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The Economic and Demographic Transition, Mortality, and Comparative Development

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This paper develops a quantifiable unified growth theory to investigate cross-country comparative development. The calibrated model can replicate the historical development dynamics in forerunner countries like Sweden and the patterns in cross-country panel data. The findings suggest a crucial role of the timing of the onset of the economic and demographic transition for explaining differences in development. Country specific differences in extrinsic mortality are a candidate explanation for differences in the timing of the take-off across countries and the resulting worldwide comparative development patterns, including the bi-modal distribution of the endogenous variables across countries.

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Explaining the differences in economic development across the world is a central objective of research in macroeconomics. While there exists a considerable body of investigations of the determinants of long-run development, as discussed in Section I, the empirical literature has only been loosely connected to unified growth theories that investigate non-linear development dynamics and the mechanics behind the endogenous transition from stagnation to sustained growth. The primary contribution of this paper is to carry out a systematic quantitative analysis of a prototype unified growth model, and to study the implications of non-linear dynamics and of delays in the development process, for cross-country comparative development. The underline hypothesis is that different countries follow a similar non-linear development process, as suggested by the striking similarities in the economic and demographic transition, but crucially differ in the actual timing of take-off to sustained growth. The analysis investigates, in particular, the role of differences in country-specific extrinsic mortality as a potential determinant of comparative development patterns.

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The analysis proceeds in three steps. Section II proposes a simple unified growth theory that can be used to quantitatively investigate the patterns of comparative development in the relevant economic and demographic variables. The framework is based on an occupational choice model with unskilled and skilled human capital and endogenous differential fertility. Bidirectional feedbacks between the education composition of the population, technological progress and demographic change (in particular mortality) eventually trigger a growth take-off. The paper contributes a simple prototype unified growth model with analytical predictions that are in line with stylized facts of long-run economic development, in terms of income and education, and demographic conditions, in terms of adult longevity, child mortality, and (gross and net) fertility.

In Section III.A the model is calibrated to the long-run development patterns of Sweden, the textbook case of long-run economic and demographic development. The calibration strategy involves setting the (time-invariant) parameters of the model by targeting data moments on the balanced growth path in 2000 and before the onset of the transition around 1800. The simulated model produces the endogenous evolution of the economy over a long period of time (from year 0 to year 2000) that includes the onset of the transition and the convergence to the balanced growth path. Section III.B compares the simulated and the actual data for Sweden over the period 1760-2000. The targeted moments (most of them referring to 2000 and a subset referring to year 1800) are matched by construction. The comparison between simulated and actual data along the full transition from stagnation to sustained growth, which is not matched by construction, documents the ability of the model to reproduce the stylized historical data patterns, including those that had been more difficult to match quantitatively (like, e.g., the reduction in net fertility).

Section IV explores the ability of the model to account for cross-country comparative development patterns. This analysis effectively constitutes an exploration of the quantitative implications of a prototype unified growth model since no data moment of the cross-country analysis are targeted for the calibration. The results are therefore informative on the model's fit to empirical patterns "out of sample". Section IV.A investigates the hypothesis that all countries follow a similar non-linear development path with a main difference being the timing of the transition. The simulated correlations between education and other equilibrium variables, including adult longevity, fertility, and income per capita are compared to the respective correlations in cross-country data in 1960 and 2000. Despite the underlying non-linear development dynamics, the simulated and empirical data display monotonic, and almost linear, cross country correlations between the equilibrium share of educated individuals and all other central variables. The model also reproduces other stylized cross-sectional data patterns whose mechanics are debated in the literature, such as a hump-shaped relationship between life expectancy and the subsequent change in the education composition, and the well documented concave relationship between income per capita and life expectancy known as the "Preston Curve".

Section IV.B quantitatively evaluates the role of exogenous cross-country differences in mortality ("baseline longevity") by performing controlled quantitative exercises. We simulate a counterfactual economy that differs from the benchmark model only in terms of the exogenous mortality environment, and that is calibrated by targeting data moments for the countries with the highest mortality in 2000 (rather than for Sweden in 1800). The results document that empirically reasonable cross-country differences in baseline longevity can result in substantial delays (of more than a century) of the economic and demographic transition, despite leaving the cross-sectional relationships, including the Preston Curve, essentially unaffected. The analysis also illustrates that the timing of development of the countries belonging to the different continents (with Europe on one extreme and Africa on the other) is coherent with their average exposure to human pathogens. The results provide a link between the unified growth literature and the empirical literature on the fundamental determinants of long-run development and contribute to the empirical debate about the role of life expectancy for development.

Finally, Section IV.C presents the results from the simulation of an artificial world composed of countries that differ in terms of baseline longevity, but that are otherwise identical to the benchmark calibration for Sweden. The results deliver cross-country distributions of all variables of interest that match the actual worldwide distributions, which are bi-modal in 1960 and rather uni-modal in 2000. The findings show that the unified growth framework provides a natural explanation for the changing bi-modality of the distributions in the different central variables due to the changes of all variables during the transition to the balanced growth path. The quantitative results also suggest the existence of an acceleration in the development path of today's developing countries, compared to the historical experience of the European forerunners, in terms of demographic conditions (mortality and fertility) and (to a lower degree) economic development.

The analytical derivations and proofs are relegated to the Appendix. The details of the calibration, the data sources and additional material are made available in the Online Supplementary Material.

I. Related Literature

Addressing the research question of this paper requires a model that is able to reproduce the main stylized facts of the economic and demographic transitions and that is suitable for a quantitative analysis of the long run development path, including the endogenous transition phase. The general equilibrium framework presented below is based on an occupational choice model with unskilled and skilled human capital. Education and fertility decisions crucially depend on economic and demographic conditions (in particular technology and mortality) whose dynamics are modeled through intergenerational externalities. The resulting prototype unified growth framework builds on Galor and Weil (2000), Cervellati and Sunde (2005) and Soares (2005).[1] The model features differential fertility across different education groups, similar to de la Croix and Doepke (2003), and produces qualitative predictions that are in line with the stylized facts.[2]

The paper contributes to the existing literature of quantitative studies of long-term development. A quantitative analysis that exploits comparative statics around the balanced growth path of Barro-Becker-type models as in most of the existing literature (see Jones, Schoonbroodt, and Tertilt, 2011, for a survey) is unsuitable for the purpose of this paper, which requires studying the endogenous take-off of the transition from quasi-stagnation to growth. The logic of analysis is closer in spirit to the unified growth studies presented by Lagerlof (2003), Doekpe (2004), Strulik and Weisdorf (2008), and de la Croix and Licandro (2012), which simulate the dynamic long-run evolution of an economy, including the demographic dynamics. One advantage of the prototype model presented in this paper is that it can reproduce the patterns of the economic and demographic transition (in terms of both fertility and mortality) with a parsimonious set of time invariant parameters that have a clear economic interpretation and that can be calibrated targeting observable data

[1]See also Doepke (2004) for an investigation of the interplay between technology and fertility; de la Croix and Licandro (1999), Kalemli-Ozcan, Ryder and Weil (2000), and Boucekkine, de la Croix and Licandro (2002, 2003) for investigations of the role of mortality; and Aiyar, Dalgaard, and Moav (2008) for a modeling of technology dynamics in pre-transitional economies. Changes in mortality have also been modelled as resulting from rational investments in health, see de la Croix and Licandro (2012) and Dalgaard and Strulik (2014).

[2]In particular, the theory can rationalize the observed drop of net fertility below the pre-transitional levels which is one of the defining elements of the demographic transition as conceptualized by demographers, see e.g. Chesnais (1992). This fact has proved difficult to rationalize with fertility theories based on the quantity-quality trade-off or child mortality, see Kalemli-Ozcan (2003) and Doepke (2005). Building on an occupational choice framework, rather than only a quantity-quality, the proposed unified growth theory can generate changes in net fertility without having to impose restrictive assumptions on the utility function, see also Mookherjee, Prina and Ray (2012).

moments.[3]

The calibrated model is suitable not only for the analysis of the evolution of one economy over time, as the literature cited above, but also of the investigation of the implied cross-country patterns at different moment in time. The solution of the prototype unified growth model presented below is always interior, so that the take-off is not generated by the exit from corner solutions of the dynamic system. This technical feature permits conducting smooth comparative statics on the main variables of interest, such as baseline mortality, and investigating their role for the differential delays in take-off. In this context, the paper contributes to the few quantitative papers in the literature that investigate the timing of the transition, such as Ngai (2004) and Chakraborty et al. (2010). To our knowledge, this paper offers the first application of a unified growth framework for the quantitative analysis of cross-country comparative development patterns.

The quantitative results shed new light on some unsettled empirical questions. Even moderate differences in extrinsic mortality can have relevant implications for comparative development patterns by delaying the economic and demographic transition. This result complements the evidence on the deep determinants of long-run development, see e.g. Spolaore and Wacziarg (2013) and Ashraf and Galor (2013)Ashraf and Galor (2013). By demonstrating that such differences in mortality have non-monotonic effects on education but leave the cross-sectional patterns between central variables, such as income, life expectancy, education, and fertility essentially unchanged, the results offer a rationale for the mixed empirical findings based on panel data and linear regression frameworks.[4] The results also show that the unified growth framework can generate the concave relationship between life expectancy and income per capita, the so called Preston Curve, whose underlying mechanisms are still not well understood.[5] Finally, the quantitative analysis provide a natural explanation for the observation of the bi-modality in the distribution of income, see Azariadis and Stachurski (2005) and life expectancy, see Bloom and Canning, (2007), across the world, and rationalizes the existence of similar bi-modalities in the distributions of education and fertility.

II. A Prototype Unified Growth Model

This section presents the theoretical framework with the functional forms that are applied in the calibration in Section III, even though they are not needed for the analytical results in Section II.B. The functional forms are specified in line with the previous literature and the available evidence, and to minimize the number of parameters.

A. Set up

Population Structure. — The economy is populated by a discrete number of generations of individuals denoted by $t \in \mathbb{N}^+$. There are two relevant subperiods in the life of an

[3]Previous quantitative works not based on a unified growth framework have investigated the long-run development dynamics by exogenously calibrating the overtime change of relevant variables (like technology or mortality) to match the empirical time series, see Eckstein, Mira and Wolpin (1999), Kalemli-Ozcan (2002), de la Croix, Lindh and Malmberg (2008), Bar and Leukhina (2010) and Cervellati and Sunde (2013). Our approach involves calibrating the time invariant parameters of the functional relationships that produce the (endogenous) evolution of the relevant variables (like technology and mortality) that are, therefore, not matched by construction.

[4]The evidence on the effect of life expectancy on growth is mixed in linear regression frameworks, see, e.g., Acemoglu and Johnson (2007) and Lorentzen, McMillan and Wacziarg (2008). These findings can be reconciled by explicitly accounting for the existence of non-monotonic dynamic relationships between life expectancy, population dynamics, and education that are consistent with the economic and demographic transition, see Cervellati and Sunde (2011a, 2011b and 2014).

[5]This relationship has been documented for the first time by Preston (1975)). See Deaton (2003) and Bloom and Canning, (2007b) for a discussion of the debate. To our knowledge, the only other theory that can provide a theoretical rationale for the Preston Curve is by Dalgaard and Strulik (2014).

individual: childhood, of length $K_t = k$, and adulthood, with duration T_t. Each individual of generation t survives to age k with probability $\pi_t \in (0, 1)$. Surviving children become adults, survive with certainty until age $k + T_t$, and then die. The variable T_t represents both life expectancy at age k and the maximum duration of adulthood.[6] In the model, T_t is a summary statistic of the effective time available during adulthood and can be also interpreted as an "health augmented" time endowment of adults.

Reproduction is asexual and takes place at age $m \geq k$, which is the length of a generation. A cohort of adults consists of a mass of agents of size $N_{t+1} = N_t \pi_t n_t$ where n_t is the average (gross) fertility of the parent cohort. Every individual of cohort t is endowed with an innate ability $a \in [0, 1]$, which is randomly drawn from a distribution $f(a)$. For the calibration of the model we assume a (truncated) normal distribution of ability with mean μ and standard deviation σ.

Preferences and Production. — During childhood individuals are fed by their parents and make no choices. At the beginning of adulthood, those individuals that survive childhood make decisions about their own education and their fertility to maximize their (remaining) lifetime utility. Individuals derive utility from own consumption, c, and the quality, q, of their (surviving) offspring, πn. The lifetime utility of an individual of generation t is additively separable and given by,

$$\int_0^{T_t} \ln c_t(\tau)\, d\tau + \gamma \ln(\pi_t n_t q_t) \quad (1)$$

where $\gamma > 0$ is the weight of the utility that parents derive from their surviving children relative to their own lifetime consumption as adults.[7]

We set the subjective discount rate to zero and assume that individuals perfectly smooth consumption within their adult period of life, $c_t(\tau) = c_t$ for all τ. This allows abstracting from the path of consumption during the life cycle which is of no primary relevance for the investigation of the long-term, intergenerational evolution of the economy.[8] The key feature of this formulation is that individuals can smooth consumption over their adult life, but they cannot perfectly substitute the utility from their own consumption with utility derived from their offspring.[9]

The inputs of production are *skilled* human capital, denoted by s, and *unskilled* human capital, denoted by u. We treat human capital as inherently heterogenous across generations. In line with the literature on vintage human capital, this reflects the view that individuals acquire their skills in environments characterized by the availability of a particular technology. The aggregate stocks of human capital of each type, H_t^u and H_t^s, supplied by generation t are used to produce the unique consumption good with a constant

[6]This modeling of child survival and adult longevity follows Soares (2005). It is formally iso-morphic to a "perpetual youth" modeling, where longevity is one over the age independent adult survival probability. Considering a deterministic longevity simplifies the set up by abstracting from uncertainty and, as discussed below, by allowing for a direct match between the simulated and empirical data for child mortality and life expectancy at age five. In the quantitative analysis, $1 - \pi_t$ corresponds to child mortality, and T_t corresponds to life expectancy at age five (so that $k = 5$). Assuming a constant death rate before age 5, life expectancy at birth is $\pi_t(5 + T_t)$.

[7]The utility formulation follows Soares (2005). As in Becker and Lewis (1973) parents derive utility from the quality of their children, which allows studying the change in the quantity-quality trade-off in the simplest way.

[8]See also Rogerson and Wallenius (2009) for a similar assumption, which is equivalent to assuming a small open economy facing a zero discount rate.

[9]The actual formulation of the utility function and the fact that longevity implicitly affects the weight of the utility from consumption and children is irrelevant for the results. As shown in a previous version of the paper, one could equivalently assume that individuals derive utility from average per period lifetime consumption and children as in Galor and Weil (2000).

returns to scale technology

$$Y_t = A_t \left[(1 - x_t)\,(H_t^u)^\eta + x_t\,(H_t^s)^\eta\right]^{\frac{1}{\eta}}, \tag{2}$$

where $\eta \in (0, 1)$. Generation t only operates the technological vintage t, which is characterized by the relative productivity of skilled human capital, $x_t \in (0, 1)$, and total factor productivity (TFP) A_t. The production function (2) is a specialized (CES) formulation of the vintage production function by Chari and Hopenhayn (1991). As in Boucekkine, de la Croix, and Licandro (2002), the vintage of technology is linked to generation-specific knowledge in terms of skilled and unskilled human capital.[10]

The returns to human capital are determined in general equilibrium on competitive markets and equal marginal productivity,

$$w_t^s = \frac{\partial Y_t}{\partial H_t^s}, \quad w_t^u = \frac{\partial Y_t}{\partial H_t^u}. \tag{3}$$

The level of human capital acquired by each individual is increasing in the level of innate ability, a, $h^j(a)$ with $dh^j(a)/da \geq 0$ for $j = \{u, s\}$. Individual ability is relatively more important in producing skilled human capital. This delivers a natural equilibrium sorting of the population into skilled and unskilled. For simplicity, we make the assumption that ability only matters for skilled human capital. An individual with ability a acquires $h^s(a) = e^{\alpha a}$ units of human capital if she decides to become skilled, and $h^u(a) = e^{\alpha\mu}$ if she decides to be unskilled. An individual that decides to become skilled, respectively unskilled, pays a fix cost, measured in term of adult time, of $\underline{e}^s > \underline{e}^u \geq 0$.[11]

Raising a child involves a time cost $r_t = \tilde{r}_t + \underline{r}$ where $\underline{r} > 0$ is a fix time cost that needs to be spent and $\tilde{r}_t \geq 0$ is the extra time that can be spent voluntarily in addition.[12] The time spent with a child increases the child's quality according to,

$$q_t(\underline{r}, \tilde{r}_t, g_{t+1}) = [\tilde{r}_t\delta(1 + g_{t+1}) + \underline{r}]^\beta \tag{4}$$

where $g_{t+1} = (A_{t+1} - A_t)/A_t$, $\beta \in (0, 1)$, and $\delta > 0$. The functional form (4) implies a complementarity between technical progress and the effectiveness of the extra time invested in children's (the quality time $\tilde{r}_t$). This formulation captures in the simplest way that faster technological progress increases the incentives to invest more time in raising children, as in Galor and Weil (2000).

The time available during adulthood is limited by adult longevity T_t, or by some exogenous limit to the number of years in the labor market (e.g., due to retirement), $R > 0$.[13] The effective time available for productive activities during adulthood is therefore bounded

[10] Vintage models that relax the assumption that human capital is perfectly homogenous across different age cohorts are empirically appealing in the context of long term development, where cohorts of workers of different age acquire knowledge of different technologies. This vintage structure is not needed for the main mechanism and the analytical results, but it allows for a transparent quantitative analysis as the optimal choices of acquiring human capital by generation t do not depend on the optimal choices of the (unborn) generations of workers that will enter in the labor market in the future.

[11] More complex skills may involve more costly processes of skill acquisition and maintenance. The crucial feature for the mechanism is that workers who decide to be skilled face a lower effective lifetime that is available for market work during their adulthood.

[12] Both increase quality but with different relative intensity. The cost $\underline{r}$ can be interpreted as the minimum investment required for the children to survive to adulthood and may include feeding (or dressing) the child. The extra investment $\tilde{r}_t$ can be interpreted as pure quality time that is not needed for survival like, e.g., talking, playing or reading a book with the child.

[13] The assumption of a limit R, which may be due to compulsory retirement or some other effective limitation to labor force participation at old ages is not needed for the main results but adds a realistic feature for the analysis of the quantitative role of bounds to productive life when longevity increases to old ages. In the quantitative analysis, the parameter R is calibrated exogenously to match the effective retirement age.

from above by $\overline{T}_t = \min\{T_t, R\}$. An individual with education $j = \{u, s\}$ cannot use more than the available time and cannot spend more than the total earnings for total consumption.[14] The budget constraint conditional on being skilled or unskilled, $j = \{u, s\}$, is thus given by

$$T_t c_t = \left(\overline{T}_t - \underline{e}^j - \pi_t n_t r_t\right) w^j_t h^j_t(a) \ . \tag{5}$$

The problem of an individual with ability a born in generation t is to choose the type of human capital to be acquired, $j \in \{u, s\}$, the number of children, n_t, and the time invested in raising each child, r_t, so as to maximize utility (1) subject to (5).

ADULT LIFE EXPECTANCY AND CHILD SURVIVAL. — In line with the available evidence, we consider a differential impact of human capital and income on adult and child mortality.[15]

Adult longevity of generation t is assumed to be increasing in the share of skilled individuals in the parent generation,

$$T_t = \Upsilon(\lambda_{t-1}) = \underline{T} + \rho\lambda_{t-1} \tag{6}$$

where $\underline{T}$ is the baseline longevity that would be observed in the economy in the absence of any skilled human capital, and $\rho > 0$ reflects the scope for improvement.[16] Since $\lambda \in (0, 1)$, the maximum level of adult longevity is given by $\overline{T} = \underline{T} + \rho$.

The child survival probability π_t depends on living conditions at the time of birth, as reflected by per capita income and parental education,

$$\pi_t = \Pi(\lambda_{t-1}, y_{t-1}) = 1 - \frac{1-\underline{\pi}}{1+\kappa\lambda_{t-1}y_{t-1}} \tag{7}$$

with $\kappa > 0$ and where $1 > \underline{\pi} > 0$ is the baseline child survival that would be observed in an economy with $\lambda_{t-1}y_{t-1} = 0$.[17]

TECHNOLOGY. — Technological progress, that takes place with emergence of a new vintage of technology characterized by TFP, A_t, and a higher relative weight of skilled human capital in the production process, x_t, is skill biased. The relative productivity of skilled human capital in production, x_t, increases with the share of skilled workers in the previous generation, λ_{t-1}, and with the scope for further improvement, $1 - x_{t-1}$,

$$\frac{x_t - x_{t-1}}{x_{t-1}} = X(\lambda_{t-1}, x_{t-1}) = \lambda_{t-1}(1 - x_{t-1}) \ . \tag{8}$$

[14]See Appendix A for the time and resource constraints.

[15]Environmental factors, in particular macroeconomic conditions, are crucial determinants of individual health. Child and adult mortality appear to be affected by the macro environment in different ways, however. Cutler et al. (2006) suggest that human capital is more important for adult longevity than per capita income since adult longevity depends on the ability to cure diseases and is related to the level of medical knowledge. Better living conditions in terms of higher incomes, but also in terms of access to water and electricity, are relatively more important for increasing the survival probability of children, see Wang (2003) for a survey.

[16]This reduced form modeling allows going beyond the assumption that changes in mortality are fully exogenous (as in, e.g. Jones and Schoonbroodt, 2010) in the simplest and most parsimonious way. The evolution of longevity could be made endogenous to human capital by extending the model to the consideration of optimal investments in health along the lines of de la Croix and Licandro (2012).

[17]Larger total income Y_{t-1} improves the probability of children reaching adulthood while population size N_{t-1} deteriorates living conditions and reduces child survival rates. Considerable evidence documents the negative effect of population density and urbanization on child mortality, especially during the early stages of the demographic transition, see Galor (2005).

For any λ_t, improvements are smaller as x_t converges to its upper limit at $x = 1$.

Finally, improvements in total factor productivity, A_t, are increasing with the share of skilled workers in the previous generation,[18]

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = G(\lambda_t) = \varphi\lambda_t \quad , \ \varphi > 0 . \tag{9}$$

B. Analytical Results

This section derives analytical results, in terms of optimal decisions, intra-generational general equilibrium and the dynamic evolution of the economy over time, that are needed for the interpretation of the quantitative analysis,.

EQUILIBRIUM FERTILITY. — The first order conditions uniquely identify the optimal fertility and the time spent raising children conditional on the type of human capital acquired by each individual.[19] The resulting average fertility in the population is given by

$$n_t^* = N(T_t, \lambda_t, \pi_t) = \frac{\gamma}{(T_t + \gamma)\, r_t^* \pi_t} \left[(1 - \lambda_t)(\overline{T}_t - \underline{e}^u) + \lambda_t(\overline{T}_t - \underline{e}^s)\right] \tag{10}$$

where λ_t denotes the share of individuals of generations t that acquire skilled human capital and r_t^* is the optimal time invested in children.[20]

To facilitate the interpretation of the quantitative results in Section III, let us briefly comment on the role of demographic variables for gross and net fertility. Gross fertility is decreasing in π_t through a *substitution effect* but net fertility is independent of π_t. The effect of adult longevity on fertility is more complex. Higher adult longevity T_t increases gross fertility as long as $T_t < R$ due to a positive *income effect*, but decreases gross fertility when $T_t \geq R$ as the income effect turns negative.[21] In addition, a higher T_t reduces fertility by a *differential fertility* effect if, as characterized next, it increases the share of skilled workers, λ_t, who have fewer children, see Skirbekk (2008) for evidence. The existence of differential fertility implies that increases in adult longevity may materialize in a reduction of both gross and net fertility. An indirect effect arises from the effect of the share of skilled individuals on technological progress.[22]

INTRA-GENERATIONAL GENERAL EQUILIBRIUM. — Agents with higher ability have a comparative advantage in acquiring skilled human capital. For any vector of wages there exists a unique ability threshold for which the indirect utilities from acquiring the two types of

[18]This can be seen as a reduced form of endogenous growth models such as Aghion and Howitt (1992), where φ can be interpreted as the average size of an innovation and the labor involved in research is increasing in λ_t.

[19]See Lemma 1 in the Appendix for the optimal individual choices and their derivation.

[20]As characterized in Lemma 1 in the Appendix, the optimal time investment in children is given by,

$$r_t^* = max \left\{ \underline{r}, \frac{1 - [1/(\delta(1 + g_{t+1}))]}{1 - \beta} \underline{r} \right\} \tag{11}$$

Along the lines of Galor and Weil (2000), when technical progress g_{t+1} is too low parents may optimally decide not to invest any extra time in raising their children beyond the minimum level, so that $r_t^* = \underline{r}$. Provided a positive extra time is invested in raising children, faster technological progress g_{t+1} increases r_t^* and reduces optimal fertility for unskilled and skilled individuals.

[21]Increases in longevity above R (so that $\overline{T}_t = R$) reduce fertility. A longer expected time in retirement requires devoting more income to consumption (to keep a constant consumption over the life cycle) thereby lowering fertility.

[22]Fertility is decreasing with the time invested in children, in line with a standard quantity-quality trade-off. The quantity-quality trade-off is not directly affected by adult longevity and child mortality, however, and the optimal time spent raising each child does not depend on the type of human capital acquired by parents. Parents substitute, however, quantity for quality in the face of technological progress, which depends on λ_t. Higher longevity therefore reduces fertility also indirectly changing the future parental investments in the quality of children.

human capital are equal. The corresponding unique share λ_t of individuals that find it optimal to acquire skilled human capital is increasing in the relative wage w_t^s/w_t^u, decreasing in $\underline{e}^s$, increasing in adult longevity T_t, and is unaffected by child mortality π_t.[23]

The general equilibrium of generation t is given by share of skilled individuals λ_t^* where individual optimal choices and market wages are jointly determined.

PROPOSITION 1: *For any* $\{T_t \in (\underline{e}^s, \infty), \pi_t \in (0,1), x_t\}$ *there exists a unique*

$$\lambda_t^* = \Lambda(T_t, x_t)\,. \tag{12}$$

and $\{H_t^{j*}, w_t^{j*}\}$ *for* $j = u, s$, *for which individual optimal education decisions are consistent with market wages. The equilibrium share of skilled individuals* λ_t^* *is an increasing function of* T_t, *with slope zero for* $T \rightarrow \underline{e}^s$ *and* $T \rightarrow \infty$.

The Proof of Proposition 1, and the explicit characterization of the function $\lambda_t^* = \Lambda(T_t, x_t)$ are reported in the Appendix. The key state variables affecting λ_t^* are adult longevity T_t and the relative importance of human capital in the production function, x_t. An increase in T_t leads to an increase in the share of skilled individuals λ_t^*. The effect of T_t on λ_t^* is non-linear, however. When T_t is low the locus $\Lambda(T_t, x_t)$ is convex and large increases in T_t are needed to induce a significant fraction of individuals to acquire skilled human capital since the fix cost $\underline{e}^s > \underline{e}^u$, prevents a large part of the population from receiving sufficient lifetime earnings when becoming skilled. When T_t is very large, the locus $\Lambda(T_t, x_t)$ is concave making large improvements in T_t necessary to induce further increases in λ_t due to the decreasing returns to human capital of either type, which drive down the relative wage w^s/w^u.[24] To shorten notation in the following we denote by λ_t the equilibrium share of skilled workers.

DEVELOPMENT DYNAMICS. — The dynamic path is given by a sequence $\{T_t, x_t, \lambda_t, A_t, \pi_t, n_t\}$ for $t = [0, 1, .., \infty)$, which results from the evolution of the nonlinear first-order dynamic system,

$$\begin{cases} T_t &= \Upsilon(\lambda_{t-1}) \\ x_t &= X(x_{t-1}, \lambda_{t-1}) \\ \lambda_t &= \Lambda(T_t, x_t) \\ A_t &= A_{t-1}(1 + G(\lambda_{t-1})) \\ \pi_t &= \Pi(T_{t-1}, x_{t-1}, \lambda_{t-1}, A_{t-1}) \\ n_t &= N(T_t, \lambda_t, \pi_t) \end{cases} \tag{13}$$

Notice that the system is block recursive. Baseline longevity $\underline{T}$ and the past level of the share of skilled workers, λ_{t-1}, determine adult longevity T_t, which in turn affects the current share of skilled workers and technological change. Total factor productivity, A_t, child mortality, π_t, and fertility, n_t, only depend on past levels of the variables and do not affect the evolution of the dynamic system (13) in terms of T_t, λ_t and x_t.[25]

[23]See Lemma 2 in the Appendix.

[24]Characterizing the second derivative of $\lambda(T_t)$ analytically is not possible at this level of generality. That there is only one inflection point (so that $\lambda(T_t)$ is increasing and s-shaped) in the parametrization used in the calibration in Section III can be shown numerically and can be established analytically when imposing assumptions on the shape of the ability distribution (like, e.g., a uniform distribution).

[25]Adult longevity T_t is as in (6), while the evolution of x_t is characterized by (8). The share of skilled, λ_t, in turn is determined by the intra-generational equilibrium implied by Proposition 1. TFP, A_t, evolves as in (9), while child survival probability, π_t, evolves according to (7) and also depends on y_{t-1} and, therefore on T_{t-1}, x_{t-1}, λ_{t-1}, and A_{t-1}. Fertility is determined in (10). A noteworthy feature of the dynamic system (13) is that all variables are characterized by interior solutions with the speed of their dynamics changes vary over time until the balanced growth path is reached. This is convenient for the quantitative analysis since it allows smooth comparative statics.

The development process involves reinforcing feedbacks between increases in human capital, and increases in adult longevity and technological progress. The different phases of development are illustrated in Figure A3 in the Online Appendix.

PROPOSITION 2: [ECONOMIC AND DEMOGRAPHIC TRANSITION] *For a sufficiently low* x_0*, the development path is characterized by:*
(i) *An initial phase with with* $\lambda = 0$*, low longevity,* $T = \underline{T}$*, high child mortality* $\pi = \underline{\pi}$*, slow income growth, and gross fertility given by,*

$$n = \gamma \frac{\underline{T} - \underline{e}^u}{(\underline{T} + \gamma)\, \underline{r}\, \underline{\pi}} . \tag{14}$$

(ii) *A final phase of balanced growth in income per capita, with* $T = \overline{T}$*, low child mortality* $\pi = 1$ *and* $\lambda = 1$ *with*[26]

$$n = \gamma \frac{min\{\overline{T}, R\} - \underline{e}^s}{T + \gamma \;\; r} . \tag{15}$$

(iii) *An endogenous transition from (i) to (ii).*

Adult longevity and the share of skilled individuals affect the timing of the transition to the balanced growth path whereas fertility and child mortality do not affect the dynamics of the economy. A lower baseline adult longevity $\underline{T}$ implies a later onset of the economic and demographic transition, since higher levels of technology x_t are required to induce the endogenous disappearance of the initial phase and the take-off to a balanced growth path.

III. Quantitative Analysis of Long-Run Development

A. Benchmark Calibration

The calibration of a unified growth framework requires setting the time invariant parameters of a model that produces a dynamic evolution that is not limited to the balanced growth path but includes the transition in the different variables. More specifically, calibrating the model proposed in Section II requires setting the values of fifteen parameters that characterize the utility and production function $\{\gamma, \eta\}$ technological progress φ, adult longevity $\{\underline{T}, \rho\}$, child survival $\{\underline{\pi}, \kappa\}$, skill acquisition $\{\underline{e}^u, \underline{e}^s, \alpha\}$, the distribution of ability $\{\mu, \sigma\}$, and the quality of children $\{\beta, \underline{r}, \delta\}$. In addition, we allow for the possibility that individuals retire at some exogenously given age R. Finally, the age at reproduction m (corresponding to the length of one generation) and two initial conditions for technology, A_0 and x_0, need to be specified. For a given set of parameters and initial conditions the evolution of all variables of interest is determined endogenously by the model along the development path, for all periods $t = \{0, 1, ..\infty\}$ and it involves a phase of quasi-stagnant development, which is eventually followed by the endogenous transition and convergence to the balanced growth path.

For the calibration, we use data for Sweden as the prototypical example of the economic and demographic transition. Data of comparably high quality are available for Sweden since the mid 18th Century, which makes it a natural benchmark for evaluating the quantitative fit of the model in terms of long-term development patterns.[27] Some parameters are set (exogenously) by matching directly observable counterparts in the data for Sweden

[26]The optimal investment in children, $\overline{r}$, corresponds to the balanced growth path rate of technological change, $g_{t+1} = \varphi$. See Lemma 3 in the Appendix.

[27]An earlier version demonstrated that the model equally well captures development dynamics in England.

or following the parametrization of existing quantitative studies. A second set of parameters is calibrated by solving the equilibrium conditions of the model and matching them to observable data moments on the balanced growth path. In the model, the balanced growth path is reached when all individuals get involved in formal education, $\bar{\lambda} = 1$; for Sweden this corresponds to the year 2000.[28] The calibration of the parameters of some functions requires solving systems of simultaneous equations by exploiting information on data moments at two point in time. In these cases we target data moments both on the balanced growth path (in 2000) and before the onset of the transition (in 1800).

When comparing the simulated data to the actual time series data for Sweden, the targeted moments (most of them referring to 2000 and a subset referring to year 1800) will be matched by construction. The comparison between simulated and actual data along the transition from (quasi-)stagnation to sustained growth (and in particular in the period 1750-2000 for which we have complete time series data) is not matched by construction and is thus informative for evaluating the fit of the model to the data.

Below we give a brief description of the calibration of the model. Table A1 contains summary information about the data moments used as targets, the data sources and the calibrated parameters. For space limitations, the details of the calibration, the data sources and the discussion of the sensitivity of the parametrization to alternative calibration strategies are reported in the Online Appendix.

PARAMETERS SET EXOGENOUSLY. — The length of generations, m, the age of retirement R and the fix (time) cost of education are set to match observable counterparts. The elasticity of substitution in the production function, η is set in line with the literature.

Length of a Generation. The length of a generation is set to $m = 20$ years. Across countries the average age of women at first birth before the demographic transition is approximately 20 years.[29] Twenty year frequencies also allow for a direct match of the simulated data with cross-country panel data without the need for interpolation.

Age of retirement. The average effective retirement age was around 64 in Sweden in 2000. Since R is the number of years before retirement as perceived at the end of childhood, at age $k = 5$, we set $R = 59$.[30]

Human Capital. To set the fix (time) cost of education, $\{\underline{e}^u, \underline{e}^s\}$, we target the average years of schooling in Sweden (for the cohort age 25-35), which was 12 years in 2000, see Lutz et al. (2007). The earliest available data suggest approximately one year of schooling around the onset of the transition.[31] This implies setting $\underline{e}^s = 12$ and $\underline{e}^u = 0$.

Production Function. The elasticity of substitution between skilled and unskilled workers is set to $1/(1 - \eta) = 1.4$ in the literature (see Acemoglu, 2002) so that $\eta = 0.285$.

PARAMETERS SET BY SOLVING THE MODEL. — The parameters of the function driving the evolution of *TFP*, the ability distribution and the preferences for fertility are set by solving

[28]The enrolment shares in Sweden have essentially reached 100% in primary and lower secondary education after 1980 and 1995, respectively.

[29]Mean age at first birth in Sweden around 1800 was slightly higher, see Dribe (2004), while age at first birth is still below 20 in pre-transitional countries in Africa nowadays, see Mturi and Hinde (2007).

[30]The data source is OECD, see http://www.oecd.org/dataoecd/3/1/39371913.xls. In spite of substantial changes health status at old age (which may facilitate old age labor supply) and the introduction of welfare programs (that anticipated retirement), the average age of retirement was relatively stable across historical cohorts in western countries. See Hazan (2009) and Strulik and Vollmer (2013). In an earlier version, the analysis abstracted from the possibility of retirement before death, with similar results.

[31]The estimates are slightly lower when referring to the entire population alive in Sweden in 2000 since older cohorts are included (for instance 11.4 in the data of Barro and Lee, 2001 and 11.5 years in Ljungberg an Nilsson (2009)). Regarding pre-transitional education levels, the estimates differ somewhat more. Ljungberg an Nilsson (2009) report 1.03 years of schooling in the total Swedish population aged 15-65 in 1870, and 0.1 average standard school years of the population aged 7-14 around 1810-1820, considering absenteeism and length of school years.

the model moments and the corresponding matching data moments for Sweden for 2000 (i.e., on the balanced growth path).

Technological Progress. The parameter of *TFP*, φ, is set to match the average annual growth rate of income per capita on the balanced growth path (which equals the growth rate of technological change). The average growth rate in Sweden over the period 1995-2010 has been about 2.4 percent per year. This implies targeting a growth factor of 1.61 over a twenty-year period. Given the function (9), and with $\lambda = 1$ along the balanced growth path, we set $\varphi = 0.61$.[32]

Ability Distribution. The parameter α, which determines the importance of ability for the acquisition of individual human capital, and the moments of the ability distribution $\{\mu, \sigma\}$ are calibrated by targeting data on the income distribution in Sweden in 2000. The income distribution on the balanced growth path only involves the production of skilled workers, since $\lambda = 1$. The empirical income distribution is approximately log-normal between the 5th and 95th percentile, with slightly thicker tails.[33] The individual (per period) income of a skilled worker is given by $w^s_t e^{\alpha a}$, which implies that individual log income is given by $\ln w^s_t + \alpha a$. The assumption of a normal distribution of ability (truncated to lie within a finite interval) and the exponential production function of human capital together imply that for $\lambda = 1$ the distribution of income in the model is also approximately log-normal with thicker tails due to the truncation. With $a \in [0, 1]$, the observed difference between the lowest and the highest income in the data is matched by setting $\alpha = 6.1$.[34] Matching the data moments implies setting $\alpha\mu = 3$ and $\alpha\sigma = 0.4$, which for $\alpha = 6.1$, implies $\mu = 0.49$ and $\sigma = 0.066$. A similar calibration would be obtained from the typical distribution of cognitive ability as a proxy for ability in acquiring skilled human capital.[35]

Preferences. The parameter γ is calibrated targeting gross fertility $n = 1$ along the BGP, which is also equivalent to targeting the net reproduction rates approximately at replacement levels, setting child survival to $\pi = 0.996$ consistent with Sweden in 2000.[36] The time spent in raising children is determined endogenously in the model and changes overtime with the growth rate of income and technology. We set a target for the number of years spent raising a child in 2000 of $r = 5$.[37] With $\lambda = 1$, $\pi = 0.996$, $R = 59$, and $r = 5$ this delivers $\gamma = 9$.

The parameters, relating to the evolution of adult longevity, child mortality and fertility, are set by solving systems of simultaneous equations and targeting data for 2000 and for 1800, which represents the latest pre-transitional period for which reliable data is available

[32]See, e.g., the ERS Dataset (www.ers.usda.gov) or historical statistics from the Bank of Sweden (www.historicalstatistics.org/). Targeting estimates of growth in multifactor productivity, labor productivity or the Solow residual deliver similar magnitudes for φ.

[33]We use micro data from the ECHP dataset for individual incomes of full-time employees aged 25 to 45, which corresponds to the two last cohorts in the dynamic simulation, and equivalently to the two first generations with $\lambda = 1$ in the data. Incomes are converted to US-$ using an average exchange rate of 9 Kroner for one US-$ in 2000. The relevant data moments extracted from this data set are broadly consistent with other data sources based on register data and alternative surveys for gross earnings, see, Domeij and Floden (2010).

[34]The distribution of log incomes has mean 9.7, standard deviation 0.4, and the lowest and highest observed log-incomes are 6.7 and 12.8, respectively, which implies a maximum spread of 6.1. The moments of the income distribution for the age cohort 25-65 are essentially the same, with the lowest, mean, and highest levels of log income being 6.7, 9.7, and 12.8, respectively, and with a standard deviation of 0.41. The data moments are also close to the ones typically used for the calibration of dispersion in permanent incomes in other OECD countries. For instance, Erosa et al. (2011) match a variance of log permanent earnings in the US of 0.36. Robustness checks show that the results are fairly insensitive to varying the dispersion.

[35]The distribution of cognitive ability (or IQ), which is generally measured in the literature as a truncated normal with mean 100 and standard deviation 15, see, e.g., Neisser et al. (1996), would imply a very similar parametrization when normalized to a support $a \in [0, 1]$, with $\mu = 0.5$ and $\sigma = 0.075$.

[36]Total fertility rates (TFR) in Sweden were on average 1.8 children per woman over the period 1980-2000, with substantial fluctuations. In 1990, the TFR was 2.13, whereas in 2000 it was 1.54 (World Development Indicators). These figures suggest that a gross fertility of 1 (which would correspond to a TFR of 2) along the balanced growth path is a reasonable target. Targets in the range from 0.75 to 1.1 deliver very similar results, however.

[37]The target is line with the estimates by (Haveman and Wolfe 1995). This is equivalent to setting a target for the share of work life that is spent in raising a child is about 15 percent which is in line with Doepke (2004) and de la Croix and Doepke (2003).

for all the variables of interest. As discussed above, baseline longevity affects the timing of the take off while child mortality and fertility do not affect the timing of the transition as consequence of the block-recursiveness of the system (13) and the lack of scale effects.

Adult longevity. Ideally the level of $\underline{T}$, which represents baseline, country-specific, mortality should be calibrated exogenously. Given the lack of reliable historical data, the calibration of these two parameters is done by solving the two equations of the type $T_t = \underline{T} + \rho\lambda_{t-1}$ at two points in time using data for 2000 and 1800. This appears a reasonable strategy since the share of educated individuals is still very small until the onset of the transition in 1800. To solve the system we consider a share of skilled workers of $\lambda = 0.1$ which roughly corresponds to the enrolment rates in early 19th Century Sweden.[38] Life expectancy at age five in Sweden was approximately 76 in 2000 and 48 around 1800.[39] With these targets, the parameters of the function (6) are set to $\underline{T} = 45$ and $\rho = 31$.

Child survival probability. Child mortality in Sweden fluctuated around one third in the period 1760-1800 and was about 0.004 in 2000. Targeting a child survival probability 0.67 and 0.996 for 1800 and 2000, respectively, and using condition (7) delivers a baseline child survival probability of $\underline{\pi} = 0.5$ and a $\kappa = 0.005$.[40]

Production function of children's quality. The parameters $\{\beta, \underline{r}, \delta\}$ are calibrated by targeting the levels of gross fertility for Sweden in 1800 and 2000, and the growth rate of technology in 1900, which is taken as the period of the exit from the corner solution of zero investments in children's quality in light of the pronounced drop in fertility in Sweden around this time.[41] To calibrate the parameters of the function, (4), we use the optimal time investment by parents in children and the minimum growth rate of technology $\underline{g}$ for which parents spend some positive extra time in raising children. Given the targets we obtain $\{\beta = 0.23, \underline{r} = 4.7, \delta = 3.54\}$.

INITIAL CONDITIONS. — We finally also need to determine the initial conditions in terms of the initial importance of skilled human capital in the production function, x_0, and the initial level of total factor productivity A_0. Given these initial productivity parameters, the dynamic system (13) generates the endogenous evolution of all variables of interest along the development path for $t = \{0, 1, .., \infty\}$. The initial importance of skilled human capital in the production function, x_0, only affects the number of generations before the take-off in the simulation. Choosing x_0 sufficiently low implies simulating the model before the onset of the phase transition that triggers the convergence to the balanced growth path. Setting $x_0 = 0.04$ the simulation converges to the balanced growth path, (which is assumed to be reached when λ exceeds 0.999) in 100 generations thereby covering the period from year

[38]Even though the available data sources provide slightly heterogeneous information on enrolment rates in the early 19th Century Sweden, the estimates range from about 5 to about 15 percent, see de la Croix et al. (2008) and Ljungberg and Nilsson (2009). The precise value of λ before the transition is therefore of little importance and the results for alternative parameters obtained by assuming for 1800 levels of λ up to 0.3 are essentially the same.

[39]The average of life expectancy at age five in the period 1760-1840 was 48.38, in the period 1790-1810 it was 48.06 (Human Mortality Data Base available at http://www.mortality.org/). Similar figures are documented for England, France and Italy, see Woods (1997) and Bideau et al. (1997) and Lewis and Gowland (2007). Also note that, as discussed below, in 2000 child mortality is around 0.004, which implies the convergence of life expectancy at 5 plus five years of 80.74, and of life expectancy at birth of 80.45.

[40]The data on child survival are available at: http://www.mortality.org. The levels of income per capita needed for the computation of the parameters are taken from the historical statistics from the Bank of Sweden (www.historicalstatistics.org/), converted to US-$ using an average exchange rate in 2000 of 9 Kroner for one US-$. The income levels used for the calibration of condition (7) are 22,717 and 884 US-$, which correspond to the GDP per capita of Sweden in 2000 and 1800, respectively, in US-$ per 2000.

[41]Gross fertility in Sweden in 1800 and 2000 was $n = 2.3$ and $n = 1$, respectively. The data are from Keyfitz and Flieger (1968) and World Development Indicators. As documented in the demographic literature, and as clearly visible in the time series reported below, a noticeable drop in gross fertility occurs in Sweden around 1900. We take 1900 to be the period of the exit from the corner solution of the quantity-quality trade-off. Estimates of TFP and income per capita growth around 1900 vary between 0.7 and 1.7 percent per year, see Krantz and Schoen (2007), Schoen (2008) and Greasley and Madsen (2010). We set the level of $\underline{g}$ to 1.2 percent per year with a corresponding growth factor over a 20-year generation of 0.27.

0 A.D. until 2000.[42] The initial level of TFP is a scale parameter that does not affect the endogenous evolution of the system (13), but only affects the level of production. We set $A_0 = 15$ to match the level of log GDP per capita in the year 2000.[43]

B. Time Series Results

The dynamic evolution of the model economy is characterized by a long period of slow development followed by a (comparatively) rapid transition to a sustained growth path takes that place over a time horizon of about 200 years.[44]

Figure 1 restricts attention to the period 1750-2000 and compares the simulated data to the corresponding time series of historical data from Sweden. To interpret the results, recall that the data in 2000 are matched by construction as they reflect the balanced growth path to which many parameters are calibrated. Panels (a) and (b) of Figure 1 report the evolution of life expectancy at birth (T0) and at age five (plus five years, T5), and child mortality rates, respectively. The calibration targets life expectancy at age five as well as child mortality at two points in time (in 1800 and 2000). During the transition, however, the values of these variables are generated by the simulated model and are not constrained to match data moments by construction. The model performs well in matching the evolution of adult longevity over the entire period, both in terms of levels and in terms of the duration of the transition. Also life expectancy at birth (which was not targeted) is matched well. Figures 1(c) and (d) plot the share of skilled individuals, λ, against the primary school enrolment rate and against the (shorter) series of average school years, respectively. Neither data series constitutes a perfect empirical counterpart for λ, but both reflect the education acquisition in the population. The model dynamics resemble the evolution of the enrolment rates in primary education and tend to lead slightly the dynamics of average school years. Given that the model does not account for institutional changes, like the emergence of school systems, the model's dynamics fit the data well.

Figure 1(e) depicts gross and net fertility. The model was calibrated by targeting three moments that are apparent in this figure: the levels of gross fertility before and after the transition (1800 and 2000) as well as the exit from the corner solution of zero investment in child quality around 1900. The simulation matches the initial and terminal levels, as well as the intermediate transition. The eventual reduction in net fertility, which, as discussed in Section I, has been difficult to rationalize in previous quantitative studies, is matched in the model due to the presence of the differential fertility effect that is absent in models based on the quantity-quality trade-off.[45] Compared to the historical data, the model does not match, however, the quantitative increase in net fertility that is observed during the early phase of the demographic transition.

Finally, Figure 1(f) depicts the evolution of income per capita. The level of initial technology and the elasticity of technological progress were calibrated to match the level and growth rate of income per capita in 2000. The evolution of income per capita matches the data series over the entire period including the acceleration during the transition.

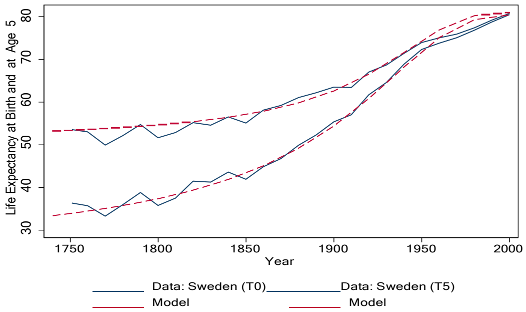
[42]Setting a smaller x_0 only implies increasing the number of generation before the take-off, that is, it only implies starting the simulation further back in time.

[43]The data are from www.historicalstatistics.org.

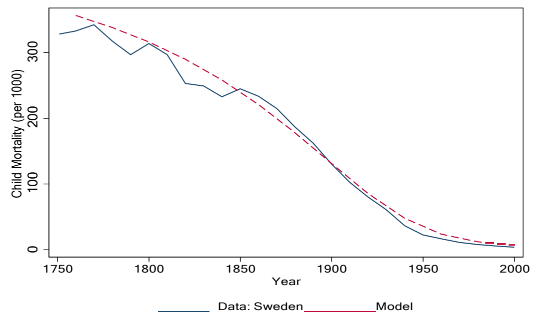
[44]When considered over the entire simulation period from year 0 to 2000, the simulated data for the variables of interest exhibit a lengthy phase of slow development, followed by the endogenous take-off around 1800. This is illustrated in Figure A2 in the Online Supplementary Material.

[45]The change in the quantity-quality trade-off is small and the observed drop in gross and net fertility is mainly due to the differential fertility effect and the negative income effect that emerges when life expectancy reaches old ages. The endogenous cost of raising children is actually very similar before the onset of the transition and on the balanced growth path, with levels of 4.7 and 5, respectively. Assuming a fixed cost of raising children at post-transition levels leaves the benchmark time series of fertility essentially unchanged. This is not the case for the quantity-quality function calibrated targeting data moments for the high fertility countries (see below).

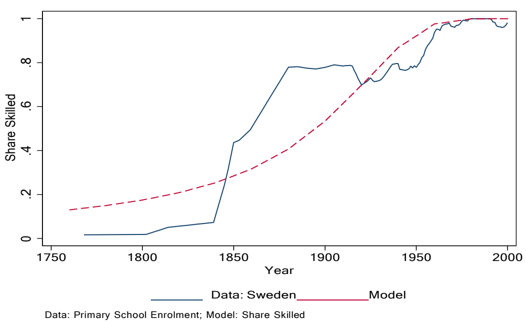
FIGURE 1. LONG-RUN DEVELOPMENT: SIMULATION OF BENCHMARK CALIBRATION OF THE MODEL AND HISTORICAL DATA FOR SWEDEN 1750-2000



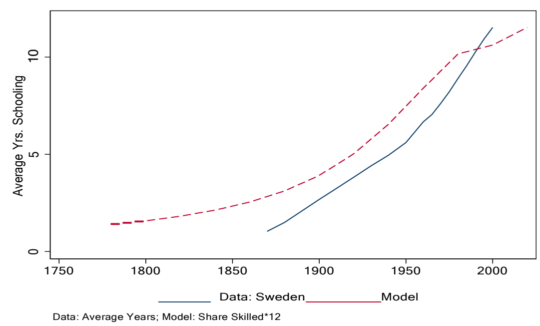
(a) Life Expectancy at Birth



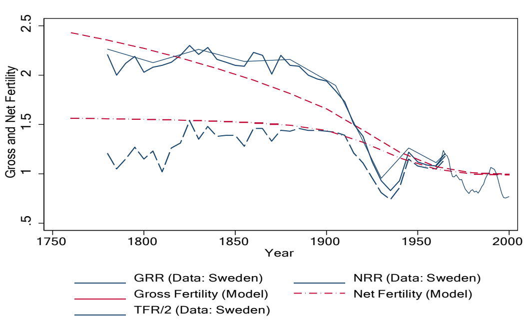
(b) Child Mortality Rate



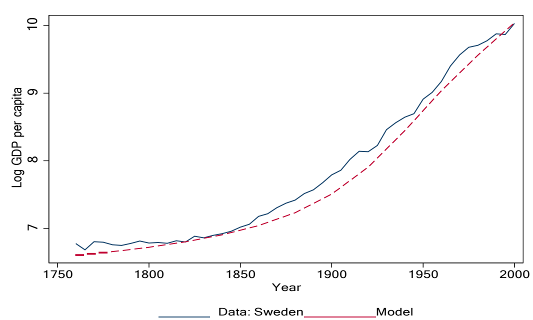
(c) Primary School Enrolment, λ



(d) Average Years of Schooling, λ



(e) Gross and Net Reproduction Rates



(f) log GDP per capita

IV. Accounting for Comparative Development

This section investigates to what extent the model can account for cross-country comparative development patterns. The analysis is motivated by the observation that the stylized patterns of long-run development dynamics are very similar across countries and times, including forerunners like Sweden and England as well as countries that entered their demographic and economic transition much later than the European forerunners. Demographers such as Kirk (1996) notice that "in non-European countries undergoing the demographic transition in the mid 20th century, the regularities are impressive".

The analysis proceeds in three steps. In Section IV.A we compare the data obtained from the benchmark calibration to cross-country panel data for the period 1960-2000. The analysis provides a first investigation of the possibility that today's differences in cross-country comparative development might be accounted for by different delays in the economic and demographic transitions. The role of differences in the country-specific, extrinsic mortality environment is investigated in Section IV.B by performing controlled variations in baseline adult longevity, $\underline{T}$. The analysis allows quantifying the role of mortality differences for the timing of the take-off from quasi-stagnation to sustained growth and for resulting comparative development patterns. Section IV.C pushes the analysis one step further by simulating an artificial world composed by countries that are identical in all dimensions except baseline longevity. The simulated data are compared to the empirical worldwide distribution of adult longevity, child mortality, education, fertility and income (in 1960 and 2000). These exercises can be viewed as an attempt to address the open question whether one can capture the cross-sectional variation as resulting from delays in the time of the take-off as driven by differences in extrinsic mortality. The results therefore represent a joint investigation of the general mechanics generated by the model and of the specific role of differences in baseline longevity.

The results in this section effectively constitute an exploration of the quantitative implications of the prototype unified growth model that has been calibrated to the long-run development patterns of Sweden. No data moment of the cross-country analysis that follows has been explicitly used as target for the calibration of the model. The results can therefore be used to judge the ability of the model to fit the data "out of sample".

A. Simulated Data and Cross-Country Panel Data

We begin the analysis by evaluating the ability of model, calibrated for Sweden, to account for comparative development patterns. If the mechanism driving the transition process is generally valid one would expect that, at each point in time, different countries are in different phases of their (otherwise similar) development process.

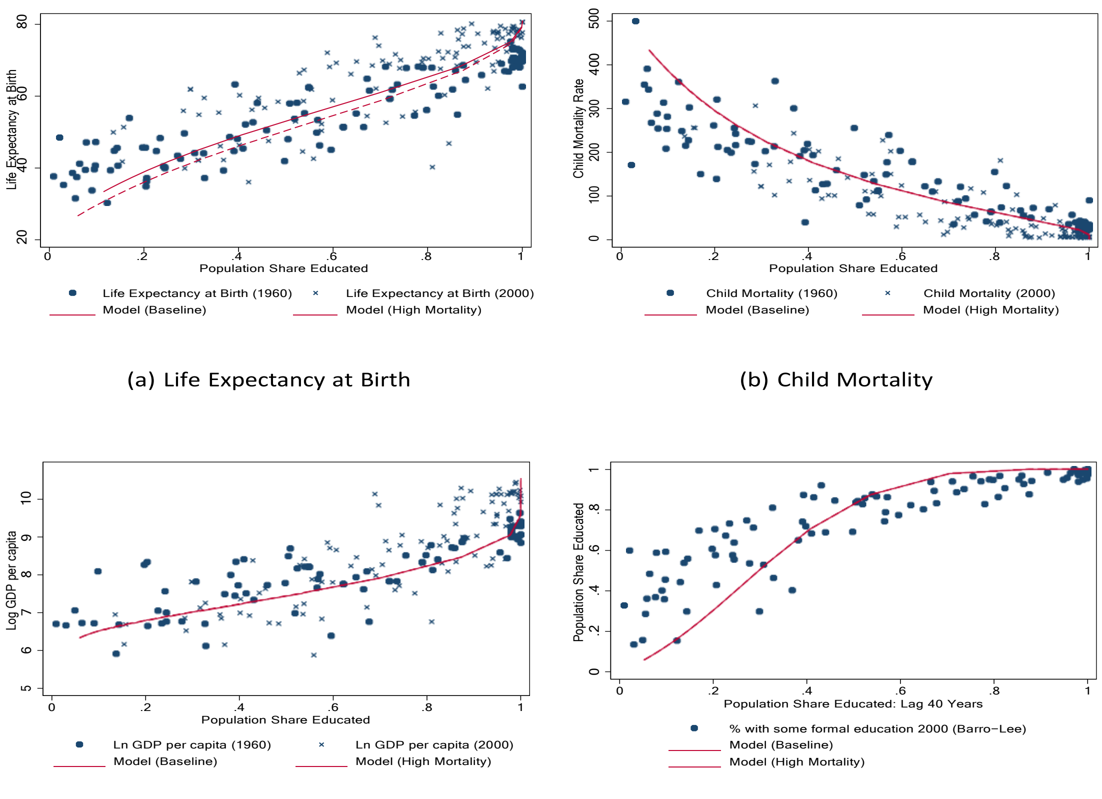
Figure 2 presents the data generated by the simulation of the calibrated model (as depicted in Figure 1), but plotted against the key variable driving the transition, the share of skilled workers λ, at the respective point in time (rather than as time series). These simulated data are plotted together with corresponding cross country data for 1960 and 2000.[46] Panels (a), (b) and (c), plot the data on life expectancy at birth, child mortality and income per capita against the share of educated individuals, λ. The cross-sectional interpretation of the calibrated data fits the cross-country data patterns quantitatively well and the relation appears stable over the 40-year horizon. For low levels of λ, that correspond to the less developed countries, the actual data exhibit higher life expectancy and child survival probabilities than predicted by the model especially for 2000.

Figure 2(d) plots the share of skilled against the value of the same variable 40 years (two generations) earlier. In the data, this corresponds to plotting the share of educated individuals in 2000 against that in 1960. Again the calibration performs comparably better for countries with a larger lagged share of educated individuals while it underestimates the improvements in education for countries with low λ in 1960. This suggests that, compared to Sweden or other European countries for the same level of initial share of educated individuals, the developing countries have experienced an acceleration in education acquisition over the last forty years.

Even though the role of baseline mortality will be explored in more detail in the next Section, it is useful to illustrate the role of differences in $\underline{T}$ for the cross-sectional patterns

[46]As empirical counterpart of λ across countries we consider the share of the total population with some formal education, generated as one minus the fraction of the population with "no schooling education" in the total population. See the Online Appendix for further information.

FIGURE 2. EDUCATION, MORTALITY AND INCOME [SIMULATION AND DATA (1960 AND 2000)]



(a) Life Expectancy at Birth

(b) Child Mortality

(c) Log GDP per capita

(d) λ 1960 and 2000

already at this point. To this end, Figure 2, also plots the simulation of the same calibrated model but with a baseline longevity that is five years lower than in the benchmark calibration (i.e., $\underline{T} = 40$).[47] The results show that the cross sectional patterns and the correlations between λ and life expectancy, child mortality and income per capita are essentially identical to those generated by the benchmark calibration with higher baseline longevity.

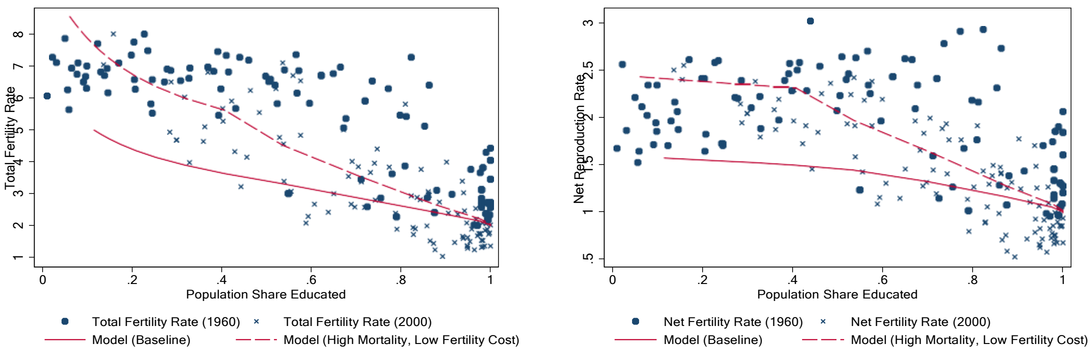
Figure 3(a) and (b) present the results for gross and net fertility.[48] The benchmark model matches the fertility levels for the more developed countries (the ones with a relatively large λ) that have undergone the demographic transition around, or shortly after, the period of the demographic transition in Sweden, but it substantially underestimates the fertility levels for pre-transitional countries with low levels of λ. Sweden, like other European countries, displays pre-transitional fertility levels that are particularly low in a worldwide perspective. The literature has offered several hypotheses that try to explain why historically the cost of raising children was comparatively high in pre-industrial Europe. To explore the quantitative implications of lower costs for children, we consider an alternative calibration of the quantity-quality trade-off by targeting data moments for pre-transitional countries with the highest recorded fertility in 2000. This alternative parametrization of the quantity-quality trade-off substantially improves the match

[47]As discussed in Section IV.B a five year difference in baseline mortality is in line with the empirical evidence on pre-transitional life expectancy in the lowest and highest mortality countries.

[48]Since reproduction in the model is asexual, the level n refers to the gross reproduction rate (the number of daughters for each woman). In order to compare this number to the data on total fertility rates, we multiply the gross reproduction rate n by two.

between simulated model and the data.[49]

FIGURE 3. EDUCATION AND FERTILITY [SIMULATION AND DATA (1960 AND 2000)]



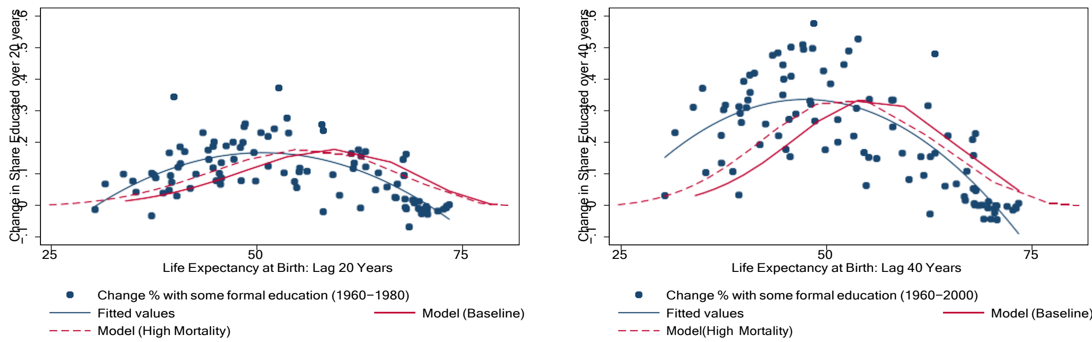
(a) Total Fertility Rate (b) Net Reproduction Rate

The theory also predicts the existence of non-linear dynamics that link economic and demographic variables during the process of long term development. The non-linearity of the equilibrium locus Λ, characterized in Proposition 1, implies that the changes in λ are largest in the intermediate range, where the locus has its steepest slope. The increase in the share of skilled workers is relatively large in countries with intermediate levels of adult longevity, but relatively small in pre-transitional and post-transitional countries. The model therefore predicts a non-monotonic relationship between life expectancy and subsequent changes in λ. As discussed in Section I the existence of a non linear effect of life expectancy on education has relevant implications for empirical investigations. Panels (a) and (b) of Figure 4 depict the relationship between life expectancy in 1960 and the change in the share of individuals with no formal education over the following twenty and forty years in the data (including a quadratic regression line), in comparison to the respective data from the benchmark calibration. The model matches the data well although it somewhat underestimates the improvements in the change in education in countries with lower initial life expectancy. Compared to the historical experience of Sweden, education improvements in the poorest countries were comparatively large in the period 1960-2000.

Another direct implication of the development dynamics of the model is the existence of concave relationship between life expectancy and income. During the early phases of development, low longevity induces little human capital accumulation as proxied by the population share skilled, λ, and consequentially income is low. As longevity improves, incentives to become skilled increase, and incomes rise. As development continues, however, further improvements in life expectancy lose momentum as the skill composition, adult longevity and child survival converge to their natural upper bounds, whereas income remains on a sustained growth path. This prediction is in line with the stylized fact known as Preston Curve in demography (see Preston, 1975). Figure 5 shows the that the patterns implied by the simulation closely match the empirical Preston Curve in the data

[49]That fertility levels have been comparatively low in Europe compared to other regions is well documented and the reasons have been investigated recently, see, e.g., Moav (2005), Strulik and Weisdorf (2014), and Voigtländer and Voth (2014). To explore the role of the cost of raising children for the high fertility countries, we calibrate an alternative ("low fertility cost") quantity-quality function that accounts for the fact that the average total fertility rates of the highest fertility countries was around 7, or above, in 2000, as compared to about 5 for pre-transitional Europe. Changing the target for the pre-transitional fertility to n = 3.5 and re-calibrating the parameters accordingly delivers $\{\beta' = 0.75, \underline{r}' = 3, \delta' = 1.06\}$ The kink in the simulated data in Figures 3(a) and (b) corresponds to the exit from the corner solution of the extra time invested in children. Recall that the cross-sectional patterns depicted in Figure 2 are unaffected by the actual calibration of the quantity-quality trade-off because the dynamic system (13) is block recursive and does not involve any scale effect.

FIGURE 4. LIFE EXPECTANCY AND CHANGES IN EDUCATION [SIMULATION AND DATA]

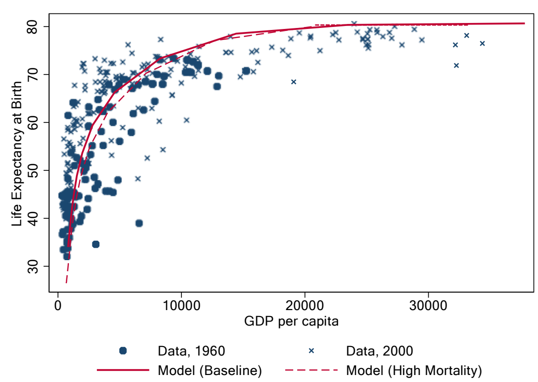


(a) Change in λ over 20 years

(b) Change in λ over 40 years

(in 1960 and 2000). The prototype unified growth theory can therefore be insightful on the mechanics behind its emergence that are still debated, as discussed in Section I.

FIGURE 5. THE PRESTON CURVE



B. *Different Levels of Extrinsic Mortality and Comparative Development*

In this Section, the calibrated model is used to investigate the quantitative role of differences in the mortality environment for comparative development. Sweden (and generally European countries) have a comparably favorable mortality environment, which is reflected in a relatively low exposure to infectious diseases, whereas the less developed countries of today are often located in areas with a harsher mortality environment. A permanently higher exogenous exposure to infectious diseases implies faster aging and lower life expectancy under similar (economic) living conditions.

We calibrate an alternative scenario of baseline adult longevity, $\underline{\underline{T}}$, that reflects the worst mortality environment of all countries. This calibration targets data moments for pre-transition countries with the highest observed adult mortality in 2000, which corresponds to targeting a life expectancy at age five of 45 years (compared to 48 years reflecting Sweden around 1800 just before the transition). This level is in line with the lowest available measure in 2000 for life expectancy at birth and implies setting $\underline{\underline{T}}$ = 40 (as compared to the baseline $\underline{T}$ = 45).[50]

[50]Retaining a target of 76 years for life expectancy at age five on the balanced growth path, and using condition (6), this implies setting a $\underline{\underline{T}}$=40 and $\bar{\rho}$ = 36 (rather than $\underline{T}$=45 and ρ = 31 as in the benchmark calibration).

The results reported in Figure 2 have shown that a five year lower baseline mortality leaves the cross-sectional patterns generated by the simulated model essentially unaffected. From the dynamic system (13), a lower baseline adult longevity implies a lower population share of skilled individuals in equilibrium for any given level of technology and education of the previous generation, and therefore a delayed take-off. To investigate the quantitative importance of this prediction, we replicate the analysis with the baseline adult longevity re-calibrated to $\underline{T} = 40$ to reflect countries with the highest baseline mortality, while keeping the remaining parameters of the benchmark calibration unchanged. This counterfactual exercise isolates the role of adult longevity by simulating the same model that has been calibrated for data moments of Sweden and investigating the effects of changes in the baseline longevity to levels that reflect those of the highest mortality countries.

Figure 6 plots life expectancy at birth, the share of skilled individuals, total fertility rates, and income per capita for the benchmark calibration and for the alternative calibration low baseline longevity. The delay in the transition, spans about 7 generations or 140 years, as consequence of imposing $\underline{T} = 40$ rather than $\underline{T} = 45$. The joint consideration of the simulation in Figures 2 and 6 therefore suggests that differences in baseline mortality may be relevant to explain the delay in comparative development, but their effect is hard to detect by estimating linear regressions with cross-country panel data. In fact, apart from the timing of the take-off, the different countries experience a very similar development process. This is illustrated in the figure by also plotting the dynamic simulation for a country with intermediate baseline mortality that converges to the balanced growth path in 2040 (rather than 2000).[51]

Figure 6 also plots the average development empirical trajectories for different continents (for the period 1960-2010). The development dynamics of Europe and Western Offshoots are captured by the baseline calibration rather well. On the other end of the spectrum, African countries display a substantially delayed development in all four dimensions. By construction, Africa as a whole still displays a somewhat better development performance than the calibration for $\underline{T} = 40$ for the worst conceivable scenario with the highest disease burden in the world. The development dynamics of Asia and Latin America lie between Europe and Africa. This pattern is consistent with estimates of the extrinsic mortality environment. Europe and Western Offshoots display the lowest, and African countries display the highest, disease burden in the world, while Asian and Latin American countries display an intermediate level of disease exposure.

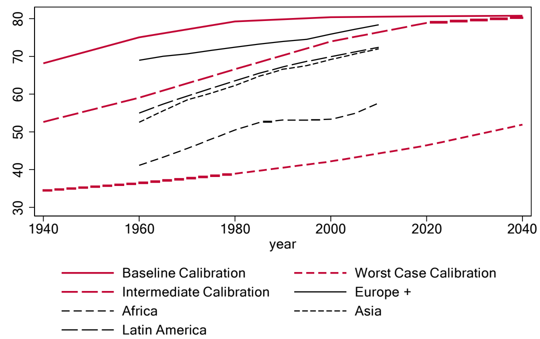
C. Accounting for the Worldwide Distribution of Comparative Development

The analysis so far supports the view that the dynamic evolution of the different countries is characterized by a similar process that involves a long period of slow development followed by a rapid transition to a sustained growth path. A main difference across countries appears to be the actual timing of the take-off. A direct implication of this view is that, at a given point in time, relatively few countries are observed during their transition

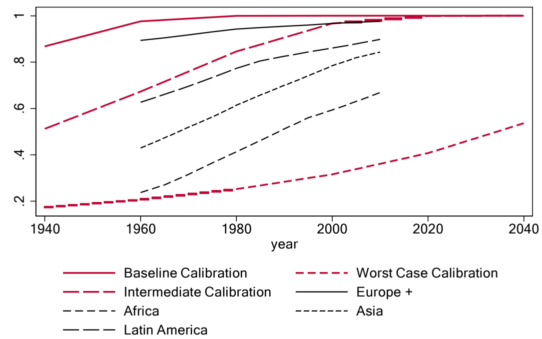
The data source is UN Population Statistics (www.unstats.un.org). Data on life expectancy at five for earlier periods are missing for many countries, including most Sub-Saharan Africa countries in 1960. Alternatively, the available information on child mortality and life expectancy at birth in 1960 can be used to derive an estimate of life expectancy at age five. This delivers a very similar target for the highest mortality countries. In 1960 life expectancy at birth was as low as 33 years in some countries like Afghanistan, and child mortality one third. Assuming a constant death rate below the age of 5, these numbers imply a life expectancy at age five between 44 and 45 years. In some countries, like Swaziland life expectancy at birth is just above 30 years still today (CIA World Factbook). This suggests that 45 is possibly a conservative estimate of baseline adult longevity in the worst conceivable mortality environment.

[51]As discussed in more detail in the next section, we estimate a distribution of baseline mortality using information on the number of multi-host vector-transmitted pathogens to simulate the distribution of baseline longevity for a world of artificial countries. The baseline longevity of the intermediate country plotted in Figure 6 corresponds to the median of the estimated distribution (which is 43.25).

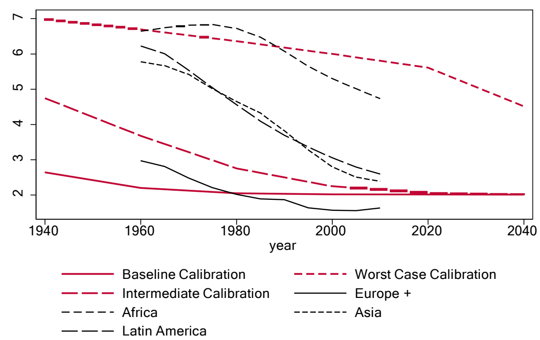
FIGURE 6. THE ROLE OF LOWER BASELINE LONGEVITY FOR COMPARATIVE DEVELOPMENT



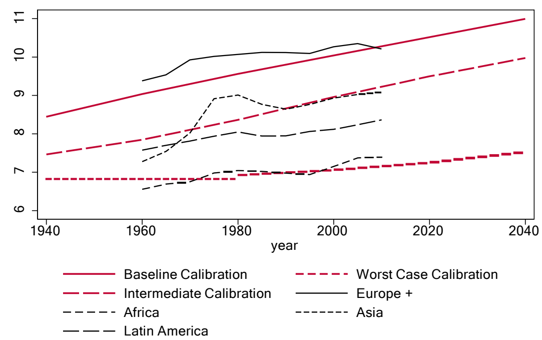
(a) Life Expectancy at Birth



(b) Population Share with Formal Education, λ



(c) Total Fertility Rates



(d) log GDP per capita

(since for most of its history each country is either pre-transitional or post-transitional, while the transition is comparably quick). As a result one should therefore expect the cross-sectional distribution of all variables of interest to display two modes corresponding to the mass of countries that are still pre-transitional or on the balanced growth path, respectively, as characterized in Proposition 2.[52] While intuitive, this implication has not been derived and empirically investigated in the existing literature.

We simulate an artificial world composed of countries that are identical in all parameters except for baseline adult longevity $\underline{T}$. To create a meaningful world-wide distribution of baseline mortality in the range [$\underline{T}$ = 40, $\underline{T}$ = 45], which is needed to simulate a cross-sectional distribution of the variables of interest, we exploit information on cross-country differences in historical disease prevalence. These data have been collected from sources from the early 20th century and therefore reflect extrinsic mortality across the world before major health innovations, and their worldwide dissemination, which took place with the so-called epidemiological revolution after World War II. In particular, the calibration exploits information on whether a particular (multi-host vector transmitted) infectious disease had been detected in a country. This means that the analysis does not rely on the spread of the disease or the number of infected cases, which were potentially endogenous to development already in the 19th century.[53] We simulate a world of countries that only differ in terms

[52]The precise shape of the distribution depends on the actual distributions of the underlying country-specific characteristics that drive the delay in the take-off. Nonetheless, the bi-modality should be detectable regardless of the particular distribution as long as sufficiently many countries are still pre-transitional.

[53]Multi-host vector borne pathogens are closely connected to local country-specific biological and climatological conditions and have not been eradicated in any country. The historical distribution of these pathogens is therefore a good proxy for the country-specific extrinsic mortality, see Cervellati, Sunde and Valmori (2012). A distribution of baseline mortality parameters for 113 economies is created using the empirical distribution of pathogens observed

of their baseline adult longevity, which is distributed in the range $[\underline{T} = 40, \underline{T} = 45]$. The data generated by the simulation of the artificial world are then pooled and used to estimate the cross-country simulated distribution of all variables of interest and compared to the corresponding distributions obtained from cross-county data in 1960 and in 2000. In interpreting the results it is useful to keep in mind that the only assumed difference across countries is baseline mortality.

FIGURE 7. DISTRIBUTIONS: EDUCATION [SIMULATION AND DATA (1960 AND 2000)]

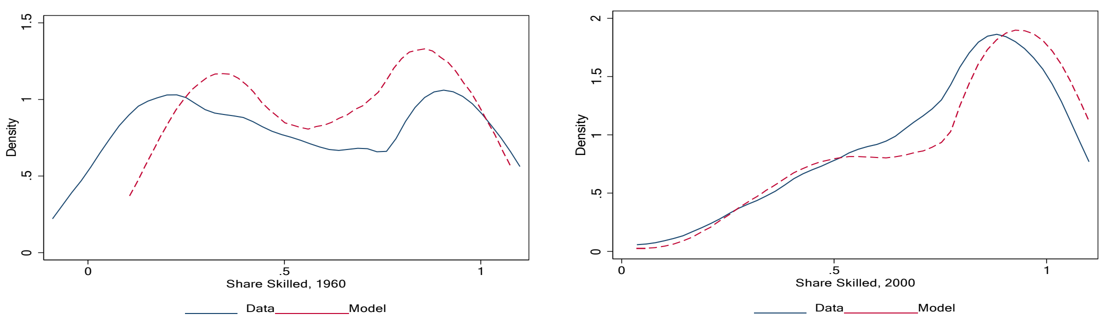


Figure 7 plots the simulated distributions of education for the years 1960 and 2000, and contrasts them to the respective distributions of the actual cross-country data by ways of kernel density estimates. Figure 8 does the same for the distributions of life expectancy and child mortality. For all variables the expected bi-modality is clearly apparent in 1960 both in the simulated and the actual data and the empirical patterns are broadly matched in terms of the support, the shape of the distribution and location of the modes. By 2000 both the simulated and the empirical distributions tend to be more unimodal since more countries have undergone the transition.[54] An interesting observation, that is in line with some of the insights from the analysis in the previous Section, is that by 2000 the bimodality in the actual distribution is less visible compared to the simulated world. This again suggests that the model underestimates the timing of the take-off in the less developed countries over the last decades. In other words, there is an anticipation in the take-off that accelerates development in these countries compared to the historical path followed by the European forerunners. This effect, which is particularly visible for child mortality and adult longevity, is compatible with the existence of possible spillovers from the developed to the developing countries, which are not considered in the simulated world.

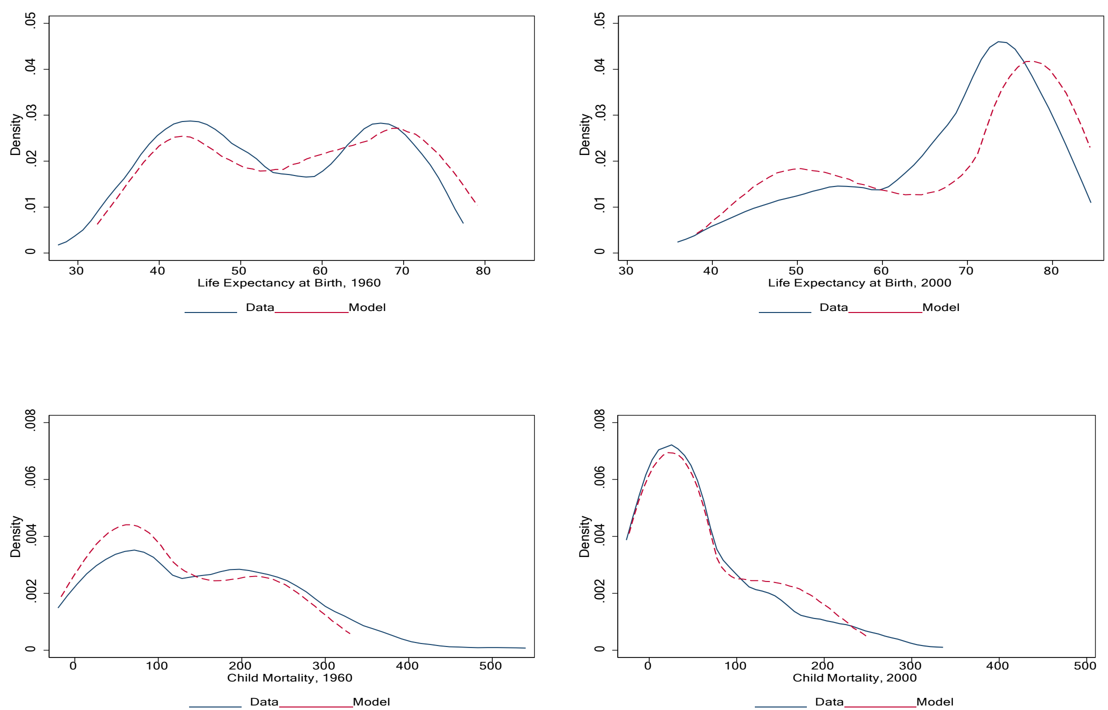
Most of the countries display fertility patterns resembling the high fertility countries, rather than Europe. We therefore simulate the artificial world by considering as benchmark the alternative parametrization of the quantity-quality function that was calibrated targeting data moments for these countries as described above. Figure 9 presents the results for total fertility rates and net reproduction rates. The simulation fits the data by roughly capturing the peaks at low and high levels of fertility, as well as the shape of the distribution and its change over the 40-year horizon.[55] Also in this case the disappearance of the low peak is faster in the real data compared to the artificial world.

in each country. The details, including the number of diseases in different continents relative to the world average and the artificial distribution, are reported in the Online Appendix.

[54]The bi-modality of the simulated distribution is not due the actual calibrated distribution of baseline mortality. Alternatively, we have performed the exercise considering a uniform distribution of baseline mortality, with a very similar pattern of bi-modality and of changes in the distributions over time.

[55]The actual calibration of the production function of children's quality is irrelevant for the kernel distributions of all variables apart from gross and net fertility. Unreported kernel distributions generated with the calibration for Sweden display a similar fit to the actual data for the most developed countries, but underestimate the location of the peak for high fertility. This suggests that differences in the cost of raising children across countries are potentially

FIGURE 8. DISTRIBUTIONS: MORTALITY [SIMULATION AND DATA (1960 AND 2000)]



Finally, Figure 10 depicts the world-wide distribution of incomes per capita. Compared to the demographic variables the worldwide income distribution is matched less both in terms of the support of the distribution and in terms of change overtime. Notice that the model is limited in capturing the world income distribution partly by construction. The model is calibrated to Sweden (which was not among the most developed countries even in an historical perspective). Also, the artificial world does not account for any other relevant country-specific determinants of cross country comparative development that have been studied in the literature.[56]

V. Concluding Remarks

This paper proposes a simple prototype theory of the economic and demographic transition that generates the endogenous evolution of mortality, education, fertility, and income. The model is calibrated to historical data for Sweden and allows for a first systematic quantitative analysis the implications of a unified growth model for long run growth, development delays and their implications for cross-country comparative development. The results document the ability of the unified growth framework to rationalize both historical and cross country development patterns. The findings provide support for the view that all countries follow similar non-linear development processes, characterized by a long period of quasi-stagnation followed by rapid economic and demographic transitions. The analysis documents the ability of the unified growth framework to account for cross-country

more important for the cross-country differences in pre-transitional fertility levels than differences in mortality.

[56]The model does not consider other determinants of cross country income differences, like e.g. differences in physical capital, natural resources or institutions that have been shown to be empirically relevant, nor does it consider possible cross-country spill-overs or transfers of technology and innovations. Also, while the samples used for the density plots in Figures 8 and 9 are balanced for the observation periods 1960 and 2000, for GDP the sample for 1960 only contains 72 countries due to data availability, but 90 countries in 2000, so that the density plots obtained from data are not perfectly comparable.

FIGURE 9. DENSITY DISTRIBUTIONS OF FERTILITY [SIMULATION AND DATA (1960 AND 2000)]

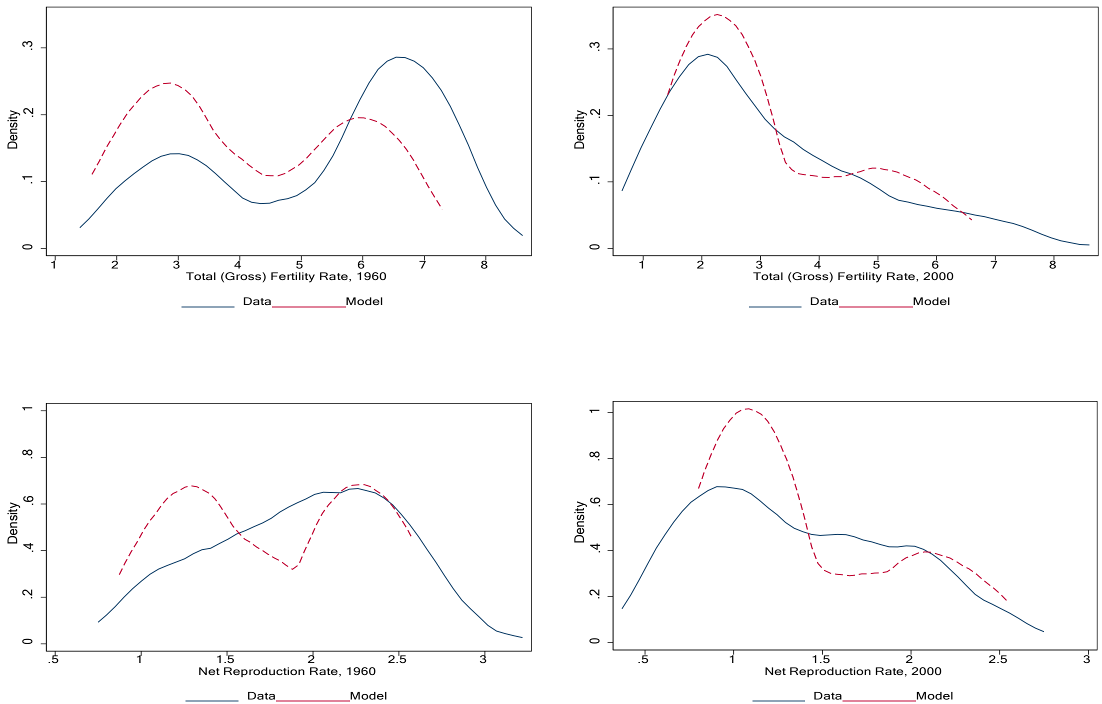
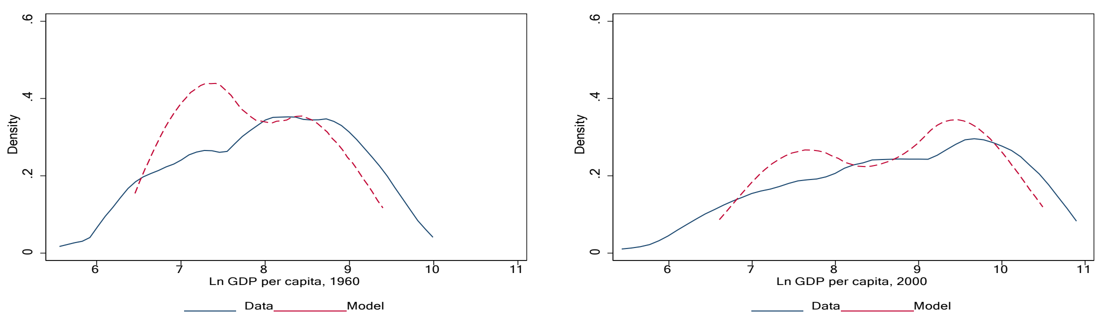


FIGURE 10. DISTRIBUTION OF INCOME PER CAPITA [SIMULATION AND DATA (1960 AND 2000)]



development patterns, suggesting that the differential timing of the take-off is a crucial determinant of comparative development differences.

The role of country specific differences in extrinsic mortality (i.e., in particular, in the exposure to pathogens) for the timing of the take-off has been isolated by performing counterfactual exercises. The results show that even moderate differences in mortality can be relevant for comparative development differences by inducing sizable delays in the take-off of growth, although they leave the cross-country correlations essentially unaffected. The analysis also provides valuable insights on the design of empirical investigations from the perspective of unified growth theory, rather than balanced growth. The results show, in particular, that linear empirical specifications may lead to misleading conclusions about the empirical the role of relevant determinants of long run growth.

The findings suggest some directions for further research. The unified growth framework can be applied to compare the quantitative relevance of alternative country specific determinants of comparative development beyond the role of extrinsic mortality investigated in this paper. Also, while instructive regarding the main mechanics, the analysis has completely abstracted from cross-country spill-overs, for instance in technological and medical knowledge, or other interactions between countries at very different stages of development. Compared to the historical experience of the European forerunners, the development of some (but not all) developing countries appears to be characterized by an acceleration, as documented by the differences between the simulated and the real world after 1960. Extending the unified growth framework to the explicit consideration of cross-country spill-overs therefore appears a fruitful direction for future research.

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Appendix: Derivations and Proofs

With the assumptions made in Section II.A, the utility can be expressed as,

$$\text{(A1)} \qquad U(c_t, \pi_t n_t q_t) = T_t \ln c_t + \gamma \ln(\pi_t n_t q_t) \; .$$

The time budget faced by an individual is given by

$$\text{(A2)} \qquad \overline{T}_t \geq l_t + \underline{e}^j + \pi_t n_t r_t \, .$$

In addition, the individual faces a resource constraint

$$\text{(A3)} \qquad l_t w_t^j h_t^j(a) \geq T_t c_t \, ,$$

where l_t is the total time spent working. Given the utility function (A1) both constraints will be binding at the optimum. Combining (A2) and (A3) delivers the budget (5) in the text. Maximizing utility (A1) subject to (5) is equivalent to maximizing

$$\text{(A4)} \qquad T_t \ln \left[(1/T_t) \left(\overline{T}_t - \underline{e}^j - \pi_t n_t r_t \right) w_t^j h_t^j(a) \right] + \gamma \ln(\pi_t n_t q_t) \; .$$

LEMMA 1: *For any* $\left\{ w_t^j, T_t, \pi_t, g_{t+1} \right\}$, *the optimal fertility of an individual acquiring human capital* $j = \{u, s\}$ *is given by,*

$$\text{(A5)} \qquad n_t^j = \frac{\gamma \left(\overline{T}_t - \underline{e}^j \right)}{(T_t + \gamma)\, r_t^j \pi_t}$$

where r_t^j *is given by,*

$$\text{(A6)} \qquad r_t^j = r_t^* = \max \left\{ \underline{r}, \frac{1 - [1/(\delta(1 + g_{t+1}))]}{1 - \beta} \underline{r} \right\}$$

Proof of Lemma 1. Consider an individual acquiring human capital of type $j = u, s$. Taking the first order condition of (A4) with respect to n_t and restricting to an interior solution gives (A5), while taking the first order condition with respect to r_t^j gives,

$$\text{(A7)} \qquad -T_t \pi_t n_t r_t^j + \gamma \left(\overline{T}_t - \pi_t n_t r_t^j - \underline{e}^j \right) q_r(\cdot) r_t^j / q(\cdot) \geq 0.$$

Using (A5) to simplify (A7) implies $\left[q_r(r_t^j, g_{t+1}) r_t^j / q(r_t^j, g_{t+1}) \right] \geq 1$. Given the functional form (4) this implies (A6).[57]

[57] From (A6) there is a unique $\underline{g} > 0$ (implicitly given by $r_t^*(\underline{g}) = \underline{r}$) such that for any $g_{t+1} > \underline{g}$ then $r_t^* > \underline{r}$ and $dr_t^*/dg_{t+1} > 0$.

LEMMA 2: *For any* $\{w^s_t, w^u_t, T_t, \pi_t\}$ *there exists a unique* $\tilde{a}_t$ *implicitly defined by*

$$\text{(A8)} \qquad \frac{h^s_t(\tilde{a}_t)}{h^u_t} = \left(\frac{\bar{T}_t - e^u}{\bar{T}_t - e^s}\right)^{\frac{T_t+\gamma}{T_t}} \frac{w^u_t}{w^s_t}$$

such that all individuals with $a \leq a_t$ *optimally choose to acquire unskilled human capital* $j = u$ *while all individuals with* $a > \tilde{a}_t$ *acquire skilled human capital* $j = s$.

Proof of Lemma 2. The optimal type of human capital maximizes the indirect utility obtained from $j = u, s$. Evaluating the indirect utility substituting for η^j with $j = u, s$ from (A5) and noting that $r^u_t = r^s_t = r^*_t$ from (A6) implies that the optimal type of skill depends on,

$$\text{(A9)} \qquad (\bar{T}_t - e^u)^{(T_t+\gamma)} (w^u_t h^u_t)^{T_t} \gtrless (\bar{T}_t - e^s)^{(T_t+\gamma)} (w^s_t h^s_t(a))^{T_t} .$$

Since the indirect utility obtained by acquiring skilled human capital increases with ability, there exists a unique $\tilde{a}_t$ such that all individuals with $a < a_t$ optimally choose to acquire u, while those with $a > a$ optimally choose to obtain s. Solving (A9) as equality gives (A8).

Proof of Proposition 1. The aggregate levels of human capital are given by,

$$\text{(A10)} \qquad H^u_t = N_t \int_0^{\tilde{a}_t} h^u_t f(a)da \quad \text{and} \quad H^s_t = N_t \int_{\tilde{a}_t}^1 h^s_t(a) f(a)da .$$

From (3), the ratio of competitively determined wages is

$$\text{(A11)} \qquad \frac{w^u_t}{w^s_t} = \frac{1-x_t}{x_t}\left(\frac{H^s_t}{H^u_t}\right)^{1-\eta} = \frac{1-x_t}{x_t}\left(\frac{\int_{\tilde{a}_t}^1 h^s(a)f(a)da}{\int_0^{\tilde{a}_t} h^u f(a)da}\right)^{1-\eta} .$$

Substituting (A11) into (A8) gives the general equilibrium ability threshold

$$\text{(A12)} \qquad \frac{h^u_t \left(\int_{a_t}^1 h^s(a)f(a)da\right)^{1-\eta}}{h^s(\tilde{a}_t)\left(\int_0^{\tilde{a}_t} h^u_t f(a)da\right)^{1-\eta}} = \frac{x_t}{1-x_t}\left(\frac{\bar{T}_t - e^s}{\bar{T}_t - e^u}\right)^{\frac{T_t+\gamma}{T_t}} .$$

Since there is a one-to-one relationship between the share of skilled workers λ_t and the threshold ability a_t, this also characterizes implicitly the equilibrium share of skilled individuals, λ_t, where H^u_t is decreasing in λ_t and H^s_t is increasing in λ_t. Rearrange (A12) to get the equilibrium relationship between $\tilde{a}_t$ and T_t expressed as

$$\text{(A13)} \qquad G(\tilde{a}_t)^{1-\eta} F(x_t) - \left(\frac{\bar{T}_t - e^s}{\bar{T}_t - e^u}\right)^{\frac{T_t+\gamma}{T_t}} = 0$$

where $\bar{T}_t := min\{T_t, R\}$, $F(x) := ((1 - x_t)/x_t)$ and

$$\text{(A14)} \qquad G(\tilde{a}_t) = \frac{(h^u)^{\frac{1}{1-\eta}} \int_{\tilde{a}_t}^1 h^s(a) f(a)\, da}{h^s(\tilde{a}_t)^{\frac{1}{1-\eta}} \int_0^{\tilde{a}_t} h^u f(a)\, da}$$

with $G'(\tilde{a}_t) < 0$. Notice that $\left(\overline{T_t} - \underline{e}^s\right) / \left(\overline{T_t} - \underline{e}^u\right) \in (0, 1)$ for $T_t \in (\underline{e}^s, \infty)$. For any x_t, the function (A13) is therefore defined over the range $a \in (\underline{a}(x_t), 1]$ where[58]

$$\text{(A15)} \qquad \underline{a}(x_t) : G(\underline{a}(x_t))^{1-\eta} F(x_t) = 1$$

Applying calculus, $\partial\underline{a}(x_t)/\partial x_t < 0$ with $\lim_{x\to 0} \underline{a}(x) = 1$ and $\lim_{x\to 1} \underline{a}(x) = 0$. Accordingly for any x_t there exists a level $\lambda(x_t) < 1$ which represents the maximum share of the population that for each generation t would acquire skilled human capital in the case in which $T_t \to \infty$. By totally differentiating (A13) we have,

$$\text{(A16)} \qquad \frac{d\tilde{a}_t}{dT_t} = \frac{d\left(\frac{\overline{T_t} - e_s}{T_t - \underline{e}^u} \frac{T_t + \gamma}{T_t}\right) / dT_t}{(1-\eta)\, G(\tilde{a}_t)^{-\eta}\, G'(\tilde{a}_t)\, F(x_t)} < 0$$

which is negative since $G'(\tilde{a}_t) < 0$. For $T_t = \underline{e}^s$ we have $\tilde{a}_t = 1$ which implies $G(a_t) = 0$ so that $G(a_t)^{-\eta} = \infty$. Since $G'(1)$ is a finite number we have that the denominator of (A16) goes to infinity as $T_t \to \underline{e}^s$. In turns the numerator has a limit at zero. For $T_t \to \infty$ we have $\tilde{a}_t \to \underline{a} < 1$ so that the denominator of (A16) is a finite number while the numerator has a limit at zero. Hence $\lim_{T_t \to \underline{e}^s} \frac{d\tilde{a}_t}{dT_t} = \lim_{T_t\to\infty} \frac{d\tilde{a}_t}{dT_t} = 0$ which also implies that the equilibrium locus (12) is convex for $T_t \to \underline{e}^s$ and concave for $T_t \to \infty$.

LEMMA 3: *TFP, A_t, and the relative productivity of skilled human capital x_t increase monotonically over generations with* $\lim_{t\to\infty} x_t = 1$, $\lim_{t\to\infty} A_t = +\infty$ *and* $\lim_{t\to\infty} g_t = \varphi$.

Proof of Lemma 3. From Proposition 1 for any $T_t > \underline{e}^s$ and any $x_t > 0$, we have $\lambda_t > 0$. From (8) this implies $x_t > x_{t-1}$ for all t with $\lim_{t\to\infty} x_t = 1$; from (9), $g_t > 0$ and $\lim_{t\to\infty} A_t = \infty$ for any $A_0 > 0$. In the limit as $\lambda_t \to 1$, $g_t = \varphi$ from (9).

Proof of Proposition 2. The equilibrium relationship linking a_t and T_t is given in (A13). For any T_t, a_t is an implicit function of x_t. Recall that by implicit differentiation of (A12) $\partial a_t/\partial x_t < 0$ which implies that the equilibrium share of skilled individuals is increasing in x_t : $\partial\lambda_t/\partial x_t > 0$ for any T_t. Consider part *(i)*. If $x_0 \to 0$ and $A_0 \to 0$ then $\underline{a}(0) \to 1$, for all $T \in (\underline{e}^s, \infty)$ which implies $a \to 1$ and $\lambda \to 0$. In this case the two loci Λ and Υ cross only once for $\lambda \to 0$ and $T \to \underline{T}$ and the average fertility is given by n^u as implied by (A5) evaluated at $T = \underline{T}$. Under these conditions, from (2) the level of income per capita is (arbitrarily) low which, from (7) and (14) implies $\pi_0 \simeq \underline{\pi}$. Part *(ii)* follows directly from Lemma 3, where $A_\infty \to \infty$, $x_\infty \to 1$, $\lambda_\infty \simeq 1$, $T = \overline{T}$. From (9) this also implies that $g_\infty = \varphi$. Finally, since $A_\infty \to \infty$, it follows that $y_\infty \to \infty$ and from (7), $\pi_\infty \to 1$ so that fertility is given as in (15). Part *(iii)* follows from combining Part *(i)*, Part *(ii)*, and Lemma 3.

Figure A3 in the Online Appendix depicts the evolution of the conditional system given by equations (6) and (12) for the case in which the latter function has a unique inflection point. From *(i)* and *(ii)* the conditional system has a unique steady state for x_0 and x_∞ as illustrated in Figure A3 Panels (a) and (c).

[58]Since the denominator of (A13) has a discontinuity at $\underline{a}$ and the function takes negative values for any $a \leq \underline{a}(x_t)$.

TABLE A1—SUMMARY INFORMATION ON CALIBRATION OF PARAMETERS

Parameter		**Value**	**Matched Moment (Information Source)**
Benchmark Calibration			
Parameters Set Exogenously			
Year of convergence to balanced growth path		2000	First generation with $\lambda > 0.999$
Length of one generation	m	20 years	Average age at first birth (Dribe, 2004, Mturi and Hinde, 2007)
Years before retirement (at age 5)	R	59	Average effective age of retirement in Sweden (OECD)
Production function	η	0.2857	Elasticity of Substitution between skilled and unskilled labor (Acemoglu, 2002)
Parameters Set Endogenously			
TFP growth	φ	0.61	Average growth GDP per capita 1995-2010 (ERS Dataset, Sweden)
Time cost for unskilled/skilled education	$\{\underline{e}^u, \underline{e}^s\}$	{0,12}	Years of schooling in 1820 and 2000 (Lutz et al. 2007/Ljungberg-Nilsson, 2009)
Productivity of ability for Human Capital	α	6.1	Spread of log income distribution 2000 (ECHP)
Mean/standard deviation of ability distribution	$\{\mu, \sigma\}$	{0.49,0.066}	Mean and variance of log income in 2000 (ECHP)
Baseline adult longevity/scope for improvement	$\{\underline{T}, \rho\}$	{45,31}	Average LE at 5 in 1760-1800 and 2000 (Human Mortality DataBase)
Minimum child survival and elasticity parameter	$\{\underline{\pi}, \kappa\}$	{0.5, 0.005}	Child survival probability in 1800 and 2000 (Human Mortality Data Base)
Preferences	γ	9	Gross (total) fertility around 2000 (World Development Indicators)
Function quality of children	$\{\beta, \underline{r}, \delta\}$	{0.23, 4.7, 3.54}	Pre/Post-transitional Fertility, TFP growth 1900 (Keyfitz and Flieger, 1968, World Development Indicators, Historical Statistics Sweden)
Initial Conditions			
Initial importance of skilled human capital	x_0	0.04	Initial year of calibration, generations before balanced growth is reached
Initial TFP	A_0	15	Level of log GDP per capita Sweden 2000 (Historical Statistics Sweden)
Cross-Country Analysis			
Parameters Set Endogenously			
Baseline adult longevity/scope for improvement	$\{\underline{T}, \rho'\}$	{40,36}	Minimum observed life expectancy at age 5 across country in 2000 (UN)
Function quality of children (high fertility)	$\{\beta', \underline{r}', \delta'\}$	{0.75, 3, 1.00}	Highest fertility rates 1960 (World Development Indicators)
Distribution of baseline adult longevity			Worldwide distribution of Human Pathogens (Murray and Schaller, 2010)

Appendix: For Online Publication

A1. Calibration: Data Sources and Details

Length of a generation. The mean age at first birth for the average country is set to 20 years. in Sweden around 1800 was slightly higher, see (Dribe 2004), while age at first birth is still below 20 in pre-transitional countries in Africa nowadays, see (Mturi and Hinde 2007).

Age of retirement. Data from http://www.oecd.org/dataoecd/3/1/39371913.xls.

Technological Progress. The parameter of *TFP*, φ, is set to match the average annual growth rate of income per capita on the balanced growth path (which equals the growth rate of technological change), see main text. Data sources: ERS Dataset (www.ers.usda.gov) or historical statistics from the Bank of Sweden (www.historicalstatistics.org). Targeting TFP (multi-factor productivity), labor productivity, or the Solow Residual around 2000, instead of income, would deliver very similar values for φ.[59]

Production Function. The elasticity of substitution between skilled and unskilled workers is set following the literature. See for instance (Acemoglu 2002).

Human Capital. To calibrate some parameters we target a 10 percent pre-transitional share of skilled individuals. The alternative available data sources provide slightly heterogeneous information on enrolment rates in the early 19th Century Sweden, with estimates ranging from about 5 to about 15 percent, see de la Croix et al. (2008) and (Ljungberg and Nilsson 2009). The target of the precise value of λ before the transition used for the computation is of little importance for the the obtained parameters, however. The results for alternative parameters obtained by targeting levels of λ up to 0.3 are essentially the same. The average years of schooling in Sweden was 12 years in 2000 (for the cohort age 25-35). Data from Lutz et al. (2007). The earliest available data suggest around 1 year of schooling on average before or around the onset of the transition. The estimates are slightly lower when referring to the entire population alive in Sweden in 2000 since older cohorts are included (for instance 11.4 in the data of Barro and Lee, 2001 and 11.5 years in (Ljungberg and Nilsson 2009)). Regarding pre-transitional education levels, the estimates differ somewhat more. (Ljungberg and Nilsson 2009) report 1.03 years of schooling in the total Swedish population aged 15-65 in 1870, and 0.1 average standard school years of the population aged 7-14 around 1810-1820, considering absenteeism and length of school years.

Ability Distribution. We estimate the income distribution for Sweden in 2000 using micro data from the ECHP dataset for individual incomes of full-time employees aged 25 to 45, which corresponds to the two last cohorts in the dynamic simulation, and equivalently to the two first generations with $\lambda = 1$ in the data. The income used to estimate the parameters of the ability distribution are converted in US-$ using an average exchange rate of 9 Kroner for one US-$ in 2000. The income distribution is approximately log-normal between the 5th and 95th percentile of the data, with slightly thicker tails. The distribution of log incomes has mean 9.7, standard deviation 0.4, and the lowest and highest observed log-incomes are 6.7 and 12.8, respectively, which implies a maximum spread of 6.1. The

[59]See, e.g., OECD Statistics (https://stats.oecd.org/Index.aspx?DataSetCode=MFP) or Donghyun et al. (2012, Table 7).

moments of the income distribution for the age cohort 25-65 are essentially the same, with the lowest, mean, and highest levels of log income being 6.7, 9.7, and 12.8, respectively, and with a standard deviation of 0.41. The ECHP data are based on surveys and refer to total net income from work, which might explain the small differences between the log income per capita from macro data, which is approximately 10 in 2000, and the mean log income from the micro data that is about 9.7. The relevant data moments extracted from this data set are broadly consistent with other data sources based on register data and alternative surveys for gross earnings, see, (Domeij and Floden 2010). The data moments are also close to the ones typically used for the calibration of dispersion in permanent incomes in other OECD countries. For instance, Erosa et al. (2011) match a variance of log permanent earnings in the US of 0.36. Robustness checks show that the results are fairly insensitive to varying the dispersion. It is worth noting that the distribution of cognitive ability (or IQ), which is generally measured in the literature as a truncated normal with mean 100 and standard deviation 15, see, e.g., Neisser et al. (1996), would imply a very similar parametrization when normalized for a support $a \in [0, 1]$, with $\mu = 0.5$ and $\sigma = 0.075$.

Adult longevity. The average of life expectancy at age five in the period 1760-1840 was 48.38, in the period 1790-1810 it was 48.06. Data from the Human Mortality Data Base available at http://www.mortality.org/. Similar figures are documented for England, France and Italy, see (Woods 1997) and Bideau et al. (1997) and (Lewis and Gowland 2007). In 2000 child mortality in Sweden was around 0.004, which explains the convergence of life expectancy at 5 plus five years of 80.74, and of life expectancy at birth of 80.45.

Child survival probability. Data from http://www.mortality.org. The levels of income per capita needed for the computation of the parameters of the function of child survival are extracted from the database of historical statistics of the Bank of Sweden that is freely available online at www.historicalstatistics.org. The data are converted to US-$ using an average exchange rate in 2000 of 9 Kroner for one US-$. The income levels used for the calibration of condition (7) are 22,717 and 884 US-$, which correspond to the GDP per capita of Sweden in 2000 and 1800, respectively, in US-$ per 2000.

Preferences. Total fertility rates (TFR) in Sweden were on average 1.8 children per woman over the period 1980-2000, with substantial fluctuations. In 1990, the TFR was 2.13, whereas in 2000 it was 1.54 (World Development Indicators). A gross fertility of 1 (which would correspond to a TFR of 2) along the balanced growth path is a reasonable target. Targets in the range from 0.75 to 1.1 deliver very similar results. Concerning the cost of raising children, the target $r = 5$ in 2000 is set in line with the estimates by (Haveman and Wolfe 1995). This is equivalent to setting a target for the share of work life that is spent in raising a child is about 15 percent which is in line with Doepke (2004) and de la Croix and Doepke (2003). The weight of children relative to own lifetime consumption changes with T_t, as in Soares (2005). For $\gamma = 9$ the relative weight of children compared to per period consumption, γ/T_t, drops from around 0.18 before the transition to around 0.12 in the steady state.

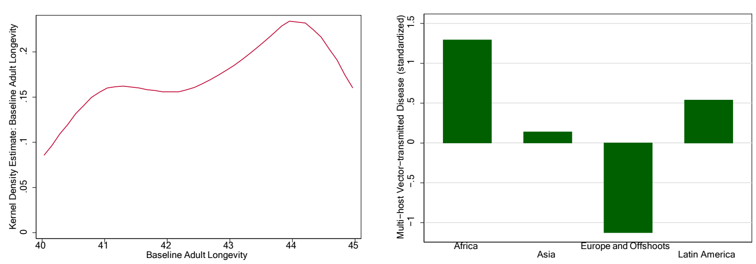
Production function of children's quality. Gross fertility in Sweden in 1800 and 2000 was $n = 2.3$ and $n = 1$. A clear drop in gross fertility occurs around 1900. The data are from (Keyfitz and Flieger 1968) and World Development Indicators. The level of TFP and income per capita growth around 1900 vary between 0.7 and 1.7 percent per year. The largest estimates are based on indexed data and include land, see (Krantz and Schön 2007), (Schön 2008) and (Greasley and Madsen 2010). Estimates of TFP and income per capita growth around 1900 vary between 0.7 and 1.7 percent per year. For the calibration we consider the average, 1.2. As an alternative calibration that does not rely on information about the growth rate of technology during the transition, one can also use information on the share of skilled around 1900 and compute the growth rate that is implied by (9). According to estimates by (Ljungberg and Nilsson 2009) average years of schooling for the cohort aged 7-14 was around 4 in 1900. Given $\{\varphi = 0.61, \underline{e}^u = 0, \underline{e}^s = 12\}$ this implies targeting a level of $\underline{g} = 0.2745$, which delivers essentially the same parametrization.

Initial Conditions. The time axis is set with reference to the convergence to the post-transitional balanced growth path (in terms of λ converging to 1) in 2000. This implies that the choice of $x_0 = 0.04$ determines the beginning of time in the calibration in the stagnation period. This parametrization also implies that the income share of unskilled human capital in total production is larger than 99.9% at the beginning of the simulation, and still above 95% in 1800 just before the transition. The initial level of technology is set targeting the level of GDP per capita in Sweden in 2000 equal to 10.03. Data are from www.historicalstatistics.org.

Cross-country differences in life expectancy. For background evidence on the role of a higher exposure to diseases in leading to a faster deficit accumulation and earlier death see, e.g., Mitnitski et al. (2001) and Searle et al. (2007). Research based on the investigation of skeletons documents that adult longevity during the Mesolithic period was lower in more difficult mortality environments, see Boldson and Paine (2000). As alternative scenario, we target a life expectancy at age five at 45 years (compared to 48 years reflecting Sweden around 1800 just before the transition). The data source is UN Population Statistics available at www.unstats.un.org. Data on life expectancy at five for earlier periods are missing for many countries, including most Sub-Saharan Africa countries in 1960. Alternatively, the available information on child mortality and life expectancy at birth in 1960 can be used to derive an estimate of life expectancy at age five. This delivers a very similar target for the highest mortality countries. In 1960 life expectancy at birth was as low as 33 years in some countries like Afghanistan, and child mortality one third. Assuming a constant death rate below the age of 5, these numbers imply a life expectancy at age five between 44 and 45 years. In some countries, like Swaziland life expectancy at birth is just above 30 years still today (data from the CIA World Factbook). This suggests that 45 is possibly a conservative estimate of baseline adult longevity in the worst conceivable mortality environment. Retaining a target of 76 years for life expectancy at age five on the balanced growth path, this implies setting a $\underline{T}$=40 and ρ = 36 (rather than $\underline{T}$=45 and ρ = 31 as in the benchmark calibration).

Cross-country differences in disease environment. The data in the historical disease prevalence across 113 countries is taken from (Murray and Schaller 2010). For each pathogen we construct a binary indicator of whether or not a disease has been present at severe or epidemic levels at least once in the history up to the early 20th century. The diseases include leishmanias, schistosomes, trypanosomes, leprosy, malaria, typhus, filariae, dengue, and tuberculosis. Six of these diseases fall into the class of multi-host vector-transmitted diseases, which are particularly difficult to prevent or eradicate even today because the pathogens survive in multiple hosts (both humans and animals), and which are bound to specific transmission vectors, like mosquitos, which require a particular geographical habitat. The endemicity of the class of multi-host vector-transmitted diseases is fairly insensitive to economic development and globalization, and thus an informative measure of cross-country differences in the extrinsic mortality environment, see Smith et al. (2007). Cervellati, Sunde and Valmori (2012) document the health relevance of the number these pathogens in terms of predicting life expectancy and the likelihood of outbreaks of epidemics. The frequency distribution of the counts of pathogens for all countries of the world is used as distribution of baseline adult longevity within the support [40, 45]. The resulting distribution, depicted in Figure A1 in terms of a kernel density plot, is modestly skewed. The frequency of simulated countries with baseline longevity $\underline{T} = 45$ corresponds to the frequency of countries with the lowest observed number of multi-host vector-transmitted diseases ever diagnosed (which includes Sweden). Conversely, the frequency of simulated countries with baseline longevity $\underline{\underline{T}} = 40$ corresponds to the frequency of countries with the highest number of multi-host vector-transmitted pathogens (which include several Sub-Saharan African countries). The distribution on the full support (40, 45) is created by a linear intrapolation of the frequency distribution of the counts of multi-host vector-transmitted diseases on a grid of 0.25 diseases. Figure A1(a) plots the resulting distribution of baseline longevity for the 113 countries of the Murray-Schaller (2010) data. Figure A1(b) plots the number of diseases in different continents relative to the world average (standardized).

FIGURE A1. THE WORLD-WIDE DISTRIBUTION OF MULTI-HOST VECTOR-TRANSMITTED DISEASES



(a) Synthetic Distribution of $\underline{T}$: Kernel Density Estimate

(b) Data: Diseases Relative to World Average

A2. *Data Sources for Time Series and Cross-Section*

Time Series for Sweden. Life expectancy and fertility data are taken from the Human Mortality Database (http://www.mortality.org), (Keyfitz and Flieger 1968) (up to 1960) and World Development Indicators (after 1960), respectively. The Data for GDP, population and GDP per capita is provided by the internet portal for historical Swedish statistics, www.historia.se and the Swedish Central Statistical Office, www.scb.se. The data on schooling are from (de la Croix, Lindh, and Malmberg 2008) while the data on average years of schooling are from (Ljungberg and Nilsson 2009).

Cross Country Panel Data. We use data from Barro and Lee as benchmark since they are used more frequently and go back to 1960. The other data sources are Human Mortality Database (www.mortality.org), the UN Population Statistics (different historical volumes of the UN Demographic Yearbook, www.unstats.un.org), the World Development Indicators at:
(http://data.worldbank.org/data-catalog/world-development-indicators).

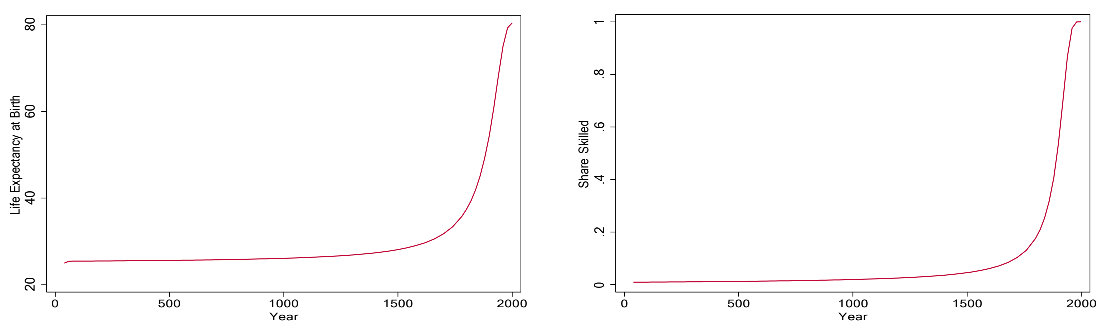
All results are qualitatively and quantitatively very similar using alternative measures like the fraction of total population with at least completed lower secondary education, or the fraction restricted to different age cohorts such as, e.g., age 20-24 years from (Lutz, Goujon, and Sanderson 2007).

Kernel Distribution. For comparability, the distributions of real data are based on a homogenized sample of 90 countries, for which information on the share of skilled individuals, life expectancy at birth, child mortality, total fertility rate, and the net reproduction rate is available for 1960 and 2000. The results are similar when using unrestricted samples for the different variables.

A3. Illustration of the Simulated Development Path

Figure A2 depicts the simulated data for the equilibrium share of individuals acquiring skilled human capital and of life expectancy at birth that is obtained from the benchmark calibration. The figure plots the evolution of these variables over the entire simulation period and illustrates the lengthy phase of slow development followed by the endogenous take-off.

FIGURE A2. LONG-RUN DEVELOPMENT: SIMULATION OF BENCHMARK CALIBRATION



(a) Life Expectancy at Birth *(b)* Share Skilled λ

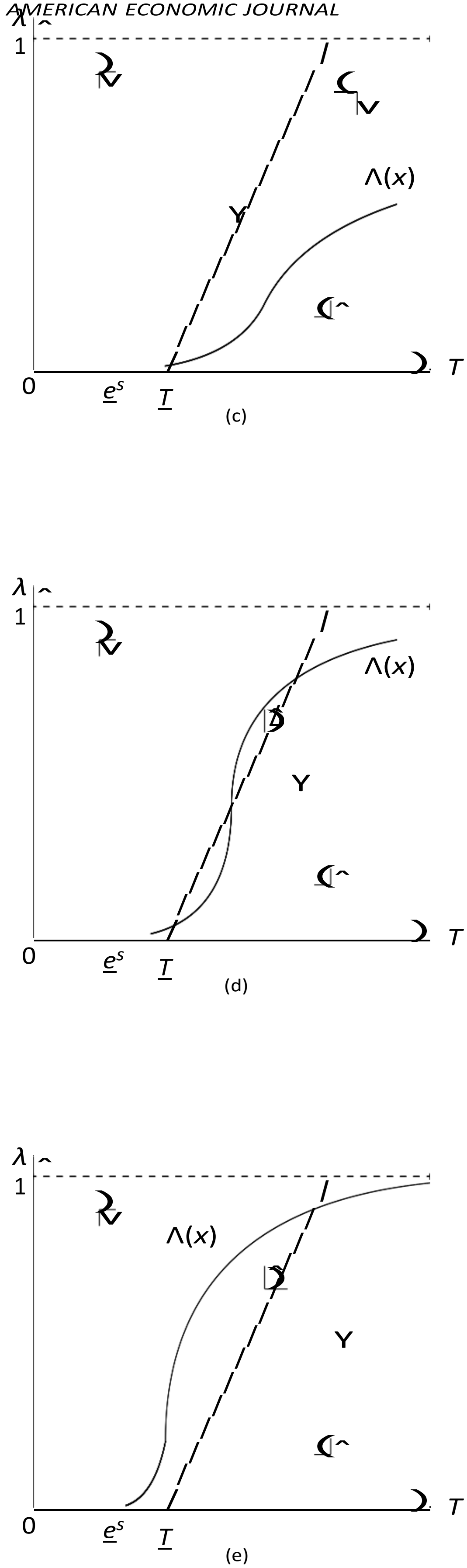


FIGURE A3. THE PROCESS OF DEVELOPMENT

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Bibliography for Online Appendix

The following references refer exclusively to the data sources and articles cited in the Online Appendix.

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