

¹ Appendix S2 - The parametric Rao's Q

² variation over the planet

³ From zero to infinity: minimum to maximum diversity of the planet by

⁴ spatio-parametric Rao's quadratic entropy

⁵

⁶ February 7, 2021

7 We applied the algorithm to a Copernicus Proba-V NDVI (Normalized
8 Difference Vegetation Index) long term average image (June 21st 1999-2017)
9 at 5 km grain, also provided in the **rasterdiv** package as a free **Rasterlayer**
10 dataset which can be loaded by the function **data()** (Figure S1). The para-
11 metric Rao's Q algorithm can also be applied to multispectral data; in such
12 a case distances are calculated in the multisystem created by the values of
13 the pixels in each axis/band. The moving window passing throughout the
14 whole image will return M_{Q_α} matrices/layers where α is the value chosen in
15 the R function **paRao**.

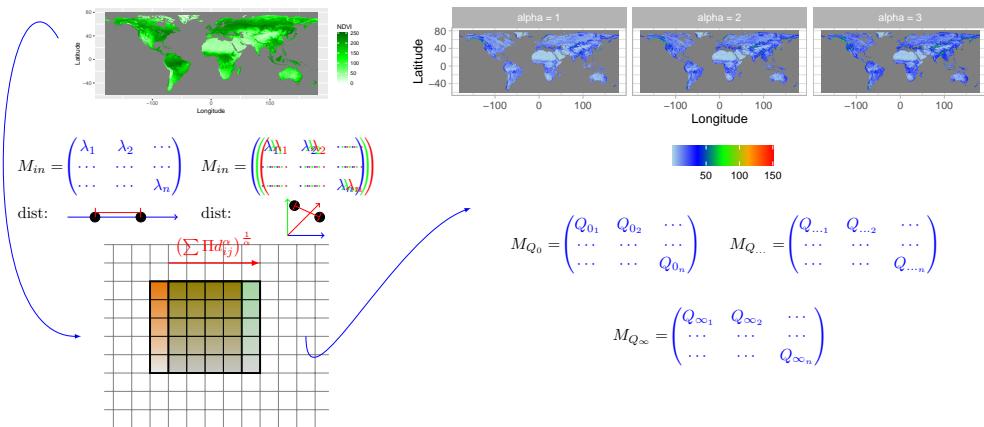
16 With $\alpha \rightarrow 0$ the \prod in Equation 6 (in the main text of the manuscript)
17 leads to zeroes throughout the whole map (Figure S2). Increasing α will
18 increase the weight of higher distances among different values until reaching
19 the maximum distance value for $\alpha \rightarrow \infty$. In this case the maximum turnover
20 is reached and areas with maximum β -diversity will be apparent. In this
21 case, a multitemporal set is used (long term average NDVI from June 21st
22 1999-2017). Hence, areas with the highest spatial and temporal turnover are
23 enhanced, namely major mountain ridges. We expect that using single frame
24 images would lead to the enhancement of the spatial component of diversity.

25 Since the whole process is based on distances in a spectral space between
26 pairs of pixels in terms of their “spectral characters” or in the “spectral
27 space”, it is important to notice some cornerstone aspects on the use of

28 distances from satellite images, especially when comparing different images
29 or the same image in different times. In satellite images, the measure of
30 distances could be impacted by: i) the use of different sensors with different
31 radiometric resolutions, as an example an 8-bit ($2^8 = 256$ values) with respect
32 to a 16-bit ($2^{16} = 65536$ values) image, or ii) the radiometric calibration
33 which has been performed, e.g. with a non-linear transform. Therefore,
34 care should be taken when making use of distances in remote sensing data,
35 explicitly taking into account how the vector of proportions between pixels
36 belonging to some defined classes (e.g., digital numbers, DNs) was obtained.

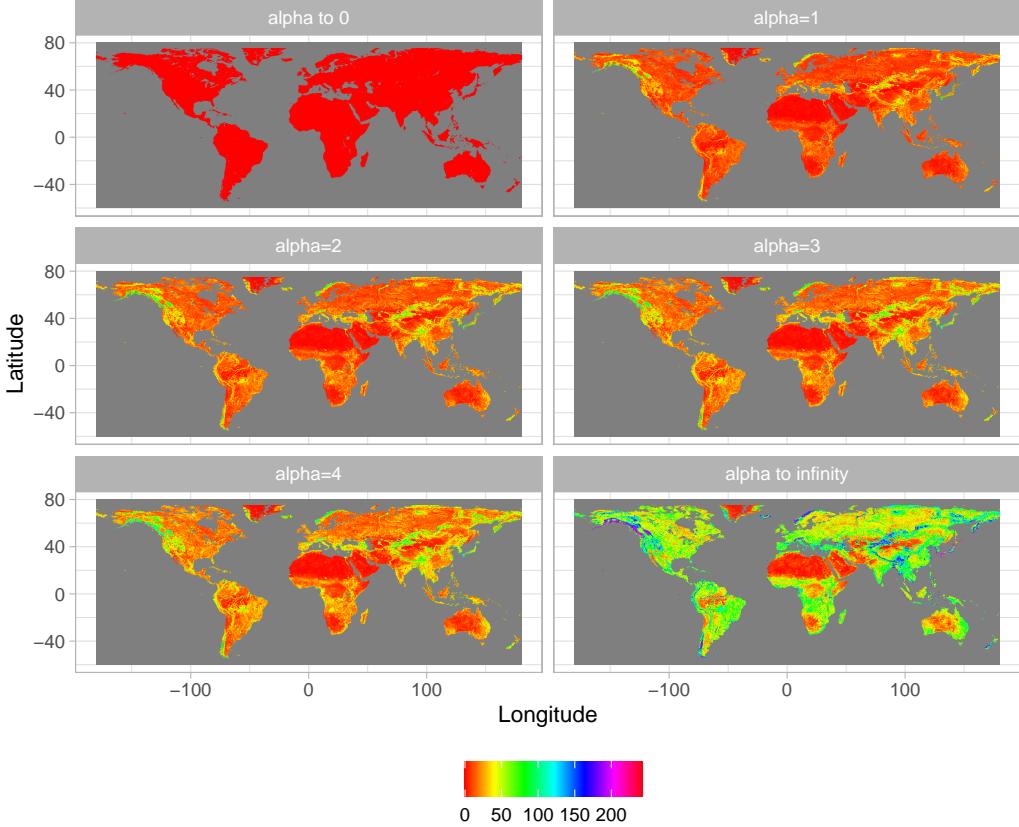
37 The complete code of the function can be directly seen in R by typing the
38 **paRao** function name. Moreover, a complete R coding session, to perform
39 the above described analysis is provided at the end of this Appendix.

40 1 Figures



41 Figure S1. Starting from Copernicus Proba-V NDVI (Normalized Differ-
 42 ence Vegetation Index) long term average image (June 21st 1999-2017) at
 43 5km grain, parametric Rao's Q is calculated in a moving window. In this
 44 paper NDVI was used as a single layer to calculate distances on one axis, but
 45 several layers can be used as well. In this example, three layers (blue, green
 46 and red matrices) are shown to calculate distances. The algorithm is based
 47 on a moving window passing throughout the whole image, calculating the
 48 Rao's Q_α and saving the output in the central pixel. In this example a mov-
 49 ing window of 5x5 pixels is passing (red arrow) from one position (orange)
 50 to the other (green). The output is a stack of layers each of which represents

⁵¹ a different mean of the whole generalized mean spectrum of Equation 5 (in
⁵² the main text of the paper).



53 Figure S2. Output of the application of the algorithm shown in Figure
 54 S1, achieved by applying different α values: from 0 to 4 until $\alpha \rightarrow \infty$. The
 55 higher the value of the parameter α , the higher the weight of highest dis-
 56 tances among pixel values, until reaching the maximum potential β -diversity
 57 (maximum distance) at $\alpha \rightarrow \infty$.

58 2 Code

59 2.1 paRao function

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60  function (x, dist_m = "euclidean", window = 9, alpha = 1,           1
61    method = "classic",
62    rasterOut = TRUE, lambda = 0, na.tolerance = 0, rescale =
63    FALSE,
64    diag = TRUE, simplify = 3, np = 1, cluster.type = "SOCK",      3
65    debugging = FALSE)
66 {
67   is.wholenumber <- function(x, tol = .Machine$double.eps
68   ^0.5) abs(x -
69     round(x)) < tol
70   if (!(is(x, "matrix") | is(x, "SpatialGridDataFrame") |      7
71   is(x,
72     "RasterLayer") | is(x, "list"))){                                9
73     stop("\nNot a valid x object.")
74   }
75   if (is(x, "SpatialGridDataFrame")){
76     x <- raster(x)
77   }
78   else if (is(x, "matrix") | is(x, "RasterLayer")){
79     rasterm <- x
80   }
81   else if (is(x, "list")){
82     rasterm <- x[[1]]                                              13
83   }
84   if (na.tolerance > 1 | na.tolerance < 0){                           15
85     stop("na.tolerance must be in the [0-1] interval.
86 Exiting...")                                                       17
87   }
88   if (any(!is.numeric(alpha))){
89     stop("alpha must be a numeric vector. Exiting...")                  21
90   }
91   if (any(alpha < 0)){
92     stop("alphas must be only positive numbers. Exiting
93 ...")                                                               23
94   }
95   if (method == "classic" & is(x, "RasterLayer")){
96     isfloat <- FALSE
97     if (!is.wholenumber(rasterm@data@min) | !is.
98     wholenumber(rasterm@data@max) |
99       is.infinite(rasterm@data@min) | !is.wholenumber(
100      median(getValues(rasterm),
101        na.rm = T))) {                                              25
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```

```

102     message("Input data are float numbers. Converting      35
103     x data in an integer matrix...")
104     isfloat <- TRUE
105     mfactor <- 100^simplify
106     rasterm <- getValues(rasterm) * mfactor           37
107     rasterm <- as.integer(rasterm)
108     rasterm <- matrix(rasterm, nrow(x), ncol(x),
109     byrow = TRUE)
110     gc()                                                 41
111   }
112   else {
113     rasterm <- matrix(getValues(rasterm), ncol = ncol
114 (x),
115     nrow = nrow(x), byrow = TRUE)                      45
116   }
117   message("Matrix check OK: \nParametric Rao output
118 matrix will be returned")
119 }
120 else if (method == "classic" & (is(x, "matrix") | is(x, "
121 list"))) {
122   isfloat <- FALSE
123   if (!is.integer(rasterm)) {
124     message("Input data are float numbers. Converting
125 x in an integer matrix...")
126     isfloat <- TRUE
127     mfactor <- 100^simplify
128     rasterm <- as.integer(rasterm * mfactor)           55
129     rasterm <- matrix(rasterm, nrow(x), ncol(x),
130     byrow = TRUE)
131     gc()                                                 57
132   }
133   else {
134     rasterm <- as.matrix(rasterm)
135   }
136   message("Matrix check OK: \nParametric Rao output
137 matrix will be returned")
138 }
139 else ("The class of x is not recognized. Exiting...")
140 if (window%%2 == 1) {
141   w <- (window - 1)/2
142 }
143 else {
144   stop("The size of the moving window must be an odd
145 number. Exiting...")
146 }
147 if (np == 1) {
148   if (method == "classic") {
149     out <- lapply(alpha, paRaoS, rasterm = rasterm, w
150 = w,

```

```

151             dist_m = dist_m, na.tolerance = na.tolerance,
152             diag = diag, debugging = debugging, isfloat = 75
153             isfloat,
154             mfactor = mfactor)
155         }
156         else if (method == "multidimension") {
157             out <- lapply(alpha, mpaRaoS, x = x, rasterm =
158 rasterm,
159             w = w, dist_m = dist_m, na.tolerance = na.
160 tolerance,
161             rescale = rescale, lambda = lambda, diag = 81
162 diag,
163             debugging = debugging)
164         }
165         if (rasterOut == T & class(x) == "RasterLayer") {
166             outR <- lapply(out, raster, template = x) 85
167             return(outR)
168         }
169         else {
170             return(out)
171         }
172     }
173     else if (np > 1) {
174         if (method == "multidimension") { 93
175             stop("Multidimensional paRao not yet implemented,
176 set 'np=1'. Exiting...")
177         }
178         else {
179             message("\n##### Starting 97
180 parallel calculation ######")
181             if (debugging) {
182                 cat("#check: Before parallel function.") 99
183             }
184             if (cluster.type == "SOCK" || cluster.type == "
185 FORK") {
186                 cls <- makeCluster(np, type = cluster.type,
187 outfile = "", 101
188                 useXDR = FALSE, methods = FALSE, output = "
189 ")
190             }
191             else if (cluster.type == "MPI") { 105
192                 cls <- makeCluster(np, outfile = "", useXDR =
193 FALSE,
194                 methods = FALSE, output = "") 107
195             }
196             else {
197                 message("Wrong definition for 'cluster.type'.
198 Exiting...")
199             }

```

```

200     doParallel::registerDoParallel(cls)
201     on.exit(stopCluster(cls))
202     gc()
203     out <- lapply(alpha, paRaoP, rasterm = rasterm, w  113
204     = w,
205             dist_m = dist_m, na.tolerance = na.tolerance,
206             diag = diag, debugging = debugging, isfloat = 115
207             isfloat,
208             mfactor = mfactor)
209     if (rasterOut == T & class(x) == "RasterLayer") { 119
210         outR <- lapply(out, raster, template = x)
211         return(outR)
212     }
213     else {
214         return(out)
215     }
216 }
217 }
218 }
```

2.2 Application of the paRao function to a synthetic set

```

220 # install rasterdiv
221 install.packages("rasterdiv")
222
223 library(raster)
224 library(rasterdiv)
225
226 # generate matrix
227 synth <- raster(ncol = 8, nrow = 8, xmn = 1, xmx = 6, ymn = 4
228     1, ymx = 6)
229 values(synth) <- rpois(ncell(synth), lambda=3)
230
231 # paRao function, using the code in the manuscript
232 synth.parao <- paRao(synth, alpha = c(0:4,30^9), dist_m = " 12
233     euclidean", window = 9, na.tolerance = 0.5, simplify = 3,
234     diag = T, rasterOut = T)
```

2.3 Application of the paRao function to the 8bit cop- NDVI dataset

```

237 library(rasterdiv)
238
239 st <- paRao(copNDVI, alpha = c(0:4,Inf),
240     dist_m = "euclidean", window = 9, na.tolerance = 0.5, 3
241     simplify = 3, diag = TRUE, rasterOut = TRUE)
```

242 2.4 Output plot

```
243 library(raster)                                     2
244 library(ggplot2)
245 library(rasterVis)
246 library(RColorBrewer)                                4
247
248 var.labs=c("layer.1" = "alpha to 0", "layer.2" = "alpha=1", "  6
249   layer.3" = "alpha=2", "layer.4" = "alpha=3", "layer.5" = "
250   alpha=4", "layer.6" = "alpha to infinity")
251
252 gplot(st, maxpixels=500000) +                      8
253   geom_raster(aes(fill = value), color = "black") +
254   labs(x="Longitude",y="Latitude", fill="") +          10
255   scale_fill_gradientn(colors=rainbow(100)) +
256   coord_equal() +                                     12
257   theme_light() +
258   facet_wrap(~ variable, ncol = 2, labeller = labeller(
259     variable = var.labs))+                         14
260   theme(legend.position = "bottom") +
261   NULL                                              16
```