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Common Ownership, Competition and Corporate Governance *

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Abstract

This paper presents a theoretical framework for determining the ownership stakes held by financial investors in companies competing in the same product market, commonly referred to as the level of common ownership. In our model, these investors are primarily motivated by the anticipation of capital gains resulting from the impact of common ownership on product market competition, which enhances profitability for the firms involved. However, common ownership also undermines effective corporate governance by diminishing blockholders' incentives to engage in value-enhancing behaviors, such as managerial monitoring. These adverse effects on corporate governance act as limiting factors, ultimately determining the equilibrium level of common ownership.

Keywords: Antitrust; Common Ownership; Corporate Governance.

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1 Introduction

This paper presents a theoretical analysis of the drivers of common ownership, consistent with the emerging consensus regarding its anticompetitive effects.

Common ownership occurs when financial investors hold stakes in companies that compete in the same product market. The traditional view used to be that these investors aim to simply diversify risk and are passive, not seeking to influence the strategies of their portfolio companies. However, a new consensus is emerging in the field of industrial organization, which asserts that companies with common ownership engage in less intense competition compared to traditional profit-maximizing entities. The reduced competition leads to higher prices and profits, thereby enhancing the value of the companies involved. Taking this perspective, it appears that there are potential capital gains to be reaped, and hence, there must exist incentives for common ownership that extend beyond simple risk diversification.[1]

This paper examines these additional incentives. The starting point of our analysis is the observation that would-be common owners can not realize any capital gain by acquiring shares exclusively from dispersed shareholders. In fact, the acquisition of shares from dispersed shareholders is subject to the free-riding problem explained in Grossman and Hart (1980): far-sighted dispersed shareholders demand a price for their shares that reflects their ex-post value, thus incorporating any benefit from common ownership.

Yet, many firms have shareholders with substantial ownership stakes alongside a multitude of dispersed shareholders.[2] These larger blockholders may internalize the acquisition externality. That is, they might be willing to tender their shares for a price lower than the ex-post value, understanding that without doing so, the acquisition would not take place.

However, the acquisition of stakes by financial investors from such blockholders comes at a cost, as it exacerbates agency problems within companies. This is because when financial investors acquire shares from blockholders intending to mitigate the intensity of competition, they reduce the blockholders' residual ownership stake. This reduction weakens the blockholders' incentives to engage in value-enhancing behaviors, such as monitoring the managers. When financial investors are an imperfect substitute for blockholders in this capacity, a trade-off emerges: common ownership

[1] This suggests that common ownership levels should exceed universal ownership levels, as indeed empirically demonstrated by Amel-Zahed et al. (2022). While common ownership refers to financial investors holding shares in competing firms, universal ownership refers to them holding shares across all firms in the economy. It is universal ownership that can be explained by risk diversification motives, and any common ownership level surpassing universal ownership requires additional explanations.

[2] Amel-Zadeh et al. (2022) document the importance of such blockholders (i.e., large shareholders who are not purely financial investors) in the ownership structure of American firms. They show that in more than 10 percent of the S&P 500 single-class firms in the period 2003-2020, the single largest shareholder is indeed a blockholder (which according to their classification may be an activist financial investor, a non-financial large shareholder, or a corporate insider). Furthermore, such blockholders exist also in many other companies, where the largest shareholder is a non-activist financial investor. Most of these blockholders are "mavericks," i.e., relatively undiversified investors who hold a large stake in one firm only.

reduces the intensity of competition, which is beneficial for profits, but simultaneously degrades blockholders' incentives, which is detrimental to profits. Leaving risk diversification motives aside, the equilibrium level of common ownership must strike a balance between these conflicting effects.

Financial investors may be an imperfect substitute for blockholders due to a lack of ability or incentives to engage in value-enhancing behaviors. To account for these possibilities, we develop a specific model of corporate governance where each firm competing in a market has a manager who can appropriate private benefits of control. The firms' blockholders can monitor the managers to limit these opportunistic behaviors, but monitoring entails private costs. In this scenario, financial investors may be less efficient at monitoring than blockholders, or they may have the incentive to free-ride on the blockholders' efforts.[3]

It is important to note that in our model, financial investors do not necessarily acquire shares exclusively from large shareholders. Once financial investors have obtained a stake in a company from its blockholders, they can capture some advantages resulting from reduced competition by acquiring shares from dispersed shareholders, even if these shareholders cannot be exploited. Our analysis indeed demonstrates that if investors engage in common ownership, it is always optimal to acquire shares from both large blockholders and dispersed shareholders. However, there exists a limiting force for the latter acquisition, too: as the stake financial investors acquire from dispersed shareholders increases, the value of shares held by blockholders also increases, and financial investors must pay a higher price to acquire them.

In this framework, we identify several factors that determine the equilibrium level of common ownership, with the intensity of product market competition and the quality of corporate governance rules and institutions being among the most significant. Specifically, we demonstrate that as competition becomes more intense or the market more fragmented, the degree of common ownership increases. Additionally, the greater the need for monitoring managers, or, more generally, the scope for blockholders to engage in value-enhancing behaviors, the lower the degree of common ownership.

These findings imply that, in comparison to the market portfolio, large institutional investors should allocate more weight to sectors where market concentration is lower and the intensity of competition is higher, thereby providing more room for common ownership to enhance firm value. They also suggest that improvements

[3] In previous versions of this article, we have considered different models of corporate governance that generate a similar trade-off. First, we analyzed the case where the controlling blockholders themselves can appropriate private benefits at the expense of smaller shareholders. If such appropriation is inefficient, as in Burkart et al. (1998), the extent of overlapping ownership will be limited because smaller blockholders have less incentive to internalize the deadweight losses resulting from rent extraction. Additionally, we considered the case where blockholders make investments that can generate a competitive advantage for their respective firms. However, these investments come with private costs, as in Anton et al. (2023). For example, investments may reduce production costs or increase product quality. In this case, financial investors are imperfect substitutes for the blockholders because they have fewer incentives to make such investments, since a portion of their income comes from the profits of rival firms, which are damaged by those investments. The findings presented in this article apply to these alternative frameworks as well.

in corporate governance, which reduce the necessity for manager monitoring, benefit shareholders but have adverse effects on consumers, as they result in greater common ownership and ultimately higher prices.

While we mainly focus on the case where investors are external to the industry (we refer to this case as *external* common ownership), we also consider the case where a company's blockholder directly acquires a stake in a rival company (we denote this case as *internal* common ownership). As long as each blockholder has an advantage in monitoring the company it controls, the trade-off between softer competition and less effective governance also arises in this latter case. However, there are differences between internal and external common ownership. In particular, internal common ownership may be profitable even if the acquisition is solely from dispersed shareholders, as the benefits arising from reduced competition are captured by the acquirer through the initial stake it already owns in one of the firms.

The remainder of the paper is organized as follows. Section 2 provides a review of the relevant literature. In Section 3, we introduce the baseline model of common ownership with two symmetric firms operating in the product market. Section 4 analyzes the equilibrium ownership structure when financial investors exclusively acquire shares from large shareholders. Expanding on this, Section 5 considers the more general case of acquiring shares from both large and dispersed shareholders. Section 6 analyzes two extensions of the basic model: one involving more than two firms operating in the market and another where the two firms are asymmetric. In Section 7, we investigate the case of internal common ownership. Finally, Section 8 concludes the paper by discussing empirical and policy implications and possible extensions. Proofs are relegated to Appendix A, while Appendix B presents some specific examples of the product market equilibrium that confirm certain regularity properties invoked in the main text.

2 Relation to the literature

This paper borrows from two strands of the literature, one on industrial organization and the other one on corporate governance.

From the industrial organization literature, we borrow the notion that common ownership mitigates the intensity of product market competition. This notion was first put forward by Rotemberg (1984) and O'Brien and Salop (2000). These papers assume that companies act in the interest of their shareholders and that any heterogeneity in shareholders' interests is accounted for by forming a weighted average of their payoffs, with weights given by their respective ownership shares. As a result, under common ownership, each firm maximizes a linear combination of its own and rivals' profits. The higher the relative weight given to rivals' profits, which is alternatively referred to as the "lambda" or "kappa," the less aggressively firms compete in product markets, and hence the higher prices and profits.

While these early papers were relatively neglected for some time, economic research on common ownership has recently surged, driven by the observation that the degree of common ownership has significantly increased in recent decades. For example, Backus et al. (2021b) calculate the weight that S&P 500 companies would place on rivals' profits in their objective function under the proportional-control assumption and show that the average weight tripled in the last decades, from 0.2 in the 1980s to almost 0.7 in 2017.[4]

Recent theoretical literature extensively analyzes the impact of common ownership on competition among firms while proposing various mechanisms for managers to internalize minority shareholder interests. For example, Azar (2017) develops a theory where a company's management proposes a strategic plan to its shareholders and dislikes their disapproval or opposition.[5] Anton et al. (2023) study a mechanism based on managerial incentives. They argue that firms with common owners tolerate managerial slack to a higher degree to keep prices and profits high.[6] Schmalz (2021) reviews these and other possible governance mechanisms whereby common ownership may affect competitive outcomes.[7].

Most of the theoretical literature treats the degree of common ownership as exogenously given, with two notable exceptions, which, however, focus on different effects than those analyzed in this paper. First, Piccolo and Schneemeier (2020) endogenize common ownership levels in a model where ownership is initially fully dispersed. In their model, shares are exchanged due to the co-existence of both informed and liquidity traders. Informed traders may exploit liquidity traders because they possess superior information about market conditions, and because they internalize the lessening-of-competition effect of common ownership that arises when they hold stakes in different competing companies. In their model, common ownership has no negative effect on corporate governance, and the extent of common ownership is limited by the volume of noise trading, behind which informed traders may conceal themselves. Second, Spiegel (2013) and Levy et al. (2018) study the incentive for partial integration among vertically related firms. Focusing mostly on the case of cross-ownership, they argue that such partial integration may lead to input or customer foreclosure more than full integration. The higher profitability of the partial acquisition relies on the exploitation of the passive shareholders of the target company, who effectively subsidize the foreclosure. In our framework of horizontal competition, in contrast, passive shareholders generally benefit from common ownership.

It is worth noting that the impact of common ownership on product market competition remains in fact a topic of contention. Some scholars continue to adhere to the traditional view that most financial investors are passive and do not intervene

[4]The literature has also documented the extent of the phenomenon in specific industries. For example, Huse, Ribeiro and Verboven (2024) show that common-ownership links constitute between 31% and 39% of the equity ownership of automobile manufacturers. In this industry, cross-ownership links amount to 6%–9%.

[5]Azar (2020) further argues the anticompetitive effects of common ownership are mitigated when managers are entrenched and they only disappear in the extreme case where managers are fully insulated from shareholders' dissent.

[6]Frey et al. (2023) find, in a laboratory experiment, that common ownership leads to compensation packages for managers that reward them for reducing output and thus mitigating competition.

[7]Shekita (2021) analyzes empirically the channels through which common ownership influences firm behavior. Studying 30 cases of common ownership, he documents three main corporate governance mechanisms – voice and engagement, executive compensation, and voting – that affect firm decision-making.

in their portfolio companies, thus they cannot facilitate anticompetitive behavior. See e.g. Bebchuk et al. (2017) for a recent articulation of this view. As theory remains unsettled, the debate has shifted to empirical terrain.

However, the empirical effects of common ownership are also controversial. In a pioneering contribution, Azar, Schmalz and Tecu (2018) show that common ownership increases prices in the U.S. airline industry. Their findings are confirmed in the analysis carried out by Park and Seo (2019). However, Kennedy et al. (2017) and Dennis et al. (2022), using a different structural model of the US airline industry or different measures of investor control of airlines operating in bankruptcy, do not find evidence that common ownership raises airline prices.[8] Azar, Schmalz and Tecu (2022) address these critiques and argue that, in fact, they do not invalidate their main finding.

Moving beyond the airline industry, He and Huang (2017) find evidence suggesting that institutional cross-ownership facilitates explicit forms of product market collaboration (e.g., within-industry joint ventures, strategic alliances, or within-industry acquisitions) and improves innovation productivity and operating profitability.[9] On the other hand, Backus et al. (2021a) find little support for markup effects of common ownership in the ready-to-eat cereal industry, and Koch et al. (2021) find that common ownership is neither robustly positively related with industry profitability nor is it robustly negatively related with measures of non-price competition.

Despite the ongoing controversies, a consensus seems to be emerging that common ownership does reduce product market competition.[10] Some authors have ventured to quantify the welfare effects of common ownership. For example, Ederer and Pellegrino (2022) estimate that the welfare cost of common ownership, measured as the ratio of deadweight loss to total surplus, has increased more than tenfold in about 20 years, from 0.3% in 1994 to over 4% in 2018.

The second strand of the literature that is relevant to our paper deals with the impact of the ownership structure on corporate governance and firm value. Jensen and Meckling (1976) argue that the ownership structure that maximizes total firm value is the one where the entrepreneur or the manager is the sole owner as it reduces agency costs. One component of the agency cost is monitoring costs. Manne (1965),

[8]In a similar vein, Lewellen and Lowry (2021) contend that the effects that are commonly attributed to common ownership are caused by other factors, such as differential responses of firms (or industries) to the 2008 financial crisis. Controlling for these factors, they find little robust evidence that common ownership affects firm behavior.

[9]Theory indeed shows that overlapping ownership may affect not only prices but also other strategic choices of the firms, such as for instance investment in R&D: see e.g. Lopez and Vives (2019).

[10]While most of the literature focuses on a single industry, Azar and Vives (2021) analyze common ownership in a general equilibrium oligopoly model and distinguish between intra-sectoral common ownership, which reduces the intensity of competition, and inter-sectoral common ownership, which enhances it. In an attempt to empirically test this prediction, Azar and Vives (2022) reexamine the US airline industry. They find that common ownership by the Big Three (BlackRock, Vanguard, and State Street), used as a proxy for inter-sectoral common ownership, is associated with lower airline prices. In contrast, common ownership by other investors, used as a proxy for intra-sectoral common ownership, is associated with higher prices. This article focuses only on the intra-sectoral case.

on the other hand, presents an opposing viewpoint, suggesting that even firms with dispersed ownership can maximize share value in the presence of an active market for corporate control, as deviations from value maximization would trigger a disciplinary takeover that ousts the less competent or opportunistic manager. However, Grossman and Hart (1980) demonstrate the flaws in this argument, revealing that atomistic shareholders tend to free-ride on each other, resulting in the failure of value-increasing tender offers. Shleifer and Vishny (1986) show that large shareholders can mitigate the free-rider problem highlighted by Grossman and Hart. In fact, a large shareholder, while not profiting from shares acquired in a tender offer, can benefit from the toehold owned prior to the takeover.

A common theme among these papers is the idea that more concentrated ownership can lead to higher firm value. However, the degree of ownership concentration is constrained by various factors. One such factor is that increased concentration often entails reduced diversification: see Demsetz and Lehn (1985). Furthermore, Burkart et al. (1997), building on Aghion and Tirole (1997), highlight that increased monitoring by a large shareholder can dampen managerial initiative. The optimal ownership structure strikes a balance between the manager's incentives to exert effort and the large shareholder's monitoring.[11]

3 The model and preliminary results

In this section, we outline the assumptions of our baseline model of common ownership and derive some preliminary results.

3.1 Model assumptions

As mentioned in the introduction, we focus on financial investors whose acquisition of stakes in competing firms is motivated by the anticipated increase in firm value due to reduced competition. Additionally, we assume the presence of blockholders who are initially large enough to internalize the acquisition-price externality and to monitor the managers. For simplicity, we consider a single financial investor, and we assume that initially in each company, there is a single blockholder, along with a mass of dispersed shareholders.

3.1.1 Agents

Consider two symmetric firms, denoted as $i = 1, 2$, that compete in the same product market.[12] Initially, firm i is owned by blockholder $\mathcal{B}_i$, who possesses a fraction β of the firm's shares, and a multitude of dispersed shareholders that collectively hold

[11] For a comprehensive examination of corporate governance and its influence on firm ownership and firm value, see Shleifer and Vishny (1997).

[12] In section 6, we will extend the analysis to encompass more than two firms and explore asymmetries among them.

the remaining fraction $1-\beta$.[13]

A financial investor, denoted as $\mathcal{I}$, may acquire a stake s_i of firm i from its initial owners. It is assumed that $\mathcal{I}$'s portfolio is already well diversified.[14] Therefore, $\mathcal{I}$ will proceed with the acquisition with the sole purpose of realizing the capital gains that may arise due to the impact of common ownership on the intensity of competition.

We allow the investor to acquire shares from both the blockholder and dispersed shareholders. Denote the stake acquired from the blockholder as s_i^B and from the dispersed shareholders as s_i^D. Thus, $s_i = s_i^B + s_i^D$, and after the acquisition, $\mathcal{B}_i$ will retain a remaining ownership share of $\beta - s_i^B$.

Firm i is run by a manager denoted as $\mathcal{M}_i$. If not supervised, the manager diverts a fraction ξ of the firm's profits for her personal benefit.[15] However, managers' ability to divert resources to themselves is limited by the monitoring activities of shareholders. We assume that monitoring efforts are solely undertaken by blockholders. It is indeed common in corporate governance literature to assume that dispersed shareholders free-ride on the monitoring efforts of larger shareholders and do not contribute any efforts of their own. Similarly, we assume that financial investors do not actively monitor the manager due to their limited monitoring capabilities or because they, too, free-ride on the blockholders' efforts. However, we could allow for monitoring by financial investors as long as they are imperfect substitutes for blockholders in this capacity. The assumption that financial investors do not engage in monitoring at all is made solely for simplicity.

Blockholder $\mathcal{B}_i$'s monitoring, denoted as m_i, reduces the manager's private benefits from $\xi\pi_i$ to $\xi(1-m_i)\pi_i$. The private cost of monitoring, $C(m_i)\pi_i$, is assumed to be proportional to the firm's profit. This facilitates the analysis by making $\mathcal{B}_i$'s optimal choice of m_i independent of product market competition. The function $C(m_i)$ is assumed increasing and convex, with $C(0) = 0$. To ensure the existence of an interior solution, we assume $C'(0) = 0$ and $C'(1) > \xi$.

3.1.2 Payoffs

Under these assumptions, blockholder $\mathcal{B}_i$'s payoff can be expressed as:

$$B_i = \left(\beta - s_i^B\right)\left[1-\xi(1-m_i)\right]\pi_i - C(m_i)\pi_i + P_i^B. \tag{1}$$

The first term on the right-hand side denotes the value of $\mathcal{B}_i$'s remaining stake $(\beta - s_i^B)$, the second term represents the cost of monitoring, and P_i^B represents the revenue obtained from the sale of the stake s_i^B. Similarly, the investor's payoff is given by:

$$I = \sum_{i=1}^{2}\left\{s_i\left[1-\xi(1-m_i)\right]\pi_i - P_i^B - P_i^D)\right\}, \tag{2}$$

[13] As a rule, we use Latin letters to denote endogenous variables, calligraphic letters for agents, and Greek letters for exogenous parameters. The one exception is profits, which are denoted by π.

[14] See Shy and Stenbacka (2020) for a model that instead emphasizes the diversification motive for common ownership.

[15] We use feminine pronouns for managers, masculine pronouns for blockholders, and neutral pronouns for financial investors.

where P_i^D is the total payment to dispersed shareholders for the shares s_i^D.

Regarding managers, we assume that they appropriate whatever private benefits they can while being subject to monitoring by blockholders. Additionally, managers are responsible for making decisions related to product market competition. In this respect, we adopt the proportional-control assumption of Rotemberg (1984) and O'Brien and Salop (2000), which posits that managers aim to maximize a weighted average of shareholders' payoffs, with the weights determined by their respective ownership shares:

$$\tilde{O}_i = \left(\beta - s_i^B\right) B_i + \theta s_i I. \tag{3}$$

The payoff of dispersed shareholders does not appear in (3) because the term representing a generic dispersed shareholder, $\mathcal{D}_h$, who holds a share ε_{hi} of firm i, is $\varepsilon_{hi}(\varepsilon_{hi}\pi_i)$. Therefore, when $\varepsilon_{hi} \approx 0$, this term becomes second-order and negligible. Consequently, the interests of dispersed shareholders do not influence managerial decisions.

Note that (3) slightly generalizes the proportional-control assumption by allowing the level of influence of the financial investor to be lower than that of the blockholder, as indicated by parameter $\theta \leq 1$. As discussed in Section 2, several mechanisms have been proposed to explain why managers may, to some extent, consider the interests of financial investors who hold minority stakes in their companies. Our findings are not dependent on a specific mechanism and remain applicable as long as $\theta > 0$.[16]

While we could develop our analysis using the formulation (3) with qualitatively similar results, it is convenient to simplify it by replacing B_i with $\left(\beta - s_i^B\right)\pi_i$ and I with $\sum_{j=1}^{2} s_j \pi_j$. This simplification means that when managers evaluate the shareholders' payoffs, they do not subtract the profits they appropriate (otherwise, they would have to assign a negative weight to these profits). Additionally, the managers ignore the monitoring costs (which are non-monetary costs borne privately by blockholders) and the acquisition prices (which only have a redistributive impact among blockholders and investors).

This results in the following simplified objective function::

$$O_i = \left(\beta - s_i^B\right)^2 \pi_i + \theta s_i \sum_{j=1}^{2} s_j \pi_j, \tag{4}$$

which rewrites as:

$$O_i = \pi_i + \lambda_i \pi_j, \tag{5}$$

where:

$$\lambda_i = \frac{\theta s_i s_j}{\left(\beta - s_i^B\right)^2 + \theta s_i^2}. \tag{6}$$

is the weight assigned by the managers of firm i to the rival firm's profit. This weight is zero if either s_i, s_j, or both, vanish.

[16]It is worth noting that the assumption of $\theta > 0$ is not necessary in the model of section 7, where the effects discussed still arise even if $\theta = 0$.

3.1.3 Product market competition

For the sake of generality, we adopt a reduced-form model for product market competition. Each firm i selects a strategic variable x_i (such as price or quantity), and these choices determine the firms' profits $\pi_i(x_i, x_j)$. (For ease of notation, we treat x_i as a scalar, but the analysis remains the same if it were a vector). As the firms are ex-ante symmetric, the functions $\pi_i(x_i, x_j)$ are assumed to be symmetric as well. Furthermore, we assume that these functions are quasi-concave and twice continuously differentiable within the relevant range. In Appendix B, we provide specific models of product market competition that satisfy these assumptions.

3.1.4 Acquisition prices

Due to the free-riding among dispersed shareholders, the payment to them, P_i^D, must be equal to the ex-post share value. By contrast, the acquisition prices P_i^B are established through negotiations between the investor and the blockholders.

We assume that with probability α, the investor makes an offer to the blockholders; with the complementary probability of $1 - \alpha$, these roles are reversed, and it is the blockholders who simultaneously and independently make an offer to the investor. Therefore, α represents a measure of the investor's bargaining power.

We allow the proposer(s) to offer payment schedules of the type $P_i^B(s_i^B, s_j^B)$, with the choice of the stakes left to the receiver. This formulation includes take-it-or-leave-it offers as a special case, where the requested (respectively offered) payment is very large (respectively very low) for all stakes except those that the proposer wants to enforce. When blockholders make the offers and the ensuing game has multiple equilibria, we restrict attention to the equilibrium that is Pareto-dominant for the blockholders.

Note that our formulation allows the contract between $\mathcal{I}$ and $\mathcal{B}_i$ to depend on the agreement between $\mathcal{I}$ and $\mathcal{B}_j$. As we shall see, however, in the baseline model, the results remain unchanged even if such conditioning were not possible.

3.1.5 Timing

The game proceeds in three stages. In the first stage, investor $\mathcal{I}$ chooses the stakes $s_i^D \geq 0$ to be acquired from dispersed shareholders and engages in negotiations with the blockholders regarding the stakes to be acquired, $s_i^B \geq 0$, and the acquisition prices, P_i^B. (Note that both stakes s_i^D and s_i^B can be zero.) In the second stage, firms engage in product market competition, which determines the equilibrium profits π_i. Lastly, in the final stage of the game, blockholders select their monitoring efforts m_i, and the payoffs are realized.

3.2 Preliminary results

We are interested in the subgame perfect equilibrium of the game, and thus we solve the model in reverse order. In this subsection, we present the equilibrium in the last two stages of the game. The analysis of acquisition prices and the equilibrium ownership structure will be addressed in the subsequent sections.

3.2.1 Monitoring

In the final stage of the game, blockholder $\mathcal{B}_i$ selects m_i to maximize his payoff, B_i. In this stage, the values of s_i^B, P_i^B and profits π_i are pre-determined, so the blockholder's objective function reduces to:

$$\left\{\left(\beta - s_i^B\right)\left[1 - \xi(1 - m_i)\right] - C_i(m_i)\right\}\pi_i.$$

Since our assumptions ensure an interior solution, the equilibrium level of monitoring is determined by the first-order condition:

$$C'(m_i) = \left(\beta - s_i^B\right)\xi. \tag{7}$$

Note that this level of monitoring is inefficiently low from the shareholders' aggregate perspective. From this viewpoint, the optimal monitoring would be determined by the condition $C'(m) = \xi$.

As mentioned earlier, our specification of monitoring costs implies that the optimal level of monitoring, m_i^*, does not depend on π_i. The convexity of $C(m_i)$ implies that it increases with the blockholder's residual ownership share, $\beta - s_i^B$. To highlight this dependence, we will write $m_i^* = m^*(\beta - s_i^B)$.

3.2.2 Product market equilibrium

When firms compete in the product market, manager $\mathcal{M}_i$ chooses x_i to maximize $O_i = \pi_i + \lambda_i \pi_j$. To keep things simple, we assume the existence of a unique interior Nash equilibrium, which is characterized by the following first- and second-order conditions:

$$\frac{\partial \pi_i}{\partial x_i} + \lambda_i \frac{\partial \pi_j}{\partial x_i} = 0 \tag{8}$$

$$\left(\frac{\partial^2 \pi_i}{\partial x_i^2} + \lambda_i \frac{\partial^2 \pi_j}{\partial x_i^2}\right) < 0. \tag{9}$$

Equilibrium profits depend on the weights λ are thus denoted as $\pi_i^*(\lambda_i, \lambda_j)$.

4 Equilibrium ownership structure

We now characterize the ownership structure of the firms and examine how it is influenced by the underlying economic parameters.

In our model, the investor has the option to acquire shares from dispersed shareholders, blockholders, or both. To gradually develop an understanding of the drivers of common ownership, in this section, we focus on the scenario where $\mathcal{I}$ exclusively acquires share from the blockholders $\mathcal{B}_1$ and $\mathcal{B}_2$. Given the restriction $s_i^D = 0$, we have $s_i^B = s_i$, allowing us to simplify the notation by omitting the superscripts. Furthermore, since the weights λ_i now depend only on the stakes s_i, we will denote equilibrium profits $\pi_i^*(\lambda_i, \lambda_j)$ as $\pi_i^*(s_i, s_j)$. In the following section, we will analyze the acquisition from dispersed shareholders.

4.1 The negotiation stage

To proceed, we provide an implicit characterization of the stake levels (s_i^*, s_j^*) that, in equilibrium, the investor will agree to acquire from the blockholders.

Proposition 1 *For any $\alpha \in [0,1]$, the equilibrium ownership structure maximizes the joint payoff of the investor and the blockholders:*

$$S = I + \sum_{i=1}^{2} B_i = \sum_{i=1}^{2} \{\beta\,[1 - \xi(1 - m^*(\beta - s_i))] - C\,(m^*(\beta - s_i))\}\,\pi_i^*(s_i, s_j) \quad (10)$$

It is instructive to sketch here a proof of the result for the case $\alpha = 1$, where the investor makes offers to the blockholders. First of all, it appears that since offers may be contingent, $\mathcal{I}$ would not commit to purchasing the target stake in firm i from $\mathcal{B}_i$ unless an agreement with $\mathcal{B}_j$ is also reached. This is because the lessening-of-competition effect of common ownership vanishes if either stake, s_i or s_j, becomes zero. Therefore, the outside option for blockholder $\mathcal{B}_i$ in this scenario is:

$$\bar{B}_i = \beta\,[1 - \xi(1 - m^*(\beta))]\,\pi_i^*(0,0) - C(m^*(\beta))\pi_i^*(0,0), \quad (11)$$

and thus is independent of the stakes (s_i, s_j).

Next, note that whatever the stakes, the investor will set acquisition prices that leave the blockholders with their reservation payoffs,[17] collecting all the value in excess of them, $S - \sum_{i=1}^{2} \bar{B}_i$. But since $\bar{B}_i$ is independent of (s_i, s_j), the equilibrium stakes (s_i^*, s_j^*) must maximize S.

The same result is obtained when offers cannot be conditioned, and hence $\mathcal{I}$ must purchase from $\mathcal{B}_i$ even without an agreement with $\mathcal{B}_j$. The reason for this is that in this case, the outside option for blockholder $\mathcal{B}_i$ is given by:

$$\overline{\overline{B}}_i(s_j) = \beta\,[1 - \xi(1 - m^*(\beta))]\,\pi_i^*(0, s_j) - C(m^*(\beta))\pi_i^*(0, s_j). \quad (12)$$

However, it appears from (6) that both weights λ_i and λ_j become zero as soon as either stake, s_i or s_j, becomes zero. This implies that $\pi_i^*(0, s_j) = \pi_i^*(0,0)$. Consequently, the two reservation payoffs actually coincide: $\overline{\overline{B}}_i(s_j) \equiv \bar{B}_i$.

[17] Thus, the acquisition prices when $\alpha = 1$ are:

$$\begin{aligned} P_i^L \;\; = \;\; & \{\beta\,[1 - \xi(1 - m^*(\beta))] - C(m^*(\beta))\}\,\pi_i^*(0,0) + \\ & - \{(\beta - s_i)\,[1 - \xi(1 - m^*(\beta - s_i))] - C(m^*\,(\beta - s_i))\}\,\pi_i^*(s_i, s_j). \end{aligned}$$

The complete proof of the proposition in the appendix shows that the result continues to hold even if $\alpha < 1$: the parameter α only affects the acquisition prices and does not impact the equilibrium ownership structure.[18]

4.2 The monitoring-competition trade-off

We will now demonstrate that the choice of the ownership structure entails a trade-off between reduced competition in the product market and diminished monitoring.

Due to the symmetry of firms, we focus on the case where the equilibrium ownership structure is symmetric, $s_1^* = s_2^* = s^*$, and refer to s^* as the equilibrium degree of common ownership. Proposition 1 then implies that s^* maximizes:

$$S = \nu^*(s)\Pi^*(s). \tag{13}$$

where

$$\nu^*(s) = \beta \left\{1 - \xi \left[1 - m^*(\beta - s)\right]\right\} - C\left[m^*(\beta - s)\right] \tag{14}$$

and

$$\Pi^*(s) = \pi_1^*(s, s) + \pi_2^*(s, s). \tag{15}$$

The first factor in expression (13), ν^*, represents the large shareholders' aggregate payoff per unit of profit, taking into account monitoring costs and managers' private benefits. We denote this term as effective cash flow rights. The second factor, Π^*, corresponds to industry profits.

A change in s affects both factors. Formally, from equation (13), we obtain:

$$\frac{dS}{ds} = \nu^*\Pi^*\left(\frac{\partial \nu^*}{\partial s}\frac{1}{\nu^*} + \frac{\partial \Pi^*}{\partial s}\frac{1}{\Pi^*}\right) \tag{16}$$

We now demonstrate that the two terms in brackets have opposite signs. The first term, representing the impact of a change in s on effective cash flow rights, captures the marginal cost of common ownership in terms of reduced monitoring. On the other hand, the second term, which is the semi-elasticity of industry profits with respect to s, represents the marginal benefit in terms of softer product market competition.

[18] The acquisition prices when the investor makes the offer, P_i^L, are reported in the previous footnote. When instead the blockholders make the offers, which occurs with a probability of $1-\alpha$, the investor's payoff is set to its reservation value, which is zero. Therefore:

$$P_i^H = s_i\left[1 - \xi(1 - m^*(\beta - s_i))\right]\pi_i^*(s_i, s_j).$$

Consequently, the average acquisition prices are:

$$\begin{aligned} P_i^B &= \alpha P_i^L + (1-\alpha) P_i^H \\ &= \alpha\left\{\beta\left[1 - \xi(1 - m^*(\beta))\right] - C(m^*(\beta))\right\}\pi_i^*(0,0) + \\ &\quad -\alpha\left\{\beta\left[1 - \xi(1 - m^*(\beta - s_i))\right] - C(m^*(\beta - s_i))\right\}\pi_i^*(s_i, s_j) + \\ &\quad +s_i\left[1 - \xi(1 - m^*(\beta - s_i))\right]\pi_i^*(s_i, s_j). \end{aligned}$$

4.2.1 The corporate-governance effect

From equations (14) and (7), we obtain

$$\begin{aligned}\frac{\partial \nu^*}{\partial s} &= \left[\xi\beta - C'(m^*(\beta - s)\right] \frac{\partial m^*(\beta - s)}{\partial s} \\ &= -\frac{\xi\left[\xi\beta - C'(m^*(\beta - s))\right]}{C''(m^*(\beta - s))} \leq 0. \end{aligned} \tag{17}$$

The derivative is zero when $s = 0$. Intuitively, an increase in the degree of common ownership reduces monitoring, as the blockholder retains a smaller residual stake and thus has less incentive to exert effort. However, at $s = 0$ monitoring is at the efficient level from the perspective of the large shareholders, so the decrease in monitoring has only a second-order effect. Formally, when $s = 0$ we have $\xi\beta = C'(m^*(\beta))$, causing the numerator of (17) to vanish. In contrast, when $s > 0$, monitoring becomes inefficiently low, and any increase in s, which further reduces monitoring, leads to a decrease in effective cash flow rights, ν^*.

4.2.2 The softening-of-competition effect

An increase in the degree of common ownership leads to a rise in industry profits Π^* by reducing competition in the product market.

Lemma 1 *If $\theta > 0$,[19] industry profits monotonically increase with the degree of common ownership s:*

$$\frac{\partial \Pi^*}{\partial s} \geq 0.$$

The derivative is strictly positive for $0 < s < \beta$ and becomes zero at $s = 0$ and $s = \beta$.

Industry profits increase because, as s increases, each firm assigns greater importance to the rival's profits and adopts a less aggressive stance. This effect is well-known in the literature on common ownership. Nevertheless, it is important to highlight that the marginal effect vanishes when common ownership is very low ($s = 0$) or very high ($s = \beta$).

At $s = 0$, the marginal effect vanishes because the weight

$$\lambda = \frac{\theta s^2}{(\beta - s)^2 + \theta s^2} \tag{18}$$

depends on the product of the two stakes. Hence, the impact of an increase in s on λ is second order at $s = 0$. At $s = \beta$, the marginal effect disappears for two reasons. First, as s approaches β, the impact of s on λ vanishes. This can be seen from the derivative:

$$\frac{\partial \lambda}{\partial s} = \frac{2\theta s\beta\,(\beta - s)}{\left[(\beta - s)^2 + \theta s^2\right]^2}. \tag{19}$$

[19] When $\theta = 0$, $\lambda = 0$ irrespective of the value of s, and common ownership does not impact profits.

Second, when $s = \beta$, the weight λ equals one, indicating perfect collusion between firms. Consequently, near this point, industry profits are close to their maximum, implying that a slight change in s has a second-order effect.

This suggests that the impact of increasing the level of common ownership on industry profits is most significant for intermediate levels of s. Appendix B demonstrates that under various commonly employed models of the product market, the derivative $\frac{\partial \Pi^*}{\partial s}$ exhibits an inverted U-shape.

4.3 The limits to common ownership

From the above, it appears that at $s = \beta$, the positive effect of common ownership on industry profits vanishes, while the negative effect on monitoring reaches its peak. This implies that as the degree of common ownership rises, the negative effect on monitoring must eventually surpass the positive effect on profits. Consequently, we have:

Proposition 2 *Common ownership is always partial:* $0 \leq s^* < \beta$.

While the equilibrium degree of common ownership is always limited in our model, it may not always be positive. Finding simple conditions to ensure that $s^* > 0$ is, however, challenging. The reason for this difficulty is that at $s = 0$, both the marginal cost and the marginal benefit of increasing s vanish, as previously discussed. Therefore, local conditions alone are insufficient to determine when the point $s = 0$ is a global maximum of the function $S(s)$.

Specifically, the point $s = 0$ can be a global maximum, a local maximum (but not global), or a local minimum. This is because, considering that the marginal cost of common ownership always exceeds the marginal benefit at $s = \beta$, only three possibilities can arise, except for degenerate cases. First, the marginal cost curve, $MC = -\frac{\partial \nu^*}{\partial s}\frac{1}{\nu^*}$, may lie entirely above the marginal benefit curve, $MB = \frac{\partial \Pi^*}{\partial s}\frac{1}{\Pi^*}$, implying that $s = 0$ is a global maximum, as shown in Figure 1. Second, the marginal cost curve may intersect the marginal benefit curve an even number of times, starting from above. In this case, $s = 0$ is a local maximum, but not necessarily a global one, as illustrated in Figure 2. Third, the marginal cost curve may intersect the marginal benefit curve an odd number of times, starting from below, implying that $s = 0$ is a minimum, as depicted in Figure 3.

Depending on the specific combination of parameter values, any of these scenarios can occur. But while local conditions may be sufficient to identify when $s = 0$ is a local minimum, global conditions are necessary to determine when it becomes a global maximum.

Also, note that in the intermediate case depicted in Figure 2, when both local maxima become global maxima, any perturbation in the underlying parameters will cause a jump in the equilibrium degree of common ownership from 0 to a strictly positive level, or vice versa. This necessitates extra care in the comparative statics analysis.

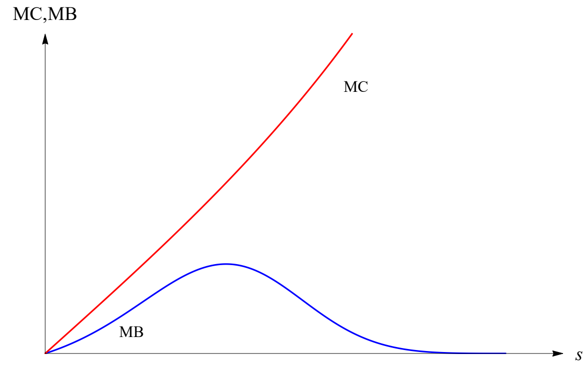

Figure 1: The case where the marginal cost of common ownership always exceeds the marginal benefit, and hence $s = 0$ is a global maximum. The picture is drawn for the case of a quadratic monitoring cost function (19) and the product market specification of Example 1 in Appendix B, with $\gamma = 2$, $\beta = \frac{4}{5}$, $\theta = \frac{4}{5}$, $\delta = \frac{3}{5}$, and $\xi = \frac{3}{4}$.

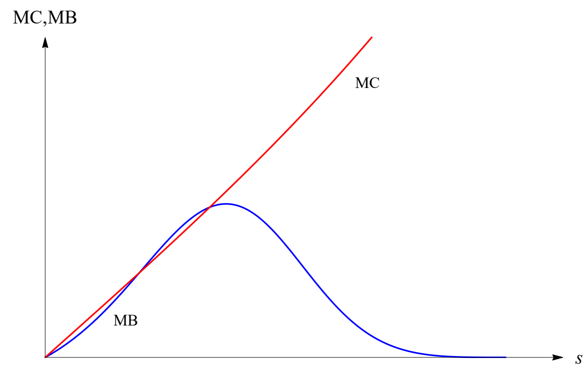

Figure 2: The case where the Marginal Cost curve cuts the Marginal Benefit curve twice, starting from above, and hence $s = 0$ is a local but not necessarily a global maximum. (Whether $s = 0$ is a local maximum or not depends on which of the two areas between the curves is larger.) The picture is drawn under the same assumptions as Figure 1, except that now $\xi = \frac{19}{30}$.

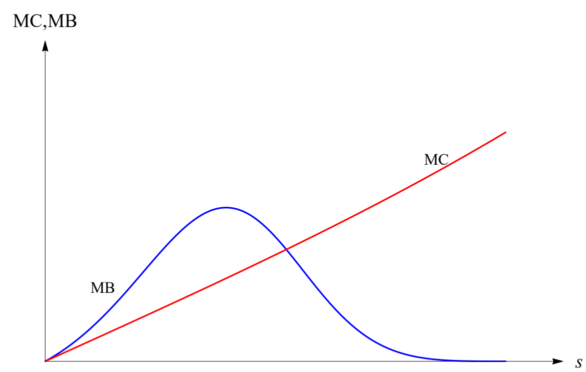

Figure 3: The case where the Marginal Cost curve cuts the Marginal Benefit once from below, and hence $s = 0$ is a global minimum. The picture is drawn under the same assumptions as Figure 1, except that now $\xi = \frac{1}{2}$.

4.4 Comparative statics

We will now analyze the role of factors determining the equilibrium level of common ownership. Some of these factors, related to corporate governance, affect only the residual income per unit of profit, ν^*, while others, related to product market competition, affect only industry profits, Π^*. We will explore each category in turn.

4.4.1 Corporate governance

Generally speaking, corporate governance rules and institutions play a crucial role in reducing shareholders' expropriation by the manager. In legal systems that offer greater protection to shareholders, managers have fewer opportunities to extract private rents, and monitoring is easier and more cost-effective.

To quantify this latter effect, it is convenient to use a quadratic specification of the monitoring-cost function:

$$C(m) = \frac{1}{2}\gamma m^2, \tag{20}$$

where the parameter γ measures the costliness of monitoring, while the parameter ξ captures the opportunity to extract private rents.[20] Thus, similar to Burkart et al. (2003), in our model legal protection may reduce manager expropriation both by directly limiting opportunities to extract private rents, as captured by parameter ξ, and more indirectly by decreasing the cost of monitoring γ.

Both parameters ξ and γ exhibit an inverse relationship with the "quality" of corporate institutions. This inverse relationship implies that they both diminish the effective cash flow rights, $\nu = \beta\left[1 - \xi\left(1 - m\right)\right] - \frac{1}{2}\gamma m^2$, for any given monitoring level m. However, the impact of ξ and γ on the equilibrium level of common ownership differs. A lower ξ leads to an increase in common ownership, but a lower γ results in a decrease in common ownership.

Proposition 3 *The equilibrium level of common ownership s^* monotonically decreases with ξ and monotonically increases with γ.*

This result implies that better corporate governance may yield contrasting effects on the equilibrium level of common ownership, depending upon its specific form. When augmented legal protection directly curtails managers' capacity to expropriate shareholders, the necessity for managerial monitoring diminishes. This reduces the cost of common ownership, i.e., its negative impact on the incentive for monitoring, and increases its equilibrium level. From this viewpoint, superior legal protection should positively correlate with common ownership. However, if better corporate governance makes monitoring more cost-effective, it becomes more important to maintain monitoring incentives. This renders common ownership more costly, reducing its equilibrium level.

In essence, the crucial factor is whether improvements in corporate governance serve as a complement or substitute for monitoring. When they act as a substitute by

[20] With this specification, condition $C'(1) > \xi$, which guarantees an interior solution for m^*, becomes $\gamma > \xi$.

directly limiting managers' rent extraction opportunities at no cost to shareholders, enhanced legal protection fosters greater common ownership. But if improvements in corporate governance facilitate monitoring, they tend to be associated with reduced common ownership.

In practice, discerning whether a specific change in corporate governance law augments or diminishes the optimal level of monitoring, and consequently results in increased or decreased common ownership, may prove challenging. Nevertheless, it is important to acknowledge the potential subtle implications of enhancements in corporate governance, which may confer benefits upon shareholders but could detrimentally impact consumers by leading to increased common ownership and, ultimately, higher prices.

Finally, observe that while the effect of changes in ξ and γ on s^* is monotone, it is not necessarily smooth. As noted, in our model, an infinitesimal change in the exogenous parameters may cause a discrete jump in the equilibrium degree of common ownership. However, Proposition 3 shows that s^* always jumps upward (respectively, downward) as γ (respectively, ξ) increases. Similar remarks apply to Proposition 4 below.

4.4.2 Product market competition

Let us now explore the factors that influence how common ownership affects industry profits Π^*. One such factor is the degree of influence exerted by the financial investor, θ, which affects $\frac{d\lambda}{ds}$ and consequently $\frac{\partial \Pi^*}{\partial s}$. Another factor is the level of product market competition. Although this variable has not been explicitly defined so far, it undoubtedly plays a role in determining how Π^* depends on s.[21]

There are various ways to parameterize the intensity of product market competition. For the sake of our comparative statics, we will represent the intensity of competition by introducing a generic parameter σ that increases the semi-elasticity $\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}$:

$$\frac{\partial}{\partial \sigma}\left(\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}\right) > 0. \tag{21}$$

Intuitively, a higher level of competition results in lower profits Π^* for firms when $\lambda = 0$ (i.e., when each firm solely maximizes its own profits). When instead $\lambda = 1$, firms effectively do not compete, and they always achieve monopoly profits. Hence, the stronger the competition, the greater the increase in industry profits when transitioning from $\lambda = 0$ to $\lambda = 1$. Condition (21) asserts that this holds not only for the transition from $\lambda = 0$ to $\lambda = 1$ but also for any small increment in λ: it is essentially a monotonicity condition. Appendix B demonstrates that parameter σ may capture various commonly utilized measures of competition intensity in standard models, such as an increase in the degree of product substitutability or a switch from Cournot to Bertrand. All of these measures align with our definition (21).

Proposition 4 *The equilibrium level of common ownership s^* monotonically increases with θ and with the intensity of product market competition σ.*

[21] On the other hand, neither of these factors affects ν^*.

The underlying intuition is as follows. With a higher θ, indicating greater influence of the financial investor in strategic decisions, the softening-of-competition effect resulting from common ownership becomes more pronounced. Consequently, investors have a stronger incentive to acquire stakes in competing firms. Similarly, the impact of common ownership on competition reduction becomes more significant in the presence of intense competition. For instance, when the two companies operate in separate markets, there is no competition even at $\lambda = 0$, rendering common ownership devoid of any benefits.

5 Dispersed shareholders

We now return to the general model, where the financial investor can purchase shares not only from the blockholders but also from dispersed shareholders. Following Grossman and Hart (1980), we assume that dispersed shareholders are forward-looking and atomistic, meaning that they will only sell at a price that fully reflects the ex-post value of their shares. We will demonstrate that while the atomistic nature of dispersed shareholders makes it impossible for the financial investor to directly benefit from buying shares from them, such acquisitions can yield an indirect gain, as the value of the shares acquired from the blockholders may increase.

5.1 The investor's payoff

Let us reconsider the negotiation between the investor and the blockholders in this new scenario. To simplify the exposition, we focus on the case $\alpha = 1$.

Since the price paid to dispersed shareholders must be equal to the value of the shares acquired from them, the investor's payoff (2) becomes, denoting equilibrium profits as $\pi_i^*(s_i^B, s_i^D, s_j^B, s_j^D)$:

$$I = \sum_{i=1}^{2} s_i^B \left[1 - \xi(1 - m^*(\beta - s_i^B))\right] \pi_i^*(s_i^B, s_i^D, s_j^B, s_j^D) - \sum_{i=1}^{2} P_i^B. \tag{22}$$

With $\alpha = 1$, the prices paid to the blockholders, P_i^B, must precisely guarantee their reservation payoff. Through conditional offers, the investor can commit to acquiring a stake from $\mathcal{B}_i$ only upon reaching an agreement with $\mathcal{B}_j$. Consequently, the reservation payoff for blockholder $\mathcal{B}_i$ is the value of its stake if the investor purchases only from dispersed shareholders:[22]

$$\bar{B}_i = \nu^*(0)\pi_i^*(0, s_i^D, 0, s_j^D). \tag{23}$$

[22]The distinction between conditional and unconditional offers now becomes significant. This is because when the investor acquires stakes in both firms from dispersed shareholders, the property $\pi_i^*(0, s_i^D, s_j^B, s_j^D) = \pi_i^*(0, s_i^D, 0, s_j^D)$ does not necessarily hold, and the blockholder's disagreement payoff with conditional and unconditional offers may differ. This complicates the analysis of unconditional offers, as discussed in greater detail in the next footnote, but does not alter the main new economic effect discussed in this section.

Therefore, acquisition prices are:

$$P_i^B = \left\{(\beta - s_i)\left[1 - \xi(1 - m^*(\beta - s_i^B))\right] - C(m^*\left(\beta - s_i^B\right))\right\} \pi_i^*(s_i^B, s_i^D, s_j^B, s_j^D) - \bar{B}_i,$$

which allows us to rewrite the investor's payoff as:

$$\begin{aligned} I &= \sum_{i=1}^{2} \nu^*(s_i^B)\pi_i^*(s_1^B, s_1^D, s_2^B, s_2^D) + \\ &\quad - \sum_{i=1}^{2} \nu^*(0)\pi_i^*(0, s_i^D, 0, s_j^D). \end{aligned} \tag{24}$$

The first line of this expression represents the joint payoff of the large shareholders, S. In the case examined in the previous section, the equilibrium profit in the second line would be $\pi_i^*(0,0,0,0)$, rendering the second line a constant. Consequently, the equilibrium ownership structure would maximize S (Proposition 1). However, in this case, the second line, which reflects the blockholders' reservation payoff, depends on the investor's acquisition strategy, implying that maximizing I is not equivalent to maximizing S. This distinction is important because, as we will see, the mechanism limiting the extent of acquisitions from dispersed shareholders is precisely related to its effects on the blockholders' reservation payoff.

5.2 Acquisition from dispersed shareholders

Focusing again on a symmetric equilibrium, where $s_1^B = s_2^B = s^B$ and $s_1^D = s_2^D = s^D$, the investor's payoff becomes, with self-explanatory notation:

$$I = \nu^*(s^B)\Pi^*(s^B, s^D) - \nu^*(0)\Pi^*(0, s^D). \tag{25}$$

Upon inspection of (24), it becomes apparent that the choice of s^B involves the same trade-off as in the previous section, with the only difference being that equilibrium profits are evaluated at $s^D \geq 0$.[23]

Let us therefore focus on the choice of s^D. The marginal effect of s^D on the investor's payoff is:

$$\frac{\partial I}{\partial s^D} = \nu^*(s^B)\frac{\partial \Pi^*(s^B, s^D)}{\partial s^D} - \nu^*(0)\frac{\partial \Pi^*(0, s^D)}{\partial s^D}. \tag{26}$$

[23] With unconditional offers, the investor payoff would be:

$$I = \nu^*(s^B)\Pi^*(s^B, s^D) - \nu^*(0)\tilde{\Pi}^*(s^B, s^D),$$

where $\tilde{\Pi}^*(s^B, s^D) = \pi_i^*(0, s^D, s^B, s^D) + \pi_j^*(s^B, s^D, 0, s^D)$. The choice of s_i^B would then entail an additional cost, which is the increase in the price paid for the stake acquired from $\mathcal{B}_j$, similar to the effect of increasing s^D discussed in what follows. The presence of this additional cost implies that the equilibrium level of common ownership is higher with conditional than unconditional offers, and the investor prefers to make conditional offers whenever possible.

The first term of the derivative represents the marginal benefit of increasing s^D, which, similar to s^B, corresponds to higher profits resulting from reduced competition.[24] The second term represents the marginal cost. Unlike s^B, this cost does not derive from a deterioration in corporate governance. Instead, it arises from the fact that when the investor purchases shares from dispersed shareholders in firm i, the value of the shares owned by the blockholders increases, and thus the investor must pay a higher price to acquire a stake from them.

We have:

Proposition 5 *In equilibrium, the investor purchases shares from dispersed shareholders, $s^D > 0$, if and only if it acquires shares from the blockholders, $s^B > 0$.*

Proposition 5 establishes two results. First, it asserts that acquiring shares from dispersed shareholders is not profitable when $s^B = 0$. This is evident from equation (24), which implies that $I(s^B, s^D) \equiv 0$ whenever $s^B = 0$. The benefit from acquiring shares from dispersed shareholders is solely indirect and may arise only when $s^B > 0$.

Second, the proposition states that once $s^B > 0$, the financial investor will indeed always acquire a portion of its shares from dispersed shareholders. This result is reminiscent of findings in the literature on takeovers, where once a raider holds a toehold in the target company, buying additional shares from dispersed shareholders can be profitable through the increased value of the toehold. However, the toehold here is not pre-existent but is created endogenously.[25]

This endogeneity implies that the profitability of acquiring shares from dispersed shareholders relies on the value of the toehold increasing more than the cost of obtaining it. To see why this is indeed true, note that starting at $s^D = 0$, a small increase in s^D has a first-order effect on the intensity of competition, and hence on equilibrium profits, when $s^B > 0$. However, such a small increase in s^D has a second-order effect when $s^B = 0$,[26] and thus it has no impact on the blockholder's outside option. Consequently, the cost of acquiring shares from dispersed shareholders vanishes at $s^D = 0$, while the benefit remains positive.

To summarize, the nature of the benefits from common ownership does not depend on whether shares are acquired from blockholders or dispersed shareholders: in both cases, the acquisition leads to softer competition and higher profits. However, the costs of common ownership differ: purchasing shares from a blockholder reduces

[24] Note, however, that the softening-of-competition effect of the acquisition of shares from dispersed shareholders is lower compared to acquiring shares from blockholders. This can be observed by examining the expression for the weight:

$$\lambda = \frac{\theta \left(s^B + s^D\right)^2}{\left(\beta - s^B\right)^2 + \theta \left(s^B + s^D\right)^2},$$

which reveals that acquiring shares from the blockholder not only increases the weight of the common owner but also decreases the weight of the blockholder, with a more significant impact on lambda.

[25] If the financial investor already owns a toehold, then it has an additional incentive to acquire shares both from the blockholders and from dispersed shareholders.

[26] This is evident from the expression for λ reported in footnote 24.

monitoring while acquiring shares from dispersed shareholders raises the price that must be paid to blockholders.

6 Extensions

The analysis thus far has assumed that there are only two firms in the market and that these firms are symmetric. We will now explore the implications of relaxing these simplifying assumptions.

6.1 More than two firms

We begin with the case of more than two firms competing in the same product market, denoting the number of firms as N and assuming that they are all identical. In particular, each firm i has a blockholder $\mathcal{B}_i$ owning a stake β and monitoring the firm's manager.

Similar to the previous section, for simplicity, we focus on the case $\alpha = 1$, where the financial investor presents an offer to all N blockholders.[27] Since all firms are symmetric, we continue to assume that the investor seeks to acquire the same stake in all firms, denoted as s. Under these assumptions, it is easy to verify that at the product market competition stage, each firm maximizes $\pi_i + \lambda \sum_{j \neq i} \pi_j$, where λ is still given by expression (17).

If the investor may condition its offers on collective acceptance, the blockholders' reservation payoff is still given by the value of their stake in the absence of common ownership.[28] The investor can then extract all the gains created by common ownership, and, following the same steps used in the case of two firms, we can conclude that the investor chooses the stake to be acquired, s, so as to maximize:

$$S = \nu^*(s)\Pi^*(s) \tag{27}$$

where $\nu^*(s)$ is still given by (13) and $\Pi^*(s) = \sum_{i=1}^{N} \pi_i^*(s, s, ..., s)$. Changing s entails the same trade-off as in the baseline model, and the first-order condition to determine the optimal stake s^* is still given by (15).

What is the impact of changing the number of firms N on the level of common ownership? By the implicit function theorem, using the second-order condition for a maximum and observing that $\nu(s)$ and λ do not depend on N, we see that the derivative of s^* with respect to N has the same sign as:

$$\frac{\partial \nu^*}{\partial s}\frac{\partial \Pi^*}{\partial N} + \nu(s^*)\frac{\partial^2 \Pi}{\partial \lambda \partial N}\frac{d\lambda}{ds}.$$

[27] The analysis can be extended to allow for acquisition from dispersed shareholders along the same lines as in the previous section.

[28] With unconditional offers, increasing s positively affects $\mathcal{B}_i$'s reservation payoff, as firm i's equilibrium profit in the case where only $\mathcal{B}_i$ rejects the investor's offer would be $\pi^*(0, s, s, ..., s) > \pi^*(0, 0, ..., 0)$. This introduces an additional cost of increasing the level of common ownership, similar to that discussed in the previous section.

Let us examine the first term initially. We know that $\frac{\partial \nu^*}{\partial s} < 0$, and in most models of product market competition, as long as $\lambda < 1$, an increase in the number of firms intensifies competition and decreases total equilibrium profits, i.e., $\frac{\partial \Pi^*}{\partial N} \leq 0$. Therefore, the first term is positive.

Turning to the second term, we follow the same logic as in subsection 4.4.2. When $\lambda = 1$, firms do not compete and collectively reap monopoly profits, so that $\frac{\partial \Pi^*}{\partial N} = 0$. When instead $\lambda = 0$, we know that an increase in the number of firms reduces industry profits: $\frac{\partial \Pi^*}{\partial N} < 0$. Therefore, it seems reasonable to assume that the negative effect of increasing the number of firms on industry profits becomes smaller and smaller as λ increases; that is, that $\frac{\partial^2 \Pi}{\partial \lambda \partial N} \geq 0$.[29] Since $\frac{d\lambda}{ds} \geq 0$, we can conclude that the second term is positive as well; in other words, that the equilibrium level of common ownership in an industry increases with the number of competing firms: $\frac{ds^*}{dN} \geq 0$.

Intuitively, the higher level of common ownership in more fragmented industries stems from two concurring economic effects. Firstly, as the number of firms operating in the market increases, total industry profits decrease. Therefore, the negative impact of reduced monitoring, triggered by acquiring a larger stake, becomes less detrimental to the financial investor. Secondly, the marginal effect of softer competition induced by common ownership on profits increases when more firms operate in the market. By combining these two effects, it appears that the financial investor is incentivized to acquire a larger stake in each firm as the industry becomes more fragmented.

6.2 Asymmetric firms

We now return to the case of two firms, $N = 2$, to analyze the consequences of possible asymmetries between them. Specifically, we are interested in the scenario where one firm, say firm 2, holds a competitive advantage over its competitor, firm 1, either through lower costs, higher demand, or a combination of the two. Generally, such a comparative advantage implies that the more efficient firm will be larger in size and earn higher profits than its rival. We aim to understand whether the financial investor will acquire a greater share in the more efficient firm or the less efficient one, and how the degree of asymmetry affects the overall level of common ownership.

For simplicity, we focus again on acquisitions solely from the blockholders. Following the same steps as in the baseline case, it can be shown that the equilibrium levels of common ownership, s_1^* and s_2^*, must maximize the aggregate payoff of the large shareholders:

$$S = \nu^*(s_1)\pi_1^*(s_1, s_2) + \nu^*(s_2)\pi_2^*(s_1, s_2). \tag{28}$$

Naturally, however, the equilibrium will no longer be symmetric when $\pi_1^*(s_1, s_2) \neq \pi_2^*(s_1, s_2)$.

We aim to ascertain, first of all, which of s_1^* and s_2^* is higher. To this end, let us

[29] Appendix B shows that this property holds, for instance, in a simple model of Cournot competition with homogeneous products.

use (27) to calculate the difference between the derivatives of S with respect to s_1 and s_2, $\frac{dS}{ds_1} - \frac{dS}{ds_2}$, evaluated at $s_1 = s_2 = s$:

$$\frac{dS}{ds_1} - \frac{dS}{ds_2} = \frac{d\nu^*}{ds}(\pi_1^* - \pi_2^*) + \nu^*(s)\left(\frac{\partial \Pi^*}{\partial \lambda_1} - \frac{\partial \Pi^*}{\partial \lambda_2}\right)\left(\frac{\partial \lambda_1}{\partial s_1} - \frac{\partial \lambda_1}{\partial s_2}\right). \tag{29}$$

If this difference is positive, then by the second order condition it follows that s_1^* must be higher than s_2^*.

The first term on the right-hand side of (29) is positive as $\frac{d\nu^*(s)}{ds} < 0$ and $\pi_1^* - \pi_2^* < 0$ by assumption. As for the second term, observe that $\nu^*(s) > 0$, and that $\frac{\partial \lambda_1}{\partial s_1} - \frac{\partial \lambda_1}{\partial s_2} > 0$ when $s_1 = s_2 = s < \frac{1}{2}$. Furthermore, most models of product market competition imply that $\frac{\partial \Pi^*}{\partial \lambda_1} - \frac{\partial \Pi^*}{\partial \lambda_2} > 0$.[30] Intuitively, total profits increase more if it is the less efficient firm that internalizes the rival's profits more, rather than the opposite. This is because when firms compete ($\lambda = 0$), the market share of the less efficient firm is generally too large from the viewpoint of joint profit maximization.

From this, we can conclude that when the investor acquires less than 50% of the shares of competing companies, in equilibrium, it will acquire a higher stake in the less efficient firm and a lower stake in the more efficient one. This conclusion rests on three economic effects. First, since the negative impact of common ownership on monitoring is proportional to firms' profit, it is more pronounced for the firm with higher profits – an effect captured by the first term on the right-hand side of (28). This implies that the common owner should acquire a greater share of the less efficient firm. Second, as noted, it proves generally more effective to mitigate the intensity of competition by tempering the aggressiveness of the less efficient firm, as opposed to the more efficient one. Finally, to make the less efficient firm more accommodating, it is more effective to increase the stake held by the financial investor precisely in that firm when stakes are below 50%.[31] The last two effects also imply that the financial investor should acquire a greater stake in the less efficient firm, thereby adding to the first effect.[32]

[30] See Appendix B for an example with Cournot competition and linear demand.

[31] This conclusion is reversed when stakes are above 50%, in which case the sign of (29) becomes uncertain.

[32] The issue of whether the overall level of common ownership is higher or lower when firms become more asymmetric is more challenging. To make some progress, we have considered a simple model of Cournot competition with homogeneous products, linear demand, and constant but asymmetric marginal costs. Even for this very simple case, analytical solutions seem out of reach. However, numerical calculations, available upon request, indicate that as the cost gap between the two firms increases, initially, the stake acquired by the financial investor in the less efficient firm increases, while that in the more efficient firm decreases. The total stake increases, and so does the average lambda, indicating an increase in the overall level of common ownership. As the cost gap increases further, the equilibrium stake in the more efficient firm also starts increasing. This is due to the complementarity between s_i and s_j, which implies that as one stake increases, the marginal effect on the intensity of competition of increasing the other rises. In this case, it is clear that the overall level of common ownership unambiguously increases.

7 Internal common ownership

In this section, we consider a scenario in which a firm's blockholder acquires shares in a rival firm. We will refer to this scenario as internal common ownership, in contrast to the common ownership by an external investor that we have analyzed thus far.

7.1 Assumptions

We return to the model of Section 3, with the sole difference being the absence of an external investor. Instead, we allow the blockholders to acquire a stake in the competing company.

We maintain the assumption that each blockholder possesses a unique ability to monitor the manager of its own firm. In other words, blockholder $\mathcal{B}_i$ does not have the capability, or the incentive, to monitor the manager of firm j; that responsibility solely lies with blockholder $\mathcal{B}_j$.[33]

Internal common ownership differs from external common ownership in that it can be profitable even if only one blockholder engages in a rival's partial acquisition. In contrast, an external investor must acquire stakes in both competing companies to achieve a reduction in competition. Furthermore, internal common ownership can be profitable even if $\theta = 0$, meaning that managers exclusively prioritize the interests of their controlling shareholder.

Consequently, we can simplify the analysis by setting $\theta = 0$, and by assuming that only blockholder $\mathcal{B}_1$, for instance, acquires a stake in firm 2. Let us denote the stake that $\mathcal{B}_1$ acquires from $\mathcal{B}_2$ as s_2^B, and the stake he acquires from dispersed shareholders as s_2^D, with $s_2 = s_2^B + s_2^D$.

Since $\theta = 0$, at the product market competition stage, managers now exclusively pursue the interests of their controlling blockholders. Thus, manager $\mathcal{M}_2$ maximizes π_2, while manager $\mathcal{M}_1$ maximizes $(\beta\pi_1 + s_2\pi_2)$. This is equivalent to maximizing:

$$O_1 = \pi_1 + \lambda_1\pi_2, \tag{30}$$

where:

$$\lambda_1 = \frac{s_2}{\beta}. \tag{31}$$

The weight λ_1 is now positive even if $\theta = 0$. It depends only on the aggregate share acquired, s_2, and not on its division between s_2^B and s_2^D, and it is directly proportional to s_2.[34]

7.2 Acquisition price

As before, dispersed shareholders are forward-looking and demand a price that fully reflects the ex-post value of their shares. Therefore, we can rewrite $\mathcal{B}_1$'s payoff as

[33] For our results, in fact, it would suffice that $\mathcal{B}_i$ is less efficient in monitoring firm j than $\mathcal{B}_j$.

[34] The fact that the weight λ depends only on s_2 instead of the product of the shares explains why unilateral acquisitions may now be profitable.

follows:

$$B_1 = \nu^*(0)\pi_1^*(s_2) + s_2^B \left\{1 - \xi\left[1 - m^*(\beta - s_2^B)\right]\right\} \pi_2^*(s_2) - P_2^B. \tag{32}$$

where the notation omits the variables s_1^B and s_1^D in $\pi_i^*(s_1^B, s_1^D, s_2^B, s_2^D)$, as they are both equal to zero in this section, and accounts for the property that equilibrium profits depend only on the aggregate share $s_2 = s_2^B + s_2^D$. On the other hand, the payoff of blockholder $\mathcal{B}_2$ remains as given in (1).

The first term in (32) reflects the fact that $\mathcal{B}_1$ maintains its full stake β in firm 1, while the other two terms denote the capital gain from the acquisition of the s_2^B shares from $\mathcal{B}_2$. Unlike dispersed shareholders, blockholder $\mathcal{B}_2$ may indeed be willing to sell his shares for a price lower than their ex-post value, anticipating that the acquisition may not occur otherwise. The acquisition of shares from dispersed shareholder does not generate any capital gain on its own, but it can increase that obtained from the acquisition of s_2^B.

For simplicity, we assume that in the negotiations over the acquisition price, blockholder $\mathcal{B}_1$ has all the bargaining power. This implies that the acquisition price P_2^B must make $\mathcal{B}_2$ indifferent between selling or not, yielding:

$$P_2^B = \nu^*(0)\pi_2^*(s_2^D) - \left\{(\beta - s_2^B)\left[1 - \xi(1 - m^*(\beta - s_2^B)\right] - C(m^*(\beta - s_2^B))\right\}\pi_2^*(s_2).$$

Inserting this expression into (32), the payoff of blockholder $\mathcal{B}_1$ can finally be expressed as

$$B_1 = \nu_1^*(0)\pi_1^*(s_2) + \nu_2^*(s_2^B)\pi_2^*(s_2) - \nu_2^*(0)\pi_2^*(s_2^D). \tag{33}$$

The first two terms correspond to the ex-post value of the blocks owned by the two blockholders. The third term represents $\mathcal{B}_2$'s reservation payoff. If this term were constant, $\mathcal{B}_1$ would obtain all the benefits from common ownership. But, in fact, $\mathcal{B}_2$'s reservation payoff increases with s_2^D, meaning that $\mathcal{B}_2$ may obtain a share of the surplus, thus indirectly benefiting from $\mathcal{B}_1$'s acquisition of shares from dispersed shareholders.

7.3 Equilibrium ownership structure

The equilibrium ownership structure now maximizes (33). We will first consider the case where $\mathcal{B}_1$ acquires shares solely from $\mathcal{B}_2$, followed by the case where he acquires shares only from dispersed shareholders, and finally, the case where $\mathcal{B}_1$ acquires shares from both sources.

7.3.1 Acquisition from the blockholder

When $\mathcal{B}_1$ acquires shares solely from $\mathcal{B}_2$, the trade-off that determines the equilibrium ownership structure is similar to that in the case of external common ownership: increasing s_2^B leads to softer competition in the product market, resulting in higher profits, but it also reduces the incentives to monitor, reducing the share of these profits that can be obtained by $\mathcal{B}_1$. The equilibrium level of s_2^B strikes a balance between these two effects.

Proposition 6 *In equilibrium, internal common ownership always exists:* $s_2^B > 0$.

Unlike the case of external common ownership, s_2^B is always positive in this scenario. This is because even when s_2^B approaches zero, the positive impact on profits remains first order, while the negative effect on corporate governance becomes second order because monitoring is at the efficient level when $s_2^B = 0$.

Akin to its external counterpart, internal common ownership tends to be limited by the fact that as s_2^B increases, $\mathcal{B}_2$ exerts less monitoring effort and this reduces the value of the shares $\mathcal{B}_1$ acquires from him. This effect represents the cost of common ownership. This cost still increases with s_2^B, as in the baseline model, but the marginal benefit from common ownership now does not vanish as s_2^B approaches β due to the asymmetry among the firms. Therefore, we cannot rule out the possibility that $\mathcal{B}_1$ may acquire $\mathcal{B}_2$'s entire stake, at the cost of completely eliminating the incentives to monitor. However, in the symmetric case where each blockholder acquires a stake in the competing company, it can be shown that the stakes acquired in equilibrium are bounded above by $\frac{\beta}{2}$. The reason for this is that when s^B approaches $\frac{\beta}{2}$, perfect cooperation in the product market is achieved, making the benefit from further acquisition vanish.

The comparative statics for internal common ownership are identical to those of external common ownership.

Proposition 7 *The equilibrium level of internal common ownership monotonically decreases with the managers' ability to steal* ξ *and monotonically increases with the costliness of monitoring* γ *and the intensity of product market competition* σ.

The intuition behind these findings is precisely the same as in the case of external common ownership.

7.3.2 Acquisition from dispersed shareholders

While Proposition 5 demonstrates that a financial investor external to the industry never acquires shares exclusively from dispersed shareholders, a blockholder may profitably acquire a positive stake in the rival company solely from dispersed shareholders. The intuitive reason for this is that the blockholder may benefit from softer competition through the stake he holds in his controlled firm.

This can be seen by differentiating B_1 with respect to s_2^D while assuming $s_2^B = 0$. The resulting expression,

$$\frac{dB_1}{ds_2^D} = \nu_1^*(0)\frac{\partial \pi_1^*(s_2^D)}{\partial s_2^D}, \tag{34}$$

shows that $\mathcal{B}_1$ can gain from the reduced competition even if he purchases solely from the dispersed shareholders, provided that $\frac{\partial \pi_1^*(s_2^D)}{\partial s_2^D} > 0$.

Whether this condition holds depends on the specificities of product market competition, and precisely on whether a firm directly gains from becoming more accommodating. In the case of strategic substitutes, $\frac{\partial \pi_1^*(s_2^D)}{\partial s_2^D}$ is always negative, so this possibility can never arise. However, in the case of strategic complements, the derivative can be positive if s_2^D remains below a certain threshold, denoted as $\bar{s}_2^D$, which is less than one. In this case, $\bar{s}_2^D$ represents the equilibrium level of internal common ownership when $\mathcal{B}_1$ does not acquire shares from $\mathcal{B}_2$. This level is entirely determined by factors pertaining to the product market and no longer depends on corporate governance.

7.3.3 Acquisition from both

Suppose now that $\mathcal{B}_1$ acquires shares from both $\mathcal{B}_2$ and dispersed shareholders. The trade-off involved in choosing s_2^B remains qualitatively the same as when $s_2^D = 0$, but the choice of s_2^D changes more significantly. Indeed, we have:

$$\frac{dB_1}{ds_2^D} = \nu^*(0)\frac{\partial \pi_1^*(s_2)}{\partial s_2^D} + \nu^*(s_2^B)\frac{\partial \pi_2^*(s_2)}{\partial s_2^D} - \nu^*(0)\frac{\partial \pi_2^*(s_2^D)}{\partial s_2^D}. \tag{35}$$

The last two terms represent the impact of s_2^D on the difference between $\mathcal{B}_2$'s equilibrium payoff and his reservation payoff. This is relevant for $\mathcal{B}_1$, as $\mathcal{B}_1$ appropriates all of this value thanks to his bargaining power. The sum of the last two terms may have either sign.[35] Therefore, the acquisition of shares from the blockholder may either crowd in or crowd out the acquisition from dispersed shareholders.

8 Conclusions

This paper has examined the interplay between the costs and benefits of common ownership. A growing body of theoretical and empirical literature suggests that common ownership reduces product market competition, leading to higher profits. The novel contribution of this paper is to highlight the costs of common ownership for shareholders, specifically its impact on corporate governance.

In our model, financial investors such as the Big Three, who hold stakes in multiple firms within the same industry, exert influence over firms' strategic decisions to mitigate product market competition. On the other hand, blockholders — individuals or families holding a significant block of shares in a company — have the incentive to incur private costs for monitoring managers or reducing costs. Financial investors must acquire shares from such blockholders to realize capital gains, as dispersed shareholders demand a price that fully reflects the shares' ex-post value. However, in doing so, they diminish the blockholders' incentives. The equilibrium degree of common ownership emerges as the optimal response to these conflicting effects.

Our analysis carries implications for both empirical research and antitrust policy. The model suggests that common ownership tends to be higher in more fragmented industries and, more broadly, in industries characterized by more intense competition. The empirical industrial organization literature has proposed several methods

[35] The reason for this is that $\pi_2^*(s_2)$ is typically a convex function, so when $s_2^B > 0$ we have $\frac{\partial \pi_2^*(s_2)}{\partial s_2^D} > \frac{\partial \pi_2^*(s_2^D)}{\partial s_2^D}$, but on the other hand $\nu^*(s_2^B) < \nu^*(0)$.

to measure the intensity of product market competition, making this prediction empirically testable. Additionally, the model predicts that financial investors are likely to acquire larger stakes in smaller and less profitable firms. While this prediction is also theoretically testable, identifying the less efficient competitors may pose practical challenges.

Another set of predictions from the model pertains to the impact on common ownership of changes in corporate governance laws and institutions. However, translating these predictions into empirical observations appears more challenging as the effect of such institutional changes on the extent of common ownership may hinge on nuanced institutional details that are not easily discernible.

In terms of antitrust policy, our analysis highlights that common ownership may not solely be driven by motives of risk diversification. In fact, large financial investors may prioritize mitigating product market competition over maintaining financial diversification. They may direct their investments towards industries and firms within those industries where the impact of common ownership on product market competition is more pronounced. If this holds true, then limiting the extent of common ownership may entail lower costs than suggested by the traditional view, which asserts that common ownership is mainly driven by diversification needs. Nevertheless, we acknowledge that identifying practical methods to limit the degree of common ownership presents a formidable challenge for antitrust authorities.

The model could be extended in various directions. For instance, while the paper primarily addresses common ownership, where companies operating within the same market share a common owner, a similar approach can be applied to cross-ownership. Cross-ownership occurs when companies directly hold a stake in their rivals. An intriguing issue arising in this context is under what conditions one form of overlapping ownership prevails over the other. For example, consider the case of blockholders who exercise control over a particular firm and intend to acquire a non-controlling stake in a rival firm. Will they use personal funds for this purpose, resulting in internal common ownership, or will they direct the company they control to make the acquisition, leading to cross-ownership?

Moreover, it could be interesting to relax the assumption that firms and their shareholders focus exclusively on profits. A notable trend in recent years is the emergence of socially responsible investors who prioritize goals such as environmental preservation and human rights protection alongside profit maximization. The literature that analyzes this trend has highlighted a significant concern known as the leakage problem. This refers to the situation where one firm, for instance, reduces emissions through green technology, but the environmental benefits are partially offset by increased emissions from competitors using less sustainable technologies. Common ownership may offer a potential solution to mitigate the leakage problem and enhance the effectiveness of socially responsible investment strategies. However, if the involvement of socially motivated investors leads to softer product market competition, consumers may bear the costs of social responsibility. Exploring these new trade-offs presents an exciting avenue for future research.

References

[1] Aghion, P. and J. Tirole (1997), Real and formal authority in organizations, Journal of Political Economy, 105, 1-29.

[2] Amel-Zadeh, A., F. Kasperk, and M. Schmalz (2002), Mavericks, universal, and common owners - The largest shareholders of U.S. public firms, CESifo W.P. 9926.

[3] Anton, M., F. Ederer, M. Giné and M. Schmalz, (2023), Common ownership, competition, and top management incentives, Journal of Political Economy, 131(5), 1294-1355.

[4] Azar, J. (2017), Portfolio diversification, market power, and the theory of the firm, mimeo.

[5] Azar, J. (2020), The common ownership trilemma, The University of Chicago Law Review, 87, 263-295.

[6] Azar, J., M. Schmalz, and I. Tecu (2018), Anticompetitive effects of common ownership, Journal of Finance, 73(4), 1513-1565.

[7] Azar, J., M. Schmalz, and I. Tecu (2022), A refutation of 'Common ownership does not have anti-competitive effects in the airline industry', ECGI W.P. 837.

[8] Azar, J., and Vives, X. (2021), General equilibrium oligopoly and ownership structure, Econometrica, 89(3), 999-1048.

[9] Azar, J., and Vives, X. (2022), Revisiting the anticompetitive effects of common ownership, ECGI Working Paper 827.

[10] Backus, M., C. Conlon, and M. Sinkinson (2021a), Common ownership and competition in the ready-to-eat cereal industry, NBER Working Paper 28350.

[11] Backus, M., C. Conlon, and M. Sinkinson (2021b), Common ownership in America: 1980-2017, American Economic Journal: Microeconomics, 13(3), 273-308.

[12] Bebchuk, L., A. Cohen, and S. Hirst (2017), The agency problem of institutional investors, Journal of Economic Perspectives, 31, 89-102.

[13] Bernheim, D., and M. Whinston (1986), Common agency, resource allocation, and economic influence, Quarterly Journal of Economics, 101(1), 1-31.

[14] Burkart, M., D. Gromb and F. Panunzi (1997), Large shareholders, monitoring, and the value of the firm, Quarterly Journal of Economics, 112(3), 693-728.

[15] Burkart, M., D. Gromb, and F. Panunzi (1998), Why higher takeover premia protect minority shareholders, Journal of Political Economy, 106(1), 172-204.

[16] Burkart, M., F. Panunzi, and A. Shleifer (2003), Family Firms, Journal of Finance, 58(5), 2167-2201

[17] Demsetz H., and K Lehn (1985), The structures of corporate ownership: Causes and consequences, Journal of Political Economy, 93, 1155-1177.

[18] Dennis, P., K. Gerardi, and C. Schenone (2022), Common ownership does not have anti-competitive effects in the airline industry, Journal of Finance, 77(5), 2766-2798.

[19] Ederer, F., and Pellegrino, B. (2022), A tale of two networks. Common ownership and product market rivalry, NBER Working Paper 30004.

[20] Frey, J., A. Ockenfels, and M. Schmalz (2023), Diversification vs. monopolization: a laboratory experiment, Working Paper.

[21] Grossman, S., and O. Hart (1980), Takeover bids, the free rider problem, and the theory of corporation, Bell Journal of Economics, 42-64.

[22] He, J., and J. Huang (2017), Product market competition in a world of cross ownership: Evidence from institutional blockholding, Review of Financial Studies, 30, 2674-2718.

[23] Huse, C., R. Ribeiro, and F. Verboven (2024), Common-ownership versus cross-ownership: evidence from the automobile industry, Journal of Industrial Economics, forthcoming.

[24] Jensen, M.C. and W. H. Meckling (1976), Theory of the firm: Managerial behavior, agency costs and ownership structure, Journal of Financial Economics, 3(4), 305-360.

[25] Kennedy, P., P. O'Brien, M. Song, and K. Waehrer (2017), The competitive effects of common ownership: Economic foundations and empirical evidence, SSRN Working Paper.

[26] Koch, A., M. Panayides, and S. Thomas (2021), Common ownership and competition in product markets, Journal of Financial Economics, 1239, 109-137

[27] Levy, N., Spiegel, Y., and D. Gilo (2018), Partial vertical integration, ownership structure, and foreclosure, American Economic Journal: Microeconomics, 10(1), 132-180

[28] Lewellen, K., and M. Lowry (2021), Does common ownership really increase firm coordination?, Journal of Financial Economics, 141(1), 322-344.

[29] Lopez, A., and X. Vives (2019), Overlapping ownership, R&D spillovers, and antitrust policy, Journal of Political Economy, 127(5), 2394-2437.

[30] Manne, H. (1965), Mergers and the market for corporate control, Journal of Political Economy, 73(2), 110-12.

[31] O'Brien D., and S. Salop (2000), Competitive effects of partial ownership: Financial interest and corporate control, Antitrust Law Journal, 67, 559-614.

[32] Park, A.H., and K. Seo (2019), Common ownership and product market competition: Evidence from the US airline industry, SNU Working Paper, 48(5), 617-640.

[33] Piccolo, A., and J. Schneemeier (2020), Ownership and competition, Working Paper, Indiana University

[34] Rotemberg, J. (1984), Financial transaction costs and industrial performance, Working Paper, MIT.

[35] Schmalz, M. (2021), Recent studies on common ownership, firm behavior, and market outcomes, Antitrust Bulletin, 66(1), 12-38.

[36] Shekita, N. (2021), Interventions by common owners, Journal of Competition Law & Economics, 18(1), 99-134.

[37] Shleifer, A., and R. Vishny (1986), Large shareholders and corporate control, Journal of Political Economy, 94, 461-488.

[38] Shleifer, A. and R. Vishny (1997), A survey of corporate governance, Journal of Finance, 52(2), 737-783.

[39] Shy, O., and Stenbacka, R. (2020). Common ownership, institutional investors, and welfare. Journal of Economics & Management Strategy, 29(3), 706-723.

[40] Spiegel, Y. (2013), Backward integration, forward integration, and vertical foreclosure, CEPR D.P. 9617

Appendix A: Proofs

This appendix collects the proofs omitted in the main text.

Proof of Proposition 1. We start from the case of contingent offers, and we then show that the result holds true even when offers are non-contingent, as described in section 3.1.4.

Consider first the case where the investor makes offers to the blockholders. In this case, $\mathcal{I}$ chooses its take-it-or-leave-it offers so as to maximize its net payoff I. Since the participation constraints $B_i \geq \bar{B}_i$ must bind in equilibrium, we have

$$I + B_1 + B_2 = I + \bar{B}_1 + \bar{B}_2.$$

Inspection of (11) reveals that $\bar{B}_i$ does not depend on the investor's stakes, s_i and s_j. It follows immediately that maximization of I is equivalent to maximization of $I + \bar{B}_1 + \bar{B}_2$, and hence of $I + B_1 + B_2 = S$.

Next, consider the case where the blockholders simultaneously and independently make offers $P_i(s_i, s_j)$ to the investor. The resulting game is one of common agency, as in Bernheim and Whinston (1986). Bernheim and Whinston (1986) demonstrated that the truthful equilibrium of such a game, which is Pareto-dominant for the principals when there are only two principals, is efficient (Theorem 2, p. 14). Since we are restricting attention to the Pareto-dominant equilibrium for the blockholders, we can conclude that this equilibrium, $P_i^*(s_i, s_j)$, maximizes the players' joint payoff, S.

This completes the proof of the proposition. For completeness, however, we now show that the result continues to hold even when the offers cannot be contingent.

When $\mathcal{I}$ makes the offers, the participation constraints become $B_i \geq \overline{\overline{B}}_i$, but since, as noted in the main text, $\overline{\overline{B}}_i = \bar{B}_i$, the conclusion is unchanged.

When the blockholders make the offers, they are now restricted to submitting schedules of the type $P_i(s_i)$. But it is easy to show that given any equilibrium of the common agency game with contingent offers, offering the schedule $P_i(s_i) = P_i^*(s_i, s_j^*)$ represents an equilibrium of the non-contingent game. Indeed, with these offers, the investor elects to acquire the same stakes s_i^* and s_j^* as in the contingent case, and no blockholder has a profitable deviation (otherwise, there would be a profitable deviation in the contingent-offer case as well). Intuitively, contingent offers are unnecessary because there is no opportunity for a mutually profitable agreement between the investor and a single blockholder. ■

Proof of Lemma 1. A change in s impacts industry profits through the weight firms attach to the rival's in their objective function. Therefore:

$$\frac{d\Pi^*}{ds} = \frac{\partial \Pi^*}{\partial \lambda} \frac{\partial \lambda}{\partial s}.$$

From (18), we know that $\frac{\partial \lambda}{\partial s}$ is positive if $\theta > 0$, except at $s = 0$ and $s = \beta$ where it vanishes.

Next, consider the factor $\frac{\partial \Pi^*}{\partial \lambda}$. Since $\Pi^*(\lambda) = \pi_1^*(\lambda) + \pi_2^*(\lambda)$, using the first-order conditions (8), we obtain:

$$\frac{\partial \Pi^*}{\partial \lambda} = \left(-\lambda \frac{\partial \pi_2^*}{\partial x_1} + \frac{\partial \pi_2^*}{\partial x_1}\right) \frac{\partial x_1}{\partial \lambda} + \left(\frac{\partial \pi_1^*}{\partial x_2} - \lambda \frac{\partial \pi_1^*}{\partial x_2}\right) \frac{\partial x_2}{\partial \lambda}$$

Symmetry implies $\dfrac{\partial x_1}{\partial \lambda} = \dfrac{\partial x_2}{\partial \lambda} = \dfrac{\partial x}{\partial \lambda}$, so the above expression may be rewritten as:

$$\frac{\partial \Pi^*}{\partial \lambda} = 2\left(1 - \lambda\right) \frac{\partial \pi_i^*}{\partial x_j} \frac{\partial x}{\partial \lambda}. \tag{A1}$$

By fully differentiating the first-order conditions (8), we obtain:

$$\frac{\partial x}{\partial \lambda} = -\frac{\frac{\partial \pi_j^*}{\partial x_i}}{\left(\frac{\partial^2 \pi_i^*}{\partial x_i^2} + \lambda \frac{\partial^2 \pi_j^*}{\partial x_i^2}\right)}.$$

Plugging this expression into (A1) we eventually get:

$$\frac{\partial \Pi^*}{\partial \lambda} = -2\left(1 - \lambda\right) \frac{\left(\frac{\partial \pi_j^*}{\partial x_i}\right)^2}{\left(\frac{\partial^2 \pi_i^*}{\partial x_i^2} + \lambda \frac{\partial^2 \pi_j^*}{\partial x_i^2}\right)} \geq 0,$$

which is positive by the second-order conditions (9). The derivative is strictly positive for $\lambda < 1$, i.e., for $s < \beta$. Therefore, the sign of $\dfrac{\partial \pi^*}{\partial s}$ coincides with the sign of $\dfrac{\partial \lambda}{\partial s}$. The result then follows from the observation that $\dfrac{\partial \lambda}{\partial s}$ is always non negative and vanishes only at $s = 0$ and $s = \beta$. ■

Proof of Proposition 2. From (13) we have:

$$\frac{\partial S}{\partial s} = \frac{\partial \nu^*}{\partial s} \Pi^* + \nu^* \frac{\partial \Pi}{\partial s}.$$

By Lemma 1, $\frac{\partial \Pi}{\partial s} = 0$ at $s = \beta$ and since, by (17), $\frac{\partial \nu^*}{\partial s} \leq 0$, with a strict inequality for $s > 0$, we can conclude that $s^* < \beta$.

Proof of Proposition 3. Monotonicity requires that $\dfrac{\partial s^*}{\partial \xi} < 0$ (resp., $\dfrac{\partial s^*}{\partial \gamma} > 0$) when $s^* > 0$, and that s^* jumps downwards (resp., upwards) as ξ (resp., γ) increases.

To show this, note first of all that with the quadratic specification (20) of the monitoring cost function, the equilibrium level of monitoring is (7). Therefore, keeping in mind that Π^* does not depend on ξ and γ, the derivative $\dfrac{\partial S}{\partial s}$ becomes:

$$\frac{\partial S}{\partial s} = \frac{\xi^2}{\gamma} H \Pi^*, \tag{A2}$$

where

$$H \equiv -s + \left[\gamma\beta\frac{1-\xi}{\xi^2} + \frac{1}{2}(\beta^2 - s^2)\right]\frac{\partial\Pi^*}{\partial s}\frac{1}{\Pi^*} \tag{A3}$$

To proceed, consider first the case in which $s^* > 0$. By Proposition 2, in this case s^* is an interior maximum of the function $S(s)$, and thus it must satisfy the first-order condition $H = 0$. By implicit differentiation, we then obtain:

$$\frac{\partial s^*}{\partial \xi} = -\frac{\frac{\partial H}{\partial \xi}}{\frac{\partial H}{\partial s}} < 0$$

and

$$\frac{\partial s^*}{\partial \gamma} = -\frac{\frac{\partial H}{\partial \gamma}}{\frac{\partial H}{\partial s}} < 0$$

where the sign follows from the fact that $\frac{\partial H}{\partial s} < 0$ by the second order condition, whereas $\frac{\partial H}{\partial \xi} < 0$ and $\frac{\partial H}{\partial \gamma} > 0$. (These latter inequalities follow immediately from (A3)).

Next, consider the possibility that as ξ or γ changes, s^* may jump from an interior solution where $s^* \equiv s^+ > 0$ to a corner solution where $s^* = 0$. At the switching point, we must have:

$$\Delta S \equiv S(s^+) - S(0) = \frac{\xi^2}{\gamma}K\Pi_+^*, \tag{A4}$$

where Π_0^* is industry profits at $s = 0, \Pi_+^*$ is industry profits at $s = s^+$, and

$$K \equiv \left\{\left[2\gamma\beta\frac{1-\xi}{\xi^2} + \beta^2\right]\frac{\Pi_+^* - \Pi_0^*}{\Pi_+^*} - s^{+2}\right\}. \tag{A5}$$

It follows that:

$$\left.\frac{\partial \Delta S}{\partial \xi}\right|_{\Delta S=0} \propto \frac{\partial K}{\partial \xi} < 0,$$

where the symbol $\propto$ means "has the same sign has." This implies that when $\Delta S = 0$, an increase in ξ makes ΔS become negative, causing a downward jump of the equilibrium level of common ownership from s^+ to 0.

Likewise, we have

$$\left.\frac{\partial \Delta S}{\partial \gamma}\right|_{\Delta S=0} \propto \frac{\partial K}{\partial \gamma} > 0,$$

implying that when $\Delta S = 0$, an increase in γ makes ΔS become positive, causing an upward jump of the equilibrium level of common ownership from 0 to s^+. ■

Proof of Proposition 4. We proceed as in the proof of Proposition 3. The function H and K defined in (A3) and (A5) clearly depend on the derivative $\frac{\partial\Pi^*}{\partial s}\frac{1}{\Pi^*}$, or its discrete analog $\dfrac{\Pi^*_+ - \Pi^*_0}{\Pi^*_+}$. We have:

$$\begin{aligned}\frac{\partial \Pi^*}{\partial s}\frac{1}{\Pi^*} &= \frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}\frac{\partial \lambda}{\partial s}\\ &= \frac{2\theta s\beta\left(\beta - s\right)}{\left[\left(\beta - s\right)^2 + \theta s^2\right]^2}\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}.\end{aligned}$$

It follows immediately that:

$$\frac{\partial s^*}{\partial \theta} = -\frac{\frac{\partial H}{\partial \theta}}{\frac{\partial H}{\partial s}} > 0$$

and

$$\frac{\partial s^*}{\partial \sigma} = -\frac{\frac{\partial H}{\partial \sigma}}{\frac{\partial H}{\partial s}} < 0.$$

A similar logic applies to the direction of the jump from $s^* = 0$ to $s^* = s^+$: in both cases, the jump is upwards, as $\dfrac{\Pi^*_+ - \Pi^*_0}{\Pi^*_+}$ is increasing in both θ and σ. ■

Proof of Proposition 5. Suppose first that $s^B = 0$. In this case, it appears that the derivative (28) vanishes, implying that the investor cannot gain by acquiring shares from the dispersed shareholders. On the other hand, suppose that $s^B > 0$. Let us evaluate the derivative

$$\frac{\partial I}{\partial s^D} = \nu^*(s^B)\frac{\partial \Pi^*(s^B, s^D)}{\partial s^D} - \nu^*(0)\frac{\partial \Pi^*(0, s^D)}{\partial s^D}$$

at $s^D = 0$. From

$$\lambda = \frac{\theta\left(s^B + s^D\right)^2}{\left(\beta - s^B\right)^2 + \theta\left(s^B + s^D\right)^2},$$

one sees that

$$\left.\frac{d\lambda}{ds^D}\right|_{s^D=0} = \frac{2\theta s^B\left(\beta - s^B\right)^2}{\left[\left(\beta - s^B\right)^2 + \theta\left(s^B\right)^2\right]^2}$$

which implies that $\left.\frac{d\lambda}{ds^D}\right|_{s^D=0} > 0$ if $s^B > 0$ and $\left.\frac{d\lambda}{ds^D}\right|_{s^D=0} = 0$ if $s^B = 0$. Therefore, $\left.\dfrac{\partial \Pi^*(s^B, s^D)}{\partial s^D}\right|_{s^D=0} > 0$ if $s^B > 0$, whereas $\left.\dfrac{\partial \Pi^*(0, s^D)}{\partial s^D}\right|_{s^D=0} = 0$. It follows that $\left.\frac{\partial I}{\partial s^D}\right|_{s^D=0} > 0$ if $s^B > 0$, which implies that at the optimum $s^D > 0$ if $s^B > 0$. ■

Proof of Proposition 6. Suppose that $s^D_2 = 0$. From (35) we have:

$$B_1 = \nu^*_1(0)\pi^*_1(s^B_2, 0) + \nu^*_2(s^B_2)\pi^*_2(s^B_2, 0) - \nu^*_2(0)\pi^*_2(0, 0).$$

Differentiating we get:

$$\frac{\partial B_1}{\partial s^B_2} = \frac{\partial \nu^*_2(s^B_2)}{\partial s^B_2}\pi^*_2(s^B_2, 0) + \nu^*_1(0)\frac{\partial \pi^*_1(s^B_2, 0)}{\partial s^B_2} + \nu^*_2(s^B_2)\frac{\partial \pi^*_2(s^B_2, 0)}{\partial s^B_2}.$$

The first term of the derivative is negative and represents the marginal cost of blockholder common ownership, the sum of the last two terms is positive and represents the marginal benefit.

At $s_2^B = 0$ the first term vanishes, as shown above. It follows that $\left.\frac{\partial B_1}{\partial s_2^B}\right|_{s_2^B=0} > 0$, proving that the level of common ownership is always positive also in this case. ■

Proof of Proposition 7. The proof is identical to the proofs of Propositions 3, 4 and 5, and is therefore omitted. ■

Appendix B: Examples

Here, we examine several specific models of product market competition, demonstrating that they indeed exhibit the properties postulated in the main text.

Example 1. Firms supply differentiated products, the inverse demand for which is:

$$p_i = 1 - q_i - \delta q_j,$$

where $\delta \in [0, 1]$ is a parameter that captures the degree of product differentiation: products are independent for $\delta = 0$, perfect substitutes for $\delta = 1$. Firms compete in quantities ($x_i = q_i$).

Consider first the case of zero marginal costs. It is straightforward to verify that the profit functions are well-behaved and that the equilibrium is unique. Equilibrium outputs and profits are:

$$\begin{aligned} q_i^* &= \frac{1}{2 + \delta + \delta\lambda} \\ \Pi^* &= \frac{1 + \delta\lambda}{(2 + \delta + \delta\lambda)^2}. \end{aligned}$$

First, we show that the derivative of aggregate profits Π^* with respect to the degree of common ownership s is an inverted U-shaped function of s. Using (18), one calculates

$$\frac{\partial \Pi^*}{\partial s} = \frac{4\theta\delta^2\beta s(\beta - s)^3}{\left[(2+\delta)(\beta - s)^2 + 2\theta(1+\delta)s^2\right]^3}.$$

It is easy to verify that the derivative is always non-negative but is first increasing and then decreasing in s, vanishing at $s = 0$ and $s = \beta$.

Next, we show that a natural index of the intensity of competition, which in this example is the degree of product substitutability δ, has a monotonic impact on the semi-elasticity $\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}$. Indeed, we have

$$\frac{\partial}{\partial \delta}\left[\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}\right] = \frac{\delta(1-\lambda)(4 + \delta + 3\delta\lambda)}{(1+\delta\lambda)^2(2+\delta+\delta\lambda)^2} > 0,$$

consistently with our condition (23).

Example 2. Under the same assumptions as in Example 1, suppose now that firms compete in prices ($x_i = p_i$). The Bertrand equilibrium is:

$$\begin{aligned} p_i^* &= \frac{1 - \delta}{2 - \delta - \delta\lambda} \\ \Pi^* &= \frac{(1-\delta)(1-\delta\lambda)}{(1+\delta)(2-\delta-\delta\lambda)^2}. \end{aligned}$$

Using (18), one then obtains

$$\frac{\partial \Pi^*}{\partial s} = \frac{4\theta(1-\delta)\delta^2 \beta s (\beta - s)^3}{(1+\delta)\left[(2-\delta)(\beta-s)^2 + 2\theta(1-\delta)s^2\right]^3}.$$

As in the case of quantity competition, the derivative is positive and inverted-U shaped.

As before, it is natural to take δ as a measure of the intensity of competition. This measure accords with our condition (23) in this case as well, as

$$\frac{\partial}{\partial \delta}\left[\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}\right] = \frac{\delta(1-\delta)(4-\delta-3\delta\lambda)}{(1-\delta\lambda)^2(2-\delta-\delta\lambda)^2} > 0.$$

Furthermore, it is generally recognized that competition is more intense when symmetric firms choose prices than when they choose output levels. This alternative notion of the intensity of competition also accords with (23), as

$$\left.\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}\right|_{\text{Bertrand}} - \left.\frac{\partial \Pi^*}{\partial \lambda}\frac{1}{\Pi^*}\right|_{\text{Cournot}} = \frac{2\delta^3(1+2\lambda-3\lambda^2)}{(1-\delta\lambda)(1+\delta\lambda)(2-\delta-\delta\lambda)(2+\delta+\delta\lambda)} > 0.$$

Internal common ownership. In the internal common-ownership model of Section 7, we claimed that when the blockholder acquires shares only from dispersed shareholders, the impact of common ownership on equilibrium profits depends on the product-market competition game being one of strategic substitutes or complements. To show this, consider again Example 1, where firms compete in quantities (strategic substitutes). Equilibrium profits are

$$\begin{aligned}
\pi_1^*(s_2^D) &= \frac{\left[\beta(2+\delta)+s_2^D\delta\right]\left[\beta(2+\delta)-(1+\delta)s_2^D\delta\right]}{\left[\beta(4-\delta^2)-s_2^D\delta^2\right]^2} \\
\pi_2^*(s_2^D) &= \frac{\beta^2(2+\delta)^2}{\left[\beta(4-\delta^2)-s_2^D\delta^2\right]^2}
\end{aligned}$$

Using these formulas, it appears that B_1 is always decreasing in s_2^D.

On the other hand, in Example 2 equilibrium profits are

$$\begin{aligned}
\pi_1^*(s_2^D) &= \frac{(1+\delta)\left[\beta(2+\delta)-s_2^D\delta\right]\left[\beta(2+\delta)+(1+\delta)s_2^D\delta\right]}{(1-\delta)\left[\beta(4-\delta^2)-s_2^D\delta^2\right]^2} \\
\pi_2^*(s_2^D) &= \frac{(1+\delta)\beta^2(2+\delta)^2}{(1-\delta)\left[\beta(4-\delta^2)-s_2^D\delta^2\right]^2}.
\end{aligned}$$

In this case, B_1 is always increasing in s_2^D at $s_2^D = 0$.

Example 3. We now specialize Example 1 by assuming that the product is homogeneous but extend the analysis to the case of $N > 2$ firms. Market demand now is

$p = 1 - Q$, where Q denotes aggregate output.

Firm i chooses its output q_i to maximize $\pi_i + \lambda \sum_{j \neq i} \pi_j$. It is straightforward to show that this is equivalent to maximizing $(1 - q_i - Q_{-i})[q_i + \lambda Q_{-i}]$, where Q_{-i} denoted the aggregate output of i's competitors.

In the symmetric equilibrium, where $Q_{-i} = (N-1)q_i$, we have $q_i = \frac{1}{N+1+\lambda(N-1)}$ and thus $Q = \frac{N}{N+1+\lambda)(N-1)}$. This implies $p = \frac{1+\lambda(N-1)}{N+1+\lambda(N-1)}$ and $\pi_i^* = \frac{1+\lambda(N-1)}{[N+1+\lambda(N-1)]^2}$. Therefore

$$\Pi^*(\lambda, N) = \frac{N[1+\lambda(N-1)]}{[N+1+\lambda(N-1)]^2}.$$

It is now straightforward to calculate

$$\frac{\partial \Pi^*}{\partial N} = -\frac{(1-\lambda)^2(N-1)}{[N+1+\lambda(N-1)]^3} < 0,$$

whence it follows

$$\frac{\partial^2 \Pi^*}{\partial \lambda \partial N} = \frac{(N-1)(1-\lambda)[(5-\lambda)N - (1-\lambda)]}{[N+1+\lambda(N-1)]^4} \geq 0.$$

The reader may easily check that the conclusion continues to hold when products are differentiated, and when firms compete in prices.

Example 4. Going back to the case $N = 2$, we now examine the case of Cournot competition with homogeneous products and $p = 1 - Q$, under the assumption that firm 1 has a positive marginal cost $c > 0$, whereas 2's marginal cost is 0.

Under these assumptions, it is straightforward to show that equilibrium outputs and profits are

$$\begin{aligned}
q_1^* &= \frac{1 - 2c - \lambda_1}{3 - \lambda_1 - \lambda_2 - \lambda_1\lambda_2} \\
q_2^* &= \frac{(1+c)(1-\lambda_2)}{3 - \lambda_1 - \lambda_2 - \lambda_1\lambda_2} \\
\Pi_1^* &= \frac{(1 - 2c - \lambda_1)[1 - \lambda_1\lambda_2 - c(2 - \lambda_1 - \lambda_1\lambda_2)]}{(3 - \lambda_1 - \lambda_2 - \lambda_1\lambda_2)^2} \\
\Pi_2^* &= \frac{(1+c)(1-\lambda_2)[1 + c(1-\lambda_2) - \lambda_1\lambda_2]}{(3 - \lambda_1 - \lambda_2 - \lambda_1\lambda_2)^2}.
\end{aligned}$$

Using these formulas, it is easy to calculate the difference $\frac{\partial \Pi^*}{\partial \lambda_1} - \frac{\partial \Pi^*}{\partial \lambda_2}$ at $\lambda_1 = \lambda_2 = \lambda$. We have

$$\frac{\partial \Pi^*}{\partial \lambda_1} - \frac{\partial \Pi^*}{\partial \lambda_2} = \frac{2c(2-c)}{(1-\lambda)(3-\lambda)^2} > 0,$$

confirming that aggregate profits increase more if it is the less efficient firm that internalizes the rival's profits more, rather than the opposite.