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Errata to “Global L^p estimates for degenerate
Ornstein-Uhlenbeck operators with variable
coefficients”, Math. Nachr. 286, No. 11–12, 1087
– 1101 (2013)

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In this note we want to point out and correct a mistake in our paper [1], where we consider a class of Ornstein-Uhlenbeck operators on \mathbb{R}^N

$$\mathcal{A} = \sum_{i,j=1}^{p_0} a_{ij}(x) \partial_{x_i x_j}^2 + \sum_{i,j=1}^N b_{ij} x_i \partial_{x_j},$$

together with the corresponding Kolmogorov-Fokker-Planck operators on \mathbb{R}^{N+1}

$$L = \sum_{i,j=1}^{p_0} a_{ij}(z) \partial_{x_i x_j}^2 + \sum_{i,j=1}^N b_{ij} x_i \partial_{x_j} - \partial_t$$

(here $z = (x, t)$). We will not recall here the structural assumptions on the matrix $B = (b_{ij})_{i,j=1}^N$. The matrix $A_0 = (a_{ij}(x))_{i,j=1}^{p_0}$ is a $p_0 \times p_0$ ($p_0 \leq N$) symmetric, bounded and uniformly positive definite matrix:

$$\frac{1}{\Lambda} |\xi|^2 \leq \sum_{i,j=1}^{p_0} a_{ij}(x) \xi_i \xi_j \leq \Lambda |\xi|^2 \quad (1)$$

for all $\xi \in \mathbb{R}^{p_0}$, $x \in \mathbb{R}^N$ (or $z \in \mathbb{R}^{N+1}$) and for some constant $\Lambda \geq 1$. The entries a_{ij} are assumed to satisfy a continuity condition which will be clarified in a moment. The main result in [1] is the following:

Theorem 0.1 (see [1, Thm. 1.1]) *For every $p \in (1, \infty)$ there exists a constant $c > 0$, depending on p, N, p_0 , the matrix B , the number Λ in (1) and the continuity modulus ω ,*

$$\omega(r) = \max_{i,j=1,\dots,p_0} \sup_{\substack{x,y \in \mathbb{R}^N \\ |x-y| \leq r}} |a_{ij}(x) - a_{ij}(y)|, \quad (2)$$

such that for every $u \in C_0^\infty(\mathbb{R}^N)$ one has:

$$\begin{aligned} \sum_{i,j=1}^{p_0} \left\| \partial_{x_i x_j}^2 u \right\|_{L^p(\mathbb{R}^N)} &\leq c \left\{ \| \mathcal{A}u \|_{L^p(\mathbb{R}^N)} + \| u \|_{L^p(\mathbb{R}^N)} \right\}, \\ \left\| \sum_{i,j=1}^N b_{ij} x_i \partial_{x_j} u \right\|_{L^p(\mathbb{R}^N)} &\leq c \left\{ \| \mathcal{A}u \|_{L^p(\mathbb{R}^N)} + \| u \|_{L^p(\mathbb{R}^N)} \right\}. \end{aligned}$$

This result is derived from an analogous L^p estimate holding for L on a strip $S_T = \mathbb{R}^N \times [-T, T]$:

Theorem 0.2 (see [1, Thm. 3.1]) *Let L be as above, with uniformly continuous coefficients a_{ij} . For every $p \in (1, \infty)$ there exist constants $c, T > 0$ depending on p, N, p_0 , the matrix B , the number Λ in (1), c also depending on the modulus of continuity ω*

$$\omega(r) = \max_{i,j=1,\dots,p_0} \sup_{\substack{z_1, z_2 \in \mathbb{R}^{N+1} \\ |z_1 - z_2| \leq r}} |a_{ij}(z_1) - a_{ij}(z_2)|, \quad (3)$$

such that

$$\sum_{i,j=1}^{p_0} \left\| \partial_{x_i x_j}^2 u \right\|_{L^p(S_T)} \leq c \left\{ \| Lu \|_{L^p(S_T)} + \| u \|_{L^p(S_T)} \right\}$$

for every $u \in C_0^\infty(S_T)$.

Now, in the statement of the above theorem we assumed the coefficients a_{ij} to be uniformly continuous in S_T in Euclidean sense. However, the assumption that we actually use in the proof of Proposition 3.2 (which implies the above theorem), is the global uniform continuity of the coefficients *with respect to the local quasidistance d* in \mathbb{R}^{N+1} which is introduced in the paper. Although the topology induced by d coincides with the Euclidean topology, so that continuity with respect to the two structures is the same thing, global uniform continuity is a different issue. In particular, global uniform continuity in Euclidean sense does not imply global uniform continuity with respect to d . Accordingly, also the assumption in the statement of [1, Thm. 1.1] must be corrected. This means that the coefficients $a_{ij}(x)$, which now are defined in \mathbb{R}^N , must be uniformly continuous with respect to d if they are regarded as defined in a strip S_T . Let us make precise the above corrections.

The definition (3) of the continuity modulus $\omega(r)$ (when the coefficients are defined in the strip S_T) must be changed to:

$$\omega_{S_T}(r) = \max_{i,j=1,\dots,p_0} \left\{ \sup |a_{ij}(z) - a_{ij}(\zeta)| : d(z, \zeta) < r, z, \zeta \in S_T \right\}.$$

The definition (2) of the continuity modulus $\omega(r)$ (when the coefficients are defined in \mathbb{R}^N) must be changed to:

$$\omega_{\mathbb{R}^N}(r) = \max_{i,j=1,\dots,p_0} \left\{ \sup |a_{ij}(x) - a_{ij}(y)| : \inf_{|s|<T, |t|<T} d((x, t), (y, s)) < r, x, y \in \mathbb{R}^N \right\} \quad (4)$$

(where $T > 0$ must be small enough). The global estimates proved in [1, Thm. 1.1, Thm. 3.1] hold, with the same proof, with the constant depending on these moduli.

We end with a remark and an example that should better enlighten the relation between uniform continuity in the two senses.

Remark 0.3 *In the case of time-independent coefficients, let us compare global uniform continuity in Euclidean sense with global uniform continuity w.r.t. $\omega_{\mathbb{R}^N}(r)$ in (4). Since*

$$\inf_{|s|<T, |t|<T} d((x, t), (y, s)) \leq d((x, 0), (y, 0)) = \sum_{i=1}^N |(x - y)_i|^{1/q_i}.$$

(where q_i are the positive integers defined in [1, p.1091]), for $r < 1$ we have

$$|x - y| < r \Rightarrow \inf_{|s|<T, |t|<T} d((x, t), (y, s)) < Nr^{1/q_N}$$

so that

$$\max_{i,j=1,\dots,p_0} \{\sup |a_{ij}(x) - a_{ij}(y)| : |x - y| < r\} \leq \omega_{\mathbb{R}^N}(Nr^{1/q_N}).$$

This shows that global uniform continuity w.r.t. $\omega_{\mathbb{R}^N}(t)$ implies global uniform continuity in Euclidean sense.

Example 0.4 *Let us consider the simplest example of degenerate KFP operator,*

$$\mathcal{L}u = u_{x_1x_1} + x_1u_{x_2} - u_t.$$

We have (keeping the notation in [1, p.1089])

$$E(s) = \begin{bmatrix} 1 & 0 \\ -s & 1 \end{bmatrix}$$

and, letting $y = (y_1, y_2)$,

$$E(s)y = (y_1, y_2 - sy_1).$$

Choosing, for $\varepsilon > 0$,

$$x = \left(\frac{1}{\varepsilon}, 1\right); y = \left(\frac{1}{\varepsilon}, 2\right); s = 0; t = \varepsilon$$

we have $(y, s) - (x, t) = (0, 1, -\varepsilon)$, so that $|(y, s) - (x, t)| \rightarrow 1$ as $\varepsilon \rightarrow 0$. On the other hand,

$$\begin{aligned} (y, s)^{-1} \circ (x, t) &= (x - E(t - s)y, t - s) = (x - E(\varepsilon)y, \varepsilon) \\ &= \left(\left(\frac{1}{\varepsilon}, 1\right) - \left(\frac{1}{\varepsilon}, 2 - \varepsilon\frac{1}{\varepsilon}\right), \varepsilon \right) = (0, 0, \varepsilon) \\ \left\| (y, s)^{-1} \circ (x, t) \right\| &= \|(0, 0, \varepsilon)\| = \sqrt{\varepsilon} \rightarrow 0. \end{aligned}$$

This shows that, if x, y are free to move in the whole \mathbb{R}^N ,

$$\left\| (y, s)^{-1} \circ (x, t) \right\| \rightarrow 0 \text{ does not imply } |(y, s) - (x, t)| \rightarrow 0.$$

For instance, let

$$a(x_1, x_2, t) = \sin\left(\frac{\pi}{2}x_2\right),$$

then a is uniformly continuous in \mathbb{R}^{N+1} , in Euclidean sense. Nevertheless, for (x_1, x_2, t) and (y_1, y_2, s) as above we have

$$d((x_1, x_2, t), (y_1, y_2, s)) = \sqrt{\varepsilon} \rightarrow 0$$

but

$$|a(x_1, x_2, t) - a(y_1, y_2, s)| = \left| \sin\left(\frac{\pi}{2}x_2\right) - \sin\left(\frac{\pi}{2}y_2\right) \right| = \left| \sin\left(\frac{\pi}{2}\right) - \sin\pi \right| = 1,$$

so that the function a is not uniformly continuous in any strip S_T , w.r.t. d .

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References

- [1] M. Bramanti, G. Cupini, E. Lanconelli, E. Priola: Global Lp estimates for degenerate Ornstein-Uhlenbeck operators with variable coefficients. *Mathematische Nachrichten*, Vol. 286, Issue 11-12 (2013), 1087–1101.

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