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The ‘miniature silo’ test: A simple experimental set-up to estimate the effective friction coefficient between the granular solid and a horizontally-corrugated cylindrical metal silo wall

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ABSTRACT

Corrugated walls are widely employed in the construction of metal silos. Despite the long history of testing to establish bulk material parameters for silo design, there does not appear to be an established test procedure to directly determine the effective friction coefficient for a corrugated wall other than an adaptation of a classical direct shear test. EN 1991-4: 2006 prescribes a simple weighted average formula for this coefficient based on the internal friction angle of the granular solid and a ‘flat’ wall friction coefficient. This paper describes a set of miniature-scale silo tests performed at the University of Bologna intended to establish a simple but useful procedure to directly estimate an average global friction coefficient between the granular solid and a corrugated silo wall. These ‘miniature silo’ tests are simple enough to be performed in any civil engineering laboratory with standard equipment.

Keywords: Cylindrical metal silo. Granular solid. Internal friction angle. Flat wall. Corrugated wall. Effective friction coefficient.

1. Introduction

The wall of a cylindrical silo is subjected to both normal pressures and friction shear tractions by the stored granular solid. The magnitude and distribution of such actions along the wall are both affected by the physical and mechanical properties of the stored material including the frictional properties of the contact surface with the wall. The structural designer's choice of the numerical value of the friction coefficient at the interface between the particles and the steel silo wall often represents an uncertain step of the silo design process, and one upon which the final project strongly depends. The uncertainties are particularly large in the case of horizontally corrugated wall.

The state-of-the-art EN 1991-4: 2006 [1] provides test methods (Annexes C and D) and design material properties values of various commonly stored granular solid (Annex E) to aid in the structural design of a silo.

Annex C allows for the determination of the values of both the internal friction angle of the granular material, and the friction coefficient of the interface between the granular particles and a flat wall, by means of standard laboratory tests such as direct shear tests (commonly used in the geotechnical engineering field). The internal friction angle can be obtained by filling the two cells with the granular particles, while the friction coefficient between the granular particles and the flat wall can be obtained by shearing a sample of the granular solid (upper cell) against a flat steel surface representing the silo wall (placed on the top of the lower cell).

Annex D presents an empirical formula (also found in Moore *et al.* [2]) to evaluate the effective friction coefficient, μ_{eff}, between the granular solid and the corrugated-wall interface by performing a weighted average of the tangent of the granular solid's internal friction angle, $\mu_g = \tan \phi_i$, and its friction coefficient against a flat wall surface, μ_w:

$$\mu_{eff} = (1 - a_w)\mu_g + a_w \mu_w \tag{1}$$

where $a_w = b_w / (b_w + b_i)$ is a geometric parameter which identifies the portion of interface that involves the solid moving against the wall, with b_w being the vertical length of the contact surface between the particles and the equivalent flat wall and b_i the vertical length of the particle–particle interface (Fig. 1). However, the accuracy of this empirical formula is uncertain, particularly for the case of a sinusoidal profile (for which the above-mentioned vertical lengths depend on the assumed horizontal position of the internal rupture surface), and indeed Moore *et al.* [2] suggested that μ_{eff} could be simply taken as the internal friction coefficient of the granular solid, i.e., $\tan \phi_i$.

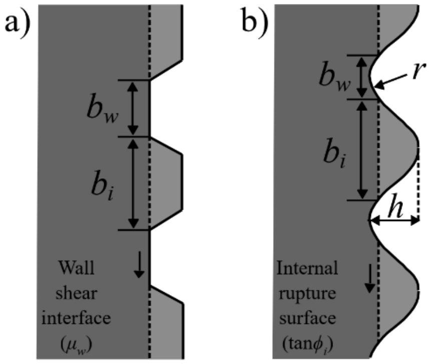


Fig. 1 – Corrugated wall interface with the granular material inside the silo.

Significant research effort has been dedicated to the determination of representative bulk granular solid properties such as the unit weight, the lateral pressure ratio, the angle of repose and the angle of internal friction as well as the friction coefficient between granular particles and flat walls under different conditions using standard testing equipment [2-19]. Other studies attempted to better reproduce the conditions experienced in practice during the filling or discharge conditions by building custom modifications of existing classical equipment.

For example, Molenda *et al.* [11] used a modified shear apparatus of large dimensions in which a corrugated steel plate could be placed. One of the objectives of their work was to compare the friction of wheat on corrugated steel of two different wave-length profiles, and to evaluate the internal friction angle of the wheat. They concluded that Eq. (1) underestimates the experimental result and is very sensitive to the corrugation wave geometry. Moore *et al.* [2] used a complicated special apparatus capable of measuring the frictional force imposed on different surfaces subjected to a constant grain pressure. A model was proposed that describes this relationship for the corrugated test surface. The obtained results tend to support the conclusions of Pieper [20] and Reimbert and Reimbert [21] that, for corrugated-wall silos, the effective wall friction coefficient can simply be taken as equal to the internal friction coefficient.

This short communication aims to propose a nonstandard but straightforward laboratory test to estimate an average effective value of the friction coefficient at the interface between the particulate solid and the silo wall, applicable to most cohesionless granular solids and wall construction. It is intended to be simple enough to be performed in even a minimally equipped structural laboratory that can aid in the validation of input parameters used in the design of horizontally corrugated metal silos, such as those obtained based on the previously presented simple formula from EN 1991-4: 2006 [1].

2. 'Miniature silo' test concept

The 'miniature silo' test is intended to be an economic, fast and practical test that should provide useful experimental support for design-oriented choices of the particle parameters and to verify the simple averaging formula adopted by EN 1991-4: 2006 [1]. The principle of the 'miniature silo' test is as follows.

A scaled open-bottom cylindrical specimen is set to rest on an even horizontal surface with no restraints and filled with a granular solid to a specified depth. The manner of filling, be it by means of a localised flow stream or distributed 'rain' filling, may strongly affect the resulting wall pressure distribution and the filling eccentricity should be kept to a minimum. After filling, the system may be idealised as shown in Fig. 2a, where a portion, $W_{g,c}$, of the total weight of the granular solid, W_g, is transmitted by frictional shear into the cylinder wall (which will then be under axial compression) and a reduced portion, $W_{g,t}$, rests directly on the table surface. The portion of the table in contact with the wall thus initially supports both $W_{g,c}$ and the weight of the cylinder, W_c. In this condition, if a Linear Variable Displacement Transducer (LVDT) is mounted at the top edge of the cylinder, it should currently detect only negligible movement corresponding to the contraction of the cylinder wall due to the compression induced by the frictional shear between the granular solid and the wall.

An upwards force F is then applied through the centroid at the top of the silo in such a way that it is distributed as evenly as possible into the top edge of the specimen. Initially, although the force increases, the bottom of the cylinder remains in direct contact with the table which supports a reduced weight of $W_c + W_{g,c} - F$. Eventually, a peak force, F_{max}, is attained at which a finite upward displacement occurs corresponding to gross plastic failure of the solid along the wall and thus fully-developed friction at the wall-particle interface. The LVDT mounted at the top edge should now detect a finite movement corresponding to the detachment of the cylinder with respect to the table. As the granular material will now begin to spill out radially from beneath the container base, this is the maximum upward force that can be applied in the test (Fig. 2b). At the instant in time of incipient movement, the contact between the cylinder and the table is lost and the uplifting force is equal to:

$$F_{\max} = W_c + W_{g,c} \tag{2}$$

with:

$$W_{g,c} = \int_0^h \int_0^{2\pi} p_t\left(z,\theta\right) r d\theta dz \tag{3}$$

where r is the radius of the cylinder, h is the content filling depth (assumed circumferentially uniform) and $p_t(z, \theta)$ is the vertical frictional traction acting on the interior wall as a function of the depth z below the idealised flat surface of the solid and the circumferential position θ. This condition may be used to estimate the global average wall friction coefficient by numerical solution based on accurately known F_{max}, W_c and an assumption regarding the pressure distribution inside the cylinder to obtain $W_{g,c}$. For example, if the classical Janssen assumptions of an axisymmetric vertical pressure state are made [22], represented by a mean vertical pressure p_v at any depth z related to the mean horizontal pressure p_h by a constant lateral pressure ratio $K = p_h / p_v$ which in turn is related to the mean frictional shear p_t by a constant wall friction coefficient μ, then the expression for $W_{g,c}$ is:

$$W_{g,c} = 2\pi r \mu K \int_0^h p_v\left(z\right) dz \tag{4}$$

Two simple axisymmetric pressure models will be illustrated here, a linear geostatic pressure distribution and the classical nonlinear Janssen pressure distribution [23] given by Eqs (5) and (6), respectively:

$$p_v\left(z\right) = \gamma z \tag{5}$$

$$p_v\left(z\right) = \gamma z_0 \left(1 - e^{-z/z_0}\right) \tag{6}$$

where:

$$z_0 = \frac{r}{2\mu K} \tag{7}$$

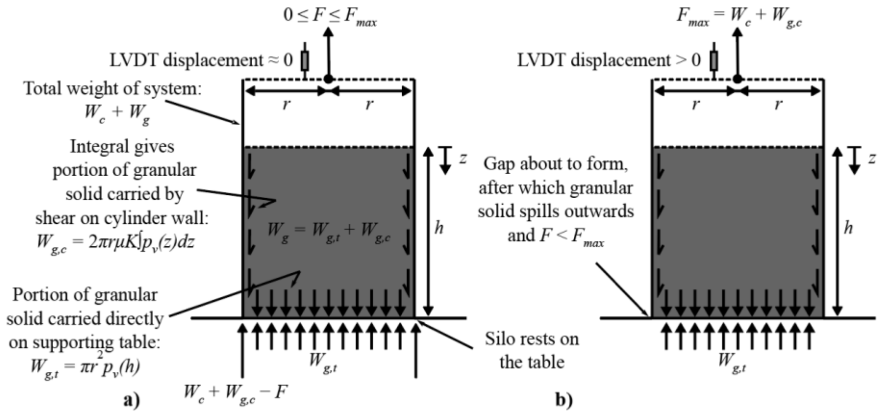


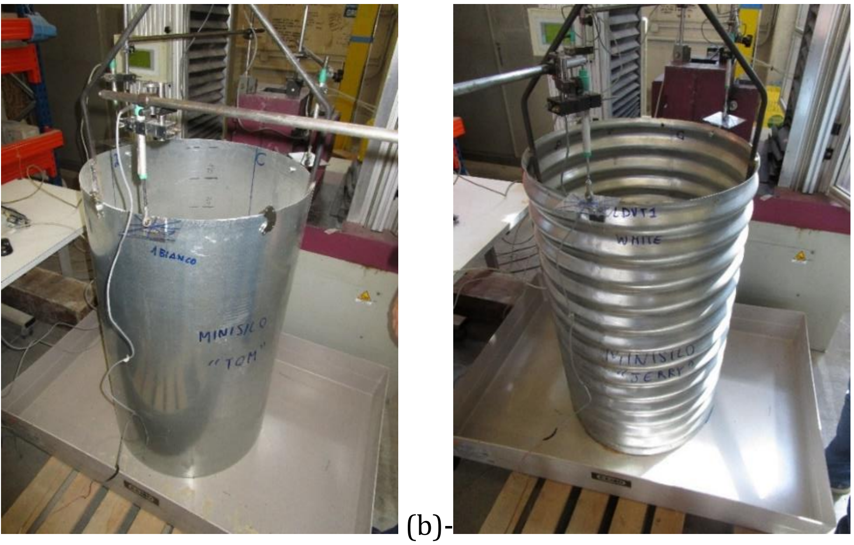
Fig. 2 – Silo specimen condition: a) before finite movement, b) incipient finite movement.

3. Application of the ‘miniature silo’ test to soft wheat

3.1. Test apparatus and materials

Two laboratory scale S350GD galvanised steel cylinders were prepared with a diameter of d = 55 cm and a cylinder height of 93 cm, one with a flat wall and the other with a corrugated wall 1 mm thick (Figs. 3a and b) and self-weights W_c = 202 and 188 N, respectively. For the corrugated ‘miniature silo’: the outer diameter was around 56 cm, the inner diameter was around 54cm, the corrugated profile was characterized by a 67.7 mm wave period and 13.5 mm wave height, while the wave radius was 18 mm (common dimensions used for steel silos). Non-circularity was not measured, though it was assumed small and unlikely to influence the test.

Soft wheat was selected as granular solid. The measured unit weight was found to be 8.04 kN/m^3 by determining the weight of the wheat in a container with known volume and weight, in order to verify the value given by the technical note of the supplier. This was found to be approximately 7% larger than the lower limit specified by EN 1991-4: 2006 [1] for the wheat, but 2% smaller than the value assigned by ANSI/ASAE S433.1: 2019 [24] for a free-flowing material. The average diameter of the particles was found to be 2.5 – 3 mm, with no more than 4% of fine material of diameter smaller than 2 mm based on the result of a particle size distribution test (i.e., sieve analysis, a well-known classical method commonly used for soil-related geotechnical studies) and manual measurements.



(a)- (b)-

Fig. 3 – a) Flat-walled and b) corrugated ‘miniature silo’ specimens.

3.2. Test procedure

Each cylinder was placed vertically in a wide base box (later used to collect the granular particles) and was filled gradually and approximately concentrically from the top by means of a funnel whose outlet was located directly above the centroid. At the end of the filling process, the top surface of the stored material was manually levelled by means of a brush to remove any conical piles of granular material.

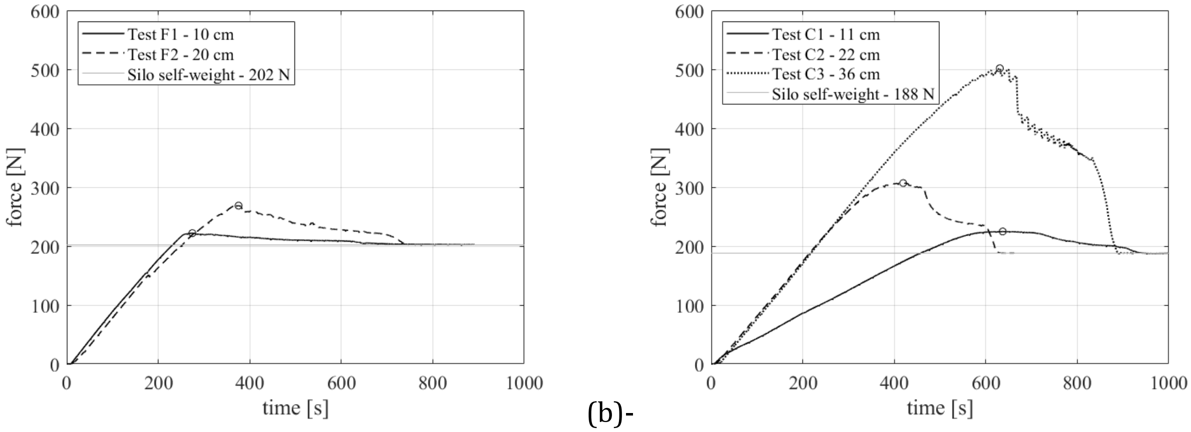
The top edge of the cylinder was equipped with steel bars to which a steel cable was anchored that could apply an uplifting force provided by a displacement-controlled machine with 1500 N capacity. A load cell with 1 N precision was placed in series along the uplifting cable. At the beginning of the test, the rate of the displacement increment was kept very low (around 0.05 mm/s, very close to the shear displacement rate suggested by Annex C.9 of EN 1991-4: 2006 for direct shear tests), corresponding (through the stiffness of the uplifting system) to a force increment of around 1 N/s to carefully detect the peak force F_{max} at which the material slip began. After the peak force was detected, the displacement rate was increased up to around 0.15 mm/s.

Each specimen was instrumented with three LVDTs spaced 120° apart along the upper circumferential edge (Fig. 3) to monitor the vertical displacements during the test.

A total of five tests were performed with different filling depths h: two tests were conducted on the flat-walled 'miniature silo' (Tests F1 and F2: h = 10 cm, h = 20 cm), while three tests were conducted on the corrugated one (Tests C1, C2 and C3: h = 11 cm, h = 22 cm and h = 36 cm). Due to significant constraints, each arrangement could be performed only once which precludes the calculation of any metrics of dispersion in the measurements.

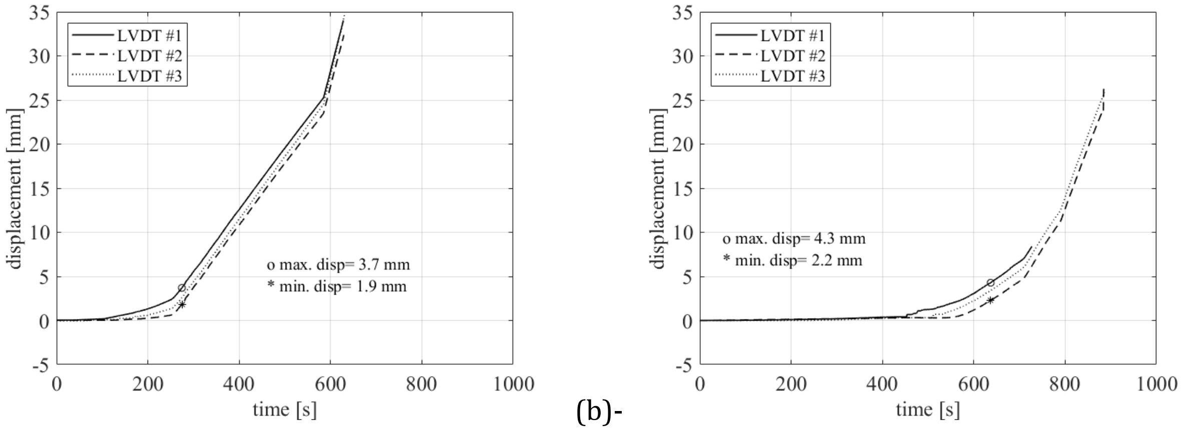
3.3. Test results

Figs. 4a and 4b display the force-time plots as obtained for all the performed tests, while Figs. 5a and 5b display the displacement-time plots for just Tests F1 and C1 respectively, as recorded by the three LVDTs installed on the upper edge of the specimens. These plots show that initially the force increases approximately linearly with only negligible vertical movement of the wall. A small movement was observed during the initial phase as the consequence of a small rotation associated with an eccentricity in the response (likely because of a mixture of slightly non-uniform lifting, non-uniform filling and thus non-uniform slip) rather than a gross vertical displacement, as testified by the displacement-time plots. The total vertical detachment of the cylinder from the table occurred at the instant in time for which the last LVDT recorded a finite displacement significantly different from zero. Afterwards, the granular particles started to flow out from the base wall edge and the force recorded by the load cell decreased down to the cylinder self-weight.



(a)- (b)-

Fig. 4 – Force-time registrations for the a) flat-walled and b) corrugated 'miniature silo'.



(a)- (b)-

Fig. 5 – Displacement-time registrations for the a) flat-walled 'miniature silo' during Test F1, and the b) corrugated 'miniature silo' during Test C1.

3.4. Interpretation of the results

No assumptions about the distribution of the internal pressure state with depth have so far been made. Starting from the theoretical model introduced in Section 2 and considering the previously stated classical Janssen assumptions underlying Eq. (4), the product μK can be estimated after substituting Eq. (4) into Eq. (2) as:

$$\mu K = \frac{W_{g,c}}{2\pi r \int_0^h p_v(z)\,dz} = \frac{F_{\max} - W_c}{2\pi r \int_0^h p_v(z)\,dz} \tag{8}$$

A pressure distribution must now be hypothesized. If a linear geostatic pressure distribution is chosen, substitution of Eq. (5) in Eq. (8) leads to a simple formula that directly provides an estimate for the μK value:

$$\mu K = \frac{F_{\max} - W_c}{2\pi r \int_0^h p_v(z)\,dz} = \frac{F_{\max} - W_c}{\pi r \gamma h^2} \tag{9}$$

If instead the Janssen or other nonlinear pressure distribution is assumed, substitution of Eq. (6) into Eq. (8) leads to a transcendental equation from which an estimate for the μK value may be obtained iteratively (since μK is also included in z_0):

$$\mu K = \frac{F_{\max} - W_c}{2\pi r \int_0^h p_v(z)\,dz} = \frac{F_{\max} - W_c}{2\pi r \gamma z_0 \left[h - z_0\left(1 - e^{-h/z_0}\right)\right]} \tag{10}$$

Table 1 reports the results obtained considering these two pressure distributions.

Table 1 – Results of the 'miniature silo' tests in terms of product μK.

Test data					μK estimates [-]	
Test	h [m]	F_{max} [N]	W_c [N]	$W_{g,c}$ [N]	Geostatic pressure distribution Eq. (9)	Janssen pressure distribution Eq. (10)
F1	0.10	222	202	20	0.29	0.31
F2	0.20	260	202	58	0.21	0.24
C1	0.11	225	188	37	0.44	0.50
C2	0.22	303	188	115	0.34	0.43
C3	0.36	490	188	302	0.34	0.50

At this point, an assumption on the numerical value of the lateral pressure ratio K must also be made to obtain the wall friction coefficient μ corresponding to the *specific* filling depth of the single test (or vice-versa).

By performing a certain number of tests with different filling depths, an overall estimation of the *global* average wall friction coefficient μ_{av} can then be derived based on the Mohr-Coulomb failure criterion ($\tau_{av} = \mu_{av}\sigma_{av} + c$) as the slope of the best-fitting interpolating line between the results in terms of couples ($\tau_{av} = W_{g,c} / A_{friction}$, $\sigma_{av} = \tau_{av} / \mu$, where μ is specific for each test) of average shear and normal stresses, assuming a uniform stress distribution along the whole friction area ($A_{friction} = 2\pi rh$). Table 2 and Fig. 6 report the outcomes considering the two pressure distributions, assuming K = 0.55 for illustration purposes. This value was initially chosen as the mean value of the range 0.50 - 0.60, provided by the application of Formula 4.7 and the limits stated by Annex E of EN 1991-4: 2006 [1] for the internal friction angle of wheat. It should be noted that this critical choice should be carefully made since it strongly affects the calculation of the friction coefficient.

Table 2 summarises the results obtained from the 'miniature silo' tests: (i) assuming the geostatic pressure distribution, μ_{av} turns out to be equal to 0.24 for the flat case and 0.53 for the corrugated case, (ii) assuming the Janssen pressure distribution, μ_{av} turns out to be equal to 0.28 for the flat case and 0.90 for the corrugated case. The plausibility of the obtained effective friction values is hereafter first checked against the widely used EN 1991-4: 2006 formula (Eq. (1)) and then further discussed in the next section.

Table 2 – Results of the 'miniature silo' tests in terms of μ assuming K=0.55.

Test	Uniform distribution assumption		Geostatic pressure distribution (Fig. 6a) K=0.55			Janssen pressure distribution (Fig. 6b) K=0.55		
	$A_{friction}$ [m^2]	τ_{av} [Pa]	μ (specific for each test) [-]	σ_{av} [Pa]	μ_{av} (average over all tests) [-]	μ (specific for each test) [-]	σ_{av} [Pa]	μ_{av} (average over all tests) [-]
F1	0.1728	115.7	0.52	221.1	**0.24**	0.56	206.7	**0.28**
F2	0.3456	167.8	0.38	442.2		0.43	390.3	
C1	0.1901	194.7	0.80	243.2	**0.53**	0.91	213.9	**0.90**
C2	0.3801	302.5	0.62	486.4		0.78	387.9	
C3	0.6220	485.5	0.61	796.0		0.91	533.5	

For the considered corrugated 'miniature silo' the geometric parameter is equal to a_{w} = 0.21. Consequently, the application of Eq. (1), adopting the numerical results of Table 2 leads to the following values of the internal friction coefficient of the granular particles: 0.60 (plausible, since it corresponds to ϕ_i = 31.2°) assuming the geostatic pressure distribution, and 1.06 (not reasonable) assuming the Janssen pressure distribution. It should be noted that it is $h / z_0 < 1$ for all specimens tested here, where z_0 is the Janssen 'great depth' value (Eq. 7), pointing to squat geometries for which the Janssen distribution is unlikely to be representative.

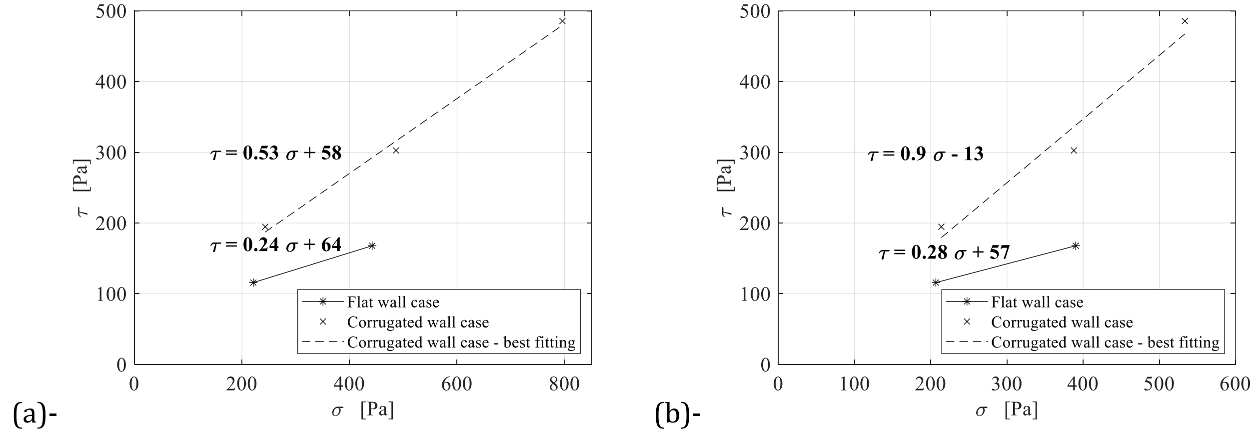


Fig. 6 – Mohr-Coulomb failure lines assuming the a) geostatic and b) Janssen pressure distributions.

4. Comparison with reference values and results obtained with standard tests

ANSI/ASAE S433.1:2019 [24] recommends the following reference design values for mechanical parameters of wheat: a particle-wall interface friction coefficient of 0.30 for a flat wall and 0.37 for a corrugated wall, with a lateral pressure ratio $K = 0.50$ regardless of any geometrical consideration. Annex E of EN 1991-4: 2006 [1] provides three values for the wall friction coefficient according to the wall roughness in the range 0.24 - 0.57. In the literature, the reported values of the internal friction coefficient of the wheat are always in the range 0.50 - 0.66 [3,10-11,14] (corresponding to an internal friction angle of 26.5° - 33.5°), while the reported values of the wall friction coefficient for wheat particles are in the range 0.18 - 0.30 [2,16-18] for a flat smooth wall and in the range 0.40 - 0.60 for a corrugated wall [2,11].

In parallel to the 'miniature silo' tests, a series of standard direct shear tests was performed on the same wheat according to EN 1991-4: 2006 [1] suggestions. The coefficient of internal friction of the granular particles was found to be $\mu_g = 0.57$. The friction coefficient at the interface between the granular material and the flat-silo wall sample gave a value $\mu_w = 0.21$. The application of Eq. (1) then led to an effective wall friction coefficient for the corrugated wall of $\mu_{eff} = 0.51$, which is quite close to the value $\mu_{av} = 0.53$ obtained from the 'miniature silo' tests assuming a simple geostatic pressure distribution model. Indeed, standards like ANSI/ASAE EP545[25], ANSI/ASAE EP446.3 [26] and EN 1991-4: 2006 [1] (see Section 5.4 therein) suggest using a linear pressure model for shallow silo design (small filling depths). In general, it is strongly recommended that several pressure models are carefully trialled to explore the stability of the value estimated using the 'miniature-silos' procedure.

5. Conclusions

This short communication aimed at proposing an original idea for a new experimental setup to help designers in the fundamental choice of the friction parameters that strongly affect the silo design. A simple experimental setup was proposed to estimate the average effective wall friction coefficient developed by the solid stored in a cylindrical silo container made up of either flat or corrugated sheets. The main results are summarised as follows:

- The friction properties of both flat and corrugated walls could be estimated quickly using the proposed test. The test apparatus is simple to construct and the procedure is easy, low-cost and fast to perform.
- The proposed test potentially overcomes the typical difficulties related to the standard tests associated with the geometrical properties of the corrugated wall and/or to the size and configurations of the shear cells.
- Assumptions about the lateral pressure ratio and the pressure distribution model are needed for the interpretation of the results. This a critical point that requires further investigations.
- The proposed experimental test led to a plausible estimate of the friction coefficient at the interface between the granular solid and the corrugated silo wall when applied to a relatively squat geometry and assuming a geostatic internal pressure distribution model. The estimate was consistent with the 'averaging formula' given by Annex D of EN 1991-4: 2006 [1].

Further research should be conducted to systematically investigate the effects of more variables (filling method, assumed pressure distribution models, filling depth, type of granular material) for a full calibration and validation of a robust interpretation method.

Acknowledgments

The main idea was inspired from the needs of the SERA-SILOS TA#18 project, in which a full-scale steel manufactured corrugated wall silo was set on a shaking table and exposed to over 250 dynamic tests (random, sinusoidal and earthquake inputs). For a sound interpretation of the results, the main friction parameters of the corrugated wall and the granular particles (product: soft wheat) had to be experimentally obtained [27].

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