Climate engineering under deep uncertainty

Vassiliki Manoussi [a, b, *], Anastasios Xepapadeas [b, c], Johannes Emmerling [a, d]

[a] *Fondazione Eni Enrico Mattei (FEEM), Corso Magenta 63, 20123 Milan, Italy*
[b] *Athens University of Economics and Business, Department of International and European Economic Studies, 76 Patission Street, 104 34 Athens, Greece*
[c] *University of Bologna, Department of Economics, 2 piazza Scaravilli, 40126 Bologna, Italy*
[d] *Centro Euromediterraneo sui Cambiamenti Climatici (CMCC), Corso Magenta 63, 20123 Milan, Italy*

A B S T R A C T

Climate engineering, and in particular solar radiation management (SRM), is attracting increasing attention as a climate policy option. However, its potentially strategic nature and unforeseen side effects provide major policy and scientific challenges. We study the role of SRM in a two-country model with the notable feature of deep uncertainty modeled as model misspecification of SRM side effects. We find that deep uncertainty leads to a reduction in SRM deployment under both global cooperation and strategic Nash behavior, and that the effect is larger if countries act strategically. Furthermore, we demonstrate that if countries have different model confidence about SRM impacts, then the more confident country will engage more strongly in using SRM, leading this country to "free drive".

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1. Introduction

Despite recent advances in international climate negotiations, such as the Paris Agreement which entered into force in 2016, skepticism remains about the potential of global cooperative action to effectively stabilize global mean temperature at a level that is not expected to cause dangerous climate change. Specifically, the targets of 2 °C or 1.5 °C increase above the pre-industrial level have been widely discussed in the scientific and policy communities. However, the relatively slow progress of global mitigation action has led to discussion of alternative policy options in order to avoid potentially substantial impacts from climate change. In particular, different climate engineering methods have been discussed as a means to avoid dangerous climate change (Heutel et al., 2016).

Climate engineering refers to the deliberate intervention in the planetary environment of a nature and scale intended to counteract anthropogenic climate change and its impacts (Shepherd, 2009). One particular technology of climate engineering is solar radiation management (SRM), which involves directly manipulating the sun's incoming radiation. Probably the

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* Corresponding author.
E-mail addresses: vamanousi@aueb.gr (V. Manoussi), xepapad@aueb.gr (A. Xepapadeas), johannes.emmerling@cmcc.it (J. Emmerling).

most popular SRM method proposes injecting sulfur aerosols into the lower stratosphere and thereby reflecting incoming radiation away from the planet, back into space (Keith, 2000; Ricke et al., 2008; Shepherd, 2009). This method mimics what occasionally occurs in nature when a volcano erupts. For instance, the Mount Pinatubo eruption in 1991 led to the injection of large volumes of sulfur into the stratosphere, and the aerosols produced in subsequent reactions cooled the planet by about 0.5 °C over the following two years (Randel et al., 1995; Robock, 2000).

One common feature of most climate engineering options is that at present, they tend to be speculative: no large-scale experiments have been conducted in order to assess their full potential or their side effects and other impacts and interactions with the earth system. In particular, in the case of SRM, its potential side effects are largely unknown (Barrett, 2008; Robock et al., 2008), and the uncertainty about it also includes its effectiveness to cool the planet globally (Emmerling and Tavoni, 2017; Moreno-Cruz and Keith, 2012). This uncertainty is due to major gaps in knowledge, limited modeling capacity, lack of theories to anticipate thresholds (Heutel et al., 2016) and the possibility of unexpected consequences in the future.

Moreover, this uncertainty must be considered as "deep" uncertainty since even defining the full potential state space and assigning probabilities to events is (almost) impossible. Deep (or Knightian) uncertainty is contrasted to risk (measurable or probabilistic uncertainty), where probabilities can be assigned to events by a subjective probability measure or a single Bayesian prior (Roseta-Palma and Xepapadeas, 2004; Vardas and Xepapadeas, 2009). Deep uncertainty or ambiguity can be modeled through extensions of the non-expected utility paradigm (e.g., maxmin, smooth ambiguity aversion) or through a manipulation of the model the decision maker considers, thus allowing for model misspecification, as in the robust control framework of Hansen and Sargent (2001) and Hansen et al. (2006).

In this paper, we use this approach, which seems an appropriate modeling framework for speculative future technologies such as SRM (Goeschl et al., 2013). That is, we consider a decision maker who cannot assign probabilities to events and therefore has limited confidence in his conceptual model and wants to find a good decision over a set or "cloud" of models that surround his benchmark model. This set of models is obtained by disturbing the benchmark model and introducing a misspecification error. These admissible disturbances reflect the set of possible probability measures that the decision maker is willing to consider. The more ambiguous the situation is considered to be by the decision maker, the larger is the set of approximate models that he will consider.

The large uncertainties in the area of climate change stem in particular from two sources: scientific uncertainty over the physics of environmental phenomena (e.g., embodied in the calibration and specification of damage functions, as in Weitzman, 2010) and socioeconomic uncertainty over the impacts on society. Apart from climate damages, the damages from SRM implementation constitute another important source of uncertainty, which we focus on in this paper. While some of the existing studies look at the potential of SRM and the implications of its use (e.g., Heutel et al., 2016; Keith, 2000; McClellan et al., 2012), the uncertainty around the damages of SRM implementation and its impact on the optimal climate policy portfolio have not yet been studied in detail.

In this paper, we focus on this aspect of SRM and model its impacts as a stochastic process and a function of its aggregate implementation. The importance of the uncertainty surrounding side effects and damages from SRM implementation has already been stressed by Robock et al. (2008), Shepherd (2009), and Moreno-Cruz and Keith (2012). These damages are related to SRM not only directly (unexpected consequences, environmental impacts of implementation) but also indirectly (disrupting precipitation patterns, ocean acidification, ozone depletion, health impacts and notably substantial regional heterogeneity of impacts). The goal of this paper is to explore the trade-offs between the avoidance of climate impacts from an increase in global temperature and the introduction of a new environmental risk from the uncertain damages of SRM implementation. In our model, uncertainty is introduced as a drift on the marginal impacts from SRM, which in general is represented by a stochastic process measurable with respect to the filtration generated by the Wiener process. This drift may be considered as a measure of the model misspecification.

In this paper, we thus consider deep uncertainty in a robust control framework, starting with the strategic modeling of SRM as in Ricke et al. (2013), Moreno-Cruz (2015), Moreno-Cruz and Smulders (2017), and Manoussi and Xepapadeas (2015). The strategic interaction is modeled in a dynamic game of climate change policy in terms of emissions and climate engineering efforts. Our main target is to determine how deep uncertainty affects the optimal decision about SRM implementation and mitigation. We formulate the problem in terms of a linear-quadratic (LQ) differential game, extending the standard LQ model of pollution control studied in Dockner and Van Long (1993), Athanassoglou and Xepapadeas (2012), or Kossioris et al. (2008). We analyze the problem in the context of a cooperative and a non-cooperative game, adding the crucial feature of deep uncertainty about impacts from SRM implementation.

The paper is, from the conceptual point of view, related to the works by Jiménez-Lizárraga and Poznyak (2007), Johnson (2011) and Li et al. (2016, 2017), who also applied the robust control framework to an LQ differential game. In the cooperative case, there is coordination between the two countries for the implementation of climate engineering in order to maximize joint welfare. In the strategic or non-cooperative case, on the other hand, each region chooses its own level of SRM and emissions independently, and we analyze this equilibrium in terms of Nash equilibrium strategies.

We derive analytical solutions for the optimal policies under deep uncertainty and then proceed to a numerical simulation of the model in order to explore the magnitude of the effects, both at the cooperative and non-cooperative solutions. We also consider asymmetry in the degree of model confidence or model misspecification. In this case we find that SRM will be used to a much higher extent by the region with a higher degree of model confidence.

The paper is organized as follows. In Section 2 we introduce a simple model of climate policy with SRM under model misspecification. In Section 3, we solve the cooperative and non-cooperative problems analytically. The main results are presented in Section 4, and the situation in which model confidence is asymmetric between countries is discussed in Section 5. Section 6 concludes.

2. A model of SRM with model misspecification

In this section, we develop an LQ dynamic model of optimal climate policy with two heterogeneous countries or regions, indexed by $i = 1, 2,$ which will be referred to hereafter as countries for simplicity.[1] We develop the model along the lines of the standard LQ model of international pollution control of Dockner and Van Long (1993). The model is framed in continuous time and all variables are indexed by the time index t. The relationship between emissions and economic output is modeled as a reduced form utility function depending on emissions as in Athanassoglou and Xepapadeas (2012).[2] Within our LQ framework, the utility function is given by the quadratic function

$$U(E_i(t)) = AE_i(t) - \frac{1}{2}BE_i^2(t), \tag{1}$$

where A and B are parameters indicating the intercept and the slope of the marginal benefits from emissions respectively. We calibrate the model based on a money-metric utility function measuring all costs and utility values in trillion USD (see Appendix A). The decision maker in each country has two intertemporal policy options: the reduction of emissions $E_i(t)$ and the use of SRM denoted by $z_i(t)$.

In formulating this climate model, we seek a simplified representation that will be compatible with the LQ structure of our coupled climate—economic model and will allow us to derive some tractable results regarding the role of SRM under deep uncertainty. The climate models underlying representative concentration pathways scenarios (IPCC, 2013) are quite complex. Since, however, these state-of-the art climate models are computationally very expensive to incorporate into economic models, in widely used integrated assessment models (IAMs) which couple the economy and the climate (e.g., Nordhaus, 2014), the climate module represents a simplification of the more complex model. In DICE 2013 in particular, the carbon cycle is based upon a three-reservoir model, in which the reservoirs represents carbon in the atmosphere, the upper oceans and the deep oceans. Then the increase in radiative forcing, which induces an increase in the global mean temperature, is determined by the well-known relationship between the current concentration of CO_2 in the atmosphere and the pre-industrial concentration in 1750 (see Nordhaus, 2014, pp.16–17).[3]

In this paper, for tractability reasons, we employ a further simplification of the climate model (see for example Hassler et al., 2016, Section 3.2.6, or Brock and Xepapadeas, 2017; Brock and Hansen, 2017), which is based on climate literature developed over the last decade (Matthews et al., 2009; 2012; Pierrehumbert, 2014). The simplification is based on linking emissions of CO_2 directly to changes in global mean temperature through the carbon-climate response (CCR), instead of linking CO_2 emissions to CO_2 concentration through carbon sensitivity and CO_2 concentration to changes in global mean temperature through climate sensitivity. The CCR is approximately constant and aggregates the climate and carbon sensitivities (including climate-carbon feedbacks) into a single metric representing the net temperature change per unit carbon emitted (Brock and Hansen, 2017; MacDougall, 2016, Fig. 3).

The relationship which is consistent with the observational record of global temperature change and anthropogenic CO_2 emissions has been named the transient climate response (TCRE) to CO_2 emissions (e.g. MacDougall et al., 2016). The TCRE embodies both the physical effect of CO_2 on climate and the biochemical effect of CO_2 on the global carbon cycle (Matthews et al., 2009). The TCRE, denoted by λ, is defined as $\lambda = \frac{\Delta T(t)}{CE(t)}$, where $CE(t)$ denotes cumulative carbon emissions up to time t and $\Delta T(t)$ the change in temperature during the same period. The constancy of λ suggests a roughly linear relationship between a change in global average temperature and cumulative emissions. This linear relationship has also been recognized by the IPCC (2013), while in the same context Collins et al. (2013) concludes that every ton of CO_2 causes about the same amount of warming, no matter when and where it is emitted.

MacDougall and Friedlingstein (2015) and MacDougall (2016) provide analytical arguments for the constancy of TCRE over a relevant range of cumulative emissions of carbon (see MacDougall and Friedlingstein, 2015, Eqs. (10) and (11), or MacDougall, 2016, Eqs. (7) and (8)). (MacDougall, 2016, p. 42) states that:

> "...TCRE arises from a combination of (1) positive carbon-climate feedbacks increasing the airborne fraction of carbon; (2) weakening radiative forcing per unit CO_2 at higher atmospheric concentrations of CO_2 and (3) contributions from non-CO_2 radiative forcing. Notably without the contribution from non-CO_2 radiative forcing the simulated TCRE remains approximately constant until 1700 Pg C of CO_2 have been emitted to the atmosphere."

Using the definition of TCRE in continuous time, the anthropogenic impact on the global temperature increase can be approximated by $T(t) - T(0) = \lambda \int_{s=0}^{t} E(s)ds,$ where $CE(t) = \int_{s=0}^{t} E(s)ds$ denotes cumulative global carbon emissions up to

[1] We assume symmetric countries in terms of climate damages and costs. However, we admit one case of asymmetry in the decision maker's attitude, i.e., we consider the case of heterogeneity in the degree of model confidence (θ_i) or ambiguity aversion between the two countries in Section 5.

[2] This function can be considered as a utility function of economic output, which itself is a function of emissions $F(E_i)$, so that $U(E_i(t)) \equiv V(F(E_i(t)))$.

[3] It should be noted that, although the simplifications of the climate model in IAMs lead to transparent results which have been used to discuss important current policy issues, they remain simplifications which may not provide unbiased results on a global scale. For example, the DICE linear carbon cycle representation has been criticized because it does not take into account non-linearities in the ocean uptake and this may result in a biased long-run projection of the temperature anomaly (Glotter et al., 2014).

time t and λ is the TCRE (see Hassler et al., 2016; Knutti and Rogelj, 2015, p. 1929). Taking the time derivative of the expression we obtain

$$\dot{T}(t) = \lambda E(t). \tag{2}$$

Using this approximation as a basis for our climate model, we introduce two further adjustments. We add a linear SRM impact, denoted by $\phi \sum_{i=1}^{2} z_i(t)$, which represents the reduction in the rate of change in global mean temperature due to SRM, where $\phi < 0$. This reduction emerges through the reduction in incoming solar radiation due to the shielding effect of SRM. This is another approximation since, for example, it is not clear whether this impact is linear. Finally, considering that a fraction of the heat stored in the atmosphere escapes, we assume that this is captured by the term $\delta T(t)$, where $\delta > 0$ is the heat dissipation parameter (see Heutel et al., 2016; Lemoine and Rudik, 2014; Nævdal and Oppenheimer, 2007). Thus, the dynamics of the temperature anomaly are assumed to evolve as:

$$\dot{T}(t) = \lambda \sum_{i=1}^{2} E_i(t) + \phi \sum_{i=1}^{2} z_i(t) - \delta T(t), \;\; T(0) = 0. \tag{3}$$

Eq. (3) is a linear approximation of climate dynamics based, for the part relating CO_2 emissions to temperature changes, on the TCRE approach and not on the forcing-response framework which is the traditional framework in coupled models of the economy and climate. There are arguments pointing to the limitations of the TCRE concept and a number of questions to be answered (see MacDougall, 2016); however, it provides a simplified framework for studying climate problems, especially when optimal control and differential games tools are used. For the present research in particular, the TCRE approach helps to keep the LQ structure that is very useful for providing tractable results for the SRM strategies adopted by individual countries, which is the main objective of the paper.

As for the costs of implementing SRM, we assume a quadratic cost function for the cost of climate engineering in each country $C(z_i)$ (see also Bickel and Agrawal, 2013; Goes et al., 2011; Gramstad and Tjøtta, 2010; Robock et al., 2009):

$$C(z_i(t)) = \frac{1}{2}\beta z_i^2(t), \;\; \beta > 0. \tag{4}$$

An important feature of this model is the impacts from global temperature increase on the one hand, and the impacts from the use of SRM on the other. We assume two types of regional damage functions, which affect welfare in each region. The first one reflects damages from the increase in the average global surface temperature, represented as usual by a convex – quadratic in our case – function in the degree of global warming since pre-industrial levels (T_0):

$$D_T(T(t)) = \tau (T(t) - T_0)^2, \tag{5}$$

where τ represents the marginal damages in each country.

The second damage function represents the impacts or side effects from the implementation of climate engineering. These impacts include potential ozone depletion, distorted precipitation patterns, negative effects on biodiversity and many others (see, e.g., Barrett et al., 2014; Robock et al., 2008).[4] We assume that these impacts depend on the total level of SRM implemented globally, and allow for potential heterogeneity between countries. Moreover, since we are interested in the deep uncertainty around these impacts, we model the impacts from SRM as a linear function of SRM deployment with a stochastic marginal impact, denoted as $u_i(t)$. The total impacts from SRM implementation in country i are then given by the linear equation

$$D_z(\mathbf{z}(t), u_i(t)) = u_i(t) \cdot \zeta \cdot \sum_{i=1}^{2} z_i(t), \tag{6}$$

where $\sum_{i=1}^{2} z_i(t)$ represents the aggregate SRM implemented in both regions. The impacts are calibrated as a percentage loss of GDP, therefore we convert them into monetary terms by the SRM damage parameter ζ (see Appendix A).

One crucial feature of the model is the consideration of uncertainty about the environmental impacts from the use of SRM. In particular, we model the marginal impacts in the linear equation (6) according to this stochastic differential equation:

$$du_i(t) = \left[\eta(1-\gamma) \sum_{i=1}^{2} z_i(t) - m u_i(t) \right] dt + \sigma d\hat{W}_i. \tag{7}$$

Note that while the damage function is linear in z_i, marginal impacts u_i are changing over time and depend on the level of SRM implemented at a region-specific marginal rate $\eta(1-\gamma)$. Therefore, marginal impacts will increase in the amount of SRM implemented, so that total impacts will have a convex shape in the total amount of SRM. Moreover, we assume that there is an adjustment rate m in marginal impacts which can be interpreted as the adaptation to SRM impacts of the

[4] Note that we do not consider impacts from CO_2 concentration increases per se, such as ocean acidification, which provide another category of potential impacts.

socioeconomic or biophysical system.[5] If $\gamma = 1$ in (7), the sulfur emitted at t is dispersed, and only the stochastic stock $\sigma d\hat{W}_i$ drives marginal impacts. If $0 < \gamma < 1$, the remaining stock of sulfur adds a trend to damages along with the stochastic shock.[6] Finally, uncertainty is introduced in each country through $\hat{W}_i(t)$, which is a Brownian motion on an underlying probability space $(\Omega, \mathcal{F}, \mathcal{G})$.

Without strategic interactions and model misspecification, solving the symmetric problem would be straightforward.[7] Now we add the strategic interaction and deep uncertainty through model misspecification of SRM impacts. Model misspecification is represented by a family of stochastic perturbations to the Brownian motion $\hat{W}_i(t)$, such that the probabilistic structure implied by the stochastic differential equation of marginal impacts from SRM (7) is distorted, and the probability measure $\mathcal{G}$ is replaced by another measure $\mathcal{Q}$. The perturbed model is obtained by performing a change of measure and replacing $\hat{W}_i(t)$ in (7) by

$$W_i(t) + \int_0^t h_i(s)dt \tag{8}$$

$$d\hat{W}_i = dW_i + h_i(t)dt, \tag{9}$$

where $\{W_i(t):\ t \geq 0\}$ is a Brownian motion and $\{h_i(t):\ t \geq 0\}$ a measurable drift distortion such that $h_i(t) = h_i(u(s) : s \leq t)$. Hence, changes to the distribution of $\hat{W}_i$ are parameterized as drift distortions to the Brownian motion. The measurable process $h_i(t)$ corresponds to any number of misspecified or omitted dynamic effects, such as a miscalculation of climate engineering damages, a miscalculation of the decay rate of sulfur in the stratosphere, or an ignorance of more complex dynamic structures including irreversibility, feedback or hysteresis effects. The distortion will be zero if $h_i(t) \equiv 0$ when the two measures $\mathcal{G}$ and $\mathcal{Q}$ coincide.

Taking into account the deep uncertainty or model misspecification, the dynamics for the environmental impacts from the SRM implementation become

$$du_i(t) = \left[\eta(1-\gamma)\sum_{i=1}^{2} z_i(t) - mu_i(t) + \sigma h_i(t)\right]dt + \sigma dW_i, \tag{10}$$

where $\{h_i(t):\ t \geq 0\}$ is the measurable drift distortion (or misspecification error), which is expressed in terms of deviations from the benchmark case. This misspecification error will serve as a measure of the policy maker's inability to choose the right model to capture the real level of the future impacts from SRM implementation.[8] The benchmark case of pure risk is defined for $h_i(t) = 0$.

We now turn to the full optimization problem under uncertainty. It is based on an LQ framework of maximizing expected discounted utility minus costs and impacts in a continuous time infinite horizon model, subject to the equations of motion (3) and (10). The multiplier robust control problem[9] for each individual country i is given by

$$V_i(t) = \max_{E_i(t), z_i(t)} \min_{h_i(t)} \int_0^\infty e^{-\rho t}\left[U(E_i(t)) - C(z_i(t)) - D_T(T(t)) + D_z(z_i(t), u_i(t)) + \frac{1}{2}\theta h_i(t)^2\right]dt, \tag{11}$$

subject to

$$dT(t) = \left[\lambda \sum_{i=1}^{2} E_i(t) + \phi \sum_{i=1}^{2} z_i(t) - \delta T(t)\right]dt$$

and

$$du_i(t) = \left[\eta(1-\gamma)\sum_{i=1}^{2} z_i(t) - mu_i(t) + \sigma h_i(t)\right]dt + \sigma dW_i\ ,\ i = 1, 2.$$

To take into account model misspecification, maximization of welfare is subject to the decisions of the minimizing agent, often referred to as an adversarial agent or Nature, which chooses $h_i(t)$. The parameter $\theta \in \Theta = \{\theta : 0 \leq \underline{\theta} < \theta < \infty\}$ constrains the minimizing choice of the $h_i(t)$ function, and therefore can be regarded as the level of preference for robustness or deep uncertainty aversion. The lower bound $\underline{\theta}$ is the so-called breakdown point[10] below which θ cannot be pushed to

[5] In the numerical part, we set it to a very low value, but its inclusion is necessary to solve for the steady state in this model.

[6] Note that there is a non-zero probability that $u_i(t)$ could turn out to be negative, reflecting the large uncertainty about impacts from SRM; positive side effects could include, e.g., reduced changes in precipitation. However, in all our calibration runs, marginal damages from SRM always turned out to be positive.

[7] This is based on a simplified maximization problem such as the one given in Eq. (11).

[8] On a side note, in Appendix B we show that, given this specification, the marginal impacts from SRM follow an Ornstein-Uhlenbeck process.

[9] For the formulation of the constrained robust control problem, see Appendix C.

[10] For the definition and the derivation of the breakdown point, see Hansen and Sargent (2008).

improve robustness. On the other hand, when $\theta \to \infty$, there are no concerns about model misspecification and deep uncertainty disappears.

Based on this general model setup, we now define both a cooperative and a non-cooperative solution to the optimal climate policy mix under model misspecification of the impacts from SRM.

3. Cooperative and strategic Nash solution

3.1. The cooperative solution

First we solve the model presented above for the case in which a global social planner chooses jointly the optimal policy for both countries. In this case, we can write the Hamilton–Jacobi–Bellman (HJB) equation for the social planner maximizing global welfare based on (11) as[11]

$$\rho V^C(u_i) = \max_{E_i, z_i} \min_h \left\{ \sum_{i=1}^{2} \left[AE_i - \frac{1}{2}BE_i^2 - \frac{1}{2}\beta z_i^2 - \left(\tau(T - T_0)^2 + u_i \zeta \sum_{i=1}^{2} z_i \right) + \frac{1}{2}\theta_i h^2 \right] + V_{u_i}^C \left[\eta(1-\gamma) \sum_{i=1}^{2} z_i - mu_i + \sigma h \right] + V_T^C \left(\lambda \sum_{i=1}^{2} E_i + \phi \sum_{i=1}^{2} z_i - \delta T \right) + \frac{1}{2}\sigma^2 V_{u_i u_i}^C \right\}. \tag{12}$$

In order to solve this problem, and given its LQ structure, we introduce a quadratic value function with respect to the two state variables u_i and T,

$$V^C(u_i, T) = \varepsilon_{0i} + \mu_{1i}u_i + \mu_{2i}u_i^2 + \nu_1 T + \nu_2 T^2 + \kappa u_i T, \tag{13}$$

with derivatives

$$V_{u_i}^C = \mu_{1i} + 2\mu_{2i}u_i + \kappa T, \quad V_{u_i u_i}^C = 2\mu_{2i} \textit{ and } V_T^C = \nu_1 + 2\nu_2 T + \kappa u_i,$$

where μ_{1i}, μ_{2i}, ν_1, ν_2, κ are the coefficients of the value function. Minimizing the right hand side of the HJB equation with respect to h, [12]we obtain the optimal value of h^* as

$$h^* = -\frac{\sigma V_{u_i}^C}{\theta_i} = -\frac{\sigma(\mu_{1i}(\theta_i) + 2u_i\mu_{2i}(\theta_i))}{\theta_i}. \tag{14}$$

Substituting this value for h in (12), we can solve the symmetric case where both regions are identical[13] to find the optimal emission level, E_i^*, as

$$E_i^* = \frac{A + \lambda V_T^C}{B}, \tag{15}$$

and the optimal level of SRM implementation, z_i^*, as

$$z_i^* = \frac{\eta(1-\gamma)V_{u_i}^C - 2\zeta u_i + \phi V_T^C}{\beta}. \tag{16}$$

These control rules are linear in the impacts per unit of SRM implementation (u_i), and have the expected sign: z_i^* is decreasing in u_i^*, and also emissions are reduced if impacts from SRM are high, since $\phi < 0$. This solution provides our benchmark scenario. In the rest of the paper we compare results with this scenario. Note that given the simple utility function of this model, although the regions may be heterogeneous in terms of damages from climate change and impacts from SRM, the social planner's objective is to optimize global welfare with respect to the optimal levels of emissions and SRM in each region. Next, we turn to the solution where both countries act strategically in their climate policy decisions.

3.2. The strategic Nash solution

In the non-cooperative strategic solution, we solve the game assuming that each country follows feedback strategies in the level of emissions and climate engineering. Feedback strategies are associated with the concept of Nash equilibrium of the differential game, which provides a time-consistent non-cooperative equilibrium. The feedback Nash equilibrium (hereafter referred to simply as Nash equilibrium) for the LQ climate change game can be obtained as the solution of the dynamic

[11] In the following, we drop the time index to avoid notation clutter.

[12] In the cooperative solution, the policy maker can have different θs for each country, but considers only one distortion of the SRM dynamics and therefore decides about one h.

[13] In symmetry, it is implied that $\tau_1 = \tau_2 = \tau$, $\eta_1 = \eta_2 = \eta$, $\theta_1 = \theta_2 = \theta$.

programming representation of the non-cooperative dynamic game. The HJB equation for the infinite horizon problem of each country $(i = 1, 2)$ based on (11) is given by

$$\rho V_i^N(u_i) = \max_{E_i, z_i} \min_{h_i} \left\{ AE_i - \frac{1}{2}BE_i^2 - \frac{1}{2}\beta z_i^2 - \left(\tau (T - T_0)^2 + u_i \zeta \sum_{i=1}^{2} z_i \right) + \frac{1}{2}\theta_i h_i^2 \right.$$
$$\left. + V_{i,u_i}^N \left[\eta(1-\gamma) \sum_{i=1}^{2} z_i - mu_i + \sigma h_i \right] + V_{i,T}^N \left(\lambda \sum_{i=1}^{2} E_i + \phi \sum_{i=1}^{2} z_i - \delta T \right) + \frac{1}{2}\sigma^2 V_{i,u_i u_i}^N \right\}. \tag{17}$$

Each country will take the emissions and the SRM level of the other country as given and solve its own optimal climate policy problem. Given the LQ structure of the problem, we consider two quadratic value functions, one for each country denoted as $V_i(u_i, T)$ (see Appendix D for more details), so that for each country we have

$$V_i^N(u_i, T) = \varepsilon_{0i} + \mu_{1i} u_i + \mu_{2i} u_i^2 + \nu_{1i} T + \nu_{2i} T^2 + \kappa_i u_i T, \quad i = 1, 2, \tag{18}$$

with derivatives

$$V_{i,u_i}^N = \mu_{1i} + 2\mu_{2i} u_i + \kappa_i T, \;\; V_{i,u_i u_i}^N = 2\mu_{2i} \; and \; V_{i,T}^N = \nu_{1i} + 2\nu_{2i} T + \kappa_i u_i.$$

The optimization with respect to E_i and z_i yields the following reaction functions in SRM and emissions of country i depending on player $j \neq i$'s choices:

$$z_i^*(E_j^*, z_j^*) = \frac{\eta(1-\gamma) V_{i,u_i}^N - 2\zeta u_i + \phi V_{i,T}^N}{\beta} \tag{19}$$

$$E_i^*\left(E_j^*, z_j^*\right) = \frac{A + \lambda V_{i,T}^N}{B}. \tag{20}$$

Moreover, from the minimization problem for h_i, we have that

$$h_i^* = -\frac{\sigma \; V_{i,u_i}^N}{\theta_i}. \tag{21}$$

Based on the definition of the parameters, it is easy to show that $\frac{\partial z_i^*}{\partial z_j^*} < 0$ and $\frac{\partial E_i^*}{\partial E_j^*} < 0$, or that both climate policy options are strategic substitutes between countries.[14] Thus, the more mitigation or SRM one country does, the lower is the incentive for the other country to implement either policy. Moreover, mitigation and SRM are strategic complements, since $\frac{\partial E_i^*}{\partial z_j^*} > 0$. That is, the country with lower emissions will implement SRM in order to compensate for the other country's increased emissions.[15]

Note that Eq. (12) implies that the parameters of the value function and the optimal Nash strategy for each country depend on the parameter θ_i. Thus, it can be used to determine a symmetric Nash equilibrium under deep uncertainty as specified above. Moreover, for $\theta_i \to \infty$, the robust Nash equilibrium converges to the one under pure risk.

4. Optimal climate policies

Now we turn to the results about the optimal policy mix in the model including model misspecification (deep uncertainty), and we explicitly compute the optimal mitigation and SRM values in the symmetric case where both countries are identical.

4.1. Optimal policies at the steady state

First, we solve the symmetric cooperative and non-cooperative problems under pure risk to calibrate the unknown parameters of the model. The values for the parameters of the utility function A and B have been calibrated following Karp and Zhang (2006), and the default calibration of marginal climate impacts τ yields a damage estimate of 5.4% of GDP for a 4.5 °C temperature increase. The parameter ϕ is calibrated such that SRM (measured in TgS) will yield a negative forcing of 0.5 W/m^2 per TgS. This estimate is based on a best guess estimate (Gramstad and Tjøtta, 2010), and relates to a range from −0.5 (Crutzen, 2006) to −2.5 (Rasch et al., 2008). For the costs of SRM, we assume a quadratic cost function at a private cost of 10 billion \$/TgS, which is within the range considered in the literature. For the calibration of γ, we approximate impacts such that 3% of GDP is lost for an SRM implementation leading to a radiative forcing of −3.5 W/m^2 (based on the estimate in Goes et al., 2011).[16]

[14] For the proof, see Appendix E.

[15] Note that mitigation is interpreted as a reduction of emissions, E_i^*.

[16] Appendix A provides the numerical values of all model parameters.

Using these numerical values, we can define the steady-state level of emissions, climate engineering and average global temperature in the symmetric cooperative solution and non-cooperative game under pure risk. The values obtained are presented below.[17]

Cooperation	$E_i^* = 22.705$GtC	$z_i^* = 1.988$TgS	$T^* = 14.655\,^{\circ}$C
Nash	$E_i^* = 29.735$GtC	$z_i^* = 2.4225$TgS	$T^* = 15.51\,^{\circ}$C

First, note that in the strategic Nash solution, global temperature, emissions, and the level of SRM are higher than in cooperation. This result is intuitive, since in the non-cooperative case, neither the externality of emissions nor the externality of SRM impacts is taken into account.

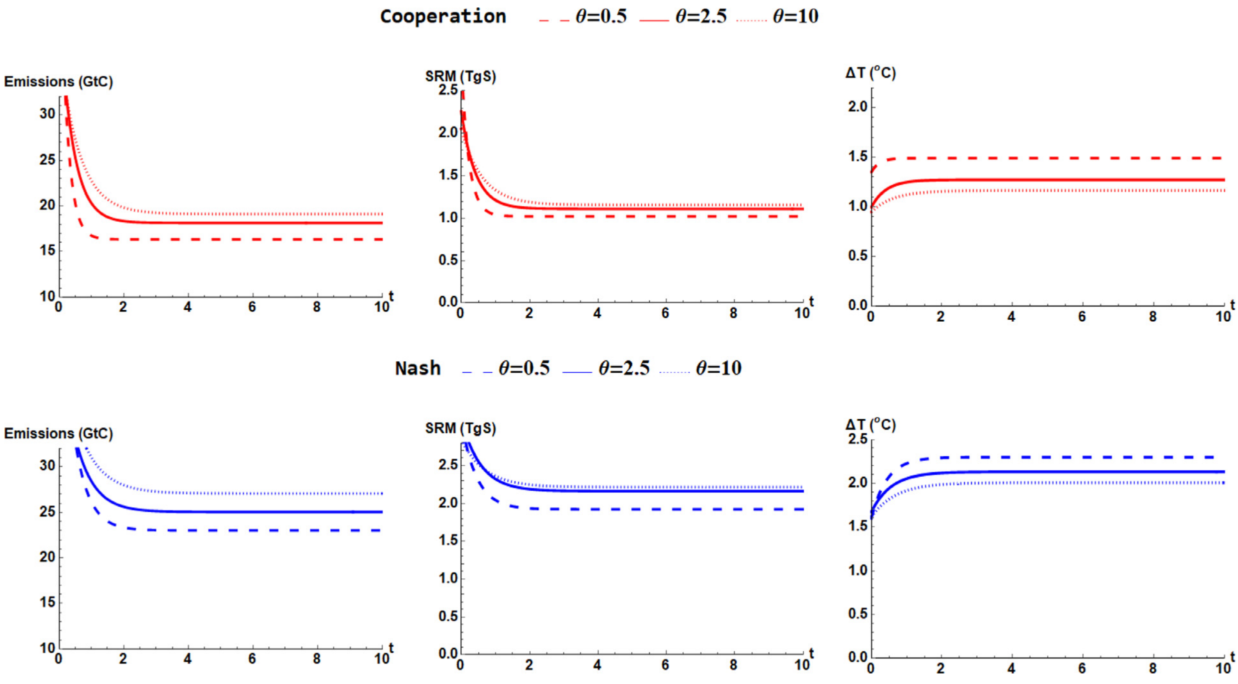

Fig. 1. Expected optimal climate policies over time for different values of θ.

Fig. 1 shows the expected optimal levels of emissions and SRM across time[18] for different levels of the robustness parameter θ. We find that all variables converge fast to their steady-state levels. The difference in optimal policies between cooperation and Nash equilibrium is substantial, as expected. Note that as θ decreases from 10 towards zero, i.e., as ambiguity increases, both emissions and SRM are decreasing due to the higher ambiguity about SRM impacts.

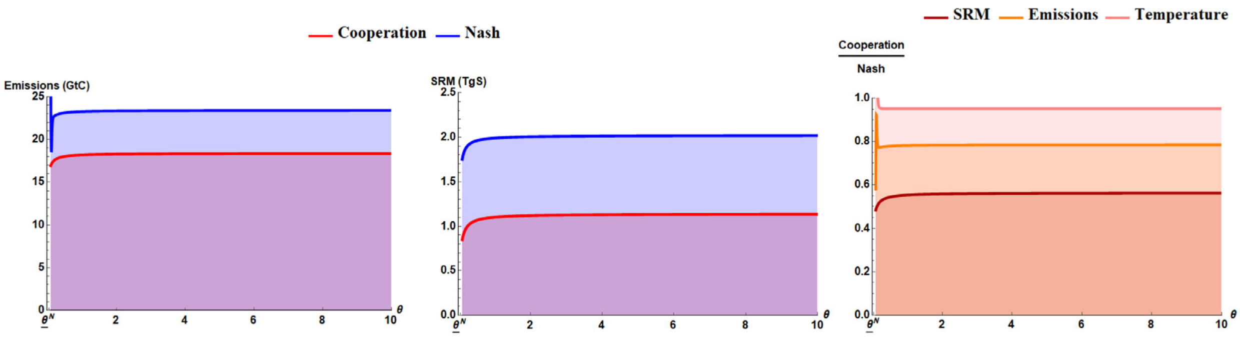

Fig. 2. Optimal climate policies (emissions and SRM) for different values of θ.

To investigate more closely the impact of the model misspecification, Fig. 2 shows the impact of changing ambiguity, as reflected in changing θ,[19] on the optimal steady states for emissions and SRM at the cooperative and non-cooperative

[17] The GHG emissions are measured in gigatons of CO_2 equivalent, the level of SRM in teragrams (or megatons) of sulfur, and the temperature as global annual average temperature in degrees Celsius.

[18] The time step of the model and in Fig. 1 is $t = 1$ year.

[19] We use a range for the value of θ of [0, 10] for all figures. This is because the results are more sensitive in this range of values since, for $\theta > 10$, the model converges fast to the steady-state values and the optimal values remain almost the same.

solutions. As expected, emissions and climate engineering are higher at the Nash equilibrium compared to cooperation. It is interesting to note, however, that as ambiguity increases (or θ decreases), the deviation between cooperative SRM and SRM in Nash equilibrium is reduced. For mitigation or emissions, on the other hand, the relative difference is approximately constant. This is shown in the third panel of Fig. 2, which depicts the ratio of the optimal level of emissions and SRM in cooperation to the value at the Nash equilibrium. Ambiguity aversion thus seems to have a slightly higher impact in the strategic Nash equilibrium case, where SRM in general is used more. This result can be attributed to the fact that here the ambiguity of SRM impacts is related directly to SRM.

A way of better understanding the implications from SRM use is through the resulting impacts from SRM implementation. Fig. 3 shows the total SRM damages as the expected percentage reduction of global GDP in the symmetric case, both for the cooperative and the non-cooperative solutions. In both cases we have that as $\theta \longrightarrow \infty$, damages converge to the case of a pure risk (without model misspecification). At the Nash solution, damages from SRM are always higher than under cooperation – about 15% of GDP in Nash compared to less than 5% in cooperation. Moreover, as θ decreases, the impacts from SRM decrease, since SRM is reduced.

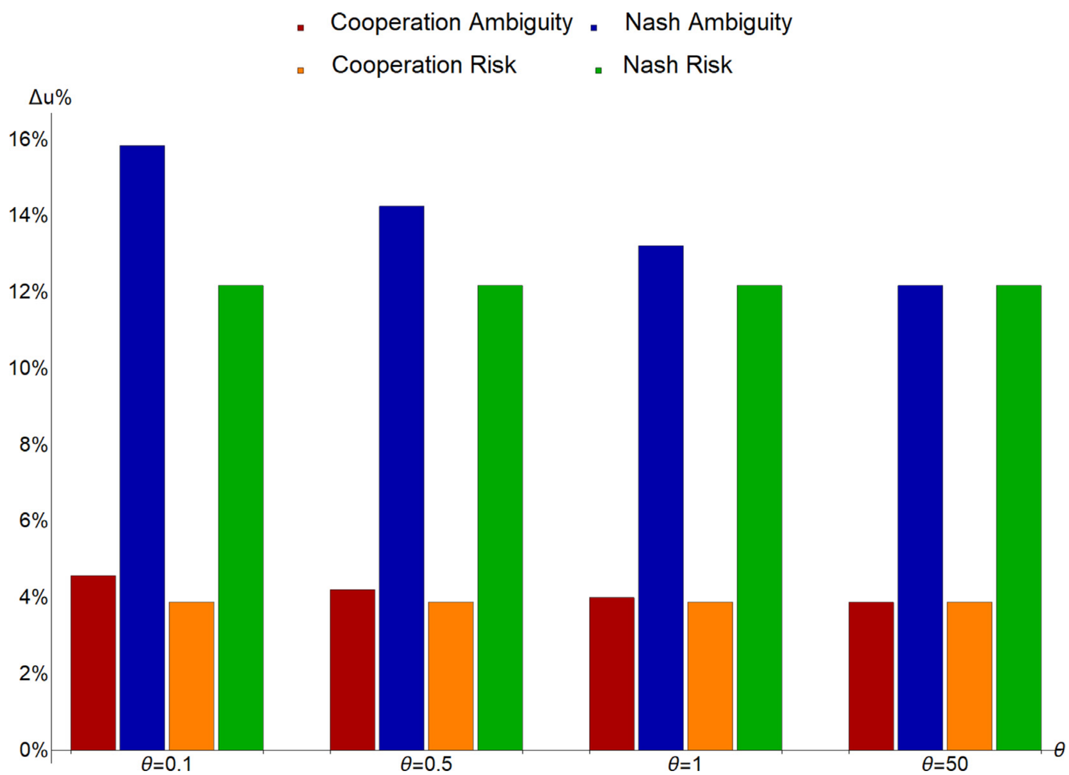

Fig. 3. Total impacts from SRM $D_z(z_i, u_i)$ as a percent of global GDP.

4.2. Quantifying the level of deep uncertainty

Fig. 4 presents a metric for the quantification of the deep uncertainty or ambiguity in terms of welfare.[20] Panel (a) shows that the expected welfare reduction in Nash is higher than in cooperation for high degrees of ambiguity ($\theta < 3$), while welfare in both cases decreases as ambiguity increases (or θ decreases). From panel (b) it can be noted that the cooperative and the Nash solutions become closer in terms of welfare as ambiguity increases, as the difference between the welfare in cooperation and Nash is lower for $\theta = 0.01$ than for $\theta = 10$. These results can be explained by the fact that as ambiguity increases, SRM at the Nash equilibrium is reduced faster than at the cooperative solution, albeit from a relatively higher level (see Fig. 3), which leads to a faster reduction in the difference in welfare.

[20] In Fig. 4, the welfare is non-monotonic for a very low value of θ, which is due to the "breakdown point" which is discussed in the next subsection.

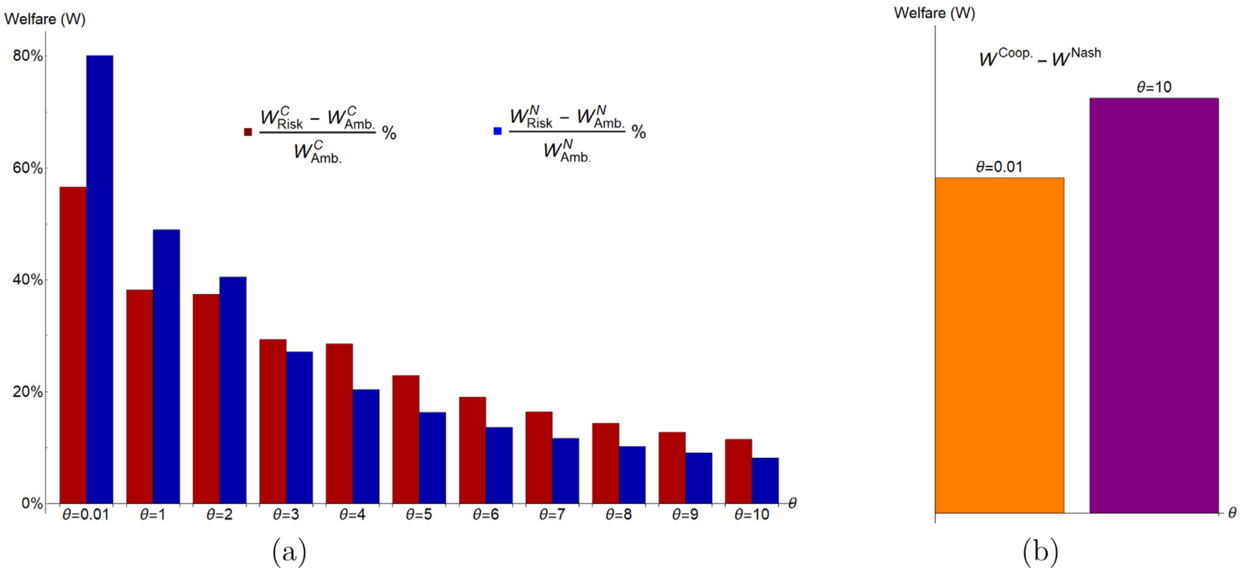

Fig. 4. Expected global welfare differences for different values of θ.

4.3. The "breakdown point"

One particular feature of the Hansen and Sargent (2008) model is the limiting case for the robustness parameter θ, when the potential model misspecification becomes so large as θ tends towards zero, that the optimization problem collapses. Thus, there exists a "breakdown point" which is the lower bound of θ, beyond which it is impossible to find a robust optimal solution. Below this point, the minimizing agent (Nature) is sufficiently unconstrained and it can push the value function to $-\infty$ despite the best response of the maximizing agent, i.e., the policy maker. So there exists a $\underline{\theta}$ above which the value function is well defined and does not lose its concavity to θ. We can easily derive this "breakdown point" in the symmetric case for both the cooperative and the strategic Nash solutions. It can be computed from the denominator of the coefficient of the value function by preventing the value functions in (13) and (18) from losing concavity ($\mu_{2i} \longrightarrow -\infty$) (see Appendix F).

For the cooperative solution, we find the breakdown point to be

$$\underline{\theta}^C = \frac{\beta\sigma^2}{2(\eta(1-\gamma))^2} \backsimeq 0.11,$$

while for the strategic Nash solution it equals

$$\underline{\theta}^N = \frac{\beta\sigma^2}{(\eta(1-\gamma))^2} \backsimeq 0.22.$$

While in general the two values cannot be compared unambiguously, we find that for our numerical calibration [21] $\underline{\theta}^C < \underline{\theta}^N$, which from the denominators of both equations can be inferred to hold for a wide range of specifications in that the cooperative value is roughly half of the strategic Nash case. This means that there is a range of robustness parameter values given by $[\underline{\theta}^C, \underline{\theta}^N]$, in which an optimal policy is possible only in the cooperative case. Within this range, countries acting non-cooperatively do not find an optimal robust policy. The differentiated breakdown point under cooperative and non-cooperative solutions is an interesting and new finding within the robust control framework, and it becomes even more relevant under asymmetry, as we show in the next section.

5. Asymmetry in model confidence

So far we have considered the case of identical countries. However, the model allows us to introduce heterogeneity between countries, which has been studied in Moreno-Cruz et al. (2012). In particular, due to our modeling approach, we are able to consider the case of heterogeneity in the degree of model confidence (θ_i) or ambiguity aversion between the two countries. Notably this provides a new insight into the behavior of decision makers that have different attitudes towards deep uncertainty in their decision making process. We saw that under cooperation, asymmetry does not affect the global

[21] See the calibration values in Appendix A.

results, so we focus on the strategic Nash solution. Model uncertainty is directly connected to the level of confidence that each policy maker has in the model. In this section we study how the difference in the degree of model confidence between the two countries affects the optimal SRM levels.

Without loss of generality, we look at the case $\theta_1 < \theta_2$. We consider the case in which the fictitious malevolent agent (Nature) in country 1 "pushes" the lower bound of the robust rule to a level where ($\theta_1 < \underline{\theta}^N$), and thus it is impossible for the policy maker in this country to seek more robustness. On the other hand, we assume that in country 2 the policy maker is more confident about the model that he uses and the regulation is possible ($\theta_2 > \underline{\theta}^N$).

Proposition

Suppose $\theta_1 < \theta_2$ in the cooperative and the Nash frameworks, and consider problems (12) and (17), respectively. If $\theta_1 < \underline{\theta}$ and $\theta_1 + \theta_2 > \underline{\theta}$, then the formulation of a robust optimal policy is possible in cooperation but not feasible in Nash.[22]

This proposition confirms our intuition. At the cooperative solution, the regulator designs policy by taking into account the sum of the two robustness parameters $(\theta_1 + \theta_2)$. Therefore, even if θ_1 is below the breakdown point but the sum is above, robust control is feasible. At the non-cooperative solution, however, if θ_1 is below the breakdown point, robust control is not possible for country 1 and therefore the Nash robust optimal policy is not feasible. In the particular case given in the proposition, we have the possibility that only under cooperation can a robust policy be found. Thus, a regulator with serious concerns about model misspecification will apply the mix of optimal policies (emissions and SRM) if she cooperates with another regulator who trusts her model more, and it is possible to have a solution for the cooperative problem.

Now we solve the model for a range of different values for the robustness parameter, and we obtain the following picture of SRM implementation between countries shown in Fig. 5. Note that we keep the value of the robustness parameter unchanged for country 1 $(\theta_1 = 1)$, and we vary the value for country 2 in both directions.

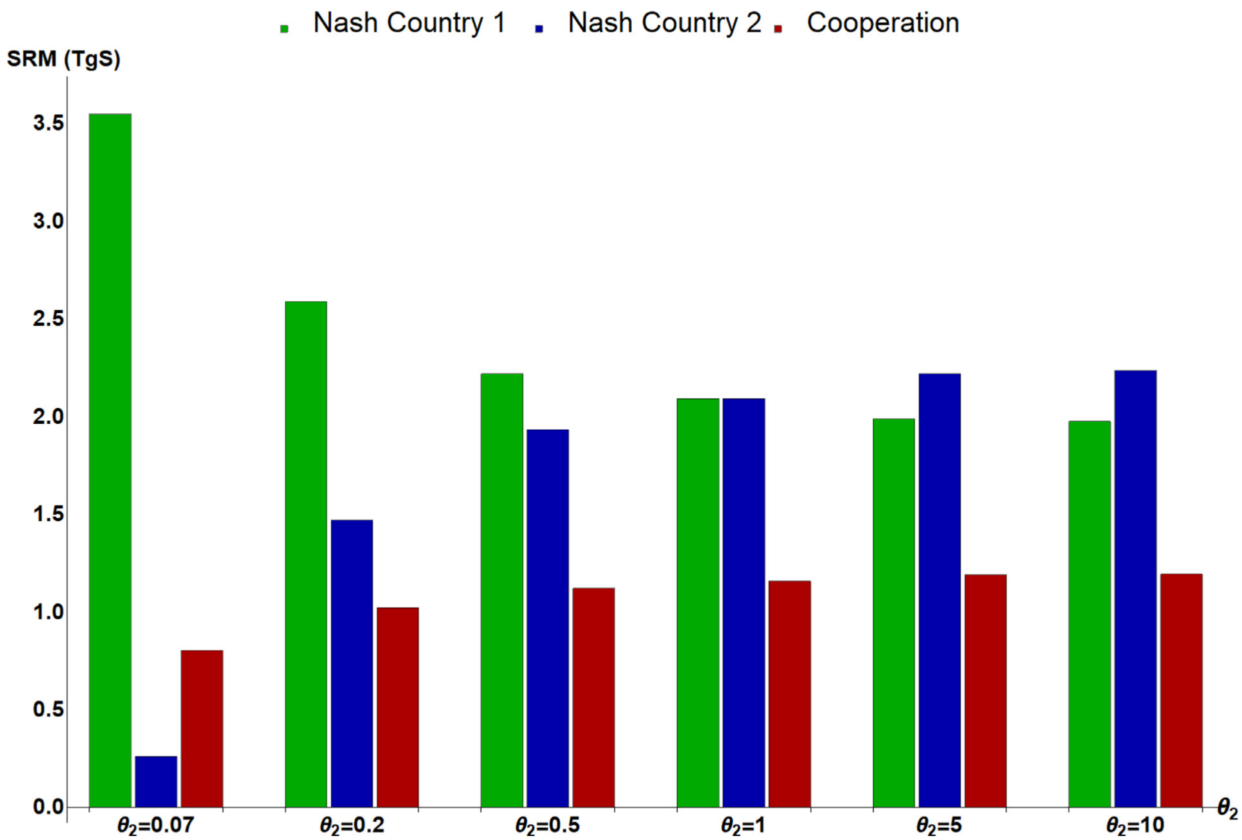

Fig. 5. Optimal SRM implementation under asymmetry in model confidence $(\theta_1 = 1)$.

First, observe that, as before, SRM is lower under cooperation in all cases. In the strategic Nash solution, the country with the lower robustness parameter, which is less confident about knowing the precise SRM impacts, will implement substantially less SRM than the more confident country. This result reaffirms the reasoning of Victor (2008) that the most confident or optimistic country will implement SRM. Note that in the symmetric case, both countries apply exactly the same amount of SRM, since both countries have the same robustness parameter $(\theta_1 = \theta_2 = 1)$. In the case where the difference in the

[22] For the proof of the Proposition, see Appendix G.

degree of confidence is high (for $\theta_2 = 0.07$ or 0.2 while $\theta_1 = 1$), the difference in the amount of optimal SRM between the two countries is large. In this case, country 2 has serious doubts about the model (the robustness parameter is low) and will use a much lower SRM level than country 1. Thus, the introduction of different levels of confidence in our model shows that model uncertainty is an important factor affecting the optimal decision of a policy maker, in particular when countries act strategically.

6. Conclusion

Climate engineering – and in particular SRM – is a controversial technology, but could be an attractive alternative solution to dealing with the consequences of global warming, due to its potentially cheap and fast applicability. The discussions about the use of SRM, however, are often focused on its potential damages or side effects. We develop a model that explores the range of SRM-induced damages in the environment. Moreover, we study the important feature of deep uncertainty surrounding SRM in a robust control framework, and we consider that countries can act cooperatively or strategically.

We develop the optimality conditions for cooperation and strategic behavior using an analytical model. Then we use numerical simulations in order to obtain better insights about deviations between cooperative and noncooperative outcomes and the relative magnitudes of the effects.

Our results suggest that deep uncertainty about impacts from SRM can be important, and leads to a reduction in SRM deployment under both cooperation and strategic behavior relative to the pure risk case. Under deep uncertainty, emissions and SRM are still higher when countries act strategically, but the deviation between SRM in the Nash and the cooperative solutions decreases when ambiguity increases. This result suggests that accounting for deep uncertainty could bring cooperative and strategic outcomes closer together. Moreover, if there is asymmetry in model confidence, or different ambiguity attitudes, the more confident country tends to increase SRM relative to the less confident country in the non-cooperative solution. Our model therefore provides some insights regarding situations in which SRM action could be undertaken unilaterally, as has been indicated by Weitzman (2015) "free driving"å argument. Under asymmetry in ambiguity attitudes, aggregate SRM – when countries act strategically – tends to be higher relative to the symmetric case. This result suggests that deep uncertainty becomes a more severe problem with heterogeneous countries in a strategic setting.

Our current model could be extended in several different directions. One possibility would be to include adaptation to climate change as a new instrument for reducing damages from temperature increase and to explore the potential trade-offs between SRM and adaptation. Another would be to consider a Ramsey-type problem where damages from temperature increase and SRM would affect output, or total factor productivity. This will lead to a fully nonlinear setup which would require more advanced numerical techniques. Finally, it might be interesting to explore whether the possibility of SRM, especially under a large number of asymmetric countries, would affect the formation and the stability of coalitions regarding international environmental agreements.

Appendix A. Numerical values of the parameters

Parameter	Description	Value	Unit
τ	Slope of social marginal damage cost from a temperature increase[a]	77.92	10^9\$/GtC
A	Intercept of marginal benefit from emissions	174	\$/tC
B	Slope of marginal benefit from emissions	1.5	$10^9\$/(\text{GtC})^2$
ϕ	Sensitivity of global mean temperature to SRM	−0.5	°C/TgS
λ	Sensitivity of temperature to emissions	0.1	°C/GtC
δ	Heat dissipation parameter	0.18	W/Km^2
ρ	Pure rate of time preference	0.01	scalar
σ	Standard deviation of $u_i(t)$	0.002	scalar
$\eta(1-\gamma)$	Marginal impacts from SRM[b]	0.085	1/TgS
m	Adjustment rate of SRM impacts	1.2	scalar
β	Slope of marginal cost from SRM[c]	40	10^9\$/TgS
ζ	Marginal SRM damage parameter[b]	1000	10^9\$/TgS
T_0	World average temperature, 2005	14	°C

[a] The marginal damage is based on the DICE model which uses a marginal damage of 0.00267% of GDP and the gross world product for 2100 of 29185 trillion USD.

[b] Firstly, $\eta(1-\gamma)$ is used to calibrate the steady state of the model, and found to be around 6% of GDP per TgS. Secondly, ζ converts the relative impacts at the calibration point of 1TgS (see Goes et al., 2011) into total USD value using the GDP used also for the utility function based on Karp and Zhang (2006).

[c] This parameter is calibrated such that at 1TgS, the average estimate of 10 billion USD per teragram of sulfur is obtained (see Bickel and Agrawal, 2013; Gramstad and Tjøtta, 2010; McClellan et al., 2012; Robock et al., 2009).

Appendix B. Evolution of the marginal SRM impacts

The marginal damages from SRM under model misspecification are

$$du_i(t) = \left\{ \left[(\eta(1-\gamma)) \sum_{i=1}^{2} z_i^*(t) - mu_i(t) \right] - \left[\frac{\sigma^2}{\theta}\mu_{1i}(\theta) + \frac{2\sigma^2}{\theta}\mu_{2i}(\theta)u_i(t) + \frac{\sigma^2}{\theta}\kappa_i(\theta)T \right] \right\} dt + \sigma dW_i(t) \tag{B.1}$$

where

$$h_i^*(\theta) = -\frac{\sigma V_{ui}^N}{\theta} = -\frac{\sigma(\mu_{1i}(\theta) + 2\mu_{2i}(\theta)u_i + \kappa_i(\theta)T)}{\theta}$$

and

$$z_i^* = \frac{\eta(1-\gamma)V_{i,u_i}^N - 2\zeta u_i + \phi V_{i,T}^N}{\beta}.$$

The Ornstein–Uhlenbeck diffusion of (B.1) is given by

$$du_i(t) = \pi(\theta)\big(\varphi(\theta) - u_i^*(t)\big)dt + \sigma dW_i, \tag{B.2}$$

where

$$\varpi(\theta) = \frac{\theta(\beta m + 2\eta(1-\gamma)(\zeta - 2\mu_2(\theta)\eta(1-\gamma)) - \phi\kappa_i(\theta))}{\theta\beta} + \frac{2\sigma^2\mu_2(\theta)}{\theta},\ \pi(\theta) = \frac{1}{\varpi(\theta)} \tag{B.3}$$

$$\psi(\theta) = \frac{2\phi\theta\eta(1-\gamma)(\nu_{1i}(\theta) + 2T\nu_{2i}(\theta)) + (T\kappa_i(\theta) + \mu_1(\theta))\big(2\theta(\eta(1-\gamma))^2 - \beta\sigma^2\big)}{\theta\beta},\ \varphi(\theta) = \frac{\psi(\theta)}{\varpi(\theta)}. \tag{B.4}$$

Appendix C. Relative entropy and the constraint robust control problem

In Hansen and Sargent (2001), the discrepancy between the two measures $\mathcal{G}$ and $\mathcal{Q}$ can be measured through the discounted relative entropy defined as

$$\mathcal{R}(\mathcal{Q}) = \int_0^\infty e^{-\rho t}\frac{1}{2}\mathbf{E}_{\mathcal{Q}}(h_i(t))^2 dt, \tag{C.1}$$

where $\mathbf{E}$ denotes the expectation operator and ρ the discount rate. To allow for the notion that even when the model is misspecified the benchmark model remains a “goodâ approximation, the misspecification error is constrained so that we only consider distorted probability measures $\mathcal{Q}$, such that the relative entropy is bounded by a value ξ:

$$\mathcal{R}(\mathcal{Q}) = \int_0^\infty e^{-\rho t}\frac{1}{2}\mathbf{E}_{\mathcal{Q}}(h_i(t))^2 dt \le \xi < \infty. \tag{C.2}$$

By modifying the value of ξ in (C.2), the decision maker can control the degree of maximum model misspecification (ξ) that he is willing to consider. In particular, if the decision maker can use physical principles or statistical analysis to formulate bounds on the relative entropy of plausible probabilistic deviations from his benchmark model, these bounds can be used to calibrate the parameter ξ (see Section 4.2).

A way to measure and quantify the effect of the model misspecification is by deriving the relative entropy. Relative entropy can be regarded as a measurement of the misspecification error. By the integration of the optimal distortion h_i^*, we determine an entropy measure and we can quantify the degree of the model misspecification that the decision maker is willing to consider. To do so, we compute the solution of the stochastic differential equation (B.2), which has a unique solution, as

$$u_i^*(t) = u(0)e^{-\pi(\theta)t} + \varphi(\theta)\big(1 - e^{-\pi(\theta)t}\big) + \sigma\int_0^t e^{-\pi(\theta)(t-s)}dW_s.$$

Given the mean and variance of $u_i^*(t)$, we can define the closed-form expression for the relative entropy of our model as

$$\mathcal{R}(\mathcal{Q}) = \int_0^\infty e^{-\rho t}\frac{1}{2}\mathbf{E}_{\mathcal{Q}}(h^*(t))^2 dt, \tag{C.3}$$

where

$$h^*(t) = -\frac{\sigma V_{u_i}^N}{\theta} = -\frac{\sigma(\mu_1(u^*(\theta)) + 2\mu_2(u^*(\theta)))}{\theta}, \tag{C.4}$$

so we have that

$$\mathcal{R}(\mathcal{Q}) = \frac{1}{2}e^{-\rho t}\int_0^\infty \mathbf{E}_{\mathcal{Q}}\left(-\frac{\sigma(\mu_1(u^*(\theta)) + 2\mu_2(u^*(\theta)))}{\theta}\right)^2 dt. \tag{C.5}$$

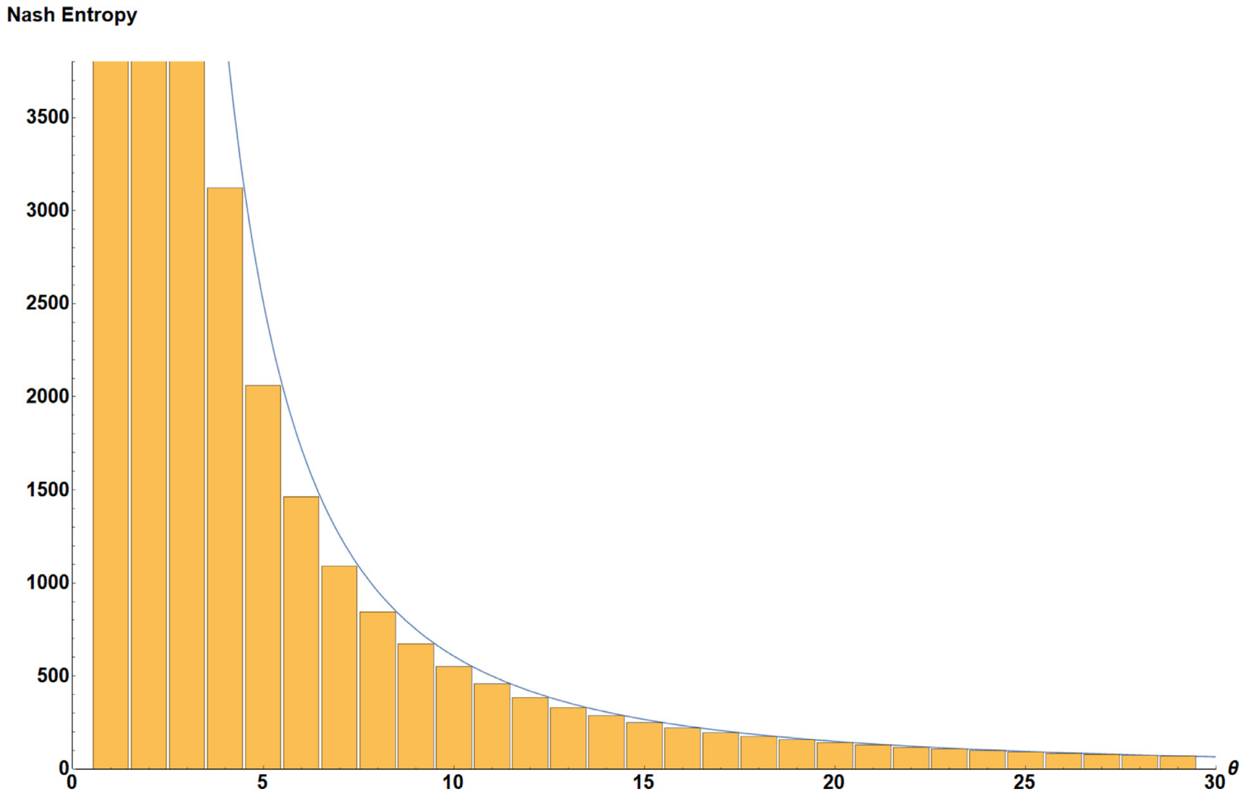

Fig. 6. Relative entropy in the symmetric Nash solution.

This measure can be seen as the relative difference between the pure risk and the deep uncertainty case. Fig. 6 shows the evolution of the entropy in non-cooperation for different values of θ. For low levels of the robustness parameter θ, the misspecification error is high and hence the ambiguity effect on the optimal policy is large. Thus a range of values, in which the ambiguity aversion and the misspecification error are relevant in our model, is the range until a value of around ten. In the context of model misspecification, one can derive two robust control problems, the constraint and the multiplier robust control problem, which represent equivalent ways of defining the problem. Following Hansen et al. (2006), we define the (alternative) constraint robust control problem by

$$\max_{E_i(t),z_i(t)} \min_{h_i(t)} \mathbf{E_0} \int_0^{\infty} e^{-\rho t}[U(E_i(t)) - C(z_i(t)) - D_T(T(t)) + D_z(z_i(t), u_i(t))]dt \;,\; i = 1, 2, \tag{C.6}$$

subject to

$$dT(t) = \left[\lambda \sum_{i=1}^{2} E_i(t) + \phi \sum_{i=1}^{2} z_i(t) - \delta T(t)\right]dt$$

$$du_i(t) = \left[\eta(1-\gamma)\sum_{i=1}^{2} z_i(t) - mu_i(t) + \sigma h_i(t)\right]dt + \sigma dW_i \tag{C.7}$$

$$\int_0^{+\infty} e^{-\rho t}\frac{1}{2}\mathbf{E}_{\mathcal{Q}}(h_i(t))^2 dt \leq \xi \;\; u_0 = u(0).$$

As shown in Hansen et al. (2006), if we assume that there exists a solution (E^*, z^*, h^*) to the robust multiplier problem, then (E^*, z^*) also solves the constraint robust control problem with $\xi = \xi^* = \mathcal{R}(\mathcal{Q}^*)$ and there exists a θ^* such that the robust multiplier and constraint problems have the same solution. Thus, according to Theorem 6.8.1 from Hansen and Sargent (2008), the problems (C.7) and (11) have the same solution, which directly associates an entropy bound $\mathcal{R}(\mathcal{Q}, \theta)$ with a given value of the ambiguity parameter θ.

Appendix D. Properties of the value function

The value function of the symmetric Nash problem $\left(\theta_i=\theta_j=\theta\right)$ satisfying (12) has the following simple quadratic form:

$$V_i^N(\theta,u_i,T)=\varepsilon_{0i}(\theta)+\mu_{1i}(\theta)u_i+\mu_{2i}(\theta)u_i^2+\nu_{1i}(\theta)T+\nu_{2i}(\theta)T^2+\kappa_i(\theta)u_iT, \tag{D.1}$$

where the coefficients can be computed as

$$\mu_{1i}(\theta)=\frac{\theta(\beta\lambda\kappa_i(A+\lambda\nu_{1i}))+B\left(\beta\lambda\kappa_iE_j^*+\left(\beta z_j^*+\phi\nu_{1i}\right)(2\mu_{2i}\eta(1-\gamma)-\zeta+\phi\kappa_i)\right)}{B\left(\theta(\beta(m+\rho)+\eta(1-\gamma)(\zeta-2\eta(1-\gamma)\mu_{2i}+\phi\nu_{2i}))+2\beta\sigma^2\mu_{2i}\right)}$$

$$\Rightarrow \mu_{1i}(\theta)\le 0$$

$$\mu_{2i}(\theta)=\frac{\theta(2\eta(1-\gamma)(\zeta-\phi\kappa_i)+\beta(2m+\rho))-\beta\sqrt{R_1^N}}{4\left((\eta(1-\gamma))^2\theta-\beta\sigma^2\right)}\le 0,$$

where

$$R_1^N=\frac{1}{\theta\beta^2}\times\left(\theta B(2\eta(1-\gamma)(\zeta-\phi\kappa_i)+\beta(2m+\rho))^2-4\left(\theta(\eta(1-\gamma))^2-\beta\sigma^2\right)\left(\beta\lambda^2\kappa_i^2+B(\zeta-\phi\kappa_i)^2\right)\right)$$

and

$$\nu_{1i}(\theta)=\frac{1}{\theta\left(B(\beta(\delta+\rho))-\phi(\eta(1-\gamma)\kappa_i+2\phi\nu_{2i})-2\beta\lambda^2\nu_{2i}\right)}$$
$$\times\left\{2\theta\beta\lambda A\nu_{2i}+B\left[\theta\eta(1-\gamma)\left(\beta\kappa_iz_j^*+\kappa_i\mu_{2i}\eta(1-\gamma)+2\phi\mu_{2i}\nu_{2i}\right)\right.\right.$$
$$\left.\left.+\beta\left(2\theta\left(\lambda\nu_{2i}E_j^*+\tau T_0+\phi\nu_{2i}z_j^*\right)-\sigma^2\mu_{2i}\kappa_i\right)\right]\right\}$$

$$\Rightarrow \nu_{1i}(\theta)\le 0$$

$$\nu_{2i}(\theta)=\frac{B(\beta(2\delta+\rho)-2\eta(1-\gamma)\phi\kappa_i)-\beta\sqrt{R_2^N}}{4\left(\beta\lambda^2+B\phi^2\right)}\le 0$$

where

$$R_2^N=\frac{1}{\theta B\beta^2}\left(\theta B(\beta(2\delta+\rho)-2\eta(1-\gamma)\phi\kappa_i)^2-4\left(\kappa_i^2\left(\theta(\eta(1-\gamma))^2-\beta\sigma^2\right)-2\theta\tau\beta\right)\left(\beta\lambda^2+B\phi^2\right)\right).$$

Finally,

$$\kappa_i(\theta)=\frac{\beta\left(m+\delta+\rho-\dfrac{2\mu_{2i}\lambda^2}{B}+\dfrac{\eta(1-\gamma)(\zeta-2\eta(1-\gamma)\mu_{2i})-2\phi^2\nu_{2i}}{\beta}\right)-\sqrt{R_3^N}}{2\eta(1-\gamma)\phi},$$

where

$$R_3^N=\frac{1}{\theta^2B^2\beta^2}\times[8\phi^2\theta^2B^2\eta(1-\gamma)\nu_{2i}(\zeta-2\eta(1-\gamma)\mu_{2i})$$
$$+(B(m\beta\theta+2\beta\sigma^2\mu_{2i}+\theta(\beta(\delta+\rho)+\eta(1-\gamma)(\zeta-2\eta(1-\gamma)\mu_{2i})-2\phi\nu_{2i}))-2\theta\beta\lambda^2\nu_{2i})^2].$$

Appendix E. Strategic substitutability of climate policy options

The value function for each country is

$$V_i^N(u_i,T)=\varepsilon_{0i}+\mu_{1i}u_i+\mu_{2i}u_i^2+\nu_{1i}T+\nu_{2i}T^2+\kappa_iu_iT,\quad i=1,2,$$

with derivatives $V_{i,u_i}^N=\mu_{1i}+2\mu_{2i}u_i+\kappa_iT,\ V_{i,u_iu_i}^N=2\mu_{2i}$ *and* $V_{i,T}^N=\nu_{1i}+2\nu_{2i}T+\kappa_iu_i.$

The optimal values of E_i and z_i in Nash can be expressed as reaction functions in SRM and emissions of country i depending on player $j \neq i$:

$$z_i^*\left(E_j^*, z_j^*\right) = \frac{\eta(1-\gamma)V_{i,u_i}^N - 2\zeta u_i + \phi V_{i,T}^N}{\beta} \tag{E.1}$$

$$E_i^*\left(E_j^*, z_j^*\right) = \frac{A + \lambda V_{i,T}^N}{B}. \tag{E.2}$$

From (E.1) and (E.2) we have

$$\frac{\partial z_i^*}{\partial z_j^*} = \frac{\eta(1-\gamma)\frac{\partial \mu_{1i}}{\partial z_j^*} + \phi\frac{\partial \nu_{1i}}{\partial z_j^*}}{\beta} < 0 \tag{E.3}$$

$$\frac{\partial E_i^*}{\partial z_j^*} = \frac{\lambda\frac{\partial \nu_{1i}}{\partial z_j^*}}{B} < 0, \tag{E.4}$$

where

$$\frac{\partial \mu_{1i}}{\partial z_j^*} = \frac{\theta\beta(2\mu_{2i}\eta(1-\gamma) - \zeta + \phi\kappa_i)}{\theta(\beta m + \eta(1-\gamma)(2\eta(1-\gamma)\mu_{2i} - \zeta + \phi\kappa_i)) + \beta\left(\theta\rho + 2\sigma^2\mu_{2i}\right)} < 0,$$

$$\frac{\partial \nu_{1i}}{\partial z_j^*} = \frac{B\beta(\eta(1-\gamma)\kappa_i + 2\phi\nu_{2i})}{B(\beta(\delta+\rho) - \phi(\eta(1-\gamma)\kappa_i + 2\phi\nu_{2i})) - 2\beta\lambda^2\nu_{2i}} < 0$$

and

$$\frac{\partial E_i^*}{\partial E_j^*} = \frac{\lambda\dfrac{\partial \nu_{1i}}{\partial E_j^*}}{B} < 0, \tag{E.5}$$

where

$$\frac{\partial \nu_{1i}}{\partial E_j^*} = \frac{2B\beta\lambda\nu_{2i}}{B(\beta(\delta+\rho) - \phi(\eta(1-\gamma)\kappa_i + 2\phi\nu_{2i})) - 2\beta\lambda^2\nu_{2i}} < 0.$$

Appendix F. The "breakdown point"

The coefficients of the value functions under symmetry for cooperation and Nash respectively are

$$\mu_2^C\left(\theta^C\right) = \frac{\theta^C(4\eta(1-\gamma)(2\zeta - \phi\kappa_i) + \beta(2m+\rho)) - \beta\sqrt{R^C}}{4\left(2(\eta(1-\gamma))^2\theta^C - \beta\sigma^2\right)},$$

where

$$R^C = \frac{1}{\theta^C\beta^2}\left(\theta^C B(4\eta(1-\gamma)(2\zeta - \phi\kappa_i) + \beta(2m+\rho))^2 - 8\left(2\theta^C(\eta(1-\gamma))^2 - \beta\sigma^2\right)\left(\beta\lambda^2\kappa_i^2 + B(2\zeta - \phi\kappa_i)^2\right)\right),$$

and

$$\mu_{2i}^N\left(\theta^N\right) = \frac{\theta^N(2\eta(1-\gamma)(\zeta - \phi\kappa_i) + \beta(2m+\rho)) - \beta\sqrt{R_1^N}}{4\left((\eta(1-\gamma))^2\theta^N - \beta\sigma^2\right)}.$$

In order to avoid $\mu_2^C\left(\theta^C\right), \mu_{2i}^N\left(\theta^N\right) \longrightarrow \infty$, we need to ensure that

$$4\left(2(\eta(1-\gamma))^2\theta^C - \beta\sigma^2\right) \neq 0,$$

and

$$4\left((\eta(1-\gamma))^2\theta^N - \beta\sigma^2\right) \neq 0.$$

Following Hansen and Sargent (2008) and Hansen et al. (2006), straightforward algebra shows that in cooperation the value function is well-defined for $\theta^C > \frac{\beta\sigma^2}{2(\eta(1-\gamma))^2}$ and diverges for $\theta^C = \frac{\beta\sigma^2}{2(\eta(1-\gamma))^2}$, and in Nash the value function is well-defined for $\theta^N > \frac{\beta\sigma^2}{(\eta(1-\gamma))^2}$ and diverges for $\theta^N = \frac{\beta\sigma^2}{(\eta(1-\gamma))^2}$.

Hence the Hansen-Sargent breakdown points in cooperation and Nash are equal to

$$\underline{\theta}^C = \frac{\beta\sigma^2}{2(\eta(1-\gamma))^2} < \theta^C \text{ and } \underline{\theta}^N = \frac{\beta\sigma^2}{(\eta(1-\gamma))^2} < \theta^N.$$

Appendix G. Proof of asymmetry in model confidence

Cooperation

In the cooperative solution, the policy maker could have different θs for each country, but he considers only one distortion of the SRM dynamics (10) and therefore decides about one h^*. Different concerns about model misspecification are expressed by different θs. From the minimization of problem (12), we have that $h^* = -\frac{\sigma V_u^C}{\theta_1+\theta_2}$. At the cooperative solution, $\theta^* = \theta_1 + \theta_2$, so even if θ_1 is below the minimum threshold ($\underline{\theta}$) the sum of the policy maker's robustness parameters might be above the lower bound, and regulation in this case is possible at the cooperative solution ($\underline{\theta} < \theta^*$). Thus a policy maker with high concerns about model misspecification ($\theta_1 < \underline{\theta}$) will apply the mix of optimal policies (emissions and SRM) if he cooperates with another policy maker who trusts his model more ($\theta_2 > \underline{\theta}$). If the countries cooperate, there is a higher possibility to have a solution of the problem ($\theta_1 + \theta_2 > \underline{\theta}$), even if the policy maker of one of the countries has high concerns about his model.

Nash

In the non-cooperative Nash solution, the policy maker of each country decides independently and Nature chooses a different misspecification error $\left(h_i^*\right)$ for each agent. Thus the different θs for each agent consider different distortions by Nature to the SRM dynamics (10). From the minimization of problem (12), we have that $h_i^* = -\frac{\sigma V_{ui}^N}{\theta_i}$. It is obvious that if θ_1 is below the threshold regulation ($\underline{\theta}_1 < \theta$) at the Nash equilibrium, the regulation might not be possible $\left(h_i^* : V_i^N(t) \longrightarrow -\infty\right)$.

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