IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 20 August 2022, accepted 8 September 2022, date of publication 12 September 2022,
date of current version 22 September 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3206368

TOPICAL REVIEW

Topology Optimization for Electromagnetics: A Survey

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ABSTRACT The development of technologies for the additive manufacturing, in particular of metallic materials, is offering the possibility of producing parts with complex geometries. This opens up to the possibility of using topological optimization methods for the design of electromagnetic devices. Hence, a wide variety of approaches, originally developed for solid mechanics, have recently become attractive also in the field of electromagnetics. The general distinction between gradient-based and gradient-free methods drives the structure of the paper, with the latter becoming particularly attractive in the last years due to the concepts of artificial neural networks. The aim of this paper is twofold. On one hand, the paper aims at summarizing and describing the state-of-art on topology optimization techniques while on the other it aims at showing how the latter methodologies developed in non-electromagnetic framework (e.g., solid mechanics field) can be applied for the optimization of electromagnetic devices. Discussions and comparisons are both supported by theoretical aspects and numerical results.

INDEX TERMS Topology optimization, electromagnetic modelling, additive manufacturing, electromagnetic design, neural networks.

I. INTRODUCTION

Originally developed for solid mechanics engineering problems [1], numerical Topology Optimization (TO) has recently become attractive also in designing electromagnetic devices [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]. When performing topology optimization, usually the resulting structures have complex and irregular geometries that are difficult to manufacture using conventional machining techniques. Nowadays, however, these limits can be overcome thanks to developments in additive manufacturing and 3D printing technologies [12], [13], [14].

Differently from parameterized shape optimization problems which act only on the design domain boundaries, topology optimization deals with the modification of the material distribution in the domain, i.e., material can be removed or added. This capability is of considerable interest in applications where there is no prior knowledge on the optimal arrangement or shape of the material in the device.

General optimization problems can be tackled following two main approaches [15]: gradient-based and gradient-free techniques, each one characterized by pros and cons. Gradient-based methodologies typically exhibit a faster convergence speed, but suffer for local optima trapping. Gradient-free methods on the other hand are more flexible since they do not require the knowledge of the objective function derivatives with respect to the design variables (sensitivities), usually not easy to compute, sand generally require a higher computational effort, especially in the case of stochastic algorithm. Alternative to these methods, deterministic gradient-free ones, such as the simplex gradient method or the simplex-base simplicial Nelder-Mead method, can be adopted [16].

Denoting with $\rho(\mathbf{x})$ the material distribution at any point of the design domain $D \subset \mathbb{R}^d$, the TO problem can

The associate editor coordinating the review of this manuscript and approving it for publication was Debdeep Sarkar.

be written as

$$\text{find } F^* = \min_{\rho} F(\mathbf{u}(\rho), \rho) \quad \text{under given constraints,} \tag{1}$$

where F is the objective function and problem constraints may be of the equality and/or inequality type. Since the objective function depends on the solution $\mathbf{u}(\rho)$ of an electromagnetic problem, (1) is a Partial Differential Equation (PDE)-constrained optimization problem (i.e., an optimization problem where at least one of the constraints is expressed as a partial differential equation) where the governing PDE can be written as

$$\mathcal{L}(\mathbf{u}(\rho), \rho) = 0. \tag{2}$$

As in solid mechanics topology optimization, also in electromagnetic problems an additional volume inequality constraint can be added to the problem (1), i.e.,

$$G(\rho) = \int_D \rho(\mathbf{x}) d\mathbf{x} - V_{trg} \leq 0, \tag{3}$$

where V_{trg} is the desired maximal volume of the device.

The basic ingredient for the solution of problem (1) is the numerical solution of the PDE governing the underlying physics of the objective function. This, if volume discretization methods are used, requires the discretization (i.e., the meshing) of the computational domain $D \subset \mathbb{R}^d$ with N_e elements, usually triangles or quadrilateral for $d = 2$ and tetrahedra or hexahedra for $d = 3$. A material property is then assigned to each mesh element. Then, using the discretization, the PDE is represented as a system of equations by exploiting classical Finite Element Methods (FEM), Integral Equation Methods (IEM), or other approaches [17], [18], [19], [20].

For topology optimization problems, the design variable related to the ith mesh element $\rho_i \in \mathbb{S}$, with $i = 1, \ldots, N_e$, defines the elemental material property g with the map

$$\mathbb{S} \to \mathbb{R} \tag{4}$$

$$\rho \mapsto g(\rho). \tag{5}$$

A first distinction between TO approaches, concerns in the space $\mathbb{S}$ where the design variable ρ_i is defined, that can be continuous or binary, i.e., [21]

$$\text{if} \begin{cases} 0 \leq \rho_i \leq 1 & \to \text{continuous TO} \\ \rho_i = \{0, 1\} & \to \text{binary TO,} \end{cases} \tag{6}$$

where the discrete problem, also termed ON/OFF method [22], is the most common approach for topology optimization. When the design variable lies in the continuous space, the intermediate values (0,1) are ''gray scales'', which have no clear material specification. A further distinction between sub-classes of TO approaches follows from the definition of the so called hard-kill methods. In these methods a finite amount of material is gradually removed or added, based on heuristic criteria, which may or may not be related to sensitivity information [23].

The solution of the functional minimization depends on the underlying technique, i.e., if a gradient-based or gradient-free approach is adopted.

When facing engineering problems, the optimal design of components often requires conflicting demands to be satisfied for their best functionality [24], [25], [26]. This reflects into the need of solving multi-objective optimization problems, where the scalar objective functions F_i are formally collected in vector form and the optimal solutions lie on the so-called Pareto front.

When only two conflicting objectives are considered it may be convenient to rewrite the vectorial multi-objective optimization problem into a scalar one through the convex combination

$$F = \alpha_1 F_1 + \alpha_2 F_2, \tag{7}$$

where $\alpha_i \in \mathbb{R}$ are weights in [0,1]. These weights can be retrieved using the Adaptive Weighted Sum (AWS) scheme, which for some problems complies also with non-convex Pareto fronts [27]. Since the AWS scheme at each iteration treats only scalar optimization problems, this vectorial-scalar transformation may be conveniently applied when using ''black-box'' numerical optimization tools for which only a scalar objective is required as input. For example the optimization toolbox of MATLAB® can be used for the single objective optimization and the results combined following AWS scheme.

The aim of this work is to address in detail the characteristics of the different TO methods presented so far and to provide practical application examples. Moreover, it is shown how numerical tools initially developed for e.g., mechanical applications, can be also applied to electromagnetic problems. In this respect, section II highlights the main peculiarities of TO approaches developed in the solid mechanics field, and the ones that can be efficiently used for the electromagnetics problems.

In particular the gradient-based method described in the following are: homogenization method III-C1, density-based method III-C2, level-set method III-C3, phase-field III-C4 method, Bi-directional Evolutionary Structural Topology Optimization (BESO) method III-C5, Topology Optimization of Binary Structures (TOBS) method III-C6 and the Two-Step method III-C7, which is a bridge between gradient-based and gradient-free approaches. The methods considered in the latter class are: boolean methods IV-A1, binary methods IV-A2, Normalized Gaussian Network Methods (NGnet) IV-A3, deterministic methods IV-B, deep learning methods IV-C and the Proportional Topology Optimization (PTO) method IV-D.

The remainder of the paper is organized as follows: section III describes the gradient-based methods requiring the computation of objective function sensitivities. To this aim, a brief introduction of the so-called adjoint variable approach is given in section III-A. Next, methods that do not require the evaluation of the gradient of the objective function, thus avoiding the knowledge of classical sensitivities, are described in section IV. Section V briefly

TABLE 1. List of gradient-based approaches discussed.

Method	References	Characteristics	Used in electromagnetics
Homogenization	[28] [29]	first TO approach, continuous ρ	✓
Density-based	[30] [31] [32] [33] [34]	simple concept, used in many physical fields, continuous ρ, suited for multi-material TO, built in commercial software	✓
Level-Set	[35] [36] [37] [38] [39] [40] [40]	evolve boundary of design domain, hole seeding for topology change, used for multi-material TO	✓
BESO with Sensitivity	[41] [42]	hard-kill method, binary ρ, heuristic search algorithm	✓
TOBS	[43] [44] [45] [46] [47]	combines continuous and binary logic, binary ρ, integer search algorithm,	
Two-Step	[48]	sensitivity information used to increase performance of stochastic algorithms	✓

summarizes other methods proposed for the TO of electromagnetic devices, which are not described in detail in this work. Section VI demonstrates the feasibility of the proposed methods for solving the topology optimization of two electromagnetic devices. In the first test-case a comparison between selected approaches belonging to the two classes is performed. In section VII, a critical discussion of the presented methods is given, then section VIII summarized the conclusions of the work.

II. PECULIARITIES OF TOPOLOGY OPTIMIZATION FOR ELECTROMAGNETICS

The topology optimization, although it was mainly developed in the structural mechanics framework, can be conceived in a transversal way based on physical problems of different nature, for example fluid dynamics [49], heat transfer [50], photonics design [51] and so on. The investigation of TO for electromagnetics (EM) is discussed in this paper. The wide spectrum of TO applications may lead to the erroneous conclusion that any method can be used ''as is'' independently of the underlying physics. In fact each physical problem has specific requirements to be satisfied. For instance, when dealing with solid mechanics, the optimized topology has to comply with connectivity properties. In this class of problems, for which the compliance is the objective function of the TO, the structural robustness from the mechanical point of view is guaranteed by the TO problem itself. In designing electromagnetic devices the concept of structural connection, is sometimes misleading because the connectivity between the different parts may or may not be wanted. Mostly of the TO algorithms developed in the solid mechanics are not general purpose and sometimes intrusive modifications are required to use them for electromagnetic problems. Generally, the gradient-based methods such as the SIMP described in section III-C2 can be used as general purpose, conversely, the level-set of section III-C3 requiring the topological derivative information, is more challenging. The PTO algorithm, developed for structural mechanics problems and briefly described in section IV-D, uses deeply the assumption that the objective function is an energy density and cannot be considered as general purpose because if the electromagnetic problem does not uses the energy density (e.g., of magnetic nature) as objective function, the algorithm cannot be applied ''as is''.

From the computational point of view, as briefly stated in the introductory section, the numerical solution of the PDE governing the physical problem is a key part in the optimization process. In fact, the objective function evaluation requires the solution of the discretized system many times during the optimization algorithm. The FEM is extensively used in solving mechanical and structural problems. The main advantage of this approach is the sparsity pattern of the arising system matrices, which reduces the memory requirements for their storage. In most cases, this approach is used even for the numerical solution of the PDE governing the electromagnetic problem. When modelling antenna propagation or eddy currents the need of discretizing the non-conductive parts, for example the air domains, can be circumvented by using e.g., IEM. These methods require only the discretization of the non-electromagnetic neutral domains, but the arising system matrices are fully populated (dense), thus increasing the computational burden for their storage up to $\mathcal{O}(N^2)$, where N is the number of unknowns, and to $\mathcal{O}(N^3)$ for the system solution (although some sophisticated technique is used [52]). In principle the TO method can be based both on FEM and IEM approaches for the solution of the EM problem. However, the development of the code in the two cases is different and also the general performance. As an example consider a TO problem which changes the material from air to magnetic or the other way round. By using FEM the number of degrees of freedom of the whole discretized domain (air + magnetic) remains unchanged, conversely, by using e.g., an IEM when subtracting or adding magnetic material, the number of unknowns is respectively reduced or

augmented, this because the IEM deals only with the non-air domain.

When dealing with electromagnetics, several sub-classes of problems can be defined: electrostatic, magnetostatic, electro- and magneto-quasistatic, full-wave with and without propagation. The solution of the electromagnetic field for each sub-class can be tackled by different techniques requiring different computational effort. In particular, the full wave propagation problem, thus the objective function derived from it, is highly demanding from the computational point of view. Since the TO algorithm requires the objective function to be evaluated many times, it is clear that the reduction of the number of evaluations is mandatory to reduce the computational burden.

The aim of this paper is to introduce the TO methods suited in electromagnetics. As described in what follows, some of them, directly derived from the solid mechanics world, are used without substantial modifications. As an example the novel TOBS algorithm described in section III-C6, and recently applied to solid mechanics and fluid dynamics problems, seems to be useful also in the electromagnetics field. The issue of structural connectivity will be addressed in the context of the so called stochastic methods IV-A.

III. GRADIENT-BASED METHODS

In this section, the gradient-based approaches for TO listed in Table 1, all using the information provided by the derivative of objective function with respect to the design variables, are described.

A. MATHEMATICAL PRELIMINARIES

When solving PDE-constrained topology optimization using a gradient-based method, evaluation of the sensitivity of the objective function w.r.t. the design variables is required [30], [53], [54]

$$\text{Sensitivity} := \frac{dF}{d\boldsymbol{\rho}}. \tag{8}$$

The so-called Adjoint Variable Method (AVM) briefly described in this section, is usually applied for sensitivity calculations [55], [56]. In principle, the AVM approach can be applied to the continuous PDE problem, but here we focus our attention to its discretized version.

Using the chain rule, the total derivative of the objective function $F(\mathbf{u}(\boldsymbol{\rho}), \boldsymbol{\rho})$ is expressed as

$$\frac{dF}{d\rho_i} = \frac{\partial F}{\partial \rho_i} + \frac{\partial F}{\partial u_j}\frac{du_j}{d\rho_i}, \tag{9}$$

while for the kth residual equation of the discretized system (2) we have

$$0 \equiv \frac{d\mathcal{L}_k}{d\rho_i} = \frac{\partial \mathcal{L}_k}{\partial \rho_i} + \frac{\partial \mathcal{L}_k}{\partial u_j}\frac{du_j}{d\rho_i}. \tag{10}$$

Multiplying the previous equation by λ_k and adding the result to (9) we obtain

$$\frac{dF}{d\rho_i} = \underbrace{\left[\frac{\partial F}{\partial \rho_i} + \lambda_k \frac{d\mathcal{L}_k}{d\rho_i}\right]}_{\text{term 1}} + \underbrace{\left[\frac{\partial F}{\partial u_j} + \lambda_k \frac{\partial \mathcal{L}_k}{\partial u_j}\right]\frac{du_j}{d\rho_i}}_{\text{term 2}}. \tag{11}$$

Computing the second term of (11) is a complex task since the derivative of the solution array with respect to the design variables is required. However, if the coefficients λ_k are chosen in such a way that

$$\frac{\partial F}{\partial u_j} + \lambda_k \frac{\partial \mathcal{L}_k}{\partial u_j} = 0, \tag{12}$$

the second term of (11) vanishes. Thus, the evaluation of the objective sensitivities is a two step process, first requiring the solution of the the so-called adjoint problem defined by (12), from which the adjoint field $\boldsymbol{\lambda}$ is obtained. Then, the latter is used for the evaluation of the first term of (11), which is equal to the objective sensitivities $dF/d\rho_i$.

As an example, considering for simplicity a system whose coefficients depend only on the design parameters $\boldsymbol{\rho}$, the discretized PDE can be written as

$$\mathbf{A}(\boldsymbol{\rho})\mathbf{u} = \mathbf{b}, \tag{13}$$

thus, the partial derivatives of the residual appearing in (11) are expressed as

$$\frac{\partial \mathcal{L}_k}{\partial u_j} = \mathbf{A}_{kj}, \quad \frac{\partial \mathcal{L}_k}{\partial \rho_i} = \frac{\partial \mathbf{A}_{kl}}{\partial \rho_i} u_l. \tag{14}$$

The arising adjoint system (12) is written as

$$\mathbf{A}^\top \boldsymbol{\lambda} = -\frac{\partial F}{\partial \mathbf{u}}, \tag{15}$$

and, once the solution array $\boldsymbol{\lambda}$ is obtained, the sensitivity of the objective function is given by

$$\frac{dF}{d\rho_i} = \frac{\partial F}{\partial \rho_i} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{A}}{\partial \boldsymbol{\rho}}\mathbf{u}. \tag{16}$$

The adjoint variable method can be extended to non-linear [55], [57] and time-domain problems such as transient eddy current ones [58].

B. INTERPOLATION, FILTERING, AND PROJECTION SCHEMES

In this section, the interpolation functions mapping the design variables $\boldsymbol{\rho}$ to the elemental material property (5) are defined. To mitigate the oscillatory behaviour of material properties, thus improving the numerical stability, filtering techniques are described together with the projection schemes used to reduce the "gray scales".

1) INTERPOLATION SCHEMES

The Material Interpolation Schemes (MIS) assigning the material property to each element are described in this section. Starting from a discretization of the computational domain Ω, the continuous variable $\boldsymbol{\rho}$, $0 \leq \rho \leq 1$ is assigned

to each mesh cell. Analyzing the two-material case, elements with $\rho = 1$ are filled with material property 1 (p_{mat1}), while material property 2 (p_{mat2}) is assigned to cells for which $\rho = 0$. When ρ is between 0 and 1, the cell is partially filled by material and is said to be in the "grey scale". This situation should be avoided since in the majority of the applications it is not possible to partially fill a cell (i.e., a portion of space) with the desired material (unless the density of the material can be also controlled, however this is not the case in the majority of applications).

Denoting with $g(\mathbf{x})$ the material property at the point $\mathbf{x} \in \Omega$, e.g., the relative permeability $\mu_r(\mathbf{x})$ for magnetic problems, the classical MIS interpolates g from the continuous density $\rho(\mathbf{x})$, using a power-law [59]

$$g(\rho) = p_{mat1} + (p_{mat2} - p_{mat1})\rho^{\alpha}, \tag{17}$$

with the property $g(0) = p_{mat1}$ and $g(1) = p_{mat2}$. In (17), $\alpha > 1$ is a penalization parameter. Due to the continuous nature of ρ, "gray" scales regions (i.e., with $\rho \in (0, 1)$) are generated during the optimization procedure. These "gray scales" regions correspond to unmanufacturable layouts since, as previously discussed, in the physical realization of the device it is necessary to decide whether or not to insert the material in the grey regions.

Increasing α helps the optimization process to avoid the generation of these "gray scale" regions assigning the material property 1 over a wide range of ρ and the material property 2 only when ρ is near to one. However, for too high values of α convergence problems may arise [59]. Moreover, the classical MIS approach shows an asymmetry trend which favours values of the material property associated with low ρ. This issue may be solved using different interpolation schemes, like the uniform sequence proposed in [59] for which

$$g(\rho) = p_{mat1} + \frac{p_{mat2} - p_{mat1}}{\alpha} \sum_{i=1}^{\alpha} \rho^i. \tag{18}$$

In solid mechanics TO, the Rational Approximation of Material Properties (RAMP) proposed in [60] is usually adopted. Given the parameter $q \geq 0$, the RAMP interpolation is expressed as

$$g(\rho) = p_{mat1} + \frac{\rho}{1 + q(1 - \rho)} (p_{mat2} - p_{mat1}). \tag{19}$$

In [61], a different interpolation method is proposed by D. Lukàš through the scheme

$$g(\rho) = p_{mat1} + \frac{(p_{mat2} - p_{mat1})}{2} \left(1 + \frac{\arctan(p(2\rho - 1))}{\arctan(p)}\right), \tag{20}$$

where the "gray scales" depend only on the parameter p, which cannot be too large, in order to avoid convergence problems [33]. Lastly, the so-called reluctivity based mapping proposed for electromagnetic TO is defined [62]

$$g(\rho) = \left[\frac{1}{p_{mat1}} + \left(\frac{1}{p_{mat2}} - \frac{1}{p_{mat1}}\right)\rho\right]^{-1}. \tag{21}$$

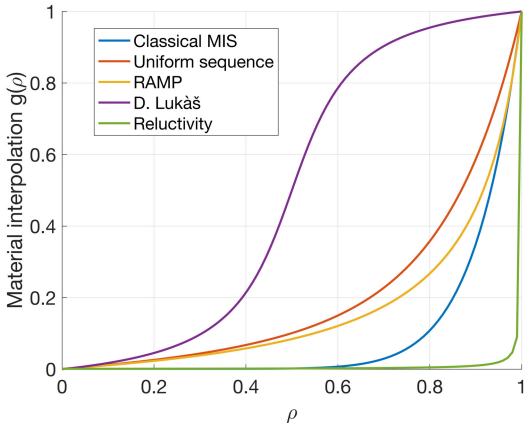

FIGURE 1. Material density interpolation schemes for $p_{mat1} = 10^{-3}$, $p_{mat2} = 1$ and $\alpha = q = 10$: classical polynomial MIS (17), uniform sequence (18), RAMP (19), the Lukàš variant with $p = 5$ (20) and the reluctivity approach (21). In Lukàš approach the transition threshold between two material properties is $\rho = 0.5$ in the limit of high p.

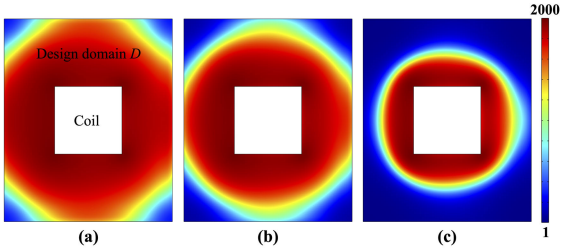

FIGURE 2. Material distribution during optimization problem produced by different values of penalization parameter in classical MIS: $\alpha = 3$ (a), $\alpha = 5$ (b) and $\alpha = 7$ (c). The simple underlying topology optimization problem aim at maximizing the magnetic energy adding iron ($p_{mat2} = 2000$) in the air ($p_{mat1} = 1$) design domain surrounding the square axialsymmetric coil. As can be seen, increasing α, the relative permeability spread over the interval [1,2000] is reduced.

A graphical comparison of the aforementioned interpolation schemes is reported in Fig. 1, while an illustrative example showing the spread of material property g when different values of the penalization parameter α of classical MIS are used is reported in Fig. 2.

The advantage of using a continuous design variable instead of a discrete binary one, lies in the possibility of computing derivatives for sensitivity analysis [30]. Although it would be better if the optimization problem was binary (i.e., $\rho \in \{0, 1\}$), it is in any case convenient to work continuously with the drawback of having to manage the gray scale issue.

2) FILTERING TECHNIQUE

Using ρ directly as input of the MIS, may produce an oscillatory behaviour of the material distribution over the finite element discretization [63], therefore a spatial filtering function has to be adopted. The density filter for each mesh element i can be written as [64]

$$\tilde{\rho}_i = \frac{\sum_{j \in N_{i,j}} \rho_j w(\mathbf{x}_j)}{\sum_{j \in N_{i,j}} w(\mathbf{x}_j)}, \tag{22}$$

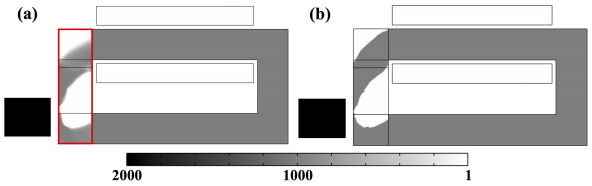

FIGURE 3. Comparison of material density with Helmholtz filter (a) and without filtering (b), for the topology optimization of a magnetic actuator. The color bar refers to the value of μ_r which can vary within [1,1000] in the design domain, whose boundary is highlighted in red.

where $N_{i,j}$ is the set of neighbourhoods of the ith cell within the filter radius R and $w(\mathbf{x}_j)$ is a weight function between cells i, j:

$$w(\mathbf{x}_j) = R - ||\mathbf{x}_i - \mathbf{x}_j||. \tag{23}$$

Particular care must be paid when selecting the filter radius R as high values may lead to sub-optimal designs.

Alternatively to the density filter given in (22), the so-called Helmholtz filter can be adopted [65], [66]

$$-R_h^2 \Delta\tilde{\rho} + \tilde{\rho} = \rho, \tag{24}$$

where R_h is again a filtering parameter playing a similar role to R and, in finite element settings, usually retrieved from the maximum element size of the mesh. Usually, R_h and R are related with $R_h = R/2\sqrt{3}$ [64].

The effect of filtering on the material density ρ is illustrated for the case of the Helmholtz filter (24) in Fig. 3.

3) PROJECTION SCHEMES

From the filtered design variables $\tilde{\rho}$, the projected variables $\hat{\rho}$ are obtained using a smoothed Heaviside function, having the form [67]

$$\hat{\rho} = H_c(\tilde{\rho}) = \begin{cases} 1 & b < \tilde{\rho} \\ c & -b \le \tilde{\rho} \le b \\ 0 & \tilde{\rho} < -b \end{cases} \tag{25}$$

where the constant c is

$$c = \frac{1}{2} + \frac{15}{16}\left(\frac{\tilde{\rho}}{b}\right) - \frac{5}{8}\left(\frac{\tilde{\rho}}{b}\right)^3 + \frac{3}{16}\left(\frac{\tilde{\rho}}{b}\right)^5. \tag{26}$$

Parameter b, with $0 \le b \le 1$, in (25) and (26) refers to the transition width. For decreasing values of b, the material density converges to binary {0, 1} representation [68].

A different Heaviside projection [64] reads

$$\hat{\rho} = H_\eta(\tilde{\rho}) = \frac{\tanh(\beta\eta) + \tanh(\beta(\tilde{\rho} - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}, \tag{27}$$

where β controls the sharpness of the projection and η is a threshold level. The shape of the projection function for different β parameters is shown in Fig. 4.

Summarizing, the design variables ρ in the continuous setting, before being used as input of MIS are firstly filtered and projected. Usually, this simple approach lacks on local convergence, thus a modified robust approach based on

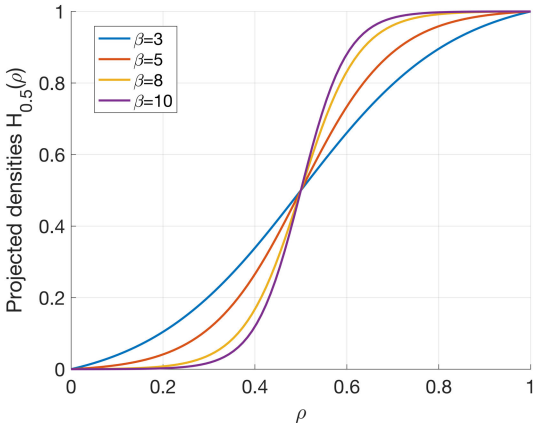

FIGURE 4. Shape of projection function (27) with $\eta = 0.5$ for $\beta = 3, 5, 8, 10$.

dilated, intermediate, and eroded designs with $\eta_d \le \eta_i \le \eta_e$ in (27) can be adopted as shown in [64].

C. METHODS

In this section, the gradient-based methods listed in Table 1 are described. At the beginning, the homogenization method III-C1, as the first TO approach, is depicted. Next, the extensively used density III-C2 and level-set III-C3 methods, together with the phase-field III-C4 method are analyzed. The Bi-directional Evolutionary Structural Topology Optimization (BESO) with sensitivity information and the novel Topology Optimization of Binary Structures Approach (TOBS) are briefly described in section III-C5 and section III-C6, where the latter, to the author's knowledge, has not yet been applied for electromagnetic TO. Finally, a method which uses the sensitivity information to improve the performance of a gradient-free technique is described.

1) HOMOGENIZATION METHOD

Firstly introduced by Bendsøe and Kikuchi in 1988 [1], the homogenization method, was applied to a variety of structural topology optimization problems [69] and, with less impact, to the design of electromagnetic devices [28], [29], [70], [71].

In the homogenization method, each finite element constituting the discretized design domain D, is composed of an infinite number of microstructures usually formed by rectangular cells with rectangular holes [2]. The size of each hole and its rotation angle are the design variables of the optimization. In homogenization logic, a cell becomes "solid" if the the hole vanishes, conversely is "void" if the hole has the same dimension of the cell. Thus, the material is changed only in the microscale cell, and not in the whole mesh element. In the optimization process, the material is transferred between different parts of the design domain giving, at the end, the optimal material distribution. This approach currently seems to be abandoned in favor of the others described below.

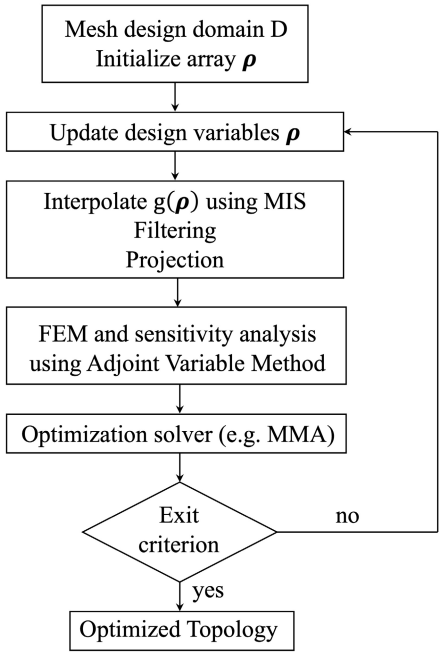

FIGURE 5. Possible flowchart of TO using density method. The exit criterion groups convergence of objective function and fulfillment of additional constrains, for example volume ones.

2) DENSITY METHOD

The density method originally developed for solid mechanics topology optimization [72], recently, due to its simplicity, has gained great interest also for electromagnetic topology optimization even for multi-material problems [30], [31], [32], [33], [34].

In a discretized domain, the density method aim at minimizing the objective function by identifying the type of material to be assigned to each element.

The density methods are based on the interpolation, filtering and projection schemes described in section III-B. Originally, the density method was developed using the simple power-law (17) as material penalization. This choice leads to the so-called Solid Isotropic Material with Penalization (SIMP) approach [73]. As already depicted in section III-B, the choice of an appropriate MIS penalizing the material property is a critical aspect from the numerical point of view. In the context of density methods, a wrong selection of material penalization function may drive to numerical issues, such as singularities in the system matrices, due to bad scaling of the coefficients.

The density method can be realized also within commercial tools and a schematic flowchart of the operations is shown in Fig. 5.

3) LEVEL-SET METHOD

The Level-Set Method (LSM) is a standard boundary-based method for topology optimization. The level-set approach was applied for the topology optimization of different

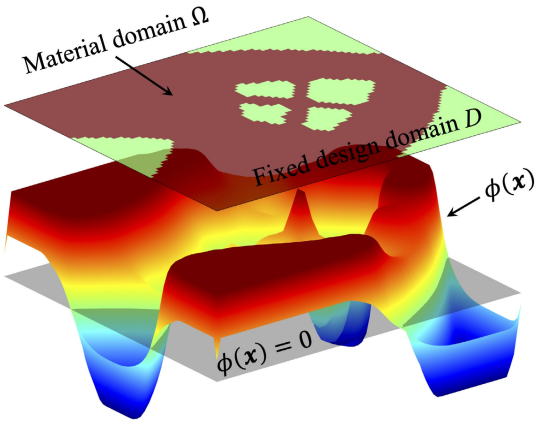

FIGURE 6. Illustration of design domain $D \subset \mathbb{R}^2$, material domain Ω and level-set function $\phi(\mathbf{x})$ for the compliance minimization of the classical cantilever test case.

electromagnetic devices [35], [36], [37], [38], also in cases when multiple materials were involved [39], [40]. In this latter case the method is called Multiple Level-Set Method (MLSM). Referring to the illustrative example of Fig. 6, $D \subset \mathbb{R}^d$ is the design domain and Ω the material region, whose boundary $\partial\Omega$ is identified by Γ. The level-set function $\phi(\mathbf{x}) : \mathbb{R}^d \to \mathbb{R}$, defines the boundary Γ as [74]

$$\phi(\mathbf{x}) \begin{cases} > 0 & \forall \mathbf{x} \in \Omega \backslash \Gamma \\ = 0 & \forall \mathbf{x} \in \Gamma \\ < 0 & \forall \mathbf{x} \in D \backslash \Gamma. \end{cases} \tag{28}$$

Performing topology optimization with LSM, means tracking the evolution of the level-set function $\phi(\mathbf{x})$ solving the Hamilton-Jacobi partial differential equation for a fictitious time t [75]

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = 0, \tag{29}$$

where $\mathbf{v}$ is the velocity field

$$\mathbf{v} = V_N \frac{\nabla \phi}{||\nabla \phi||}, \tag{30}$$

with V_N the scalar velocity normal to the interface Γ, derived from the design sensitivities [35], [36]. The optimization proceeds until convergence, when the velocity V_N becomes zero, stopping the propagation of the level-set function.

When the level-set function is a signed distance function, the level-set function at iteration $k+1$, or equivalently at time instant $t + \Delta t$, is

$$\phi(t + \Delta t) = \phi(t) - \Delta t V_N, \tag{31}$$

where Δt is the fictitous time step size chosen in such a way that the Courant–Friedrichs–Lewy (CFL) condition is fulfilled [76]. Following [77], the time step at each iteration k, can be expressed as

$$\Delta t^{(k)} = \zeta^{(k)} l_{min} / V_{max}^{(k)}, \tag{32}$$

where l_{min} is the minimum edge length of mesh elements, $V_{max}^{(k)}$ is the maximum absolute value of advection velocity at kth iteration and

$$\zeta^{(k)} = \begin{cases} \zeta^{(k-1)} & (F(\phi^{(k-1)}) \leq F(\phi^{(k-2)})) \\ \tau\zeta^{(k-1)} & (F(\phi^{(k-1)}) > F(\phi^{(k-2)})) \end{cases} \tag{33}$$

for $k > 2$, with $\tau < 1$ a regularization parameter.

Equation (29) is usually solved by means of stabilization techniques which involve the inclusion of reaction and diffusion terms.

Following [74], [78], a simple diffusion term can be used and controlled by the parameter σ, with the aim of adjusting the amount of numerical diffusion to be added. Equation (29) is then reformulated as

$$\begin{cases} \phi(t+\Delta t) = \phi(t) - \Delta t(\beta\bar{F}' - \sigma\nabla^2\phi) & \text{in } D \\ \phi = 0 & \text{on } \partial D, \end{cases} \tag{34}$$

where $\bar{F}'$ is the derivative of the augmented objective function which incorporate possible volume constraints, and β is the normalization parameter

$$\beta = \frac{\int_D d\Omega}{\int_D |F'| d\Omega}. \tag{35}$$

Alternatively, as shown in [79], [80], and [81], an approach involving Radial Basis Functions (RBF) may be applied for the time evolution of the level-set function.

One major criticality of the level-set method, relies in the difficulty of adding new holes in the design domain or, in other words, change the material property of the elements. To overcome this issue, the initial design domain should be seeded with holes or a mechanism for hole nucleation during the optimization process would need to be adopted [82]. Potentially, this issue can be circumvented by coupling the level-set method with the concept of topological derivative [83], [84].

Considering a design-domain dependent objective function $F(D)$, its topological derivative $d_T F$ represent the sensitivity with respect to the insertion of an infinitesimal hole ω_ε around point $\mathbf{x}_0 \in D$ [85]:

$$F(D\backslash\bar{\omega_\varepsilon}) - F(D) = f(\varepsilon) d_T F(\mathbf{x}_0) + \mathcal{O}(f(\varepsilon)), \tag{36}$$

where $f(\varepsilon) \to 0$ for $\varepsilon \to 0$. Topology variations are allowed in regions where the values of $d_T F(\mathbf{x})$ are small. The derivation of topological derivatives for linear and nonlinear magnetostatic problems can be found in [83] and [86].

Following [87], in a discretized setting, if φ_{max} and φ_{min} are maximum and minimum values of the nodal sensitivity field, introducing the parameter η_i as

$$\eta_i = \frac{\varphi_i - \varphi_{max}}{\varphi_{max} - \varphi_{min}}, \tag{37}$$

hole seeding can proceed by following these steps [88]:

1) Compute the sensitivity field for each mesh element;
2) Define a maximum value for the field and truncate it to all void elements;
3) Map sensitivity values to mesh nodes;
4) Compute nodal array $\boldsymbol{\eta}$ with (37) and allow hole-nucleation at a number of selected nodes for which $\eta_i < \eta_{selected}$;
5) Generate the hole perturbing the design variables in the selected nodes.

A large value of $\eta_{selected} \in [0, 1]$ can speed up the convergence, but at the cost of increasing the probability of reaching a non-optimal solution [87]. A MATLAB® numerical implementation of the aforementioned logic for the compliance minimization problem can be found at `https://www.topopt.mek.dtu.dk/`.

4) PHASE-FIELD METHOD

The phase-field method is a variation of the standard level-set approach aiming at regularizing the topology optimization problem. In the phase-field approach the level-set function is re-defined as [89], [90]

$$\begin{cases} 0 < \phi(\mathbf{x}) \leq 1 & \forall \mathbf{x} \in \Omega\backslash\Gamma \\ \phi(\mathbf{x}) = 0 & \forall \mathbf{x} \in \Gamma \\ -1 \leq \phi(\mathbf{x}) < 0 & \forall \mathbf{x} \in D\backslash\Gamma, \end{cases} \tag{38}$$

and the regularization is achieved adding a fictitious interface energy term to the objective function, that is

$$\bar{F}(\phi) = F(\phi) + \underbrace{\int_D \varepsilon ||\nabla\phi||^2 d\Omega}_{\text{fictitious interface energy}}. \tag{39}$$

This means that the design domain is subdivided into two sub-regions where the phase state is "1" for the material region, "-1" for the void region and the diffusion region $-1 < \phi < 1$ in a layer of thickness ε [91], [92].

5) BI-DIRECTIONAL EVOLUTIONARY STRUCTURAL TOPOLOGY OPTIMIZATION

The Bi-directional Evolutionary Structural Topology Optimization (BESO) is an evolutionary method developed by Querin *et al.* in 1998 [93], dealing with binary design variables. In classical Evolutionary Structural Optimization (ESO), material can only be removed from the structure [94], while in Additive Evolutionary Structural Optimisation (AESO) algorithms, material can only be added [95]. Conversely, BESO enables the bi-directional addition or removal of material.

These algorithms in general belong to the class of hard-kill methods, however as described in [41], the sensitivity information φ_i can be used to change the elemental density as

$$\begin{aligned} 0 &\to 1 \quad \text{if } \varphi_i > \varphi_{add}^{th} \\ 1 &\to 0 \quad \text{if } \varphi_i \leq \varphi_{rem}^{th}, \end{aligned} \tag{40}$$

where φ_{rem}^{th} and φ_{add}^{th} are real numbers used as threshold values to decide whether add or remove.

6) TOPOLOGY OPTIMIZATION OF BINARY STRUCTURES APPROACH

The Topology Optimization of Binary Structures (TOBS) method is a gradient-based optimization technique based on the binary design variable $\rho \in \{0, 1\}$, proposed by R. Sivapuram and R.Picelli [43]. Within the TOBS, the optimization problem is solved by means of sequential approximations, through linearization of objective and constraint functions. At the kth iteration, the problem to be solved is written as [44]

$$\begin{aligned} \text{Minimize} \quad & \frac{\partial F(\boldsymbol{\rho}^k)}{\partial \boldsymbol{\rho}} \cdot \Delta \boldsymbol{\rho}^k \\ \text{Subject to} \quad & \frac{\partial G_i(\boldsymbol{\rho}^k)}{\partial \boldsymbol{\rho}} \cdot \Delta \boldsymbol{\rho}^k \leq \Delta G_i(\boldsymbol{\rho}^k) \;\; i \in [1, N_g] \\ & ||\Delta \boldsymbol{\rho}^k||_1 \leq \beta N_d \\ & \Delta \rho_j^k \in \{-\rho_j^k, 1 - \rho_j^k\} \;\; j \in [1, N_d], \end{aligned} \tag{41}$$

where N_G is the number of constraints and N_d is the number of design variables. The number of flips $0, 1 \rightarrow 1, 0$ at each iteration is controlled by the parameter $\beta \leq 1$, that is, only a fraction β of cells are allowed to change state. Due to the integer nature of the design variables, the problem (41) is solved using Integer Linear Programming (ILP) [43]. As shown in [96], even if TOBS employs binary design variables, the sensitivities can be evaluated using a density-based approach, thus exploiting the techniques described in III-B.

Recently, the TOBS method was applied for the compliance maximization of fluid structures [45], [46], [47], but up to now, to the author's knowledge, not to electromagnetic problems.

7) TWO-STEP TOPOLOGY OPTIMIZATION METHODS

Two-step topology optimization method combine a global stochastic search algorithm with a local method based on sensitivity analysis. These algorithms were developed to overcome the poor performances of stochastic algorithms like the Genetic Algorithm (GA), in finding engineering-feasible designs at the end of the optimization procedure [48]. To increase the efficiency in finding optimal topologies, a local search method based on sensitivities information is added giving rise to the following two-step algorithm:

1) The first step coincides with the global search performed by the stochastic algorithm;
2) The configuration achieved in the first step is further optimized using methods involving sensitivity analysis.

In [48], the local search is performed evolving the material boundary using the level-set equation, while in [97] an approach involving only the Normalized Gaussian network (NGnet) basis functions both for global and local search is proposed.

IV. GRADIENT-FREE METHODS

In this section, the use of gradient-free methods avoiding the computationally expensive evaluation of objective function sensitivities, is discussed for the TO of electromagnetic devices. The section generally distinguishes between stochastic IV-A and deterministic IV-B approaches. The novel deep learning methods based on neural networks are briefly described in section IV-C, followed by the recently proposed proportional topology optimization (PTO) method IV-D for solid mechanics.

A list of the described methods is reported in Table 2.

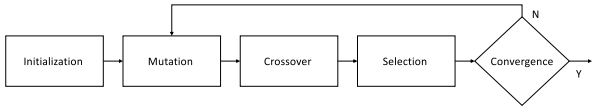

FIGURE 7. Basic flowchart of Differential Evolution algorithm.

A. STOCHASTIC METHODS

The so-called stochastic approaches, which avoid the computationally expensive calculation of objective function sensitivity, such as Genetic Algorithm (GA), Differential Evolution (DE), Particle Swarm Optimization (PSO), Simulated Annealing (SA) and Bat algorithm were extensively applied in literature for the design optimization of a wide class of electromagnetic components [117], [118], [119], [120], [121], [122], [123]. Just to summarize the algorithmic structure of one of these evolutionary approaches, the flowchart of DE is illustrated in Fig. 7. In principle, these algorithms belong to the class of hard-kill methods, where the property of each material cell is changed heuristically. In addition, restricting the space of design variables to the binary case, algorithms can be also classified as ON/OFF methods [72]. As an example, when dealing with magnetostatic problems for which the material can be chosen among iron and air, "1" encode iron elements and "0" air elements.

1) BOOLEAN METHODS

When dealing with binary search spaces as it is the case of ON/OFF methods, the so-called boolean algorithms can be adopted [98]. Referring to the scheme of Fig. 7, the optimization begins with a random binary population of Np individuals, then the standard subtraction, addition, and multiplication embedded in the algorithm blocks, are replaced with the logical operators "XOR" ($\otimes$), "OR" ($\oplus$) and "AND" ($\odot$) [99], [100]. As an example, defined with $\mathbf{x}_i^G \in \mathbb{R}^D$ the ith population individual composed of D optimization parameters at generation G, the typical mutation operator of DE scheme

$$\mathbf{v}_i^{G+1} = \mathbf{x}_{r1}^G + F(\mathbf{x}_{r2}^G - \mathbf{x}_{r3}^G) \tag{42}$$

for $i = 1, \ldots, Np$, is replaced with the following [124]

$$\mathbf{v}_i^{G+1} = \mathbf{x}_{r1}^G \odot F \otimes (\mathbf{x}_{r2}^G \otimes \mathbf{x}_{r3}^G), \tag{43}$$

where $r1 \neq r2 \neq r3$ are randomly chosen and F is the mutation factor.

TABLE 2. List of gradient-free methods discussed.

Method	References	Characteristics	Used in electromagnetics
Boolean	[98] [99] [100]	simple logic, uses logical operators ($\otimes, \oplus, \odot, \ldots$)	✓
Binary	[101] [102]	same algorithmic structure of standard evolutionary, convert continuous $\boldsymbol{\rho}$ to binary using e.g., a sigmoid function	✓
μGA	[103]	based on genetic approach of GA, uses small population number w.r.t. GA	✓
Immune Algorithm (IA)	[104] [105]	combines local and global search	✓
NGnet	[106] [107]	smooth design using Gaussian basis, used in combination with GA	✓
Nelder-Mead	[108]	deterministic approach used as local search in combination with global stochastic	✓
Deep Learning	[109] [110] [111] [112] [113] [114] [115]	reduced number of function evaluations, create surrogate model, suitable for large-scale optimization	✓
PTO	[116]	simple heuristic logic, design variables $\boldsymbol{\rho}$ proportional to global quantity	

2) BINARY METHODS

In the class of binary methods we group all the methods that have the same algorithmic structure of the continuous versions, but for which the design variables defining the material property of each cell, later used for the field solution, lies in the binary space. Looking at the flowchart of Fig. 7 this can be achieved introducing for instance a sigmoid function $f(x) = 1/[1 + \exp(-x)]$ [101], [125] acting on each population individual $\mathbf{x}_i$, after the crossover step

$$y_{i,j} = \begin{cases} 0 & \text{if } U(0,1) < f(x_{i,j}) \\ 1 & \text{otherwise,} \end{cases} \tag{44}$$

transforming the continuous design variables to discrete ones. Another approach based on the algorithmic structure of DE and preserving its benefits, relies in the definition of a probability estimation operator starting from the mutated vector (42) [102], [126]

$$P(x_{i,j}^{G+1}) = \{1 + \exp[-2b(MO - 0.5)/(1 + 2F)]\}^{-1}, \tag{45}$$

where MO defines the mutated vector and $b > 0$ is the bandwidth factor. Applying the probability operator, the binary representation of the mutant vector $v_{i,j}^{G+1}$ becomes

$$v_{i,j}^{G+1} = \begin{cases} 0 & \text{if } U(0,1) \leq P(x_{i,j}^{G+1}) \\ 1 & \text{otherwise.} \end{cases} \tag{46}$$

It is worth remembering that the advantage of ON/OFF methods relies in the fact that no ''gray'' regions are generated in the domain, however due to the random nature of the classical evolutionary algorithms, single spots of solid or empty material can appear in the design. This reflects the well known checkerboard-like situation characterizing the class of ON/OFF topology optimization problems [127].

To circumvent this criticality, different approaches were presented in literature, most of these involving the inclusion of penalty regularization terms and techniques to guarantee elements connectivity [103], [128]. Among them, we mention the approaches applied to Immune Algorithms (IA) [104], [105], [129], [130], [131] and microGA (μGA) algorithms, with the latter recently applied for the topology optimization of wireless power transfer (WPT) devices [132].

3) NORMALIZED GAUSSIAN NETWORK METHODS

The Normalized Gaussian network (NGnet) introduced by Sato et. al. in 2015 allows smooth shapes without introducing additional filtering [106], [107]. Considering a finite element discretization of the computational domain, the material property g to be assigned to each mesh element is determined by the value of the shape function $f(\mathbf{x})$ defined as

$$\begin{aligned} f(\mathbf{x}) &= \sum_{i=1}^{N_g} w_i b_i(\mathbf{x}) \\ b_i(\mathbf{x}) &= G_i(\mathbf{x}) / \sum_{j=1}^{N_g} G_j(\mathbf{x}) \end{aligned} \tag{47}$$

where G_i is the Gaussian function centered at the ith element barycenter, b_i is the ith normalized function and w_i is the ith weight. For the two-material case (e.g., iron and air), the elemental material is expressed as [133]

$$g \leftarrow \begin{cases} \text{iron} & f(\mathbf{x}) \geq 0 \\ \text{air} & f(\mathbf{x}) < 0, \end{cases} \tag{48}$$

but extensions are available also in multi-material problems [134]. The weight vector $\mathbf{w} = [w_1, \ldots, w_{Ng}]$ is determined by an evolutionary algorithm (e.g., μGA) in such a way the optimization problem (1) is satisfied. That is, the original problem becomes a parametric optimization problem [132]. An illustrative example of the procedure determining the shape function $f(\mathbf{x})$, is shown in Fig. 8.

When the genetic algorithm is used as search tool, the initial configuration may have a high number of non-survival individuals [110], thus increasing the computational cost of the method as stated in [111].

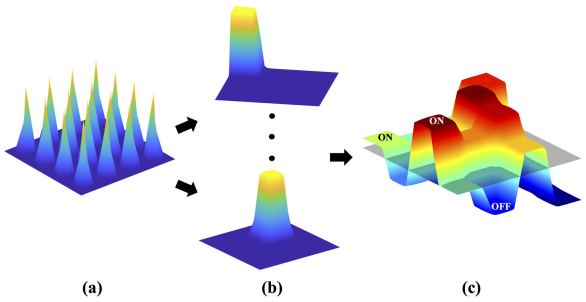

FIGURE 8. Example of NGnet procedure over a 2D domain discretized with 16 regular quadrilateral elements. Gaussians $G_i(\mathbf{x})$ are shown in (a), two examples of their normalized version $b_i(\mathbf{x})$ in (b) and the shape function $f(\mathbf{x})$ from the weighted sum in (c). In the latter, elements label with "ON" refers to $f(\mathbf{x}) \geq 0$ values, while those labelled with "OFF" to $f(\mathbf{x}) < 0$.

B. DETERMINISTIC METHODS

In this section, we briefly discuss the deterministic (direct search) gradient-free methods for TO. Starting from a candidate point solution, the direct search algorithm find the best points within a set around the starting one, for which the objective function is lowered. The search directions are not stochastic and can be retrieved using line search methods, simplex methods (e.g, Nelder-Mead), simplex gradients methods and so on [135].

For TO a direct search method is usually used in symbiosis with a stochastic method thus defining hybrid algorithms. In such cases the deterministic method plays the role of local search, while the stochastic one is used for the global search [108], [136]. An example of flowchart of hybrid stochastic-deterministic algorithm can be seen in Fig. 9.

From the author's knowledge, its seems that electromagnetic TO literature discusses these approaches in a little extensive way.

C. DEEP LEARNING METHODS

In recent years, deep learning techniques gained great interest for the optimization of electromagnetic devices [109]. Due to the stochastic nature of evolutionary methods like GA, the topology optimization based solely on these techniques suffers from high computational costs since every objective function (fitness) evaluation requires the solution of an electromagnetic problem. Machine learning techniques trained with the data generated by the electromagnetic simulations, were used to obtain surrogate models with the aim of speeding up the computations. In this setting, surrogate models based on Artificial Neural Networks (ANNs) [137], [138], [139], [140], [141], Kriging methods [142], [143], response surface methods [144], and Space Mapping methods [145] have been built, even for multi-objective problems [146], [147], [148].

A particular subclass of ANNs, usually applied in computer vision, called Convolutional Neural Networks (CNNs) [149] has recently gained particular attention due to its capability in dealing with huge number of design variables

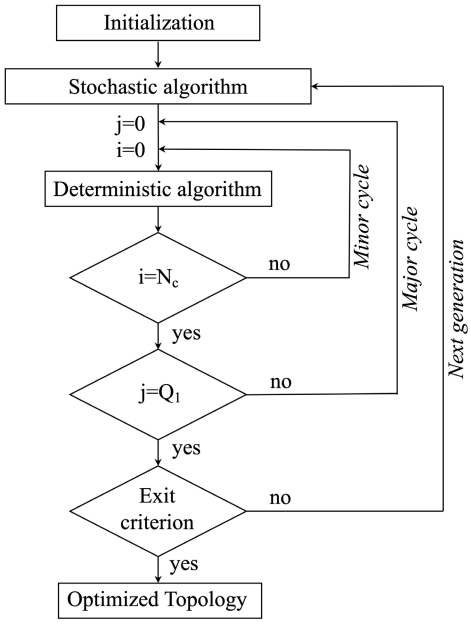

FIGURE 9. Schematic flowchart of TO using hybrid stochastic-deterministic method where N_c is the number of cells and Q_1 the number of major cycles. Following [108], GA can be used as stochastic method and the deterministic Nelder-Mead for the local search.

(for example in real problems where the mesh has million of degrees of freedom) [150]. CNNs were applied to a large class of electromagnetic problems, showing their general capabilities [110], [111], [112], [113], [114], [115].

D. PROPORTIONAL TOPOLOGY OPTIMIZATION METHOD

The Proportional Topology Optimization (PTO) is an heuristic non-sensitivity based method for solid mechanics applications proposed by Biyikli and To [116]. Even in this method, the material property is interpolated according to the MIS approach (17), while the elemental densities during the iteration procedure are updated as

$$\rho_e(k+1) = \alpha\rho_e(k) + (1-\alpha)\rho_e^{opt}, \tag{49}$$

where ρ_e^{opt} is the optimized density proportional to the objective function, and α is a history parameter controlling the ratio of dependence of elemental density to its older value from the previous iteration [116].

With the focus of increasing robustness and capability of approaching binary distributions, some improvements have been recently proposed [151].

A MATLAB® implementation of the PTO algorithm applied to classical solid mechanics examples is available at `http://www.ptomethod.org/`. Up to now, to the author's knowledge, this method has not been used for the topology optimization of electromagnetic devices.

V. OTHER METHODS

Since the research on TO for electromagnetics is rapidly evolving, a variety of approaches were developed in the recent

years, and some of them are summarized in this section. In [152] the Allen-Cahn equation is used to update the design variables using the phase-field method of section III-C4. In [153] a method based on the $\alpha - \beta$ Swap Move theorem to improve the hole generation capability of boundary-based TO methods (e.g., the level-set described in III-C3) is proposed. In [154] the min-cut theorem is used to define a new methodology for 3D problems. A two-step algorithm for multi-material problems coupling the combinatorial optimization for the global search with the variable design region method [155] for the local search is proposed in [156]. A hybrid parametric and topology optimization algorithm to optimize both the shape of the magnet and the flux barriers of a permanent magnet motor is proposed in [157]. The Monte Carlo tree search (MCTS) in combination with the hybrid parametric-topology optimization is proposed in [158] for the design of permanent magnet motors. The Gabor filtering technique, alternative to the NGnet approach of section IV-A3, usually adopted in image processing, is applied in [159]. The geometry projection method using elementary bars parametrized by the location of the endpoints of its medial axis, is applied in [160] for the optimization of a WPT device.

Finally, it is worth mentioning that high frequency electromagnetic problems can be tackled with novel methodologies, such as the one described in [161], using the Characteristic Mode Analysis (CMA).

VI. RESULTS

In this section, numerical results obtained with selected methods described in the previous sections are reported. The first test case compares gradient and non-gradient methods for the design of a magnetic actuator.

For the gradient-based method, the SIMP and level-set approaches are selected, due to their known applicability in electromagnetic TO. In addition, the novel TOBS approach is also investigated. It is worth noting that, up to the author's knowledge, this is the first time that TOBS is used for electromagnetic TO.

Concerning the choice of the selected gradient-free approaches, the standard Binary-DE algorithm is selected as it is an example of a basic and naive approach. Then, due to the poor capabilities of the standard Binary-DE of ensuring structural connectivity and to mitigate the checkerboards pattern, a modified version with added topology constraints is also adopted. As expected, this modified Binary-DE shows much better performances but it looses generality since the required modifications and the added constraints are highly problem-dependent and cannot be always applied in general TO optimization. Lastly, the novel deep learning-based Self-directed Online Learning Optimization (SOLO) method is used since it is a promising method to reduce the number of function evaluations required by stochastic gradient-free TO.

The second test-case compares the novel TOBS approach and SIMP method for the TO of the ferrite domain in a standard WPT1-Z1 device for wireless power transfer. This example is chosen to show the capability of the proposed approaches of treating 3D non-trivial geometries which will be of great interest for future industrial applications.

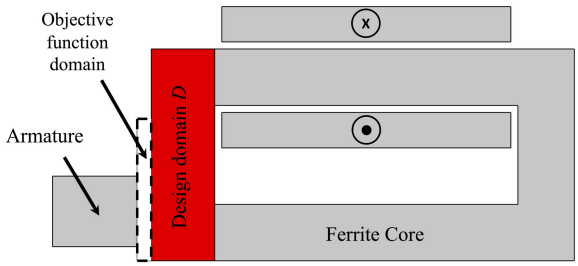

FIGURE 10. 2D layout of magnetic actuator retrieved from [35]. The armature have $\mu_r = 2000$, while the ferrite core $\mu_r = 1000$. The relative permeability of the design domain can be $\mu_r^{min} = 1$ or $\mu_r^{max} = 1000$. The current density in the coil is 2 A/mm^2.

A. MAGNETIC ACTUATOR

The 2D linear magnetic actuator showed in Fig. 10 is used as a benchmark to compare the selected TO approaches. In such test case the goal of the topology optimization is the maximisation of the magnetic energy W_m in the objective function domain, while keeping the amount of ferrite material in the design domain D below a fraction $V_{frac} = 0.6$ of the full design domain

$$\begin{aligned} &\text{find } W_m^* = \max W_m := \frac{1}{2}\int \mathbf{J}\cdot\mathbf{A}d\Omega \\ &\text{s.t. } \frac{V_{ferrite}}{V_D} \leq V_{frac}. \end{aligned} \tag{50}$$

In the following sections some selected gradient-based III and gradient-free IV approaches are used to solve problem (50) and the results are compared. The objective function improvement is measured with the following metric

$$\text{Objective Increase} = \log_{10}\left(\frac{W_m^* - W_m^0}{W_m^0}\right), \tag{51}$$

where W_m^0 is the magnetic energy when the whole design domain is filled with ferrite.

1) GRADIENT-BASED RESULTS

Due to their general interest and wide application, even in electromagnetics, the density based III-C2 and level-set III-C3 approaches are used to solve TO problem. Moreover, the novel TOBS method III-C6 which to the author's knowledge, has not yet been used in electromagnetics, is considered. The density-based approach is implemented in COMSOL® under the topology optimization node and the classical MIS approach (17) used for the material interpolation. Then, then minimization problem is solved using the Method of Moving Asymptotes (MMA) [162]. The developed level-set method combines COMSOL® with MATLAB ® environments. The physics is solved within COMSOL®, while the optimization proceeds in MATLAB

TABLE 3. Gradient-based TO methods comparison.

Method	Objective Increase	Obj. Fun. Evaluations
SIMP	2.65	90
Level-Set	2.53	36
TOBS	2.51	24

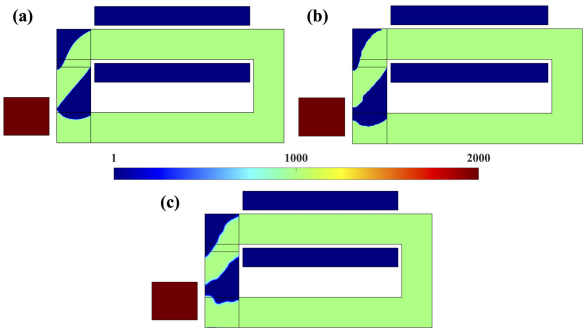

FIGURE 11. Final material distributions achieved by the different gradient-based methods under examination: SIMP (a), TOBS (b), Level-Set (c). The color bar refers to the value of the relative permeability.

® following the code which uses RBFs for the time evolution of the level-set function [81].

The TO scheme using TOBS is described in the paper by R. Picelli *et al.* [163] and uses COMSOL® for the physical and sensitivity analyses in combination with MATLAB ® for the optimization problem. Here, the binary elemental densities are evolved using Integer Linear Programming (ILP). An online repository demonstrating TOBS approach for structural mechanics, is available at `https://github.com/renatopicelli/tobs`.

Optimization results are reported in Table 3 and the final material distribution is illustrated in Fig. 11.

2) GRADIENT-FREE RESULTS

Here, the Binary-DE (BDE) IV-A2 and deep learning approach IV-C are considered to solve problem (50).

A MATLAB® script for the BDE was implemented based on the continuous algorithm, and converted to ON/OFF method through a sigmoid function. Here, only the volume constraint is added without domain connectivity check. A BDE with the improved capability of satisfy both volume constraint and domain connectivity in the same fashion of μGA described in [103], is coded in MATLAB®.

Lastly, the recently developed Self-directed Online Learning Optimization (SOLO) [164], based on deep learning method IV-C, was used. SOLO uses a Deep Neural Network (DNN) to reduce the objective function evaluations, thus requiring less computational effort. An online repository of the SOLO method with some explanatory examples using COMSOL® plus MATLAB® and Python scripts, is available at `https://github.com/deng-cy/deep_learning_topology_opt`. For the field solution and thus the objective function evaluation,

TABLE 4. Gradient-free TO methods comparison.

Method	Objective Increase	Obj. Fun. Evaluations
BDE	2.48	10^4
Constrained BDE	2.55	10^4
SOLO	2.38	10^3

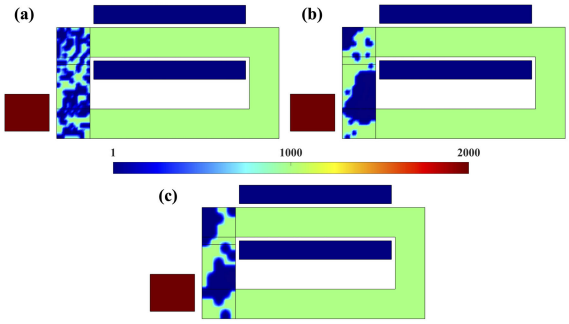

FIGURE 12. Final material distributions achieved by the different gradient-free methods under examination: Binary-DE (a), μGA (b), SOLO (c). The color bar refers to the value of the relative permeability.

TABLE 5. WPT1-Z1 device parameters.

Parameter	Value
Air-gap	100 mm
Ferrite width	440 mm
Ferrite length	600 mm
Ferrite μ_{FE}	3300
Ferrite σ	0.16667 S/m
Turns GA coil	15
Turns VA coil	8 (upper layer), 9 (lower layer)

proprietary FEM MATLAB® scripts are used to speed-up the computation bypassing communication with COMSOL®.

The number of iterations of standard and constrained DBE are selected as $G_{max} = 100$ and the number of individuals $Np = 100$, thus the number of objective function evaluations is $G_{max} \times Np = 10^4$. The proposed SOLO method is based on the binary version available at the linked repository and runs for 100 algorithm iterations with 10 additional training samples for a total of 10^3 objective function evaluations.

The results are reported in Table 4 and the final material distribution illustrated in Fig. 12.

B. WPT1-Z1 DEVICE

The wireless power transfer device termed as WPT1-Z1, according to the description provided by the SAE international standard [165], is here considered as 3D test case. Referring to Fig. 13, the design domain D lies below the Ground Assembly (GA) coil. There, the relative magnetic permittivity is the subject of topology optimization, that is $\mu_r \in \{1, \mu_{FE}\}$. The geometrical parameters are reported in Table 5.

The topology optimization aims at maximising the coupling coefficient k, while keeping the amount of ferrite below

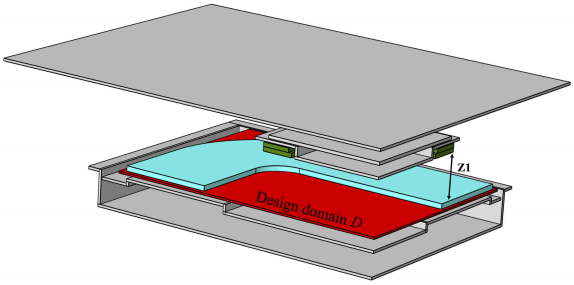

FIGURE 13. CAD view of WP1-Z1 3D device. Due to symmetry, only a quarter of the design is shown. The design domain *D* highlighted in red, correspond to the ferrite in the Ground Assembly (GA). GA and Vehicle Assembly (VA) coils are colored in cyan and green, respectively, while gray parts are aluminum shielding.

TABLE 6. Results for WPT1-Z1 TO.

Method	Objective Increase [%]	Obj. Fun. Evaluations
TOBS	12.5%	18
SIMP	13.9%	18

half of the volume of the design domain, i.e.,

$$\text{find } k^* = \max k := \frac{M}{\sqrt{L_{GA}L_{VA}}}$$
$$\frac{V_{ferrite}}{V_D} \leq V_{frac} = 0.5, \tag{52}$$

where M is the mutual inductance between GA and VA coils and L_{GA}, L_{VA} their self-inductances. The problem is firstly solved with the TOBS method (Sec. III-C6), using combined COMSOL® plus MATLAB® algorithms and the obtained ferrite topology is illustrated in Fig. 14. Even if TOBS produces binary design variables, for graphical reasons and for direct comparison with the continuous approach, the material property is linearly interpolated. As a comparison, the SIMP method III-C2 fully developed within COMSOL® Multiphysics software is used to solve the problem and the final ferrite topology reported in Fig. 15. Table 6 summarizes the numerical results. There, the objective increase percentage is computed with respect to the case of fully-filled ferrite design domain, which has $k = 0.3619$. The TOBS approach increases the objective function to $k^* = 0.4071$ after 18 iterations, while SIMP results in $k^* = 0.4123$ after 18 iterations of MMA. In the latter case, due to the continuous nature of the design variable ρ, the value of the relative permittivity is intrinsically spread over the interval $[1, \mu_{FE}]$ and post-processing is required to cut the final layout.

VII. DISCUSSION

The information carried by the sensitivity of the objective function highly increases the capability of gradient-based approaches of obtaining good results after few objective function evaluations, as reported in Table 3. However, these approaches may become extremely costly and intrusive from the computational point of view since they require the derivative of the system matrix. If the analytical expression of the derivative is not available, this may be evaluated using the finite difference method by adding a small perturbation to

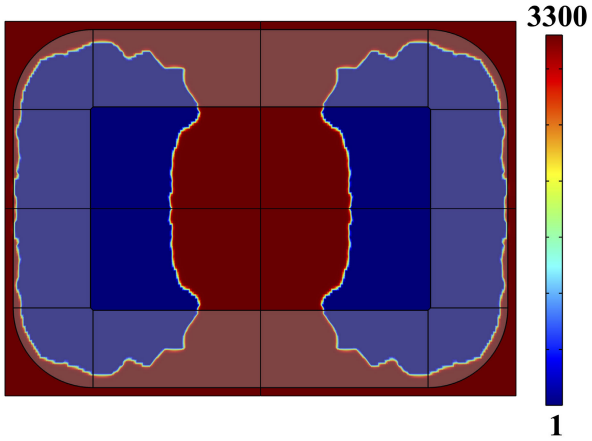

FIGURE 14. Top view of final ferrite topology obtained with the TOBS method. TOBS deals with binary ρ but for graphical reasons the achieved material distribution is linearly interpolated. Light blue area represents GA coil. The color bar refers to the value of the relative permeability.

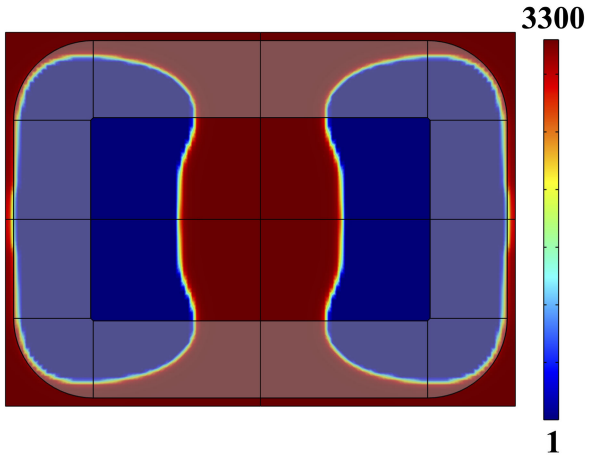

FIGURE 15. Top view of final ferrite topology obtained with the COMSOL® density method using the SIMP approach with $\alpha = 5$. Light blue area represents GA coil. The color bar refers to the value of the relative permeability.

the design variable [55], thus requiring multiple constructions of the system of equations. When a density method is used (e.g., SIMP), the selection of MIS function penalizing the material property, is extremely important to reduce numerical instabilities. In addition, when the objective function exhibits many local minima, these approaches may remain stuck and the true global minimum may not be reached. The latter is a well-known problem when dealing with gradient-based optimization techniques.

To overcome this issues gradient-free approaches may be adopted IV. As highlighted in the test-case results of Table 4, one of the main disadvantages of these methods is the high number of objective function evaluations, which depend on the population size. Reducing the population size in general may produce non-satisfactory results thus this approach cannot be followed to improve the performances. Due to the stochastic nature of these approaches it is difficult to ensure domain connectivity or avoid checkerboards layouts, and the situation is even worse in cases of final topologies where multiply disconnected components are generated. As expected,

the naive DE approach fails to find a connected structure and the checkerboard pattern is highly evident. To overcome this issue, the user can ensure the connectivity between selected parts using specific techniques, but this approach is extremely problem dependent and unfeasible in cases when no prior information is known on the final topology. An example for this can be found in the WPT TO of [132] where the final layout may have different disconnected components. If the TO has to result in a single domain, a structurally feasible layout can be ensured coupling the electromagnetic TO optimization with a structural one, as shown in [166].

The higher number of objective function evaluations required by gradient-free approaches may be reduced using novel deep learning approaches, for example the tested SOLO method. As stated in [164], SOLO is able to drastically reduce the number of objective function evaluations. When the evaluation of objective function becomes computationally expensive due to the solution of the underlying system of equations, the possibility of reducing its evaluation is extremely important, even at the cost of increasing the computational time.

Lastly we highlight the fact that, in general, the gradient-free methods can be used more easily for multi-objective optimizations.

VIII. CONCLUSION

A survey on Topology Optimization (TO) methods for electromagnetic applications is proposed. The state-of-art approaches already used in electromagnetics are reviewed together with novel techniques recently developed for mechanical and fluid-dynamics problems.

The paper follows the general distinction between gradient-based and gradient-free methods. Although numerical TO is highly problem dependent, so that it is difficult to assert that an approach is better or worse than another, a simple 2D example of magnetic actuator and a realistic 3D example of wireless power transfer device are considered as benchmarks for the selected methods. The test-case is used to analyze the different aspects of the proposed approaches, with the purpose of highlighting general pros and cons of the gradient-based and gradient-free techniques.

From a practical perspective of a potential user that should choose most suited optimization method, a trade-off between generality, computational burden, and manufacturability of the final result should be considered. For instance, if the ''gray scales'' are not a problem, the standard density-based approach III-C2 can be preferred, due to its implementation within commercial software, thus simplifying the application of TO techniques in the industrial world. If a clear transition boundary between the material is needed, the user has to choose ON/OFF optimization methods. In this respect the novel gradient-based TOBS method, which uses concepts of standard density-based techniques but is based on binary design variables, seems to be very interesting. Unfortunately, the computation of sensitivities may represent a major limitation of this approach due to its intrusive nature and more involving computational skills are required by the user unless the algorithm is coupled with commercial software that already implement the computation of sensitivity maps.

If the sensitivity of the objective function is extremely costly from the computational viewpoint, a gradient-free approach may be preferred due to its simplicity, requiring only objective function evaluation. With these methods some forethought must be used to ensure domain connectivity, thus avoiding checkerboards patterns. Moreover, when using stochastic-based methods, the number of objective function evaluations is highly increased with respect to gradient-based ones. When the evaluation of objective function becomes very costly from the computational point-of-view, as is the case of large systems of equations, the novel approaches using neural networks can be adopted to reduce the computational burden through the creation of surrogate models.

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