Corrigendum

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Integral formulas for a class of curvature PDE's and applications to isoperimetric inequalities and to symmetry problems

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In the proof of Theorem 1.2 (isoperimetric estimates) in [2], we used inequality (15) without actually knowing the sign of the eigenvalues of $\partial\bar{\partial}f$, where f is a solution of (14); we thank Professor Zbigniew Błocki for having pointed out it to us.

So, Theorem 1.2 in [2] is true for $j = 1$, since in that case inequality (15) holds without any hypothesis on the sign of the eigenvalues of the matrix A (the case $j = 1$ in the statement of Theorem 1.2 corresponds to $j = 2$ in formula (15)).

However, here we show that once one has the case $j = 1$, by using some Gärding inequalities, one can prove formula (2) in Theorem 1.2 also for $j > 1$.

We use the same notation as in [2]. Let $K_{\partial\Omega} = K^{(1)}_{\partial\Omega}$. Then we have the following:

Theorem (Isoperimetric estimate). *Let Ω be a bounded domain of $\mathbb{C}^{n+1}$ with boundary a real hypersurface of class C^∞. If $K_{\partial\Omega}$ is positive, then*

$$\int_{\partial\Omega} \frac{1}{K_{\partial\Omega}(x)}\, d\sigma(x) \geq 2(n+1)|\Omega|, \tag{1}$$

where $|\Omega|$ is the Lebesgue measure of Ω. If $K_{\partial\Omega}$ is constant, then equality holds in (1) *if and only if Ω is a ball of radius $\frac{1}{K_{\partial\Omega}}$.*

Proof. As in [2], Theorem 1.2, case $j = 1$. □

Here we recall some inequalities from [1]. Let $\lambda(A) = \{\lambda_1, \dots, \lambda_n\}$ be the vector of eigenvalues of an $n \times n$ Hermitian matrix A. For $k \in \{1, \dots, n\}$ we define the normalized k-th elementary symmetric function of the eigenvalues of A as

$$s_k(A) = \frac{\sigma_k(A)}{\binom{n}{k}}.$$

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We also denote by $\Gamma_k \subseteq \mathbb{R}^n$ the connected component of the set $\{\lambda \in \mathbb{R}^n : s_k(A) > 0\}$ which contains the vector $(1, \dots, 1)$. We have that $\Gamma_k \subseteq \Gamma_i$ if $i \leq k$; moreover, if $\lambda(A) \in \Gamma_k$, then it holds

$$(s_k(A))^{\frac{1}{k}} \leq s_1(A). \tag{2}$$

Now, we have the following:

Corollary. *Let $j \in \{2, \dots, n\}$. Let Ω be a bounded domain of $\mathbb{C}^{n+1}$ with boundary a real hypersurface of class C^∞. If $K^{(j)}_{\partial\Omega}$ is positive, then*

$$\int_{\partial\Omega} \left(\frac{1}{K^{(j)}_{\partial\Omega}(x)} \right)^{\frac{1}{j}} d\sigma(x) \geq 2(n+1)|\Omega|, \tag{3}$$

where $|\Omega|$ is the Lebesgue measure of Ω. If $K^{(j)}_{\partial\Omega}$ is constant, then equality holds in (3) *if and only if Ω is a ball of radius $(\frac{1}{K^{(j)}_{\partial\Omega}})^{\frac{1}{j}}$.*

Proof. Since $\partial\Omega$ is compact, there exists at least a point of ellipticity, namely there exists $p_0 \in \partial\Omega$ such that all the eigenvalues of the second fundamental form (at p_0) are strictly positive; therefore all the eigenvalues of L_{p_0} (the Levi form at p_0) are strictly positive: in particular, $\lambda(L_{p_0}) \in \Gamma_j$. Now, since the function $K^{(j)}_{\partial\Omega} := s_j(L_p)$ is positive and continuous on $\partial\Omega$, we have that $\lambda(L_p) \in \Gamma_j$, for all $p \in \partial\Omega$. Hence, by (1) and (2), we obtain

$$\int_{\partial\Omega} \left(\frac{1}{K^{(j)}_{\partial\Omega}(x)} \right)^{\frac{1}{j}} d\sigma(x) \geq \int_{\partial\Omega} \frac{1}{K_{\partial\Omega}(x)} \, d\sigma(x) \geq 2(n+1)|\Omega|. \qquad \square$$

References

[1] L. Gärding, An inequality for hyperbolic polynomials, *J. Math. Mech.* **8** (1959), 957–965.

[2] V. Martino and A. Montanari, Integral formulas for a class of curvature PDE's and applications to isoperimetric inequalities and to symmetry problems, *Forum Math.* **22** (2010), no. 2, 255–267.