

Appendices

Appendix A. Equilibrium

Appendix A.1. Mixed-purchase Scenario

Equilibrium prices. The symmetric equilibrium price p_M readily stems from the first-order condition, $\frac{\partial [p_i(1 - \frac{v-p_i}{T})]}{\partial p_i} = 0$. One can check that the second order condition is fulfilled.

Interval $\frac{v}{2} < T \leq v$. Plugging p_M into (3) and (4) yields $D_1(p_M) \equiv \frac{T - \frac{v}{2}}{T} \equiv D_2(p_M)$. Solving $\frac{T - \frac{v}{2}}{T} \in (0, 1)$ for T yields $T > \frac{v}{2}$. Plugging p_M into (5) yields $D_{1+2}(p_M) \equiv \frac{v}{T} - 1$. Plugging p_M into (2) yields $2v - T - (\frac{v}{2} + \frac{v}{2}) = v - T$, which is nonnegative when $T \leq v$.

Welfare analysis. The producer surplus is given by

$$2 \times p_M \left(\frac{v - p_M}{T} \right) = 2 \frac{v}{2} \left(\frac{v - \frac{v}{2}}{T} \right) = \frac{v^2}{2T}.$$

The transport company's equilibrium profits are

$$\begin{aligned} & \int_0^{x_1(p_M)} (mx) dx + \int_{x_1(p_M)}^{x_2(p_M)} m dx + \int_{x_2(p_M)}^1 [m(1-x)] dx \\ &= \int_0^{\frac{T - \frac{v}{2}}{T}} (mx) dx + \int_{\frac{T - \frac{v}{2}}{T}}^{\frac{v}{2T}} m dx + \int_{\frac{v}{2T}}^1 [m(1-x)] dx = \frac{mv^2}{4T^2}. \end{aligned}$$

The consumer surplus is given by

$$\begin{aligned} & \int_0^{x_1(p_M)} (v - p_M - Tx) dx + \int_{x_1(p_M)}^{x_2(p_M)} (2v - 2p_M - T) dx + \int_{x_2(p_M)}^1 [v - p_M - T(1-x)] dx \\ &= \int_0^{\frac{T - \frac{v}{2}}{T}} \left(\frac{v}{2} - Tx \right) dx + \int_{\frac{T - \frac{v}{2}}{T}}^{\frac{v}{2T}} (v - T) dx + \int_{\frac{v}{2T}}^1 \left[\frac{v}{2} - T(1-x) \right] dx = \frac{v^2}{4T}. \end{aligned}$$

Appendix A.2. Joint-purchase Scenario

Equilibrium prices. When the price charged by both firms is p_M , the location of the last consumer buying from firm 1 (respectively, 2) is $x_1(p_M) = \frac{v}{2T}$ (respectively, $x_2(p_M) = 1 - \frac{v}{2T}$). Note that $x_1(p_M) \geq 1$ and $x_2(p_M) \leq 0$ if $T \leq \frac{v}{2}$, meaning that the last consumer buying from firm 1 (2) is actually located at the endpoint 1 (0) and, most importantly, she enjoys weakly positive utility when buying only from firm 1 (2): $v - T - \frac{v}{2} = \frac{v}{2} - T \geq 0$. The profit-maximizing price chosen by firms must therefore be weakly higher than p_M ; in particular, it must be such that the consumer located at the endpoint 0 (1) is left with zero utility from the consumption of the service 1 (0); in symbols, $v - T - p_i = 0$. Solving this equation for p_i gives (8).

Though intuitive, the reasoning above does not rely on first-order conditions. Accordingly, we check that no unilateral profitable deviations from (8) exist to show that (8) is indeed an equilibrium. At this candidate equilibrium, the firms' profit is $(v - T) \times 1$. Suppose $p_j^* = v - T$ and analyze the two possible deviations available to firm j . First, if firm j sets $p_j < v - T$, one can easily check that firm j serves the entire market; however, its deviation profits are $p_j \times 1 < (v - T) \times 1$, so this downward deviation is not profitable. Second, if firm j sets $p_j > v - T$, one can easily check that firm j 's demand is less than 1; in particular, it is equal to $D_j = \frac{v - p_j}{T}$. The resulting deviation profit function, $p_j \left(\frac{v - p_j}{T} \right)$, is maximized at $p_j = \frac{v}{2}$ according to the proof in Appendix A.1. However, $\frac{v}{2} \leq v - T$ in interval $T \leq \frac{v}{2}$, so this upward deviation is not feasible. We conclude that (8) is an equilibrium of the price competition game.

Welfare analysis. The producer surplus is given by $2 \times p_J \times 1 = 2(v - T)$. The transport company's equilibrium profits are $\int_0^1 m dx = m$. The consumer surplus is given by

$$\int_0^1 (2v - T - p_J) dx = \int_0^1 [2v - T - 2(v - T)] dx = T.$$

Appendix A.3. Single-purchase Scenario

Welfare analysis. The producer surplus is given by

$$2 \times p_S \left(\frac{v - p_S}{T} \right) = 2 \frac{v}{2} \left(\frac{v - \frac{v}{2}}{T} \right) = \frac{v^2}{2T}.$$

The transport company's equilibrium profits are

$$\begin{aligned} & \int_0^{x_1(p_S)} m dx + \int_{x_1(p_S)}^{x_2(p_S)} 0 dx + \int_{x_2(p_S)}^1 m(1-x) dx \\ &= \int_0^{\frac{v}{2T}} m dx + \int_{1-\frac{v}{2T}}^1 m(1-x) dx = \frac{mv^2}{4T^2}. \end{aligned}$$

The consumer surplus is given by

$$\begin{aligned} & \int_0^{x_1(p_S)} (v - p_S - Tx) dx + \int_{x_1(p_S)}^{x_2(p_S)} 0 dx + \int_{x_2(p_S)}^1 \left[v - \frac{v}{2} - T(1-x) \right] dx \\ & \int_0^{\frac{v}{2T}} \left(v - \frac{v}{2} - Tx \right) dx + \int_{1-\frac{v}{2T}}^1 \left[v - \frac{v}{2} - (t+m)(1-x) \right] dx = \frac{v^2}{4T}. \end{aligned}$$

Appendix B. Destination Card

We solve backwards the three-stage game described in the text. Before proceeding, we anticipate that all consumers decide to use the Card in the first stage: we then verify this is true at equilibrium.

Third stage. In the third stage, the utility of any joint purchaser and any single purchaser is given by (9) and (10), respectively, because all consumers use the Card. Solving equations $U_1^{DC} = U_{1+2}^{DC}$ for x and $U_2^{DC} = U_{1+2}^{DC}$ for $1-x$ yields the demand of single purchasers of service i , $D_i^{DC} \equiv \frac{t-v+\alpha p_i}{t}$. The demand of joint purchasers is therefore $D_{1+2}^{DC} \equiv \frac{2v-t-\alpha(p_1+p_2)}{t}$.

Second stage. In the second stage, firms compete over prices. We consider the three subgame equilibrium scenarios separately. (a) Under the mixed-purchase scenario, firm i 's profit function is

$$\Pi_i^{DC} = \alpha p_i (D_i^{DC} + D_{1+2}^{DC}) = \alpha p_i \frac{v - \alpha p_i}{t}$$

and the FOC yields the symmetric equilibrium price $\frac{v}{2\alpha}$. (b) Joint-purchase scenario: following the reasoning developed in Appendix A.2, the profit-maximizing price is such that the consumers located at the endpoints enjoy zero utility from the consumption of the service at the opposite end: $v - t - \alpha p_i = 0$; solving this equation for p_i gives $\frac{v-t}{\alpha}$. (c) Single-purchase scenario: solving equations $U_1^{DC} = 0$ for x and $U_2^{DC} = 0$ for $1-x$, we get the demand for service 1, $\frac{v-\alpha p_1}{t}$, and for service 2, $\frac{v-\alpha p_2}{t}$. Therefore, the symmetric Nash equilibrium price is $\frac{v}{2\alpha}$.

First stage. In the first stage, any single consumer decides whether to use the Card or not. To show that using the Card is a dominant strategy for any single consumer, both joint purchasers and single purchasers, under any second-stage equilibrium scenario, we proceed as follows. Plugging the second-stage equilibrium prices into (9) and (10) yields the utility of any joint purchaser and any single purchaser, respectively, using the Card. Plugging the same prices into (2) and (1) gives instead the utility of any joint purchaser and any single purchaser, respectively, not using the Card. One can easily check that any consumer is better off when using the Card because she enjoys free public transport and price discounts. In conclusion, we remark that inactive consumers get zero utility both when using and not using the Card, so these two options are equally liked.

Parametric intervals and welfare analysis. Replicating the analysis of Appendix A.1, one can easily check that the parametric interval under which the mixed-purchase equilibrium arises is $\frac{v}{2} < t \leq v$ and that the producer and consumer surplus are $\frac{v^2}{2t}$ and $\frac{v^2}{4t}$, respectively. Under the joint-purchase scenario, the producer and consumer surplus are as PS_J and CS_J with t rather than T . Finally, the single-purchase-scenario producer and consumer surplus are $\frac{v^2}{2t}$ and $\frac{v^2}{4t}$, respectively.

Appendix C. Proposition 3

Interval (1). One can easily check that: (i) $PS_J^{DC} \equiv 2(v-t) > PS_J \equiv 2(v-T)$; (ii) $\Pi_{TC,J}^{DC} \equiv 0 < \Pi_{TC,J} \equiv m$; (iii) $CS_J^{DC} \equiv t < CS_J \equiv T$; (iv) $TS_J^{DC} \equiv 2(v-t) + 0 + t = TS_J \equiv 2(v-T) + m + T$.

Interval (2). Solving $PS_J^{DC} \equiv 2(v-t) > PS_M \equiv \frac{v^2}{2(t+m)}$ for t yields $\frac{v-m-\sqrt{m^2+2vm}}{2} < t < \frac{v-m+\sqrt{m^2+2vm}}{2}$. One can check that $\frac{v}{2} - m < \frac{v-m-\sqrt{m^2+2vm}}{2} < \frac{v-m+\sqrt{m^2+2vm}}{2} < \frac{v}{2}$, so inequality $PS_J^{DC} > PS_M$ is fulfilled in interval (2).

Solving $CS_J^{DC} \equiv t < CS_M \equiv \frac{v^2}{4(t+m)}$ for t yields $t < \frac{\sqrt{m^2+v^2}-m}{2}$. One can check that $\frac{v}{2} - m < \frac{\sqrt{m^2+v^2}-m}{2} < \frac{v}{2}$, so inequality $CS_J^{DC} < CS_M$ is fulfilled in interval (2) if and only if $\frac{v}{2} - m < t < \frac{\sqrt{m^2+v^2}-m}{2}$.

Finally, solving $TS_J^{DC} \equiv 2v-t = TS_M \equiv \frac{(4m+3t)v^2}{4(m+t)^2}$ for t yields $t_1 \equiv \frac{3v-2m-\sqrt{(2m+v)(2m+9v)}}{4}$, $\frac{v}{2} - m$,

$t_3 \equiv \frac{3v-2m+\sqrt{(2m+v)(2m+9v)}}{4}$. One can check that $t_1 < \frac{v}{2} - m < \frac{v}{2} < t_3$. It follows that $TS_J^{DC} \neq TS_M$ in interval (2). To determine whether TS_J^{DC} is larger or smaller, we compute $\frac{\partial(TS_J^{DC}-TS_M)}{\partial t}$, evaluate it at $t = \frac{v}{2} - m$, and get $\frac{4m+2v}{v}$. Since this value is positive and the difference $(TS_J^{DC} - TS_M)$ is continuous in t , we conclude that $TS_J^{DC} = TS_M$ at $t = \frac{v}{2} - m$ and $TS_J^{DC} > TS_M$ in interval (2).

Interval (3). First consider $\frac{v}{2} < t \leq v - m$. One can easily check that: (i) $PS_M^{DC} \equiv \frac{v^2}{2t} > PS_M \equiv \frac{v^2}{2T}$; (ii) $CS_M^{DC} \equiv \frac{v^2}{4t} > CS_M \equiv \frac{v^2}{4T}$. Rearranging $TS_M^{DC} \equiv \frac{3v^2}{4t} > TS_M \equiv \frac{(4m+3t)v^2}{4(m+t)^2}$ yields $m(3m+2t) > 0$, which is true. These findings extend to intervals $v - m < t \leq v$ and $t > v$ because the welfare values under the single-purchase scenario take the same functional forms as under the mixed-purchase scenario.

Appendix D. Robustness Checks

Priced destination card. We solve backwards the four-stage game described in the text. Before proceeding, we assume that all consumers buy the Card in the second stage: we then verify under which conditions this is true at equilibrium. Note that if this is the case, consumers decide to visit at least one firm in the fourth stage: visiting no firms would indeed yield utility $-k$, in which case consumers would be better off by not buying the Card. As a result, we can restrict our attention to the joint- and the mixed-purchase scenario only.

In the fourth stage, the utility of a joint purchaser and a single purchaser when buying the Card is as in (9) and (10), respectively, minus the price k of the Card. This implies that the demands of joint and single purchasers are as in Appendix B.

In the third stage, firms choose prices. (a) The mixed-purchase equilibrium price is still $\frac{v}{2\alpha}$, but the parametric interval shrinks to $\frac{v}{2} < t \leq v - k$ because of the Card price paid by consumers. (b) Under the joint-purchase scenario, arising in the usual interval $t \leq \frac{v}{2}$, the equilibrium price is still $\frac{v-t}{\alpha}$ and the resulting consumer equilibrium utility is $t - k$. However, when this value is negative (i.e., $t < k$), firms reduce the symmetric equilibrium price to $\frac{2v-t-k}{2\alpha}$ so that consumers get zero utility and the market is still covered.

In the second stage, consumer utility when buying the Card is: (a) $v - t - k$ for joint purchasers and $\frac{v}{2} - td_i(x) - k$ for single purchasers, under the mixed-purchase scenario; (b) $\max\{t - k, 0\}$ under the joint-purchase scenario. Consumer utility when not buying the Card is: (a) $v(\frac{2\alpha-1}{\alpha}) - T$ for joint purchasers and $v(\frac{2\alpha-1}{2\alpha}) - Td_i(x)$ for single purchasers, under the mixed-purchase scenario; (b) $2v - T - 2(\frac{v-t}{\alpha})$, under the joint-purchase scenario.

In the first stage, we verify all consumers are better off when buying the Card, provided that the Destination Management Organization sets $k \leq \min\{m, v(\frac{1-\alpha}{2\alpha})\} \cap \alpha \leq \frac{2}{3}$. Intuitively, the Card price must be relatively low and, at the same time, the discount on attractions relatively high for all consumers to be willing to buy the Card.

On the above basis, we can compare the welfare values with the priced Card to those without the Card. Before proceeding, two aspects must be highlighted. First, total surplus contains also the profits accruing to the Destination Management Organization from the sale of the Card; since the market is fully covered, this profit is given by the price k times the mass 1 of consumers. Second, this amount is transferred from consumers to the Destination Management Organization, so it factors out in the calculation of total surplus.

1. When $t \leq \frac{v}{2} - m$, the welfare comparison gives the same results as those in interval (1), Appendix C.
2. When $\frac{v}{2} - m < t \leq \frac{v}{2}$, two parametric constellations must be considered separately. (a) If $k < \frac{v}{2} - m$ or if $k > \frac{v}{2} - m \cap k \leq t \leq \frac{v}{2}$, the welfare comparison is as in interval (2), Appendix C, except for the consumer surplus CS_J^{DC} , which is now given by $t - k$. (b) If $k > \frac{v}{2} - m \cap \frac{v}{2} - m < t < k$, CS_J^{DC} becomes zero and, accordingly, lower than CS_M ; PS_J^{DC} becomes $2v - t - k$, which turns out to be higher than PS_M .
3. When $\frac{v}{2} < t \leq v - k$, the welfare comparison is as in interval (3), Appendix C, except the consumer surplus CS_M^{DC} , which is now given by $\frac{v^2}{4t} - k$. One can check that CS_M^{DC} becomes lower than CS_M when k is sufficiently high.

100% Price Discount. We focus on full market coverage because we showed that the single-purchase scenario does not add to the welfare comparison in Proposition 3. When $\alpha = 0$, firm i 's profit function is as in Section 4, but multiplied by the share $(1 - \mu)$ of consumers that still pay p_i for attraction i . As a result, one can easily check that the equilibrium prices are as in Section 4: $\frac{v}{2}$ in the mixed-purchase scenario, which arises in interval $\frac{v}{2} - m < t \leq v - m$, and $v - T$ in the joint-purchase one, which arises when $t \leq \frac{v}{2} - m$. The scenarios are defined by the purchasing decisions of consumers without the Card because firms obtain profits only from them. As for the share $(1 - \mu)$ of consumers endowed with the Card, one can check that the proportion of joint purchasers is given $\frac{2v-t}{t}$ and that this value is larger than 1 in our interval of interest, $t \leq v - m$. As a result, all consumers receiving the Card decide to joint-purchase.

On the above basis, we can compute the welfare values and compare them to those without the Card.

1. When $t \leq \frac{v}{2} - m$, we obtain: $PS_J^{DC} \equiv 2(1 - \mu)(2v - T) < PS_J$; $\Pi_{TC,J}^{DC} \equiv (1 - \mu)m < \Pi_{TC,J}$;
 $CS_J^{DC} \equiv (1 - \mu)T + \mu(2v - t) > CS_J$; and $TS_J^{DC} \equiv 2v(2 - \mu) - t > TS_J$.
2. When $\frac{v}{2} - m < t \leq v - m$, we get: $PS_M^{DC} \equiv (1 - \mu)\frac{v^2}{2T} < PS_M$; $\Pi_{TC,M}^{DC} \equiv (1 - \mu)\frac{mv^2}{4T^2} < \Pi_{TC,M}$;
 $CS_M^{DC} \equiv (1 - \mu)\frac{v^2}{4T} + \mu(2v - t) > CS_M$; and $TS_M^{DC} \equiv \frac{(4m+3t)v^2 + \mu(8T^2v - 4T^2t - 3Tv^2 - mv^2)}{4T^2} > TS_M$.