10.1007/s40032-023-00943-y.
[187] N.Y. Godi, Constructal solid and perforated fin installation in a combined microchannel heat sink for maximum heat transfer, J. Inst. Eng. (India): Ser. C 105 (2024) 17–30, https://doi.org/10.1007/s40032-023-01006-y.
[188] A. Bejan, N. Dan, Constructal trees of convective fins, J. Heat Transf. 121 (1999) 675–682, https://doi.org/10.1115/1.2826032.
[189] A. Bejan, Constructal tree-shaped paths for conduction and convection, Int. J. Energy Res. 27 (2003) 283–299, https://doi.org/10.1002/er.875.
[190] M.A. Almogbel, Constructal tree-shaped fins, Int. J. Therm. Sci. 44 (2005) 342–348, https://doi.org/10.1016/j.ijthermalsci.2004.11.002.
[191] J. Kephart, G.F. Jones, Optimizing a functionally graded metal-matrix heat sink through growth of a constructal tree of convective fins, J. Heat Transf. 138 (2016), https://doi.org/10.1115/1.4033209.
[192] E. Çetkin, Inverted fins for cooling of a non-uniformly heated domain, J. Therm. Eng. 1 (2015) 1–9, https://doi.org/10.18186/jte.12488.
[193] L. Chen, H. Feng, F. Zhang, Y. Ge, Constructal design for composite heat dissipating structure composed of an "arrow"-shaped high conductivity channel and an externally connected "T"-shaped fin, Int. Commun. Heat Mass Transf. 153 (2024), https://doi.org/10.1016/j.icheatmasstransfer.2024.107341.
[194] T. Xie, L. Chen, H. Feng, Y. Ge, Constructal design for a composite structure with an "X"-shaped high conductivity channel and a "T"-shaped fin, Int. Commun. Heat Mass Transf. 159 (2024), https://doi.org/10.1016/j.icheatmasstransfer.2024.107985.
[195] L. Chen, T. Xie, F. Zhang, H. Feng, Y. Ge, Proposal of two kinds of composite heat-dissipation structures composed of high-conductivity-channels and T-shaped fins and their constructal designs, Case Stud. Therm. Eng. 75 (2025) 107117, https://doi.org/10.1016/j.csite.2025.107117.
[196] A.-H. Wang, X.-G. Liang, J.-X. Ren, Constructal enhancement of heat conduction with phase change, Int. J. Thermophys. 27 (2006) 126–138, https://doi.org/10.1007/s10765-006-0016-7.
[197] R. Kalbasi, M.R. Salimpour, Constructal design of horizontal fins to improve the performance of phase change material rectangular enclosures, Appl. Therm. Eng. 91 (2015) 234–244, https://doi.org/10.1016/j.applthermaleng.2015.08.024.
[198] R. Kalbasi, M.R. Salimpour, Constructal design of phase change material enclosures used for cooling electronic devices, Appl. Therm. Eng. 84 (2015) 339–349, https://doi.org/10.1016/j.applthermaleng.2015.03.031.
[199] M.R. Salimpour, R. Kalbasi, G. Lorenzini, Constructal multi-scale structure of PCM-based heat sinks, Contin. Mech. Thermodyn. 29 (2017) 477–491, https://doi.org/10.1007/s00161-016-0541-y.
[200] V. Joshi, M.K. Rathod, Constructal enhancement of thermal transport in latent heat storage systems assisted with fins, Int. J. Therm. Sci. 145 (2019) 105984, https://doi.org/10.1016/j.ijthermalsci.2019.105984.
[201] A.K. Barik, P.K. Swain, Constructal invasion of fins for melting time prediction of a phase change material in a triplex-tube heat exchanger, J. Energy Storage 54 (2022), https://doi.org/10.1016/j.est.2022.105281.
[202] A. Shukla, K. Kant, P.H. Biwole, R. Pitchumani, A. Sharma, Melting and solidification of a phase change material with constructal tree-shaped fins for thermal energy storage, J. Energy Storage 53 (2022), https://doi.org/10.1016/j.est.2022.105158.
[203] A. Arshad, A phase change material-based constructal design finned heat sink: an evolutionary design for thermal management, Int. Commun. Heat Mass Transf. 161 (2025) 108379, https://doi.org/10.1016/j.icheatmasstransfer.2024.108379.
[204] S.A. Marzouk, A. Aljabr, F.A. Almehmadi, M.A. Sharaf, T. Alam, D. Dobrotă, Thermal performance enhancement of phase change material melting using innovative fins, Therm. Sci. Eng. Progr. 61 (2025), https://doi.org/10.1016/j.tsep.2025.103585.
[205] M. Eslami, M.A. Bahrami, Sensible and latent thermal energy storage with constructal fins, Int. J. Hydrog. Energy 42 (2017) 17681–17691, https://doi.org/10.1016/j.ijhydene.2017.04.097.
[206] A. Solé, Q. Falcoz, L.F. Cabeza, P. Neveu, Geometry optimization of a heat storage system for concentrated solar power plants (CSP), Renew. Energy 123 (2018) 227, https://doi.org/10.1016/j.renene.2018.02.008.
[207] F. Khosravi, K. Jafarpur, M. Eslami, Design, analysis, and comparative study of a latent heat thermal energy storage system with parallel constructal fins, Therm. Sci. Eng. Progr. 38 (2023), https://doi.org/10.1016/j.tsep.2022.101633.
[208] F. Selimefendigil, H.F. Öztop, Numerical analysis of laminar pulsating flow at a backward facing step with an upper wall mounted adiabatic thin fin, Comput. Fluids 88 (2013) 93–107, https://doi.org/10.1016/j.compfluid.2013.08.013.
[209] G. Lorenzini, S. Moretti, Numerical performance analysis of constructal I and Y finned heat exchanging modules, J. Electron. Packag. 131 (2009) 0310121–03101210, https://doi.org/10.1115/1.3144152.
[210] G. Lorenzini, S. Moretti, A bejan's constructal theory approach to the overall optimization of heat exchanging finned modules with air in forced convection and laminar flow condition, J. Heat Transf. 131 (2009) 1–18, https://doi.org/10.1115/1.3109996.
[211] G. Lorenzini, S. Moretti, Bejan's Constructal theory analysis of gas-liquid cooled finned modules, J. Heat Transf. 133 (2011) 1–10, https://doi.org/10.1115/1.4003556.
[212] E. Cetkin, Constructal vascular structures with high-conductivity inserts for self-cooling, J. Heat Transf. 137 (2015) 1–6, https://doi.org/10.1115/1.4030906.
[213] E. Cetkin, Constructal structures with and without high-conductivity inserts for self-cooling, Int. J. Heat Technol. 34 (2016) S37–S42, https://doi.org/10.18280/ijht.34S105.
[214] B.C. Feijó, M. Dos Santos Pereira, F.B. Teixeira, L.A. Isoldi, L.A.O. Rocha, J.N. V. Goulart, E.D. Dos Santos, Constructal design applied to a channel with triangular fins submitted to forced convection, Defect Diffusion Forum 372 (2017) 152–162, https://doi.org/10.4028/www.scientific.net/DDF.372.152.
[215] G. Xie, M. Asadi, B. Sunden, S. Zheng, Constructal theory based geometric optimization of wavy channels in the low Reynolds number regime, J. Electron. Packag. 136 (2014) 1–8, https://doi.org/10.1115/1.4027728.
[216] Y. Song, M. Asadi, G. Xie, L.A.O. Rocha, Constructal wavy-fin channels of a compact heat exchanger with heat transfer rate maximization and pressure losses minimization, Appl. Therm. Eng. 75 (2015) 24–32, https://doi.org/10.1016/j.applthermaleng.2014.05.027.
[217] M.M.A. Masoud, Geometrical optimization of louver-fin arrays by using, in: Proceedings of the Romanian Academy Series A - Mathematics Physics Technical Sciences Information Science, 2018, pp. 231–236.
[218] M. Asadi, B. Sunden, G. Xie, Constructal optimization of louver fin channels subjected to heat transfer rate maximization and pressure loss minimization, Heat Transf. Eng. 39 (2018) 436–448, https://doi.org/10.1080/01457632.2017.1312881.
[219] O.O. Adewumi, T. Bello-Ochende, J.P. Meyer, Constructal design of combined microchannel and micro pin fins for electronic cooling, Int. J. Heat Mass Transf. 66 (2013) 315–323, https://doi.org/10.1016/j.ijheatmasstransfer.2013.07.039.
[220] R. Wang, Z. Xie, Z. Lu, J. You, Y. Ge, Second law constructal designs of hybrid single-finned and stagger-finned microchannel heat sinks, J. Enhanced Heat Transf. 29 (2022) 115–142, https://doi.org/10.1615/JEnhHeatTransf.2022040329.
[221] A.M. de Moura, E.D. dos Santos, J.A. Souza, Numerical study of three-dimensional microchannel using the constructal law, Int. Commun. Heat Mass Transf. 146 (2023) 106891, https://doi.org/10.1016/j.icheatmasstransfer.2023.106891.
[222] J. Zhang, W. Guo, Z. Xie, X. Guan, X. Qu, Y. Ge, Optimization of fin layout in liquid-cooled microchannels for multi-core chips, Case Stud. Therm. Eng. 41 (2023) 102615, https://doi.org/10.1016/j.csite.2022.102615.
[223] Z. Lu, Z. Xie, K. Xi, D. Lin, H. Liu, Y. Ge, F. Wu, Constructal evolutionary design of liquid cooling heat sink embedded in 3D-IC based on deep neural network prediction, Int. Commun. Heat Mass Transf. 152 (2024), https://doi.org/10.1016/j.icheatmasstransfer.2024.107273.
[224] B. Wu, H. Feng, L. Chen, Y. Ge, X. Liu, Constructal design of a hybrid heat sink with rectangular microchannel and porous fin in a 3D electronic device with artificial neural-network, NSGA-II and different decision-making methods, Int. Commun. Heat Mass Transf. 158 (2024), https://doi.org/10.1016/j.icheatmasstransfer.2024.107954.
[225] H. Feng, K. Sun, L. Chen, Y. Ge, Constructal design of a nanofluid cooling channel with sidewall ribs and cavities in a rectangular heat generation body, Case Stud. Therm. Eng. 41 (2023) 102640, https://doi.org/10.1016/j.csite.2022.102640.
[226] H. Hajabdollahi, Multi-objective optimization of plate fin heat exchanger using constructal theory, Int. Commun. Heat Mass Transf. 108 (2019), https://doi.org/10.1016/j.icheatmasstransfer.2019.104283.
[227] A.F. Sotoodeh, M. Amidpour, M. Ghazi, Developing of constructal theory concept for plate-fin heat exchanger modelling, design and optimisation, Int. J. Exergy 18 (2015) 22–45, https://doi.org/10.1504/IJEX.2015.072054.
[228] M. Shafiey Dehaj, H. Hajabdollahi, Fin and tube heat exchanger: constructal thermo-economic optimization, Int. J. Heat Mass Transf. 173 (2021) 121257, https://doi.org/10.1016/j.ijheatmasstransfer.2021.121257.
[229] J.V.C. Vargas, A. Bejan, Thermodynamic optimization of finned crossflow heat exchangers for aircraft environmental control systems, Int. J. Heat Fluid Flow 22 (2001) 657–665, https://doi.org/10.1016/S0142-727X(01)00129-1.
[230] J. Bonjour, L.A.O. Rocha, A. Bejan, F. Meunier, Dendritic fins optimization for a coaxial two-stream heat exchanger, Int. J. Heat Mass Transf. 47 (2004) 111–124, https://doi.org/10.1016/S0017-9310(03)00406-X.
[231] L.E. Paniagua-Guerra, S. Sehgal, C.U. Gonzalez-Valle, B. Ramos-Alvarado, Fractal channel manifolds for microjet liquid-cooled heat sinks, Int. J. Heat Mass Transf. 138 (2019) 257–266, https://doi.org/10.1016/j.ijheatmasstransfer.2019.04.039.
[232] S. Lorente, A. Bejan, Combined "flow and strength" geometric optimization: Internal structure in a vertical insulating wall with air cavities and prescribed strength, Int. J. Heat Mass Transf. 45 (2002) 3313–3320, https://doi.org/10.1016/S0017-9310(02)00052-2.

[233] Z. Xie, L. Chen, F. Sun, Constructal optimization of a vertical insulating wall based on a complex objective combining heat flow and strength, Sci. China Technol. Sci. 53 (2010) 2278–2290, https://doi.org/10.1007/s11431-010-4003-7.
[234] L. Chen, Z. Xie, F. Sun, Multiobjective constructal optimization of an insulating wall combining heat flow, strength and weight, Int. J. Therm. Sci. 50 (2011) 1782–1789, https://doi.org/10.1016/j.ijthermalsci.2011.03.022.
[235] A.K. Pramanick, P.K. Das, Method of synthetic constraint, Fermat's principle and the constructal law in the fundamental principle of conductive heat transport, Int. J. Heat Mass Transf. 50 (2007) 1823–1832, https://doi.org/10.1016/j.ijheatmasstransfer.2006.10.003.
[236] A. Bejan, S. Lorente, Thermodynamic optimization of flow geometry in mechanical and civil engineering, J. Non-Equilibr. Thermodyn. 26 (2001), https://doi.org/10.1515/JNETDY.2001.022.1.
[237] H. Feng, L. Chen, Z. Xie, F. Sun, Constructal design for gas-turbine blade based on minimization of maximum thermal resistance, Appl. Therm. Eng. 90 (2015) 792–797, https://doi.org/10.1016/j.applthermaleng.2015.07.063.
[238] A. Beyene, J. Peffley, Constructal theory, adaptive motion, and their theoretical application to low-speed turbine design, J. Energy Eng. 135 (2009) 112–118, https://doi.org/10.1061/(ASCE)0733-9402(2009)135:4(112).
[239] S. Tescari, P. Neveu, N. Mazet, Thermochemical solar reactor: simplified method for the geometrical optimization at a given incident flux, Int. J. Chem. React. Eng. 8 (2010), https://doi.org/10.2202/1542-6580.2091.
[240] S. Tescari, N. Mazet, P. Neveu, Constructal method to optimize solar thermochemical reactor design, Sol. Energy 84 (2010) 1555–1566, https://doi.org/10.1016/j.solener.2010.06.015.
[241] P. Neveu, S. Tescari, D. Aussel, N. Mazet, Combined constructal and exergy optimization of thermochemical reactors for high temperature heat storage, Energy Convers. Manag. 71 (2013) 186–198, https://doi.org/10.1016/j.enconman.2013.03.035.
[242] T. Zhou, Z. Sun, S. Li, H. Liu, D. Yi, Design and optimization of thermophotovoltaic system cavity with mirrors, Energies (Basel) 9 (2016), https://doi.org/10.3390/en9090722.
[243] J. Ojeda, F. Méndez, Application of Bejan's constructal theory to a solar collector system. part I: the fundamentals to define the first construction, Int. J. Thermodyn. 13 (2010) 135–141. https://www.scopus.com/inward/record.uri?eid=2-s2.0-78649233685&partnerID=40&md5=826c6791b37a7d992f870753de096b87.
[244] K. Kant, R. Pitchumani, Analysis of a novel constructal fin tree embedded thermochemical energy storage for buildings applications, Energy Convers. Manag. 258 (2022), https://doi.org/10.1016/j.enconman.2022.115542.