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On the influence of net head and efficiency fluctuations over the performance of existing run-of-river hydropower plants

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ABSTRACT

The power output over a typical year is a key feature to assess the performance of a run-of-river hydropower plant.

In carrying out this analysis, net head fluctuations due to changes in the processed flow rate are frequently neglected, while variations in the efficiency of the electro-mechanical generation group are more often taken into account.

To depict the combined effect of net head and efficiency fluctuations, the concept of Global Efficiency is introduced. Great emphasis is put on discharge conditions and an approximate method is put forward for correlating the residual energy in the discharge section with the processed flow rate.

The proposed approach is applied to the existing Cavaticcio run-of-river plant, set in the historical centre of Bologna (Italy). Results show that fluctuations of the residual energy at discharge may affect the net head value even more than upstream losses. Moreover, although assuming a constant net head could provide an acceptable estimation of the annual energy production, yet the Best Efficiency Point in terms of Global Efficiency may significantly differ from what expected from the generation group efficiency curve.

This may help decision makers to assess the actual suitability of an existing generation group for a given site.

1. Introduction

In the last decades, global awareness about drawbacks and limits connected with energy supply systems based on non-renewable energy sources has been increasing; as a result, renewable energy sources, namely energy sources which are replenished naturally on human timescales [1], are gaining importance as a sustainable and cleaner alternative [2].

Hydropower is, still today, one of the most reliable renewable energy sources, since the technology on which it is based is well-known and widely experienced [3].

Unlike dam-based plants, run-of-river hydropower plants do not usually have an impoundment, although sometimes they are equipped with a little storage capacity, which is nonetheless unable to achieve a seasonal adjustment of the streamflow regime [4]. Consequently, hydropower production from this kind of plants is less reliable and usually discontinuous, being directly influenced by the streamflow regime of the exploited natural water body. On the other hand, investment costs are usually much lower than those required by dam-based plants and environmental implications are usually softer [5].

The size of run-of-river plants is usually (but not necessarily) quite modest: installed capacity is often within the range 1 ÷ 50 MW, with some remarkable exceptions [1]; however, suitable sites are much more, and often they have not been exploited yet [6,7].

These motivations partly justify the renewed interest shown, both by private and public investors, in small run-of-river hydropower plants, whose number is increasing in many countries throughout the world, especially in Europe [8,9].

The design of new run-of-river plants has been widely discussed: some Authors have focused on the input hydrological data, showing that the streamflow distribution at the intake site should be carefully assessed in order to avoid sizing errors [10,11], while different approaches have been proposed for the optimal design of the plant [12]. Beside a few methods relying on an analytical framework [13], a comparison between different design alternatives is usually performed, taking into account several technical, economic and environmental aspects [14]. Optimization algorithms have been widely applied for finding the best solution according to single [1,15,16] or multiple

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objectives [17,18].

On the other hand, the problem of performance evaluation of existing plants is much less debated in the specialized literature [19]; nonetheless, this already relevant issue is likely to gain even more importance in the coming years. Average lifetime of small hydropower plants could in fact easily exceed fifty years with little maintenance [20]: this applies especially to the hydraulic components of the plant (weirs, sluice gates, canals, conduits), since they are mainly exposed to continuous steady-state operations without high temperatures and stresses [21]. Conversely, the payback period for the initial investment and the time span considered in the preliminary financial analysis, which sometimes have significant influence on the choice of the design parameters, are often reduced to a couple of decades [22]. Therefore, in the near future we may need to decide how to sustainably manage small hydropower plants which have reached the end of their nominal lifetime, but are still provided with mechanical equipment and civil infrastructures fully operable and almost new, as far as their own lifetime is concerned.

Two main features are often considered for deciding if it is worth investing in the renewal of a plant instead of decommissioning or radically changing it:

- the amount of electric energy which is expected to be produced over a typical year, which is the main income from the operation of the power station [23];
- the turbine efficiency curve, which is supposed to describe the suitability of the mechanical equipment for the installation site [1] and which is therefore often compared with the distribution of worked flow rates observed during the past nominal lifetime (the most persistent flow rates should be processed with the highest values of efficiency).

When such analyses are developed for low-head run-of-river plants, the influence exerted by downstream and upstream conditions over the hydroelectric production is frequently neglected or roughly considered, although these may significantly affect the head available for power generation, as pointed out by Ahn et al. [24].

A detailed approach is herein proposed, aimed at assessing the effects of fluctuations of the head losses and of the residual energy at the discharge section, induced by variations of the processed flow rate. In addition, the concept of Global Efficiency curve is introduced to portray the overall efficiency of the whole plant (and not only of the electromechanical equipment) according to the processed flow rate. This concept appears to be particularly significant when improvement interventions must be evaluated during the renewal phase of an existing hydroelectric plant.

A case study shows the limits of the traditional approach and exemplifies the feasibility of the proposed method, pointing out its advantages.

2. Theory and methods

Hydropower technology relies on the energy exchange between a fluid stream and a hydraulic motor, namely the turbine [20,25]. In run-of-river plants reaction turbines are more often installed; however, also impulse turbines have some applications, especially for micro-hydropower plants in remote areas [3,26].

The turbine is coupled with an electric generator, which converts the mechanical shaft power into electric power. Sometimes a gearbox can be interposed between the turbine and the generator [1], in order to increase the shaft rotational speed, thus making it easier the coupling of the hydraulic motor with the electric generator. In the following, the system composed by the turbine, the gearbox and the electric generator will be simply denoted as “generation group”.

The electric energy produced in a unit of time thanks to the interaction between a fluid stream and the generation group is called electric power output P_e and can be evaluated as:

$$P_e = \rho \cdot g \cdot Q_w \cdot H_{net} \cdot \eta_{group} \qquad \text{(W)} \tag{1}$$

where: ρ (kg/m^3) is the density of the fluid ($\approx$ 1000 kg/m^3 for water); g (m/s^2) is the gravitational acceleration ($\approx$ 9.81 m/s^2 as a conventional standard value); Q_w (m^3/s) is the worked flow rate, that is the volume flow rate processed by the turbine; H_{net} (m) is the net head, that is the difference between the total hydraulic head H_{in} in the inlet section of the turbine and the total hydraulic head H_{out} in the outlet section (these two sections should be properly defined by the manufacturer for each turbine); η_{group} (−) is the generation group efficiency.

The electric power output P_e is not generally constant over time, mainly due to the variability of the worked flow rate Q_w, which is often also responsible for a fluctuation of the net head H_{net} and of the efficiency η_{group} around their nominal values. This topic will be further developed in the following sub-paragraphs.

2.1. Net head fluctuations

As far as low-head run-of-river plants are concerned, a constant value for the net head is often considered in the power output computations [13], thus neglecting the flow-dependant variations of head losses and of the residual energy in the discharge section. Furthermore, in the preliminary stage, the net head is sometimes even confused with the gross head, possibly reduced by a constant head loss proportional to the distance between the intake and the powerhouse, in case a long diverting infrastructure has to be considered [18,27].

Existing hydropower plants are usually characterized through a limited number of key features, which are often the nameplate parameters for the installed turbine: therefore, a nominal net head H_{nom} is often considered as a constant reference net head value.

Nonetheless, the knowledge of the function $H_{net} = f(Q_w)$, correlating the worked flow rate with the net head, could sometimes offer valuable information about the operation of the plant.

An analytical framework to deduce an expression for this function is herein proposed.

To begin, a reaction turbine is considered (Fig. 1); the upstream total hydraulic head is denoted as H_1, while the hydraulic head at the beginning of the tailrace is denoted as H_2.

The gross head H_g is therefore defined as:

$$H_g = H_1 - H_2 \tag{2}$$

Notice that the gross head is often set equal to the difference in elevation between the upstream water level ($z_1 + h_1$) and the downstream water level ($z_2 + h_2$), thus ignoring the generally negligible contribution of the upstream and downstream kinetic heads [1].

The upstream head H_1 is usually maintained constant; nonetheless, the head H_{in} at the turbine inlet section varies according to the worked flow rate, due to head losses occurring between the intake and the turbine.

The (usually constant) value of H_1 can be connected to the flow-dependant value of H_{in} by means of the following expression:

$$H_{in}(Q_w) = H_1 - \Delta H_f^u(Q_w) - \Delta H_m^u(Q_w) \tag{3}$$

where $\Delta H_f^u(Q_w)$ represents the contribution of the distributed friction losses, while $\Delta H_m^u(Q_w)$ stands for the minor losses (owing to stream intake, bends, valves, trash racks).

The term $\Delta H_f^u(Q_w)$ can be evaluated by means of a hydraulic resistance formula, such as the Darcy-Weisbach equation:

$$\Delta H_f^u(Q_w) = \frac{\lambda}{D} \cdot \frac{Q_w^2}{2 \cdot g \cdot A^2} \cdot L \tag{4}$$

where L (m) is the length of the conduit, A (m^2) is its cross-sectional area, D (m) is its diameter and λ (−) is the friction factor, which

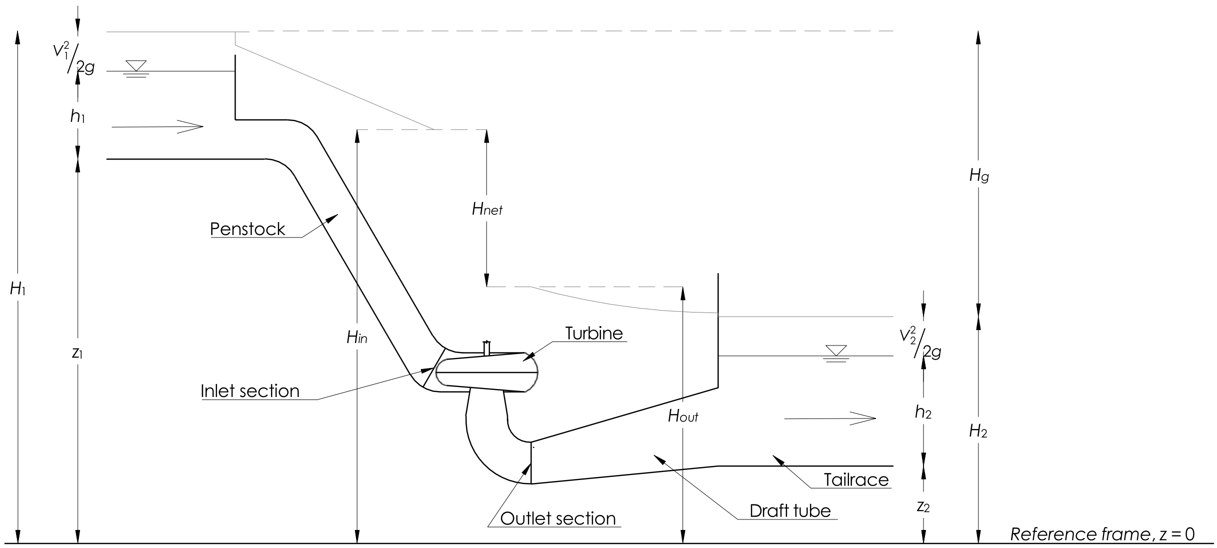

Fig. 1. Typical hydraulic scheme of a run-of-river hydropower plant in case a reaction turbine is installed. Kinetic heads and head losses have been voluntarily exaggerated for explanatory purpose.

depends on the Reynolds number and on the roughness of the conduit walls.

As for the minor losses $\Delta H_m^u(Q_w)$, they can be assumed proportional to the kinetic head:

$$\Delta H_m^u(Q_w) = \sum_{i=1}^{N} \xi_i \cdot \frac{Q_w^2}{2 \cdot g \cdot A_i^2} \tag{5}$$

where N is the overall number of minor losses along the path, while ξ_i is the minor loss coefficient associated to the i-th local resistance, whose value must be chosen according to the characteristics of the singularity responsible for the head loss.

Unlike the upstream total head H_1, the total head H_2 at the beginning of the tailrace is usually variable: in particular, it tends to increase when the processed flow rate increases, thus reducing the net head [28]. Although such an effect may have remarkable consequences on the plant performance, to our knowledge it is often neglected when dealing with run-of-river hydropower plants, as hydropower production models which take into account net head fluctuations usually only consider head losses upstream of the inlet section of the turbine [1,17].

The proper relation $H_2 = f(Q_w)$ depends on the site-specific discharge conditions: it should be obtained through *in situ* measurements, CFD or even physical modelling, possibly taking into account non-uniform outflow effects, which may arise due to the interaction between the swirling flow in the draft tube and the open channel flow at the tailrace [24].

When such an experimental approach cannot be carried out, a simplified analysis may be developed resorting to the rating curve for uniform flow in the tailrace. In fact, assuming uniform flow conditions in the discharge section, one can write:

$$H_2(Q_w) = z_2 + h_2(Q_w) + \frac{V_2^2}{2 \cdot g} \approx z_2 + h_u(Q_w) + \frac{Q_w^2}{2 \cdot g \cdot [A(h_u)]^2} \tag{6}$$

where z_2 is the elevation of the lowest point of the discharge section, while $h_u(Q_w)$ is the normal depth for the cross-section, which can be evaluated from Q_w thanks to the rating curve for uniform flow. Notice that, once the geometrical characteristics of the tailrace are known, the cross-sectional area of flow A can be evaluated as a proper function of h_u, $A = A(h_u)$.

If the outlet section of the turbine and the discharge section in the tailrace do not coincide (as shown in Fig. 1), the total hydraulic heads H_{out} and H_2 in these two sections can be put in relation as:

$$H_{out}(Q_w) = H_2(Q_w) + \Delta H_f^d(Q_w) + \Delta H_m^d(Q_w) \tag{7}$$

where $\Delta H_f^d(Q_w)$ and $\Delta H_m^d(Q_w)$ respectively stand for distributed friction head losses and minor head losses occurring along the conduits connecting the two above-mentioned sections (the draft tube is sometimes between these sections).

If an assessment of the vortex phenomena occurring at discharge is available, the related minor head losses can be included in $\Delta H_m^d(Q_w)$.

The function $H_{net} = f(Q_w)$ is finally obtained as:

$$H_{net}(Q_w) = H_{in}(Q_w) - H_{out}(Q_w) \tag{8}$$

The expression obtained for reaction turbines can be easily generalized to impulse turbines. The upstream total hydraulic head is still denoted as H_1, while z_2 now represents the height of the nozzle jet above the same reference as that one to which H_1 is referred.

The gross head H_g is defined as:

$$H_g = H_1 - z_2. \tag{9}$$

Also in this case, the upstream total hydraulic head H_1 is usually maintained constant, while the total hydraulic head H_{in} at the inlet section of the turbine varies according to the processed flow rate.

The value of H_{in} can be estimated through Eq. (3), as for reaction turbines.

What makes the biggest difference between impulse and reaction turbines is the interaction of the fluid stream with the runner, which takes place in the atmosphere for impulse turbines. Consequently, downstream conditions exert no influence on the operation of impulse turbines (unless the water level in the tailrace rises so as to interfere with the runner; however, this should not happen if the tailrace is properly designed). As a result, the net head can be evaluated as:

$$H_{net}(Q_w) = H_{in}(Q_w) - z_2. \tag{10}$$

2.2. Generation group efficiency fluctuations

The efficiency η_{group} of the generation group can be defined as the ratio of the electric power output P_e to the hydraulic power P_t yielded by the fluid stream between the inlet and the outlet section of the turbine:

$$\eta_{group} = \frac{P_e}{P_t} \tag{11}$$

where:

$$P_t = \rho \cdot g \cdot Q_w \cdot H_{net} \tag{12}$$

In order to make explicit the contributions of the different components of the generation group, it is useful to split η_{group} as:

$$\eta_{group} = \eta_{turb} \cdot \eta_{gearbox} \cdot \eta_{gen} \tag{13}$$

The turbine itself is mainly responsible for efficiency fluctuations, while the generator efficiency η_{gen} is usually assumed to be constant [1,14].

The turbine efficiency η_{turb} varies not only according to the worked flow rate Q_w, but also together with the net head H_{net} [29] and the rotational speed n (usually expressed in revolutions per minute) of the turbine [25,30].

The so-called hill chart is the most complete representation of the turbine behaviour in every possible operating condition; this is often drawn in a Cartesian plane having the unit value $n_1^{'}$ on the x axis and the unit value $Q_1^{'}$ on the y axis. It is worth recalling that unit values are defined as [25,30]:

$$n_1^{'} = \frac{n \cdot d}{\sqrt{H_{net}}} \tag{14a}$$

$$Q_1^{'} = \frac{Q_w}{d^2 \cdot \sqrt{H_{net}}} \tag{14b}$$

where the characteristic diameter d of the turbine also appears.

Once the operating conditions (namely Q_w, H_{net} and n) are given, the turbine efficiency η_{turb} can be deduced from the hill chart as:

$$\eta_{turb} = \eta_{turb}\left(n_1^{'}, Q_1^{'}\right) \tag{15}$$

The hill chart owes its name to the fact that it essentially consists in a contour plot of the function $\eta_{turb} = \eta_{turb}(n_1^{'}, Q_1^{'})$, whose shape recalls that one of a hill, which has the Best Efficiency Point (B.E.P.) at the top.

The rotational speed n is usually fixed by the coupling with the electric generator; when the net head H_{net} is constant too, an efficiency curve, like those presented in many papers [1,10,13,14,17,20], can be adopted: in this particular situation, in fact, all the operating points fall on the same vertical line (if n and H_{net} are constant, consequently $n_1^{'}$ is constant too). The hill chart can thus be replaced by a simple curve (the efficiency curve), which basically consists in a "cross section" of the hill chart obtained by means of a vertical section line.

Manufacturers seldom disclose the whole hill chart, while they more often provide the efficiency curve obtained for a constant net head corresponding to the nominal value. These curves are not usually expressed as $\eta_{turb} = \eta_{turb}(Q_1^{'})$. Rather, they are frequently represented in equivalent, but handier forms, like $\eta_{turb} = \eta_{turb}\left(\frac{Q_w}{Q_{B.E.P.}}\right)$ or $\eta_{turb} = \eta_{turb}\left(\frac{Q_w}{Q_{max}}\right)$, where $Q_{B.E.P.}$ is the flow rate corresponding to the Best Efficiency Point and Q_{max} is the maximum flow rate which can be processed by the turbine.

It can be argued that generally, as we said before, the net head is not constant indeed. However, it should be noticed that net head fluctuations around the nominal value should be quite narrow, since – if the turbine is properly designed – the nominal net head should be representative of the whole range of possible net head values (it could be the central value of the range). When these fluctuations are limited to a few percentage points around the nominal net head, while flow rate variations are far wider, it can be acceptable to use an efficiency curve instead of a complete hill chart, due to the prominent role played by flow rate variations. It should be noticed, in fact, that the unit values are a linear function of Q_w, while H_{net} contributes only with its square root.

Once the turbine efficiency η_{turb} has been determined, the group efficiency η_{group} can be obtained multiplying it by the gearbox efficiency (if a gearbox is present) and by the electric generator efficiency.

2.3. Global Efficiency curve of the plant

By analogy with the efficiency curve of the generation group, the novel concept of Global Efficiency curve is introduced, aimed at synthetically portraying the efficiency of the whole hydraulic-electrical energy transformation process, from the intake to the tailrace.

Let us define "nominal theoretical power" $P_n(Q_w)$ the following flow-related quantity:

$$P_n(Q_w) = \rho \cdot g \cdot Q_w \cdot H_{nom} \tag{16}$$

where H_{nom} is the nominal net head for the plant (see §2.1).

This can be interpreted as the power yielded to the generation group by a fluid stream having a volume flow rate Q_w in a theoretical condition in which the net head corresponds to the nominal value.

Let us suppose that an efficiency curve of the generation group $\eta_{group} = \eta_{group}(Q_w)$ can be used instead of a hill chart and let us assume that such a curve has been derived considering a constant net head value corresponding to the nominal value H_{nom}.

The electric power output P_e can be therefore put in relation with the worked flow rate Q_w as:

$$P_e(Q_w) \approx \rho \cdot g \cdot Q_w \cdot H_{net}(Q_w) \cdot \eta_{group}(Q_w) \tag{17}$$

We define as Global Efficiency of the plant, corresponding to the worked flow rate Q_w, the following ratio:

$$\eta_{glob}(Q_w) = \frac{P_e(Q_w)}{P_n(Q_w)} \tag{18}$$

which also takes into account the power dissipated by head losses and the power "lost", as retained by the discharged flow.

We can re-write the expression for $\eta_{glob}(Q_w)$ as:

$$\eta_{glob}(Q_w) = \frac{H_{net}(Q_w)}{H_{nom}} \cdot \eta_{group}(Q_w) \tag{19}$$

which differs from the definition of "plant hydraulic efficiency" given by Ahn et al. [24] in the presence of the nominal net head instead of the gross head: as stated before, for existing plants the nominal net head value is a commonly available piece of information.

According to Eq. (19), the Global Efficiency of the plant can be expressed as the product of the group efficiency η_{group} times a flow-dependant coefficient, which may increase the value of η_{group} (when $H_{net}(Q_w) > H_{nom}$) or it may decrease it. This can be explained recalling the physical meaning of the total hydraulic head: for a given flow rate Q_w, if $\eta_{glob}(Q_w) > \eta_{group}(Q_w)$ this means that the energy per unit of weight yielded in a unit of time by the fluid stream to the generation group (i.e., $H_{net}(Q_w)$) is higher than the nominal value (i.e., H_{nom}) and therefore an electric power output higher than that one in the nominal head conditions can be harnessed.

Notice that in case $H_{net}(Q_w) = \text{const.} = H_{nom}$, then $\eta_{glob}(Q_w) = \eta_{group}(Q_w)$, i.e. only the generation group is responsible for efficiency fluctuations, as expected.

It is worth pointing out that, at least theoretically, the Global Efficiency of the plant could be higher than 1.0. According to Eq. (18), in fact, $\eta_{glob}(Q_w) > 1.0$ implies $P_e(Q_w) > P_n(Q_w)$: for a given flow rate, such a situation occurs when the electric energy per unit of weight of the fluid produced in a unit of time (which is given by $H_{net}(Q_w) \cdot \eta_{group}(Q_w)$) is higher than the energy yielded, in a unit of time per unit of weight, by the fluid stream to the generation group in the nominal conditions (which is given by H_{nom}). However, this should not happen if H_{nom} is properly defined, i.e. if it is not too far from the actual net head values.

2.4. Computing the electric energy production

The mean daily electric power output from a run-of-river plant is usually changeable over a year: the mean daily flow rate processed in a day, in fact, usually varies within a range [Q_{min}; Q_{max}], according to the natural resource availability. To be precise, the processed flow rate often exhibits even intraday fluctuations. However, in the following the latter will be neglected and the flow rate processed in a day will be supposed to be constant and therefore equal to the mean daily value, which is a common assumption [1].

The extreme values Q_{min} and Q_{max} mainly depend on the size and on the technical characteristics of the turbine [14], but, to our experience, also prescriptions imposed by the abstraction licence or the hydraulic characteristics of the leat or operational rules may have an influence.

Let us suppose a sequence $\{Q_{w,1}, \ldots, Q_{w,365}\}$ of the mean daily flow rates workable by the plant over a year to be known. Let us assume, in addition, that the plant is provided with a unique turbine.

The electric energy E obtainable over the whole year can be evaluated as:

$$E = \frac{1}{1000}\sum_{i=1}^{365}\left[24 \cdot \rho \cdot g \cdot Q_{w,i} \cdot H_{net}\left(Q_{w,i}\right) \cdot \eta_{group}\left(Q_{w,i}\right)\right] \tag{20}$$

whose result is expressed in kWh, provided that the fluid density ρ is expressed in kg/m^3, the gravitational acceleration g in m/s^2, the net head H_{net} in m and the workable flow rate $Q_{w,i}$ in m^3/s.

3. Case study

The amplitude of net head and efficiency fluctuations due to changes in the worked flow rate and their impact on the energy production have been analysed for a specific hydropower plant, in order to test the effectiveness of the proposed method.

The "Cavaticcio" run-of-river plant is set in the city centre of Bologna (Italy) and it was put into operation in 1994 in order to exploit for hydropower purpose a difference in altitude existing along the bed of an artificial canal.

The hydropower plant has a nominal maximum capacity of about 1.8 MW; it is equipped with a single axial turbine endowed with adjustable blades and coupled with the electric generator by means of a speed increaser. The range of operable flow rates spans from 3.00 m^3/s to 15.00 m^3/s and, according to the group efficiency curve, the best performances are reached for a flow rate corresponding to 10.50 m^3/s [31,32]. The main characteristics are summarized in Table 1.

The Cavaticcio plant is powered by the water diverted from the Reno river and then routed to the powerhouse thanks to a medieval canal more than 6 km long, called "Canale di Reno".

Recently, a general overhaul of the plant turned out to be necessary, in order to renew obsolete or damaged components and to fix some operational problems which had emerged during the first 25 years of its life.

In fact, in spite of the maximum flow rate that can be processed by the turbine (15.00 m^3/s), the present hydraulic and geometric characteristics of the Canale di Reno prevent more than 9.00 m^3/s from being delivered, so the turbine can never work at full load. Doubts consequently arose on the effectiveness of maintaining the actual plant configuration: the operation at low discharges for long periods, in fact, was suspected of being slightly unprofitable, due to a worsening in the efficiency associated to low flow rates according to the group efficiency curve (see §3.2).

An accurate analysis through our detailed approach provided a wider picture of the Cavaticcio plant potentialities: in the following subsections the most interesting aspects are discussed.

Table 1
Main characteristics of the Cavaticcio run-of-river plant.

Characteristic	Symbol	Value
Maximum operable flow rate	Q_{max}	15.00 m^3/s
Minimum operable flow rate	Q_{min}	3.00 m^3/s
Best efficiency flow rate	$Q_{B.E.P.}$	10.50 m^3/s
Upstream total hydraulic head (maintained constant)	H_1	50.50 m a.m.s.l.
Elevation of the lowest point of the discharge section	z_2	34.53 m a.m.s.l.
Nominal net head	$H_{nom.}$	14.30 m
Turbine nominal rotational speed	n_t	290 min^{-1}
Generator nominal rotational speed	n_g	750 min^{-1}
Installed capacity	$P_{inst.}$	≈1800 kW

3.1. Analysis of net head fluctuations

In the original project of the Cavaticcio plant the nominal net head H_{nom} (14.30 m) was considered for the evaluation of the electric energy output (Table 1).

The Cavaticcio plant has a rather unconventional layout (Fig. 2), since the designers had to fulfil many urban planning constraints due to the exceptional location in the city centre.

The plant, which is mostly underground, is located more than 150 m from the Canale di Reno, which provides water for the energy production. In order to make the abstraction easier, the canal is barred by means of a weir, thus forcing the water level to rise so as to completely submerge the inlet of the pressurized conduit which drives the water to the turbine.

When the plant is operating, a remote control system automatically adjusts the opening of the turbine distributor and, if necessary, that one of an auxiliary drainpipe, in order to maintain a constant water level of 50.50 m a.m.s.l. at the intake; since the flow speed is therein extremely low and, moreover, water abstraction is carried out orthogonally to the flow main direction, we assumed that the water level and the total hydraulic head at the intake could be confused, so, for the subsequent analysis, we considered a constant upstream total hydraulic head H_1 of 50.50 m a.m.s.l.

Water available for the plant is brought near the powerhouse through a pressurized concrete conduit about 158 m long. Once the geometric features of the conduit were known, the friction head losses along the path $\Delta H_{f,cond.}$ were correlated to the flow rate.

Minor losses $\Delta H_{m,cond.}$ along this conduit occur mainly due to the curve-shaped intake, due to a totally opened gate valve and because of two bends; although these losses are very small, they were anyway taken into account, using Eq. (5) and basing the choice of the coefficients ξ_i on site inspections and indications found in the specialized literature [33,34].

At the end of the concrete conduit, water is introduced in a welded sheet-metal pressurized pipe which drives the stream directly to the inlet section of the turbine through a steep path: such a pipe can be therefore considered as a sort of penstock. Despite its short length (11.22 m, measured along its axis), the head losses $\Delta H_{f,pen}$ and $\Delta H_{m,pen}$ which occur in this pipe are of the same order of magnitude as those which take place in the concrete upstream conduit, since the cross-sectional area of this "penstock" is much smaller.

The analytical expression correlating the total hydraulic head H_{in} at the inlet section of the turbine with the worked flow Q_w finally reads:

$$H_{in}(Q_w) = H_1 - \Delta H_{f,cond.}(Q_w) - \Delta H_{m,cond.}(Q_w) - \Delta H_{f,pen}(Q_w) - \Delta H_{m,pen}(Q_w) \tag{21}$$

The outlet section of the turbine of the Cavaticcio plant is set almost at the end of the draft tube (Fig. 2), therefore head losses occurring in the diffuser are implicitly taken into account in the efficiency curve of the generation group. The outlet section of the turbine and the discharge section in the tailrace are so close to each other that the total hydraulic head in these two sections was confused, assuming $H_{out}(Q_w) \approx H_2(Q_w)$.

The water processed by the turbine is discharged, through the draft tube, in a closed conduit where it flows under free-surface conditions. This conduit has quite a constant slope of 1 ‰; the shape of its generic

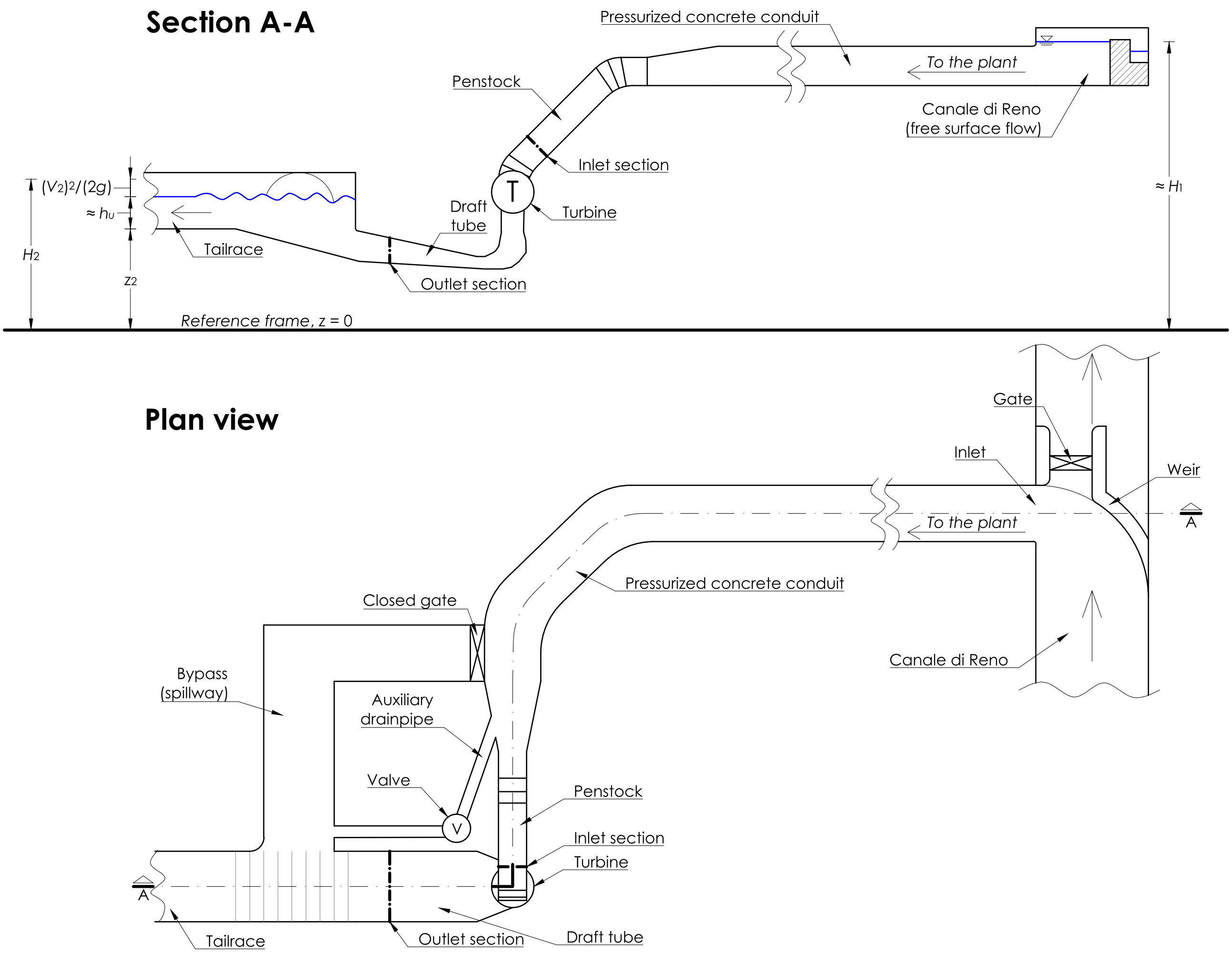

Fig. 2. Principle scheme of the Cavaticcio plant: vertical section (above) and plan view (below). Proportions and details may have not been preserved for explanatory purpose.

cross-section is shown in Fig. 3a and the rating curve for uniform flow is reported in Fig. 3b in a dimensionless form.

A qualitative analysis of the backwater curves and of the profiles expected for the specific energy along the conduit led us to rate as acceptable (though simplified) an evaluation of the net head in the discharge section $H_2(Q_w)$ using Eq. (6), which would strictly apply to uniform flow conditions; therefore, it was finally set:

$$H_{out}(Q_w) \approx H_2(Q_w) \approx z_2 + h_u(Q_w) + \frac{Q_w^2}{2 \cdot g \cdot [A(h_u)]^2} \tag{22}$$

where a proper value for h_u can be assigned to every (admissible) discharge Q_w by means of the rating curve shown in Fig. 3b.

Under Eq. (8), the net head value H_{net} corresponding to any given flow rate Q_w processed by the turbine is finally given by:

$$H_{net}(Q_w) = H_1 - z_2 - \Delta H_{f,cond.}(Q_w) - \Delta H_{m,cond.}(Q_w) - \Delta H_{f,pen.}(Q_w) - \Delta H_{m,pen}(Q_w) - h_u(Q_w) - \frac{Q_w^2}{2 \cdot g \cdot [A(h_u)]^2} \tag{23}$$

According to Eq. (23), the net head value is affected by three main terms. The first one, $\Delta H_0 = H_1 - z_2$, is a constant contribution, while the other two are flow dependant: the term ΔH_{upstr} (see Eq. (25a)) is related to the effect of upstream head losses, while the term $\Delta H_{downstr.}$ (see Eq. (25b)) describes the fluctuations of the residual specific energy in the discharge section.

In order to detect the relative weight of the two flow-related terms, the following indicators $\delta_{upstr.}$ and $\delta_{downstr.}$ were introduced:

$$\delta_{upstr.} = \frac{\Delta H_{upstr.}}{H_1 - z_2} \tag{24a}$$

$$\delta_{downstr.} = \frac{\Delta H_{downstr.}}{H_1 - z_2} \tag{24b}$$

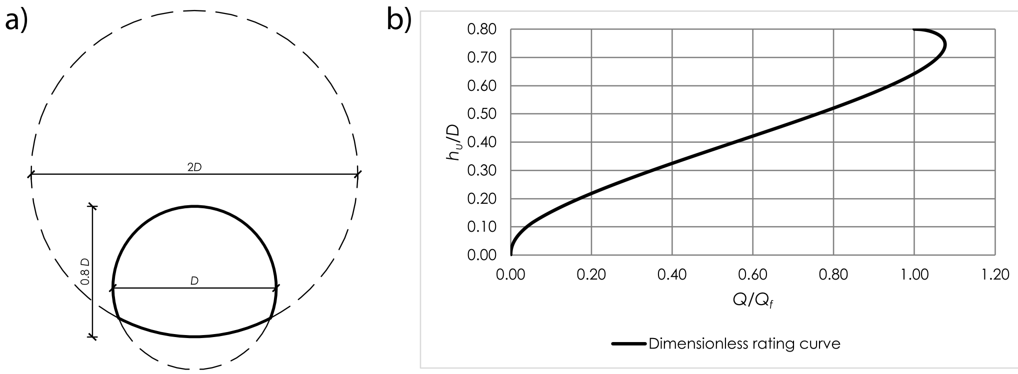

Fig. 3. (a) Shape of the generic cross-section of the tailrace ("Vigentino" shape). Notice that the height of the cross-section (and consequently the maximum admissible water level) corresponds to 0.8*D*, where *D* stands for the diameter of the top vault, which is also equal to the section width. (b) Rating curve for uniform flow for the tailrace, expressed in dimensionless form; Q_f stands for the discharge flowing in uniform flow conditions when the conduit is fully filled, while h_u is the normal depth corresponding to the discharge *Q*.

where:

$$\Delta H_{upstr.} = \Delta H_{f,cond.}(Q_w) + \Delta H_{m,cond.}(Q_w) + \Delta H_{f,pen.}(Q_w) + \Delta H_{m,pen}(Q_w) \tag{25a}$$

$$\Delta H_{downstr.} = h_u(Q_w) + \frac{Q_w^2}{2 \cdot g \cdot [A(h_u)]^2} \tag{25b}$$

The results are summarized in Fig. 4 and in Table 2.

Interestingly, the function $H_{net} = f(Q_w)$ exhibits an almost linear trend: this can be explained considering that, in our case study, the functions $\Delta H_{upstr.}(Q_w)$ and $\Delta H_{downstr.}(Q_w)$ are characterized by almost opposite concavities, which tend to compensate each other.

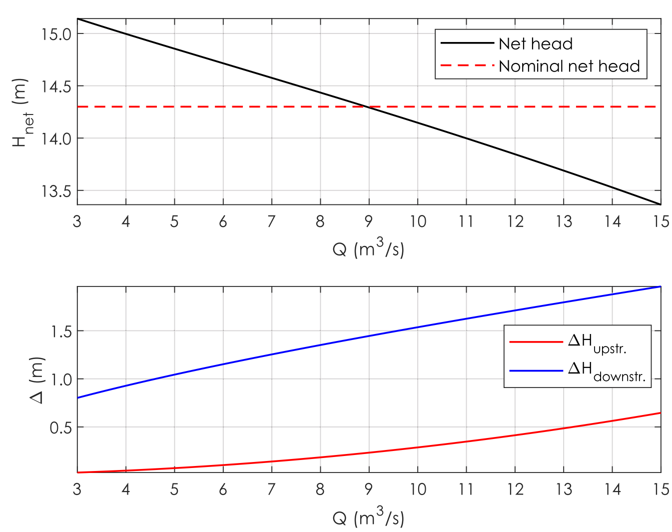

Fig. 4. Above: trend of the function $H_{net}(Q_w)$ deduced for the Cavaticcio plant; below: trend of the functions $\Delta H_{upstr.}(Q_w)$ and $\Delta H_{downstr.}(Q_w)$ for the same plant.

Table 2
Value of the indicators $\delta_{upstr.}$ and $\delta_{downstr.}$, defined within the text.

Q_w (m^3/s)	$\delta_{upstr.}$ (–)	$\delta_{downstr.}$ (–)
3.00	0.002	0.050
4.00	0.003	0.058
5.00	0.004	0.065
6.00	0.006	0.072
7.00	0.009	0.079
8.00	0.012	0.085
9.00	0.015	0.091
10.00	0.018	0.096
11.00	0.022	0.102
12.00	0.026	0.107
13.00	0.030	0.112
14.00	0.035	0.118
15.00	0.040	0.123

Net head fluctuations appear to be quite significant: we have in fact estimated a net head of more than 15.10 m corresponding to the minimum workable flow rate (3 m^3/s) and a net head of almost 13.40 m corresponding to the maximum flow rate (15 m^3/s). According to our analysis, the nominal value of the net head (14.30 m) is reached for a worked flow rate of nearly 9 m^3/s.

Net head fluctuations quite symmetrically spread around the nominal value; their amplitude is always lower than 7 % of the nominal net head value: consequently, the use of an efficiency curve instead of a hill chart was admitted for the evaluation of the generation group efficiency.

It is worth pointing out that downstream conditions affect the net head value much more than upstream head losses. Moreover, downstream specific energy fluctuations are wider than those observed for upstream head losses: in fact, considering the operable flow range 3 ÷ 15 m^3/s, upstream head losses increase by nearly 0.6 m (from 0.03 m to 0.65 m), while the residual specific energy at discharge increases by almost 1.2 m (from 0.80 m to 1.96 m).

3.2. Generation group efficiency evaluation

A curve was provided by the manufacturer, describing the trend of the efficiency η_{group} according to the worked flow rate for the group composed by the turbine, the speed increaser, the electric generator and the transformer. The curve was derived under the hypothesis of a constant net head, equal to the nominal value (14.30 m): although such an assumption does not exactly correspond to the real operating conditions, we rated the use of the curve acceptable, since net head fluctuations around the nominal value are limited to a few percentage points, as we have already pointed out.

The given efficiency curve was traced in a Cartesian plane $Q_{10} \div \eta_{group}$, where Q_{10} is defined as:

$$Q_{10} = \frac{Q_w}{Q_{max}} \cdot 10 \tag{26}$$

No analytical expression was available for the function $\eta_{group}(Q_{10})$, so we sampled the given curve according to a regular step and then we approximated the resulting *m* points through a proper functional model.

In [21] a second-degree polynomial is used as approximant function, leading to satisfactory results for Pelton, Francis and axial turbines. However, we thought that a higher degree polynomial was likely to be necessary to properly approximate the efficiency curve of a bi-regulating axial turbine, as that one we were studying, since the efficiency curve for these turbines can be interpreted as the envelope of the efficiency curves corresponding to the possible orientations of the blades. Moreover, the curve we were looking at approximating provided an overall description of the group efficiency, given by the product of the efficiencies of the turbine, of the gearbox, of the generator and of the transformer: this strengthened our convincement about the appropriateness of resorting to a higher degree polynomial.

The sampling points were therefore approximated using different polynomials of increasing degree, starting from the second degree and stopping once a satisfying result was achieved.

The polynomial coefficients were evaluated according to the least squares method, using the MATLAB® built-in function `polyfit`, which solves the solution system thanks to a QR factorization [35].

For each tested polynomial approximation, in every i-th sampling point, the relative error ε_i between the actual efficiency value η_i and the approximated value $\eta_{i,appr.}$ was evaluated as:

$$\varepsilon_i = \frac{\left|\eta_{i,appr.} - \eta_i\right|}{\min\left(\left|\eta_{i,appr.}\right|; \left|\eta_i\right|\right)} \tag{27}$$

The resulting values were collected, for each approximation, in a m-vector $\boldsymbol{\varepsilon}$, whose infinity norm

$$||\boldsymbol{\varepsilon}||_\infty = \max_{i=1,\dots,m} |\varepsilon_i| \tag{28}$$

was computed.

As acceptance criterion, a relative error not exceeding 0.5 % was considered, i.e.:

$$||\varepsilon||_\infty \leq 0.005 \tag{29}$$

As it appears from Fig. 5, the approximation accuracy rapidly increased with the degree of the polynomial: in particular, a fourth-degree polynomial fulfilled the acceptance criterion (29) and proved excellent fit to the sampling points.

In conclusion, the worked flow was correlated with the corresponding overall efficiency by means of the following analytical expression:

$$\eta_{group} = -0.0128 \cdot Q_{10}^4 + 0.3729 \cdot Q_{10}^3 - 4.1245 \cdot Q_{10}^2 + 20.6729 \cdot Q_{10} + 46.9818 \tag{30}$$

3.3. Worked flow rate characterization

The distribution of workable flow rates for the Cavaticcio plant over a typical year has been described through a median annual flow-duration curve [36,37].

The curve was elaborated starting from a mean daily streamflow time series spanning from 1997 to 2013, which was made available by

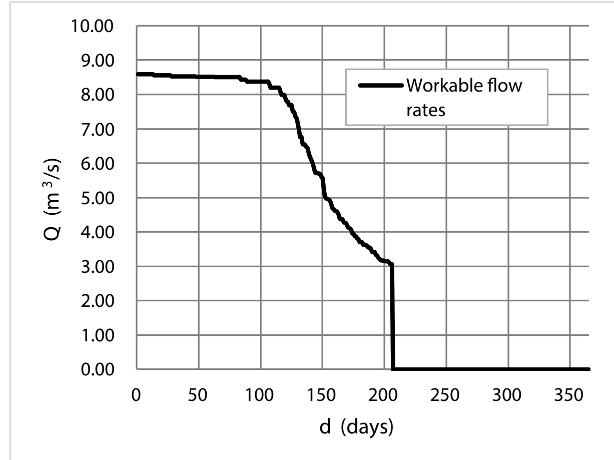

Fig. 6. Flow-duration curve describing the distribution of the flow rates workable by the turbine over a median year.

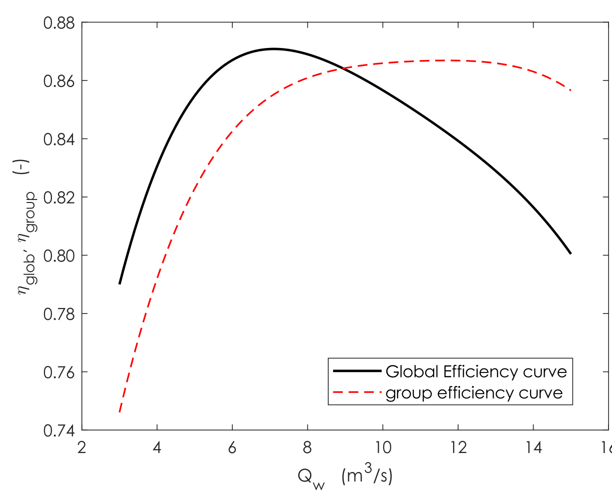

Fig. 7. Generation group efficiency curve $\eta_{group}(Q_w)$ and Global Efficiency curve $\eta_{glob}(Q_w)$ for the Cavaticcio plant.

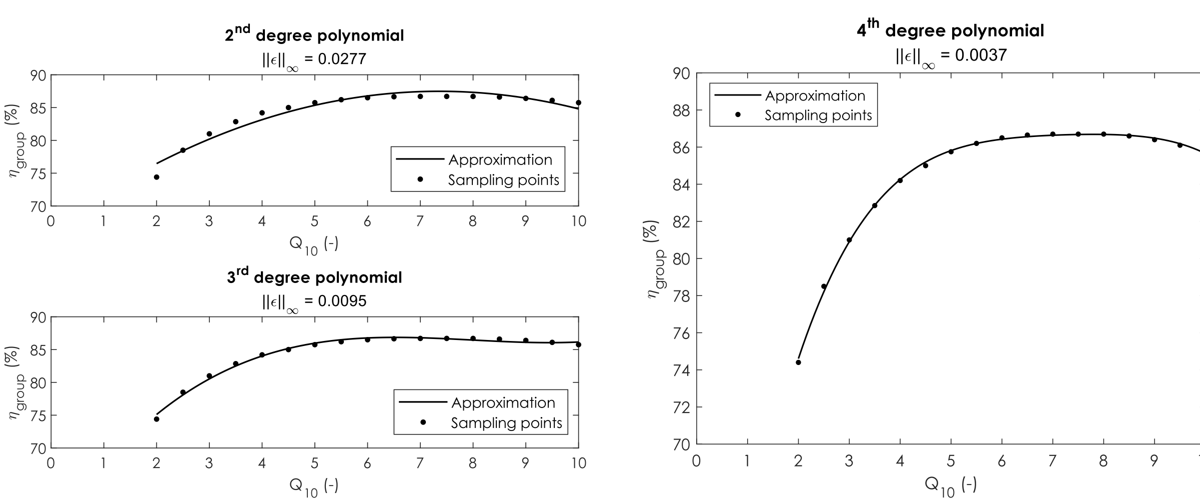

Fig. 5. Approximation of the group efficiency curve with different degree polynomials.

Table 3
Simulation plans considered for the evaluation of the electric energy revenue expected over a year from the Cavaticcio plant.

Plan	Net head H_{net}	Group efficiency η_{group}	Electric energy E
1	Variable	Variable	4152 MWh
2	Constant	Variable	4098 MWh
3	Variable	Constant	4088 MWh
4	Constant	Constant	4031 MWh

the regional Meteorological and Hydrological Service (ARPAE-SIMC) for a cross-section of the Reno river located close to the intake of the Canale di Reno; the hydraulic and geometric characteristics of the canal, the environmental flow requirements and the upstream abstractions were subsequently taken into account, in order to estimate the flow rate routed to the plant and therefore available for hydropower production; the plant operable flow range [Q_{min}; Q_{max}] and the operating rules were finally considered to define the actual turbine inflow [1,14].

The resulting curve is reported in Fig. 6. It is worth noticing that the curve is bounded above as a consequence of the Canale di Reno characteristics, which currently prevent more than 9 m^3/s from being delivered; on the other hand, the tail of the curve has been cut off according to the plant minimum operable flow rate ($Q_{min} = 3$ m^3/s).

3.4. Results and discussion

Starting from the analytical expressions correlating the net head value H_{net} and the generation group efficiency η_{group} to the worked flow rate Q_w, the Global Efficiency curve of the plant was obtained through the procedure described in §2.3.

The shape of the resulting curve is shown in Fig. 7; in the same figure, also the generation group efficiency curve is presented for comparison. As expected, the two curves intersect for a flow rate Q_w of nearly 9 m^3/s, whose associated net head value corresponds to the nominal value (see §3.1).

It is worth pointing out that the trend shown by $\eta_{glob}(Q_w)$ is much steeper than that one exhibited by $\eta_{group}(Q_w)$; moreover, the Best Efficiency Point moves leftwards, shifting from almost 10.5 m^3/s considering $\eta_{group}(Q_w)$ to nearly 7.0 m^3/s considering $\eta_{glob}(Q_w)$.

We would like to emphasize that basing the analysis of the system behaviour on the Global Efficiency curve rather than on the "conventional" generation group efficiency curve allows to better grasp the plant potentialities in the whole range of workable flow rates. In particular, as far as this case study is concerned, low flow operation appears less unprofitable than what it could be expected looking only at the group efficiency curve, while the Global Efficiency associated to higher flow rates dramatically drops.

This was a valuable result of our analysis, because it has shown, rather unexpectedly, that the turbine which is now installed in the Cavaticcio plant is quite suitable for the current operating conditions: in fact, the drop in the group efficiency corresponding to low flow rates is compensated by a significant increase in the net head. Moreover, considering the present water availability (Fig. 6), the power station even appears to operate around the Best Efficiency Point in terms of Global Efficiency.

A last analysis was developed in order to point out the effects of net head and efficiency fluctuations according to flow rate on the evaluation of the electric energy revenue over a typical year. The numerical solution method proposed in §2.4 was adopted and the flow-duration curve described in §3.3 was used to portray the distribution of workable flow rates expected in a typical year.

The computations were repeated four times, considering the net head and the group efficiency either flow dependant or constant, according to the simulation scenarios presented in Table 3.

The functions $H_{net}(Q_w)$ and $\eta_{group}(Q_w)$, introduced in §3.1 and §3.2, were used to describe the net head and the efficiency fluctuations. As a constant net head value, the nominal net head H_{nom} (14.30 m) was assumed, while the constant value for the group efficiency was set equal to 83.6 %, which is the average efficiency guaranteed by the manufacturer.

Differences turned out to be negligible between all cases. Considering Plans 1 and 2, it emerges that, assuming a constant net head value, the energy annually produced is slightly reduced. Notice that this result is consistent with the shape of the flow-duration curve shown in Fig. 6: according to this curve, in fact, the plant should operate for almost half of the time processing around 9 m^3/s, to which a net head corresponding to the nominal value is associated, while for the rest of the time the plant operates with lower flow rates, which are related to net head values higher than the nominal one.

This evidence confirms that a good assessment of the expected energy revenue over a year can be achieved even considering a constant net head value, provided that this is properly chosen.

Comparing Plans 1 and 3, a little reduction in the electric energy output is detected due to the imposition of a constant (but reasonable) value for the generation group efficiency. However, it is worth pointing out that, in our example, neglecting net head fluctuations leads to an error in the assessment of the expected energy output of nearly the same extent as that one which arises ignoring variations in the efficiency. This is quite interesting, since, in the common practice, neglecting net head fluctuations is often implicitly considered more acceptable than assuming a constant value for the generation group efficiency [13,38]: as we have shown, this should not be given for granted when the plant layout is not simple.

4. Conclusions

An assessment of the performances of an existing run-of-river plant focusing only on the turbine and on the mechanical equipment could be misleading, since it could offer a partial representation of the plant potentialities. Sometimes, in fact, head losses which occur along the pipes and conduits driving water to the turbine, together with downstream discharge conditions, may exert strong influence on the net head value.

As far as the evaluation of the expected energy revenue over a typical year is concerned, the assumption of a constant net head could lead to an acceptable result, provided that the net head value is chosen properly. Nonetheless, an accurate analysis of net head variations according to the worked flow rate could offer valuable information to cast light on the effectiveness of the plant configuration, especially in case of low-head run-of-river plants.

To detect the combined effect of net head and efficiency fluctuations we have introduced a "Global Efficiency", defined as the product of the generation group efficiency times a proper coefficient, whose effect can be either improving or not, depending on the actual net head value.

When such a detailed analysis is carried out, of course it is important to carefully account for head losses which occur upstream of the inlet section of the turbine, but, as far as reaction turbines are concerned, it is crucial not to neglect fluctuations of the residual energy at the discharge section, since they may be even more remarkable than fluctuations connected with upstream head losses.

CRediT authorship contribution statement

Cristiana Bragalli: Conceptualization, Methodology, Formal analysis, Investigation, and Writing. **Domenico Micocci:** Conceptualization, Methodology, Formal analysis, Investigation, and Writing. **Giovanni Naldi:** Conceptualization, Methodology, Formal analysis, Investigation, and Writing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] V. Yildiz, J.A. Vrugt, A toolbox for the optimal design of run-of-river hydropower plants, Environ. Model. Software 111 (2019) 134–152, https://doi.org/10.1016/j.envsoft.2018.08.018.

[2] A. Wasti, P. Ray, S. Wi, C. Folch, M. Ubierna, P. Karki, Climate change and the hydropower sector: a global review, WIREs Clim. Change 13 (2022) e757, https://doi.org/10.1002/wcc.757.

[3] A.H. Elbatran, O.B. Yaakob, Y.M. Ahmed, H.M. Shabara, Operation, performance and economic analysis of low head micro-hydropower turbines for rural and remote areas: a review, Renew. Sustain. Energy Rev. 43 (2015) 40–50, https://doi.org/10.1016/j.rser.2014.11.045.

[4] D. Egré, J.C. Milewski, The diversity of hydropower projects, Energy Pol. 30 (2002) 1225–1230, https://doi.org/10.1016/S0301-4215(02)00083-6.

[5] D. Magaju, A. Cattapan, M. Franca, Identification of run-of-river hydropower investments in data scarce regions using global data, Energy Sustain. Dev. 58 (2020) 30–41, https://doi.org/10.1016/j.esd.2020.07.001.

[6] D.E.H.J. Gernaat, P.W. Bogaart, D.P. van Vuuren, H. Biemans, R. Niessink, High-resolution assessment of global technical and economic hydropower potential, Nat. Energy 2 (2017) 821–828, https://doi.org/10.1038/s41560-017-0006-y.

[7] W.M. Tefera, K.S. Kasiviswanathan, A global-scale hydropower potential assessment and feasibility evaluations, Water Resour. Econ. 38 (2022), 100198, https://doi.org/10.1016/j.wre.2022.100198.

[8] T.B. Couto, J.D. Olden, Global proliferation of small hydropower plants – science and policy, Front. Ecol. Environ. 16 (2018) 91–100, https://doi.org/10.1002/fee.1746.

[9] E. Quaranta, K. Bódis, E. Kasiulis, A. McNabola, A. Pistocchi, Is there a residual and hidden potential for small and micro hydropower in Europe? A screening-level regional assessment, Water Resour. Manag. 36 (2022) 1745–1762, https://doi.org/10.1007/s11269-022-03084-6.

[10] L. Barelli, L. Liucci, A. Ottaviano, D. Valigi, Mini-hydro: a design approach in case of torrential rivers, Energy 58 (2013) 695–706, https://doi.org/10.1016/j.energy.2013.06.038.

[11] I.A. Niadas, P.G. Mentzelopoulos, Probabilistic flow duration curves for small hydro plant design and performance evaluation, Water Resour. Manag. 22 (2008) 509–523, https://doi.org/10.1007/s11269-007-9175-y.

[12] C. Sasthav, G. Oladosu, Environmental design of low-head run-of-river hydropower in the United States: a review of facility design models, Renew. Sustain. Energy Rev. 160 (2022), 112312, https://doi.org/10.1016/j.rser.2022.112312.

[13] S. Basso, G. Botter, Streamflow variability and optimal capacity of run-of-river hydropower plants, Water Resour. Res. 48 (2012), https://doi.org/10.1029/2012WR012017.

[14] A. Santolin, G. Cavazzini, G. Pavesi, G. Ardizzon, A. Rossetti, Techno-economical method for the capacity sizing of a small hydropower plant, Energy Convers. Manag. 52 (2011) 2533–2541, https://doi.org/10.1016/j.enconman.2011.01.001.

[15] M. Ibrahim, Y. Imam, A. Ghanem, Optimal planning and design of run-of-river hydroelectric power projects, Renew. Energy 141 (2019) 858–873, https://doi.org/10.1016/j.renene.2019.04.009.

[16] M. Nedaei, P.R. Walsh, Technical performance evaluation and optimization of a run-of-river hydropower facility, Renew. Energy 182 (2022) 343–362, https://doi.org/10.1016/j.renene.2021.10.021.

[17] J.S. Anagnostopoulos, D.E. Papantonis, Optimal sizing of a run-of-river small hydropower plant, Energy Convers. Manag. 48 (2007) 2663–2670, https://doi.org/10.1016/j.enconman.2007.04.016.

[18] S. Basso, G. Lazzaro, M. Bovo, C. Soulsby, G. Botter, Water-energy-ecosystem nexus in small run-of-river hydropower: optimal design and policy, Appl. Energy 280 (2020), 115936, https://doi.org/10.1016/j.apenergy.2020.115936.

[19] Y. Liu, L. Ye, I. Benoit, X. Liu, Y. Cheng, G. Morel, C. Fu, Economic performance evaluation method for hydroelectric generating units, Energy Convers. Manag. 44 (2003) 797–808, https://doi.org/10.1016/S0196-8904(02)00098-5.

[20] O. Paish, Small hydro power: technology and current status, Renew. Sustain. Energy Rev. 6 (2002) 537–556, https://doi.org/10.1016/S1364-0321(02)00006-0.

[21] N.G. Voros, C.T. Kiranoudis, Z.B. Maroulis, Short-cut design of small hydroelectric plants, Renew. Energy 19 (2000) 545–563, https://doi.org/10.1016/S0960-1481(99)00083-X.

[22] L. Liucci, D. Valigi, S. Casadei, A new application of flow duration curve (FDC) in designing run-of-river power plants, Water Resour. Manag. 28 (2014) 881–895, https://doi.org/10.1007/s11269-014-0523-4.

[23] A.D. Karlis, D.P. Papadopoulos, A systematic assessment of the technical feasibility and economic viability of small hydroelectric system installations, Renew. Energy 20 (2000) 253–262, https://doi.org/10.1016/S0960-1481(99)00113-5.

[24] S.-H. Ahn, X. Zhou, L. He, Y. Luo, Z. Wang, Numerical estimation of prototype hydraulic efficiency in a low head power station based on gross head conditions, Renew. Energy 153 (2020) 175–181, https://doi.org/10.1016/j.renene.2020.01.113.

[25] M. Nechleba, Hydraulic Turbines: Their Design and Equipment, Artia, Prague, 1957.

[26] S.J. Williamson, B.H. Stark, J.D. Booker, Low head pico hydro turbine selection using a multi-criteria analysis, Renew. Energy 61 (2014) 43–50, https://doi.org/10.1016/j.renene.2012.06.020.

[27] M.F. Müller, S.E. Thompson, M.N. Kelly, Bridging the information gap: a webGIS tool for rural electrification in data-scarce regions, Appl. Energy 171 (2016) 277–286, https://doi.org/10.1016/j.apenergy.2016.03.052.

[28] G. Evangelisti, Impianti Idroelettrici, Pàtron, Bologna, 1982.

[29] A.L. Diniz, P.P.I. Esteves, C.A. Sagastizabal, in: A Mathematical Model for the Efficiency Curves of Hydroelectric Units, 2007 IEEE Power Engineering Society General Meeting, 2007, pp. 1–7, https://doi.org/10.1109/PES.2007.385632.

[30] J.I. Pérez, J.R. Wilhelmi, L. Maroto, Adjustable speed operation of a hydropower plant associated to an irrigation reservoir, Energy Convers. Manag. 49 (2008) 2973–2978, https://doi.org/10.1016/j.enconman.2008.06.023.

[31] P.L. Bottino, Cavaticcio mini power plant, in: Proceedings of the Hidroenergia 1991, 2nd international conference and exhibition: 1991 Jun 12-15; Nice, France, AFME, Valbonne, 1991, pp. 408–416.

[32] P. Lamberti, Il progetto della nuova centrale idroelettrica del Cavaticcio, in: Bologna D'acqua: L'energia Idraulica Nella Storia Della Città, Compositori, Bologna, 1994, pp. 21–29.

[33] E. Marchi, A. Rubatta, Meccanica dei fluidi: principi e applicazioni idrauliche, UTET, Torino, 1981.

[34] D.S. Miller, Internal Flow: a Guide to Losses in Pipe and Duct Systems, British Hydromechanics Research Association, Bedford, 1971.

[35] MATLAB Function Reference R2021b, 2021, 16098.

[36] R.M. Vogel, N.M. Fennessey, Flow duration curves II: a review of applications in water resources planning, JAWRA J. Am. Water Resour. Assoc.. 31 (1995) 1029–1039, https://doi.org/10.1111/j.1752-1688.1995.tb03419.x.

[37] R.M. Vogel, N.M. Fennessey, Flow-duration curves. I: new interpretation and confidence intervals, J. Water Resour. Plann. Manag. 120 (1994) 485–504, https://doi.org/10.1061/(ASCE)0733-9496(1994)120:4(485).

[38] R. Montanari, Criteria for the economic planning of a low power hydroelectric plant, Renew. Energy 28 (2003) 2129–2145, https://doi.org/10.1016/S0960-1481(03)00063-6.