

Toward disciplinary boundaries: “Mixed Mathematics” experiments at the Accademia del Cimento (1657–1667)

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Summary. — The Accademia del Cimento (1657–1667), established under the patronage of the Medici family and inspired by Galileo’s legacy, carried out more than a thousand experiments, focusing on pneumatics (vacuum and air pressure) and thermology (heat, cold, and freezing phenomena). Its agenda also included studies in optics, acoustics, and mechanics, which can be associated with the contemporary concept of “Mixed Mathematics” —a term that emerged around 1660 to describe the application of numerical and geometrical principles to physical phenomena. Key experiments included investigations of the propagation of sound and light through various media, pendulum oscillations, projectile trajectories, the fall and rebound of bodies. This article provides an overview of these activities, shedding light on a period when scientific disciplines were not yet specialized, and the boundaries between what we now consider distinct fields of physics were fluid and interconnected.

1. – Introduction

In the early modern period, European scholars engaged in inquiries into “Natural Philosophy,” a term often used by historians of science to describe the study of nature before it evolved into what we now recognize as modern “science”. This transition, spanning several centuries, gradually resulted in the emergence of distinct scientific disciplines such as biology, chemistry, and physics.

Until the Renaissance, Natural Philosophy was starkly separated from Mathematics, traditionally encompassing pure geometry and arithmetic. However, early modern scholars began exploring the principles governing nature through both qualitative descriptions and quantitative measurements. The increasing use of mathematical models to understand and explain nature [1], alongside the growing importance of experiments in mathematical sciences [2], led to the diffusion of a new term: “Mixed Mathematics.” Emerging around 1660, this term described the shift from purely abstract mathematical pursuits to the application of mathematics to tangible objects and natural phenomena, integrating areas such as astronomy, architecture, optics, acoustics, mechanics, engineer-

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ing⁽¹⁾. Over the years, Mixed Mathematics gained prominence over Pure Mathematics due to its practical utility and interdisciplinary nature, facilitated by the development of new mechanical instruments⁽²⁾.

Building on the framework of Mixed Mathematics in the early modern period, this article investigates experiments that exemplify the application of mathematical reasoning to tangible phenomena, within the context of the Florentine Accademia del Cimento (1657–1667). As the first European scientific academy to prioritize experimental methods, the Cimento provides a case study for understanding the interplay between theoretical concepts and practical experimentation. The Academy began its activities in 1657 under the patronage of Prince Leopoldo de Medici (1617–1675) and Grand Duke Ferdinando II (1610–1670). Operating without a formal constitution, it relied on collaborations among university lecturers and scholars, who used to gather at Pitti Palace in Florence. Despite dissolving in 1667 due to Leopoldo’s appointment as a cardinal and other factors, the Academy’s legacy endures through the extensive experiments conducted across diverse fields.

Although historiography has mainly emphasized the Cimento’s contributions to pneumatics and thermology —central topics in its only official publication, the *Saggi di Naturali Esperienze* (1667, hereafter: *Saggi* [7])— the Academy’s original documents preserved in the Galilean collection of the National Central Library of Florence (hereafter: BNCF) reveal a much broader experimental agenda⁽³⁾. These documents encompass a variety of mathematical and physical observations, some of which remain unpublished.

This paper highlights the diversity of the Academy’s experimental pursuits in Mixed Mathematics through case studies drawn from the science of sound (sect. 2), light (sect. 3), and motion (sect. 4)⁽⁴⁾. By revisiting previously overlooked manuscript sources from the BNCF, this study seeks to shed new light on the Academy’s rich experimental narrative, situating it within the broader intellectual landscape of seventeenth-century science —a period characterized by the fluid and interconnected boundaries of what we now consider distinct fields of physics.

2. – The science of sound

The establishment of scientific academies such as the Cimento stimulated a new experimental approach to the study of sound through observations with acoustic and musical instruments and the development of mathematical frameworks⁽⁵⁾.

⁽¹⁾ According to Brown [3], Francis Bacon (1561–1626) was the first to use the term in 1605. However, Bennett [4] challenged this perspective noting John Dee’s (1527–1608) 1570 distinction between “Simple” and “Mixed” Mathematics in the preface to the first English edition of Euclid’s *Elements*, which however did not align with the historians’ interpretation of the term Mixed Mathematics in the early modern period.

⁽²⁾ The evolving terminology —particularly the replacement of “Mixed” by “Applied Mathematics” after the eighteenth century [5]— poses challenges to historical analysis, requiring nuanced interpretations of its origins and applications [6].

⁽³⁾ Fundamental contributions on the Cimento’s experimental activity are [8–10]. For the status of mathematics in and around the Cimento, see [11].

⁽⁴⁾ Mixed Mathematics observations at the Cimento also encompassed areas such as astronomy, practical mathematics, and the science of water. However, these fall beyond the scope of this paper, which aims to provide an overview of specific case studies.

⁽⁵⁾ The term *acoustique*, derived from the Greek and meaning “capable of being heard”, was introduced in 1701 by the French mathematician Joseph Sauveur (1653–1716) and subsequently disseminated across Europe.

The *Saggi* features a section titled *Esperienze intorno ai movimenti del suono*, spanning five pages, which proposes three hypotheses: the independence of sound speed from the loudness or intensity of a shot, the effect of winds, and the physical distance traveled [12]. The latter concept, known as the “equability of sound,” was reportedly coined by Vincenzo Viviani (1622–1703), Galileo’s self-proclaimed “last disciple” and an active member of the Cimento⁽⁶⁾.

While the *Saggi* attributes these studies to the Academy’s activities, the Grand Duke’s interest in acoustics dates back to the 1650s⁽⁷⁾. Experiments conducted in Florence involved measuring the time delay between the visual and auditory perception of artillery fire, an innovative method inspired by the earlier work of French Minim Friar Marin Mersenne (1588–1648). An important innovation introduced by the Florentine scholars was the use of the bifilar pendulum as a timekeeping device, which enabled the assertion that sound travels a “known distance of one [Tuscan] mile” in “five seconds.” The Tuscan mile, measuring 2,833 braccia (approximately 1,653 meters), aligns closely with the modern international mile (1,609 meters). Consequently, the speed of sound calculated in 1656 corresponded to a value between 330 and 331 meters per second, remarkably close to today’s value of 331.29 meters per second at 0°C —far more accurate than Mersenne’s earlier estimates [17].

These pioneering calculations underscore the Cimento’s significant contributions to the experimental science of sound and proved important for grounding new observations in the general field of acoustics. By the 1660s, consistent with broader European interests in natural philosophy, the Academy was particularly focused on determining whether sound propagated through water or vacuum [18]. Concerning the propagation of sound in water, some observations indicated no visible movement or vibrations in the water; however, the scholars speculated that louder sounds might produce different results. Visual representations played a crucial role in documenting these studies, appearing both in private records (*e.g.*, experimental notes, correspondence) and public context like the *Saggi* [19]. This emphasis on images extended to other natural phenomena, such as the behavior of light, pendulum oscillations, and projectile trajectories. Regarding propagation in vacuum —typically obtained in Florence using the Torricellian method— numerous tests were carried out, although most were unsuccessful. The limitations in achieving a substantial vacuum hindered definitive conclusions, yet an experiment was nonetheless included in the *Saggi* to highlight the Academy’s methodology and rigor.

Overall, the Cimento’s innovative approach to the science of sound demonstrated how theoretical insights, practical experimentation and mathematical calculations merged to improve the understanding of physical phenomena.

3. – The science of light

The *Saggi* also includes a section dedicated to the science of light, with the title *Esperienze intorno alla luce e suoi effetti*. The first set of experiments describes an un-

⁽⁶⁾ In relation to equability, the academicians experimentally demonstrated that sound propagates uniformly in all directions —a topic introduced to the Cimento by the Aristotelian scholar Carlo Rinaldini (1615–1698) [13].

⁽⁷⁾ There are records on sound probably dating to the autumn of 1656 in Viviani’s notes [14] and epistolary [15, 16], which is full of undated letters and/or whose addressee is unknown.

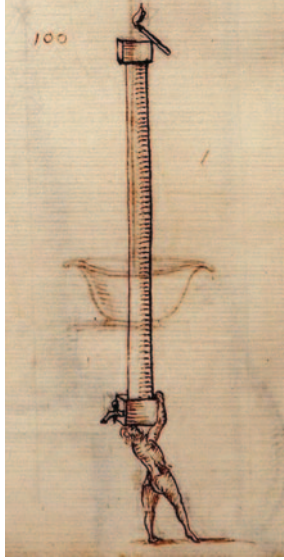


Fig. 1. – An artisan-academician observing the light of a flame through a vertical tube filled with water [20]. This figure is reproduced by kind permission of the Ministry of Culture - National Central Library of Florence (BNCF). Any further reproduction by any means must be authorized by the National Central Library of Florence (BNCF).

successful attempt to measure the speed of light, an ambitious undertaking given the technological limitations of the time [21]. Building on Galileo’s earlier work, the scholars employed various methods, such as lighting fires in sequence or simultaneously at separate locations within Medici-controlled territories, and measuring the time delay between the perception of light and sound. The physicist Giovanni Alfonso Borelli (1608–1679) even proposed a geometric method employing reflected rays in a system of mirrors [22], which was also discussed during an academic meeting in 1660 [23]. Although Galileo and Cimento’s approaches to light experiments share similarities, they differ methodologically [24]: while the first introduced what was later termed “reaction time” —related to the level of readiness and expertise of the observer— the latter never accounted for observer-induced errors. Nevertheless, the scholars acknowledged the challenges of operating pendulums with short oscillation periods. Despite their inconclusive results, these experiments illustrate the Academy’s commitment to advancing empirical inquiry and addressing one of the most complex scientific challenges of the epoch.

Unpublished manuscripts also attest to the Cimento’s interest in light behavior in different media. For example, scholars observed candle flames through water-filled tubes of varying orientations and lengths (see figs. 1 and 2, which demonstrate the extensive use of visuals), modifying the experimental setups to confirm their findings. This iterative methodology reflects the Cimento’s Dantean motto *Provando e riprovando* (“testing and retesting”), which encapsulates the Florentine approach to experimental science.

Additional observations from the *Saggi* include material ignition via reflected light, luminescence phenomena, and color changes in liquids. Although these experiments appear in the same section as the measurements of the speed of light, they differ in approach from the quantitative framework of classical optics. Instead, they explore light

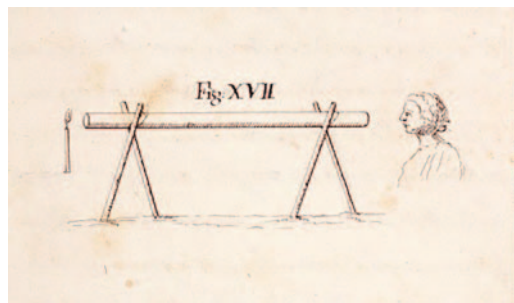


Fig. 2. – An artisan-academician observing the light of a flame through a horizontal tube filled with water [25]. This figure is reproduced by kind permission of the Ministry of Culture - National Central Library of Florence (BNCF). Any further reproduction by any means must be authorized by the National Central Library of Florence (BNCF).

as a physical phenomenon in its various appearances, behaviors, and manifestations, reflecting the era’s broad intellectual curiosity. Closely tied to alchemical traditions and chemical transformations, these investigations are primarily qualitative in nature, emphasizing observation and description over precise measurement.

In conclusion, the Academy’s diverse optical studies, both published and unpublished, illustrate the interdisciplinary strategies that characterized early modern empirical investigations into the science of light. These practices challenged traditional historiographical categories and contributed to evolving debates on the phenomenon of light and its properties during the period.

4. – The science of motion

In this last section, I turn my attention to the science of motion at the Cimento [26], with a traditional analysis of the discipline, that is with a focus on three key areas: pendulum oscillations, the free fall, and the motion of projectiles. While this scope does not fully encompass the broad spectrum of early mechanical science —particularly regarding practical mechanics and machinery design— it highlights subjects that were central to the historical actors involved and that exemplify Galileo’s legacy in Florence.

The pendulum was central to early mechanical experiments, serving both as a measuring tool (*e.g.*, as a chronometer in the study of acoustics and optics) and as a means to study oscillatory dynamics. The Cimento academicians observed the proportional relationships between wire lengths and oscillation times, as shown in parabolic arcs (fig. 3) and they addressed challenges such as wire twisting and measurement errors, recommending bifilar pendulums for improved stability. Unpublished folios in the miscellanea also contain numerous drawings of pendulums and their motions. In one experiment, the academicians employed a water-filled basin, showing that when hung from four non-parallel wires, the instrument tilted without spilling, while parallel wires caused spillage [27]. This highlighted the role of wire configuration in stability and demonstrated the dynamics of a compound pendulum, similar to simple pendulums, though only qualitative observations were recorded.

The Cimento also studied falling bodies, connecting their experimental findings to military applications and theoretical mechanics, including rebound behaviors. Using balls of different sizes, materials, and fillings, they demonstrated that discrepancies in reflec-

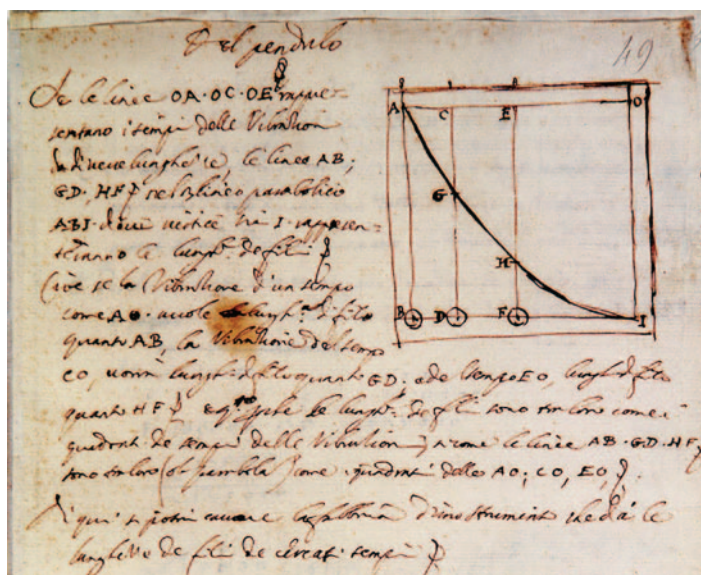


Fig. 3. – Notes on pendulum motion [28]. This figure is reproduced by kind permission of the Ministry of Culture - National Central Library of Florence (BNCF). Any further reproduction by any means must be authorized by the National Central Library of Florence (BNCF).

tion arose from both internal and external inconsistencies, reinforcing Galileo's earlier conclusions. These unpublished experiments were carried out in January 1667.

Building on such studies, the late seventeenth century saw advancements in precise measuring instruments, new projectiles, and firearm design, enabling innovative experimental setups. The Cimento incorporated ballistics into mechanical studies, underlining the strategic importance of artillery in the military efforts of the Medici; the scholars published their findings in the *Saggi*'s section *Esperienze intorno ai Proietti*, confirming Galileo's theories. Specifically, they found that projectiles fired vertically downward had a "supernatural" velocity compared to those fired horizontally and that projectiles retained their relative velocity whether fired from stationary or moving platforms.

Overall, the Cimento's investigations in experimental mechanics, though rooted in Galileo's legacy, were far from mere replications of his work. Instead, they extended his inquiries into the laws of motion, introducing new methods, using quantitative descriptions and contributing to the broader intellectual currents of the seventeenth century.

5. – Concluding remarks

This study showed the interplay between early modern empirical practices and the broader framework of Mixed Mathematics withing the context of the Cimento. Through examples from the science of sound, light, and motion, this paper highlighted how the Florentine Academy's interdisciplinary approach mirrored the complexity of seventeenth-century experimentation.

The structure of the *Saggi*, organizing experiments into sections addressing various aspects of nature, represents one of the earliest systematic attempts to order the empirical study of the natural world. This organizational effort transcended the chronological

sequence of experiments recorded in the Academy’s manuscript notes, reflecting a deliberate taxonomic impulse and a methodical approach to clustering experiments and classifying knowledge. In addition, by emphasizing fields like pneumatics and thermology, the Academy demonstrated its intellectual priorities, focusing on controlled and measurable phenomena over observational practices associated with less quantifiable areas.

The Cimento’s systematic arrangement invites further investigation of the criteria to classify natural inquiry in the early modern period, while being attentive to the risks of anachronism and the diverse historical and historiographical categories at play. The difficulty of strictly categorizing empirical observations underscores the pre-specialization context of physics, a period when natural phenomena were studied holistically rather than confined to narrowly defined fields.

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