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A novel procedure to determine the effects of debonding on case exposure of solid rocket motors

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(Article begins on next page)

1 **A Novel Procedure to Determine the Effects of Debonding on**
2 **Case Exposure of Solid Rocket Motors**

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7
8 **Abstract**

9 Solid rocket motors are complex systems which need to withstand extreme physical conditions in terms of
10 temperature, pressure, and high-density energy release. Therefore, specific attention should be brought to
11 the flaws that may occur during motor manufacturing\handling phases prior to launch. An example of such
12 flaws is debonding, usually arising at the interface between case insulation and solid grain. When debonding
13 is significant in size, it may result in the premature case exposure to combustion chamber hot gases, and,
14 in worst cases, it may even cause a complete motor failure. This work is intended to evaluate the impact of
15 propellant debonding on solid rocket motor case-insulating layer, making predictions about the most
16 unfavorable regions where the debonding could occur. Numerical simulations are performed with an in-
17 house simulation software applied to an actual solid rocket motor stage.

18
19 **KEYWORDS:** solid rocket motor flaws, grain debondings, debonding effect, solid rocket motor simulation.

20

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21 **1 Introduction**

22 In solid rocket motors, the interface region between the case and the solid grain consists of various layers
23 (Figure 1).

24 First, an insulation layer is used next to the case as a thermal coating [1] in order to protect it from the high
25 amount of thermal power released by grain combustion [2]. Then, a thin adhesive layer, known as *liner*,
26 bonds together the solid propellant and the thermal insulation. It is able, indeed, to establish chemical bonds
27 between the thermal protection layer and the grain itself. The last layer, much thicker than the previous
28 ones, is represented by solid propellant. The strength of the interface region is crucial due to the stresses
29 and strains accumulation taking place in that region. In fact, one fundamental requirement of the interface
30 is to withstand pressure loadings occurring during the phases before launch, from manufacturing process to
31 transportation [3]. However, if these stresses exceed the bond strength of the interface materials, a fracture
32 may arise through the grain which represents the weakest part between liner material and grain itself from
33 a mechanical perspective [4,5]. Hence, the propellant may separate from the insulation layer leading to
34 *debonding* [6].

35 Debonding areas are usually high-suspicion regions for two reasons. First, because the presence of
36 debonding during combustion could cause an increase in combustion chamber pressure. Due to both
37 debonding presence in the combustion chamber and debonding tip propagation, the debonding surface
38 increases, contributing with additional surface area to the burning process [7], in a similar way as for grain
39 cavities [8]. The burning surface increase leads to a pressure rise: if the pressure becomes higher than the
40 design pressure, it may cause unsustainable mechanical deformations and even motor failure. To make
41 matters worse, the mechanical expansion of the case could heighten debonding dilatation velocity [9].
42 Therefore, the higher burning surface increase implies a greater pressure rise. Second, when the burning
43 surface reaches the debonding, the debonded area could offer a path for hot gases to prematurely attack
44 case-insulating thermal protection layer. When thermal protections are exposed to the combustion chamber
45 hot gases, thermal loads coming from the combustion chamber are restrained since the insulating material
46 changes its state of matter (sublimation) absorbing most of the thermal energy close to the wall.

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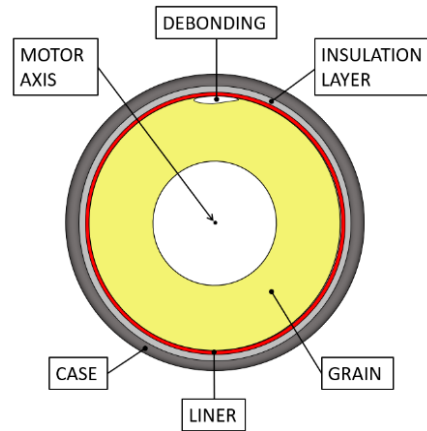


Figure 1: Propellant debonding.

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Under nominal conditions, thermal protections are designed to withstand a certain amount of thermal stresses for a theoretical time interval, since if they are not thick enough to absorb all the heat coming from the combustion chamber, they are no longer able to insulate the case [10]. When the burning surface reaches the debonding region, the thermal protection material layer is prematurely exposed to combustion chamber hot gases: if all the material is depleted before the end of grain combustion, the hot gases could reach the case. Also in this case, the ultimate consequence could be the launcher failure.

Hence, the capability to estimate the impact of debonding on solid rocket motors performance is of great importance in order to guarantee that the actual performance of the launcher closely matches the nominal one. Most works in the literature focus on the structural initiation\interaction of a propagating debonded flaw. For instance, Wu [9,11] shows a methodology to evaluate the combustion of solid propellant in a propagating debonded cavity. This model consists in linking together a 1D unsteady fluid dynamic model and a 0D viscoelastic model. The main result is an effective procedure establishing a pressure limit under which an existing debonding remains stable and no significant macrostructural damage appears. Meanwhile, Sih [12,13] uses an iso-energy density theoretical model to predict sites of potential failure initiation at the interface between liner and propellant. More in detail, local elevation of stresses and energy density leading to debonding propagation are investigated. Furthermore, in line with the aim of thoroughly examining debonded flaw propagation, in [14] a pure CFD model consisting of density-based Navier-Stokes equations is used in order to obtain the pressure distribution within the grain-liner debonding region. In addition, in [15,16] it is respectively shown how the liner properties influence the occurrence of strain critical locations, i.e., where a debonding could arise and a procedure to evaluate the interface resistance against debonded flaw initiation. As previously outlined, past literature on debonding impact on SRMs mainly focuses on the structural causes leading to debonding initiation and propagation. Some “check”

72 criteria [9] are obtained with the aim of establishing if a debonding goes through an unstable propagation
73 causing chamber over-pressurization and subsequent case burn-through. However, even if debonding
74 remains stable during grain combustion, it could have a great impact on SRMs performance in terms of
75 both combustion chamber pressure increment and premature exposure of case-insulating thermal protection
76 layer, as outlined in previous paragraphs. Additionally, to the present authors' knowledge, there are no past
77 studies regarding the impact of such debonding on case thermal exposure.

78 The aim of this work is the evaluation of the most critical zones in terms of case insulation exposure, by
79 proposing a procedure to reconstruct the SRM case exposure map of a generic-shaped debonding. Knowing
80 the most dangerous debonding positions on SRM case is fundamental at the time of the structural integrity
81 inspection of a solid rocket. In general, the occurrence of debonding is checked through radiography as a
82 non-destructive diagnostic tool for measuring debonding surface extension [17,18]. More in detail, an
83 ordinary x-ray imaging system[19–21] is used to inspect propellant bulk and case thermal protection with
84 a flaw size accuracy of about 0.5 mm [22]. Inspection tests are performed on both the overall motor and/or
85 at specific regions. Knowing the most critical case exposure regions, it is possible to drive the radiography
86 planning in the direction of a high-resolution local inspection next to critical areas, and a low-resolution
87 radiography in the remaining zones. Through the above-mentioned approach it is possible to devote most
88 time and effort to debonding in critical regions only, obtaining a more effective and optimized usage of the
89 x-ray technique in order to identify debond flaws in such regions. Furthermore, the method introduced in
90 the present work could be used to determine the effect on case exposure caused by debonding detected
91 through radiography observations. If the early exposure of the thermal protection material due to the
92 debonding region is considered unacceptable, the solid rocket stage could probably undergo specific
93 attempts of be repaired before final firing. Moreover, it is important to emphasize that the present study is
94 meant to investigate the debonding impact on SRMs integrity rather than debonding structural stability as
95 in previous literature. The proposed procedure allows for the evaluation of the risk linked to a generic
96 debonding by considering its effect in terms of case thermal protection exposure anticipation with respect
97 to the nominal condition without debonding, where the nominal case exposure condition is obtained with
98 ROBOOST [23,24], namely ROcket BOOst Simulation Tool internal ballistics software. That software has
99 been previously validated on an actual SRM, specifically, ZEFIRO 9, in [8]. Hence, ROBOOST software
100 tool is considered a reliable instrument to confirm the adequacy of the novel procedure aimed to determine
101 debonding influence on thermal protections exposure.

102

103 **2 Code Overview**

104 The first step of the proposed procedure consists in determining a map of the thermal protection exposure,
105 and a map of the angle of arrival of the combustion surface on the thermal protection layer. Both maps are
106 used to determine the effects of a debonding in terms of additional exposure of the thermal protection to
107 the hot gases that are present inside the combustion chamber and are evaluated using ROBOOST software.
108 One of the outputs that ROBOOST can provide is, in fact, the 3D representation of the burning surface
109 discretized with a triangular mesh: tracking the burning surface vertices that lay on the SRM case at each
110 simulation iteration, it is possible to determine when the corresponding case position begins to be exposed
111 to the combustion chamber hot gases, and evaluate the orientation (i.e., the direction of incidence) of the
112 burning surface motion when approaching the thermal protection surface. Identification of the burning
113 surface vertices laying on the case is possible simply looking at the points that belong to the free boundary
114 triangular facets (i.e., mesh edges) of the mesh itself.

115 A mesh edge in the triangulation is on the free boundary if it is referenced by only one triangle of the mesh
116 describing the combustion surface, implying that the set of free boundary vertices coincides with the set of
117 outer edges of the 3D triangulation. Figure 3 shows at different ROBOOST iterations, namely n_k and n_h
118 with $h > k$, mesh free boundary points moving from the positions marked in blue to the new positions
119 represented by green dots. At a specified iteration, each free boundary mesh vertex is uniquely linked to
120 both a specific point on the case surface and an iteration number. Since it is possible to evaluate the
121 corresponding web consumption (i.e., the thickness of burned propellant) at each iteration, each case
122 position can be associated with a single burning surface regression iteration, and thus, with a single value
123 of the web consumption. That value represents the web position where case exposure to the combustion
124 chamber hot gases begins.

125 As already mentioned, identifying mesh edges on the free boundary is possible because each edge is
126 referenced by a unique triangle of the mesh describing the combustion surface. The evaluation of the normal
127 vector to each of the triangles containing the mesh edges on the free boundary indicates the direction of
128 arrival of the combustion surface on the thermal protection. The inclination of the direction of arrival with
129 respect to the thermal protection surface describes the angle of arrival δ (Figure 2) of the combustion
130 process on the thermal protection surface. The angle δ can range from 0° (when the burning surface is
131 orthogonal to the local thermal protection surface, thus moving tangentially to it) to 90° (when the
132 combustion surface reaches the thermal protection layer moving along a orthogonal direction to it). As it
133 will be shown in the following sections, the evaluation of the angle δ is important in order to determine the
134 effect of debonding thickness, i.e., the extension along an orthogonal direction with respect to the thermal
135 protection surface.

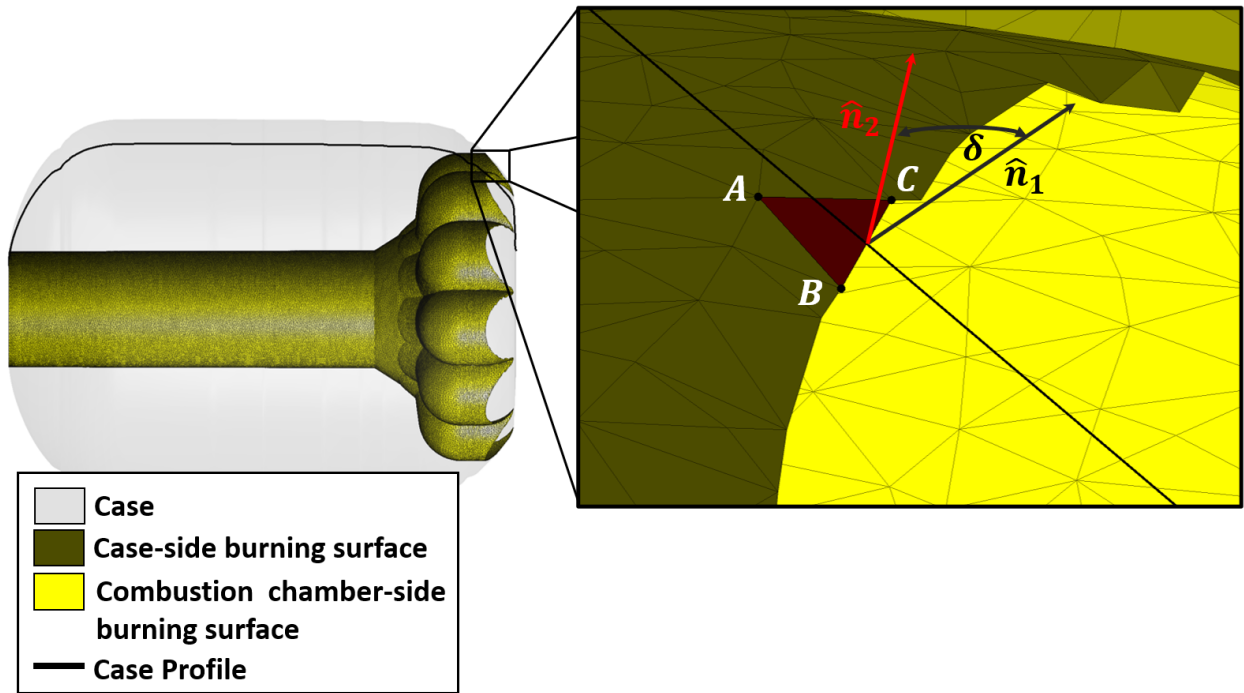


Figure 2: combustion surface-to-thermal protection arrival angle

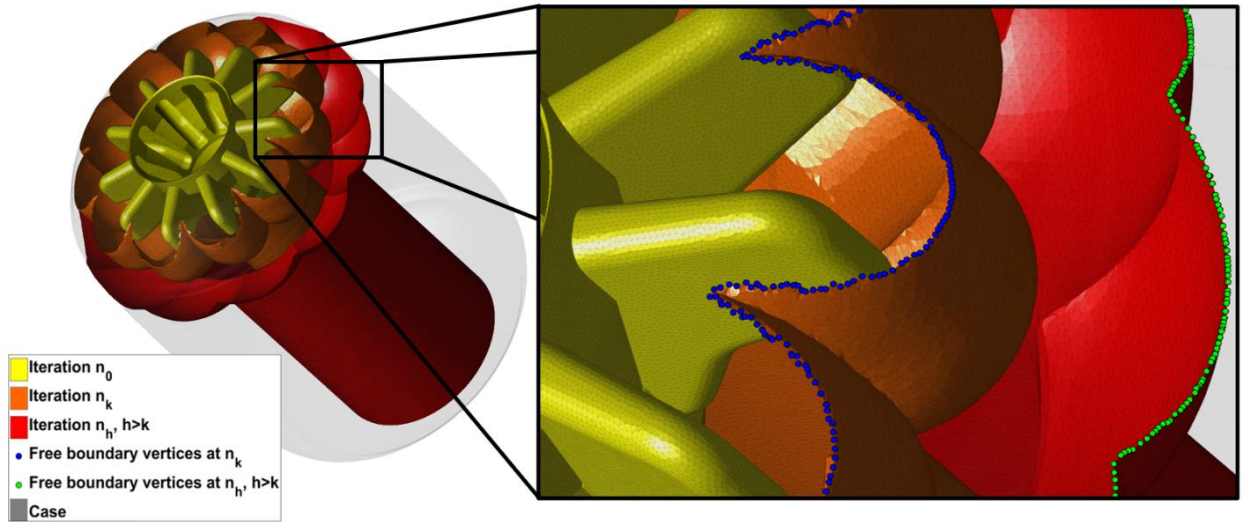


Figure 3: Mesh free boundary vertices.

141 Once the web consumption coordinate is obtained, it can be used to express thermal protection exposure
142 based on the following assumptions: web consumption value is equal to 0 (at iteration n_0) for all the free

143 boundary vertices of the initial mesh; the actual value can be obtained as the distance covered by the burning
144 surface to reach the actual positions; its maximum value is evaluated when the last portion of burning
145 surface reaches the case. Consequently, a thermal protection exposure parameter can be established as the
146 difference between the maximum web consumption and the local value associated to each case surface
147 point. The identified parameter has the same units as the web consumption, and it is named from now on
148 web exposure. Applying the evaluation to each point on the case surface, the outcome is a map of web
149 exposure values defined over the whole thermal protection describing its gradual uncovering: high web
150 exposure values correspond to case regions where the propellant is burned earlier with respect to case zones
151 with lower web exposure values. More in detail, zero web exposure values are linked to those case surface
152 zones which are exposed to combustion chamber hot gases at the burn-out phase; while maximum web
153 exposure is associated to case regions which are exposed to hot gasses at the beginning of the combustion
154 process. Hence, with the above-mentioned definitions, web exposure maps do not depend on the burning
155 rate at a specific time instant anymore, but rather they are suitable for general values of burning rate.
156 The presence of a debonding on the propellant-thermal protections interface creates a variation of the web
157 exposure, since it causes a quicker spreading of the flame along the flaw of the internal surface and within
158 its volume. It can be easily stated that the web exposure variation generated by a debonding depends on its
159 location and dimensions. The aim of the procedure described in the following paragraphs is to estimate
160 these effects.

161 *2.1 Debonding positioning effect*

162 The most suitable parameter to describe the influence of a flaw position is the gradient of the web exposure
163 which is directly linked to the local variation of the web exposure and corresponds to the rate of change of
164 the web exposure that is generated by the presence of an infinitesimal debonding located in each position
165 of the case. It must be emphasized that the gradient value does not depend on debonding shape, however,
166 it is used to show the most critical debonding in terms of its position on the case.

167 The representation of the web exposure map and its gradient is done using the reference frame represented
168 in red in Figure 4. It consists of three perpendicular unit vectors: e_{x_c} identifies the curvilinear coordinate
169 along the case profile (x_c), e_{a_z} is the azimuthal coordinate unit vector along the circumferential direction
170 (x_{a_z}) around the motor axis e_z , e_{x_n} corresponds to the perpendicular direction to the plane of the case
171 profile (x_n), while the center (O') of the local reference frame is taken on the case surface. The zero level
172 of the curvilinear coordinate x_c is assumed at the intersection between the black case profile and the vertical
173 axis (point Q in Figure 5), i.e., at the beginning of the case close to the igniter side. Hence, x_c increases
174 moving along the case from the igniter side to the nozzle one. Since exposure maps are computed on the

175 case surface, the local reference frame $O'x_cx_{az}x_n$ establishes two coordinates needed for the graphical
 176 representation of the maps. Such quantities are x_c (curvilinear coordinate) and x_{az} (azimuthal coordinate).
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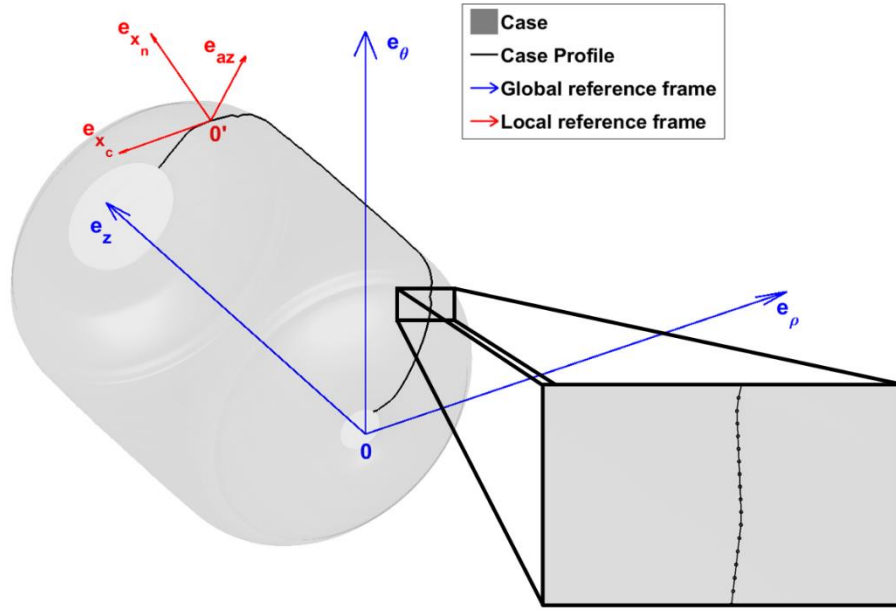


Figure 4: Reference frames.

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 180 As already mentioned, these coordinates are used to display the web exposure map and to compute the case
 181 web exposure gradient.

182 Based on the provided definitions, the gradient of the web exposure map ϕ is computed using Eq. (1).

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$$\nabla\phi = h_{x_c} \frac{\partial\phi}{\partial x_c} \vec{e}_{x_c} + h_{az} \frac{\partial\phi}{\partial x_{az,\theta}} \vec{e}_{az}$$

$$h_{x_c} = 1$$

(1)

$$h_{az} = \frac{1}{(x_c - x_{c_{0'}}) \sqrt{\frac{\beta^2}{\beta^2 + \alpha^2} + \rho_{0'}}$$

184
 185 where h_{x_c} and h_{az} are the *Lamè coefficients* respectively referred to the curvilinear coordinate and the
 186 azimuthal coordinate; $\rho_{0'}$ is the local reference frame center radial position (Figure 5) expressed in the

187 global cylindrical reference frame; x_{c_0} is the curvilinear coordinate value up to the center of the local
 188 reference frame. The detailed procedure to obtain the Lamè coefficients is shown in Appendix A. The case
 189 profile is discretized into a certain number of small linear segments along the curvilinear coordinate
 190 (segment AO' in Figure 5). That linear approximation guarantees a simpler form of the *Lamè coefficients*
 191 regarding the gradient expression without loss of accuracy. In fact, simply by increasing the number of
 192 linear discretization segments, a more refined gradient map that follows the case profile curvature more
 193 consistently is obtained.

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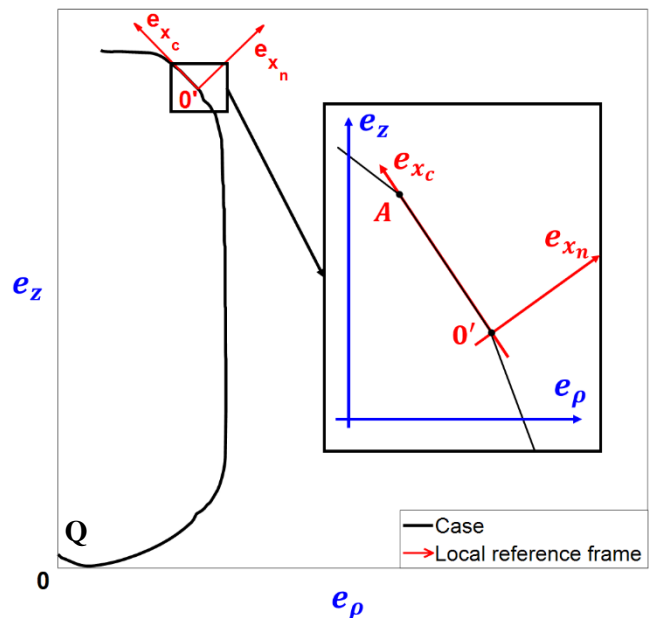


Figure 5: Reference frame for gradient computation.

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Figure 5 highlights the applied discretization process. It can be noticed that the blue reference frame in the figure is the same global reference frame of Figure 4 but it is expressed with respect to the motor axis; Eq. (1) is computed for each discretization segment of the case (segment AO' in Figure 5); α and β are defined by Eq. (2), representing the straight line passing through the two segment vertices (like A and O').

$$\alpha\rho + \beta z + \gamma = 0 \tag{2}$$

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As pointed out before, the local reference frame is sequentially moved on each segment of the case in order to evaluate segment-by-segment the gradient value of the case-insulating exposure map. Equation (3) shows

206 the formula linking x_{az} and $x_{az,\theta}$: the gradient was estimated by considering the azimuthal coordinate
 207 expressed in angle units ($x_{az,\theta}$).

208

$$x_{az} = \rho \cdot x_{az,\theta} \tag{3}$$

209

210 From here on, x_{az} will be named arc azimuthal coordinate, whereas $x_{az,\theta}$ will be identified as angular
 211 azimuthal coordinate. Finally, $\frac{\partial \phi}{\partial x_c}$ and $\frac{\partial \phi}{\partial x_{az,\theta}}$ are approximated using a second order centered scheme
 212 respectively along the curvilinear coordinate and the azimuthal direction.

213 *2.2 Debonding dimension effects*

214 The gradient map of web exposure evaluated on the thermal protection layer can highlight the most
 215 dangerous positions, independently of the flaw dimension. The effect of debonding extension along the
 216 case curvilinear coordinate x_c and azimuthal coordinate x_{az} is now investigated through a dedicated
 217 procedure using the same concept already introduced when considering the local gradient.

218 First, size and geometry of the debonding along x_c and x_{az} are chosen (in Figure 6 the displayed debonding
 219 has a square-shaped geometry with dimensions L_{x_c} and L_{az}). Then, the debonding is positioned at different
 220 points on the case exposure map (in Figure 6 case exposure map is represented with a black curve bounding
 221 the debonding). The new map, namely *exposure increase map*, is computed by associating at each point
 222 (representing the debonding center, like point P in Figure 6) the difference between the maximum and the
 223 minimum web exposure among the exposure values restrained by the debonding geometry on the web
 224 exposure map. The exposure increase map is then associated to the maximum effect a debonding positioned
 225 at each point on the case surface can cause on the thermal protection exposure time.

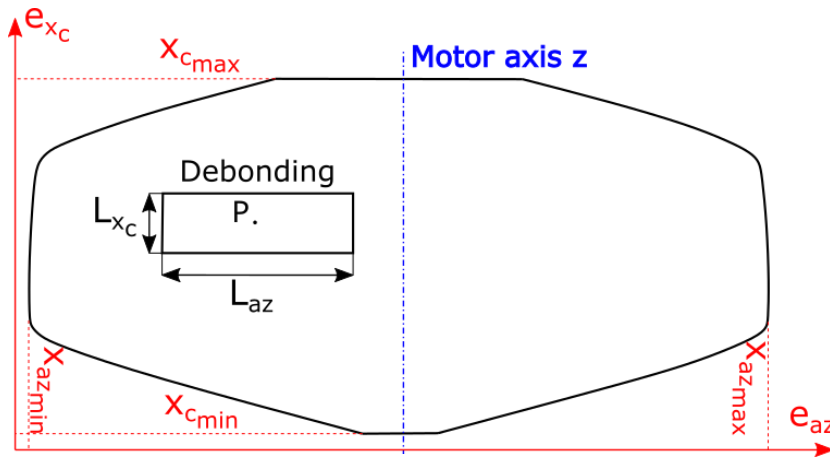


Figure 6: Debonding influence map generation.

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The debonding size along the direction e_{x_n} , i.e. the debonding thickness, usually has a negligible effect in terms of anticipating the thermal protection case exposure with respect to the nominal value. Indeed, in actual debondings, the thickness is an order of magnitude lower than the other two dimensions, namely L_{x_c} and L_{az} . For the sake of completeness and to address also those cases where thickness is not negligible, its effect and value is discussed in the next paragraphs. The maximum exposure advance due to the debonding thickness occurs when the burning surface moving toward the debonding remains parallel to the case. In a general situation the effect of thickness is associated with both the extension L_{x_n} of the debonding in the direction e_{x_n} , and the angle δ between the combustion surface regression direction and the thermal protection surface, as expressed by Equation (4):

$$\Delta exp_{th} = L_{x_n} \cdot \sin(\delta) \quad (4)$$

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Equation (4) clearly shows that the effect of thickness is directly proportional to the thickness value L_{x_n} , and depends on δ , with the largest influence associated with its 90° value. Exposure advance evaluated through Equation (4) indicates how much the minimum web exposure of the debonding footprint on the thermal protection is further anticipated due to its thickness. Based on these considerations, the exposure increase map should be evaluated as:

$$\Delta exp = \max(\text{web exp})_{deb} - \min(\text{web exp} - \Delta exp_{th})_{deb} \quad (5)$$

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It is of fundamental importance to highlight that this map (Figure 6) does not only allow for the investigation of debonding extended in one direction (e_{x_c} or e_{az}). As a matter of fact, such method also offers the possibility to deal with flaws characterized by generic aspect ratio values, where the aspect ratio is intended as the ratio between L_{x_c} and L_{az} . Even more, generic-shaped flaws can be included as well. In fact, in order to generate the debonding position influence map, it is sufficient to superimpose the debonding generic shape on the case exposure map and evaluating, as already mentioned before, the maximum difference among the exposure values bounded by the debonding itself.

The above-mentioned procedure has been validated with respect to ROBOOST software by comparing the obtained results for a series of appropriately designed flaws. Their location has been conveniently chosen in the direction of investigating both the most critical positions on the case and the impact of the debonding direction of elongation.

257 **3 Results and discussion**

258 The map generation method explained in the previous chapter has been applied to the third stage of Vega
259 launcher, namely, ZEFIRO 9 (Z9).

260 Vega is designed to launch small payloads: up to 1500 kg satellites for scientific and Earth observation
261 missions in low Earth orbits. It consists of four stages: three of them are solid propellant based, the fourth
262 is a liquid propellant engine. Z9 is 3.5 m tall, has a diameter of slightly less than 2 m, weighs 11,500 kg,
263 and burns 10,500 kg of HTPB based composite propellant. Regarding the propellant geometry
264 configuration, it has been designed with a circular section in the fore and central parts, and with a finocyl
265 shaped configuration in the rear part near the nozzle inlet (Figure 7).

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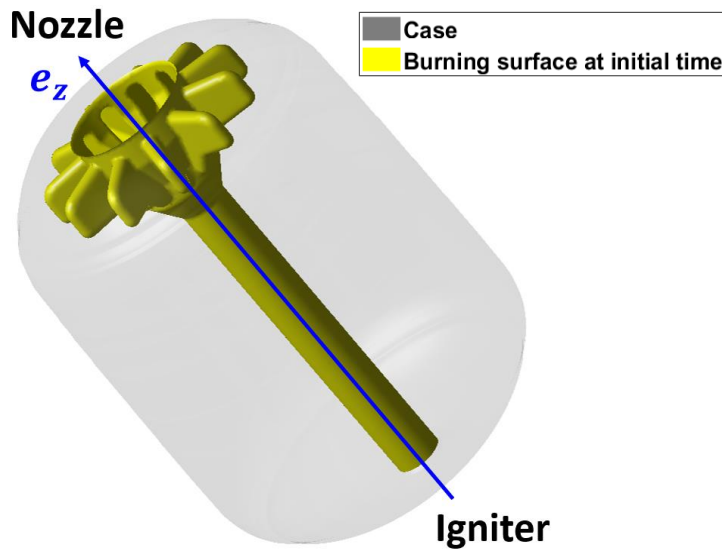


Figure 7: ZEFIRO 9 central bore.

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269 Figure 8 shows the web exposure map and its gradient in the different direction for the Z9 motor. In
270 particular Figure 8a shows the case-insulating thermal protection exposure map. The web exposure has
271 been normalized by dividing all values by its maximum value. The same procedure has been performed for
272 both the curvilinear coordinate x_c and the azimuthal coordinate x_{az} , respectively dividing them with respect
273 to the two maximum values $x_{c_{max}}$ and $x_{az_{max}}$.

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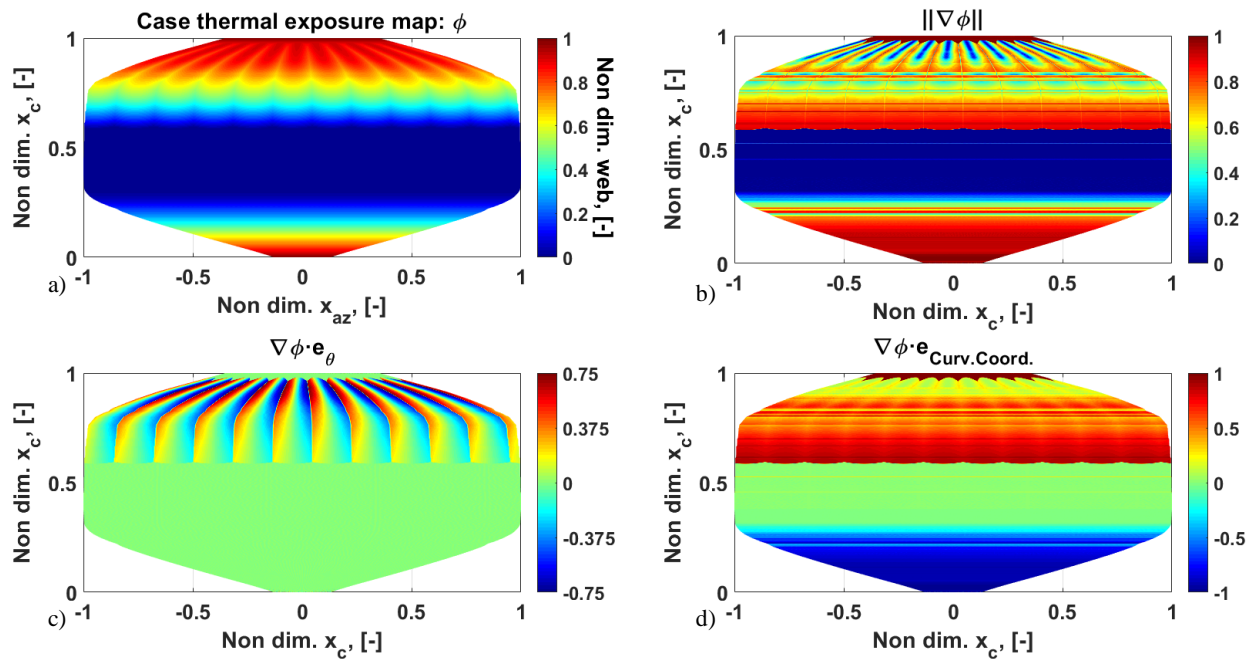


Figure 8: Maps linked to case exposure and case exposure gradient.

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277 The above-mentioned map shows the maximum exposure close to the curvilinear coordinate level
 278 corresponding to the fynocil region of the burning surface (Figure 7). Indeed, the local distance between
 279 each lobe of the fynocil configuration and the case is lower than the distance referred to the burning surface
 280 cylindrical region. Thus, it is evident that the case region with the fynocil shape is exposed to combustion
 281 chamber hot gases earlier than the other regions. However, there are other regions of the SRM which present
 282 such an early exposure. These zones are the end of the cylindrical shape and the end of the fynocil
 283 respectively in proximity of the igniter and of the nozzle. In fact, the burning surface recedes along the case
 284 meaning that, during its outward advancement in radial direction with respect to the motor axis, the
 285 propellant is gradually depleted causing the local uncovering of the case thermal protection layer. Figure
 286 8b displays the gradient norm regarding the exposure influence of debonding with an aspect ratio, computed
 287 by dividing its azimuthal coordinate and curvilinear coordinate elongation, is close to 1. Highly critical
 288 locations are depicted with red color, nearly zero gradient zones with blue color.

289 Figure 8c and d are respectively linked to the projection of case exposure map gradient along the azimuthal
 290 coordinate direction (e_{az}) and the curvilinear coordinate direction (e_{x_c}). In Figure 8c the green regions
 291 correspond to a zero-value gradient implying that a low impact on thermal exposure occurs for debonding
 292 mainly elongated along the azimuthal coordinate. The above-mentioned statement can be explained in
 293 accordance with the burning surface motion toward the case, indicating that the burning surface reaches the

294 case remaining parallel to it. The same considerations are valid for green regions in Figure 8d, where the
295 gradient trend is linked to debonding mainly elongated along the curvilinear coordinate direction. On the
296 other hand, regions with high gradient values (dark red and dark blue regions) regard case portions where
297 any debonding could significantly impact the thermal protection exposition in terms of web exposure
298 anticipation.

299 The observation of these maps allows determining the most critical regions where diagnostic procedures
300 should focus to identify flaws and also highlight the most dangerous direction along which the debonding
301 extension should be checked.

302 The following step is to evaluate the effect caused by the dimension of the flaws following the procedure
303 previously described. According to the local reference frame directions (red reference frame in Figure 6),
304 three sets of debonding are evaluated. The first set is characterized by the main elongation along the
305 azimuthal coordinate, while the second set is linked to the curvilinear coordinate elongation. The above-
306 mentioned debonding shapes are characterized by a large elongation along a specified main direction linked
307 to curvilinear or azimuthal coordinate. Since the procedure developed in this work allows the analysis of a
308 generic debonding shape regardless of the direction of the main extension, a third debonding with the same
309 magnitude along the two main directions, i.e., curvilinear, and azimuthal, has been investigated.

310 *3.1 First set of debonding*

311 The length of the main dimension of the flaw has been chosen equal to 207 mm . The reason for this choice
312 is associated with the evaluation of the minimum distance between two zero level regions containing the
313 maximum absolute peak (positive or negative) in Figure 8c. This extension of the flaw is in fact capable of
314 generating the highest thermal protection exposure increment in the azimuthal direction and, for this reason,
315 it is particularly interesting.

316 Based on this choice, the dimensions of the first set of debonding are as follows: $L_{az} = 207\text{ mm}$, $L_{x_c} =$
317 10 mm and $L_{x_n} = 5\text{ mm}$. L_{x_c} and L_{x_n} have been set respectively to 10 mm and 5 mm with the aim of
318 obtaining a debonding mainly extended along e_{az} . Another debonding (debonding 1a in Figure 9) was
319 considered with the same position, curvilinear and azimuthal extension of debonding 1 (Figure 9) but with
320 thickness L_{x_n} equal to 15 mm in order to investigate the web exposure induced by debonding thickness
321 increment.

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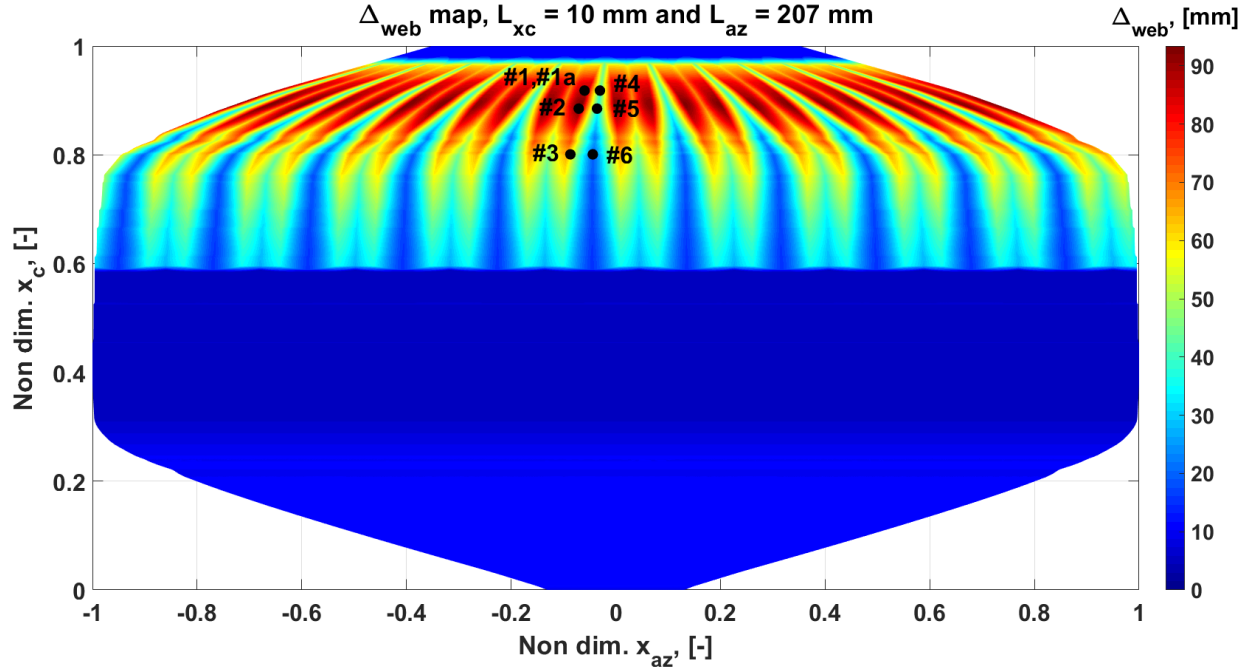


Figure 9: Debonding influence map regarding debonding elongation along e_{az} .

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325 The exposure increase map shown in Figure 9 is obtained following the procedure outlined in the previous
 326 section for the first set of flaws. Each point on the map represents the exposure increase that would be
 327 generated by a debonding characterized by the pre-defined geometry and centered on that point, with respect
 328 to the nominal motor. Eleven green-to-red colored regions are shown in the higher portion of Figure 9: each
 329 region corresponds to an evenly spaced lobe associated with the propellant finocyl configuration of Z9.
 330 Each of the identified regions contains two light red strips that are characterized by a dark red spot. The
 331 location of that spot corresponds to the maximum exposure increase (76 mm) and coincides with the
 332 maximum gradient position in Figure 8c.

333 In order to validate the obtained results, seven debonding positions (black dots in Figure 9, marked from 1
 334 to 6) have been chosen, located at 3 levels of x_c and 2 levels of $x_{az,\theta}$ ($x_{az,\theta} = -16.55^\circ$ and $x_{az,\theta} = -8^\circ$):
 335 in particular debonding 1, 1a, 2, 3 belong to level $x_{az,\theta} = -16.55^\circ$, while debonding 4, 5, 6 belong to level
 336 $x_{az,\theta} = -8^\circ$.

337 Each position is subsequently implemented inside the tool ROBOOST to perform a complete regression
 338 simulation, obtaining the corresponding 3D evolution of the burning surface. After running the simulations,
 339 the web exposure map is determined following the approach already explained in the previous paragraphs
 340 and schematized also in Figure 3. Web exposure maps with and without debonding are then compared, and
 341 the differences are shown in Figure 10. The results obtained for the different positions investigated are

342 reported in the same figure for the sake of compactness. Each result is presented within a box, while in the
343 remaining thermal protection surface there are not any effects and the exposure difference is equal to 0
344 (blue background). Each of the boxes is connected to a white dot, marking the real position where the
345 content of the box should be located, and contains the exposure increase generated by each debonding
346 position together with the corresponding color scale ranging from 0 to the maximum value observed. Within
347 each box, the blue regions are associated with an exposure increase that is equal to 0, while the surfaces
348 affected by the debonding are marked by other colors, where dark red is associated with the highest value.
349 The shape of the regions affected by the debonding is significantly different as it can be seen comparing
350 the content of the boxes. The debonding set at $x_{az,\theta} = -16.55^\circ$ (debonding 1, 1a, 2 and 3) presents only
351 one affected region for each debonding, while debonding set at $x_{az,\theta} = -8^\circ$ (debonding 4, 5 and 6) is
352 characterized by two (almost symmetrical) affected regions. This difference is due to the fact that for
353 positions 1, 1a, 2 and 3 the combustion surface reaches the debonded surface on one of the two sides of the
354 flaw (as shown in Figure 11a), while for positions 4, 5 and 6 the combustion surface intersects the debonded
355 surface at its midpoint and, from that condition on, two combustion-regression fronts simultaneously spread
356 from the above-mentioned midpoint towards debonding side points (Figure 11b). Furthermore, since the
357 thickness of debonding 1a is larger than the thickness of debonding 1, the burning surface reaches debonding
358 1a first, implying a higher web exposure increment (96 mm) compared to debonding 1 (87 mm).
359

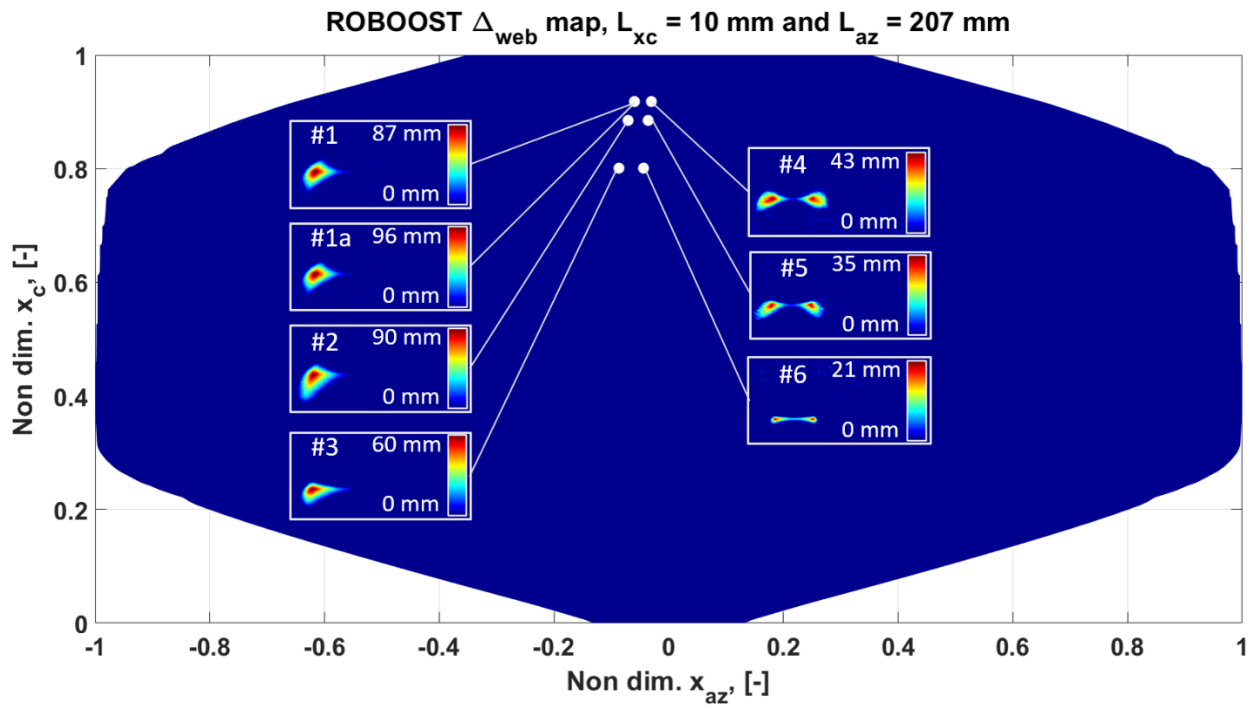


Figure 10: ROBOOST simulations regarding debonding elongated along azimuthal direction.

360

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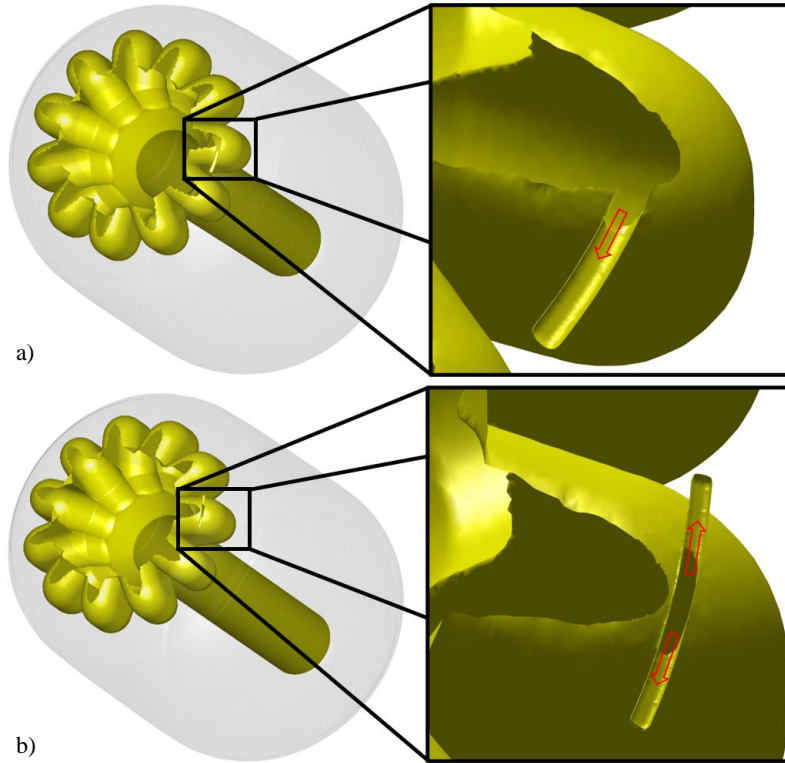


Figure 11: ROBOOST regression of debonding 1 (a) and 4 (b).

362
363
364
365
366
367

The maximum exposure increase obtained for each simulation can now be compared with the value determined with the geometrical approach developed in this work, as shown in Figure 12.

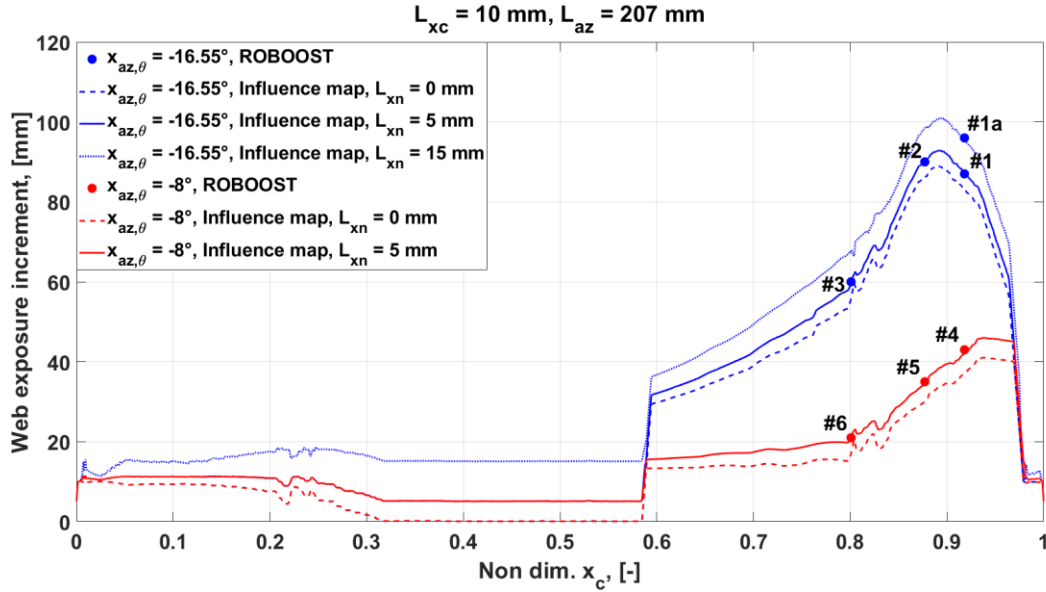


Figure 12: Comparison between ROBOOST results and debonding influence map with debonding main elongation along e_{az}

368

369

370 Each curve is obtained by representing the exposure increase map (Figure 9) at fixed values of the angular
 371 azimuthal coordinate ($x_{az,\theta} = -16.55^\circ$ and $x_{az,\theta} = -8^\circ$ for simulations 1, 1a, 2, 3 and 4, 5, 6 respectively)
 372 and considering the effect of different debonding thickness values ($L_{x_n} = 5 \text{ mm}$ for debondings 1 to 6 and
 373 $L_{x_n} = 15 \text{ mm}$ for debonding 1a). Moreover, the exposure increases map trends at $L_{x_n} = 0 \text{ mm}$ were
 374 included to cover the instance of earlier exposure of a region of the thermal protection case surface due to
 375 hot gas penetration between the thermal protection layer and the grain. In fact, combustion chamber hot gas
 376 could produce a crack progressively igniting the grain surface corresponding to the flaw zone. It must be
 377 highlighted that such mechanism is different from the one linked to a debonding with a non-zero thickness
 378 value. The former is characterized by the earlier exposure of the thermal protection layer caused by a nearly
 379 zero thickness flaw produced by hot gases, while the latter originates from an already present debonding with
 380 a finite thickness, leading to a more significant web exposure increment (Figure 12)

381 Figure 12 shows a good agreement between the estimation of the debonding effects evaluated through the
 382 methodology developed in this paper and the exposure increase determined for the 7 simulations run with
 383 ROBOOST. The maximum difference is equal to 0.78 mm, corresponding to a percentual error of 1.8 %.

384 3.2 Second set of debonding

385 The same analysis has been performed also on the second set of debonding, characterized by a major
 386 elongation along the curvilinear coordinate. The same debonding dimensions discussed before are inverted

387 so that $L_{x_c} = 207 \text{ mm}$ and $L_{az} = 10 \text{ mm}$, while thickness $L_{x_n} = 5 \text{ mm}$ is unchanged. The exposure
 388 increase map obtained following the procedure previously introduced is shown in Figure 13.
 389 Observation of Figure 13 marks out higher exposure increase values up to nearly 201 mm with respect to
 390 93 mm of the maximum exposure linked to Figure 9, meaning that a debonding with $L_{x_c} \gg L_{az}$ can be
 391 more critical than a debonding with $L_{x_c} \ll L_{az}$ in the case regions of maximum exposure.
 392 Similarly to what was done for the previous set of debonding, also this set was validated through some
 393 simulations run on ROBOOST (marked by the black dots in Figure 13 and the white dots in Figure 14), in
 394 order to obtain the corresponding exposure increase values. The results of these simulations are represented
 395 for the sake of compactness in the same Figure 14
 396 Figure 15 shows the comparison between the evaluations coming from the procedure proposed in this paper
 397 and the simulations performed with ROBOOST. The maximum difference is equal to 1 mm, corresponding
 398 to a percentual error of 0.8 %.
 399

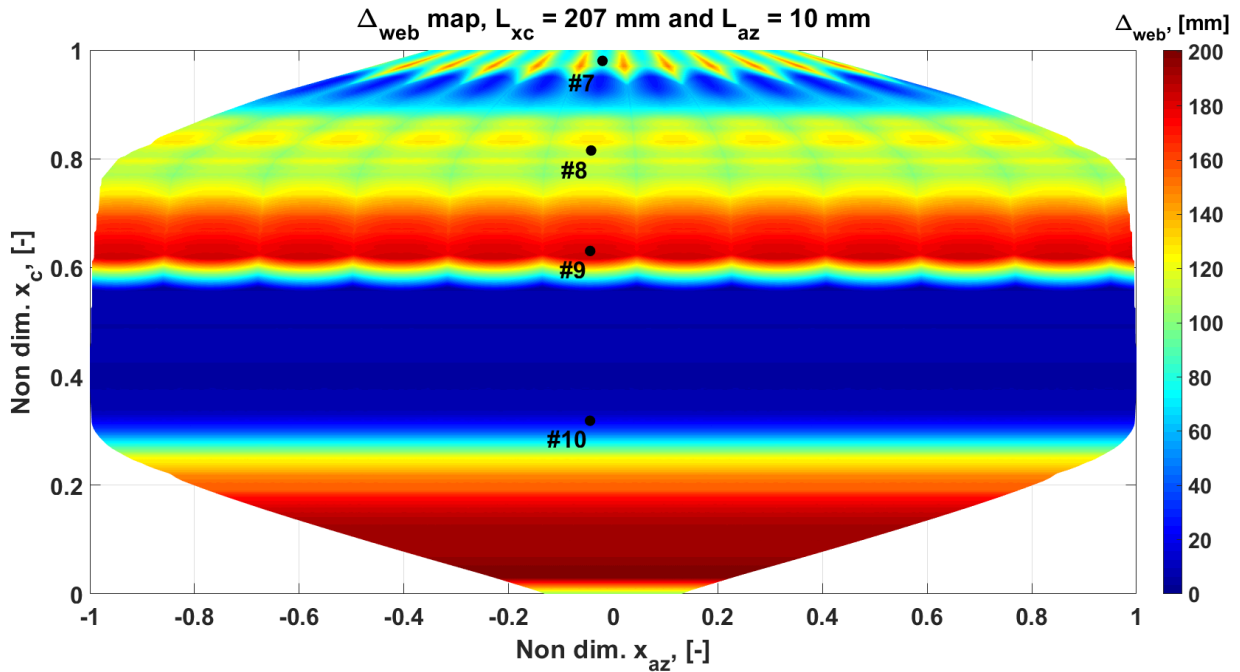


Figure 13: Influence map determined for debonding mainly developed along e_{x_c}

400
 401

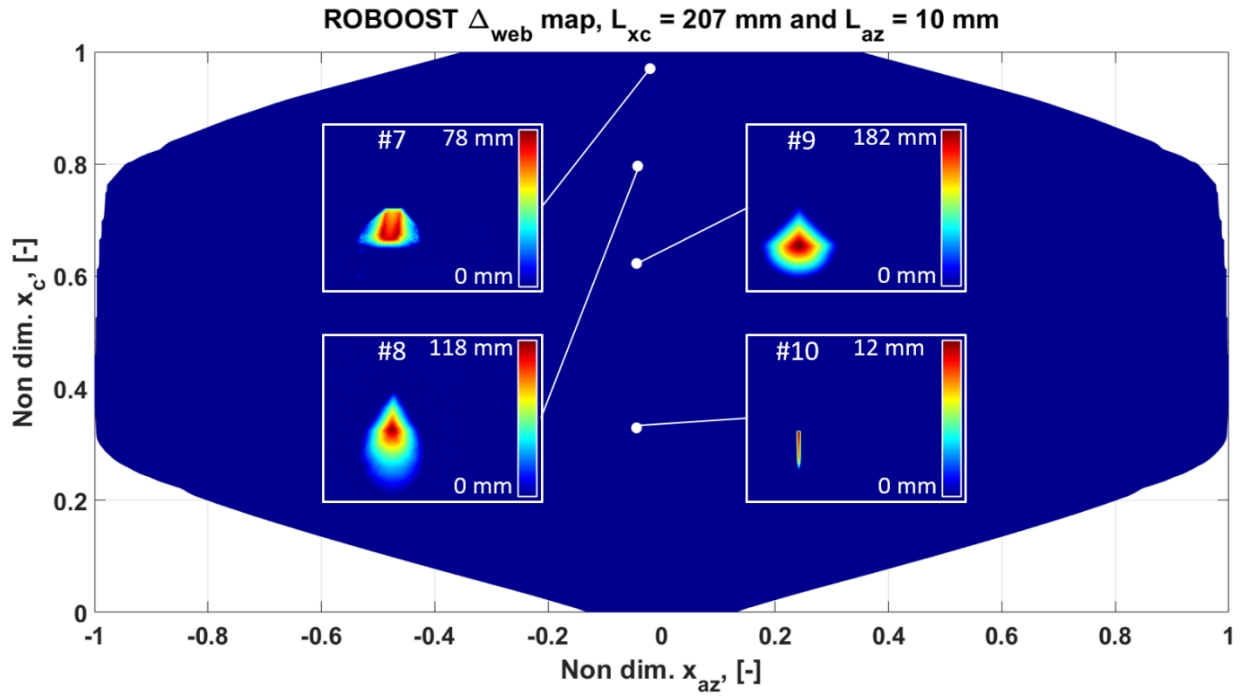


Figure 14: ROBOOST simulations regarding debonding developed along curvilinear coordinate direction.

402

403

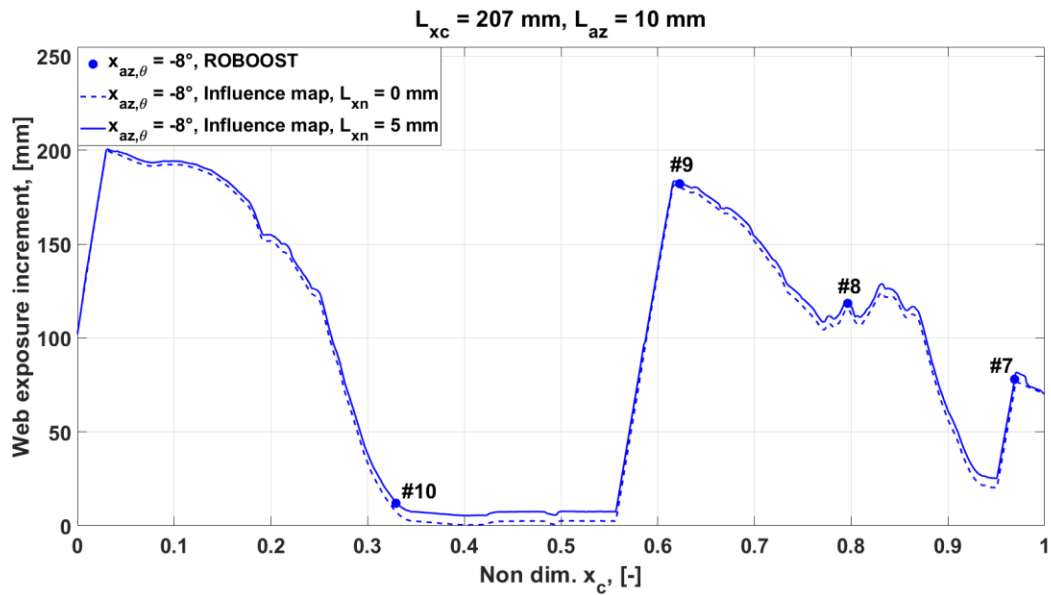


Figure 15: Comparison between ROBOOST results and debonding influence map with debonding main elongation along e_{x_c}

404

405

406 Previous debondings are respectively identified by the main elongation along azimuthal direction or along
 407 the curvilinear coordinate. However, the procedure outlined in the present work is also valid for debondings
 408 extended along both curvilinear and azimuthal directions. According to the above-mentioned statement, a
 409 debonding with the following dimensions was chosen: $L_{x_c} = 207 \text{ mm}$, $L_{az} = 207 \text{ mm}$ and $L_{x_n} = 5 \text{ mm}$.
 410 First, the corresponding debonding influence map was determined with the method used to compute
 411 previous contour plots (Figure 9, Figure 13). Then, the maximum web exposure increment of the debonding
 412 was identified at $x_{az,\theta} = -18.25^\circ$ and $x_c/x_{c_{max}} = 0.6$. Finally, ROBOOST simulation was performed
 413 evaluating the impact of such debonding in the maximum web exposure increment position on the thermal
 414 protection case, with an exposition of 207 mm. Figure 16 shows a good agreement between the result
 415 obtained through ROBOOST and the debonding influence map prediction. The difference is equal to 1.1
 416 mm, corresponding to a percentual error of 0.5 %.

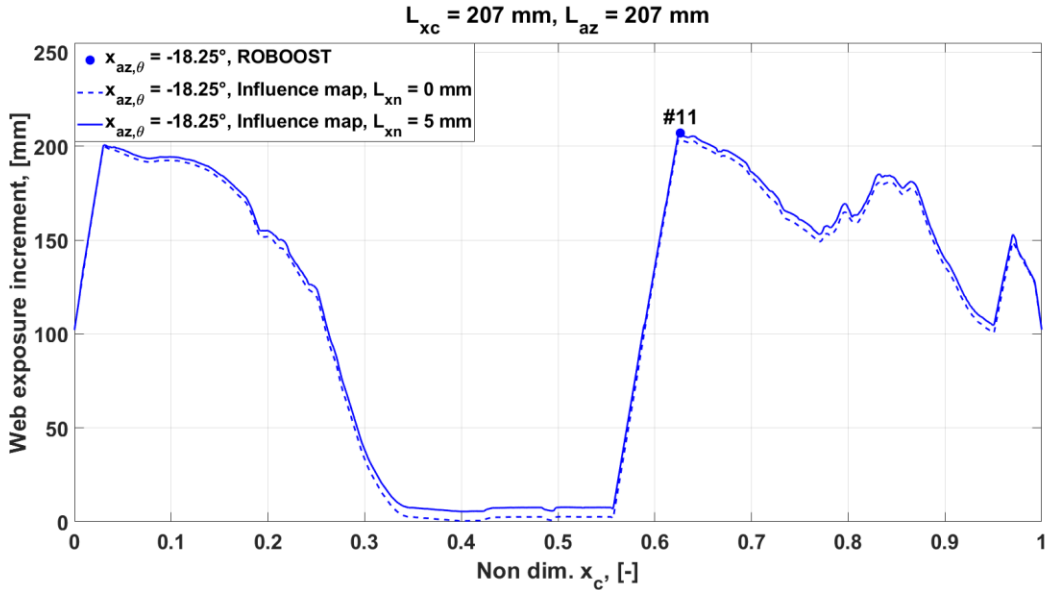


Figure 16: Comparison between ROBOOST result and debonding influence map with debonding elongation along e_{x_c} and e_{az} .

418
 419
 420 All results previously discussed were obtained with ROBOOST software installed on a calculator with the
 421 following features: 16 Gb RAM, Intel Core i7-7th generation CPU machine with 3.10 GHz and NVIDIA
 422 Quadro M1200 graphic card. ROBOOST simulations lasted 24 hours each, while exposure increase maps
 423 lasted 1 hour each.

424 **4 Conclusions**

425 A novel procedure has been introduced to examine the effect of generic-shaped debondings on case-
 426 insulating thermal protection material in terms of web exposure increment. By means of such method, the
 427 most critical debonding positions on SRM case can be determined. Knowing the most critical exposure
 428 regions can drive flaw inspection tests toward critical areas only, implying an optimized usage of x-ray
 429 techniques. In the present work, debonding influence maps are evaluated on Z9 considering three sets of
 430 debonding flaws, where the first one corresponds to debonding elongated along azimuthal direction, the
 431 second one to debonding elongated along curvilinear coordinate direction and the last one to a debonding
 432 extended of the same amount in both directions. Finally, those contour maps are validated by comparing
 433 debonding web exposure increments with ROBOOST results obtaining a maximum percentage error of
 434 1.8%, meaning that the method proposed can be considered satisfactory.

435 **Appendix A**

436 The abovementioned thermal protection exposure maps were obtained in a curvilinear reference frame
 437 described in 0. This curvilinear coordinate system can be described by three orthogonal level surfaces:

438

$$439 \begin{cases} w_1 = f(x, y, z) \\ w_2 = g(x, y, z) \\ w_3 = z(x, y, z) \end{cases}$$

440

441 Where w_1, w_2 and w_3 , are the curvilinear coordinate, x, y, z are the cartesian coordinate. In the curvilinear
 442 reference frame, the gradient is written as follows

443

$$444 \nabla\varphi = \sum_{j=1}^3 \frac{1}{h_j} \frac{\partial\varphi}{\partial w_j} \hat{h}_j = \sum_{j=1}^3 \frac{\vec{h}_j}{\|\vec{h}_j\|^2} \frac{\partial\varphi}{\partial w_j}$$

445

$$\vec{h}_j = \frac{\partial\vec{r}}{\partial w_j}$$

446

$$h_j = \left\| \frac{\partial\vec{r}}{\partial w_j} \right\|$$

447

448 Where h_j are the Lamè coefficients, \vec{r} is the position vector and in the cartesian coordinate it is written as
 449 follows

450

451

$$\vec{r} = x e_x + y e_y + z e_z$$

452

453 Where e_x, e_y, e_z are the cartesian basis. To evaluate the Lamè coefficients, the vector \vec{r} needs to be written

454 as function of the curvilinear coordinate (w_1, w_2, w_3) .

455

456

$$\vec{r} = k_x(w_1, w_2, w_3) e_x + k_y(w_1, w_2, w_3) e_y + k_z(w_1, w_2, w_3) e_z$$

457

458

$$\begin{cases} x = k_x(w_1, w_2, w_3) \\ y = k_y(w_1, w_2, w_3) \\ z = k_z(w_1, w_2, w_3) \end{cases}$$

459

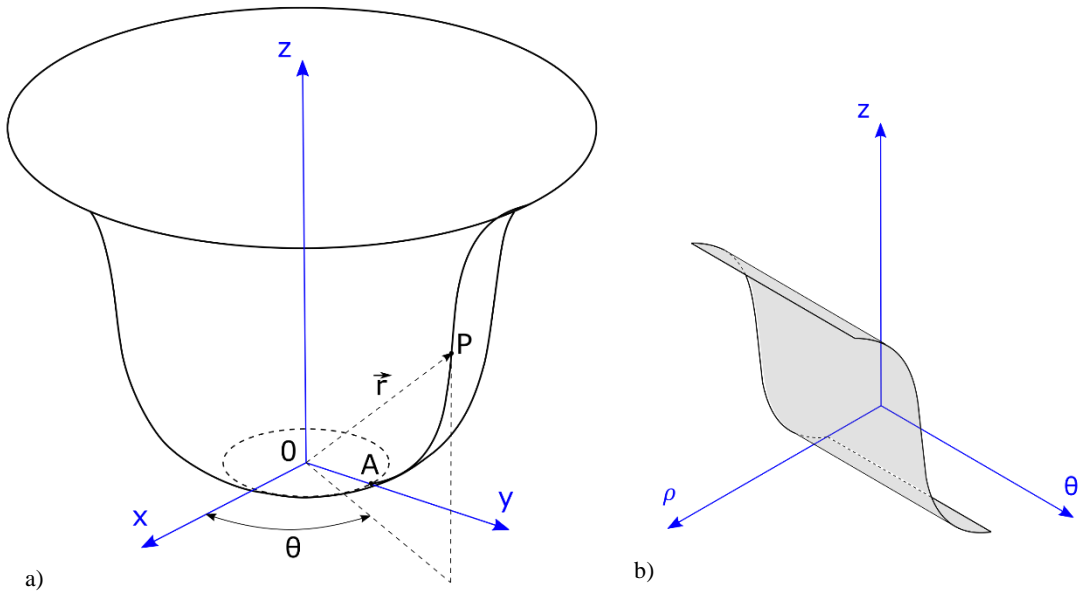


Figure 17: Surface of revolution

460

461

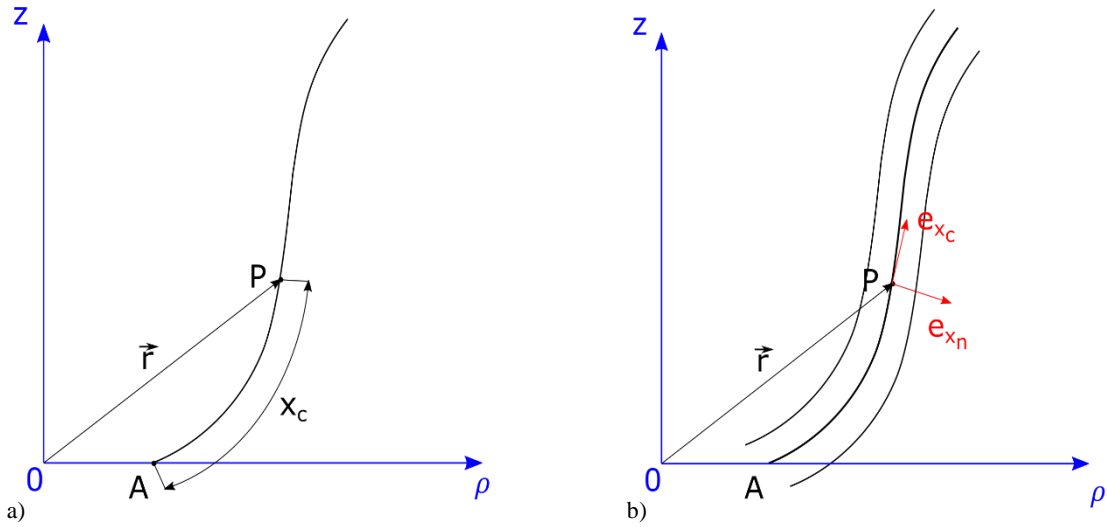


Figure 18: Curvilinear coordinate system

462

463

464 Figure 17a shows a general revolution surface obtained rotating the red curve along the z -axis. w_2 is defined
 465 as the angle θ between the projection of \vec{r} on xy -plane and the x -axis. Figure 17b shows the same surface
 466 in cylindrical coordinate. Figure 18 shows the definition of w_1 and w_3 : w_1 is defined as the length of red
 467 curve from the point A to P, w_3 is the last coordinate defined by the basis always orthogonal to the case
 468 surface.

469

470
$$w_1 = x_c = \int_A^P ds$$

471
$$w_2 = \theta$$

472
$$w_3 = x_n$$

473

474 Functions k_x , k_y and k_z in the general form are quite complicated to be obtained, but the gradient needs to
 475 evaluate on the case surface, therefore k_x , k_y and k_z could be written considering small values of x_n . The
 476 black solid curves in Figure 18b are the isoline for the curvilinear coordinate x_n . For small values of x_n , a
 477 general point (x, y, z) could be written as follows

478
$$\begin{cases} x = k_x(x_c, \theta, x_n) = \rho(x_c) \cos(\theta) + \hat{n}_x(x_c, \theta)x_n \\ y = k_y(x_c, \theta, x_n) = \rho(x_c) \sin(\theta) + \hat{n}_y(x_c, \theta)x_n \\ z = k_z(x_c, \theta, x_n) = a(x_c) + \hat{n}_z(x_c, \theta)x_n \end{cases}$$

479

480 Where \hat{n} is the normal to the case. Therefore, it is straightforward the evaluation \vec{r} and the Lamé
 481 coefficients.

482

$$483 \quad \vec{r} = [\rho(x_c) \cos(\theta) + \hat{n}_x(x_c, \theta)x_n] \hat{e}_1 + [\rho(x_c) \sin(\theta) + \hat{n}_y(x_c, \theta)x_n] \hat{e}_2$$

$$484 \quad \quad \quad + [a(x_c) + \hat{n}_z(x_c, \theta)x_n] \hat{e}_3$$

$$485 \quad \vec{h}_1 = \frac{\partial \vec{r}}{\partial w_1} = \frac{\partial \vec{r}}{\partial x_c} = \left[\frac{\partial \rho}{\partial x_c} \cos(\theta) + \frac{\partial \hat{n}_x}{\partial x_c} x_n \right] e_x + \left[\frac{\partial \rho}{\partial x_c} \sin(\theta) + \frac{\partial \hat{n}_y}{\partial x_c} x_n \right] e_y + \left[\frac{\partial a}{\partial x_c} + \frac{\partial \hat{n}_z}{\partial x_c} x_n \right] e_z$$

$$486 \quad \|\vec{h}_1\|^2 = \left(\frac{\partial \rho}{\partial x_c} \right)^2 + \left(\frac{\partial a}{\partial x_c} \right)^2 + \left[\left(\frac{\partial \hat{n}_x}{\partial x_c} \right)^2 + \left(\frac{\partial \hat{n}_y}{\partial x_c} \right)^2 + \left(\frac{\partial \hat{n}_z}{\partial x_c} \right)^2 \right] (x_n)^2 + 2 \left[\frac{\partial \rho}{\partial x_c} \cos(\theta) \frac{\partial \hat{n}_x}{\partial x_c} + \frac{\partial \rho}{\partial x_c} \sin(\theta) \frac{\partial \hat{n}_y}{\partial x_c} + \right.$$

$$487 \quad \left. \frac{\partial a}{\partial x_c} \frac{\partial \hat{n}_z}{\partial x_c} \right] x_n$$

$$488 \quad \vec{h}_2 = \frac{\partial \vec{r}}{\partial w_2} = \frac{\partial \vec{r}}{\partial \theta} = \left[-\rho(x_c) \sin(\theta) + \frac{\partial \hat{n}_x}{\partial \theta} x_n \right] e_x + \left[\rho(x_c) \cos(\theta) + \frac{\partial \hat{n}_y}{\partial \theta} x_n \right] e_y + \left[\frac{\partial \hat{n}_z}{\partial \theta} x_n \right] e_z$$

$$489 \quad \|\vec{h}_2\|^2 = \rho(x_c)^2 + \left[\left(\frac{\partial \hat{n}_x}{\partial \theta} \right)^2 + \left(\frac{\partial \hat{n}_y}{\partial \theta} \right)^2 + \left(\frac{\partial \hat{n}_z}{\partial \theta} \right)^2 \right] (x_n)^2 + 2 \left[-\rho(x_c) \sin(\theta) \frac{\partial \hat{n}_x}{\partial \theta} + \rho(x_c) \cos(\theta) \frac{\partial \hat{n}_y}{\partial \theta} \right] x_n$$

$$490 \quad \vec{h}_3 = \frac{\partial \vec{r}}{\partial w_3} = \frac{\partial \vec{r}}{\partial x_n} = \hat{n}_x(x_c, \theta) e_x + \hat{n}_y(x_c, \theta) e_y + \hat{n}_z(x_c, \theta) e_z$$

$$491 \quad \|\vec{h}_3\|^2 = \hat{n}_x(x_c, \theta)^2 + \hat{n}_y(x_c, \theta)^2 + \hat{n}_z(x_c, \theta)^2 = 1$$

492

493 Imposing $x_n \rightarrow 0$ the Lamé coefficients becomes

494

$$495 \quad \vec{h}_1 = \frac{\partial \vec{r}}{\partial w_1} = \frac{\partial \vec{r}}{\partial x_c} = \frac{\partial \rho}{\partial x_c} \cos(\theta) e_x + \frac{\partial \rho}{\partial x_c} \sin(\theta) e_y + \frac{\partial a}{\partial x_c} e_z$$

$$496 \quad \|\vec{h}_1\|^2 = \left(\frac{\partial \rho}{\partial x_c} \right)^2 + \left(\frac{\partial a}{\partial x_c} \right)^2 \quad (4)$$

497

$$498 \quad \vec{h}_2 = \frac{\partial \vec{r}}{\partial w_2} = \frac{\partial \vec{r}}{\partial \theta} = -\rho(x_c) \sin(\theta) e_x + \rho(x_c) \cos(\theta) e_y$$

$$499 \quad \|\vec{h}_2\|^2 = \rho(x_c)^2$$

$$500 \quad \vec{h}_3 = \frac{\partial \vec{r}}{\partial w_3} = \frac{\partial \vec{r}}{\partial x_n} = \hat{n}_x(x_c, \theta) e_x + \hat{n}_y(x_c, \theta) e_y + \hat{n}_z(x_c, \theta) e_z = \hat{n}$$

$$501 \quad \|\vec{h}_3\|^2 = 1$$

502

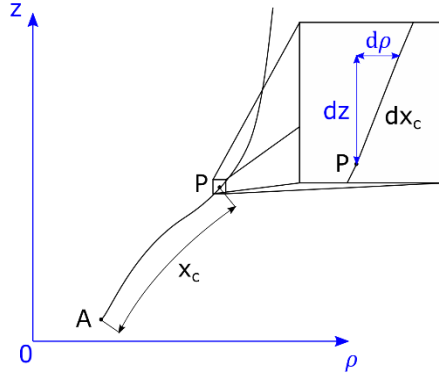


Figure 19: geometric relation

503

504

505 Using Figure 19, it possible to prove that $\left(\frac{\partial \rho}{\partial x_c}\right)^2 + \left(\frac{\partial a}{\partial x_c}\right)^2 = 1$.

506

507
$$d\rho = \frac{\partial \rho}{\partial x_c} dx_c, da = \frac{\partial a}{\partial x_c} dx_c$$

508

509 Applying the Pythagoras theorem on the triangle shown in Figure 19.

510

511
$$dx_c = \sqrt{d\rho^2 + da^2} = \sqrt{\left(\frac{\partial \rho}{\partial x_c}\right)^2 + \left(\frac{\partial a}{\partial x_c}\right)^2} dx_c$$

512 Therefore

513

514
$$\left(\frac{\partial \rho}{\partial x_c}\right)^2 + \left(\frac{\partial a}{\partial x_c}\right)^2 = 1$$

515

516 The previous results could be substituted into Eq. (4).

517
$$\nabla \varphi = \sum_{j=1}^3 \frac{\vec{h}_j}{\|\vec{h}_j\|^2} \frac{\partial \varphi}{\partial w_j} = \frac{\partial \varphi}{\partial x_c} \frac{\vec{h}_1}{\|\vec{h}_1\|^2} + \frac{\partial \varphi}{\partial \theta} \frac{\vec{h}_2}{\|\vec{h}_2\|^2} + \frac{\partial \varphi}{\partial x_n} \frac{\vec{h}_3}{\|\vec{h}_3\|^2} =$$

518
$$= \frac{\partial \varphi}{\partial x_c} \left[\frac{\partial \rho}{\partial x_c} \cos(\theta) e_x + \frac{\partial \rho}{\partial x_c} \sin(\theta) e_y + \frac{\partial a}{\partial x_c} e_z \right] + \frac{\partial \varphi}{\partial \theta} \frac{[-\rho(x_c) \sin(\theta) e_x + \rho(x_c) \cos(\theta) e_y]}{\rho(x_c)^2}$$

519
$$+ \frac{\partial \varphi}{\partial x_n} \hat{n}$$

$$= \frac{\partial \varphi}{\partial x_c} \left[\frac{\partial \rho}{\partial x_c} \cos(\theta) e_x + \frac{\partial \rho}{\partial x_c} \sin(\theta) e_y + \frac{\partial a}{\partial x_c} e_z \right] + \frac{\partial \varphi}{\partial \theta} \frac{1}{\rho(x_c)} [-\sin(\theta) e_x + \cos(\theta) e_y] + \frac{\partial \varphi}{\partial x_n} \hat{n}$$

520

$$\hat{e}_{x_c} = \frac{\partial \rho}{\partial x_c} \cos(\theta) e_x + \frac{\partial \rho}{\partial x_c} \sin(\theta) e_y + \frac{\partial a}{\partial x_c} e_z$$

521

$$\hat{e}_\theta = -\sin(\theta) e_x + \cos(\theta) e_y$$

$$\nabla \varphi = \frac{\partial \varphi}{\partial x_c} \hat{e}_{x_c} + \frac{\partial \varphi}{\partial \theta} \frac{1}{\rho(x_c)} \hat{e}_\theta + \frac{\partial \varphi}{\partial x_n} \hat{n} \quad (5)$$

522

523

524 Finally, it is possible to introduce x_{ac} coordinate: $x_{ac} = \theta \rho(x_c)$, therefore

525

$$\frac{\partial \varphi}{\partial \theta} = \frac{\partial \varphi}{\partial x_{ac}} \frac{\partial x_{ac}}{\partial \theta} = \frac{\partial \varphi}{\partial x_{ac}} \rho(x_c)$$

526

527 The Eq. 4 could be specialized for a discretized shape of the case. The parametric curve defined by
528 $(\rho(x_c), a(x_c))$ could be discretized in segments which could be described by the following law:

529

$$\alpha \rho + \beta z + \gamma = 0 \rightarrow z = -\frac{\alpha}{\beta} \rho \rightarrow dz = -\frac{\alpha}{\beta} d\rho$$

530 Where α and β are the two coefficients obtained by the linearization.

531

$$x'_c = \int_C^D ds = \int_C^D \sqrt{(d\rho)^2 + dz^2} = \int_{\rho_C}^{\rho_D} \sqrt{1 + \left(\frac{\alpha}{\beta}\right)^2} d\rho = \sqrt{\frac{\beta^2 + \alpha^2}{\beta^2}} (\rho_D - \rho_C)$$

532

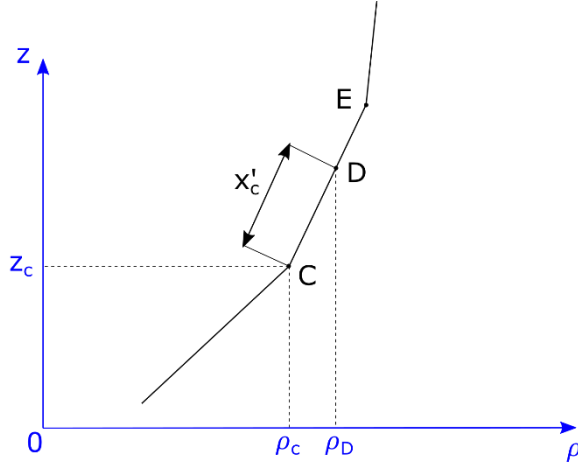


Figure 20: Case profile discretization

539

540

541

$$\rho_D = \frac{x'_c}{\sqrt{\frac{\beta^2 + \alpha^2}{\beta^2}}} + \rho_C = \sqrt{\frac{\beta^2}{\beta^2 + \alpha^2}} x'_c + \rho_C$$

542

543 This result could be substituted into the Eq. (5), where $\rho = \rho_D$:

544

545

$$\nabla\varphi = \frac{\partial\varphi}{\partial x_c} \hat{e}_{x_c} + \frac{\partial\varphi}{\partial\theta} \frac{1}{\sqrt{\frac{\beta^2}{\beta^2 + \alpha^2} x'_c + \rho_C}} + \frac{\partial\varphi}{\partial x_n} \hat{n}$$

546

547 Only the gradient component along \hat{e}_{x_c} and \hat{e}_θ are important, because the thermal protection maps belong

548 to the case surface, therefore the previous gradient becomes

549

550

$$\nabla\varphi = \frac{\partial\varphi}{\partial x_c} \hat{e}_{x_c} + \frac{\partial\varphi}{\partial\theta} \frac{1}{\sqrt{\frac{\beta^2}{\beta^2 + \alpha^2} x'_c + \rho_C}} \hat{e}_\theta$$

551

552 Nomenclature

553 *Latin*

554	$a(x_c)$	= case profile height function, [m]
555	ds	= infinitesimal length, [m]
556	e_{az}	= unit vector defining the orthonormal basis (e_{x_c}, e_{az}, e_{x_n})
557	e_x	= unit vector defining the orthonormal basis (e_x, e_y, e_z)
558	e_{x_c}	= unit vector defining the orthonormal basis (e_{x_c}, e_{az}, e_{x_n})
559	e_{x_n}	= unit vector defining the orthonormal basis (e_{x_c}, e_{az}, e_{x_n})
560	e_y	= unit vector defining the orthonormal basis (e_x, e_y, e_z)
561	e_z	= unit vector defining the orthonormal basis (e_x, e_y, e_z) and (e_ρ, e_ϑ, e_z)
562	e_ϑ	= unit vector defining the orthonormal basis (e_ρ, e_ϑ, e_z)
563	e_ρ	= unit vector defining the orthonormal basis (e_ρ, e_ϑ, e_z)
564	h_{az}	= Lamè coefficient linked to the azimuthal coordinate x_{az}
565	h_j	= Lamè coefficients
566	h_{x_c}	= Lamè coefficient linked to the azimuthal coordinate x_c
567	\hat{h}_j	= unit vector regarding Lamè coefficients
568	\vec{h}_j	= vector regarding Lamè coefficients
569	L_{az}	= debonding size along e_{az} , [m]
570	L_{x_c}	= debonding size along e_{x_c} , [m]
571	L_{x_n}	= debonding size along e_{x_n} , [m]
572	\hat{n}	= unit vector normal to the case surface
573	\hat{n}_x	= x-axis component of the unit vector normal to the case surface
574	\hat{n}_y	= y-axis component of the unit vector normal to the case surface
575	\hat{n}_z	= z-axis component of the unit vector normal to the case surface
576	\vec{r}	= position vector, [m]
577	w_1	= general curvilinear coordinate
578	w_2	= general curvilinear coordinate
579	w_3	= general curvilinear coordinate
580	x	= coordinate of the orthonormal basis defined by e_x, e_y , and e_z [m]
581	x_{az}	= coordinate (expressed in length units) of the orthonormal basis defined by e_{x_c}, e_{az} , and e_{x_n} ,
582	[m]	
583	$x_{az,\theta}$	= coordinate (expressed in angle units) of the orthonormal basis defined by e_{x_c}, e_{az} , and e_{x_n} , [°]
584	$x_{az_{max}}$	= maximum exposure map extension along e_{az} , [m]

585 $x_{az_{min}}$ = minimum exposure map extension along e_{az} , [m]
586 x_c = coordinate of the orthonormal basis defined by e_{x_c} , e_{az} , and e_{x_n} , [m]
587 $x_{c_{max}}$ = maximum exposure map extension along e_{x_c} , [m]
588 $x_{c_{min}}$ = minimum exposure map extension along e_{x_c} , [m]
589 x_{c_0} = coordinate x_c computed at local reference frame origin, namely $0'$, [m]
590 y = coordinate of the orthonormal basis defined by e_x , e_y , and e_z [m]
591 z = coordinate of the orthonormal basis defined by e_x , e_y , and e_z [m]

592 *Greek*

593 α = coefficient of the line equation linked to the case envelope discretization
594 β = coefficient of the line equation linked to the case envelope discretization
595 ρ = coordinate of the orthonormal basis defined by e_ρ , e_θ , and e_z
596 $\rho_{0'}$ = radial position of the local reference frame origin expressed in global reference frame
597 cylindrical coordinates
598 $\rho(x_c)$ = case profile radius function
599 ϕ = exposure map regarding the case-insulating thermal protection material
600 φ = generic scalar value

601 *Acronyms*

602 *CFD* = Computational Fluid Dynamics
603 *ROBOOST* = Rocket BOOst Simulation Tool
604 *SRM* = Solid Rocket Motor
605 *ZEFIRO 9* = ZERo First stage ROcket 9
606

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611 **Declaration of competing interest**

612 The authors declare that they have no known competing financial interests or personal relationships that
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