Exploratory Factor Analysis: A Practical Guide for Psychological Research

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Exploratory Factor Analysis (EFA) is a multivariate statistical method that aims to provide a parsimonious and simple representation (i.e., explanation) of the associations between a set of observed variables (i.e., manifest variables, indicators) through the smallest number of hypothesized latent variables or factors (i.e., constructs, dimensions) (e.g., Gorsuch, 1983; Mulaik, 1990). Given a set of observed variables in relation to one another, the EFA method aims to identify coherent groupings of these variables that are relatively independent of each other. Each of these groupings, according to the theoretical model of EFA, is caused by a latent factor. Given that, EFA aims to determine the number, nature, and inter-relations of latent variables, i.e., common factors, that explain these patterns of correlations between a set of observed variables (see, for example, Tabachnick & Fidell, 2019). To conduct an EFA, it is necessary to make several key decisions (see, for example, Fabrigar et al., 1999; Watkins, 2018; Norris & Lecavalier, 2010). This chapter aims to present easy-to-use guidelines and provide an example that demonstrates their application to facilitate the completion of an EFA. Our aim is to help readers conducting EFA and critically reviewing papers using EFA.

Theoretical Model

EFA is based on Spearman (1904) and Thurstone's (1935, 1940) theoretical model of Common Factors, according to which each observed variable is equal to the linear function of one or more common latent factors and a single unique factor. Common factors are statistical constructs introduced to account for the shared common variance among observed variables. Although common factors are often described as underlying causes of observed variables, this interpretation has been extensively debated (Borsboom et al., 2003). From a purely statistical standpoint, latent factors can be conceptualized as abstractions that encapsulate patterns of covariance among observed measures. The single unique factor, on the other hand, affects only the specific variable and does not contribute to the relationships with other variables. The unique factor can be decomposed into two components: a specific latent factor, representing reliable variance unique to the observed variable, and a measurement error component, which represents the random error inherent in every measurement. Accordingly, the total variance of an observed variable can be partitioned into common variance (communality), which represents the variance explained by the common factors, and unique variance, equal to reliable specific variance plus measurement error variance:

$$\sigma_2 = h_2 + (s_2 + e)$$

The theoretical model of Common Factors is based on the idea that each psychological concept is a construct comprising distinct dimensions that can only be measured indirectly through behavioral indicators, i.e., aspects of behavior believed to be caused by these dimensions (e.g., Edwards & Bagozzi, 2000; Tucker & MacCallum, 1997). According to this concept, the indicators are the observed variables, and the dimensions are the latent factors that EFA aims to identify.

When a correlation matrix of observed variables is considered, along with the variances of the observed variable equal to 1 on the positive diagonal of the matrix, a number of common factors can be identified through the use of EFA. Each observed variable is a linear combination of these common factors, which in turn allows for the reproduction of the matrix of observed correlations. Since these common factors account only for the common variance between observed variables, i.e., for their correlation, and because each variables contains unique variance not shared with other observed variables, the correlation matrix reproduced by EFA model represent only an approximation of the observed correlational matrix and cannot be expected to be identical in reduced-factor solutions. As will be seen below, EFA analyzes the shared variance of multiple variables, which is summarized by communality. Unique variance, including measurement errors, is not explained by the common factors and remains as residual variance.

The difference between the original observed matrix and the reproduced one provides an index of the goodness of fit of the identified factorial solution. The smaller the residual between the original matrix and the reproduced one, the greater the amount of correlation between the observed variables explained by the identified factors. Therefore smaller residuals indicate that a greater amount of the common variance is explained by the factors. The difference between the observed and reproduced matrices also depends on the amount of unique variance, and since this is also an expression of measurement error, the amount of common variance explained also depends on the reliability of the observed variables.

Exploratory Factor Analysis and Confirmatory Factor Analysis

EFA has the same objectives as Confirmatory Factor Analysis (CFA) (and the same theoretical model), i.e., given a correlation matrix between the observed variables, to identify a smaller number of latent factors that allow this correlation matrix to be reproduced. However, unlike the CFA, EFA does not require alternative factorial solutions to be hypothesized a priori, based on theories or previous investigations, to verify their validity in explaining the data. EFA aims to explore the data to identify the best factorial solution, whereas CFA aims to verify with the data which of the hypothesized factorial solutions is the best (e.g., Bollen, 1989; Brown, 2015; Kline, 2016).

For both EFA and CFA, each observed variable is modeled as a linear function of one or more latent factors and a unique factor specific to each variable. Although both EFA and CFA are based on the same factor model and estimate the same type of parameters (saturation coefficients between the observed variables and the latent factors, also known as factor loading, factor covariances, and unique variances) they differ in how the factor structure are defined. In EFA the structure is derived from the data, in CFA is specified in advance. This allows CFA to verify if a factor structure derived from a theoretically defined model provides an adequate representation of the observed correlations, rather than a simple exploration of the data-driven structure. Therefore, although the reproduced matrix obtained with the CFA remains an approximation of the observed matrix, CFA allows to formally evaluate the quality of this approximation.

Due to the differences between EFA and CFA, it is recommended that, when the sample size is adequate, participants be randomly divided into two groups. One group should undergo EFA, and the validity of the resulting solution should be verified using CFA on the other group (cross-validation).

Exploratory Factor Analysis and Principal Component Analysis

EFA differs from Principal Component Analysis (PCA). Given a set of observed variables in relation to each other, both methods aim to identify coherent groupings of variables that are relatively independent of each other. However, while EFA assumes that these groupings are caused by latent factors, PCA assumes that they are an expression of principal components, i.e., a linear combination of observed variables. PCA does not aim to represent the correlations between variables more parsimoniously by identifying a reduced number of latent factors. PCA is essentially a method of data reduction. It aims to reduce a large set of observed variables to a small number of variables that are linear functions of specific observed variables, thereby preserving as much of the information conveyed by the original set of variables as possible (Widaman, 1993).

EFA distinguishes between the variance of the observed variables in common variance (caused by common factors) and in unique variance (caused by a single latent factor and measurement error). It assumes that the observed variables are explained by a linear function of one or more common factors, which account for the correlations between the variables (i.e., the variance each variable shares with the others), as well as by the action of a single factor, which accounts for the unique variance of the factor (i.e., the variance each variable does not share with the others). On the contrary, PCA does not differentiate between common variance and unique variance; the object of investigation is the total variance of the observed variables. According to the model underlying PCA, the observed variables are caused by a linear function of principal components, i.e., a linear combination of the observed variables. The objective of EFA is to identify a small number of factors that can reproduce the variance shared by the observed variables, i.e., common variance modeled through latent factors. The objective of PCA is to identify a small number of components that explain and reproduce the variance between the observed variables, i.e., total variance.

Both methods of data analysis involve identifying common factors, or principal components, from the correlation matrix of observed variables. When performing an EFA, the reduced matrix with an estimated communality on the positive diagonal is analyzed. This estimate represents the variance of each variable that can be explained by the latent common factors. It replaces the variance of each variable on the positive diagonal of the original correlation matrix. In contrast, PCA analyzes the entire matrix. As with the original correlation matrix, the variance of each observed variable is located on the positive diagonal. This represents the total variance of each variable that can be explained by the principal components.

Therefore, in the context of EFA, the reproduced correlation matrix can only be considered an approximation of the observed matrix because the factor solution only models the common variance. In contrast, for PCA, particularly when as many principal components are extracted as there are observed variables, the reproduced matrix corresponds to the observed one. As we will see, in the case of PCA, the solution is unique, while in the case of EFA, the solution is indeterminate, i.e., the true latent factor scores of individuals are unobservable and cannot be uniquely recovered from the observed indicators, but can only be estimated (Rhemtulla & Savalei, 2025).

In cases where the observed variables have low communality (< .40) and the factors are not defined by more than three variables, the factorial solutions obtained with these two methods of data analysis are different (Thompson, 1992).

Exploratory Factor Analysis Prerequisites

The EFA method requires specific prerequisites to be detected.

Screening Data Prior to Analysis. Before conducting any analyses, it is essential to screen the observed variables collected and entered into a data file. This includes checking for missing data and, for interval-scale variables, checking for univariate and multivariate normality of the distribution and for univariate and multivariate outliers among participants. Tabachnick and Fidell (2019) described these steps appropriately. Here, we would like to reiterate that for interval-scale variables, the univariate distribution must first be verified for normality by computing the asymmetry and kurtosis values and considering indices within the range of -1.00 to 1.00. However, according to Fabrigar and colleagues (1999), asymmetry values below 2 and kurtosis values below 7 may be considered indices of a normal distribution. Then, the normality of the multivariate distribution must be verified using the Mardia's test.

Eventually, normalization of the distribution or another transformation must be applied to try to achieve a normal distribution. The presence of univariate outliers, i.e., participants with a z-value greater than 3.29, and multivariate outliers, i.e., participants with a Mahalanobis distance probability lower than 0.001, must also be verified. According to Tabachnick and Fidell (2019), any outliers should be eliminated and the process should be restarted with the remaining participants by verifying the normality of the distribution and the presence of outliers.

The correlation (covariance) matrix between the observed variables must be suitable for the use of EFA, i.e., specific prerequisites must be met. For screened data, these prerequisites concern the participants, the observed variables, and the correlation matrix between the observed variables.

Participants. The participants must be sufficiently numerous, with the right degree of homogeneity/heterogeneity, and free of any uni- and multivariate outliers in the case of interval-level observed variables.

With regard to sample size, different and sometimes conflicting indications have been reported in the literature, concerning both the absolute number of participants and the ratio between participants and observed variables. According to the first criterion, for example, Guilford (1954) recommended having at least 200 participants. Gorsuch (1974) believed that 200 could be defined as a large group and less than 50 as a small group. Comrey and Lee (1992) argued that groups of 50 participants are to be considered small, 100 mediocre, 200 adequate, 300 good, 500 very good, and 1000 excellent. According to the second criterion, i.e., the ratio of participants to observed variables, recommendations vary from a ratio of 3:1 to 6:1 (Cattell, 1978) to a ratio of 20:1 (Hair et al., 1995). Stevens (1996), for example, believes that there should be 5 to 10 participants for each observed variable.

Statistical simulations have found that the optimal sample size cannot be defined in absolute terms or even on the basis of the variable/participant ratio. On the contrary it depends on the factorial solution extracted and, in particular, on the communality of the observed variables, the loadings of the observed variables on the factors (i.e., saturation coefficients, which are similar in meaning to regression coefficients), the number of observed variables per factor, and the number of factors (e.g., MacCallum et al., 1999, 2001; Marsh & Hau, 1999; Velicer & Fava, 1998).

MacCallum and colleagues (1999) showed that sample sizes of 100 to 200 participants may be sufficient when communalities are approximately .50, and factors are well determined, that is, when factors are loaded by several observed variables. Under these conditions, the latent structure tends to be relatively stable, even with moderate sample sizes. When communalities are low and there are only a few factors, each defined by just three or four observed variables, a minimum of about 300 participants is needed. In unfavorable conditions involving low communalities and many weakly determined factors, sample sizes well above 500 participants are recommended. The influence of sample size is reduced when communalities are consistently high (≥ .60) and factors are strongly defined. In such cases, samples smaller than 100 may be acceptable (de Winter et al., 2009).

Mundfrom and colleagues (2005) presented empirical guidelines in the form of tables showing the minimum sample size required for EFA given different communalities and the ratio of variables to factors. They found that minimum sample sizes appear to be smaller for higher levels of communality and for higher ratios of observed variables to factors. When the ratio of observed variables to factors exceeds six, the minimum sample size begins to stabilize, regardless of the number of factors or the level of communality.

Recently, Lorenzo-Seva and Ferrando (2024) developed the Simple Empiric NumEriCAl estimate of sample size in factor analysis (SENECA) tool. SENECA aims to determine, given an observed dataset, the sample size needed to achieve a particular threshold value for the residuals between the observed and reproduced correlation matrices. SENECA is implemented in R (R script "SenecaEstimate.r") and in the free software they developed to compute factor analysis (Ferrando & Lorenzo-Seva, 2017; https://psico.fcep.urv.cat/utilitats/factor/).

It is useful to remember that in the case of participants who are more homogeneous than the general population, the correlation coefficients between variables are attenuated due to the restriction of their range of variability. As a result, it is likely that the correlation coefficients (both those between observed variables and factors, i.e., saturation coefficients, and those between factors) are underestimated in the factorial solution. Furthermore, combining distinct groups of participants for whom the factor structure is likely to be different into a single large group compromises the internal validity of the survey, i.e., the possibility of identifying the real factor structure.

Variables. All observed variables must be valid and reliable indicators of the phenomenon to be measured. Only in this way are the conditions in place for there to be relationships between the variables and, therefore, the possibility of achieving high communality (i.e., if reliability is high, then the unique variance of each variable is low). If it is possible to hypothesize the number and nature of the factors that explain the relationships between the variables, it is necessary that for each factor there are at least 3 to 5 "pure" observed variables, marker variables. These markers should have a factor loading of |.70| or higher (Fabrigar et al., 1999).

Correlation Between Observed Variables Different correlation coefficients must be used to compute the correlation matrix to be analyzed based on the level of measurement and/or normality of the distribution score of the observed variables. The *Pearson correlation* must be used if the observed variables are at the interval level (i.e., a score on a Likert-type scale of five or more steps) and are linearly and multivariate normally distributed. The *policoric correlation* should be used when the observed variables are at the interval level, but are not normally distributed, or when they are on an ordinal scale (e.g., a score on a Likert-type scale with four or fewer steps). The *tetracoric correlation* should be used for dichotomous or politomic observed variables that deviate severely from normality (e.g., Holgado-Tello et al., 2010; Wirth & Edwards, 2007).

A fundamental condition for the use of EFA is that the observed variables are related, i.e., that there are correlation coefficients equal to at least |.30|. Furthermore, the correlation coefficient must be less than |.90| to avoid collinearity between the observed variables, i.e., variables with such high correlations that they can be assumed to measure the same aspect of the phenomenon (Tabachnick & Fidell, 2019). It is useful to remember that sample size affects the precision and stability of the correlation coefficient. Larger samples produce estimates that are more stable and better approximate the population value of the correlation, resulting in narrower confidence intervals.

There are two statistical analyses available for investigating the factorability of the correlation matrix: the Bartlett's sphericity test (Bartlett, 1954), and the Kaiser-Meyer-Olkin (KMO; Kaiser, 1974) sample adequacy test.

The *Bartlett's sphericity test* (1954) allows one to check for significant differences between the observed correlation matrix and the identity matrix, which contains ones on the diagonal and zeros off the diagonal and assumes no relationships between variables. A statistically significant Chi^2 value must be obtained.

The *KMO* sample adequacy test compares the correlation matrix with the corresponding partial correlation matrix, where each coefficient between pairs of variables has been purified with respect to the effects of all other variables. In the partial correlation matrix, high magnitude coefficients would indicate that the relationships among variables are not adequately accounted for by common latent factors. This comparison detects the extent to which correlations are a function of the variance shared across all variables rather than the variance shared by particular pairs of variables. The test detects if individual bivariate coefficients remain high in magnitude when the shared variance attributable to the common latent factor is low (e.g., individual pairs of observed variables may be correlated without reflecting a common factor underlying multiple variables). KMO can be calculated at both the overall and item levels. KMO values range from zero to 1, with high values indicating sample adequacy according to the following cut-off values: greater than .90, excellent value; between .80 and .90, meritorious; between .70 and .80, acceptable; between .60 and .70, mediocre; and lower than .60, poor (Kaiser, 1974).

Application of the Exploratory Factor Analysis

The application of the EFA method requires that, given an adequate correlation (covariance) matrix between observed variables, a factor structure be identified through the following steps: (1) factors estimation and extraction, (2) determination of the number of factors to be extracted in the final solution and estimation of factor loadings, (3) factors rotation, and (4) interpretation of the factors, i.e., definition of their nature, and assessment of the factorial solution (e.g., Fabrigar et al., 1999).

Factors Estimation and Extraction. Given an adequate correlation matrix, EFA first involves extracting latent factors. This process implies the identification of factors, fewer in number than the observed variables, that allow the correlations between the variables to be represented parsimoniously. The goal is to estimate the relationships between the observed variables and the factors (i.e., factor loadings, regression coefficients, of the measured variables on the factors) that replicate the observed correlation matrix as closely as possible. Various estimation methods are available, each with different assumptions about the level of measurement and/or normality of the observed variables' distribution scores.

Principal Factors (Principal Axis Factoring) Method. This method is one of the most frequently used methods for factor extraction, and it does not make any requests regarding the normality of the distribution. This is an iterative method that allows the identification of factors, orthogonal to each other, that explain the greatest amount of common variance among the observed variables. The Principal Factors analyzes the correlation matrix between the observed variables, with an estimate of the communality placed on the positive diagonal. Starting estimates of communality are made based on the squared multiple correlation coefficients of each observed variable with respect to all the others.

First, the factor that accounts for the largest amount of common variance among the observed variables is extracted. Through an iterative process, the second orthogonal factor is then extracted from the residual matrix. This process continues until no further extraction is possible. The proportion of common variance explained by a factor is calculated by dividing its eigenvalue (defined as the amount of common variance in the observed variables that is accounted for by a factor) by the total communality (equal to the sum of the common variance, communality, of each observed variable or the sum of the eigenvalues of all the extracted factors) and multiplying the result by 100. Since the objective of the Principal Factors is to explain the maximum amount of common variance, it does not aim to reproduce the total variance of the correlation matrix (equal to the number of the observed variables).

Maximum Likelihood (ML) Method. This method is particularly appropriate when the observed variables are multivariate normally distributed. The ML method allows us to identify the factors whose saturation coefficients (i.e., linear regression coefficients) maximize the likelihood of reproducing the observed correlation matrix given a specific factor model. The correlation matrix is analyzed directly rather than being replaced by initial communality estimates. Since communalities and unique variances are estimated directly as model parameters, an estimate of communality is not required. However, it assumes that the number of factors to be extracted, the relation between them, and the observed variables that saturate each factor are fixed a priori.

The ML method is very useful because it provides information on the statistical significance of the estimated model parameters in the factorial solution (correlation coefficients both between observed variables and factors, i.e., factor loadings or saturations, and between factors). It also provides indices of goodness of fit of the identified factor model in explaining and representing the empirical data: chi-square statistic (χ^2), comparative fit index (CFI), root-mean-square error of approximation (RMSEA) with associated 95% confidence intervals (CI), and standardized root-mean-square residual (SRMR) (Schermelleh-Engel et al., 2003). Values higher than 0.95 for CFI, smaller than 0.05 for RMSEA, and smaller than 0.08 for SRMR suggest a reasonable fit (Hu & Bentler, 1999; Kline, 2023; Schermelleh-Engel et al., 2003). Indices of goodness and indications of statistical significance (with CI) are not provided by the Principal Factors method (Lawley & Maxwell, 1963).

Least-Squares Methods. This is a family of methods used to extract factors by minimizing the difference between observed and reproduced correlation matrices. The only difference between them is in the way residuals are weighted.

Unweighted Least Squares (ULS) assigns equal weight to all residuals between observed and reproduced correlations, regardless of differences in their sampling variability. Estimation focuses exclusively on the off-diagonal elements of the correlation matrix, and communalities are not estimated directly but are derived from the final solution (Comrey & Lee, 1992).

Because ULS does not rely on distributional assumptions for the observed variables, it is particularly well suited for ordinal and dichotomous indicators and for data that substantially violate the assumption of multivariate normality. ULS performs well when applied to polychoric or tetrachoric correlation matrices, even under severe skewness or kurtosis. Its robustness makes ULS especially advantageous in small to moderate samples, where sampling variability and non-normality may compromise more statistically efficient but less robust estimation methods.

However, because ULS does not account for differences in the estimation precision of correlations, that is, it does not explicitly model the variances of correlations or the covariances among residuals, it yields consistent but not asymptotically efficient parameter estimates; consequently, even in very large samples, its estimates are not maximally precise, and reliable global fit statistics are generally unavailable.

Full Weighted Least Squares (WLS) weights residuals by the inverse of their asymptotic variances and covariances, using the full inverse asymptotic covariance matrix of the observed correlations as the weight matrix (i.e., it uses both the diagonal variance terms and the off-diagonal covariance terms; Browne, 1984). By explicitly accounting for differences in the precision with which correlations are estimated, full WLS can be asymptotically efficient when its regularity conditions are met and the asymptotic covariance matrix is accurately estimated.

This method is appropriate for approximately continuous non-normal variables, or for ordinal/dichotomous indicators and polychoric/tetrachoric correlations, and under conditions in which stable and accurate estimates of the asymptotic covariance matrix are available. These conditions typically require very large sample sizes. In practice, however, obtaining a stable and well-conditioned estimate of this matrix typically requires very large samples, especially when correlations are based on ordinal/dichotomous indicators (e.g., polychoric/tetrachoric correlations).

Diagonally Weighted Least Squares (DWLS) represents an intermediate approach that explicitly addresses violations of normality while retaining some efficiency gains associated with weighted estimation (Flora & Curran, 2004). Like full WLS, DWLS weights residuals according to the estimated sampling variances of the correlations; however, it relies only on the diagonal elements of the weight matrix and ignores covariances among residuals. This simplification substantially improves numerical stability and reduces sample-size requirements, as DWLS avoids the need to estimate and invert a large and potentially ill-conditioned weight matrix. Consequently, DWLS is well suited for ordinal or dichotomous variables analyzed using polychoric or tetrachoric correlations, as well as for continuous data exhibiting substantial departures from univariate and multivariate normality. Although slightly less efficient than full WLS under ideal conditions, DWLS provides a favorable balance between robustness and efficiency in most applied psychometric contexts.

In practice, the various extraction methods tend to yield broadly comparable solutions, and the choice among them should be guided by the nature of the data, the extent of distributional violations, and the inferential needs of the researcher. If the factorial solution is stable, the use of different extraction methods does not lead to significant differences; it is therefore recommended to use different extraction methods and verify that there are no differences between the solutions (Fabrigar et al., 1999).

Determination of the Number of Factors to Retain and Estimation of the Factor Loadings of the Observed Variables. With EFA, as many factors can be extracted as there are observed variables (obtaining the reproduced correlation matrix, which is the best approximation to the real observed correlation matrix). However, since the objective of EFA is to provide a parsimonious representation of the correlations between the observed variables, it is necessary to extract fewer factors than the number of observed variables. The number of factors to be extracted must be decided taking into account that we want to achieve a solution that is both parsimonious (i.e., consisting of a relatively small number of factors) and plausible, truthful (i.e., consisting of a sufficient number of factors to explain the covariance between the variables). Various methods have been proposed for this purpose.

Kaiser Criterion (Guttman, 1954; Kaiser, 1960). It has been among the most widely used, despite not being the best method available (Fabrigar et al., 1999). According to this criterion, given the complete correlation matrix between the observed variables, with variance equal to 1 on the positive diagonal, all factors with eigenvalues (i.e., the amount of total variance explained by each factor) greater than 1 are retained. In other words, all the factors that explain a greater amount of total variance than that of a single observed variable (which is equal to 1) must be retained. However, since EFA aims to explain common variance, not total variance, this method should be applied to the reduced matrix (with variable communalities instead of variance equal to 1 on the positive diagonal). For this reason, its use in EFA has been questioned, as it is based on total variance rather than common variance, which leads to an overestimation of the number of retained factors.

Scree Test of Eigenvalues (Cattell, 1966). According to this criterion, the eigenvalues of the factors extracted from the analysis of the reduced matrix must be represented in a line graph where the x-axis shows the factor number in descending order of eigenvalue and the y-axis shows the magnitude of the eigenvalue. The point at which the line changes slope indicates the number of factors that can be extracted. Only the factors above this point should be extracted. Although subject to subjective judgments (Fabrigar et al., 1999), it seems to be quite useful in identifying the range of factors to be extracted (especially in optimal cases of high communality and at least 4 variables per factor).

Parallel Analysis (Horn, 1965). According to this method, factors are retained when their eigenvalues are greater than those obtained by EFA in random simulations of correlation matrices with the same number of variables, sample size, and observed data. This method appears to be one of the best (Fabrigar et al., 1999).

As in other cases, the reduced matrix must be analyzed, and the eigenvalues must be relative to the common variance, not the total variance.

Minimum Average Partial correlation (MAP) (Velicer, 1976). This procedure evaluates a sequence of partial correlation matrices obtained after successively partialling out the variance accounted for by the extracted factors from the observed correlation matrix, and at each step computes the average squared partial correlation among variables. The optimal number of factors is identified at the point where this average squared partial correlation reaches its minimum, indicating that most common variance has been extracted and that the remaining associations largely reflect unique variance.

Comparison of the Goodness of Factorial Solutions with Progressively Higher Number of Factors. When using the ML extraction method to identify the number of factors to be extracted, it is necessary to verify the goodness of factorial solutions with a progressively higher number of factors (from zero) and by comparing the goodness of fit indices. Since the compared factorial solutions are nested (i.e., the factorial solution with fewer factors can be obtained from the factorial solution with more factors by imposing constraints), the following differences in their fit indexes may be employed as criteria for comparing model fit: Δ CFI, Δ RMSEA, and Δ SRMR. The minimum difference thresholds between indices that indicate statistical significance and allow us to identify the most parsimonious model (i.e., the solution with the lowest number of factors) with the best fit indexes are a Δ CFI of 0.01, a Δ RMSEA of 0.015, and a Δ SRMR of 0.01 (Kline, 2023).

Analysis of the Matrix of Standardized Residuals between Observed and Reproduced Correlations. Since the goal of EFA is to represent the correlations between observed variables in a parsimonious way, another method consists of extracting a progressive number of factors until the standardized residuals between the observed and reproduced correlation matrices are minimal, well distributed, and less than $|.10|$.

The most effective strategy for determining the optimal number of factors to extract is to utilize diverse methods to delineate the range of factors and to subsequently undertake the following steps of the EFA to compare the various solutions that may be obtained. Indeed, only by rotating the factors and then identifying the simplest solution and interpreting the nature of the factors will it be possible to identify the final number of factors (e.g., Balboni et al., 2018). Determining the number of factors cannot be based solely on the application of statistical strategies, but also requires an analysis of the solution based on the criteria of simplicity (factorial solutions with variables that saturate a single factor and with high magnitude are preferable) and interpretability of the solution (solutions whose factors can be defined in an unambiguous and theoretically plausible way are preferable). This analysis can only be performed after the factors have been rotated.

Unrotated Factor Loading Matrix. The extraction of a number of factors allows the production of the unrotated factor loading matrix, in which the factor loadings (i.e., the saturations, the linear regression coefficients of the observed variables with respect to each extracted factor) are reported for each observed variable. In the case of the unrotated factorial solution, these coefficients correspond to the correlation coefficient between the observed variables and the factors.

As will be seen below, the definition of the nature of the factors, their interpretation, and the definition of the dimensions of the construct represented by each factor is achieved by identifying for each factor the observed variables that have high linear regression coefficients with that factor (at least .32, which corresponds to 10% of the common variance accounted for by the factor). As can be seen in a matrix of unrotated factors, there may be several cases of observed variables loading more than one factor. This makes the interpretation of the factorial solution complex. In contrast, in order to interpret the factors, i.e., to identify their nature and define the dimension of the phenomenon they represent, the solution must be simple (Thurstone, 1947). This means the solution should consist of observed variables that load highly on only one factor and factors loaded by a few observed variables with high factor loading magnitude for only that factor.

For this reason, the unrotated factorial solution must be rotated in the factor space, which has as many dimensions and axes as the extracted factors, to identify the simplest solution. The factors of the factor space (i.e., the axes) are rotated to bring them closer to the location of the observed variables. Since the factors are closer to the observed variables, the space between them is quite empty, and the solutions become simpler. The goal of rotation is precisely to maximize some high loadings and minimize others to achieve simple structure. The simplest solution is the one with the highest number of observed variables that saturate only one factor and the highest number of factors that are highly saturated by a defined number of observed variables.

Factors Rotation. To obtain the simple structure, the rotation constrains the factors to be uncorrelated (orthogonal rotation) or allows the factors to be correlated (oblique rotation). In both cases, the rotation process redistributes the common variance across the individual factors. Compared to the unrotated solution, the rotated factorial solution shows factor loadings of different magnitudes and a different distribution of common variance across factors. What does not vary, however, is the total common variance explained by the factor solution (i.e., the sum of the total common variance accounted for across all retained factors), as well as the communality of the observed variables, and the total communality. In other words, what does not vary is the goodness of the factorial solution in representing the phenomenon, i.e., the correlations between the observed variables.

Orthogonal Factors Rotation. Different methods of orthogonal rotation are available that differ in how they pursue simple structure across variables and factors. These methods include Varimax, Quartimax, and Equamax.

The *Varimax* rotation (Kaiser, 1958) is the most well-known and widely used method (Fabrigar et al., 1999). This method aims to optimize the simplicity of the solution by maximizing the magnitude of the factor loadings within each factor. High saturations increase, while low saturations decrease. The Varimax method produces factors with high factor loadings on a small set of variables. The common variance is redistributed across factors so as to achieve a clearer separation of variables within each factor. The Varimax method tends to produce well-differentiated factors. Therefore, it is useful when the factors are assumed to be truly independent.

The *Quartimax* rotation (Carroll, 1953; Harman, 1976) aims to simplify the solution by maximizing the magnitude of the factor loadings within each observed variable, i.e., across factors for each observed variable. Typically, a strong general factor emerges because many observed variables are pushed toward a common factor, sometimes at the expense of clear differentiation among specific factors. Therefore, it is useful when exploring the existence of a general dimension.

The *Equamax* rotation (Kaiser, 1958) is a compromise between the Varimax and Quartimax objectives, seeking a middle ground between simplifying the factor loadings across factors (columns) and simplifying the factor loadings across observed variables (rows). Consequently, the solutions obtained typically exhibit a moderate degree of factor distinctiveness and a moderate tendency toward a general factor, without a pronounced emphasis on either aspect as observed in Varimax or Quartimax analyses. Though rarely preferred, it is sometimes helpful when factors have similar common variances.

The orthogonal factors rotation produces the *Factor Matrix*, which contains the factor loadings of the observed variables, i.e., the linear regression coefficients of each variable with respect to each rotated factor. In the case of orthogonal rotations, as well as for the unrotated solution, these saturation coefficients correspond to the correlation coefficients between the observed variables and the factors.

A comparison of this matrix with one obtained using unrotated factors reveals that the rotation has simplified the factorial solution. Each observed variable tends to saturate only one factor, and each factor tends to be saturated by a limited number of variables. This simplification of the solution enables the next step: defining the nature of the factors and interpreting them.

Oblique Factors Rotation. Similar to the case of orthogonal rotation, the factors are rotated in the factor space in order to make the solution as simple as possible. However, unlike with an orthogonal rotation method, the rotations performed to identify the simplest solution are not constrained to be orthogonal (i.e., do not have to be 90 degrees). In other words, the rotation yielding the simplest solution is identified without any constraints on the angle of the rotation. Various methods are available, such as Promax and Direct Quartimin. Some methods require defining the maximum degree of obliquity by setting a parameter. If the solution is stable, the use of different rotation methods does not cause significant differences (Fabrigar et al., 1999).

Direct Oblimin is an oblique rotation method that minimizes cross-loadings and estimates factor correlations in one step (Jennrich & Sampson, 1966). The extent to which factor correlations are permitted is governed by the parameter δ. When $\delta = 0$ (the default setting), moderate correlations are permitted. When $\delta > 0$, more orthogonality is encouraged. When $\delta < 0$, stronger factor correlations are encouraged. Because the rotation is performed directly on the loading matrix, Direct Oblimin provides stable and interpretable simple-structure solutions, particularly when moderate to substantial correlations among latent factors are expected. It is theoretically principled but may present issues in computational efficiency.

Promax begins with an orthogonal Varimax solution. The Varimax loadings are then transformed by raising them to a specified power (typically $\kappa = 4$) to construct a target matrix that accentuates salient loadings and attenuates trivial ones. Subsequently, an oblique Procrustes rotation is applied to rotate the original loading matrix toward this target, yielding a simple-structure solution while allowing for nonzero correlations among factors (Hendrickson & White, 1964). Because Promax does not directly optimize an oblique rotation criterion, but instead approximates simple structure through transformation of an orthogonal solution, it is considerably faster and numerically more stable in large datasets. It prioritizes computational efficiency and scalability.

When sample sizes are small to moderate and careful interpretation of factor correlations is central, Direct Oblimin is often recommended. Conversely, Promax is well suited for large samples and high-dimensional solutions, where computational burden and convergence issues may arise (Costello & Osborne, 2005; Fabrigar et al., 1999).

In all cases, however, since the factors are allowed to be related to each other, the relationship of each observed variable with each factor is an expression of both the direct relationship of the variable with that factor and the indirect relationship of the variable with all other factors, which is conveyed by virtue of the correlation of the factor with all the others. For this reason, performing an oblique rotation involves producing two distinct matrices: the Structure Matrix, which contains the correlation coefficients between the observed variables and the factors, and the Pattern Matrix, which contains the factor loading, i.e., the linear regression coefficients of the variable with the factor.

Structure Matrix. The reported correlation coefficients between the observed variables and the factors express the overall relationship of the variable with the factor, or the strength of the association between the factor and the variable, as well as the indirect relationship of the variable with all other factors. The latter is how much all other factors, by virtue of their relationship with the factor in question, contribute to explaining the variable, or the relationship of the variable with other factors conveyed by the relationship between factors.

Pattern Matrix. Reported factor loadings (i.e., saturation or linear regression coefficients of a variable with a factor) express the relationship between a variable and a factor by removing indirect relationships of the variable with other factors, as expressed through factor correlations. This matrix corresponds to the Factor Matrix with correlations between variables and factors obtained with unrotated and orthogonally rotated factorial solutions.

In addition, a *Correlation Matrix* with the correlation coefficients between the factors is also produced.

The Structure Matrix is equal to the product of the Pattern Matrix and the Correlation Matrix. As the relationships between the factors increase, the correlation coefficients tend to be higher than the corresponding linear regression coefficients. As will be seen below, the interpretation of the factors is based on both the structure and pattern matrices.

The oblique rotation method allows factors to be related if they are, but does not force factors that are not related to be related. Consequently, it is the most appropriate method to use. Indeed, if the simplest factorial solution involves related factors, this method will allow it to be identified, while if it involves unrelated factors, the solution identified will be similar to that obtained with an orthogonal rotation method. Consequently, there is agreement in suggesting that oblique rotation should always be used as the first rotation and only if the correlations between the factors are less than |.32| should an orthogonal rotation be performed (Fabrigar et al., 1999).

The complexity of psychological phenomena makes it unlikely, or at least rare, that dimensions of the same construct, i.e., latent factors, are unrelated to each other. Therefore, it is considered wrong to use an orthogonal rotation without checking whether there is a relationship between factors (e.g., Fabrigar et al., 1999; Henson & Roberts, 2006).

Interpretation of Factors and Assessment of the Factorial Structure. Given the Factor Matrix (also labeled as factor loading matrix) obtained from an orthogonal rotation, as well as the two Pattern and Structure Matrices obtained from an oblique rotation, it is necessary to interpret the factors and assess the factorial solution.

Interpreting the factors involves identifying which dimension of the construct or psychological phenomenon each factor represents. To do so, the observed variables with high loading for each factor must be identified. Then, based on the psychological content they are intended to measure, the nature of each factor must be defined.

However, since different factorial solutions may be produced for a given phenomenon, based on the simple factor criteria, the simplest solution must be detected. To this end, given the range of factors defined by applying different methods to determine the number of factors, it is necessary to compare factorial solutions with different numbers of extracted factors (e.g., Balboni et al., 2018). Each of these solutions should be rotated obliquely, and possibly orthogonally if there is no correlation between factors, to identify the optimal solution.

The optimal solution is parsimonious, simple, and easily interpretable. Therefore, we must determine the solution with the fewest factors identified and saturated by a limited number of observed variables that saturate only that factor and whose nature is easily identifiable. In other words, parsimony, simplicity, and plausibility in defining factors guide us in determining the number of factors to extract within the range determined by applying different methods.

Different criteria should be applied to detect the simplest factorial solution among those produced.

1. *Magnitude and reliability of factor loadings of the observed variables.* Each factor must have at least three observed variables with a primary loading $\geq |.32|$ (i.e., approximately 10% of explained common variance; overdetermined) and secondary loadings $< |.32|$. In the case of an orthogonal rotated factorial solution, the Factor Matrix (in which the Pattern and Structure Matrices coincide) must be inspected. For factorial solutions with oblique rotation, the Pattern Matrix is generally interpreted, although the Structural Matrix must also be considered to ensure that anomalous results were not produced. For instance, a large pattern coefficient and a low structure coefficient could indicate a weak direct relationship of the variable with the factor, accompanied by stronger indirect associations with other correlated factors. Conversely, a small pattern coefficient and a large structure coefficient could indicate a variable that is strongly influenced by other factors (Graham et al., 2003). The obtained factor loadings should be reliable, meaning they are unlikely to be the results from sampling error. Larger sample sizes allow for a more confident interpretation of smaller loadings, whereas small samples require more conservative thresholds (MacCallum et al., 1999). When available, confidence intervals or bootstrap estimates may be used.
2. *Simple factorial solution.* Ideally, observed variables should exhibit a simple structure, loading strongly on one factor and weakly on all others. A variable should be excluded if it exhibits cross-loadings unless the cross-loadings are theoretically plausible.
3. *Observed variable communality.* Each observed variable must have a communality greater than .20 (i.e., variance accounted for by the factorial solution).

Adequate communalities indicate that a variable is well explained by the retained factorial solution.

4. *Observed variable-total score correlation.* The correlation between each observed variable score and the total score (corrected, i.e., not considering the score of the observed variable) should be at least .30 and less than .70 to avoid redundancy.
5. *Factor reliability.* Cronbach's alpha, or the corresponding ordinal version (Zumbo et al., 2007), can be used for an observed variable at the interval or ordinal level (e.g., a Likert scale with a maximum of four steps) if the items form an essentially unidimensional scale and the assumption of approximate tau-equivalence is valid. This occurs when the items have similar factor loadings and contribute equally to the latent construct. McDonald's omega is preferable when the loadings differ substantially (McNeish, 2018; Revelle & Condon, 2019). In both cases, reliability estimates of .70 or higher are generally considered acceptable for newly developed scales. Higher thresholds (.80-.90) are recommended for more demanding research applications or high-stakes decisions (Nunnally & Bernstein, 1994).
6. *Factor stability.* According to Stevens (1996), Guadagnoli and Velicer (1988), factor solution stability depends not only on sample size and the participant-to-variable ratio, but also on component saturation (the magnitude of the factor loadings) and the number of variables per factor. Below is a breakdown of the stability criteria:
   (a) High Saturation: A factor is considered highly stable across replicated samples, regardless of the number of indicators, even with a sample size as small as 50 participants, if at least three variables have loadings in the |.80| range.
   (b) Moderate Saturation: A factor is considered highly stable across replicated samples of at least 150 participants if the factor contains at least four variables with loadings greater than |.60|.
   (c) Low Saturation: A factor can still be considered stable with a sample size of 150 participants if it is "overdetermined", meaning it has at least 10 variables loading on it with a magnitude of |.40| or higher.

   If a factor is defined by a few variables with low loadings around .40, the required sample size to achieve stability increases to 300–400 participants.
7. *Factorial solution interpretability.* The factor solution must be interpretable and coherent with the theoretical background. To this end, the dimensions of the constructs assessed by each factor must be defined. The nature and name of each factor should be based on the observed variables that comprise it and the magnitude of their saturation (.30 is minimal, .40 is important, and .50 is high) (Hair et al., 1995). Analyzing the Factor Matrix in the case of an orthogonal rotated factor solution and the Pattern Matrix in the case of an oblique rotated factor solution allows us to identify variables with a saturation greater

than .32 that contribute to the definition of the factor. Additionally, examining the Structure Matrix and comparing it with the Pattern Matrix enables us to pinpoint variables that indirectly contribute to defining the factor through other factors.

8. *Factor validity.* Evidence of discriminant validity with respect to other factors must be investigated. To this end, a study should be conducted to investigate the relationships between these factors and the other constructs, i.e., external criteria, with which the reference theory indicates they have varying degrees of association. The goal is to verify that the observed relationships align with the hypothesized ones.

Once the optimal factorial solution has been identified, it is necessary to describe its goodness in reproducing the correlations between the observed variables. To this end, the following must be indicated:

- *Communality of the observed variables*, i.e., the variance of the observed variables accounted for by the latent factors. Communality is calculated differently depending on whether the rotation has been orthogonal or oblique. In the case of orthogonal rotation, the communality of each observed variable is calculated from the Factor Matrix (i.e., the Pattern/Structure Matrix) and is equal to the sum of the loading coefficients squared. In the case of oblique rotation, on the other hand, it is necessary to multiply the Pattern Matrix by the Structure Matrix element by element in order to obtain a new matrix that expresses the direct and indirect effects of the factors on the variables. The communality is then equal to the sum of the coefficients obtained in this matrix.
- *Total communality*, i.e., the common variance expressed by the factorial solution, is equal to the sum of the communalities of each observed variable. Total communality does not change as a result of the rotation of the solution.
- *Common variance explained by each factor* (changes following rotation and is calculated differently depending on whether the rotation is orthogonal or oblique). In the case of orthogonal rotation, it is equal to the sum of the loading coefficients squared divided by the number of variables*100. In the case of oblique rotations, it is necessary to multiply the Pattern Matrix by that Structure Matrix, element by element in order to obtain a new matrix that expresses the direct and indirect effects of the factors on the variables. Given this matrix, the common variance explained by each factor is equal to the sum of these coefficients (not squared) divided by the number of variables*100. In the case of non-rotated solutions, the eigenvalue is equal to the sum of the loading coefficients squared; the common variance explained by the factor is equal to this value divided by the number of observed variables*100.
- *Total common variance* is equal to the sum of the common variance explained by each factor; multiplied by the number of variables, it is equal to the total communality.

Factor scores are the scores that each individual would have achieved in each factor if the factor had been measured directly. Various methods are available to determine these factor scores, but various criticisms have been made regarding the use of these scores, to the point of suggesting that they should not be used (Fabrigar et al., 1999).

Example of Exploratory Factor Analysis for Ordinal Variables Using R

Here, we provide a step-by-step example of how to perform an EFA in R using the Psych package (Revelle, 2025). The data and results presented in this chapter have been modified and simplified. They are provided solely for illustrative purposes to guide readers through the main decisions and procedures involved in EFA. Accordingly, the reported numerical results should not be interpreted as empirical findings.

We used data obtained from a scale designed to measure the level of agreement with the ethical principle of Protecting Children's Data and Privacy in the context of AI-based telerehabilitation for children with neurodevelopmental disorders (Castellani et al., 2024). This principle concerns four ethical requirements: Consent; Data Agency; Privacy; Data Quality and Auditable Use. The scale has 12 items (three for each requirement), labeled Q01–Q12 in the dataset, which describe how the ethical principle can be applied to AI-based telerehabilitation. Professionals involved in designing, using, or evaluating AI-based telerehabilitation systems responded to each item on a four-step Likert scale (1 = strongly disagree; 4 = strongly agree).

Each requirement addressed a distinct aspect of protecting children's data and privacy. However, the underlying theoretical assumption was that all requirements reflected a single, overarching principle. Therefore, the EFA was implemented with the objective of evaluating whether the empirical structure of the collected data supported this unidimensional hypothesis.

Participants. Initially, the sample size was evaluated. Following the exclusion of outliers and the assurance that no missing data were present in the items, i.e., observed variables, the final dataset comprised 172 participants. To assess the adequacy of this sample size for EFA, the ratio between participants and observed variables was considered. This was equivalent to 172 participants/12 variables, yielding an approximate ratio of 14:1. This ratio exceeded the thresholds commonly recommended in the literature. Specifically, it exceeded the minimum ratio of 3:1 or 6:1 (Cattell, 1978) and the minimum of 5–10 participants per variable (Stevens, 1996). Consequently, the number of participants could be regarded as sufficient. However, this evaluation is based on a priori criteria. A more precise assessment of the adequacy of the sample size becomes possible once the most appropriate factorial solution has been selected.

Factorability of the Correlation Matrix. Given that the items were measured on a 4-step Likert scale, a polychoric correlation matrix was employed, as this is more appropriate for ordinal scales (Holgado-Tello et al., 2010). The factorability was assessed using the following criteria: 1) multiple inter-item correlations $\geq |.30|$; 2) no evidence of collinearity, commonly operationalized as correlations exceeding $|.90|$ between items; 3) a significant Bartlett's test of sphericity (Bartlett, 1954); and 4) a Kaiser–Meyer–Olkin (KMO) index, both overall and at the item level, interpreted as: >.90, excellent value; between .80 and .90, meritorious; between .70 and .80, acceptable; between .60 and .70, mediocre; and < .60, poor (Kaiser, 1974).

The following R code illustrates the steps used to assess the factorability assumptions prior to conducting an EFA.

```

# ==========================================
# Step 1 - Load required packages
# ==========================================
# Run this installation line only the first time:
install.packages(c("haven", "psych"))

# If your dataset is in Excel (.xlsx) format instead of SPSS:
# install.packages("openxlsx")
# library(openxlsx) # use read.xlsx() to import Excel files

library(haven)  # load haven (to read SPSS .sav files)
library(psych)  # load psych (to compute correlations and
    factor analysis tools)

# ==========================================
# Step 2 - Import the dataset
# ==========================================
file_path <- "path/to/your/dataset.sav"  # path to the SPSS
    file

full_data <- read_sav(file_path) # import and read the SPSS
    dataset into R as the object full_data

# At this point, 'full_data' may contain many variables.
# In this example, we focus only on the items of our scale [Q01
    :Q12].

# ==========================================
# Step 3 - Select the items
# ==========================================

efa_data <- data.frame(
  Q01 = full_data$Q01,
  Q02 = full_data$Q02,
  Q03 = full_data$Q03,
```

```
  Q04 = full_data$Q04,
  Q05 = full_data$Q05,
  Q06 = full_data$Q06,
  Q07 = full_data$Q07,
  Q08 = full_data$Q08,
  Q09 = full_data$Q09,
  Q10 = full_data$Q10,
  Q11 = full_data$Q11,
  Q12 = full_data$Q12
) # create a new dataframe named efa_data including only the
    scale items

N <- nrow(efa_data) # check the number of participants (rows)
print(N)  # print number of participants in the console. This
    should match the sample size used in the study

# ==========================================
# Step 4 - Prepare data for polychoric correlations
# ==========================================
# In this example, each item is a 4-step Likert response.
# We need to ensure that the responses are treated as ordinal
    categories (1 - 4).

efa_data <- as.data.frame(
  lapply(
    efa_data,
    function(x) ordered(as.numeric(x), levels = c(1, 2, 3, 4))
  )
)

# ==========================================
# Step 5 - Polychoric correlation matrix
# ==========================================

pc <- psych::polychoric(efa_data) # compute polychoric
    correlations
R  <- pc$rho  # this is the polychoric correlation matrix named
     R

rownames(R) <- colnames(R) <- colnames(efa_data)  # make sure
    row and column names of R match the item names: assign item
     names to rows/columns

round(R, 2) # show the correlation matrix rounded to 2 decimals

# ==========================================
# Step 6 - Bartlett's Test of Sphericity
# ==========================================

bart <- psych::cortest.bartlett(R, n = N) # compute Bartlett's
    test of sphericity
```

```
print(bart) # print test results

# ==========================================
# Step 7 - Kaiser-Meyer-Olkin (KMO) index
# ==========================================

kmo <- psych::KMO(R) # compute Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy
print(kmo) # print KMO values in the console
```

The analysis showed that most inter-item correlations were $\geq |.30|$, with none exceeding $|.90|$, indicating the absence of multicollinearity. Bartlett's test of sphericity was significant ($\chi^2_{(66)} = 1165.22, p < .001$). The KMO index indicated adequate sampling overall (.88), with acceptable to excellent values at the item level (.79 –.92). Overall, the evaluated criteria confirmed that the correlation matrix was suitable for EFA.

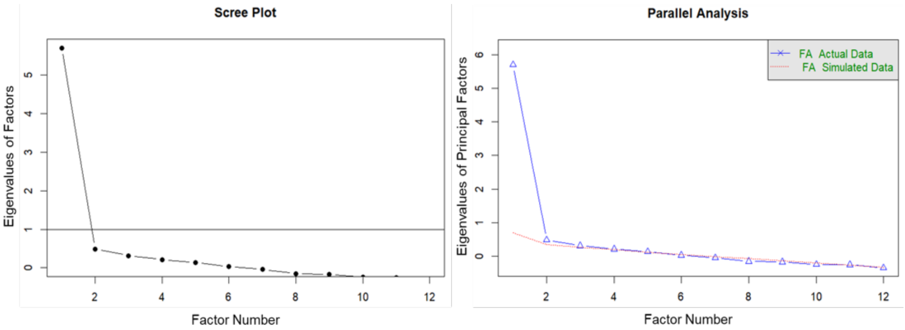
Application of the Exploratory Factor Analysis. The next step was to select an appropriate factor extraction method. For this analysis, we used unweighted least squares (ULS), which is particularly suitable for ordinal data (Comrey & Lee, 1992). Then, we determined the number of factors to extract by applying two criteria jointly: the scree test of eigenvalues (Cattell, 1966) and parallel analysis. For the latter, we used the 95$^{\text{th}}$ percentile of the eigenvalues from randomly simulated correlation matrices as the decision threshold (Horn, 1965; Watkins, 2005).

The following R code illustrates the procedures that were implemented to conduct both the scree test and the parallel analysis.

```

# ==========================================
# Step 8 - Determine the number of factors (Scree Test and Parallel Analysis)
# ==========================================

# 8a) Scree plot based on the polychoric correlation matrix
# The plot will appear in the graphics device (Plots pane)

psych::scree(
  R,              # polychoric correlation matrix used as input
  factors = TRUE, # show eigenvalues from factor analysis
  pc     = FALSE, # do not show eigenvalues from principal components
  main   = "Scree Plot" # title of the plot
)

# 8b) Parallel Analysis with ULS extraction ---
# The plot will appear in the graphics device (Plots pane)

```

```
pa <- psych::fa.parallel(
  R,              # polychoric correlation matrix used as input
  n.obs = N,      # number of observations (sample size)
  fm   = "uls",     # extraction method: Unweighted Least Squares
  fa   = "fa",      # type of analysis: factor analysis
  quant = .95,    # percentile used to define the random
    eigenvalue cutoff
  main   = "Parallel Analysis" # title of the plot
)
```

The criteria used to determine the number of factors did not converge. Specifically, the parallel analysis suggested retaining five factors, whereas the scree test indicated a two-factor solution, with an elbow between the second and third eigenvalues (see Figure 5).



5. Scree Plot and Parallel Analysis based on the Polychoric Correlation Matrix.

In light of the discrepancy between the results of the scree test and the parallel analysis, an examination of alternative factor solutions was undertaken. Accordingly, factor solutions ranging from five to one factor were considered. The five-factor solution was included because the parallel analysis suggested it, whereas the one-factor solution was tested in line with the theoretical assumption of one dimension underlying the scale.

For all solutions, we applied the oblique Oblimin rotation method, which allows factors to correlate (Fabrigar et al., 1999; Henson & Roberts, 2006). Even if the ethical requirements were to emerge as distinct factors, it would still be theoretically reasonable to expect them to be associated, as they represent different requirements of the same overarching principle related to Protecting Children's Data and Privacy.

Each factor solution was evaluated according to the following criteria: (1) at least three items per factor with primary loadings $\geq |.32|$ and secondary loadings $< |.32|$ in the Pattern Matrix; (2) extracted communalities $\geq .20$ for each item; and (3) a simple and theoretically interpretable factor structure (Costello & Osborne, 2005; Slocum-Gori & Zumbo, 2011; Tabachnick & Fidell, 2019; Thurstone, 1947).

The R code below illustrates how to implement these analyses, estimating the one-to five-factor solutions.

```r

# ==========================================
# Step 9 - EFA solutions (from 5 to 1 factors)
# ==========================================

efa_results <- list()       # list to store all models
for (nf in 5:1) {  # loop from 5 to 1 factor (testing multiple solutions)

  cat("\n=====================================\n")
  cat(" FACTOR SOLUTION WITH", nf, "FACTORS\n")
  cat("=====================================\n")

  model <- psych::fa( # run EFA on the polychoric correlation matrix
    R,                 # polychoric correlation matrix
    nfactors = nf,     # number of factors to extract
    n.obs = N,       # sample size
    fm    = "uls",   # Unweighted Least Squares extraction method
    rotate   = if (nf == 1) "none" else "oblimin" # oblique Oblimin rotation was applied to solutions with two or more factors; no rotation was applied to the one-factor solution
  )
  efa_results[[paste0("F", nf)]] <- model  # save model in a list

  # --- Pattern Matrix ---
  cat("\nPattern Matrix:\n")
  pattern <- as.matrix(unclass(model$loadings)) # convert loadings object to numeric matrix
  print(round(pattern, 3)) # print with 3 decimals

  # --- Structure Matrix ---
  cat("\nStructure Matrix:\n")
  structure <- as.matrix(unclass(model$Structure)) # convert to plain numeric matrix
  print(round(structure, 3)) # print with 3 decimals

  # --- Communalities per item ---
  cat("\nCommunalities per item:\n")
  print(round(model$communality, 3)) # print with 3 decimals

  # --- Total communality (sum of communalities per item) ---
  total_comm <- sum(model$communality)
  cat("\nTotal communality:\n")
  print(round(total_comm, 3)) # print with 3 decimals


```

```r
# --- Variance accounted for by the factor solution ---

# The fa() function returns a table (Vaccounted) summarizing
  the variance accounted for by the extracted factors.

print(round(model$Vaccounted, 3))


# --- Variance explained by each factor (%) ---

# In the Vaccounted table, "Proportion Var" represents the
  proportion of total item variance explained by each factor.
   Values are converted to percentages.

cat("\nVariance explained by each factor (%):\n")
print(round(100 * model$Vaccounted["Proportion Var", ], 2))


# --- Total common variance explained (%) ---

# In the Vaccounted table, total common variance explained
  corresponds to the last value of the "Cumulative Var" row
  for multifactor solutions. For the one-factor solution,
  total common variance explained coincides with the value
  reported in the "Proportion Var" row.

cat("\nTotal variance explained (%):\n")
if ("Cumulative Var" %in% rownames(model$Vaccounted)) {
  print(round(100 * tail(model$Vaccounted["Cumulative Var",
  ], 1), 2))
} else {
  print(round(100 * model$Vaccounted["Proportion Var", 1], 2)
  )
}


# --- Common variance explained by each factor (%) ---

# In the Vaccounted table, "Proportion Explained" represents
  the relative contribution of each factor to the total
  common variance explained by the factor solution. In the
  one-factor solution, this quantity is not applicable and is
   therefore not reported.

if ("Proportion Explained" %in% rownames(model$Vaccounted)) {
  cat("\nCommon variance explained by each factor (%):\n")
  print(round(100 * model$Vaccounted["Proportion Explained",
  ], 2))
}

# --- Factor Correlation Matrix (Phi) ---
```

```
  # Phi is only defined when the number of factors is greater
    than 1
  # If nf = 1, model$Phi is NULL. This condition prevents R
    from returning an error

  if (!is.null(model$Phi)) {
    cat("\nFactor Correlation Matrix (Phi):\n")
    print(round(model$Phi, 3))
  } else {
    cat("\nFactor Correlation Matrix (Phi): Not applicable for
    1-factor solution\n")
  }
}
```

The five- to two-factor solutions showed adequate communalities; however, none met the minimum criteria of having at least three items per factor, with primary loadings $\geq |.32|$ and secondary loadings < |.32|. Moreover, these solutions exhibited multiple cross-loadings and did not display a simple, theoretically interpretable structure. In contrast, the one-factor solution (total explained variance = 48%) showed satisfactory primary loadings (range = .48 –.82) and communalities (range = .23 –.67), along with a simple and interpretable structure. Overall, these results supported a unidimensional structure consistent with the underlying theoretical assumption.

Based on the unidimensional solution emerging as the most appropriate from the analysis, it was also possible to further assess the adequacy of the number of participants by referring to the empirical guidelines proposed by Mundfrom and colleagues (2005). According to these guidelines, for a structure with one factor, 12 variables, and communalities ranging from .23 to .67 (i.e., wide communalities; .20 – .80), at least 30 participants are required to obtain a stable estimate of the factor solution. Since the number of participants in this study (n = 172) exceeded this threshold, it could be considered adequate to support the reliability of the factor analysis conducted.

Item-total Correlation, Reliability and Stability. As a final step, the one-factor solution was further evaluated by examining item–total correlations, as well as indices of reliability and factor stability. Corrected item–total correlations were examined to assess the extent to which each item was related to the overall construct, as shown in the R syntax reported below.

```

# ============================================================
# Step 10 - Corrected Item - Total Correlations for the one-
    factor solution
# ============================================================

# The alpha() function from the psych package can be applied
# directly to a correlation matrix.
```

```
# When the input matrix is polychoric, the resulting
    reliability estimates and item statistics are ordinal.

a_ord <- psych::alpha(R, n.obs = N) # R = polychoric
    correlation matrix; n.obs  = sample size
citc_polychoric <- a_ord$item.stats$r.drop # In psych::alpha(),
     the corrected item - total correlation is reported in the
    column called r.drop (item-rest correlation)
print(round(citc_polychoric, 3)) # Print the corrected item -
    total correlations rounded to three decimals

# NOTE: The alpha() function returns a comprehensive set of
    reliability and item - level statistics (e.g., item - total
     correlations, item - rest correlations, alpha if item
    deleted, etc.). To inspect the full output, including all
    available statistics, simply print the entire alpha object:
print(a_ord)
```

All items showed corrected item–total correlations values above .30 and below .70, indicating a satisfactory item–construct relationship (Boateng et al., 2018).

The internal consistency of the unidimensional scale was then assessed using the ordinal alpha coefficient, which provides more accurate reliability estimates for ordinal response data than traditional Cronbach's alpha (Zumbo et al., 2007). Following the guidelines of the European Federation of Psychologists' Associations (2013), the reliability coefficient was interpreted as inadequate when $r < .70$, acceptable when $.70 \leq r < .90$, and excellent when $r \geq .90$. Although ordinal alpha can also be obtained using the psych package in R, several tools are available for this purpose. In the present study, ordinal alpha was computed using the free online tool Ordinal Alpha and Parallel Analysis developed by Olvera Astivia (2018), by uploading the dataset containing the twelve items (see Figure 6).

The results showed that reliability was excellent (ordinal $\alpha = .91$), further supporting the adequacy of the one-factor solution.

Finally, based on Stevens' criteria (1996), the solution could be considered highly stable, as more than four variables loaded on the factor with coefficients greater than |.60|.

Table 23 summarizes the results for the final one-factor solution, including factor loadings, communalities, corrected item–total correlations, total common variance explained, and ordinal alpha.

Information to Include in an Exploratory Factor Analysis Paper/Report

To ensure the rigor, replicability, and transparency of your research, scientific papers reporting on EFA should include the following information:

1. **Study Design and Data Suitability**
    - *Screening data prior to analysis*: Describe the rate of missing data and the han-

psychometroscar

Ordinal Alpha and Parallel Analysis

This shiny app will:

- Give you the polychoric (or tetrachoric, in case of binary data) correlation matrix

- Do Parallel Analysis and a scree plot based on the polychoric (or tetrachoric) correlation matrix

- Calculate ordinal alpha as recommended in:

Zumbo, B. D., Gadermann, A. M., & Zeisser, C.. (2007). Ordinal Versions of Coefficients Alpha and Theta For Likert Rating Scales. *Journal of Modern Applied Statistical Methods, 6,* 21-29.

It currently takes in certain SPSS files (so .sav file extensions from older versions of SPSS, say around 2013 or less), Microsoft Excel files (so .xls file extensions) and comma-delimited files (so .csv extensions). If your data is in none of those files, please change it before using the app (it's super easy), or it will give you an error. Also notice that the app will use ALL of the variables in the file uploaded, so make sure you upload a file that only has the variables (test items in most cases) which you want to correlate/calculate alpha for. You'll need to provide a clean dataset for it to work. So if you have missing values, you'll need to manually remove them before submitting it. If there are outliers, those need to be dealt with before using the app.

Please notice that in accordance to research, if you have 8 (or more) Likert responses, the app will give you an error saying you have enough categories to safely treat your variables as continuous, so you don't really need to use this app. You can see why in Rhemtulla, Brosseau-Liard & Savalei (2012).

YOU CAN CLICK HERE TO ACCESS THE APP

6. Homepage of the Online Tool Used to Compute Ordinal Alpha.

Item	Loading	Communality	Corrected Item - Total Correlation
Q01	0.53	0.28	0.50
Q02	0.82	0.67	0.69
Q03	0.48	0.23	0.45
Q04	0.76	0.57	0.63
Q05	0.81	0.66	0.67
Q06	0.78	0.61	0.64
Q07	0.65	0.42	0.62
Q08	0.70	0.49	0.67
Q09	0.61	0.37	0.58
Q10	0.60	0.36	0.56
Q11	0.80	0.64	0.66
Q12	0.64	0.41	0.62
Total Communality		5.70	
Total Common Explained Variance (%)		48	
Ordinal Alpha		0.91	

23. Exploratory Factor Analysis: Item Loadings, Communalities, Corrected Item-Total Correlations, Explained Variance, and Ordinal Alpha.

dling strategy (e.g., listwise or pairwise deletion, imputation, and rationale). For interval-scale variables, present the results of checking for univariate and multivariate normality of the distribution and for univariate and multivariate outliers among participants. For ordinal items, include response distributions.

- *Justification of variables*: Provide a rationale for the measured observed variables included in the analysis, ensuring they adequately represent the domain of interest.
- *Participant details*: Justify the type and number of participants, confirming that the sample size is adequate.
- *Data characteristics*: Report descriptive statistics of the observed variables.
- *Correlation matrix*: Identify the type of correlation matrix analyzed (e.g., Pearson, polychoric, or tetrachoric) (If space permits provide the complete correlation matrix).
- *Factoriability of the correlation matrix*: Inspect the magnitude of the correlation coefficients, which should be between |.30| and |.90|. Report Bartlett's test of sphericity and the Kaiser–Meyer–Olkin (KMO) measure of sampling adequacy at the level of the observed variables and the total score.

2. **Factor Extraction, Retention, and Rotation**
   - *Extraction method*: Specify the mathematical procedure used to estimate the model (e.g., Maximum Likelihood, Principal Factor, or Ordinary Least Squares).
   - *Retention criteria*: Detail how the number of factors to retain was determined. Best practice recommends using multiple criteria.
   - *Unrotated factor loading matrix*: Report the factor loadings (if space permits) and eigenvalues for all factors. Ideally, include the eigenvalue of the first factor not retained, which allows readers to evaluate the decision.
   - *Rotation method*: Specify the rotation method used (e.g., Varimax for an orthogonal rotation or Direct Oblimin for an oblique rotation) and justify the choice.
3. **Final Factor Solutions and Interpretation**
   - *Pattern and structure matrixes*: Provide the complete Pattern and Structure Matrixes (or the pattern/structure matrix for orthogonal solutions). Do not omit low coefficients.
   - *Salience definition*: Clearly define the cutoff value used to determine a "meaningful" loading (e.g., |.32|).
   - *Communalities*: List the final communality coefficients for each observed variable.
   - *Observed variable-total score correlation*: Provide the observed variable-total score correlation coefficients.
   - *Common variance explained*: Report the percentage of common variance accounted for by each factor and by the factorial solution.

- *Interfactor correlations*: If an oblique rotation was used, report the correlations among the factors.
- *Factor labeling*: Provide the names/labels given to the factors and the theoretical rationale for their interpretation based on the salient items.
- *Reliability and stability*: Report reliability estimates (e.g., Cronbach's alpha or McDonald's omega) and stability for each of the identified factors.

Conclusions

EFA is a relevant technique that has been used in psychology for many years. However, correctly using it requires thoughtful judgment on the part of the researcher regarding several analytical decisions. This chapter aimed to outline the steps for properly applying EFA and drawing valid conclusions from its results. We also provide an example with R syntax to guide readers in conducting an EFA properly. Several other open-access software programs, such as JASP and Jamovi, are available with tutorials. For example, see QuantFish with Christian Geiser at http://www.youtube.com/@QuantFish. Finally, readers may also find The Quantitude Podcast with Patrick Curran and Greg Hancock useful. The podcast is available on the Quantitude homepage: http://bit.ly/4mcI1Ui.We hope we have been able to help readers truly understand what the EFA is, how it works, and how it can be applied for different purposes.

References

Balboni, G., Perrucci, V., Cacciamani, S., & Zumbo, B. D. (2018). Development of a scale of sense of community in university online courses. *Distance Education*, *39*(3), 317–333. https://doi.org/10.1080/01587919.2018.1476843

Bartlett, M. S. (1954). A further note on the multiplying factors for various χ^2 approximations in factor analysis. *Journal of the Royal Statistical Society: Series B (Methodological)*, *16*(2), 296–298. https://doi.org/10.1111/j.2517-6161.1954.tb00174.x

Boateng, G. O., Neilands, T. B., Frongillo, E. A., Melgar-Quiñonez, H. R., & Young, S. L. (2018). Best practices for developing and validating scales for health, social, and behavioral research: A primer. *Frontiers in Public Health*, *6*, 149. https://doi.org/10.3389/fpubh.2018.00149

Bollen, K. A. (1989). *Structural equations with latent variables*. Wiley.

Borsboom, D., Mellenbergh, G. J., & Van Heerden, J. (2003). The theoretical status of latent variables. *Psychological Review*, *110*(2), 203–219. https://doi.org/10.1037/0033-295X.110.2.203

Brown, T. A. (2015). *Confirmatory factor analysis for applied research* (2nd ed.). Guilford Press.

Browne, M. W. (1984). Asymptotically distribution-free methods for the analysis of covariance structures. *British Journal of Mathematical and Statistical Psychology*, *37*(1), 62–83. https://doi.org/10.1111/j.2044-8317.1984.tb00789.x

Carroll, J. B. (1953). A stochastic model for the detection of changed items in multiple-choice tests. *Psychometrika*, *18*, 303–316.

Castellani, A., Benassi, M., & Balboni, G. (2024). Ethical principles in artificial intelligence telerehabilitation for neurodevelopmental disorders: Development of a questionnaire for survey research [Conference session, July 2–5]. *Proceedings of the 14th International Test Commission (ITC) Conference*.

Cattell, R. B. (1966). The scree test for the number of factors. *Multivariate Behavioral Research*, *1*(2), 245–276. https://doi.org/10.1207/s15327906mbr0102_10

Cattell, R. B. (1978). *The scientific use of factor analysis in behavioral and life sciences*. Plenum Press.

Comrey, A. L., & Lee, H. B. (1992). *A first course in factor analysis* (2nd ed.). Lawrence Erlbaum Associates.

Costello, A. B., & Osborne, J. W. (2005). Best practices in exploratory factor analysis: Four recommendations for getting the most from your analysis. *Practical Assessment, Research & Evaluation*, *10*(7), 1–9. https://doi.org/10.7275/jyj1-4868

de Winter, J. C. F., Dodou, D., & Wieringa, P. A. (2009). Exploratory factor analysis with small sample sizes. *Multivariate Behavioral Research*, *44*(2), 147–181. https://doi.org/10.1080/00273170902794206

Edwards, J. R., & Bagozzi, R. P. (2000). On the nature and direction of relationships between constructs and measures. *Psychological Methods*, *5*, 155–174. https://doi.org/10.1037/1082-989X.5.2.155

European Federation of Psychologists' Associations. (2013). *Review model for the description and evaluation of psychological and educational tests – version 4.2.6*. Retrieved December 30, 2025, from https://mlp.fi/wp-content/uploads/2020/09/4.-DISCEFPA_TestReviewModel2020_Report.pdf

Fabrigar, L. R., Wegener, D. T., MacCallum, R. C., & Strahan, E. J. (1999). Evaluating the use of exploratory factor analysis in psychological research. *Psychological Methods*, *4*, 272–299. https://doi.org/10.1037/1082-989X.4.3.272

Ferrando, P. J., & Lorenzo-Seva, U. (2017). Program factor at 10: Origins, development and future directions. *Psicothema*, *29*(2), 236–240. https://doi.org/10.7334/psicothema2016.304

Flora, D. B., & Curran, P. J. (2004). An empirical evaluation of alternative methods of estimation for confirmatory factor analysis with ordinal data. *Psychological Methods*, *9*(4), 466–491. https://doi.org/10.1037/1082-989X.9.4.466

Gorsuch, R. L. (1974). *Factor analysis*. W. B. Saunders.

Gorsuch, R. L. (1983). *Factor analysis* (2nd ed.). Erlbaum.

Graham, J. M., Guthrie, A. C., & Thompson, B. (2003). Consequences of not interpreting structure coefficients in published cfa research: A reminder. *Structural Equation Modeling*, *10*, 142–153. https://doi.org/10.1207/S15328007SEM1001_7

Guadagnoli, E., & Velicer, W. F. (1988). Relation of sample size to the stability of component patterns. *Psychological Bulletin*, *103*(2), 265–275. https://doi.org/10.1037/0033-2909.103.2.265

Guilford, J. P. (1954). *Psychometric methods* (2nd ed.). McGraw-Hill.

Guttman, L. (1954). Some necessary conditions for common-factor analysis. *Psychometrika*, *19*(2), 149–161. https://doi.org/10.1007/BF02289162

Hair, J. F., Anderson, R. E., Tatham, R. L., & Black, W. C. (1995). *Multivariate data analysis* (4th ed.). Prentice Hall.

Harman, H. H. (1976). *Modern factor analysis* (3rd ed.). University of Chicago Press.

Hendrickson, A. E., & White, P. O. (1964). Promax: A quick method for rotation to oblique simple structure. *British Journal of Statistical Psychology*, *17*, 65–70. https://doi.org/10.1111/j.2044-8317.1964.tb00244.x

Henson, R. K., & Roberts, J. K. (2006). Use of exploratory factor analysis in published research: Common errors and some comment on improved practice. *Educational and Psychological Measurement*, *66*(3), 393–416. https://doi.org/10.1177/0013164405282485

Holgado-Tello, F. P., Chacón-Moscoso, S., Barbero-García, I., & Vila-Abad, E. (2010). Polychoric versus pearson correlations in exploratory and confirmatory factor analysis of ordinal variables. *Quality & Quantity*, *44*(1), 153–166. https://doi.org/10.1007/s11135-008-9190-y

Horn, J. L. (1965). A rationale and test for the number of factors in factor analysis. *Psychometrika*, *30*(2), 179–185. https://doi.org/10.1007/BF02289447

Hu, L.-t., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling*, *6*(1), 1–55. https://doi.org/10.1080/10705519909540118

Jennrich, R. I., & Sampson, P. F. (1966). Rotation for simple loadings. *Psychometrika*, *31*(3), 313–323. https://doi.org/10.1007/BF02289465

Kaiser, H. F. (1958). The varimax criterion for analytic rotation in factor analysis. *Psychometrika*, *23*(3), 187–200. https://doi.org/10.1007/BF02289233

Kaiser, H. F. (1960). The application of electronic computers to factor analysis. *Educational and Psychological Measurement*, *20*(1), 141–151. https://doi.org/10.1177/001316446002000116

Kaiser, H. F. (1974). An index of factorial simplicity. *Psychometrika*, *39*, 31–36. https://doi.org/10.1007/BF02291575

Kline, R. B. (2016). *Principles and practice of structural equation modeling* (4th ed.). Guilford Press.

Kline, R. B. (2023). *Principles and practice of structural equation modeling* (5th ed.). Guilford Press.

Lawley, D. N., & Maxwell, A. E. (1963). *Factor analysis as a statistical method*. Butterworths.

Lorenzo-Seva, U., & Ferrando, P. J. (2024). Determining sample size requirements in efa solutions: A simple empirical proposal. *Multivariate Behavioral Research*, *59*(5), 899–912. https://doi.org/10.1080/00273171.2024.2342324

MacCallum, R. C., Widaman, K. F., Preacher, K. J., & Hong, S. (2001). Sample size in factor analysis: The role of model error. *Multivariate Behavioral Research*, *36*(4), 611–637. https://doi.org/10.1207/S15327906MBR3604_06

MacCallum, R. C., Widaman, K. F., Zhang, S., & Hong, S. (1999). Sample size in factor analysis. *Psychological Methods*, *4*(1), 84–99. https://doi.org/10.1037/1082-989X.4.1.84

Marsh, H. W., & Hau, K.-T. (1999). Confirmatory factor analysis: Strategies for small sample size. In R. H. Hoyle (Ed.), *Statistical strategies for small sample research*. Sage Publications.

McNeish, D. (2018). Thanks coefficient alpha, we'll take it from here. *Psychological Methods*, *23*(3), 412–433. https://doi.org/10.1037/met0000144

Mulaik, S. A. (1990). *Foundations of factor analysis*. Chapman & Hall/CRC.

Mundfrom, D. J., Shaw, D. G., & Ke, T. L. (2005). Minimum sample size recommendations for conducting factor analyses. *International Journal of Testing*, *5*(2), 159–168. https://doi.org/10.1207/s15327574ijt0502_4

Norris, M., & Lecavalier, L. (2010). Evaluating the use of exploratory factor analysis in developmental disability psychological research. *Journal of Autism and Developmental Disorders*, *40*, 8–20. https://doi.org/10.1007/s10803-009-0816-2

Nunnally, J. C., & Bernstein, I. H. (1994). *Psychometric theory* (3rd ed.). McGraw-Hill.

Olvera Astivia, O. L. (2018). *Ordinal alpha and parallel analysis*. Retrieved December 30, 2025, from https://psychometroscar.com/2018/07/31/ordinal-alpha-and-parallel-analysis/

Revelle, W. (2025). *Psych: Procedures for psychological, psychometric, and personality research* [R package version 2.5.6]. Northwestern University. https://CRAN.R-project.org/package=psych

Revelle, W., & Condon, D. M. (2019). Reliability from α to ω: A tutorial. *Psychological Assessment*, *31*(12), 1395–1411. https://doi.org/10.1037/pas0000754

Rhemtulla, M., & Savalei, V. (2025). Estimated factor scores are not true factor scores. *Multivariate Behavioral Research*, *60*(3), 598–619. https://doi.org/10.1080/00273171.2024.2444943

Schermelleh-Engel, K., Moosbrugger, H., & Müller, H. (2003). Evaluating the fit of structural equation models: Test of significance and descriptive goodness-of-fit measures. *Methods of Psychological Research*, *8*(2), 23–74.

Slocum-Gori, S. L., & Zumbo, B. D. (2011). Assessing the unidimensionality of psychological scales: Using multiple criteria from factor analysis. *Social Indicators Research*, *102*(3), 443–461. https://doi.org/10.1007/s11205-010-9682-8

Spearman, C. (1904). "General intelligence," objectively determined and measured. *The American Journal of Psychology*, *15*(2), 201–293.

Stevens, J. P. (1996). *Applied multivariate statistics for the social sciences* (3rd ed.). Lawrence Erlbaum Associates.

Tabachnick, B. G., & Fidell, L. S. (2019). *Using multivariate statistics* (7th ed.). Pearson.

Thompson, B. (1992). *Exploratory and confirmatory factor analysis: Understanding concepts and applications*. American Psychological Association.

Thurstone, L. L. (1935). *The vectors of mind*. University of Chicago Press.

Thurstone, L. L. (1940). Current issues in factor analysis. *Psychological Bulletin*, *37*, 189–236.

Thurstone, L. L. (1947). *Multiple-factor analysis: A development and expansion of "the vectors of mind"*. University of Chicago Press.

Tucker, L. R., & MacCallum, R. C. (1997). *Exploratory factor analysis* [Unpublished manuscript], University of North Carolina at Chapel Hill.

Velicer, W. F. (1976). Determining the number of components from the matrix of partial correlations. *Psychometrika*, *41*(3), 321–327. https://doi.org/10.1007/BF02293557

Velicer, W. F., & Fava, J. L. (1998). Effects of variable and subject sampling on factor pattern recovery. *Psychological Methods*, *3*, 231–251. https://doi.org/10.1037/1082-989X.3.2.231

Watkins, M. W. (2005). Determining parallel analysis criteria. *Journal of Modern Applied Statistical Methods*, *5*(2), 344–346. https://doi.org/10.22237/jmasm/1162354020

Watkins, M. W. (2018). Exploratory factor analysis: A guide to best practice. *Journal of Black Psychology*, *44*(3), 219–246. https://doi.org/10.1177/0095798418771807

Widaman, K. F. (1993). Common factor analysis versus principal component analysis: Differential bias in representing model parameters? *Multivariate Behavioral Research*, *28*(3), 263–311. https://doi.org/10.1207/s15327906mbr2803_1

Wirth, R. J., & Edwards, M. C. (2007). Item factor analysis: Current approaches and future directions. *Psychological Methods*, *12*(1), 58–79. https://doi.org/10.1037/1082-989X.12.1.58

Zumbo, B. D., Gadermann, A. M., & Zeisser, C. (2007). Ordinal versions of coefficients alpha and theta for likert rating scales. *Journal of Modern Applied Statistical Methods*, *6*(1), 4. https://doi.org/10.22237/jmasm/1177992180