

A simultaneous system of dynamic spatial stochastic frontier models with dependent error components and inefficiency determinants

S. Emili, F. Galli¹ *

Department of Statistical Sciences “Paolo Fortunati”, University of Bologna, Via delle Belle Arti 41, Bologna, Italy

ARTICLE INFO

JEL classification:

C33
R15
Z1

Keywords:

Multi-output stochastic frontier models
Spatial and temporal dependence
Copula functions
Inefficiency determinants
Cultural and creative industry

ABSTRACT

In this paper, we develop a system of simultaneous stochastic frontier models with inefficiency determinants, spatio-temporal effects and correlated inefficiency as well as correlated random errors among frontiers. The dependence among the errors of the different equations can stem from either shocks external to the system, interrelated inefficiency mechanisms, or a combination of both. Estimation is performed using a copula-based quasi-maximum likelihood approach. Simulation results confirm the good finite sample properties of the proposed estimator. To demonstrate the effectiveness of the proposed model and estimation technique in empirical settings, we analyse the key role of some sustainability-related factors in determining the efficiency level of Italian cultural and creative sectors.

1. Introduction

Stochastic frontier models (SF) have been first proposed by Aigner et al. (1977) and Meeusen and van Den Broeck (1977) in the context of productivity and efficiency analysis. The main feature of this class of models consists of splitting the error into two independent components: the inefficiency error term and the random disturbance capturing respectively deviations from the optimal frontier due to inefficiency mechanisms and random shocks. Based on economic and geographic theories as well as on empirical evidence on spatial phenomena such as agglomeration and clustering (Porter, 1998, 2000; Delgado et al., 2014; Andersson and Lööf, 2011; Ellison et al., 2010; Tveteras and Battese, 2006), in the last two decades, SF models have been extensively expanded in order to account for cross-sectional dependence among producers. In particular, taking advantage of spatial econometric techniques, authors introduced in the baseline SF model different spatial terms such as the spatial lag of the dependent variable (Glass et al., 2016), of the inputs (Adetutu et al., 2015), of the inefficiency determinants (Galli, 2023b), of the two error terms (Schmidt et al., 2009; Orea and Alvarez, 2019) or of different combinations of these terms (Galli, 2023a).

Although standard SF models as well as spatial SFs are typically designed to handle only a single output in relation to a set of input factors, a major challenge in productivity and efficiency analysis lies in accounting for multi-output production processes. This issue arises from the existing interrelationships and dependencies that strongly characterize economic systems, stemming from the multisectoral and multiproduct nature of production processes (Badinger and Egger, 2016). For instance, different economic sectors are interconnected through common production networks due to existing input–output linkages (Acemoglu and Azar, 2020), while a complex network of production sharing activities impacts the productivity performance of individual national economies (Liu and Sickles, 2023).

In this sense, multi-output SF models were developed in order to jointly model complex production systems characterized by distinct frontier functions by taking simultaneity in production processes into account (a situation in which two or more outcome

* Corresponding author.

E-mail addresses: silvia.emili2@unibo.it (S. Emili), federica.galli14@unibo.it (F. Galli).

<https://doi.org/10.1016/j.spasta.2025.100947>

Received 4 March 2025; Received in revised form 21 November 2025; Accepted 26 November 2025

Available online 29 November 2025

2211-6753/© 2025 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

variables influence each other at the same point in time, hereinafter also referred as simultaneous or mutual effect). One of the earliest contributions in this area is by [Ferreira and Steel \(2007\)](#) which developed a system of stochastic frontier equations using a multivariate skewed distribution to model the composed error term, estimating the system through Bayesian methods. Subsequently, [Carta and Steel \(2012\)](#) and [Lai and Huang \(2013\)](#) advanced a copula-based approach to capture the dependence among inefficiency terms across multiple outputs. [Liu et al. \(2020\)](#) further extended this line of work by formulating a simultaneous system of stochastic frontier models that allows for dependence not only between the random noise and inefficiency components within individual observations, but also across all equations, using copula functions.

Recently, [Schmidt and Kneib \(2023\)](#) introduced a flexible distributional approach for multi-output SF models based on the GAMLSS framework. In their model, all distributional parameters can be specified as functions of exogenous covariates, which allows for straightforward extensions to a spatial setting through the incorporation of spatial smoothers. Their framework does not explicitly model spatial spillovers or propagation mechanisms within and between the different outputs through structured spatial lag components. Unlike spatial autoregressive models or systems employing spatial weight matrices, their specification does not account for how the production level in one unit might directly influence neighbouring ones. However, there is a growing recognition among researchers that inter-industry linkages are shaped not only by economic dependencies – such as vertical and horizontal relationships – but also by spatial proximity (among others, [Ertur and Koch, 2007](#); [Badinger and Egger, 2016](#)). Spatial effects can arise from interactions among neighbouring producers within the same sector as well as from relationships between different sectors within the broader industry, both of which significantly influence producers' performance across various geographic levels. Spatial propagation mainly acts through mechanisms and factors like international trade and foreign direct investment, technology diffusion, imitation and human capital externalities ([Delgado et al., 2014](#); [Ellison et al., 2010](#)).

To this end, [Emili and Galli \(2025\)](#) developed a multi-output stochastic frontier model with temporal and spatial components to analyse the aggregate production of agricultural-related industries in European OECD countries. In their framework, multi-output SF models are extended to capture autoregressive spatial effects and input spillovers; however, they impose independent inefficiency mechanisms across equations by assuming a diagonal covariance matrix for the inefficiency term, which leads to a straightforward maximum likelihood estimation procedure. While such an assumption might be justified in certain empirical applications, it can be overly restrictive in contexts where inefficiency is likely to be correlated across equations.

In this paper, we extend the modelling framework proposed by [Emili and Galli \(2025\)](#) in two directions. First, we allow for potential error dependence across different frontier functions. Such dependence may arise from interrelated inefficiency mechanisms, from correlated responses to external shocks, or from a combination of both ([Lai and Huang, 2013](#)). This extension enables us to capture the propagation of inefficiency and shocks through interconnected production processes. While [Emili and Galli \(2025\)](#) assume only correlated random error such that a maximum likelihood procedure is feasible and well suited to estimating the model's parameters, relaxing the assumption of independent inefficiency components across frontiers requires the development of a multivariate copula-based estimation approach. Moreover, as pointed out by [Skevas \(2025\)](#), disregarding this specific source of dependence could lead to distortions in technical efficiency estimates and their determinants. The second extension concerns the incorporation of exogenous variables that explain mean inefficiency levels across producers. In the presence of systematic efficiency differences among decision-making units (DMUs), it is crucial to explicitly model the factors driving these variations by including exogenous determinants of inefficiency ([Battese and Coelli, 1995](#); [Wang and Schmidt, 2002](#); [Schmidt et al., 2009](#)).

The main features of this extended model include the possibility of accounting for (i) simultaneous effects across different frontier functions, (ii) time and spatial effects, (iii) exogenous variables that explain the mean inefficiency level of the DMUs, and (iv) Gaussian-type dependence between the inefficiency and/or the random error terms of the different frontiers. Estimation is achieved by adapting the copula-based quasi-maximum likelihood approach commonly used in the multi-output SF framework to accommodate mutual output effects, simultaneous and time-lagged spatial effects, alongside an inefficiency model. This flexible tool may assist researchers in a wide range of empirical applications both at the micro-scale of DMUs and at the macro scale of regions or countries providing relevant policy suggestions.

According to the spatio-temporal and cross-sectoral dynamics characterizing the cultural and creative industry, in this paper, we apply the proposed model to the Italian cultural and creative sectors. The DMUs are grouped into core cultural firms and cultural and creative units, as identified by the Measuring CCS Consortium (<https://www.measuring-ccs.eu/the-measuring-ccs-consortium-publishes-the-final-report/>) and we use balance sheet data for the two sectors at the NUTS-2 level in the time period 2013–2022. Given the key role of this industry in driving the economic growth of the country, the understanding of the sustainable territorial features influencing the inefficiency mechanisms can lead to a more effective, long-lasting growth of the sector without depleting resources or harming the environment. Therefore, we introduce in the system of simultaneous spatio-temporal SF model different local sustainability determinants to investigate the role of economic, social and environmental sustainability aspects in the inefficiency mechanisms of the two sectors in line with [Battese and Coelli \(1995\)](#). Based on the estimated model parameters, direct and indirect effects can be computed for each sustainability determinant in order to measure its direct contribution to regional efficiency degrees as well as the impact of sustainable characteristics of neighbouring areas. Nevertheless, our empirical analysis depicts only one among the diverse potential applications of the model introduced in this work, exemplifying its capacity to assist policymakers across a wide spectrum of topics.

In what follows, we introduce the model specification and identifiability conditions in Section 2, we present the quasi ML estimation procedure and the computation of marginal effects in Section 3, we test the finite sample properties of our model via simulations in Section 4, we make an example using real data on Italian core cultural and cultural and creative industries in Section 5, and in Section 6 we conclude.

2. Model

The specification of the model is outlined in Eq. (1)–(3). To facilitate understanding of the model components, we refer to our empirical application on the cultural and creative industries as a concrete example.

The system in Eq. (1) consists in M different equations identifying the different interdependent frontier functions. For the general m th equation, with $m = 1, \dots, M$, n_m units are observed over T time periods. For simplicity, in the subsequent sections, we assume n_m to be constant across equations (henceforth n to simplify the notation), and that the set of exogenous covariates X_t affecting the outputs is the same across all equations. The same holds for the set of inefficiency determinants d_t , without any loss of generality. Note that d_t and X_t refer to distinct sets of covariates. The model is specified as

$$Bz_t = \Psi Wz_t + \Gamma Wz_{t-1} + Pz_{t-1} + \Pi vec(X_t) + v_t - u_t \quad (1)$$

$$(v_{1,it}, v_{2,it}, \dots, v_{M,it})' \sim \mathcal{N}_M(0_{M \times 1}, \Sigma_v) \quad (2)$$

$$(u_{1,it}, u_{2,it}, \dots, u_{M,it})' \sim \mathcal{N}_M^+(Yvec(d_{it}), \Sigma_u) \quad (3)$$

for $i = 1, \dots, n$, and $t = 1, \dots, T$. In particular, $\mathcal{N}_M^+$ in Eq. (3) refers to an M -variate truncated normal distribution. In Eq. (1), $z_t = (z'_{1,t}, z'_{2,t}, \dots, z'_{M,t})'$ represents the $(nM \times 1)$ vector of production outputs observed at time t , and W is an $(nM \times nM)$ matrix collecting a set of $(n \times n)$ spatial weight matrices such that

$$W = \begin{bmatrix} W_1 & W_{12} & \dots & W_{1M} \\ W_{21} & W_2 & & \\ \vdots & & \ddots & \\ W_{M1} & W_{M2} & \dots & W_M \end{bmatrix}$$

Parameters associated with mutual impacts among the M output variables are collected in $B = (B \otimes I_n)$, where B is an $M \times M$ matrix containing $M(M-1)$ free parameters and M ones on the main diagonal. These entries (identified henceforth with lower case letters b) capture instantaneous – or equivalently, simultaneous – output effects across the different equations (i.e. vertical linkages between sectors; in the application, how the output of the core cultural sector influences that of the cultural and creative industries within a given area during the year, and vice versa). Autoregressive effects and time lagged effects across equations within areas are captured through $P = P \otimes I_n$, by the main diagonal and off-diagonal parameters respectively. Indeed, while autoregressive effects can be clearly associated to some features of both the demand and the nature of goods and services provided by a certain sector, we also suspect possible rigidities in the production processes, admitting that an increase in the performances of one of the two sectors can affect the performance of the other only one year later. Matrices $\Psi = \Psi \otimes I_n$ and $\Gamma = \Gamma \otimes I_n$ refer to instantaneous and temporal lagged spatial terms. Specifically, Ψ and Γ capture the influence of neighbouring units respectively at time t and $t-1$ within the same production process (main diagonal of the coefficient matrices) and across different production processes (off-diagonal elements). Referring to the example of the cultural and creative sectors, the parameters in Ψ capture mutual spatial spillovers between the outputs of the two groups, that is, how the output in a given area at time t is influenced by the output levels of neighbouring areas in the same period (as before, spatial effects within the same sector collected on the main diagonal, spillovers between different sectors on the off-diagonal elements). A similar interpretation applies to the Γ matrix, which accounts for time-lagged spatial effects, namely, how the output in a given area at time t is influenced by the output levels of neighbouring areas in the previous period.

In Eq. (1) we assume the same set of exogenous regressors, implying the same set of inputs (e.g. labour for core cultural sector, labour for the creative industry sector, capital for core cultural sector, etc.) for all production equations. In our case, X_t is an $(n \times k)$ matrix of input factors with associated $(Mn \times kn)$ parameter matrix $\Pi = (\Pi \otimes I_n)$, such that $vec(X_t) = (X'_{1,t}, X'_{2,t}, \dots, X'_{k,t})'$ is of dimension $(kn \times 1)$. Extension to the case of different sets of covariates across equations is described in the Supplementary Materials S.1.

The composite error ε_t , in the case of a production frontier, is given by $v_t - u_t$ since the inefficiency level u_t contributes to decreasing the units' level of output. The discussion below straightforwardly applies also to a cost frontier, i.e. $\varepsilon_t = v_t + u_t$, with some minor changes in signs. The random disturbance v_t is distributed as a multivariate normal with full covariance matrix $\Sigma_v = \Sigma_v \otimes I_n$. In the baseline SF framework, the inefficiency component u_t is typically assumed to follow a distribution with non-negative support, as inefficiency can only take values greater than or equal to zero. To allow for greater flexibility, here u_t follows an M -variate truncated normal distribution with mean $Yvec(d_t)$ and full covariance matrix $\Sigma_u = \Sigma_u \otimes I_n$. Based on the inefficiency model proposed by Battese and Coelli (1995), the mean of the inefficiency error term is therefore modelled as a function of s exogenous inefficiency determinants collected in the $(n \times s)$ matrix d_t , (included in Eq. (3) as $vec(d_t) = (d'_{1,t}, \dots, d'_{s,t})'$ of dimension $(sn \times 1)$), with associated parameter matrix $Y = (Y \otimes I_n)$ of dimension $(Mn \times sn)$. As previously mentioned, for the sake of simplicity (but without any loss of generality), we assume a common set of determinants d_t for all inefficiency terms across the M equations, following the same rationale applied to the exogenous variables in X_t . These variables typically refer to contextual characteristics that influence the inefficiency level of DMUs. In the case of the cultural and creative sectors, we consider the sustainable performance of Italian regions, measured in terms of economic, social, and environmental sustainability, as a potential determinant of regional inefficiency for both sectors.

Under standard regularity conditions, the model described in Eq. (1)–(3) remains stable in both spatial and temporal dimensions, such that generated spatial effects do not amplify indefinitely across the network or the geography (Yang and Lee, 2017, 2010; Lee and Yu, 2010). Stability is primarily determined by the parameter space, which encompasses the parameters in B , Ψ , Φ , and P .

To ensure stability, this parameter space must be compact, ensuring that the spectral radius of the Jacobian term remains below 1. The Jacobian is defined as $S = (B' \otimes I_n - (\Psi \otimes \mathcal{W})) = (B' - \Psi \mathcal{W})$. Additionally, for stability to hold, the solutions to the equation $|S^{-1}(\Phi \mathcal{W} + P)|$ must lie within the unit circle, guaranteeing stability in both spatial and temporal dimensions. Furthermore, spatial stability requires specific assumptions about each spatial weight matrix in $\mathcal{W}$. First, each matrix is a non-stochastic, row-normalized spatial weight matrix with zero diagonal elements. Second, its row and column sums are uniformly bounded in absolute value as n approaches infinity. Lastly, for all possible parameter configurations, S must be non-singular, with S^{-1} remaining bounded in row and column sum norms. For a more in-depth discussion on stationarity conditions in dynamic spatial panel data models, refer to Yang and Lee (2017, 2019).

In general, the identification of the system in Eq. (1)–(3) falls within the framework of parameter identification in a system of spatial dynamic panel simultaneous equations (e.g. Yang and Lee, 2017, 2019; Elhorst and Emili, 2022). Identification is achieved if it can be deduced from the quasi-reduced form of the model in Eq. (1), and given by $z_t = B^{-1}\Psi \mathcal{W} z_t + B^{-1}\Gamma \mathcal{W} z_{t-1} + B^{-1}P z_{t-1} + B^{-1}\Pi \text{vec}(X_t) + B^{-1}v_t - B^{-1}u_t$ (Yang and Lee, 2019). However, while in presence of spatial simultaneous effects only general conditions are required for identification (see among others Lee and Yu, 2016; Malikov and Sun, 2017; Yang and Lee, 2017), when considering possible simultaneous effects among variables in a system (i.e. impacts related to a situation in which two or more outcome variables influence each other at the same point in time, collected in B in the model specification) a further discussion is needed. This discussion draws on a vast literature—spanning over fifty years—on identification strategies in structural vector autoregressive (SVAR) models (e.g. Kilian and Lütkepohl, 2017). The identification of the instantaneous parameters, which encodes the instantaneous relations among endogenous variables (in this study collected in B), typically requires structural restrictions beyond the information contained in the reduced-form VAR. Common strategies include short-run (e.g., Cholesky decompositions), long-run (Blanchard and Quah, 1989), sign restrictions (Uhlig, 2005), and narrative or proxy-based identification (Stock and Watson, 2012; Mertens and Ravn, 2013). While the inclusion of exogenous covariates could be read as belonging to this last class of identification strategies, the introduction of some regressors in the model specification may improve estimation but it does not in itself resolve the identification problem unless incorporated within a broader structural framework. In the absence of theoretically motivated restrictions or valid instruments, the system remains underidentified. For further details on identification strategies and extensions to the case of panel SVAR models, see Herwartz and Wang (2024).

In this study, given $S = (B' - \Psi \mathcal{W})$, the reduced form of model in Eq. (1) is defined by

$$\begin{aligned} z_t &= S^{-1}\Gamma \mathcal{W} z_{t-1} + S^{-1}P z_{t-1} + S^{-1}\Pi \text{vec}(X_t) + S^{-1}v_t - S^{-1}u_t \\ &= \Gamma^\dagger \mathcal{W} z_{t-1} + P^\dagger z_{t-1} + \Pi^\dagger \text{vec}(X_t) + v_t^\dagger - u_t^\dagger \end{aligned} \quad (4)$$

where $v_t^\dagger$ is normally distributed with covariance matrix $S^{-1}\Sigma_v S^{-1'}$ and $u_t^\dagger$ is a truncated multivariate normal with mean $\mathbf{Y}^\dagger \text{vec}(d_t) = S^{-1}\mathbf{Y} \text{vec}(d_t)$, and covariance matrix $S^{-1}\Sigma_u S^{-1'}$.

As pointed out by Yang and Lee (2019), a straightforward (and very common) identification strategy for a system of M equations is given by imposing a certain number of zero restrictions. In this case the necessary and sufficient condition for identifying any particular equation relates to the rank of matrix parameters once the m th equation is excluded. An equation is identified if it is possible to construct at least one non-zero determinant of order $(M - 1)$ from the coefficients excluded from that equation but present in other equations of the model. According with Eq. (1) and Eq. (4), a straightforward solution for identification can be achieved by considering exclusive restrictions in B . For instance, when $M = 2$, one zero restriction is a necessary and sufficient condition for identification. More generally, let R_m be a matrix with r rows, as the number of possible restrictions, and number of columns equal the number of free parameters in the m th equation, collected in the vector θ_m . R_m is specified to collect all the r exclusive restrictions for the coefficients in θ_r , such that $R_m \theta_m = 0_{r \times 1}$. The necessary and sufficient rank condition for identifying the m th equation is $\text{rank}(R_m b_m) = M - 1$, with necessary order condition $r \geq M - 1$.

3. Estimation

The main challenge in the class of multi-output SF models revolves around the need to model the dependence among the different frontier equations, where relevant computational issues arise from the need to handle a multivariate truncated distribution for the inefficiency error term. To address this issue, most studies have relied on copula-based approaches, proposing various estimation techniques such as Bayesian methods (Ferreira and Steel, 2007), likelihood based procedures (Carta and Steel, 2012; Lai and Huang, 2013; Liu et al., 2020), and penalized maximum likelihood estimation via smoothers (Schmidt and Kneib, 2023).

According to Lai and Huang (2013) and Tsay et al. (2012), in the baseline case of M dependent frontier functions with correlated inefficiency components, the joint probability density function (PDF) of the composite errors can be rewritten as follows

$$f(\varepsilon_{1,it}, \dots, \varepsilon_{M,it}) = \prod_{m=1}^M f_m(\varepsilon_{m,it}) \times c(F_1(\varepsilon_{1,it}), \dots, F_M(\varepsilon_{M,it}); \Omega). \quad (5)$$

Specifically, the first term on the right-hand-side of Eq. (5) refers to the marginal PDFs of the errors of the separate SF regressions. In other words, this first term of the joint PDFs of the errors relies on the diagonal counterpart of the variance-covariance matrices of v_t and u_t , i.e. “temporarily” restricting the covariances to zero. Then, the second term models the dependence among the M frontiers through the copula function $c(F_1(\varepsilon_{1,it}), \dots, F_M(\varepsilon_{M,it}); \Omega)$ where $F(\cdot)$ represents the cumulative distribution function (CDF) of the composite error of each frontier equation and Ω is the $(M \times M)$ correlation matrix of the marginal CDFs. This dependence refers to the covariances in Σ_u and Σ_v meaning that inefficiency can be correlated among the M frontiers as well as the random error. In the subsections below, we first outline the analytical form of $\prod_{m=1}^M f_m(\varepsilon_{m,it})$ in Eq. (5), then we focus on the second term of the equation (i.e. the copula component, accounting for dependence among the M frontiers) and finally, we present the log-likelihood function.

3.1. Marginal PDFs of the composite errors

To formalize the first term of the joint PDF in Eq. (5) we focus on two main assumptions: first, the standard assumption in SF literature of independence between u_i and v_i , and second, since we are considering marginal PDFs, the independence of the errors among the M frontiers (this second assumption is relaxed in this study through the copula term in Eq. (5), while it still holds for the analysis of the first part of Eq. (5)). Then, we can write the PDF of the convolution of the two error terms as the product of the marginal distributions of u_i and v_i , where the covariance matrices of u_i and v_i are given by diagonal matrices.

First, the probability density function of v_i is given by the PDF of an M -variate normal random variable with a mean vector of zeros and variance-covariance matrix Σ_v^* , where $\Sigma_v^* = \text{diag}(\Sigma_v) = (\sigma_{v,1}^2, \sigma_{v,2}^2, \dots, \sigma_{v,M}^2)$. Second, the probability density function of u_i for the separate M frontiers can be derived from the PDF of a multivariate truncated normal as

$$f(u_i) = (2\pi)^{-\frac{Mn}{2}} |\Sigma_u^*|^{-\frac{1}{2}} \prod_{i=1}^n (\Phi_M(C_u^{-1} Y \text{vec}(d_{it})))^{-1} \exp \left\{ -\frac{1}{2} (u_i - Y \text{vec}(d_{it}))' \Sigma_u^{*-1} (u_i - Y \text{vec}(d_{it})) \right\}, \quad (6)$$

where C_u denotes the Cholesky decomposition of Σ_u^* , $Y = Y \otimes I_n$, and Φ_M is the CDF of the M -dimensional standard normal distribution. Here, since we are separately considering the M frontier equations, Σ_u^* is defined as $\Sigma_u^* = \text{diag}(\Sigma_u) = (\sigma_{u,1}^2, \sigma_{u,2}^2, \dots, \sigma_{u,M}^2)$ and $\Sigma_u^* = \Sigma_u^* \otimes I_n$. Moreover, the product of n M -variate normal CDFs results from the denominator of the multivariate truncated normal distribution (Horrace, 2005, Definition 2) according with the assumption of independence among n observations, with mean $Y \text{vec}(d_{it})$ for $\text{vec}(d_{it}) = (d_{1,it}, \dots, d_{M,it})'$ and variance matrix Σ_u^* .

The joint PDF of v_i and u_i can be written as

$$f(u_i, v_i) = (2\pi)^{-Mn} |\Sigma_u^*|^{-\frac{1}{2}} |\Sigma_v^*|^{-\frac{1}{2}} \prod_{i=1}^n (\Phi_M(C_u^{-1} Y \text{vec}(d_{it})))^{-1} \exp \left\{ -\frac{1}{2} [(u_i - Y \text{vec}(d_{it}))' \Sigma_u^{*-1} (u_i - Y \text{vec}(d_{it})) + v_i' \Sigma_v^{*-1} v_i] \right\}. \quad (7)$$

Since $\varepsilon_i = v_i - u_i$, we can substitute $v_i = \varepsilon_i + u_i$ in Eq. (7). Then, integrating out u_i , the resulting PDF is given by

$$f(\varepsilon_i) = (2\pi)^{-Mn} |\Sigma_u^*|^{-\frac{1}{2}} |\Sigma_v^*|^{-\frac{1}{2}} |\Sigma_\mu^{-1}|^{\frac{1}{2}} \prod_{i=1}^n (\Phi_M(C_u^{-1} Y \text{vec}(d_{it})))^{-1} \prod_{i=1}^n \Phi_M(C_\mu \mu_{it}) \exp \left\{ -\frac{1}{2} [(Y \text{vec}(d_{it}))' \Sigma_u^{*-1} (Y \text{vec}(d_{it})) + \varepsilon_i' \Sigma_v^{*-1} \varepsilon_i - \mu_i' \Sigma_\mu \mu_i] \right\}, \quad (8)$$

where $\Sigma_\mu = \Sigma_v^{*-1} + \Sigma_u^{*-1}$, $\Sigma_\mu = \Sigma_\mu \otimes I_n$, $\mu_i = ((Y' \text{vec}(d_{it})' \Sigma_u^{*-1} - \varepsilon_i' \Sigma_v^{*-1}) \Sigma_\mu^{-1})'$ and C_μ is the Cholesky decomposition of Σ_μ^{-1} .

3.2. Copula density

The second component of Eq. (5) is made by copula functions. This class of tools is commonly used in multi-output SF models in order to consider the dependence among the M frontier equations (see Lai and Huang, 2013; Lai, 2020; Marra and Radice, 2017; Marra et al., 2017). Indeed, according to Sklar's theorem, the joint CDF of $\varepsilon_{1,it}, \dots, \varepsilon_{M,it}$ can be expressed as $C(F_1(\varepsilon_{1,it}), \dots, F_M(\varepsilon_{M,it}); \Omega)$, where $C(\cdot)$ is the copula function of the error terms and Ω is an $M \times M$ matrix measuring the dependence among the M CDFs $F(\cdot)$. Since, for the maximum likelihood estimation we need the joint PDF of the errors, the corresponding copula density can be derived as

$$c(F_1(\varepsilon_{1,it}), \dots, F_M(\varepsilon_{M,it}); \Omega) = \frac{\partial^M C(F_1(\varepsilon_{1,it}), \dots, F_M(\varepsilon_{M,it}); \Omega)}{\partial F_1(\varepsilon_{1,it}) \cdot \dots \cdot \partial F_M(\varepsilon_{M,it})} \quad (9)$$

and $f(\varepsilon_{1,it}, \dots, \varepsilon_{M,it})$ can be written as the product of the copula density and all the marginals distributions of the errors as shown in Eq. (5). In general, different choices can be made to select the copula function depending on the structure of dependence between the errors. In this work, following previous literature on multi-output SF models, we select a Gaussian copula (see Cherubini et al., 2004 for further details), allowing to capture Gaussian-type dependence among the composite errors of the different frontier equations. The Gaussian copula density is given by

$$c(F_1(\varepsilon_{1,it}), \dots, F_M(\varepsilon_{M,it}); \Omega) = \frac{1}{|\Omega|^{\frac{1}{2}}} \exp \left(-\frac{1}{2} \zeta_{it}' (\Omega^{-1} - I_M) \zeta_{it} \right), \quad (10)$$

where I_M is a $M \times M$ identity matrix, ζ_{it} is a $1 \times M$ vector with elements $(\Phi^{-1}(F_1(\varepsilon_{1,it})), \dots, \Phi^{-1}(F_M(\varepsilon_{M,it})))$ and the off-diagonals of Ω capture the correlation among each pair of units of ζ_{it} . In particular, Φ^{-1} indicates the inverse of the CDF of the univariate normal. For each frontier equation $m = 1, \dots, M$, given the PDF of $\varepsilon_{m,it}$, $f_m(\varepsilon_{m,it})$, the corresponding CDF has no closed form evaluation, and typically requires some numerical integration procedures or the use of simulations to approximate it (Greene, 2003, 2010). Lai and Huang (2013) suggest using the approximation proposed by Tsay et al. (2012) in the case $\varepsilon_{m,it}^* = v_{m,it} + u_{m,it}$. The method uses some mathematical approximation functions to obtain the closed form of the CDF of ε_{it} . Note that, for the case of a production frontier ($\varepsilon_{m,it} = v_{m,it} - u_{m,it}$), the CDF of $\varepsilon_{m,it}$ can be derived from the case of a cost frontier since the distribution of $\varepsilon_{m,it}$ is the same distribution of $-\varepsilon_{m,it}^*$ (Amsler et al., 2019). The CDF of ε_{it}^* for a generic frontier function m at the point Q_{it} is given by

$$F(Q_{it}) = \int_{-\infty}^{Q_{it}} f(\varepsilon_{it}^*) d\varepsilon_{it}^* = \frac{2}{\sigma} I(Q_{it}), \quad (11)$$

where $I(\cdot)$, according to [Tsay et al. \(2012\)](#), can be approximated with an error less than 10^{-5} , by

$$I(Q_{it}) \approx \exp\left(\frac{a^2 c_1^2}{4b^2 - 4a^2 c_2}\right) \frac{1}{b\sqrt{b^2 - a^2 c_2}} \left(1 - \operatorname{erf}\left(\frac{-ac_1 + \sqrt{2}Q(b^2 - a^2 c_2)\operatorname{sign}(Q)}{2\sqrt{b^2 - a^2 c_2}}\right)\right) + \frac{\operatorname{erf}\left(\frac{bQ}{\sqrt{2}}\right)}{2b} \frac{1 + \operatorname{sign}(Q)}{2} \quad (12)$$

and $\sigma^2 = \sigma_u^2 + \sigma_v^2$, $\lambda = \sigma_u/\sigma_v$, $a = \lambda/\sigma$, $b = 1/\sigma$, $c_1 = -1.09500814703333$, $c_2 = -0.75651138383854$, $\operatorname{sign}(Q_{it}) = -1, 0, 1$ depending respectively on $Q_{it} <, =, > 0$ and $\operatorname{erf}(\cdot)$ is the error function.

3.3. Log-likelihood function

Given [Tsay et al. \(2012\)](#)'s approximation for the unavailability of a closed form for the CDF of the composite error, and according to the uncertainty related to the empirical reliability of the Gaussian copula density, the log-likelihood function presented below can be referred to as a quasi-loglikelihood function. Starting from Eq. (5), the final quasi log-likelihood function can be written as

$$\begin{aligned} \log L(\theta) &= T \log(|S|) + \sum_{i=1}^n \log(c(F_1(\varepsilon_{1,it}), \dots, F_M(\varepsilon_{M,it}); \Omega)) + \sum_{i=1}^n \sum_{m=1}^M \log(f_m(\varepsilon_{m,it})) \\ &= T \log(|S|) - \frac{nT}{2} \log|\Omega| - \frac{1}{2} (\zeta'_t(\Omega^{-1} - I_{nM})\zeta_t) - MnT \log(2\pi) \\ &\quad - \frac{nT}{2} \log|\Sigma_u^*| - \frac{nT}{2} \log|\Sigma_v^*| - \frac{nT}{2} \log|\Sigma_\mu| + \sum_{i=1}^n \log(\Phi_M^{-1}(C_u^{-1} Y \operatorname{vec}(d_{it}))) \\ &\quad + \sum_{i=1}^n \log(\Phi_M(C_\mu \mu_{it})) - \frac{1}{2} ((Y \operatorname{vec}(d_t))' \Sigma_u^{*-1} (Y \operatorname{vec}(d_t)) + \varepsilon'_t \Sigma_v^{*-1} \varepsilon_t - \mu'_t \Sigma_\mu \mu_t), \end{aligned} \quad (13)$$

where θ is the vector of all parameters to be estimated, $\Omega = (\Omega \otimes I_n)$, $\varepsilon_t = \mathbf{B}z_t - \Psi \mathcal{W} z_t - \Gamma \mathcal{W} z_{t-1} - \mathbf{P} z_{t-1} - \Pi \operatorname{vec}(X_t)$ and $|S| = |\mathbf{B}' - \Psi \mathcal{W}|$ is the determinant of the Jacobian matrix deriving from the transformation from ε_t to z_t .

The ML estimator, under standard regularity conditions (see [Serfling, 1980](#)), is consistent, asymptotically efficient and asymptotically normal, i.e. $\sqrt{n}T(\hat{\theta} - \theta_0) \sim \mathcal{N}(0, I^{-1}(\theta_0))$, where $I(\theta_0)$ is the Fisher Information matrix and θ_0 is the vector of true parameters. The asymptotic properties of the ML estimator for stochastic frontier models with spatial autoregressive terms have been studied by [Jin and Lee \(2020\)](#), which demonstrated that all model parameter estimators are $\sqrt{n}$ consistent and asymptotically normal when inefficiency exists. When there is no inefficiency, some parameters have slower convergence rates but this case is seldom met in practice.

In the case of multi-output SF models estimated through copula-based maximum likelihood techniques, [Lai and Huang \(2013\)](#) highlighted that the derived quasi-maximum likelihood estimator is still consistent but it is necessary to compute the 'sandwich' estimator to obtain correct standard errors. The parameter estimates can be found numerically maximizing the quasi log-likelihood function, where the entries of the vector ζ_t are appropriately approximated by Eq. (11)–(12). MATLAB code is available upon request.

Starting from the estimated parameters, technical efficiency scores for each unit i at time t in equation m can be obtained as $\exp(-E(u_{m,it}|\varepsilon_{m,it}))$, where the expected value of $u_{m,it}$ given $\varepsilon_{m,it}$ can be computed as proposed by [Battese and Coelli \(1995\)](#). As highlighted by [Lai and Huang \(2013\)](#), the technical efficiency scores obtained from the joint estimation are not directly comparable to those from the separate estimation since, in the joint model, the predicted composite errors reflect the dependence structure across equations.

3.4. Marginal effects of inputs and inefficiency determinants

After model estimation, marginal effects can be computed distinguishing between input variables and inefficiency determinants. The starting point is represented by the reduced form of the model in Eq. (4). Then, following [Elhorst et al. \(2021\)](#), we can compute instantaneous and point-in-time effects, and cumulative effects. For the first typology of impacts we calculate the partial derivative $\operatorname{ME}_{z_{X,t+h}} = \frac{\partial z_{t+h}}{\partial X_t} = (\mathbf{P}^\dagger + \Gamma^\dagger \mathcal{W})^h \Pi^\dagger = \mathbf{S}^{-h} (\Pi + \Gamma \mathcal{W})^h \mathbf{S}^{-1} \Pi$. With $h = 0$ the derivative provides information on the instantaneous effects to a unitary change in the input level (hereinafter also called short-run effects), while point-in-time impacts are obtained for $h > 0$ (henceforth, long-run for brevity). As h increases, this quantity tends to zero. Finally, cumulative effects can be calculated as $\sum_{h=0}^H \operatorname{ME}_{z_{X,t+h}}$.

A general representation of marginal effects for a system of dynamic spatial panel data model is a matrix of dimension $(Mn \times kn)$. According to [Elhorst and Emili \(2022\)](#) these marginal effects can be rewritten as

$$\operatorname{ME}_{z_{X,t+h}} = \begin{bmatrix} \operatorname{ME}_{z_1 X_1,t+h} & \operatorname{ME}_{z_1 X_2,t+h} & \dots & \operatorname{ME}_{z_1 X_k,t+h} \\ \operatorname{ME}_{z_2 X_1,t+h} & \ddots & & \\ \vdots & & & \\ \operatorname{ME}_{z_M X_1,t+h} & \dots & \dots & \operatorname{ME}_{z_M X_k,t+h} \end{bmatrix} = \begin{bmatrix} \frac{\partial z_{1,t+h}}{\partial X_{1,t}} & \frac{\partial z_{1,t+h}}{\partial X_{2,t}} & \dots & \frac{\partial z_{1,t+h}}{\partial X_{k,t}} \\ \frac{\partial z_{2,t+h}}{\partial X_{1,t}} & \ddots & & \\ \vdots & & & \\ \frac{\partial z_{M,t+h}}{\partial X_{1,t}} & \dots & \dots & \frac{\partial z_{M,t+h}}{\partial X_{k,t}} \end{bmatrix}$$

where the average of the diagonal elements of the $(n \times n)$ general block $\operatorname{ME}_{z_m X_{j,t+h}}$, with $m = 1, \dots, M$, $h = 0, 1, \dots$ and $j = 1, \dots, k$, represents direct effects (i.e. impacts occurring within a certain area) while the average of the off-diagonal elements refers to indirect effects (i.e. impacts from neighbouring units).

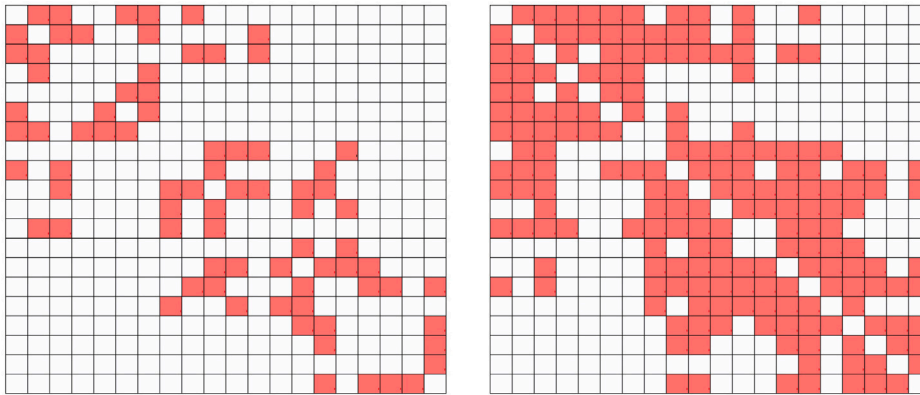


Fig. 1. First and second order contiguity matrix for MC experiment.

In line with the literature on marginal effects of inefficiency determinants in single equation SFs (Wang and Schmidt, 2002; Wang, 2002; Galli, 2023b), to quantify the impact of a unitary change of the determinants of inefficiency on the inefficiency term u_t , we focus on the partial derivative of the model for u_t with respect to d_t . In particular, as in the case of $ME_{zX_{t+h}}$, we consider the reduced form of the system and we focus on the model for inefficiency. While in the structural form we can assume that $u_t = Y \text{vec}(d_t) + w_t$, with $w_t \sim \mathcal{N}_M^+(0_{nM \times 1}, \Sigma_u)$, its reduced counterpart (implied by the pre-multiplication of all the terms in the system by S^{-1}) can be written as

$$\begin{aligned} S^{-1}u_t &= S^{-1}Y \text{vec}(d_t) + S^{-1}w_t, \\ u_t^\dagger &= Y^\dagger \text{vec}(d_t) + w_t^\dagger, \quad w_t^\dagger \sim (0_{nM \times 1}, S^{-1}\Sigma_u S^{-1}). \end{aligned} \quad (14)$$

Then, starting from Eq. (14) we can compute the marginal effects to a unitary change in the inefficiency determinants as the partial derivative of $u_t^\dagger$ with respect to the vector d_t , finally obtaining $ME_{ud,t} = \frac{\partial u_t^\dagger}{\partial d_t} = \frac{\partial(S^{-1}Y \text{vec}(d_t))}{\partial d_t} = S^{-1}Y = Y^\dagger$. The result is a $(Mn \times sn)$ matrix, where for each block $ME_{u_m d_t}$, with $m = 1, \dots, M$ and $\cdot = 1, \dots, s$, the average of the diagonal elements represents direct effects (i.e. impacts occurring within a certain area) while the average of the off-diagonal elements refers to spatial effects (i.e. impacts from neighbouring units).

4. Simulation study

In order to test the finite sample properties of the proposed system of spatial SF models, in this section, we conduct a Monte Carlo (MC) experiment. In particular, according to the dimensions of the empirical example below on Italian regional creative and cultural industries, we set $M = 2$, $n = 20$, and $T = 30, 50$, aiming to provide some intuitions on the reliability of our results. The data generating process (DGP) is given by

$$\begin{bmatrix} I_n & 0_{n \times n} \\ 0.10I_n & I_n \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} 0.25W_a & -0.30W_b \\ -0.10W_b & 0.50W_a \end{bmatrix} \begin{bmatrix} z_{1t-1} \\ z_{2t-1} \end{bmatrix} + \begin{bmatrix} 0.15W_a & -0.10W_b \\ 0.05W_b & 0.10W_a \end{bmatrix} \begin{bmatrix} z_{1t-1} \\ z_{2t-1} \end{bmatrix} + \begin{bmatrix} 0.15I_n & -0.05I_n \\ -0.10I_n & 0.25I_n \end{bmatrix} \begin{bmatrix} z_{1t-1} \\ z_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \quad (15)$$

where W_a and W_b enter the model after row-standardization. Fig. 1 shows the two spatial matrices, corresponding to a first-order and a second-order binary contiguity matrix, which are among the most common in the literature (LeSage and Pace, 2007). Although the matrices are random, they are designed such that each unit has, on average, 20% of all units as neighbours in W_a and 50% in W_b . This matrix combination, ensuring both the matrix exogeneity and model parsimony, also reflects the choice of spatial weight matrices used in the empirical application.

Focusing on the two error components, v_t is generated from a bivariate normal distribution as $\mathcal{N}_2(0_{2 \times 1}, \Sigma_v)$. To generate the inefficiency component, we consider the equation $u_t = (Y \otimes I_n) \text{vec}(d_t) + w_t = Y \text{vec}(d_t) + w_t$, with two inefficiency determinants, common to both equations. In particular, the observations of d_t are generated from a bi-variate standard normal distribution and remain constant across the different MC iterations; the associated parameter vector is given by $Y = (0.2, 0.1, -0.05, 0.3)'$. Following Battese and Coelli (1995), $w_t \sim \mathcal{N}_2^+(0_{2n \times 1}, \Sigma_u)$, with lower truncation point $-Y \text{vec}(d_t)$.

Following the approach proposed by Lai and Huang (2013), we assign values to Σ_v and Σ_u based on alternative specifications. The objective is to illustrate the effect of increasing the share of random noise in the composite error and the correlation between the errors of the different equations on the performance of joint estimation. Specifically, in C1 we set $\text{vech}(\Sigma_v) = (0.5, 0, 0.8)$ and $\text{vech}(\Sigma_u) = (1.5, 0, 1.9)$, in C2 $\text{vech}(\Sigma_v) = (1.5, 0.5, 1.9)$ and $\text{vech}(\Sigma_u) = (0.5, 0.3, 0.8)$, in C3 $\text{vech}(\Sigma_v) = (0.5, 0.3, 0.8)$ and $\text{vech}(\Sigma_u) = (1.5, 0.85, 1.9)$, and in C4 $\text{vech}(\Sigma_v) = (1.5, 1.3, 1.9)$ and $\text{vech}(\Sigma_u) = (0.5, 0.4, 0.8)$, where vech is the column stacking operator for symmetric matrices, such that it stacks the elements on and below the main diagonal only. Given Σ_v and Σ_u , we capture the dependence between the two frontier functions through ρ , which represents the value on the off-diagonal elements of

Table 1
Monte Carlo simulation results.

	Case C1, $\rho = 0.00$					Case C2, $\rho = 0.31$				
	DGP	T=30		T=50		DGP	T=30		T=50	
		Est.	sd _{MC}	Est.	sd _{MC}		Est.	sd _{MC}	Est.	sd _{MC}
b_1	0.10	0.12	0.19	0.13	0.13	0.10	0.15	0.16	0.16	0.13
ψ_{11}	0.25	0.24	0.05	0.25	0.04	0.25	0.24	0.05	0.24	0.04
ψ_{21}	-0.10	-0.10	0.15	-0.11	0.11	-0.10	-0.13	0.16	-0.14	0.11
ψ_{12}	-0.30	-0.30	0.06	-0.30	0.05	-0.30	-0.29	0.08	-0.29	0.06
ψ_{22}	0.50	0.50	0.05	0.50	0.04	0.50	0.50	0.05	0.51	0.03
γ_{11}	0.15	0.15	0.05	0.15	0.04	0.15	0.15	0.06	0.16	0.04
γ_{21}	0.05	0.05	0.09	0.04	0.08	0.05	0.04	0.10	0.05	0.08
γ_{12}	-0.10	-0.11	0.07	-0.10	0.06	-0.10	-0.11	0.08	-0.10	0.06
γ_{22}	0.10	0.11	0.06	0.10	0.05	0.10	0.10	0.07	0.11	0.05
p_{11}	0.15	0.15	0.04	0.15	0.03	0.15	0.14	0.04	0.15	0.03
p_{21}	-0.10	-0.10	0.06	-0.10	0.04	-0.10	-0.11	0.06	-0.11	0.05
p_{12}	-0.05	-0.05	0.03	-0.05	0.02	-0.05	-0.05	0.03	-0.05	0.03
p_{22}	0.25	0.25	0.04	0.25	0.03	0.25	0.25	0.05	0.25	0.04
$\sigma_{v,1}^2$	0.50	0.49	0.09	0.50	0.07	1.50	1.47	0.13	1.52	0.12
$\sigma_{v,2}^2$	0.80	0.80	0.14	0.79	0.10	1.90	1.83	0.24	1.84	0.16
$\sigma_{u,1}^2$	1.50	1.48	0.29	1.46	0.22	0.50	0.56	0.29	0.50	0.24
$\sigma_{u,2}^2$	1.90	1.95	0.41	1.92	0.30	0.80	0.93	0.32	0.92	0.28
ρ	0.00	-0.02	0.16	-0.02	0.12	0.31	0.23	0.14	0.23	0.11
v_{11}	0.20	0.19	0.11	0.19	0.08	0.20	0.22	0.14	0.21	0.13
v_{21}	0.10	0.10	0.15	0.09	0.11	0.10	0.06	0.16	0.06	0.14
v_{12}	-0.05	-0.05	0.11	-0.06	0.09	-0.05	-0.12	0.13	-0.07	0.10
v_{22}	0.30	0.30	0.13	0.32	0.10	0.30	0.39	0.18	0.34	0.15

	Case C3, $\rho = 0.48$					Case C4, $\rho = 0.75$				
	DGP	T=30		T=50		DGP	T=30		T=50	
		Est.	sd _{MC}	Est.	sd _{MC}		Est.	sd _{MC}	Est.	sd _{MC}
b_1	0.10	0.19	0.18	0.17	0.12	0.10	0.21	0.14	0.21	0.12
ψ_{11}	0.25	0.24	0.05	0.25	0.04	0.25	0.26	0.05	0.26	0.04
ψ_{21}	-0.10	-0.15	0.13	-0.14	0.10	-0.10	-0.12	0.11	-0.11	0.08
ψ_{12}	-0.30	-0.28	0.05	-0.28	0.04	-0.30	-0.31	0.06	-0.31	0.04
ψ_{22}	0.50	0.52	0.04	0.52	0.03	0.50	0.51	0.04	0.51	0.03
γ_{11}	0.15	0.16	0.05	0.16	0.04	0.15	0.16	0.05	0.15	0.05
γ_{21}	0.05	0.05	0.09	0.05	0.07	0.05	0.03	0.08	0.04	0.07
γ_{12}	-0.10	-0.12	0.07	-0.11	0.06	-0.10	-0.11	0.07	-0.10	0.05
γ_{22}	0.10	0.11	0.06	0.11	0.05	0.10	0.10	0.05	0.11	0.05
p_{11}	0.15	0.15	0.04	0.15	0.03	0.15	0.13	0.05	0.13	0.04
p_{21}	-0.10	-0.12	0.06	-0.11	0.04	-0.10	-0.14	0.07	-0.14	0.05
p_{12}	-0.05	-0.05	0.03	-0.05	0.02	-0.05	-0.04	0.04	-0.04	0.03
p_{22}	0.25	0.25	0.04	0.25	0.03	0.25	0.27	0.05	0.27	0.04
$\sigma_{v,1}^2$	0.50	0.52	0.08	0.53	0.07	1.50	1.45	0.13	1.48	0.10
$\sigma_{v,2}^2$	0.80	0.77	0.14	0.79	0.11	1.90	1.71	0.36	1.70	0.31
$\sigma_{u,1}^2$	1.50	1.56	0.28	1.56	0.23	0.50	0.61	0.30	0.54	0.24
$\sigma_{u,2}^2$	1.90	1.98	0.46	1.93	0.34	0.80	0.90	0.30	0.86	0.28
ρ	0.48	0.37	0.14	0.39	0.10	0.75	0.64	0.08	0.65	0.07
v_{11}	0.20	0.22	0.11	0.22	0.09	0.20	0.26	0.14	0.23	0.12
v_{21}	0.10	0.04	0.14	0.04	0.11	0.10	0.04	0.17	0.00	0.14
v_{12}	-0.05	-0.14	0.11	-0.14	0.11	-0.05	-0.14	0.14	-0.17	0.12
v_{22}	0.30	0.39	0.14	0.39	0.09	0.30	0.44	0.14	0.40	0.14

$\Sigma_\varepsilon = \Sigma_v + (\frac{\pi-2}{\pi})\Sigma_u$, where π refers to the standard mathematical constant. According to the values specified for Σ_v and Σ_u , the resulting correlation coefficients ρ are $\rho = \{0, 0.31, 0.48, 0.75\}$.

For each simulation exercise, we run 1000 repetitions. To select the starting values, we adopt the approach proposed by [Liu et al. \(2020\)](#). Specifically, we use the estimates from the single-equation stochastic frontier models obtained via the STATA package `sfsd` ([Du et al., 2025](#)). For the correlation parameter, we rely on MATLAB's built-in pseudo-random number generator to draw from a standard uniform distribution, which we then scale to the support $[-1, +1]$. By default, the random number generator employs the Mersenne Twister algorithm. For each parameter, we report its DGP value, the mean estimate over the 1000 iterations and sd_{MC} computed as $\sqrt{E(X^2) - (E(X))^2}$.

The estimation procedure is quite computationally intensive. For example, with $T = 30$ and $\rho = 0.31$, estimation takes approximately 163.72 s on a laptop equipped with 12-core CPU and 32 GB of RAM. Increasing T to 50 raises the computation time to 240.32 s.

The results in [Table 1](#) indicate that all parameters are estimated quite accurately and with increasing precision as the sample size increases. Among all, the larger biases are detected for the σ_u^2 parameters which are also those with greater sd_{MC} values. However,

as found by Lai and Huang (2013), the relative bias of σ_u^2 tends to decrease when the values of σ_u^2 exceed those of σ_v^2 , due to the smaller influence of the statistical noise in the composite error (C2 and C4). A similar pattern also applies to the estimates of the inefficiency determinants. Parameters related to spatio-temporal effects are estimated with negligible bias in all cases. The only exception concerns b_1 (i.e. the parameter related to the instantaneous output effect) which is estimated more accurately for smaller ρ values.

The performance of the proposed model is then assessed against a benchmark estimation procedure, as represented by Emili and Galli (2025), here extended to account for inefficiency determinants. This procedure aims to capture dependence only in the random error (assuming independence among inefficiency terms of different equations in the system) and relies on a ML approach instead of a copula-based technique. To this end, we perform Monte Carlo simulations for $T = 30$ and $\rho = \{0, 0.31, 0.48, 0.75\}$, corresponding to the cases outlined above. The comparison of the results reported in Table S2.1 in the Supplementary Material section S.2 with those presented in Table 1 indicates that, while for $\rho = 0$ the performance of the proposed approach is generally comparable to that of Emili and Galli (2025), neglecting the correlation between the inefficiency error terms leads to poorer outcomes, both in terms of bias and estimation variability. This pattern becomes increasingly evident as the correlation value rises.

To further compare the proposed model with respect to the benchmark, we also consider two key aspects in spatial SF analysis: (i) technical efficiency scores (TE) and (ii) predictive performance of the model. First, based on the estimation results for $T = 30$ reported in Table 1 and Table S2.1 in the Supplementary Materials, we calculate TE scores and associated bias for both $z_{1,i}$ and $z_{2,i}$. For each equation, the bias in case C1 is 0.0033 and 0.0099 for the proposed model and 0.0035 and -0.0058 using the benchmark. In case C2, it equals 0.0063 and 0.0171, against 0.0656 and -0.0308 ; 0.0017 and -0.0209 vs -0.0979 and -0.1498 in C3; in C4 the bias is 0.0288 and 0.0109 vs 0.0442 and -0.0281 for the benchmark. These results indicate that accounting for cross-equation error dependence in the inefficiency components is particularly important for obtaining accurate estimates of efficiency scores.

Second, to evaluate the predictive performance of the proposed model, we performed a leave-one-out experiment. In spatial contexts, standard leave-one-out cross-validation (LOOCV) may underestimate prediction errors, as the excluded observation is still spatially correlated with neighbouring units included in the estimation. To address this issue, we implement a Spatial Leave-One-Out Cross-Validation (SLOOCV) procedure, which excludes not only the target observation but also its spatially contiguous neighbours when re-estimating the model (Le Rest et al., 2014; Roberts et al., 2017). In this exercise, for $m = 2$ equations, $N = 30$, and $T = 30$, the DGP is given by Eq. (15) with covariance and correlation values defined by C3, representing an intermediate correlation scenario. For 100 iterations, we evaluate the predictive performance of our proposal and the one of Emili and Galli (2025), augmented to deal with inefficiency determinants, for $\hat{z}_T = \hat{S}^{-1} \hat{\Gamma} \mathcal{W} z_{T-1} + \hat{S}^{-1} \hat{P} z_{T-1} - \hat{S}^{-1} \hat{Y} \text{vec}(d_T)$, where $\hat{S} = (\hat{B}' - \hat{\Psi} \mathcal{W})$. For each unit, estimated parameters and predictions are obtained once the target unit and its neighbours are excluded from the estimation sample. The comparison of the predictive performances is evaluated by means of Mean Absolute Percentage Error (MAPE). For the proposed approach values of $z_{1,i}$ in Eq. (15) we observe a MAPE equal 0.1412, while applying the estimation procedure in Emili and Galli (2025), MAPE is 0.1485. Referring to $z_{2,i}$, MAPE is equal -0.2102 for the proposed approach while for the benchmark proposal the prediction error measure is -0.2190 . This result aligns with previous insights on the goodness of estimated parameters in Ψ , Γ , P and B for both the two approaches.

5. Empirical application

5.1. Data, model and variables

To demonstrate how the proposed modelling approach can be applied in practice, we consider the interesting case study of the cultural and creative sectors in Italy, as a key issue for local, national and international economic growth and sustainable development (European Commission, 2021).

The cultural and creative industry is implicitly characterized by spatio-temporal dynamics (see, among others, the cultural districts definition by Le Blanc, 2010) and by a clear multi-sectoral and multi-product nature (see the Measuring CCS Consortium's Final Report 2022, at <https://www.measuring-ccs.eu/the-measuring-ccs-consortium-publishes-the-final-report/>). These characteristics make the application of the proposed model to CCS coherent and potentially influential in policy planning, particularly for identifying significant sustainable determinants of efficiency mechanisms. Therefore, in line with the works of Benita (2019) for Mexico and Hong et al. (2014), Zeng et al. (2016) for China, this empirical application aims to investigate how different regional characteristics contribute to the efficiency performance of Italian CCS, with a specific focus on sustainability-related aspects.

To this end, we consider balance sheets data on Italian cultural and creative industries collected through the AIDA Bureau Van Dijk database aggregated at the NUTS-2 level and different indicators on equitable and sustainable well-being ("Benessere equo e sostenibile", BES) at the regional level provided by the Italian National Institute of Statistics (ISTAT) in the period 2013–2022 which is the longest time horizon available for AIDA data. According to the final report of the Measuring CCS Consortium, Italian cultural and creative industries can be classified into two categories, namely core cultural and cultural and creative industries (CCI). The former group concerns core cultural firms performing activities related to education, museums and heritage sites, libraries and archives, visual arts and writing, and performing arts, while the latter includes manufacturing and services related to cultural industries (e.g. radio, TV, recorded music, book and press, artistic craft, etc.) and creative sectors (advertising, design, architecture and videogames). The model is estimated on a final dataset of 18 statistical units (Italian regions, excluding the two islands), observed for two sectors, over a 10-year time horizon. A detailed description of the sectoral composition, spatial distribution, and descriptive statistics is provided in the Supplementary Material, section S.3.

Taking advantage of the system of simultaneous equations introduced in this study, we separately model the regional production processes of the two groups of firms (in the sense that we do not aggregate the two production processes in a unique one), but considering possible simultaneous interdependencies among them and evaluating the role of different sustainability determinants on their efficiency level. The rationale is that, given the specificities of the production processes of firms in the two groups (among others, the former more related to the production of services and the latter to products), we separately estimate two equations taking into account different dependence mechanisms. Specifically, we consider (i) intra and inter group spatial and temporal correlation, (ii) simultaneity effects among the outputs of the two groups, (iii) cross-group input effects, and (iv) the overall level of dependence between the error components of the two frontier functions. The empirical specification is given by

$$Bz_t = \Psi Wz_t + \Gamma Wz_{t-1} + Pz_{t-1} + \Pi_L vec(L_t) + \Pi_K vec(K_t) + v_t - u_t \quad (16)$$

$$\mu_t^* = Y_1 Inc_t + Y_2 LifeExp_t + Y_3 Online_t + Y_4 Energy_t, \quad (17)$$

where Eq. (16) represents a system of two Cobb–Douglas production functions (Cobb and Douglas, 1928), and Eq. (17) is the inefficiency model given $u_{it} \sim \mathcal{N}_2^+(\mu_{it}^*, \Sigma_u)$ with $u_{it} = (u_{1,it}, u_{2,it})'$ and

$$\mu_{it}^* = Y vec(d_{it}) = \begin{bmatrix} Y_1 & Y_2 & Y_3 & Y_4 \end{bmatrix} \begin{bmatrix} Inc_{it} \\ LifeExp_{it} \\ Online_{it} \\ Energy_{it} \end{bmatrix}.$$

In particular, z_t is the logarithm of value added of core cultural and cultural and creative industries by region while L_t and K_t are the logarithms of the input variables, i.e. number of workers and fixed asset. The spatial weight matrix W is given by a combination of a first and second-order binary contiguity row-normalized spatial weight matrix, the former on the off-diagonal blocks elements of W , and the latter on the main diagonal blocks of W . This specification not only ensures identification (Elhorst and Emili, 2022), but considers the possibility of distinguishing between intra and inter sectoral transmission channels. Specifically, the idea is that it is easier for spatial effects to materialize within the same group (second-order contiguity matrix) than across different groups (first-order contiguity matrix) due to a similar technological and knowledge base (Malerba et al., 2014).

To select the sustainability determinants, we rely on the widely accepted conceptualization of sustainability, which is defined by three interconnected pillars: economic, environmental, and social (for a review see Purvis et al., 2019). Thus, to model the mean of the inefficiency error term (Eq. (17)), we consider the average disposable income per capita (*Inc*) to proxy economic development, life expectancy at birth (*LifeExp*) and the percentage of municipalities with entirely online services (*Online*) to proxy social sustainability, and the percentage of electricity from renewable sources (*Energy*) for environmental sustainability. Some descriptive statistics on these variables are shown in Table S.3 in the Supplementary Material section S.3.

5.2. Estimation results and efficiency scores

The estimation results for the model proposed in this study are reported in Table 2 and are denoted as M1 in the following. Table 2 also includes the estimates obtained for three nested specifications: M2, obtained from a model without inefficiency determinants but allowing for both error and inefficiency dependence among equations; M3, corresponding to the specification that includes inefficiency determinants but assumes no error dependence in the inefficiency component (an augmented version of Emili and Galli (2025), accounting for some explanatory variables of inefficiency); and M4, represented by Emili and Galli (2025)'s original proposal, where neither inefficiency determinants nor correlation in the inefficiency error terms are considered. Thus, correlation in M3 and M4 is only based on dependence in the random error term while assuming uncorrelated inefficiency components. Before estimating the full system, each equation was first estimated separately using existing routines such as Du et al. (2025). The resulting parameter estimates were then employed as starting values for the joint system estimation.

Simultaneous effects among the outputs of the two groups appear to be statistically significant in most of the four models. Overall, while the output of core cultural industries (z_{1t}) positively affects CCI in the same area, a negative simultaneous effect occurs in the opposite direction. Considering the spatial and temporal dimensions, in general, the two industries are characterized by both dynamics, with opposite effects for within and between-group relationships; while signs are preserved across the different specifications, coefficients appear larger in magnitudes when inefficiency dependence is set to zero (i.e. M3 and M4). The estimated correlation indicates positive error dependence between the two production frontiers. Interestingly, correlation is higher in magnitude (0.23 in M2 and 0.15 in M1) when considering possible dependence in the inefficiency error term compared to models in which correlation is only based on random error dependence (0.05 in M3 and 0.01 in M4). This indicates that overall correlation is mainly driven by inefficiency-related factors, suggesting the key role of considering this possible source of error dependence. Moreover, while correlation values are statistically different from zero at 1% significance level in M1 and M2, they are not significant in M3 and M4, suggesting that correlation related to the random error does not significantly contributes to the overall level of dependence. For parameter estimates of input factors, we detect a general agreement across models, with mainly positive and significant coefficients. Finally, the estimates for the regional sustainability determinants are all negative, suggesting a possible positive effect on efficiency for both groups. When comparing the four nested specifications based on the loglikelihood value (LL) and AIC and BIC criteria, we find that the proposed model specification (M1) is the one that maximizes the LL and that is preferable in terms of AIC and BIC.

Due to the inclusion of the spatial autoregressive and simultaneous terms (Ψ and B in Eq. (1)), direct and indirect effects for both the input variables and the determinants in the inefficiency model should be computed from parameter estimates. The short and

Table 2
Estimation results.

	M1		M2		M3		M4	
	z_{1t}	z_{2t}	z_{1t}	z_{2t}	z_{1t}	z_{2t}	z_{1t}	z_{2t}
z_{1t}	–	0.19*** (0.03)	–	0.10** (0.05)	–	0.26*** (0.01)	–	0.34*** (0.04)
z_{2t}	–0.21** (0.09)	–	–0.26*** (0.02)	–	–0.05*** (0.01)	–	–0.02 (0.06)	–
$W'z_{1t}$	0.31*** (0.07)	–0.13** (0.06)	0.32*** (0.07)	–0.10 (0.07)	0.47*** (0.01)	0.07*** (0.02)	0.45*** (0.09)	0.03 (0.08)
$W'z_{2t}$	–0.05** (0.02)	0.11*** (0.03)	–0.04 (0.03)	0.14** (0.04)	0.11*** (0.01)	0.41*** (0.02)	0.05 (0.10)	0.37*** (0.08)
$W'z_{1t-1}$	–0.28*** (0.07)	0.20*** (0.06)	–0.29*** (0.08)	0.19*** (0.07)	–0.45*** (0.04)	–0.03 (0.03)	–0.44*** (0.10)	–0.02 (0.08)
$W'z_{2t-1}$	0.11*** (0.02)	–0.07*** (0.02)	0.09*** (0.02)	–0.09*** (0.03)	–0.06** (0.03)	–0.44*** (0.02)	–0.04 (0.10)	–0.37*** (0.08)
z_{1t-1}	0.37*** (0.05)	–0.28*** (0.05)	0.31*** (0.07)	–0.21*** (0.06)	0.66*** (0.05)	–0.14*** (0.04)	0.67*** (0.06)	–0.15** (0.06)
z_{2t-1}	–0.28*** (0.03)	0.20*** (0.03)	–0.27*** (0.02)	0.23*** (0.02)	–0.36*** (0.02)	0.77*** (0.02)	–0.42*** (0.08)	0.74*** (0.07)
L_{1t}	0.32*** (0.05)	0.30*** (0.04)	0.38*** (0.05)	0.24*** (0.01)	0.25*** (0.02)	0.10*** (0.00)	0.24*** (0.06)	0.06 (0.10)
L_{2t}	–0.05* (0.03)	0.70*** (0.05)	0.01 (0.04)	0.71*** (0.05)	–0.01** (0.01)	0.23*** (0.01)	–0.02 (0.06)	0.26** (0.03)
K_{1t}	0.14*** (0.03)	0.42*** (0.05)	0.18*** (0.03)	0.42*** (0.08)	0.13*** (0.01)	0.12*** (0.00)	0.16*** (0.03)	0.12 (0.08)
K_{2t}	0.00 (0.02)	0.34*** (0.04)	0.01 (0.01)	0.31*** (0.05)	0.07*** (0.00)	0.11*** (0.01)	0.06 (0.04)	0.11 (0.08)
Inc_t	–0.06 (0.07)	–0.12* (0.07)	–	–	–0.04*** (0.01)	–0.07 (0.05)	–	–
$LifeExp_t$	–0.12*** (0.03)	–0.23*** (0.05)	–	–	–0.07*** (0.02)	–0.13*** (0.00)	–	–
$Online_t$	–0.01 (0.07)	–0.05 (0.11)	–	–	–0.01 (0.01)	–0.01 (0.04)	–	–
$Energy_t$	–0.08** (0.03)	–0.16* (0.09)	–	–	–0.05*** (0.01)	–0.09*** (0.03)	–	–
σ_v	0.04	0.02	0.03	0.02	0.05	0.02	0.02	0.01
σ_u	0.26	0.11	0.15	0.01	0.29	0.25	0.10	0.13
ρ	0.15		0.23		0.05		0.01	
LL	–263.03		–512.22		–375.86		–698.52	
AIC	596.06		1078.40		821.72		1451.03	
BIC	707.81		1164.60		933.48		1537.24	

Note: The matrix W in the table is followed by a general subscript $\cdot$ to represent both $W_a z_{1t}$ (in the equation of z_{1t}) and $W_b z_{1t}$ (in that of z_{2t}).

* p -value ≤ 0.10 .

** p -value ≤ 0.05 .

*** p -value ≤ 0.01 .

long-run marginal effects (ME) of the input factors and inefficiency determinants are reported in Supplementary Materials, section S.4 for the four nested models. The effects are computed based on the reduced form of the model as discussed in Section 3.4. Short-run effects represent instantaneous impacts, whereas, long-run effects capture the impact for a one-year horizon, where the length of the horizon is based on the speed at which certain MEs approach zero. In general, the core cultural group relies primarily on its own labour and capital inputs, while the CCI group also benefits from the inputs of the core cultural sector. In the long run, direct effects remain positive but weaken, whereas indirect effects become negative, likely reflecting competitive pressures arising from neighbouring regions' investments.

The comparison of the marginal effects of input factors across the four frontier systems reported in Tables S4.1–S4.4 highlights the overall robustness of the results. However, when inefficiency determinants are included in the model (M1 and M3), a slight decrease in the magnitude of the covariate impacts is observed for both direct and indirect effects. Including a correlation term in the inefficiency component mainly affects the marginal effects of the inefficiency determinants, which are slightly higher in M1 than in M3.

Given the importance of accounting for the stochastic dependence both between inefficiency and random errors, Fig. 2 presents the distribution (upper panel) and time trend (bottom panel) of the technical efficiency scores for models M1–M4, by group of firms. Efficiency scores were computed following Battese and Coelli (1995). In line with Skevas (2025) some differences arise when dependence in inefficiency terms is neglected.

The figure reveals a marked difference in the efficiency levels between core cultural industries and CCIs: the former reach lower efficiency levels (0.82 on average with M1, 0.76 with M2, 0.79 with M3, 0.88 with M4), while the latter achieve an excellent

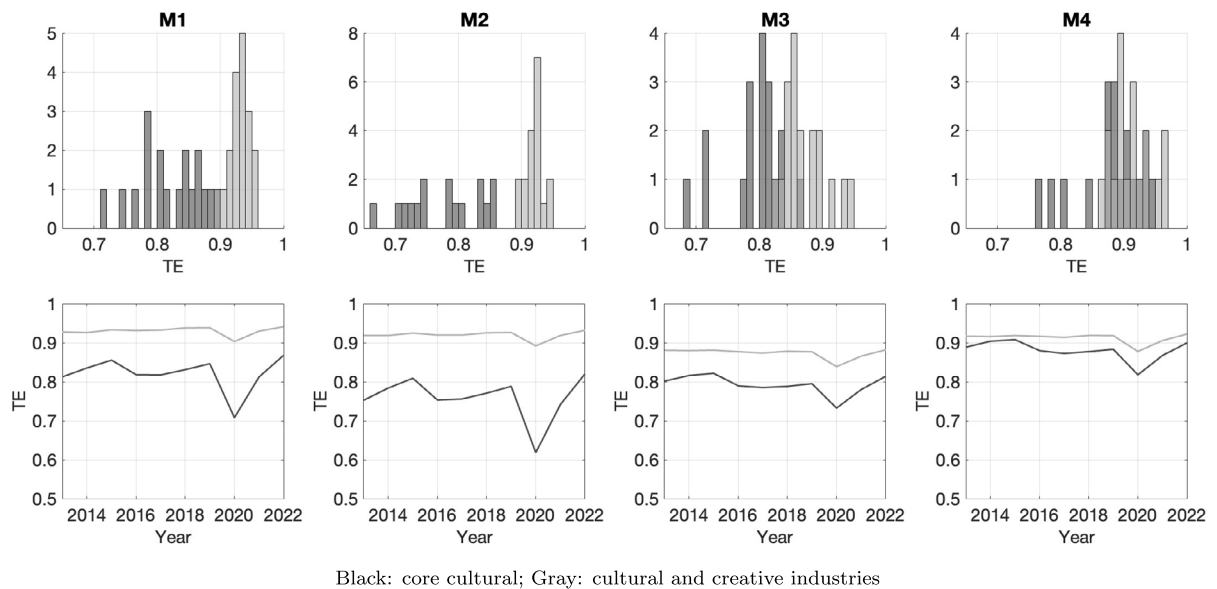


Fig. 2. Upper panel: Mean TE scores by region and group. Bottom panel: Mean TE scores in time by group.

performance (0.93 on average with M1, 0.92 with M2, 0.87 with M3, 0.91 with M4). For M1 and M2, higher variability is observed within the core cultural group, whereas very small differences in efficiency levels characterize the cultural and creative industries of the Italian regions.

Regarding temporal variations, a smoother trend can be observed for CCI compared to the core cultural group, which is more affected by year-to-year fluctuations, especially for M1 and M2. Both groups of firms experienced a decline in efficiency in 2020, likely due to the COVID-19 pandemic, with a larger negative impact on the core cultural group for models accounting for inefficiency dependence between sectors. However, for all four models, both groups managed to recover from the efficiency shock, returning to and slightly exceeding pre-pandemic levels by 2022.

6. Conclusions

In this study, we proposed a system of simultaneous spatial stochastic frontier models with dependent error components and inefficiency determinants. This model specification allows to consider Gaussian-type error dependence representative of both possible common channels of shock transmission among sectors and/or interrelated inefficiency mechanisms. Moreover, the model is enriched by means of inefficiency determinants, simultaneous output effects and spatial and non-spatial spillovers among the different frontier equations. The inefficiency model, combined with the inclusion of some spatial autoregressive terms makes it possible to compute the direct and indirect effects of the inefficiency determinants on the units' efficiency level providing interesting insights on the direct role of the inefficiency variables and impacts resulting from neighbours. This versatile tool can support researchers in various empirical applications related to multi-output interdependent production processes characterized by spatio-temporal correlation and error dependence.

Technically, the model can be estimated by using likelihood-based techniques combined with a copula approach that can be implemented in standard statistical software. Simulation studies confirm the satisfactory final sample properties of the proposed estimator even for small sample sizes. However, one of the main limitations of multi-output SF models lies in their empirical applicability when dealing either with a large number of equations (for parameters proliferation), or with large sample sizes (for computational burden associated with inverting a large matrix to compute the log-determinant). However, numerous applications are well-suited to contexts with a relatively small number of frontiers and moderate sample sizes. This has been highlighted, among others, by Emili and Galli (2025) for the agribusiness industry, Huang et al. (2018) for financial and non-financial holding banks, Lai and Huang (2013) for tourism, and Schmidt and Kneib (2023) for agriculture.

As an example, we evaluate the superiority of the proposed model with respect to some alternatives considering Italian data on the cultural and creative sectors, differentiating between core cultural and cultural and creative industries. The estimates of the model proposed in this study are compared with estimates obtained from (i) a model without inefficiency determinants but allowing for correlation between the error terms; (ii) a specification that includes inefficiency determinants but assumes zero correlation; (iii) Emili and Galli (2025)'s original proposal, where neither inefficiency determinants nor correlation terms are considered. Results show that, while controlling for a potential correlation between the error terms leads to estimates, marginal effects and technical efficiency scores with similar patterns, the omission of this component has several relevant effects on the estimates of the remaining specifications.

Future research on this topic may consider using alternative specifications for the inefficiency model referring to both the mean and the variance of the inefficiency error term by taking advantage of the scaling property (Alvarez et al., 2006; Wang and Ho, 2010). Moreover, as proposed by Orea and Alvarez (2019), among others, in the single-equation SF framework, additional spatial terms can be introduced either in the random error or in the inefficiency component. Introducing this type of spatial structure, however, also affects the identification of the model. Therefore, particular attention must be paid to the identification scheme adopted and to its empirical implications, as well as to the interpretation of the underlying phenomena. For example, one may assume different spatial structures for the inefficiency term and for the idiosyncratic error (Elhorst et al., 2021). Finally, although the distributions assumed for the two error components are consistent with the main strand of the empirical literature, extending the framework to alternative distributional forms would be of considerable interest. Such extensions, however, require additional assumptions and more elaborate identification strategies. We view this as a promising direction for future research.

Funding

Funded by the European Union - NextGenerationEU under the National Recovery and Resilience Plan (PNRR) - Mission 4 Education and research - Component 2 From research to business - Investment 1.1 Notice Prin 2022 - DD N. 104 del 2/2/2022, from title [TOWARDS LOCAL DEVELOPMENT THROUGH A SUSTAINABLE VALORIZATION OF CULTURAL HERITAGE], proposal code [2022M87K4F] - CUP [J53D23009430006].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Supplementary materials

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.spasta.2025.100947>.

References

- Acemoglu, D., Azar, P., 2020. Endogenous production networks. *Econometrica* 88 (1), 33–82.
- Adetutu, M., Glass, A., Kenjegaliev, K., Sickles, R., 2015. The effects of efficiency and TFP growth on pollution in Europe: A multistage spatial analysis. *J. Prod. Anal.* 43 (3), 307–326.
- Aigner, D., Lovell, C., Schmidt, P., 1977. Formulation and estimation of stochastic frontier production function models. *J. Econometrics* 6 (1), 21–37.
- Alvarez, A., Amsler, C., Orea, L., Schmidt, P., 2006. Interpreting and testing the scaling property in models where inefficiency depends on firm characteristics. *J. Prod. Anal.* 25, 201–212.
- Amsler, C., Schmidt, P., Tsay, W., 2019. Evaluating the CDF of the distribution of the stochastic frontier composed error. *J. Prod. Anal.* 52 (1/3), 29–35.
- Andersson, M., Löf, H., 2011. Agglomeration and productivity: evidence from firm-level data. *Ann. Reg. Sci.* 46 (3), 601–620.
- Badinger, H., Egger, P., 2016. Productivity spillovers across countries and industries: New evidence from OECD countries. *Oxf. Bulletin Econ. Stat.* 78 (4), 9305–9049.
- Battese, G., Coelli, T., 1995. A model for technical inefficiency effects in a stochastic frontier production function for panel data. *Empir. Econ.* 20 (2), 325–332.
- Benita, F., 2019. On the performance of creative industries: Evidence from Mexican metropolitan areas. *Pap. Reg. Sci.* 98 (2), 825–842.
- Blanchard, O., Quah, D., 1989. The dynamic effects of aggregate demand and supply disturbances. *Am. Econ. Rev.* 4 (79), 655–673.
- Carta, A., Steel, M., 2012. Modelling multi-output stochastic frontiers using copulas. *Comput. Statist. Data Anal.* 56 (11), 3757–3773.
- Cherubini, U., Luciano, E., Vecchiato, W., 2004. *Copula Methods in Finance*. Wiley, New York.
- Cobb, C., Douglas, P., 1928. A theory of production. *Am. Econ. Rev.* 18 (1), 139–165.
- Delgado, M., Porter, M., Stern, S., 2014. Clusters, convergence, and economic performance. *Res. Policy* 43 (10), 1785–1799.
- Du, K., Galli, F., Wang, L., 2025. A command to fit spatial stochastic frontier models with inefficiency spillovers. *Stata J.* 25 (1), 189–211.
- Elhorst, J.P., Emili, S., 2022. A spatial econometric multivariate model of Okun's law. *Reg. Sci. Urban Econ.* 93, 103756.
- Elhorst, J.P., Gross, M., Tereanu, E., 2021. Spillovers in space and time: where spatial econometrics and Global VAR models meet. *J. Econ. Surv.* 35 (1), 192–226.
- Ellison, G., Glaeser, E., Kerr, W.R., 2010. What causes industry agglomeration? Evidence from coagglomeration patterns. *Am. Econ. Rev.* 100 (3), 1195–1213.
- Emili, S., Galli, F., 2025. Modelling spatio-temporal dynamics in multi-output stochastic frontiers for the European agribusiness industry. *J. Agric. Biol. Environ. Stat.* 30, 540–575.
- Ertur, C., Koch, W., 2007. Growth, technological interdependence and spatial externalities: theory and evidence. *J. Appl. Econometrics* 22 (6), 1033–1062.
- European Commission, 2021. EU Research and Innovation – Driving a green, digital and innovative European Cultural Heritage – Building our Future from the Past. Publications Office.
- Ferreira, J., Steel, M., 2007. Model comparison of coordinate-free multivariate skewed distributions with an application to stochastic frontiers. *J. Econometrics* 137 (2), 641–673.
- Galli, F., 2023a. A spatial stochastic frontier model including both frontier and error based spatial cross-sectional dependence. *Spat. Econ. Anal.* 18 (2), 239–258.
- Galli, F., 2023b. A spatial stochastic frontier model introducing inefficiency spillovers. *J. R. Stat. Soc. Ser. C. Appl. Stat.* 72 (2), 346–367.
- Glass, A., Kenjegaliev, K., Sickles, R., 2016. A spatial autoregressive stochastic frontier model for panel data with asymmetric efficiency spillovers. *J. Econometrics* 190 (2), 289–300.
- Greene, W., 2003. Simulated likelihood estimation of the normal- gamma stochastic frontier function. *J. Prod. Anal.* 19, 179–190.
- Greene, W., 2010. A stochastic frontier model with correction for sample selection. *J. Prod. Anal.* 34, 15–24.
- Herwartz, H., Wang, S., 2024. Statistical identification in panel structural vector autoregressive models based on independence criteria. *J. Appl. Econometrics* 4 (39), 620–639.
- Hong, J., Yu, W., Guo, X., Zhao, D., 2014. Creative industries agglomeration, regional innovation and productivity growth in China. *Chin. Geogr. Sci.* 24 (2), 258–268.
- Horrace, W., 2005. Some results on the multivariate truncated normal distribution. *J. Multivariate Anal.* 94 (1), 209–221.

- Huang, T., Hu, C., Chang, B., 2018. Competition, efficiency, and innovation in Taiwan's banking industry — An application of copula methods. *Q. Rev. Econ. Financ.* (67), 362–375.
- Jin, F., Lee, L., 2020. Asymptotic properties of a spatial autoregressive stochastic frontier model. *J. Spat. Econ.* 1 (2), 1–40.
- Kilian, L., Lütkepohl, H., 2017. *Structural Vector Autoregressive Analysis*. Cambridge University Press.
- Lai, H., 2020. Maximum simulated likelihood estimation of the seemingly unrelated stochastic frontier regressions. *Empir. Econ.* 60, 2943–2968.
- Lai, H., Huang, C., 2013. Maximum likelihood estimation of seemingly unrelated stochastic frontier regressions. *J. Prod. Anal.* 40, 1–14.
- Le Blanc, A., 2010. Cultural districts, a new strategy for regional development? The south-east cultural district in Sicily. *Reg. Stud.* 44 (7), 905–917.
- Le Rest, K., Pinaud, D., Monestiez, P., Chadoeuf, J., Bretagnolle, V., 2014. Spatial leave-one-out cross-validation for variable selection in the presence of spatial autocorrelation. *Glob. Ecol. Biogeogr.* 23 (7), 811–820.
- Lee, L.-F., Yu, J., 2010. A unified transformation approach for the estimation of spatial dynamic panel data models: stability, spatial cointegration and explosive roots. In: Ullah, A., Giles, D. (Eds.), *Handbook of Empirical Economics and Finance*. Chapman and Hall/CRC, pp. 395–432.
- Lee, L.-F., Yu, J., 2016. Identification of spatial durbin panel models. *J. Appl. Econometrics* 31 (1), 133–162.
- LeSage, J., Pace, R., 2007. A matrix exponential spatial specification. *J. Econometrics* 140, 190–214.
- Liu, W., Sickles, R., 2023. Industry specific productivity and spillovers through input-output linkages: evidence from Asia-Pacific value chain. *Rev. Income Wealth* 70 (2), 345–369.
- Liu, J., Wang, M., Ma, J., Rahman, S., Sriboonchitta, S., 2020. A simultaneous stochastic frontier model with dependent error components and dependent composite errors: An application to Chinese banking industry. *Mathematics* 8 (2), 238.
- Malerba, F., Mancusi, M., Montobbio, F., 2014. Innovation, international R&D spillovers and the sectoral heterogeneity of knowledge flows. *Rev. World Econ.* 149 (4), 697–722.
- Malikov, E., Sun, Y., 2017. Semiparametric estimation and testing of smooth coefficient spatial autoregressive models. *J. Econometrics* 199 (1), 12–34.
- Marra, G., Radice, R., 2017. Bivariate copula additive models for location, scale and shape. *Comput. Statist. Data Anal.* 112, 99–113.
- Marra, G., Radice, R., Bärnighausen, T., Wood, S., McGovern, M., 2017. A simultaneous equation approach to estimating hiv prevalence with nonignorable missing responses. *J. Amer. Statist. Assoc.* 112 (518), 484–496.
- Meeusen, W., van Den Broeck, J., 1977. Efficiency estimation from Cobb-Douglas production functions with composed error. *Internat. Econom. Rev.* 18 (2), 435–444.
- Mertens, K., Ravn, M., 2013. The dynamic effects of personal and corporate income tax changes in the United States. *Am. Econ. Rev.* 4 (103), 1212–1247.
- Orea, L., Alvarez, I., 2019. A new stochastic frontier model with cross-sectional effects in both noise and inefficiency terms. *J. Econometrics* 213 (2), 556–577.
- Porter, M., 1998. Clusters and the new economics of competition. *Harv. Bus. Rev.* 76, 77–90.
- Porter, M., 2000. Location, competition, and economic development: local clusters in a global economy. *Econ. Dev. Q.* 14, 15–34.
- Purvis, B., Mao, Y., D., R., 2019. Three pillars of sustainability: in search of conceptual origins. *Sustain. Sci.* 14 (3), 681–695.
- Roberts, D.R., Bahn, V., Ciuti, S., Boyce, M.S., Elith, J., Guillera-Aroita, G., Hauenstein, S., Lahoz-Monfort, J.J., Schröder, B., Thuiller, W., Warton, D.I., Wintle, B.A., Hartig, F., Dormann, C.F., 2017. Cross-validation strategies for data with temporal, spatial, hierarchical, or phylogenetic structure. *Ecography* 40 (8), 913–929.
- Schmidt, R., Kneib, T., 2023. Multivariate distributional stochastic frontier models. *Comput. Statist. Data Anal.* 187, 107796.
- Schmidt, A., Moreira, A., Helfand, S., Fonseca, T., 2009. Spatial stochastic frontier models: accounting for unobserved local determinants of inefficiency. *J. Prod. Anal.* 31 (2), 101–112.
- Serfling, R., 1980. *Approximation Theorems of Mathematical Statistics*. Wiley, New York.
- Skevas, I., 2025. Technical and environmental inefficiency measurement in agriculture using a flexible by-production stochastic frontier model. *J. Agric. Econ.* 76 (1), 1–236.
- Stock, J., Watson, M., 2012. Disentangling the channels of the 2007–2009 recession. *Brookings Pap. Econ. Act.* 1 (43), 81–156.
- Tsay, W., Huang, C., Fu, T., Ho, I., 2012. A simple closed-form approximation for the cumulative distribution function of the composite error of stochastic frontier models. *J. Prod. Anal.* 39, 259–269.
- Tveteras, R., Battese, G., 2006. Agglomeration externalities, productivity, and technical inefficiency. *J. Reg. Sci.* 46 (4), 605–625.
- Uhlig, H., 2005. What are the effects of monetary policy on output? Results from an agnostic identification procedure. *J. Monet. Econ.* (52), 381–419.
- Wang, H., 2002. Heteroscedasticity and non-monotonic efficiency effects of a stochastic frontier model. *J. Prod. Anal.* 18, 241–253.
- Wang, H., Ho, C., 2010. Estimating fixed-effect panel stochastic frontier models by model transformation. *J. Econometrics* 157 (2), 286–296.
- Wang, H., Schmidt, P., 2002. One-step and two-step estimation of the effects of exogenous variables on technical efficiency levels. *J. Prod. Anal.* 18, 122–144.
- Yang, K., Lee, L.-F., 2010. Estimation of dynamic panel spatial vector autoregression: Stability, spatial cointegration and explosion. Working paper, Shanghai University of Finance and Economics, Shanghai, China.
- Yang, K., Lee, L.-F., 2017. Identification and QML estimation of multivariate and simultaneous equations spatial autoregressive models. *J. Econometrics* 196 (1), 196–214.
- Yang, K., Lee, L.-F., 2019. Identification and estimation of spatial dynamic panel simultaneous equations models. *Reg. Sci. Urban Econ.* 76, 32–46.
- Zeng, S., Hu, M., Su, B., 2016. Research on investment efficiency and policy recommendations for the culture industry of China based on a three-stage DEA. *Sustainability* 8 (4), 324.