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A review of mechanical models of dike propagation: Schools of thought, results and future directions

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A review of mechanical models of dike propagation: schools of thought, results and future directions

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Abstract

Magma transport in brittle rock occurs by diking. Understanding the dynamics of diking and its observable consequences is essential to deciphering magma propagation in volcanic areas. Furthermore, diking plays a key role in tectonic phenomena such as continental rifting and plate divergence at mid-ocean ridges. Physics-based models of propagating dikes usually involve coupled transport of a viscous fluid with rock deformation and fracture. But the behaviour of dikes is also affected by the exchange of heat with the surroundings and by interaction with rock layering, pre-existing cracks, and the external stress field, among other factors. This complexity explains why existing models of propagating dikes are still relatively rudimentary: they are mainly 2D, and generally include only a subset of the factors described above. Here, we review numerical models on dike propagation focusing on the most recent studies (from the last 15–20 years). We track the influence of two main philosophies, one in which fluid dynamics are taken to control the behavior and the other which focuses on rock fracturing. It appears that uncertainties in the way that rock properties such as fracture toughness vary from laboratory to field scale remains one of the critical issues to be resolved. Finally, we present promising directions of research that include emerging approaches to numerical modeling and insights from hydraulic fracturing as an industrial analogue.

Keywords: Dike propagation, fluid-filled fractures, lubrication theory, Weertman cracks, Boundary element method, layered media, induced

seismicity, volcano deformation, rifting, hydraulic fracture.

1	Contents	
2	1 Introduction	5
3	2 Geometrical properties of dikes and relationship to magma and rock rheology	6
4	3 Observations	10
5	3.1 Field observations	10
6	3.1.1 Structural geology	10
7	3.1.2 Crustal deformation	12
8	3.1.3 Dike-induced seismicity	15
9	3.2 Analogue laboratory experiments	19
10	4 Dike Propagation Modelling	22
11	4.1 Introduction	22
12	4.2 The Weertman school: buoyancy-driven magma-filled dikes . .	23
13	4.2.1 Formulation for static fractures	24
14	4.2.2 Formulation for moving fractures	25
15	4.2.3 Recent extensions	26
16	4.2.4 Strengths and limitations	28
17	4.3 The lubrication-theory school: dynamics controlled by magma flow	30
18	4.3.1 Model formulation: Magma Flow	31
19	4.3.2 Elastic Deformation	33
20	4.3.3 Propagation Condition	33
21	4.3.4 Boundary Conditions	34
22	4.3.5 Strengths and Limitations	34
23	4.4 Critical analysis of the two approaches	36
24	4.4.1 Dimensional Analysis	36
25	4.4.2 Fracture Toughness and Dike Propagation Regime . . .	39
26	4.4.3 Magma influx from the reservoir and the dike tail . . .	44
27	4.4.4 Where the Two Schools Reconcile	46
28	4.5 Numerical Models	51
29	5 Results including interaction of dikes with the surroundings	55
30	5.1 External stress field	55
31	5.2 Load of a volcanic edifice	56
32	5.3 Unloading	59
33	5.4 Extensional and compressional tectonics	61

34	5.5	Rigidity layering	62
35	5.6	Density layering	64
36	5.7	Dike–dike interaction	66
37	5.8	Scaling and dike sequences in rifting episodes	68
38	5.9	Fracturing, faulting, induced seismicity	69
39	5.10	Coupling of magma chambers and dikes, connectivity, induced deformation	73
40	5.11	Dike arrest	76
41	5.12	Predicting vent locations and times of intrusions and eruptions	78
42	5.13	Segmentation	79
43	5.14	Heat exchange, cooling	80
44	5.15	Volatile exsolution and fragmentation in dikes	82
45	5.16	Coupling to the asthenosphere	84
46	6	Investigation along with industrial hydraulic fractures	87
47	6.1	Growth barriers	88
48	6.2	Induced seismicity	92
49	6.3	Networks and swarms	93
50	7	Outlook: Perspectives and challenges	96
51	8	Acknowledgements	98

52 **1. Introduction**

53 The motivation to improve and extend models of dikeing comes from sev-
54 eral key scientific areas. Firstly, most basaltic eruptions occur in the form
55 of dikes; the characteristics of these volcanic events are thus determined by
56 the dynamics of the dikes. Secondly, plate divergence and crustal accre-
57 tion at mid-ocean ridges and continental rift-zones occurs mostly in form
58 of repeated episodes of dikeing. Thirdly, models of dikes can be applied to
59 industrially important processes related to fluid transport and storage in the
60 crust, such as hydraulic fracturing of hydrocarbon reservoirs and formation
61 of diamond-bearing kimberlite deposits. The barriers to progress in mod-
62 elling dikes stem from the mathematical complexity and/or computational
63 impracticality of models that account for every possible mechanism affecting
64 dike propagation. At their most basic, propagating dikes can be considered
65 as a hot, compressible fluid flowing between cold elastic walls. However,
66 non-linear rock behavior such as fracture and plasticity must be considered
67 to model the motion of the fracture tip. Furthermore, magma rheology and
68 buoyancy change during propagation due to gas exsolution and crystalliza-
69 tion. Hence some of the most difficult problems in mathematical physics
70 are closely linked to the dynamics of dikes. To make the problem tractable,
71 most models of propagating dikes are simplified by some combination of re-
72 ducing the problem to two-dimensions, linearizing the behavior of the rock
73 when it deforms and breaks, neglecting thermal processes, linearizing and/or
74 neglecting the changes in magma rheology, and simplifying the geometry or
75 neglecting altogether the pre-existing structures within the host rock.

76 Nevertheless, in recent years our understanding of dikes has increased
77 significantly, due both to a wealth of geophysical data now available and to
78 progress in modelling. Some of the questions that have engaged researchers
79 during the last two decades include: What is the three-dimensional shape of
80 propagating dikes? What factors control their dynamics, and in particular
81 their geometry? How can we explain the details of the seismicity or deforma-
82 tion field associated with dike emplacement? What is the effect of an external
83 stress field? What are the effects of the free surface, layering or topography?
84 What is the role played by the coupling with a magma reservoir?

85 In this review, we present an overview on how those questions have been
86 addressed and indicate short and long-term perspectives on what questions
87 might be answered in the future. In particular we present:

- 88 1. Geometrical and dynamical properties of dikes and the main observa-

- 89 tions available to constrain these (Sec. 2 and 3);
- 90 2. Different schools of thought in deriving models of dikes, with consider-
91 ation of their strengths and limitations (Sec. 4);
- 92 3. The main results achieved in modelling the interplay of propagating
93 dikes with a range of external factors in various tectonic settings (Sec. 5);
- 94 4. Applications to hydraulic fracturing and other industry-related prob-
95 lems (Sec. 6);
- 96 5. Perspectives on future research direction and open problems (Sec. 7).

97 **2. Geometrical properties of dikes and relationship to magma and** 98 **rock rheology**

99 Magma-filled dikes have different manifestations often associated with
100 different magma compositions and tectonic settings. In some active volcanic
101 environments, such as Kilauea volcano in Hawai'i, magma may intrude in
102 the host rock as a propagating dike and then flow through a stable, tabu-
103 lar conduit. This last manifestation of stable flow is sometimes referred to
104 as a dike. None of the models presented in this review correspond to this
105 stable configuration; see instead Montagna and Gonnermann (2013), for ex-
106 ample. Here we consider the unstable configuration of a dike with a tip that
107 propagates through more or less intact rock.

108 The thickness of dikes is much less than their breadth and length (Fig. 2).
109 This aspect ratio is shared with sills, which are similar to dikes but em-
110 placed horizontally along bedding planes rather than cutting through them.
111 It distinguishes dikes and sills from other magma-filled bodies, such as di-
112 apirs, laccoliths or cylindrical conduits. Modern geophysical monitoring has
113 enabled an accumulation of measurements of dike characteristics. These
114 observations show that the thickness (also referred to as width or open-
115 ing) typically ranges from tens of centimeters to several meters, while the
116 length L (here defined as their dimension along the direction of propagation,
117 which is sometimes referred to as height if propagation is vertical) ranges
118 from several hundreds meters to a few kilometers, or even several tens of
119 kilometers for horizontally propagating intrusions (Tryggvason, 1984, 1986;
120 Toda and Stein, 2002; Wright et al., 2006). The third dimension of dikes
121 (here called breadth b) is generally of the same order of magnitude as their
122 length, but somewhat smaller, especially for horizontally propagating dikes
123 (Fig. 2A,B). Anomalously large dikes, tens of meters thick and hundreds to
124 thousands of kilometers long, can be identified in the field, sometimes in the

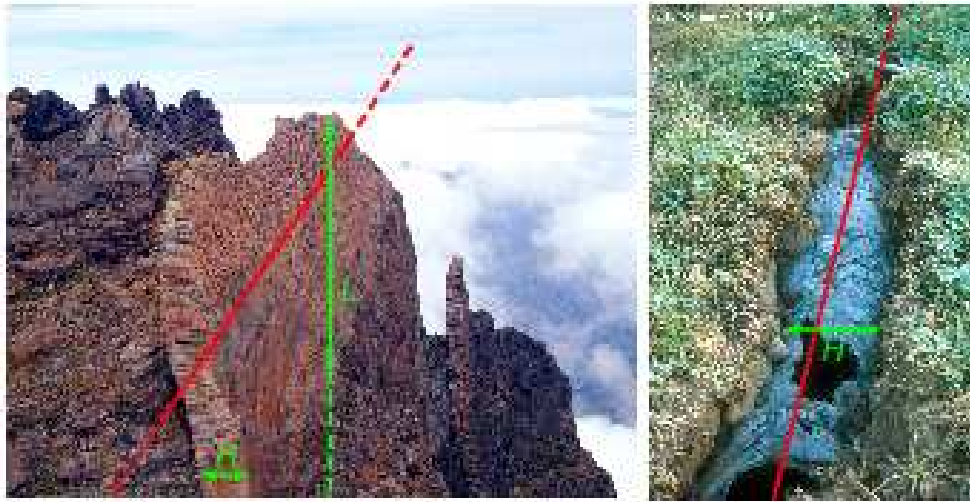


Figure 1: Left and right: The width of the dike, H , is much smaller than the other horizontal extension highlighted by the red line. The left picture was taken by N. Villeneuve at Piton des Neiges, the right picture by J.-C. Komorowski at Nyragongo. The origin of 2-D approximations in modeling dykes comes from field observations.

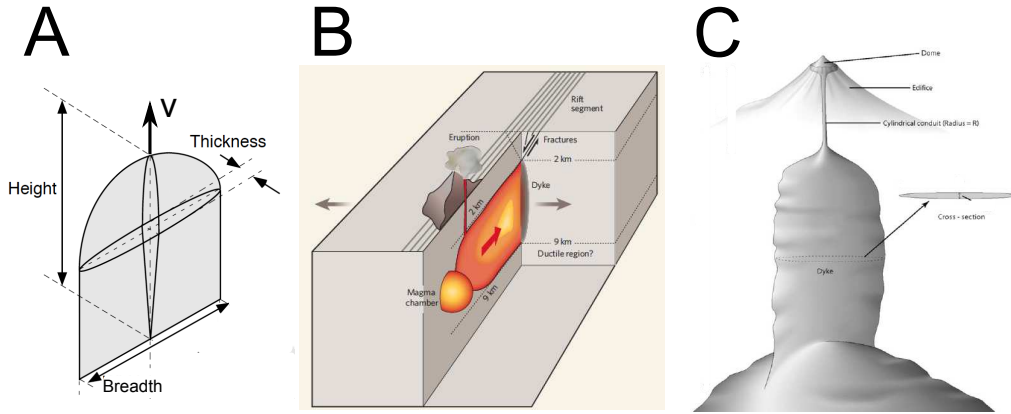


Figure 2: A) Simplified image of a vertically propagating dike (Modified from Fig. 3 in Watanabe et al. (2002), with permission). B) Dike propagating horizontally from a magma chamber (Fig. 2 in Sigmundsson (2006), with permission). C) Dike-like root developing from a magma reservoir into a cylindrical conduit in the volcanic edifice (Fig. 1 from Costa et al. (2007), with permission).

125 form of swarms (Pollard, 1987; Ernst et al., 1995; Jolly and Sanderson, 1995;
 126 Fialko and Rubin, 1999).

127 In general, dikes will open against the minimum compressive stress, σ_3 ,
 128 and propagate on a plane perpendicular to σ_3 (Anderson, 1951). In 2D, they
 129 propagate in the direction of maximum compressive stress, σ_1 . When both
 130 horizontal stresses are similar ($\sigma_{hmin} \sim \sigma_{Hmax}$), intrusions will still have a
 131 thin aspect ratio but will orient randomly; this can occur, for instance, near
 132 a free surface. Examples of this behaviour are observed in laboratory exper-
 133 iments that entail fluid injection into solidified gelatin. For a simple gelatin
 134 block with hydrostatic stress conditions, $\sigma_{hmin} \sim \sigma_{Hmax}$; if the gelatin is
 135 brittle, fluid-filled cracks form vertically and in an essentially random hori-
 136 zontal direction, apparently according to small perturbations within the spec-
 137 imens. If the gelatin is viscoelastic (viscoelastic properties arise for specific
 138 types of gelatin), then the fluid-filled fractures assume a diapir-shaped aspect
 139 (Sumita and Ota, 2011). In nature, dikes may also form in $\sigma_{hmin} \sim \sigma_{Hmax}$
 140 stress conditions: for example at volcanoes such as Etna, where in conse-
 141 quence of several dike injections in a specific direction, σ_{Hmax} may become
 142 very similar to σ_{hmin} , or the principle stresses may rotate. The dike pat-
 143 tern becomes radial in such stratovolcanoes, where it is also linked to the
 144 gravitational load of the edifice (Nakamura, 1977; Acocella and Neri, 2009).

145 Dikes take a tabular shape because they are fractures driven by internal

146 fluid pressure, opening in brittle materials. Other types of conduits, such as
147 low-aspect-ratio cylindrical pipes, are energetically disfavored in the brittle
148 crust, or need very long time scales and sustained, high temperatures to
149 stabilize. They are observed for volcanoes such as Montserrat, where the
150 viscosity of the magma is very high. Recent models (Costa et al., 2007,
151 2012) suggest that close to the surface, where the viscosity gradient of dacitic
152 magmas is steep, flat-bottle shaped volcanic conduits might form, with a deep
153 dike-like root transporting low-viscosity magma developing into a cylindrical
154 conduit where the magma viscosity becomes high. This configuration may
155 help to explain observed patterns of deformation (Fig. 2C).

156 What is the minimum rock/magma viscosity contrast such that rock re-
157 sponds in a brittle rather than ductile manner? Rubin (1993a) calculated
158 self-similar solutions of fluid-filled, pressurized cracks in viscoelastic mate-
159 rials to study the rheological conditions promoting ascent by fracture versus
160 viscous deformation of the host rock. He concluded that if the ratio
161 of rock to magma viscosity η_r/η_m is larger than 10^{11} – 10^{14} , the crust be-
162 haves elastically at the time scale of magmatic intrusion, such that basaltic
163 magmas and low-viscosity rhyolitic magmas ($\eta_m \leq 10^4 - 10^6$ Pa.s) will gen-
164 erate dikes in crustal rocks. If the viscosity contrast is smaller than 10^6 –
165 10^8 , then magma transport occurs via equi-dimensional diapirs inducing
166 ductile flow in the host rock. For intermediate viscosity contrasts (10^6 –
167 $10^8 \leq \eta_r/\eta_m \leq 10^{11}$ – 10^{14}) the form of transport is hybrid, with an emergent
168 tabular aspect ratio; there is progressively more fracture and less ductile de-
169 formation of the host rock with increasing viscosity. Sumita and Ota (2011)
170 describe their experimental study on the aspect ratio of buoyancy-driven
171 fluid-filled fractures in a host material with a rheological transition from
172 ductile to brittle. They find the fluid migrates as a hybrid of a diapir (the
173 head) and a dike (the tail). The diapir is a bulging crack fracturing the
174 agar at its propagating tip and closing at its tail to form a dyke. A small
175 amount of fluid is left along its trail and the fluid decelerates with time.
176 Sumita and Ota (2011) study how the shape and velocity of a constant-
177 volume fluid batch change as the agar concentration, C , and the density
178 difference between the fluid and the agar, $\Delta\rho$, vary (Fig. 3 and supplementary
179 videos at <http://www.sciencedirect.com/science/article/pii/S0012821X11000562>).
180 As C decreases, the medium becomes ductile and the 3D shape of the fluid
181 batch changes from dike-like (with a blade-like section as seen from above) to
182 a meandering or a bifurcating dike, and finally to a diapir–dike hybrid (the
183 section as seen from above becomes a cusped ellipse). A similar transition is

184 also observed when $\Delta\rho$ increases under a fixed C . The experiments suggest
185 that fluids may migrate as a diapir/dike hybrids around the depth where a
186 transition from brittle to ductile rheology occurs and that fluid migration
187 of various styles can coexist at the same depth, if the fluids have different
188 buoyancy.

189 It is generally agreed that in the crust, dikes have sufficient energy to
190 propagate upward through intact rock; pre-existing fractures are not needed
191 to transport magma. Analogously, a pre-existing fracture network is not re-
192 quired for a volcanic eruption to occur. However, if buoyant magma enters
193 into pre-existing zones of weakness it may exploit those paths, provided that
194 the weak zones are oriented in a favourable way relative to the stress field
195 (Delaney et al., 1986; Ziv et al., 2000). This phenomenon is sometimes ob-
196 served (Valentine and Krogh, 2006; Hooper et al., 2011), however it is not
197 universal because faults are generally oriented in the stress field differently
198 from fluid-filled fractures.

199 **3. Observations**

200 *3.1. Field observations*

201 Observations of propagating dikes represent a constraint on models and a
202 means to identify open questions. In this section we review the main hurdles
203 related to using data from different disciplines to constrain numerical models
204 of dike propagation. For more comprehensive reviews of field and laboratory
205 observations on dikes and sills refer to Menand (2011) and Tait and Taisne
206 (2013).

207 *3.1.1. Structural geology*

208 While a wealth of observations on frozen dikes in the field is available
209 (e.g. Gudmundsson, 1983), structural geology has rarely been used in com-
210 bination with models, due to scarce communication between these disci-
211 plines. Exceptions to this trend that make this link within a single inves-
212 tigation include Pollard (1976); Pollard and Muller (1976); Pollard (1987);
213 Valentine and Krogh (2006); Kavanagh and Sparks (2011); Geshi et al. (2012);
214 Daniels et al. (2012). Even in the absence of complementary modeling ef-
215 forts, valuable information can be obtained by studying outcrops in the
216 field. For example, a partial 3D view of frozen dikes has been obtained
217 by Kavanagh and Sparks (2011) by taking advantage of mining. However,
218 fossil structures exposed by weathering or mining represent the final, static

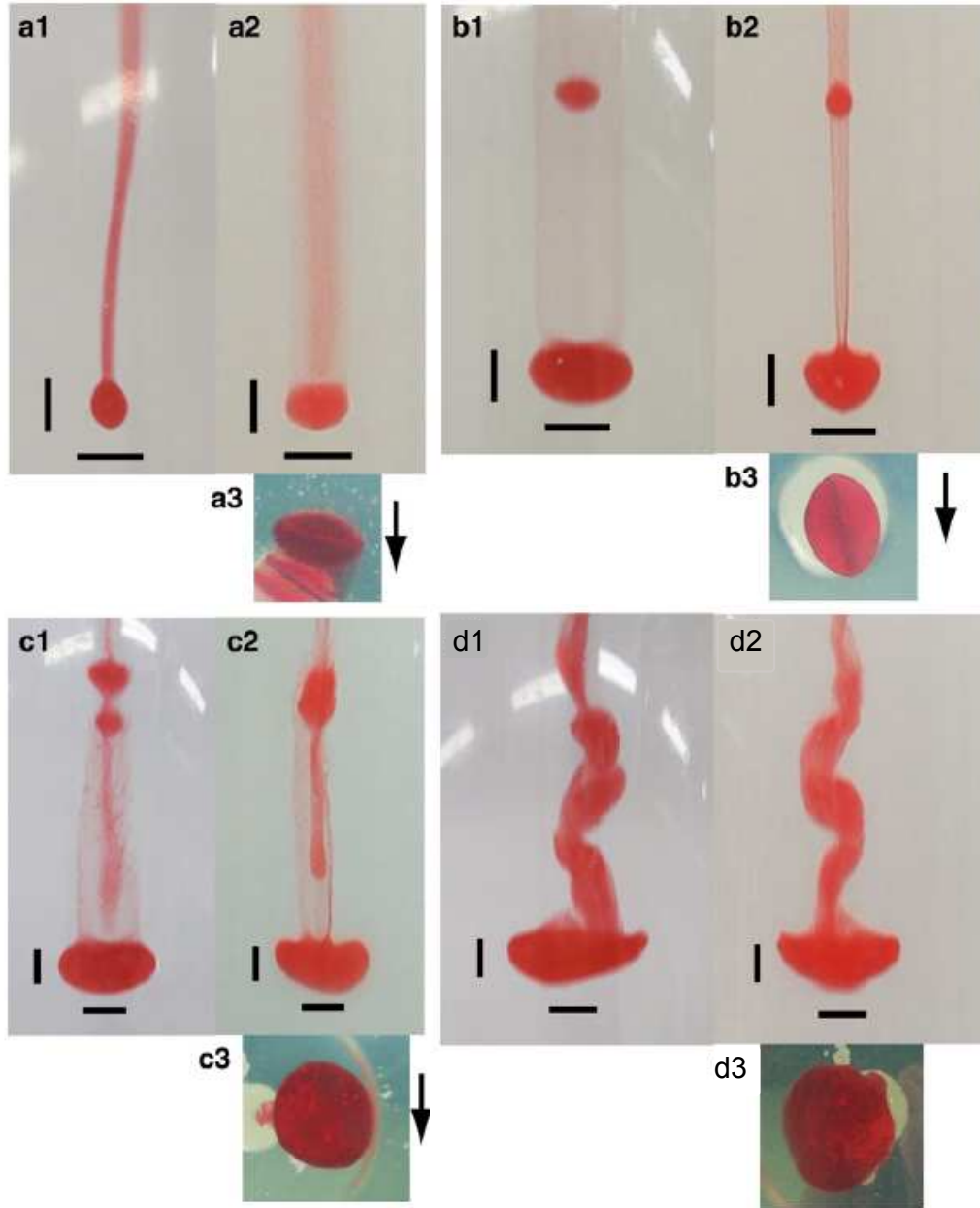


Figure 3: Examples of buoyancy-driven fluid-filled cracks propagating in Agar with different concentration C (%) and with a different density difference $\Delta\rho$ between gelatin and the aqueous solutions used for the injections. The cracks are viewed from 3 orthogonal angles (1, 2 and 3). A scale bar is 10 mm. (a) $C = 0.08$ wt.%, $\Delta\rho=31 \text{ kg m}^{-3}$, (b) $C = 0.08$ wt.%, $\Delta\rho= 300 \text{ kg m}^{-3}$ (see also video), (c) $C = 0.07$ wt.%, $\Delta\rho = 604 \text{ kg m}^{-3}$, and (d) $C = 0.06$ wt.%, $\Delta\rho=604 \text{ kg m}^{-3}$. (Figs. 6 and 7 from Sumita and Ota (2011), with permission).

219 state of magma intrusion. It is difficult to use them to constrain numerical
220 models of dike propagation because the link between the dynamic shape of
221 dikes and their final frozen state is poorly understood. Therefore, caution is
222 required when attempting to infer the elastic properties of the host rock on
223 the basis of the final, static geometry of a dike.

224 Other geological and geophysical techniques useful to evaluate dynamic
225 aspects include examining the magnetic (Kirton and Donato, 1985; Knight and Walker,
226 1988; Craddock et al., 2008; Chadima et al., 2008; Silva et al., 2010; Neres et al.,
227 2014) or flow fabric of frozen dikes (e.g. Correa-Gomes et al., 2001), gravity
228 (e.g. Carbone, 2003 and review by Battaglia et al., 2008); magnetic (e.g.
229 Del Negro et al., 2003), and magnetotelluric data (Siniscalchi et al., 2012).
230 Structural geology can also provide strong constraints, but it must be car-
231 ried out with the recognition that the resulting observations are typically a
232 sparse and dimensionally limited view of what are usually extensive, three-
233 dimensional structure. For this reason, the value of these studies can be
234 greatly increased when modelling is used to assist with interpretation of 3D
235 morphologies and probable conditions that governed emplacement.

236 *3.1.2. Crustal deformation*

237 Inversion of GPS and/or InSAR deformation data can be used to esti-
238 mate the shape and the volume of magma-filled dikes and sills and the of
239 volume change of magma chambers. Recent developments in satellite inter-
240 ferometry allow highly resolved measurements of ground deformation. This
241 enables the recognition of complex interactions between different feeding
242 sources and intrusions (Wright et al., 2006; Grandin et al., 2009; Hamling,
243 2010; Grandin et al., 2010a,b; Montgomery-Brown et al., 2010; Bagnardi and Amelung,
244 2012). In some cases evidence of deflating sources is lacking, suggesting that
245 magma was probably sourced very deep (Pallister et al., 2010).

246 The temporal period of InSAR data acquisition is much larger than the
247 time scale of dike intrusions (several hours to a few days), so that it is very
248 rare to measure an actively propagating dike. Thus, inversions of InSAR
249 data generally give information on the ground deformation accumulated over
250 the entire emplacement phase; they seldom provide information on the de-
251 tailed dynamics. There are exceptions to this pattern, such as the work of
252 Bagnardi and Amelung (2012) and Nobile et al. (2012), who obtained inter-
253 ferograms spanning the early and late intrusion phases of a sill and a dike,
254 respectively.

255 The potentially high temporal resolution of GPS or strain data in prin-

256 ciple allows for inversion with respect to the evolving shape of dikes or sills.
257 However, this has been performed only in a few cases (Aoki et al., 1999;
258 Segall et al., 2001; Irwan et al., 2006; Aloisi et al., 2006; Montgomery-Brown et al.,
259 2011). In some earlier work, forward models were used to explain temporal
260 changes in the deformation field (Okada and Yamamoto, 1991; Linde et al.,
261 1993). Peltier et al. (2005) modelled changes in direction of a propagating
262 dike using data from tiltmeters; that work shows that flank eruptions at
263 Piton de la Fournaise volcano are preceded by a relatively fast vertical dike
264 migration, ~ 2 m/s followed by a slower horizontal propagation, 0.2 m/s to
265 0.8 m/s.

266 In general, dike models for inversions of crustal deformation data are
267 calculated by discretising the dikes into a mosaic of rectangular disloca-
268 tion patches (Okada, 1985, 1992) or, for magma chambers, one or more di-
269 latational point sources (Yamakawa, 1955; Mogi, 1958). In some instances,
270 the inclusion of graben faulting is required to achieve a good match be-
271 tween the modeled and observed deformation field and consistency with seis-
272 mic observations of larger events (Wright et al., 2006; Pallister et al., 2010;
273 Nobile et al., 2012). In most studies, the dislocations and point sources
274 involved are taken to be non-interacting, though Pascal et al. (2013) sug-
275 gests that this might cause relatively large errors. Furthermore, smooth-
276 ing/regularization and positivity constraints are applied in order to obtain
277 (subjectively) realistic solutions (Wright et al., 2006; Montgomery-Brown et al.,
278 2010; Nobile et al., 2012). In some cases, physical constraints are instead ap-
279 plied, such as requiring constant pressure drop or a linear pressure gradient on
280 the dike/sill plane (Yun et al., 2006; Sigmundsson et al., 2010; Hooper et al.,
281 2011). Both using physical constraints and smoothing the opening over the
282 dike's plane results in approximately penny-shaped crack (Fig. 4). Horizon-
283 tally elongated systems are obtained for dikes propagating laterally in rift
284 systems (Montgomery-Brown et al., 2010; Nobile et al., 2012)(Fig. 4D and
285 C respectively). Vertically elongated systems are found for dikes ascending
286 to the surface from deep crustal levels (Pallister et al., 2010) (Fig. 4B). In
287 the inversion by Nobile et al. (2012) (Fig. 4C), a thin channel connecting a
288 Mogi-Yamakawa source and the dike is visible. However, the spatial resolu-
289 tion of the data is generally too low to constrain fine-scale details of dikes
290 and sills shapes in the inversions.

291 As for longer time scales, while post-seismic deformation studies have
292 contributed considerably to the understanding of the mechanics of faulting
293 during the seismic cycle, studies of post- or inter-diking deformation phases

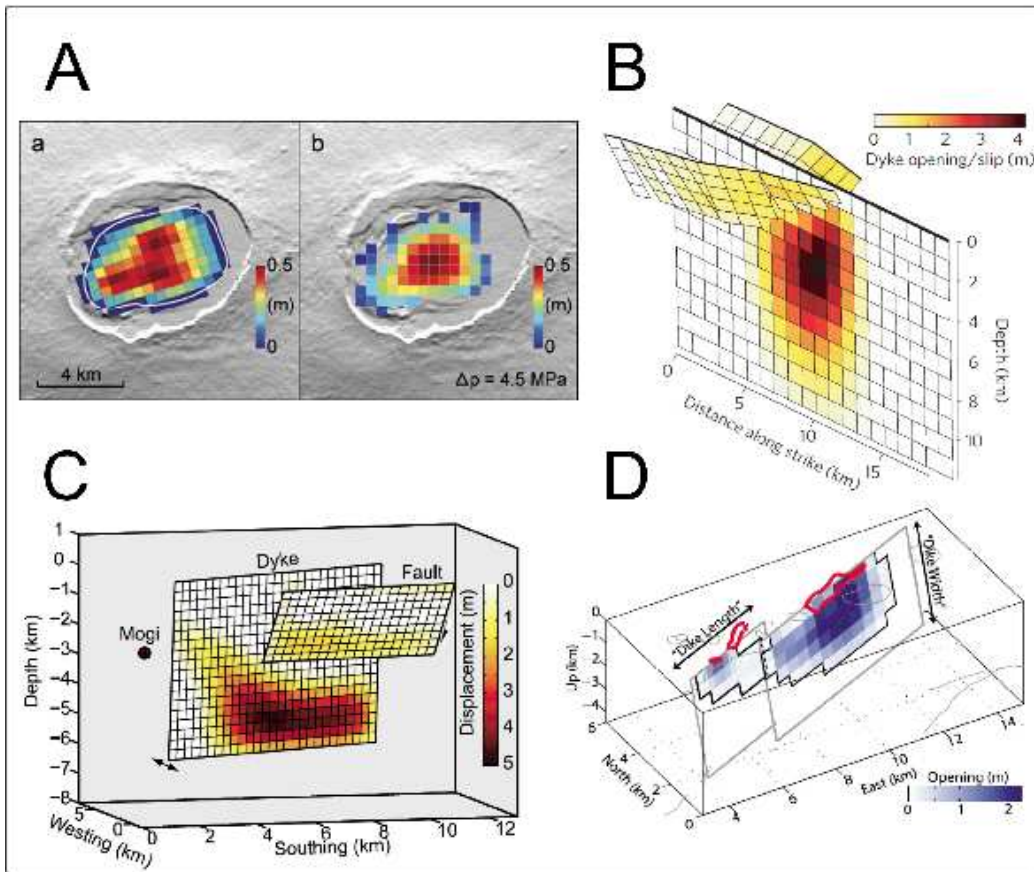


Figure 4: A) Inversion of a horizontal sill at Fernandina (Galapagos), the right panel solution was obtained with uniform pressure boundary conditions. Fig. 10 from (Yun et al., 2006), with permission. B) Inversion of a dike and related graben faulting at Harrat Lunayyir, Saudi Arabia. No feeding source was detected in this case. From Fig. 5 by (Pallister et al., 2010), with permission. C) Inversion of a dike including deflation at a Mogi-Yamakawa point source and graben faulting. From Fig. 2 by (Nobile et al., 2012), with permission. D) Inversion of an en-echelon dike during the 2007 father's day intrusion at Kilauea. From (Montgomery-Brown et al., 2010), Fig. 12, with permission.

294 are rare (Desmarais and Segall, 2007; Pedersen et al., 2009; Hughes, 2010;
295 Ali et al., 2014). Grandin et al. (2010a) studied inter-diking deformation for
296 the Manda-Hararo dike sequence. InSAR data for the inter-diking period
297 highlights deflation or inflation at the magma chambers, reflecting main-
298 tained connectivity between the deeper reservoirs, but separation between
299 the northern magma chambers and the after-dikes intruded further South.
300 Grandin et al. (2010a) also detect inflation of a ~ 25 kilometers deep reser-
301 voir, probably at the crust-mantle boundary.

302 For a review of progress in SAR imagery applied to the field of volcanol-
303 ogy, see Pinel et al. (2014).

304 *3.1.3. Dike-induced seismicity*

305 The association of magmatic intrusions with earthquake swarms is an
306 important motivation for the study of dikes. Dike-induced seismicity carries a
307 wealth of information on the physics of diking. This information has typically
308 been obtained by measuring the timing and location of events, but this can
309 be enriched by assessing focal mechanisms (Passarelli et al., 2014a), seismic
310 productivity (Rubin and Gillard, 1998; Pedersen et al., 2007) or by including
311 earthquake nucleation models in the inversion of crustal deformation data
312 (Segall et al., 2013).

313 Observations from 1975–1984 rifting episode at Krafla volcano in Ice-
314 land (Einarsson and Brandsdóttir, 1978; Brandsdóttir and Einarsson, 1979)
315 made clear that propagating dikes induce migrating seismicity, and that this
316 seismicity can be loosely associated with the propagating tip of the dike. Mi-
317 grating seismicity associated with diking has also been commonly reported
318 on and close to stratovolcanoes — for example, by Battaglia et al. (2005) be-
319 fore the 1998 eruption at Piton de la Fournaise; by Patanè et al. (2002) and
320 Aloisi et al. (2006) for the 2001 and 2002 dike intrusion at Etna, respectively;
321 by Klein et al. (1987); Gillard et al. (1996); Rubin and Gillard (1998) and
322 Rubin et al. (1998) for the 1983 dike at Kilauea; by Toda and Stein (2002);
323 Uhira et al. (2005) for the 2000 intrusion at Izu islands, Japan; Baer and Hamiel
324 (2010) for the dike event in Arrat Lunayyir, Saudi Arabia; by Dziak et al.
325 (1995) for mid-ocean ridges; and more recently for the Manda-Harraro rifting
326 episode by Ayele et al. (2009); Keir et al. (2009); Belachew et al. (2011).

327 The exceptionally long seismic phase observed in March 1998 at Piton de
328 la Fournaise presented a clear upward migration (Fig. 5B, from Battaglia et al.
329 (2005)). The data highlight a sudden decrease in the upward velocity at a
330 depth of 1.5 kilometers below the surface that could be explained by a lower

331 density of the upper-layer host rock (Taisne and Jaupart, 2009; Maccaferri et al.,
332 2011). According to Taisne and Jaupart (2009) the velocity variation may
333 correspond to a factor-of-two decrease in the density difference between rock
334 and magma. Closer to the free surface, the upward migration of the seis-
335 micity accelerates, ending with the eruption. Rivalta and Dahm (2006) at-
336 tributed this to the effect of the free surface; alternatively, late degassing
337 may decrease the magma density and induce an acceleration of the dike
338 (Taisne and Jaupart, 2011). The analysis of the seismic crises spanning 25
339 years of activity at Piton de la Fournaise volcano (Roult et al., 2012) also
340 shows that the early stage of the seismic crisis is not necessarily related to
341 magma migration. Instead, by looking at the ratio of the seismic amplitude,
342 Taisne et al. (2011a) show that the migration of the radiated seismic energy
343 that is associated with the migration of the magma was delayed with re-
344 spect to the onset of the seismic crisis. This observation suggest that magma
345 migration is preceded by a phase of rupture/damage of the magma storage
346 region.

347 Following the above considerations, a caveat regarding the relationship
348 between tip migration and seismicity is appropriate. While the data hint at
349 an equivalence between tip migration and seismicity, there is evidence that
350 the tip is not the only, and sometimes not even the primary source of seis-
351 micity. For example, Rubin et al. (1998) evaluate the seismicity induced by
352 the 1983 dike intrusion at Kilauea, which included a re-location of seismic
353 sources. Most of the hypocenters collapsed onto a few tightly spaced clusters,
354 sometimes linked to areas with high background seismicity, suggesting that
355 pre-existing weakness and high differential stress are needed to reach failure.
356 This is consistent with rate-state earthquake nucleation theory (Dieterich,
357 1994): positive Coulomb stresses are predicted to increase pre-stressing seis-
358 micity rates. Therefore, areas with high pre-diking seismic rates but low
359 dike-induced Coulomb stresses may appear more active than areas with very
360 high dike-induced stresses, such as the tips, if pre-diking seismicity there was
361 very low or below the detection threshold. This highlights that for a correct
362 interpretation of dike-induced seismicity we need both to estimate Coulomb
363 stress changes and assess pre-existing seismic rates.

364 Production of seismicity can also occur on a different time scale to the
365 migration of the dike tip. Aoki et al. (1999) noticed that the migration of
366 seismicity for the swarm accompanying the 1997 intrusion at Izu Islands,
367 Japan, had a time scale of 12 hours in contrast to the time scale of several
368 days for the vertical migration of deformation. They concluded that the

369 migration of the seismicity does not necessarily reflect the migration of the
370 dike; rather, the seismicity is linked to the evolution of the stress field asso-
371 ciated to the opening of the dike and to the stress previously stored in the
372 crust. Later, Hayashi and Morita (2003) and Morita et al. (2006) offered a
373 contrasting view based on precisely relocated earthquakes from a 1998 swarm
374 in the same region. They argue that the swarm seismicity actually marks
375 the edge of the propagating dike. As of 1997, the seismic network was not
376 good enough for an accurate assessment of the relationship between the dike
377 trajectory and seismicity. This highlights the importance of well-designed
378 and dense seismic networks for correctly inferring dike trajectories.

379 Patterns of dike-induced seismicity can have other peculiar aspects that
380 have not yet been fully explained. For example, an advancing front of mi-
381 grating epicenters (blue lines in Fig. 5A to D), often with a convex-upward
382 trend if migration is lateral, is very often trailed by a retreating front, which
383 delimits a spatio-temporal frame where the seismicity is active (green lines
384 in Fig. 5A to D). While the advancing front generally has a simple shape,
385 the retreating front sometimes shows a complex functional trend, with the
386 distance between the two changing in time (this is true in particular for
387 lateral injections). Sometimes, bi-directional migration of the seismicity is
388 observed; this is probably associated with an initial bi-lateral propagation of
389 dike (Fig. 5D), generally followed by uni-lateral propagation.

390 Tarasewicz et al. (2012) discuss another peculiar pattern of seismicity,
391 associated with the 2011 eruptive phase of Eyjafjallajökull volcano, which was
392 characterized by downward migration. They interpreted their observations in
393 terms of a downward-migrating decompression wave that started at the top
394 of the volcano with the removal of 200m of ice cap and progressed with the
395 subsequent removal of magma from a series of stacked sills into the eruptive
396 conduit and then into the atmosphere.

397 Additional information on the orientation and shape of the dikes and
398 on their pressurization level can be obtained by studying the focal mech-
399 anisms of the induced earthquakes. Fault plane solutions of large induced
400 earthquakes are often used in crustal deformation models to constrain the ori-
401 entation of co-diking faulting processes (Wright et al., 2006; Pallister et al.,
402 2010; Nobile et al., 2012). Roman et al. (2004) and Roman and Cashman
403 (2006) observed rotations of the maximum compressive stress axis during
404 isolated periods of time in volcanic areas and interpreted this in terms of
405 pressurization of dike-like conduits, possibly precursor to volcanic eruptions.

406 Volcanic seismicity often involves large non-double-couple (non-DC) com-

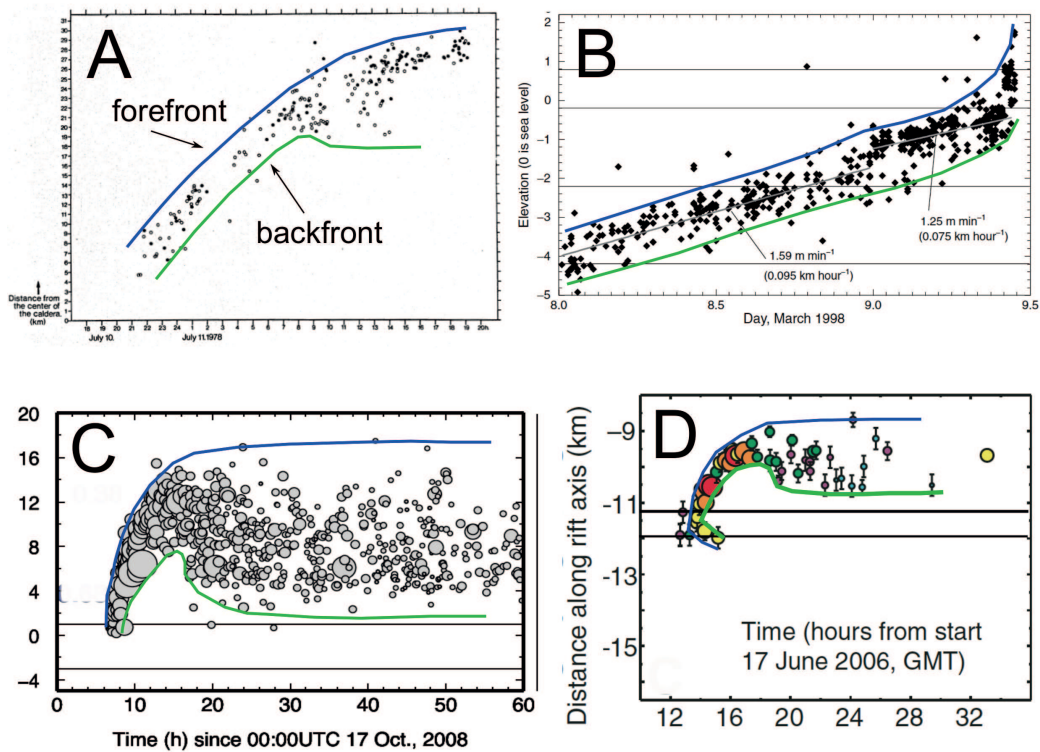


Figure 5: A) Seismicity induced by the 1978 lateral intrusion event at Krafla (Einarsson and Brandsdóttir, 1978, with permission), B) Seismicity induced by the 1998 vertical dike intrusion at Piton de la Fournaise, from Fig. 5, (Battaglia et al., 2005, with permission), C) Seismicity induced by the dike N. 11 of the rifting episode in the Manda-Harraro rift segment, Afar, Ethiopia, from Fig. 3, (Belachew et al., 2011, with permission) D) Seismicity induced by the 17 June 2006 dike of the rifting episode in the Manda-Harraro rift segment, Afar, Ethiopia, from Fig. 3, (Keir et al., 2009, with permission)

407 ponents, indicating faulting mechanisms deviating from pure shearing. Specif-
408 ically for dikes, full stress inversions of induced seismicity returns puzzling
409 results. Minson et al. (2007) found large non-DC components (both isotropic
410 and CLVD) for the 2000 intrusion at Miyakejima and proposed a mixed shear-
411 ing and opening mechanism of the faults. White et al. (2011) on the contrary
412 found for the 2007–2008 dike below Mount Upptyppingar in the Kverkfjöll
413 volcanic system (Iceland) insignificant non-DC components. In general, the
414 origin and the significance of non-DC components in volcanic areas is still
415 debated; independent evidence from crustal deformation and physics-based
416 models is needed to shed light on this aspect.

417 *3.2. Analogue laboratory experiments*

418 Analogue laboratory experiments are an approach to understanding the
419 kinematics and dynamics of diking that is complementary to natural observa-
420 tions. Although material properties differ drastically from natural systems,
421 experimental conditions can be controlled rather precisely, and it is relatively
422 easy to measure the three-dimensional propagation of an analogue dike in
423 real time. Care is obviously required in scaling the experiments and in ex-
424 trapolating results from analogue to natural systems, but insight gained in
425 the laboratory can supply hypotheses and quantitative models to be tested
426 against nature.

427 In order to observe fluid-filled cracks nucleating and propagating at lab-
428 oratory scale, brittle solids with low stiffness, such as gelatin, need to be em-
429 ployed, because the absolute dimension of the fracture depends on the rigidity
430 of the host medium (Weertman, 1971b). A sample of gelatin that is analogous
431 of the brittle crust is obtained by dissolving gelatin powder into water and
432 letting it set in a cold environment until it becomes a solid with Young mod-
433 ulus in the range of 100-50000 Pa (Takada, 1990, 1994a; Heimpel and Olson,
434 1994; Menand and Tait, 2002; Rivalta et al., 2005; Kavanagh et al., 2006;
435 Di Giuseppe et al., 2009; Kavanagh et al., 2013). One of the difficulties in
436 using gelatin as an analog material is that its rheological properties depend
437 on the history of the cooling process. Kavanagh et al. (2013) presented a
438 study of the rheology of gelatin. They also studied the evolution of gelatin
439 parameters with time of curing, with the aim of defining the scaling condi-
440 tions for experiments on magmatic intrusions. They conclude that to achieve
441 appropriate geometric, kinematic, and dynamical scaling, experiments should
442 be carried out in the temperature range 5-10°C (for the viscous component

443 to be negligible) and should employ gelatin concentrations in the range 2-
444 5 wt%. Stable values of the elastic parameters are reached after about 1 or
445 2 days, depending on concentrations. Di Giuseppe et al. (2009) published a
446 similar study on the use of gelatin for tectonic experiments.

447 Using a range of experimental techniques, fluids of different density and
448 viscosity (for example air, water, glycerine, vegetable oils) are injected into
449 gelatin, in a configuration that depends on scaling requirements to the natural
450 system of interest. Air or water-filled cracks in gelatin will have a length and
451 breadth of a few centimeters and a thickness of a few millimeters (Takada,
452 1990, 1994a; Heimpel and Olson, 1994). The geometry and kinetics of the
453 developing fracture can be observed along with the response to the inter-
454 action with different external factors: rigidity layering (Rivalta et al., 2005;
455 Kavanagh et al., 2006; Maccaferri et al., 2010), free surface (Rivalta and Dahm,
456 2006), density gradient (Lister and Kerr, 1991), external stress field (Watanabe et al.,
457 2002; Acocella and Tibaldi, 2005; Kervyn et al., 2009; Menand et al., 2010;
458 Corbi et al., 2014), dike-dike interaction (Takada, 1994a,b; Ito and Martel,
459 2002), dike-fault interaction (Le Corvec et al., 2013). The shape is affected
460 significantly by the fluid viscosity. For example, a thicker tail is observed for
461 more viscous liquids such as glycerine (Heimpel and Olson, 1994). In those
462 cases, significant amounts of fluid mass are lost in the tail during propagation
463 and a constant influx of fluid (or sustained pressure at the magma source) is
464 necessary to maintain propagation (Fig. 6).

465 An example of an air-filled crack forming and propagating in gelatin is
466 shown in the movie http://www.youtube.com/watch?v=iD1h_2T75Jk with
467 a $5\times$ speed-up over real time (Rivalta et al., 2013b). From a hole in the
468 container, air is injected slowly with a syringe in solid gelatin. The resulting
469 crack propagation is recorded with video cameras from two perpendicular
470 perspectives (left: frontal view, right: cross-section). A crack of a few cen-
471 timeters breadth and length (left), and a few mm thickness (right) opens and
472 extends while being fed with air. Plumose lines, which show the 2D pattern
473 of the propagating fracturing front, are visible in the movie as well. Gelatin
474 blocks (such as this one) that are set in the refrigerators generally have hy-
475 drostatic stress conditions, so the crack picks a random vertical orientation.
476 Here the crack develops tilted with respect to the vertical, probably due to
477 how the injection needle was inserted. When the volume reaches a critical
478 value (Weertman, 1971a,b), the crack begins to ascend by fracture propa-
479 gation through the gelatine at the upper tip. If the viscosity of the fluid is
480 sufficiently low, as in this case, the vast majority of the fluid can escape effi-

481 ciently from the crack tail and the crack pinches itself shut at the lower tip.
482 Otherwise, some fluid volume will be retained in the elongating tail during
483 propagation. The relatively high surface tension between water and air also
484 helps an effective emptying of the crack’s tail. Once the crack has formed in
485 a particular orientation, it will continue on the same plane, even if the free
486 surface is very close, as in the previous movie. This is also predicted by nu-
487 merical models (Maccaferri et al., 2010); later we discuss in more detail the
488 apparent sensitivity of the propagation direction to the initial orientation.

489 The aspect-ratio and the driving pressure of a crack containing a given
490 volume of fluid may vary if the stiffness of the gelatin changes. In movie
491 <http://www.youtube.com/watch?v=8y4U1vrk-gg> (Rivalta et al., 2013c), a
492 layered gelatin block is composed of a stiffer layer and a more compliant
493 layer superposed to it (gelatin of different stiffness can be obtained by using
494 gelatin powder of different Bloom number or by varying the concentration
495 of the same gelatin type). In this case the crack orients itself vertically, and
496 once it reaches the critical volume of fluid, it proceeds at an approximately
497 constant velocity until it comes close to the layer interface. There it accel-
498 erates upon crossing until it reaches a new constant velocity that persists
499 until it accelerates again when approaching the free surface (Rivalta et al.,
500 2005; Rivalta and Dahm, 2006). While the breadth of the fracture does not
501 change much from one medium to another, the thickness increases, the crack
502 shortens and the velocity increases by a factor of 20.

503 When the ordering of the layers is reversed, in movie <http://www.youtube.com/watch?v=MJHs1>
504 (Rivalta et al., 2013c), the crack approaches a stiff interface. During propa-
505 gation in the lower medium, an instability is visible that causes the crack tail
506 to close in spurts rather than smoothly, as the upper tip propagates (Dahm,
507 2000b). When the crack approaches the interface, it decelerates and stops,
508 because it does not carry enough volume to be overcritical also for the upper
509 gelatin type (Rivalta et al., 2005, see also Section 4.2). With continued in-
510 jections, the crack enlarges laterally until it exceeds the critical stress at the
511 upper tip and breaks through the interface. Eventually it reaches the free
512 surface, accelerating just before it.

513 Heterogeneities can disrupt the steady propagation of analogue dikes.
514 In movie <http://www.youtube.com/watch?v=h71uTwuaG7s> (Rivalta et al.,
515 2013a), the gelatin contains growths of fungi and gas bubbles. This gives
516 a chance to observe what may happen when the medium has preexisting
517 voids or fractures. Here we observe that when fluid-filled cracks propagate
518 in non-intact gelatin, they do so in spurts, and the elastic energy is released

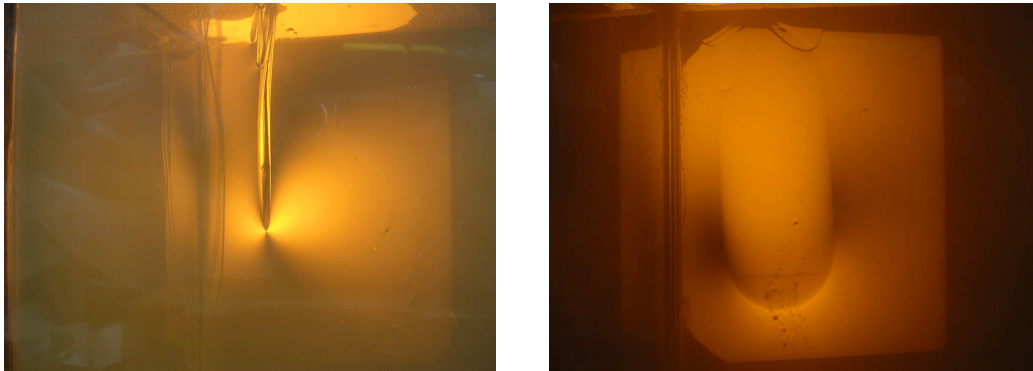


Figure 6: Stress field around a downward-propagating glycerin-filled crack in gelatin as evidenced by illuminating the tank through a polariser (left: cross-sectional view, right: frontal view).

519 in jerky leaps. These observations may be linked to seismicity during dike
 520 propagation: earthquakes may occur when the fluid-filled crack comes very
 521 close to preexisting fractures that concentrate stress leading to rock failure
 522 (see also Le Corvec et al., 2013).

523 Non-steady propagation can also result from coupling of propagation with
 524 solidification. Taisne and Tait (2011) injected, under constant volumetric
 525 flux condition, a hot, molten paraffin into a cold, brittle gelatin. They ob-
 526 serve a step-wise mode of propagation, with duration and amplitude of each
 527 step a function of dimensionless flux and temperature. This unsteady be-
 528 haviour could help to interpret the seismic burst observed during magma
 529 emplacement in cases such as Izu Peninsula, Japan (Hayashi and Morita,
 530 2003; Morita et al., 2006) or Iceland (White et al., 2011).

531 4. Dike Propagation Modelling

532 4.1. Introduction

533 The field and laboratory observations of propagating dikes described in
 534 the previous section offer a basis for validation of existing numerical models
 535 or to construct new approaches. In this section, we review the technical
 536 developments from the last twenty years in modeling propagating dikes, with
 537 a particular focus on dike ascent.

538 Nakashima (1993) pointed out that there can be two types of fluid trans-
 539 port, which he labels types I and II. Type I involves the propagation of a

540 fluid-filled crack connected with a pressurized reservoir. The dynamic of the
541 crack propagation is driven by the reservoir's excess pressure and the buoy-
542 ancy of the fluid, either positive or negative. Type II involves the propagation
543 of a fluid-filled crack isolated from any reservoir. This propagation is driven
544 only by the buoyancy of the fluid confined in the crack. While useful for
545 identifying end-member behavior, this classification itself does not address
546 the implicit question of what are the conditions allowing a dike to become
547 decoupled from its feeding source.

548 In the following, we describe two established methods or approaches that
549 may be roughly linked to the two types of dike propagation. In the first
550 section we describe the Weertman theory (Section 4.2) that (at least in its
551 original formulation) addresses isolated, self-contained ascending fluid-filled
552 cracks, driven only by buoyancy. Next, we address the lubrication theory
553 (Section 4.3), that involves viscous flow of fluid into the ascending dike.
554 Both approaches give rise to semi-analytical models when the geometry is
555 simplified to 2D, taking advantage of the sheet-like shape of dikes. Both ap-
556 proaches are in principle extensible to more complicated geometries and three
557 spatial dimensions, by using numerical methods. After the two approaches
558 are discussed, we conclude with a review of current and emerging numerical
559 methods that are applicable to either model type.

560 *4.2. The Weertman school: buoyancy-driven magma-filled dikes*

561 The Weertman approach for buoyancy-driven ascending dikes was first
562 introduced by Weertman (1971a,b, 1973) to model water-filled crevasses in
563 glaciers. Later it was applied to study magma-filled dikes at mid-ocean
564 ridges and water-filled cracks in submerged materials. Weertman cracks are
565 constant-volume batches of fluid propagating by breaking rock apart at their
566 leading tip due to concentration of buoyancy-induced stresses. In Weertman
567 models, the volume of fluid contained in the fracture is conserved during
568 propagation by assuming that the fractures pinch themselves closed at the
569 tail while they propagate. The dimension of the cracks must be large enough
570 to make surface tension/surface energy effects associated with the magma
571 unimportant, but small relative to the thickness of the crust (Weertman,
572 1971a). In their original formulation, Weertman fluid-filled fractures are re-
573 stricted to a vertical crack plane, and are considered to be filled with an
574 incompressible fluid; the viscous pressure drop within the fracture is alter-
575 nately neglected or simplified as a constant gradient over the crack plane.

576 *4.2.1. Formulation for static fractures*

577 The Weertman theory predicts that if the fluid volume injected, V , is
 578 lower than a critical value V_c , the fracture will be static because there exists
 579 no configuration in which the stress intensity factor at the tips overcomes the
 580 fracture toughness of rock, K_c . If the injected volume increases, the fracture
 581 elongates. If $V = V_c$, the stress intensity factor at the upper tip (for a buoyant
 582 fracture that propagates upward), K^+ , equals exactly K_c , so the fracture will
 583 tend to break apart the host material and ascend. As soon as propagation
 584 has started, the stress intensity factor at the lower tip, K^- , approaches zero;
 585 the fracture is assumed to close at the lower tip, forcing magma out of the
 586 tail when the stress intensity factor equals that for a broken medium, $K_c = 0$.
 587 The critical length of the fracture can be obtained from the two equations
 588 for the stress intensity factor at the tips of a fracture opening due to a linear
 589 pressure gradient (Secor and Pollard, 1975, e.g.):

$$K^+ = \sqrt{\pi a} \left(p_0 + \frac{a}{2} \frac{dp}{dz_{\text{tot}}} \right) = K_c \quad (1)$$

$$K^- = \sqrt{\pi a} \left(p_0 - \frac{a}{2} \frac{dp}{dz_{\text{tot}}} \right) = 0 \quad (2)$$

590 where p_0 is the overpressure at the mid-point of the fracture, a the fracture
 591 half-length and dp/dz_{tot} the total pressure gradient. For a liquid-filled frac-
 592 ture ascending in a hydrostatic/lithostatic stress field, $dp/dz_{\text{tot}} = \Delta\rho g$, with
 593 g the acceleration due to gravity and $\Delta\rho$ the density difference between solid
 594 and liquid. If the stress field is more complicated, then the total tectonic gra-
 595 dient should be considered (for example, for dikes propagating longitudinally
 596 driven by topographic/tectonic gradients).

597 Eq. 1 gives $p_0 = a/2 dp/dz_{\text{tot}}$ and, once substituted in Eq. 1, leads to the
 598 critical length, a_c :

$$a_c = \left(\frac{K_c}{\sqrt{\pi} \frac{dp}{dz_{\text{tot}}}} \right)^{2/3} \quad (3)$$

599 The opening of a fracture extending from $z = -a$ to $z = a$ is then given by
 600 (Weertman, 1980):

$$h(z) = \frac{(1 - \nu)K_c}{2G} \sqrt{\frac{a}{\pi}} \sqrt{1 - \left(\frac{z}{a}\right)^2} \left(1 + \frac{z}{a}\right) \quad (4)$$

601 Note that Eq. 4 compares very well to air-filled fractures from laboratory
 602 experiments (see Heimpel and Olson (1994) and Dahm (2000b)), while for

603 liquid-filled fractures a slight modification is needed for the loss of fluid to a
 604 tail with a finite residual thickness (Heimpel and Olson, 1994).

605 4.2.2. Formulation for moving fractures

606 Nunn (1996) modified the Weertman formulation in order to take into
 607 account a simplified viscous stress drop and obtain the velocity of moving
 608 Weertman fractures. The formulation is based on the observation that for
 609 $V > V_c$ the fracture propagates, fluid flows in the direction of propagation,
 610 and a pressure drop will necessarily develop due to viscous resistance within
 611 the crack. If this viscous pressure drop is approximated as a constant gra-
 612 dient dp/dz_v over the whole length of the fracture, the basic formulation of
 613 the Weertman theory can be maintained. The thinner sections of the frac-
 614 ture (front tip and constriction to the tail) will in reality experience a higher
 615 viscous pressure drop, but a constant gradient is considered a good approxi-
 616 mation over most of the crack plane, as shown in Taisne and Jaupart (2009),
 617 Fig. 7, Fig. 11 and Fig. 17, in Dahm (2000b), Fig. 6 and in Roper and Lister
 618 (2007), Fig. 2. The total pressure gradient can be separated into an external,
 619 static contribution, $\frac{dp}{dz_{ext}}$, and a dynamic contribution due to viscous motion:

$$\frac{dp}{dz_{tot}} = \frac{dp}{dz_{stat}} - \frac{dp}{dz_v} \quad (5)$$

620 The propagating fracture in this model will be longer than a critical static
 621 fracture. In other words, if the viscous pressure drop is taken into account,
 622 the initial length of fracture that is required to attain a propagating state is
 623 larger than the critical length a_c by a finite amount ($\frac{dp}{dz_{tot}}$ is in the denomi-
 624 nator of Eq. 3).

625 For example, for a dike ascending vertically in a lithostatic stress field
 626 (Nakashima, 1993; Nunn, 1996; Dahm, 2000b):

$$\frac{dp}{dz_v} = \frac{dp}{dz_{stat}} - \frac{dp}{dz_{tot}} = \Delta\rho g - \frac{K_c}{a\sqrt{\pi a}} \quad (6)$$

627 Viscous resistance within the narrow fracture means flow of fluid to the
 628 crack tip is relatively slow, which limits the rate of fracture propagation. It is
 629 then appropriate to assume that the stress intensity factor at the propagating
 630 tip K is approximately equal to the critical value of that parameter, K_c .

631 Assuming laminar Poiseuille flow within the fracture, the pressure gra-
 632 dient associated with viscous resistance will be proportional to the mean

633 magma speed, v , the viscosity, μ , and inversely proportional to the square of
634 the half-opening, h :

$$\frac{dp}{dz_v} = \frac{3\mu v}{h^2} \quad (7)$$

635 So, for a given volume of injection, a given fracture toughness and a given
636 external, static gradient, the model predicts the propagation speed of the
637 dike. The assumption of a simple Poiseuille flow is likely a significant over-
638 simplification and has been argued to lead to speed predictions that are too
639 large (Dahm, 2000b).

640 *4.2.3. Recent extensions*

641 The Weertman theory has been extended recently to include the effects of
642 a free surface. Rivalta and Dahm (2006) observed air-filled cracks in gelatine
643 experiments accelerated as they approached the free surface. To explain
644 this they invoked a change associated with the free surface in the stress
645 intensity factor at the upper tip of the fractures (Pollard and Holzhausen,
646 1979). Equivalently, this can be understood as the decreasing resistance to
647 fracture opposed by the medium with the dike approaching the free surface,
648 which makes the medium appear effectively weaker.

649 Rivalta and Dahm (2006) derived an approach incorporating into the for-
650 mulation dynamic changes in the stress intensity factor (Nakashima, 1993;
651 Nunn, 1996). The resulting modification of the Weertman theory was applied
652 to explain the acceleration of the seismicity towards the free surface docu-
653 mented by Battaglia et al. (2005) for Piton de la Fournaise. By observing
654 that the effect of the free surface should scale with the distance of the dike
655 tip to the surface, Rivalta and Dahm (2006) also formulated an approach for
656 inverting dike lengths on the basis of the hypocentral locations of the induced
657 seismicity.

658 Dahm et al. (2010) extended the Weertman theory to model the bilateral
659 migration of hypocenters sometimes observed in seismic data from hydraulic
660 fracturing. Upon injections of fluids at high pressure, the induced seismicity
661 is often found to migrate bilaterally at first, and then the migration contin-
662 ues in just one direction. They explain this with the presence of an external
663 (i.e. tectonic) stress field driving the growth in competition with the injec-
664 tion pressure. Dahm et al. (2010) divide the growth process into four stages
665 (Fig. 4.2.3):

- 666 1. Injection phase, when the excess pressure at the injection source is
667 dominating, and therefore driving the growth. Generally the two tips of

- 668 the hydrofracture advance in opposite directions with different speeds.
669 This phase ends when injection is stopped.
- 670 2. Post-injection phase during bidirectional growth, when the flow from
671 the injection has dropped to zero. Now the external pressure gradient
672 (due to buoyancy or of tectonic origin) takes over in driving the fracture
673 into one particular direction, and the fracture must adjust to a new
674 pressure balance by redistributing the volume of fluid it contains. The
675 seismicity continues to propagate in two opposite directions for some
676 time. This phase ends when, as evidenced by the migrating seismicity,
677 the slower tip stops propagating.
 - 678 3. Post-injection phase with unidirectional growth, when the fracture con-
679 tinues to elongate in one direction while it is still adjusting to the new
680 pressure balance on the crack plane. This phase ends when the frac-
681 ture toughness at the back tip becomes zero (the hydrofracture cannot
682 elongate anymore and starts to close at the tail). The elongation of the
683 fracture is now maximal.
 - 684 4. The final phase, when the fracture has now become a Weertman-Nunn
685 fracture. In this phase it is driven only by buoyancy or by tectonic
686 stress. It is propagating as an isolated batch of fluid. This propagation
687 will be very slow and can last for long periods of time.

688 Dahm et al. (2010) model the Coulomb stress change during propagation
689 and show that such a model can explain not only the advancing front of
690 the migrating seismicity, but also the retreating front (see Fig. 5), because
691 after the passage of the propagating tip, rock volumes might fall under a
692 stress shadow (a negative Coulomb Stress inhibiting seismicity) and therefore
693 experience a sudden drop in seismic rate.

694 Non-symmetric growth, described by the model of Dahm et al. (2010), is
695 consistent with other field (e.g. Maxwell et al., 2002; Reynolds et al., 2012)
696 and microseismic (e.g. Fisher et al., 2004; Daniels et al., 2007; Walker et al.,
697 2012; Reynolds et al., 2012) observations of hydraulic fractures. This model
698 of successive phases may be applied with minor changes to explain the pat-
699 tern of seismicity induced by a dike propagating from a magma chamber.
700 For example, an initial bilateral migration of the seismicity was observed
701 for several laterally propagating dikes during the Krafla and Manda Hararo
702 rifting episodes (Keir et al. (2009); Wright et al. (2012), Fig. 5D). Tectonic
703 stress taking over after the pressure gradient from the magma chamber has
704 dropped to zero may then be the cause for the bilateral propagation of seis-

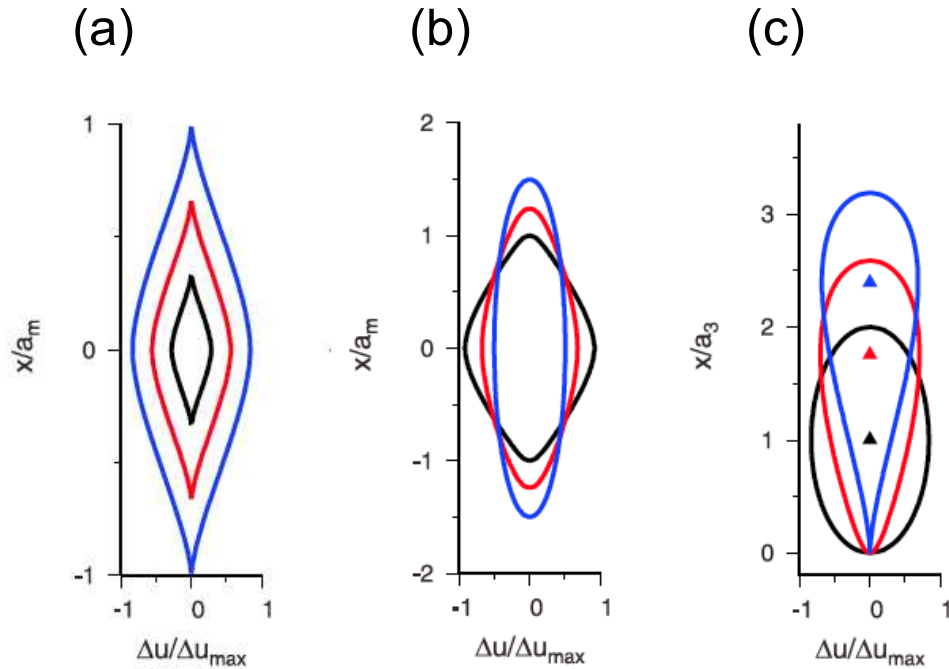


Figure 7: a) Injection phase: the fracture propagates bilaterally. The volume grows and the fracture elongates and propagates driven by the injection pressure. Here, the external (tectonic) gradient is zero. b) Bilateral post-injection propagation. The volume is now fixed. The stress intensity factor at the tips still overcomes the fracture toughness of the host rock and the fracture continues to elongate. c) Unilateral post-injection phase and solitary ascent phase: the fracture continues to elongate driven by its overpressure, until it becomes a Weertman fracture, driven only by buoyancy or by the tectonic gradient. From Dahm et al. (2010), with permission.

705 micity. Also, the discrimination of phases in the intrusion process might be
 706 relevant for dikes too: dikes are generally injected by a pressurized magma
 707 chamber.

708 4.2.4. Strengths and limitations

709 The Weertman model is a simple approach, and this represents both a
 710 strength and a limitation. The main advantage is that by neglecting or sim-
 711 plifying to a constant gradient the pressure gradient (and hence also the
 712 fluid flow within the dike), a large part of the analytical or numerical diffi-
 713 culties are avoided. This facilitates the inclusion and study of effects that
 714 are otherwise prohibitive (Sect. 5).

715 However, Lister and Kerr (1991) raise a number of issues regarding the

716 validity of the assumptions on which Weertman models are based.

- 717 1. They note that the model, in its original formulation, is inherently
718 unstable: fractures are assumed to be exactly $a = a_c$ long so that
719 $K^+ = K_c$, but as soon as they propagate they will be in fact longer
720 ($a > a_c$). As described above (Sec. 4.2.2), this important point has been
721 addressed by Nunn (1996), who introduced a constant viscous pressure
722 gradient over the crack plane, making it possible to model supercritical,
723 moving fractures and at the same time maintain the simple formulation.
- 724 2. They mention that for K_c of the order of $1 \text{ MPa m}^{1/2}$, as in labora-
725 tory measurements, the resistance to fracture is negligible with respect
726 to other contributions and it is unlikely that the size of propagating
727 dikes is determined by the fracture toughness of rock. This point is
728 very important and we discuss it at length in Sec. 4.4.2, where we
729 show that there is evidence for field-based, effective fracture toughness
730 measurements in the order $K_c^{\text{eff}} \approx 100 \text{ MPa m}^{1/2}$ or more. If such val-
731 ues are appropriate, fracture toughness is not negligible and becomes
732 an important factor controlling dike size (Sec. 4.4.1). If we employ
733 $K_c^{\text{eff}} \approx 100 \text{ MPa m}^{1/2}$ and keep the other parameters unchanged in the
734 dike length and thickness estimate by (Lister and Kerr, 1991, p. 10055,
735 Eq. 14a and 14b), we obtain a length in the order of 2 km and thickness
736 0.5 m instead of 100 m and 2 mm, respectively.
- 737 3. They stress that critically-long dikes ($a = a_c$) are very short and thin,
738 and would freeze in a short time interval. This problem is solved by
739 introducing higher values for effective fracture toughness, which will
740 imply longer and thicker critical fractures (see point 2 above) and a
741 constant pressure gradient as in Nunn (1996), because this also allows
742 for longer fractures (point 1 above). However, the criticism that Weert-
743 man fractures need to move relatively fast in cold rock (shallow crust)
744 for the approach to be valid still holds in general.
- 745 4. They point out that for dikes to pinch themselves closed at the trail-
746 ing front, the viscosity of the fluid should theoretically vanish. This
747 crude assumption may be justified only for low-viscosity magmas (we
748 discuss this point in detail in Sect. 4.4.3). There is evidence that pure
749 Weertman models work very well for low-viscosity fluids, and almost
750 perfectly for gases. For example, they match observations of propaga-
751 tion of air-filled cracks in gelatin extremely well (Dahm, 2000b). This
752 occurs because: a) the less viscous the fluid, the thinner the tail of the

753 crack, and the better the approximation of a constant volume ascent,
754 b) the less viscous the fluid, the better is the approximation of negli-
755 gible viscous stresses, c) air is a hydrophobic fluid and surface tension
756 effects assist emptying the crack tail effectively.

757 No experiment or numerical model has addressed the issue of incorpo-
758 rating surface-tension effects as a modification to the effects of viscosity. A
759 discussion of generalised trailing-front dynamics, based on numerical model-
760 ing, is included by Dahm (2000b).

761 The approximation of a vanishing fracture toughness at the trailing tip
762 is also somewhat crude: stress concentrations at the trailing tip of the dike
763 (the inlet from the tail) are visible, for example, from photoelastic images;
764 see figure 3.9 of Tait and Taisne (2013).

765 Finally, Weertman models are 2D (as most models of dike propagation).
766 This has to be kept in mind, especially when the conditions in the host
767 medium change. For example, laboratory experiments with gelatin show that
768 for density layering, fracture toughness or volume variations of the fracture,
769 the breadth of the fracture might change with time.

770 *4.3. The lubrication-theory school: dynamics controlled by magma flow*

771 Simplifying the role of viscosity as in the Weertman approach (Weertman,
772 1971a,b) (Sect. 4.2) might not be an appropriate simplification of cases where
773 viscosity is large. A better treatment of hydraulic fracturing in the oil and
774 gas industry motivated the development of a theory for coupling elastic
775 fracturing and fracture-hosted fluid flow (Khristianovic and Zheltov, 1955;
776 Perkins and Kern, 1961; Barenblatt, 1962; Geertsma and de Klerk, 1969; Nordgren,
777 1972; Spence and Sharp, 1985). The flow model was simplified using the lu-
778 brication approximation, which is appropriate for a flow with an aspect ratio
779 much greater than unity in the direction of flow. Building on this litera-
780 ture, Spence et al. (1987) modelled the effect of viscous flow on a buoyancy-
781 driven dike in 2D, assuming a steady state regime of propagation. Their
782 model considered only one particular value for the ratio of fracture tough-
783 ness to fluid viscosity. A generalization for relatively small fracture tough-
784 ness was carried out by Lister (1990b,a) and the theory was fully gener-
785 alized by Roper and Lister (2007). Even in these more recent papers, a
786 stationary solution is obtained for physical properties that are constant in
787 time and space; in other words, these models neglect variations of the fluid
788 injection and of stratification of the host rock. These limitations are ad-

789 dress by Taisne and Jaupart (2009) and Taisne and Jaupart (2011) who de-
790 velop a semi-implicit numerical scheme that allows for the study of spatial
791 and temporal changes of host-rock and/or magma properties. Furthermore,
792 most of the theoretical papers cited above focus on incompressible magma
793 (Spence et al., 1987; Lister, 1991; Lister and Kerr, 1991; Roper and Lister,
794 2007; Taisne and Jaupart, 2009). Only a few of them consider a pressure-
795 dependent density for the magma or the presence of a gas precursor (Lister,
796 1990b; Taisne and Jaupart, 2011; Maimon et al., 2012), this will be detailed
797 in Section 5.15. We discuss some results and future directions regarding
798 magma properties including compressibility and phase transitions in Sects. 4.4.3,
799 5.10 and 5.15.

800 4.3.1. Model formulation: Magma Flow

801 The theory as formulated by (e.g. Lister, 1990a) begins by expressing the
802 magma pressure P as

$$P = P_{\text{Lith}} + P_e, \quad (8)$$

803 where P_{Lith} is the lithostatic pressure, and P_e is the over-pressure that drives
804 deformation of the fracture walls. For small Reynolds numbers, the flow of
805 a viscous fluid within a thin fracture is laminar and one can use lubrication
806 theory (Batchelor, 2000); in that case the Navier-Stokes equations reduce to

$$0 = -\frac{\partial P}{\partial z} + \mu \frac{\partial^2 w}{\partial x^2} - \rho_m g, \quad (9)$$

807 where w is the vertical velocity profile of the magma within the dike (see
808 Fig. 8c), ρ_m and μ the magma density and viscosity.

809 Solution to Eq. (9) follows by first noting that the volume flux ϕ of magma
810 at depth z is given by

$$\phi(z) = \int_{-h}^h w(z, x) dx, \quad (10)$$

811 where the dike width is $2h$. Combination of equations 9 and 10 leads to:

$$\phi = -\frac{2}{3\mu} h^3 \left(\frac{\partial P}{\partial z} + \rho_m g \right). \quad (11)$$

812 Finally, substituting for P into the flux equation (11), we obtain the Poiseuille
813 equation

$$\phi = -\frac{2}{3\mu} h^3 \left(\frac{\partial P_e}{\partial z} - \Delta \rho g \right), \quad (12)$$

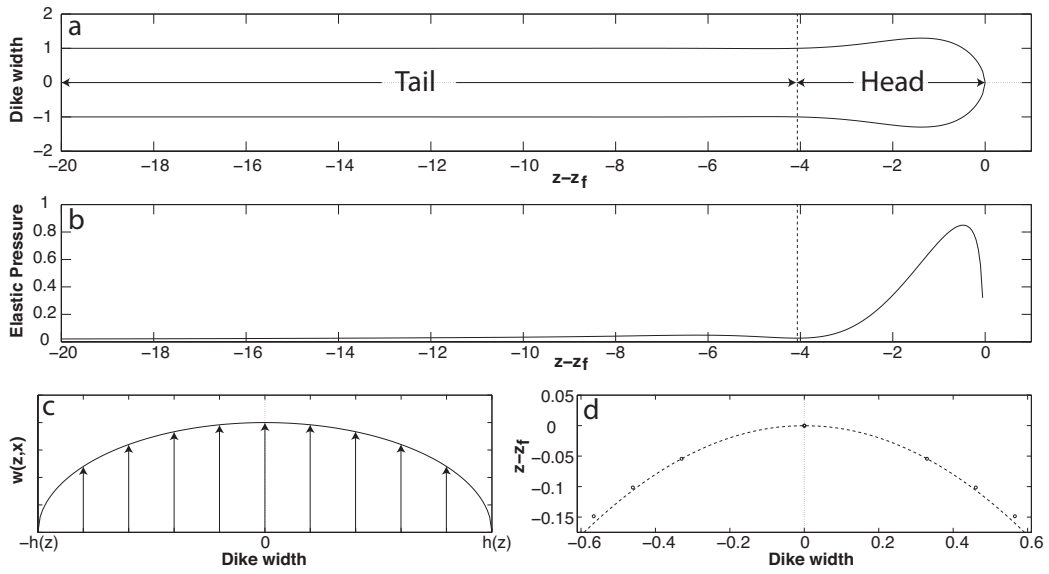


Figure 8: Panels a and b: typical dimensionless width and elastic pressure profile within a propagating dike. Panel c: arbitrary velocity profile for a laminar viscous flow used in equation 10. Panel d: Cross-section of the tip of the dike. The condition defined by Eq. 20 is respected (dashed line shows Eq. 20, and the dots the output of the numerical simulation). Dike width, depth and pressure have been normalized using scaling argument leading to equations 22, 24 and 25.

814 where $\Delta\rho = \rho_s - \rho_m$ is the magma buoyancy.

815 The formulation of the fluid flow equations is completed by enforcing
816 mass conservation according to the equation

$$2\frac{\partial\rho_m h}{\partial t} = -\frac{\partial\rho_m\phi}{\partial z}. \quad (13)$$

817 When limiting consideration to an incompressible fluid filling the crack,
818 Eq. 13 reduces to volume conservation

$$2\frac{\partial h}{\partial t} = -\frac{\partial\phi}{\partial z}. \quad (14)$$

819 4.3.2. Elastic Deformation

820 For a dike extending from a distant source ($z \rightarrow -\infty$) to a tip located at
821 $z = z_f$, half-width h and overpressure P_e are related to one another through
822 the following equation (Muskhelishvili, 1953; Weertman, 1971a):

$$P_e(z) = -\frac{G}{1-\nu}\frac{1}{\pi}\int_{-\infty}^{z_f}\frac{\partial h}{\partial\xi}\frac{d\xi}{\xi-z}, \quad (15)$$

823 where G is the shear modulus and ν is Poisson's ratio. One can invert this
824 equation to solve for the dike width as a function of P_e . Integrating by parts
825 leads to the following equation for h (Spence et al., 1987):

$$h(z) = \frac{1-\nu}{G}\frac{1}{\pi}\int_{-\infty}^{z_f}k(z_f, z, \xi)P_e(\xi)d\xi, \quad (16)$$

826 where kernel $k(z_f, z, \xi)$ is such that:

$$k(z_f, z, \xi) = \ln\left|\frac{\sqrt{z_f-z} + \sqrt{z_f-\xi}}{\sqrt{z_f-z} - \sqrt{z_f-\xi}}\right|. \quad (17)$$

827 4.3.3. Propagation Condition

828 The shape of the dike's tip is imposed in the region where $z \rightarrow z_f$ to
829 ensure that sufficient energy is available to fracture rock. It is also possible
830 to employ this known asymptotic solution to regularize the pressure near the
831 tip in computational algorithms. Just ahead of the dike tip, the singularity
832 is expressed as

$$P_e(z) \sim -\frac{K}{2\sqrt{z-z_f}} \text{ for } z > z_f, \quad (18)$$

833 where K is the stress intensity factor. In the case of a propagating dike, the
 834 stress intensity factor, K , is equal to the fracture toughness, K_c , implying
 835 that the shape of the dike near the tip (Fig. 8), is defined as (Muskhelishvili,
 836 1953; Weertman, 1971a):

$$h \sim \frac{1-\nu}{G} K_c \sqrt{2(z_f - z)}, \text{ for } z \rightarrow z_f. \quad (19)$$

837 Combining equations (16-19) leads to the following boundary condition for
 838 the dike tip (Lister, 1990a):

$$K_c \sqrt{2} = \frac{2}{\pi} \int_{-\infty}^{z_f} \frac{P_c(\xi)}{\sqrt{z_f - \xi}} d\xi. \quad (20)$$

839 4.3.4. Boundary Conditions

840 The system of equations requires three boundary conditions. Firstly, a
 841 steady state solution is made possible by assuming that magma is injected
 842 at a constant rate Q at $z \rightarrow -\infty$. Secondly, the near-tip region is assumed
 843 to be filled with vapor and so the pressure at the fluid front is equal to
 844 the saturated vapor pressure for the magma. Finally we have the asymptotic
 845 solution for h at the leading edge of the dike defined in eq. 19.

846 Note that the analytical results derived by Lister (1990a) are possible only
 847 provided the steady state solution associated with a constant-flux condition
 848 at the dike source. However, this assumption is neither necessary in a general
 849 sense nor is it universally valid. Alternately, a time-dependent input flux
 850 can be introduced with a semi-implicit method used by Taisne and Jaupart
 851 (2009, 2011) by appropriately modifying the mass conservation equation

$$\int_{z_1}^{z_f^{t+1}} (\rho_m h)^{t+1}(\xi) d\xi = \frac{Q^{t+1} + Q^t}{2} \Delta t + \int_{z_1}^{z_f^t} (\rho_m h)^t(\xi) d\xi, \quad (21)$$

852 with Q describing the mass flux at the source. This enables modelling of the
 853 temporal evolution of an injection of a constant mass of magma, assuming
 854 $Q = 0$.

855 4.3.5. Strengths and Limitations

856 The main strength of lubrication theory model is the accuracy of the
 857 solution of the viscous motion within the dike and, with it, the accuracy of
 858 predictions regarding the velocity of the dike (which is a central problem in
 859 magma propagation) and the shape of the dike. The weaknesses are related
 860 to

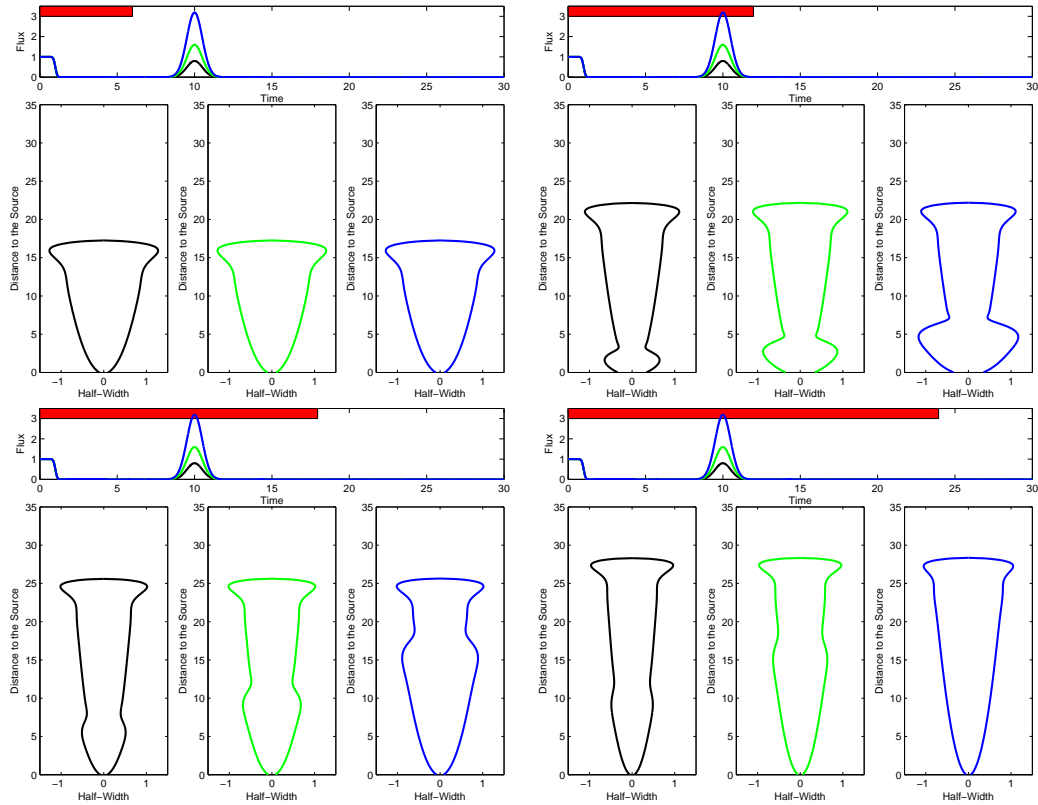


Figure 9: Example of variation of the flux at the source. Four snapshots showing the time evolution of a dike subject to a pulse of magma being injected at the source, the normalized volume of the pulses are 1, 2 and 4, respectively, for black, green and blue curves. The top curve represents the normalized flux at the source, the red bar represent the time at which the profiles are drawn.

- 861 1. The difficulty of including external influences such as inhomogeneous
862 stress fields. This could be included in the future through the use of
863 an apparent buoyancy.
- 864 2. In order to resolve the dynamic of dike propagation much effort has
865 been done on the 2D problem, neglecting the 3D effect. This introduces
866 large epistemic uncertainties: it is difficult to know what would change
867 in the results if the fluid could also flow along the third dimension.
868 From a mathematical point of view, any change in the third dimension
869 will drastically impact the dynamic obtain in 2D, since the 2D flux is
870 derived from the 3D flux divided by b .
- 871 3. Available lubrication theory models do not provide information on po-
872 tential change in the direction of propagation, but assume either purely
873 vertical or purely horizontal propagation.

874 4.4. *Critical analysis of the two approaches*

875 The approaches described in Sections 4.2 and 4.3 were seen for some time
876 as being in conflict with each other. The conflict can ultimately be cast as
877 a debate about the importance of viscous resistance to flow of the magma
878 versus fracture toughness of the hostrock. To help guide the choice of the
879 most appropriate model for a specific application, we next compare Weert-
880 man and lubrication theory models through dimensional analysis, review
881 laboratory and field measurements of rock fracture toughness, and discuss
882 the appropriateness of common assumptions and individual dike propagation
883 models.

884 4.4.1. *Dimensional Analysis*

885 As shown by Lister (1990a) and Lister and Kerr (1991), a simple method
886 to evaluate dike behavior is to consider the main force balance in the tail
887 and nose regions (see Fig. 8a). In both regions, the driving force is magma
888 buoyancy and one must determine the dominant resistance to propagation.
889 There are two relevant sources of resistance, or dissipation, in the system:
890 1) viscous flow of the magma, and 2) fracture of the rock. We may thus
891 consider two different force balances in the nose, depending on the dominant
892 resistance to propagation. The characteristic dimensions of the system are
893 the scales for the half-width of the dike tail, h , and for the length of the dike
894 head, L .

895 In the case where viscous dissipation is dominant, h^* is derived by balanc-
896 ing buoyancy and viscous pressure drop (elastic overpressure can be neglected

897 in the tail region, Fig. 8b) leading to (Lister and Kerr, 1991):

$$h^* = \left(\frac{3\mu Q}{2\Delta\rho g} \right)^{1/3}, \quad (22)$$

898 where Q represents the 2D volumetric flux of magma injected into the prop-
899 agating dike. Combination of Q and h^* defines the velocity scale

$$c^* = \frac{Q}{2h^*}. \quad (23)$$

900 L^* is derived from the balance between the driving force from buoyancy
901 and viscous resistance to flow (which is coupled with elastic deformation of
902 the rock). This leads to

$$L^* = \left[\frac{Gh^*}{(1-\nu)\Delta\rho g} \right]^{1/2} = \left(\frac{3\mu Q}{2\Delta\rho^4 g^4} \right)^{1/6} \left(\frac{G}{1-\nu} \right)^{1/2}. \quad (24)$$

903 Finally the pressure scale is defined as

$$P^* = \Delta\rho g L^* = \left(\frac{G}{1-\nu} \right)^{1/2} \left(\frac{3Q\mu\Delta\rho^2 g^2}{2} \right)^{1/6}. \quad (25)$$

904 If fracture is the limiting process, the fluid does not provide any resistance
905 to closure in the tail region and hence according to Eq. 22, which describes
906 the balance of force in the tail region regardless of the relative importance
907 of the toughness, the thickness of the tail goes to zero when viscosity and/or
908 influx goes to zero. Balancing the driving force of magma buoyancy with
909 resistance to fracturing defines a scaling length for the head of the dike, L_f ,
910 such that:

$$K_c = \Delta P \sqrt{L_f} = \Delta\rho g L_f^{3/2}, \quad (26)$$

911 where K_c is the toughness and ΔP is the magma overpressure in the nose.
912 Solving for L_f , we find that:

$$L_f = \left(\frac{K_c}{\Delta\rho g} \right)^{2/3}. \quad (27)$$

913 Comparison with Eq. 3 confirms that this approach recovers the Weertman
914 solution. It is also apparent that there is no means by which to estimate

915 velocity of the dike. By using Eqs. 25 and 27, the pressure scale can be
 916 expressed as

$$P_f = (\Delta\rho g K_c^2)^{1/3}. \quad (28)$$

917 The ratio between the two different length-scales or, equivalently, the
 918 ratio between the two pressure scales quantifies the relative importance of
 919 viscous flow versus rock fracture. It is therefore useful as a proxy for the
 920 regime of dike propagation. This ratio is given by:

$$\frac{P_f}{P^*} = \frac{L_f}{L^*} = \left(\frac{2}{3}\right)^{1/6} \left(\frac{(1-\nu)^3 K_c^4}{G^3 Q \mu}\right)^{1/6}. \quad (29)$$

921 This can also be written as a function of a dimensionless toughness ratio such
 922 that:

$$\frac{L_f}{L^*} = \left(\frac{K_c}{K^*}\right)^{2/3}, \quad (30)$$

923 where K^* is a toughness scale associated with viscous flow requirements.
 924 Combining Eqs. 29 and 30 lead to:

$$K^* = \Delta\rho g (L^*)^{3/2} = \left(\frac{G^3}{(1-\nu)^3} \frac{3\mu Q}{2}\right)^{1/4}. \quad (31)$$

925 This toughness scale does not depend on the buoyancy of magma.

926 The dominant resistance to propagation dictates the magnitude of the
 927 buoyancy force in the nose region and hence the length of that region. In turn,
 928 this sets the relevant length-scale for the equations of motion. If $L_f/L^* \gg 1$,
 929 or if $K_c/K^* \gg 1$, corresponding to large rock toughness, then viscous dissipa-
 930 tion is not limiting and hence sufficient buoyancy-induced driving force must
 931 be accumulated over the length L_f to overcome the resistance to fracture. In
 932 this case, the proper length-scale is L_f . In the other limit, for $L_f/L^* \ll 1$,
 933 or $K_c/K^* \ll 1$, it is the elastic opening of the fracture that requires the
 934 largest stresses, and one should scaling lengths with L^* . Roper and Lister
 935 (2007) have demonstrated that for $K_c/K^* < 2$ the length of the nose region
 936 scales with the viscous length-scale, L^* , and its width is comparable to h^*
 937 (see Fig. 8a in which $K_c/K^* = 1$). As shown in Fig. 2 of Roper and Lister
 938 (2007), the nose extends over a length of $\approx 4L^*$ in this regime, independent
 939 of the toughness ratio and hence independent of the fracture toughness of
 940 encasing rocks. For $K_c/K^* \gtrsim 2$, the length of the nose region is much larger,
 941 as expected. The asymptotic limit such that the nose length scales with L_f

942 is reached for $K_c/K^* \gtrsim 8$. In this limit, the width of the nose region deviates
 943 markedly from that of the tail, as shown in Fig. 3 of Roper and Lister (2007).

944 In both cases, the tail region is in the same dynamical regime character-
 945 ized by a balance between buoyancy and viscous forces, which emphasizes
 946 the fundamental role played by the nose region. Hence, there arises a second
 947 criterion for the validity of the constant-volume, zero-viscosity Weertman
 948 model. This criterion requires that the volume of residual fluid in the tail
 949 must be small relative to the volume of fluid in the head of the dike. By using
 950 Eq. 22 for the half-width of the viscous-dominated dike tail, and Eqs. 3 and 4
 951 to obtain the volume of a Weertman crack, we obtain:

$$L_t \ll \left(\frac{(1-\nu)^3 K_c^6}{G^3 Q \mu \Delta \rho^2 g^2} \right)^{1/3}, \quad (32)$$

952 where L_t is the length of the tail, i.e. the difference between the depth of the
 953 head region and the depth of the magma source.

954 In summary, the lubrication model under the zero-toughness assumption
 955 is valid for $L_f/L^* \ll 1$. On the other hand, the Weertman model applies
 956 for $L_f/L^* \gg 1$ as long as the dike tail (which is not formally treated within
 957 the Weertman model) satisfies Eq. 32. The case $L_f/L^* \gg 1$ with Eq. 32 not
 958 satisfied, i.e. a K_c -dominated dike with decreasing volume, is not covered by
 959 the classic Weertman theory but needs a tailored approach. The applicability
 960 of the individual models is therefore closely linked to the value of the rock
 961 fracture toughness, as discussed in the next section.

962 4.4.2. Fracture Toughness and Dike Propagation Regime

963 There is a long debate in the literature regarding estimates of rock fracture
 964 toughness relevant for km-sized dikes. In theory, the fracture toughness is a
 965 material property and should not vary with crack dimensions. However, non-
 966 elastic processes at the crack tip such as plastic deformation or microcracking
 967 are not necessarily invariant with respect to fracture size; in fact there is
 968 ample evidence to the contrary. The question is not whether the toughness
 969 depends on fracture size but rather how to translate the dependence into
 970 estimates of toughness at the scale of dikes and other large structures.

971 In laboratory studies, K_c for rocks is typically of the order of 1 MPa m^{1/2}
 972 (Atkinson, 1984; Atkinson and Meredith, 1987). For example, Balme et al.
 973 (2004) measured the fracture toughness of basalt samples from Iceland, Vesu-
 974 vius and Etna at up to 600° temperature and 30 MPa pressure, obtaining

975 values from 1.4 to 3.8 MPa m^{1/2}. They observe an influence of both pressure
 976 and temperature on K_c but values remain in the same order of magnitude at
 977 the scale of the experiments. However, even at laboratory scale, the size of the
 978 initial notch or ligament size has been empirically shown to give a power-law
 979 variation of K_c , supporting the idea of a size effect on the fracture toughness
 980 (e.g. Carpinteri, 1994; Shlyapobersky et al., 1998). These observations have
 981 inspired a number of theoretical treatments based on the statistical mechan-
 982 ics of crack propagation in disordered media and/or on the interaction of the
 983 near-tip stresses with the plastic zone ahead of the crack tip (e.g. Borodich,
 984 1999; Bažant, 1997; Dyskin, 1997), but without direct experimental evidence
 985 at very large scale, even the theoretical treatments are difficult to trust as
 986 tools for extrapolation.

987 One important line of research for resolving this issue is based on detailed
 988 observations of dikes in the field. Klein et al. (1987) observed that areas along
 989 Kilauea’s East Rift where seismicity is persistent appear to act as barriers
 990 for propagation. Rubin et al. (1998) remarked that in general the seismicity
 991 induced by dikes constitutes a sink of inelastic energy: it provides evidence
 992 of a release of fracture energy beyond what needed to create new surface for
 993 the propagating dike alone. Given that K_c is related to strain energy through
 994 the relation:

$$K_c^2 = 2 \frac{G}{1 - \nu} \frac{\Delta E}{\delta l}, \quad (33)$$

995 where ΔE is the variation of strain energy for an incremental fracture ex-
 996 tension δl , any dissipation of elastic energy will be mirrored into an effective
 997 (also called apparent) value of K_c , K_c^{eff} . Slip on pre-existing fractures, and
 998 the correspondent energy release during propagation, scale with the dimen-
 999 sion of the dike supporting the hypothesis of a fracture-size scaling of fracture
 1000 toughness.

1001 A variety of other work has considered the relation between the dimension
 1002 of cracks and fracture toughness. Field studies (for example Delaney and Pollard,
 1003 1981; Delaney et al., 1986; Pollard, 1987; Vermilye and Scholz, 1995) have
 1004 shown that the size of the tip process zone scales with the dimension of
 1005 the crack (Heimpel and Olson, 1994). Olson (2003) studied the scaling rela-
 1006 tionship between fracture opening and length of three sets of fractures from
 1007 Vermilye and Scholz (1995) and Delaney and Pollard (1981) and conclude
 1008 that for those data sets, the fracture aperture scales with the squared length
 1009 of the fractures. By assuming that the growth of those fractures was con-
 1010 trolled by the fracture toughness of the medium, Olson (2003) calculates the

1011 relevant “in-situ” K_c^{eff} and finds values in the range 8 to 25 MPa m^{1/2} for
1012 fractures between 2 cm and 20 m length, and 40–4000 MPa m^{1/2} for 100
1013 meter-scale dike segments around the Ship Rock volcanic plug in NW New
1014 Mexico (Delaney and Pollard, 1981).

1015 K_c^{eff} can also be estimated using inverse methods and associated numer-
1016 ical models. Jin and Johnson (2008) developed a model for the propagation
1017 of multiple parallel dikes. The model incorporates viscous flow within the
1018 dike and solves the resulting non-linear (integral) equation through a pertur-
1019 bative approach. They assume constant dike velocities of the order of 0.01 –
1020 0.1 m/s and an overpressure of about 3 MPa, and obtain compatible stress
1021 intensity factors of about 100 to 200 MPa m^{1/2}. Rivalta and Dahm (2006)
1022 also found a value of about 100 MPa m^{1/2} by considering the effects of the
1023 free surface on the migration of hypocenters for an ascending dike. Similarly,
1024 Bungler and Cruden (2011) found that K_c in the range of 500–1300 MPa m^{1/2}
1025 provides the best match between a model of laccolith emplacement and as-
1026 pect ratio data. A possible scale-dependance of K_c^{eff} has never been taken
1027 into account in any model so far.

1028 These values of K_c exceed laboratory values by as much as 3 orders of
1029 magnitude and the scaling they imply between K_c and the dike length re-
1030 mains a matter of discussion (Scholtz, 2010; Olson and Schultz, 2011). The
1031 data is probably too limited to resolve these debates at present. There is
1032 also a lack of fracture toughness data at confining pressures relevant for
1033 dikes (mid to shallow crustal levels). Most measurements (Balme et al.,
1034 2004; Schmidt and Huddle, 1977) have been carried out at confining pres-
1035 sures relevant for the upper crustal layers (<3 km depth) and there is poor
1036 constraint on the influence of the confining pressure on K_c , when the effect
1037 is probably significant (Rubin, 1993c), in particular when combined with the
1038 effect of increased temperature (Funatsu et al., 2004). Furthermore, all of
1039 these discussions, including the present one, must bear in mind that field
1040 measurements are made on solidified structures that do not necessarily re-
1041 flect dynamics of the dike during the propagation phase. This will influence
1042 the value estimated for K_c^{eff} since these measurements may overestimate the
1043 ratio of the thickness to the length, especially if there was additional inflation
1044 following the arrest of the dike tip.

1045 What is clear, though, is that the issue is central to appropriately mod-
1046 eling dike growth. If we focus on dykes driven by basaltic magmas, we can
1047 take typical values of the relevant physical properties and control variables
1048 as $\mu = 10^2$ Pa s and $Q = 2$ m³s⁻¹m⁻¹ (Thordarson and Self, 1993). As-

1049 suming a reasonable value of $G \approx 10^{10}$ Pa and $\nu = 0.25$ for the rock, we
 1050 find that the toughness scale coming from viscous dissipation (Eq. 31 and
 1051 Lister (1990a)) lead to $K^* \approx 160$ MPa m^{1/2}. One should note that this
 1052 estimate does not depend on magma buoyancy at all and is weakly sensi-
 1053 tive to the various inputs because of the small power-law exponent involved.
 1054 Therefore, for lower-end viscosity magmas such as Kimberlite (0.1 Pa·s), it
 1055 only decreases slightly to $K^* \approx 30$ MPa m^{1/2} and for lower magma fluxes
 1056 ≈ 0.02 m³s⁻¹m⁻¹ (appropriate for Piton de la Fournaise volcano on Réunion
 1057 Island, (Traversa et al., 2010)) it decreases to $K^* \approx 50$ MPa m^{1/2}. We there-
 1058 fore conclude that if $K_c^{eff} \lesssim O(100)$ MPa m^{1/2} at the scale of dikes, then the
 1059 zero toughness lubrication model is valid, while the Weertman model would
 1060 be more appropriate from $K_c^{eff} \sim O(1000)$ MPa m^{1/2}.

1061 In general, Weertman and lubrication theory models in their original for-
 1062 mulation (no viscous flow and zero fracture toughness, respectively) are there-
 1063 fore end-member models for dike propagation: 1) Viscous magma flows slowly
 1064 into the dike and the dike tip does not propagate a very long distance from
 1065 the chamber ($\lesssim 1$ km, so that the effective fracture toughness should not
 1066 significantly exceed 100 MPa m^{1/2}). These dikes may best modeled with the
 1067 lubrication theory approach. 2) Magma viscosity is very low (see for example
 1068 long carbonatite or kimberlite-filled dikes in the range 5 to 10 km long and
 1069 for which viscosity is in the range 0.1 to 1 Pa·s, Sparks et al. (2006)), with
 1070 magma-filled pockets detaching effectively from the magma reservoir where
 1071 they originated and traveling long distances (even through 200 km thick cra-
 1072 tons) in a few hours or days (as demonstrated by the degree of preservation
 1073 of diamonds in kimberlite deposits). These may be better modeled as Weert-
 1074 man fractures.

1075 The case of interest may be intermediate, so the modeling approach
 1076 should be chosen according to an assessment of the approximations or as-
 1077 sumptions that are appropriate for a specific application (Fig. 10). More-
 1078 over, as discussed above, for km-sized dikes K_c^{eff} may be in the range $O(100)$
 1079 to $O(1000)$ MPa m^{1/2} (corresponding to $L_f = [1-5]$ km). Perhaps unsurpris-
 1080 ingly, these values overlap with the limits of validity reported above. Given
 1081 this overlap, we conclude that recent extensions aimed at relaxing the strong
 1082 assumptions of no viscous flow on one hand and zero fracture toughness on
 1083 the other should be preferred (Sects. 4.2.2, 4.3 and 4.4.4).

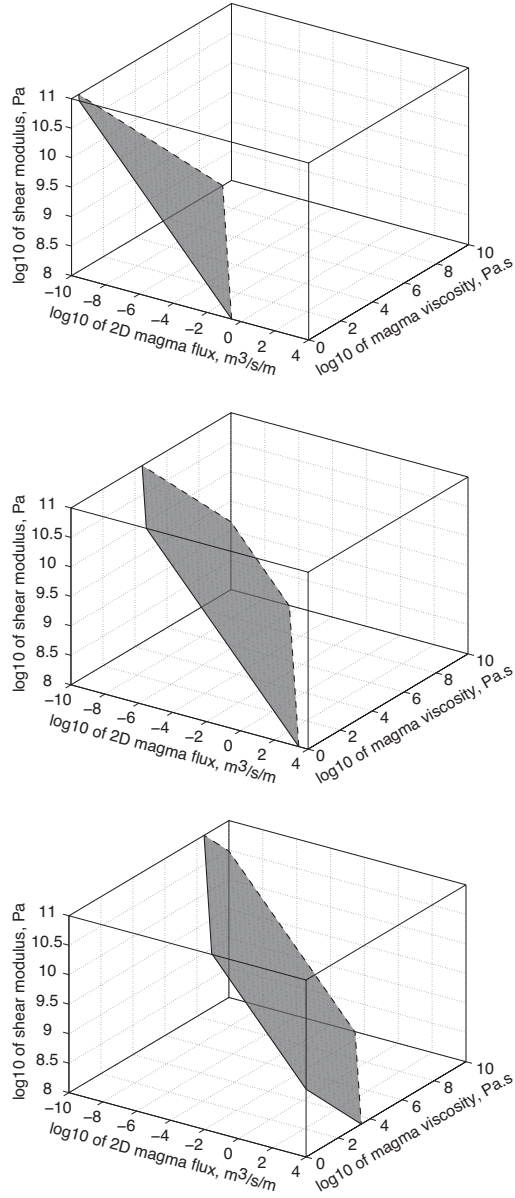


Figure 10: Surface representing $K_c/K^* = 1$. Above the surface, $K_c/K^* < 1$, represents the "lubrication" domain, below the surface, $K_c/K^* > 1$, represents the "Weertman" domain. Each panel is associated with a different value of the fracture toughness, $K_c = 1 MPa m^{1/2}$, $10 MPa m^{1/2}$ and $100 MPa m^{1/2}$

1084 *4.4.3. Magma influx from the reservoir and the dike tail*

1085 Beside the strong assumptions just discussed (no viscous flow for Weert-
1086 man models, zero fracture toughness for lubrication models), either model
1087 types rely on further simplifications to make the problem tractable. Weert-
1088 man models are conceptualised as propagating, isolated constant-volume
1089 batches of magma; Lubrication models are often implemented with simplified
1090 boundary conditions such as constant influx from below or constant pressure
1091 at the magma reservoir. In this paragraph we will discuss these assumptions
1092 and their broader implications.

1093 Three main issues are hidden behind those conceptualisations, related to
1094 magma transfer from a reservoir into a dike and within the dike tail, that
1095 have never been properly addressed. 1) What conditions allow an effective
1096 emptying of the tail (magma retained completely within the head of the
1097 propagating fracture)? 2) Under what conditions does the head separate
1098 hydraulically from the feeding magma source? 3) How does the state of
1099 magma reservoirs change during feeding?

1100 Magma is retained effectively in the head of the fracture if the volume
1101 contained in the tail is small compared to the head volume. This condition
1102 is respected if Eq. (32) holds. Therefore, if the length of the tail is smaller
1103 than the quantity L_t , the constant-volume assumption is approximately valid,
1104 and Weertman models can be applied safely. Using the parameter values
1105 above along with $\Delta\rho = 300 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$ we obtain $L_t =$
1106 $0.06, 600, 66000 \text{ m}$ for $K_c = 1, 100, 1000 \text{ MPa m}^{1/2}$, respectively. Again,
1107 we find that Weertman models require large fracture toughness values, and
1108 obtain that they can be safely applied to model dikes propagating over large
1109 distances if the fracture toughness is large, because a small portion of magma
1110 mass is lost within the tail. It is possible that the rate of loss of magma to the
1111 tail may also be compensated by volume increases caused by decompression.
1112 Volume compensation may be very effective since gas bubbles in magma grow
1113 in dimension and number during ascent, although this has never been checked
1114 quantitatively. Moreover, magma viscosity and compressibility are factors
1115 that could moderate the issue of magma loss to the tail. Lower viscosity
1116 (which can be as low as 0.1 Pa s for kimberlitic, ultrabasic, low-silica melts)
1117 means a smaller viscous pressure drop over the length of the dike. This, in
1118 turn, implies that less magma is needed to maintain the same propagation
1119 speed or shape.

1120 The issue of hydraulic connectivity between magma reservoir and dike is

1121 vastly ignored or sometimes simplified to a cylindrical channel transferring
 1122 the magma from a higher pressure reservoir to a lower pressure dike inlet. In
 1123 models where the pressure at the reservoirs and dike are solved for dynami-
 1124 cally, the shape and thickness of such a channel may have a significant effect.
 1125 We discuss how this issue has been addressed in recent models further below
 1126 (Sect. 5.10).

1127 Most lubrication theory models rely for simplicity on the assumption of
 1128 a constant pressure at the inlet. However, even if the initial volume of the
 1129 magma chamber is much larger than the total volume of magma injected in
 1130 the dike, it may be a poor approximation to consider the pressure of the cham-
 1131 ber as constant. In fact, significant pressure decreases at magma chambers
 1132 feeding dikes have been observed for lateral dike propagation events around
 1133 the world (Segall et al., 2001; Buck et al., 2006; Rivalta, 2010). The pressure
 1134 drop due to the extraction of magma from a reservoir can be calculated as
 1135 follows:

$$\frac{\Delta M/M}{\Delta p} = \frac{(\rho \Delta V + V \Delta \rho)/(\rho V)}{\Delta p} = \frac{1}{V} \frac{\Delta V}{\Delta p} + \frac{1}{\rho} \frac{\Delta \rho}{\Delta p} = \beta_c + \beta_m \quad (34)$$

1136 where p , V , β_c , are the pressure, volume and elastic compressibility of the
 1137 magma chamber and ρ , and β_m are the density and compressibility of the
 1138 magma, respectively. The compressibility of degassed basaltic magma at
 1139 crustal depths is in the range $\beta_m = 0.4\text{--}2 \times 10^{-10} \text{ Pa}^{-1}$ (Spera, 2000), while β_c
 1140 depends on the shape of the chamber and on the rigidity of the host medium
 1141 (Segall et al., 2001; Rivalta and Segall, 2008; Amoroso and Crescentini, 2009;
 1142 Rivalta, 2010). For spherical chambers and $G = 3 \text{ GPa}$ to 25 GPa , $\beta_c = 0.3\text{--}$
 1143 $3 \cdot 10^{-10} \text{ Pa}^{-1}$. This implies that extracting just 0.1% of the magma resident in
 1144 a chamber may result in a pressure drop $\Delta p = \Delta M/M \cdot 1/(\beta_c + \beta_m)$ of several
 1145 MPa or more. Hence, similar to hydraulic fracturing where the fluid influx
 1146 is mechanically controlled and the pressure varies (most often decreasing) in
 1147 response to fracture growth, the pressure at the magma chamber feeding a
 1148 dike will tend to decrease during injection.

1149 Alternatively, some models assume a constant magma influx into the
 1150 dike. The relation between source pressure and magma influx is influenced
 1151 by the force balance that drives dike propagation (Menand and Tait, 2002;
 1152 Roper and Lister, 2005). Traversa et al. (2010) showed that a finite-sized
 1153 magma chamber experiencing a pressure decrease as it feeds a dike may
 1154 lead to propagation with nearly-constant volumetric flux. However, inver-
 1155 sions from crustal deformation data indicate that real cases may show some

1156 complexity. For example, the estimated time dependent volumetric flux
1157 into the 1997 and 2007 dike at Kilauea (Segall et al., 2001, Fig. 4) and
1158 (Montgomery-Brown et al., 2011, Fig. 7) was maximum during the first
1159 hours of propagation and then decreased with time. Models assuming a
1160 constant total mass for coupled magma chamber(s) and dike(s) systems have
1161 obtained an exponentially decreasing volumetric influx into the dike (Rivalta,
1162 2010). That model however does not include fracturing. A further exten-
1163 sion of the model including a time-dependent K_c might help us interpreting
1164 observations during dike arrest.

1165 4.4.4. *Where the Two Schools Reconcile*

1166 Recently developed models provide a path to reconciliation between the
1167 Weertman and lubrication classes of dike model. They do this by relaxing
1168 the assumptions that create key differences. Two papers, one from each
1169 side of the debate (Dahm, 2000b; Roper and Lister, 2007), are particularly
1170 relevant to understand how the two approaches reconcile and in what cases
1171 the end-member approaches are valid.

1172 Dahm (2000b), representing the Weertman school, developed a numerical
1173 boundary element model (see also Sec. 4.5 below for other boundary element
1174 studies of dike propagation) for constant-mass, buoyancy-driven, fluid-filled
1175 fractures (hence in principle Weertman fractures, with no magma influx from
1176 below). However, he also included 2D magmatic flow within the crack and the
1177 consequent viscous stress drop on the crack plane. In particular, the model
1178 is of a Hagen-Poiseuille flow through a piecewise constant-width fracture (h
1179 is discontinuous along the crack) with a moving boundary (Fig. 11a). The
1180 sophistication of the model for the fluid flow is somewhere between the con-
1181 stant pressure gradient normally considered in extended Weertman models
1182 (Sec. 4.2.2) and solving the equations governing the flow, as in the lubrica-
1183 tion theory approach. If the fracture propagates with constant mass (if none
1184 of the fluid is left in the channel behind the fracture) the pressure gradient
1185 is singular at the link between head and the tail of the crack, as noticed be-
1186 fore by Spence and Turcotte (1990); Nakashima (1993); Rubin (1995). This
1187 occurs because buoyancy-propelled ascent of a magma pocket requires the
1188 magma-filled fracture to retain effectively all the enclosed magma during
1189 propagation; in contrast, lubrication theory states that in a finite time inter-
1190 val, it is impossible to fully expel viscous fluid out of a closing gap. Dahm
1191 (2000b) addresses this problem by requiring that a small quantity of fluid
1192 is left in the channel during propagation. In this way, the viscous pressure

1193 gradient is no longer singular, it is just very large at the tail (Fig. 11b). This
 1194 suggests that most of the energy during propagation is dissipated within the
 1195 tail region or, in other words, that the constriction at the tail controls the
 1196 velocity of the fracture, consistent with a result from the lubrication the-
 1197 ory. Dahm (2000b) links the numerical instability at the tail to a physical
 1198 instability observed during experimental injections in gelatin, where the tail
 1199 of air-filled cracks is observed to shut closed in jerky movements (see movie at
 1200 <http://www.youtube.com/watch?v=hHqUwHRvilU>, Rivalta et al. (2013a)). In
 1201 the approach by Dahm (2000b), it is not possible to predict from theory how
 1202 much fluid gets lost in the tail.

1203 Roper and Lister (2007), representing the lubrication school, develop a
 1204 lubrication-based model with a finite fracture toughness and solve the prob-
 1205 lem for a crack containing a constant volume of fluid (Fig. 12c). They find
 1206 that viscous effects still control how quickly the cracks propagate. They also
 1207 find that a large fracture toughness implies a teardrop-shaped crack, whose
 1208 length and width scale with $k^{2/3}$ and $k^{4/3}$, respectively, fed by a narrow tail.
 1209 They define

$$k = K_c/K^* = \left(\frac{2K_c^4}{3\mu Q m^3} \right)^{1/4}, \quad (35)$$

1210 where Q is the magma influx into the dike and $m = G/(1 - \nu)$. They show
 1211 that the head and tail are connected through a constriction of length and
 1212 width scaling with $k^{-2/5}$ and $k^{-4/15}$, respectively. These results obtained
 1213 through modeling and scaling analysis match very well with the geometry
 1214 of a Weertman fracture in general and with the results by Dahm (2000b) in
 1215 particular (compare Figs. 11 and 12).

1216 Both models were developed with the aim of extending the applicability
 1217 of the original approaches. Furthermore, Dahm (2000b) applied his model to
 1218 fluid-filled fractures in nature. He estimated a velocity of $\approx 0.1 \text{ m yr}^{-1}$ for
 1219 water-filled fractures in pressurized sediments and 0.1 m s^{-1} for magma-filled
 1220 dikes in the upper mantle (Fig. 13). The model also predicts that water- and
 1221 oil-filled fractures in sediments have a thickness of the order of 10^{-5} m and
 1222 begin to ascend spontaneously when their length exceeds about 1.2 m. For
 1223 $K_c \approx 1 \text{ GPa m}^{-1/2}$, magma-filled fractures in the upper mantle would start
 1224 to ascend spontaneously when they accumulate enough volume to reach a
 1225 length of 5 km. Their average thickness during propagation of such a dike
 1226 would be about 0.3 m. Dahm (2000b) includes a thorough comparison of
 1227 his estimates with previous models. Additionally, he finds good agreement

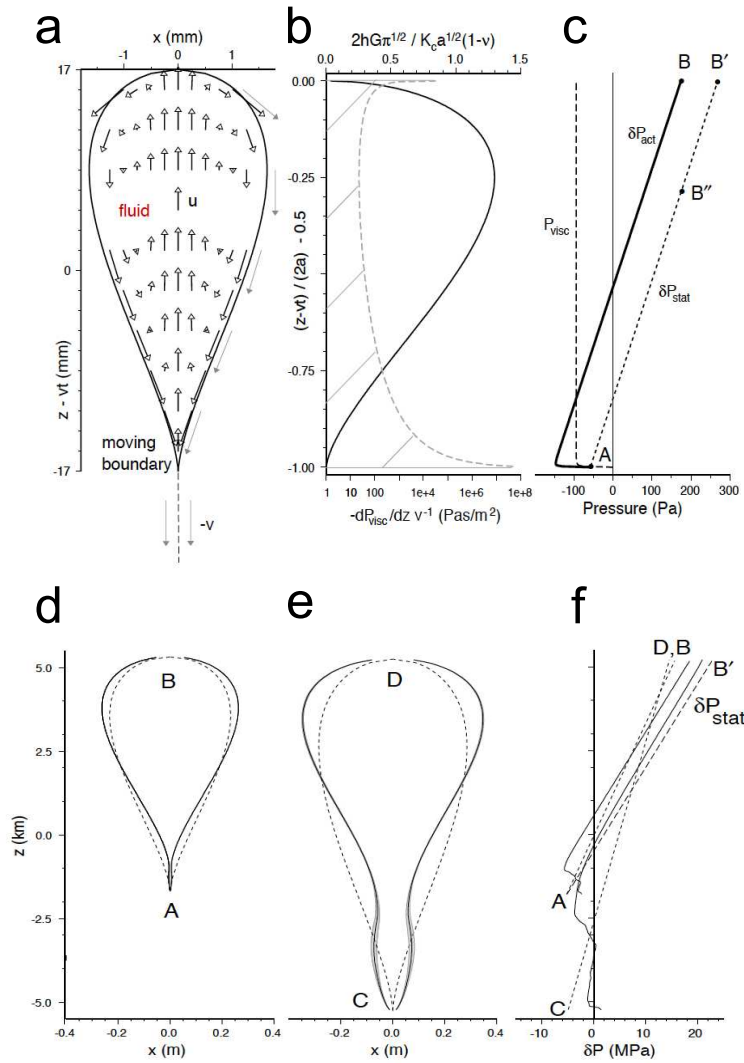


Figure 11: Fig. 3 and 6 from Dahm (2000b), with permission. a) Crack cross-section and velocity field of the fluid within the fracture. b) Solid line: crack half-width (solid line), dashed line: viscous pressure gradient. c) Total overpressure in the fracture, sum of the static overpressure and viscous pressure drop. (d) Average shapes (solid lines) derived from the first 500 iterations of the model. The dyke lengths are 1.3 (left) and 1.9 (right) times the critical length for fracture propagation. Dashed line: initial opening of the fracture at iteration 1. (e) Average (solid lines) and initial (dashed) overpressure in the fluid. The hypothetical static overpressure is indicated for the shorter fracture by a long-dashed line (A to B').

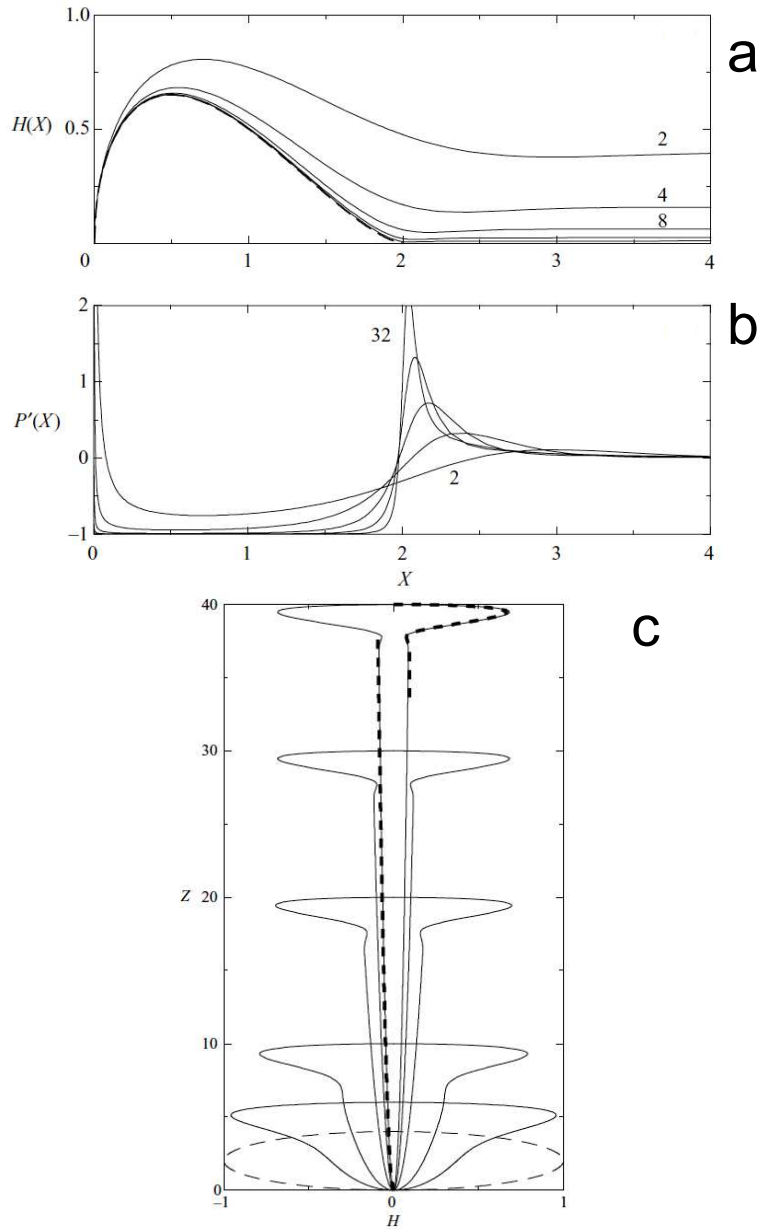


Figure 12: a) Normalized opening of fracture for different ratios between fracture toughness and viscous resistance ($k=2, 4, 8, 16$ and 32). b) Normalized pressure gradient. c) Evolution of the scaled crack width from an initial shape (long-dashed) shown at 6, 10, 20, 30 and 40 iterations. Figs. 3 and 8 from Roper and Lister (2007), with permission.

1228 with some field data from both water-filled fractures in sediments and dikes
1229 in the upper mantle. A wider comparison with estimates from nature would
1230 be desirable, especially now that the physical understanding of propagation
1231 of fluid-filled fractures in brittle materials and the role played by magma and
1232 rock parameters has advanced, and that industrial operations can provide a
1233 much wider dataset for a comparison.

1234 Roper and Lister (2007) apply their model to discuss the validity and ap-
1235 plicability of analog experiments using gelatin (Sect. 3.2). They observe that
1236 laboratory experiments are in the regime $K \gg 1$ (fracture toughness dom-
1237 inated). They compare their theoretical results to experimental studies by
1238 Takada (1990) and Heimpel and Olson (1994). As they notice, in laboratory
1239 experiments using large viscosity fluids, head-and-tail structures are clearly
1240 visible. The viscous control of propagation rate can be deduced from the
1241 increasing propagation rates in a given gel with decreasing-viscosity fluids.
1242 They also discuss the fracture criterion in gelatin, observing that there is
1243 no analytical solution for a three-dimensional Weertman pulse (with stress
1244 intensity equal to K_c along its upper boundary and 0 on the point of closing
1245 along its lower boundary). They observe that a numerical solution would be
1246 expected to have the same scalings as their equation (6.5) with vertical and
1247 lateral extent $O((K_c/\Delta\rho g)^{2/3})$ and width $O((K_c/\Delta\rho g)^{1/3}K_c/m)$, thus giving
1248 a critical volume $V_c = O((K_c/\Delta\rho g)^{5/3}K_c/m)$. They observe that 1) a buoy-
1249 ant crack with volume less than V_0 should not propagate and, 2) a crack with
1250 low viscosity and large toughness should propagate with a head of approxi-
1251 mately the fixed shape and volume of such a pulse. They seem not to find
1252 confirmation of this in the data by Takada (1990) and Heimpel and Olson
1253 (1994) and conclude that the failure mechanisms in gelatin are significantly
1254 different from those of more rigid brittle solids such as ceramics or rock.
1255 In particular, they suggest a rate-dependent fracture resistance. While the
1256 correctness of points 1) and 2) for gelatin experiments have been indeed con-
1257 firmed in numerous other experiments (see Sect. 3.2), further investigations of
1258 scaling relationships and fracture processes in gelatin and comparison to the-
1259 ory would help our understanding of the applicability of laboratory analogs
1260 to water- and magma-filled fractures in the Earth's crust and mantle.

1261 The studies by Dahm (2000b) and Roper and Lister (2007) testify how
1262 predictions from the Weertman and the lubrication schools converge when
1263 restrictive assumptions are relaxed. Both approaches remain limited in that
1264 they consider only dikes propagating straight and they lack flexibility in in-
1265 cluding external effects because of the large computational effort necessary

1266 to obtain a very detailed modeling of the fluid motion in the fracture. For
1267 the purpose of addressing questions regarding the interaction of dikes with
1268 external factors and studying the behavior of dikes in different tectonic set-
1269 tings, numerical models simplifying strongly the motion of fluid within the
1270 fracture have instead proved very flexible, as described in the next section.

1271 *4.5. Numerical Models*

1272 As presented above, semi-analytical solutions from the classical presen-
1273 tations by Weertman (1971a,b) and lubrication models (Spence et al., 1987;
1274 Lister, 1990a) are appropriate in the limits of: 1) negligible fluid viscosity
1275 or fracture toughness, respectively, and, for both types of models, 2) sim-
1276 ple geometries typically limited to homogeneous, infinite media subjected
1277 to a relatively simple (i.e. locally uniform) lithostatic stress. Additional
1278 contributions to both confining and internal pressure due, for example, to
1279 gradients in the topography or bubble nucleation within the dike will lead to
1280 more complicated geometries and propagation behavior and cannot be easily
1281 treated with the semi-analytical approaches. Furthermore, a growing con-
1282 sensus in the debate on effective fracture toughness (Sect. 4.4) suggests that
1283 both viscous flow and rock fracture toughness must be included to produce
1284 a broadly applicable dike model. In this context, two hybrid styles of models
1285 are particularly promising: 1) Weertman-type models that relax the constant
1286 volume assumption and include fluid flow, and 2) Lubrication-theory based
1287 models that allow for a non-zero fracture toughness. These “mixed” models
1288 require numerical solutions even for simple geometries.

1289 Probably the most common numerical approach to solving dike propaga-
1290 tion models is based on the boundary element method (BEM), see Crouch and Starfield
1291 (1983). This method is designed to incorporate the coupling between magma
1292 pressure and rock deformation; it requires discretization of the dike bound-
1293 ary and any other non-analytical boundary in the medium. Such models are
1294 built by taking advantage of analytical solutions for elementary dislocations;
1295 these are appropriately superposed to represent a pressurized, opening crack.
1296 The most important advantage of this method relative to others that require
1297 meshing of the entire domain, such as the classical Finite Element Method,
1298 is that re-meshing as the dike propagates is relatively simple and computa-
1299 tionally inexpensive. This is because re-meshing simply requires that new
1300 elements are added at the dike tip.

1301 Early dike propagation models of this type include Dahm (2000a) and
1302 Muller et al. (2001). Both methods are used to derive dike trajectories re-

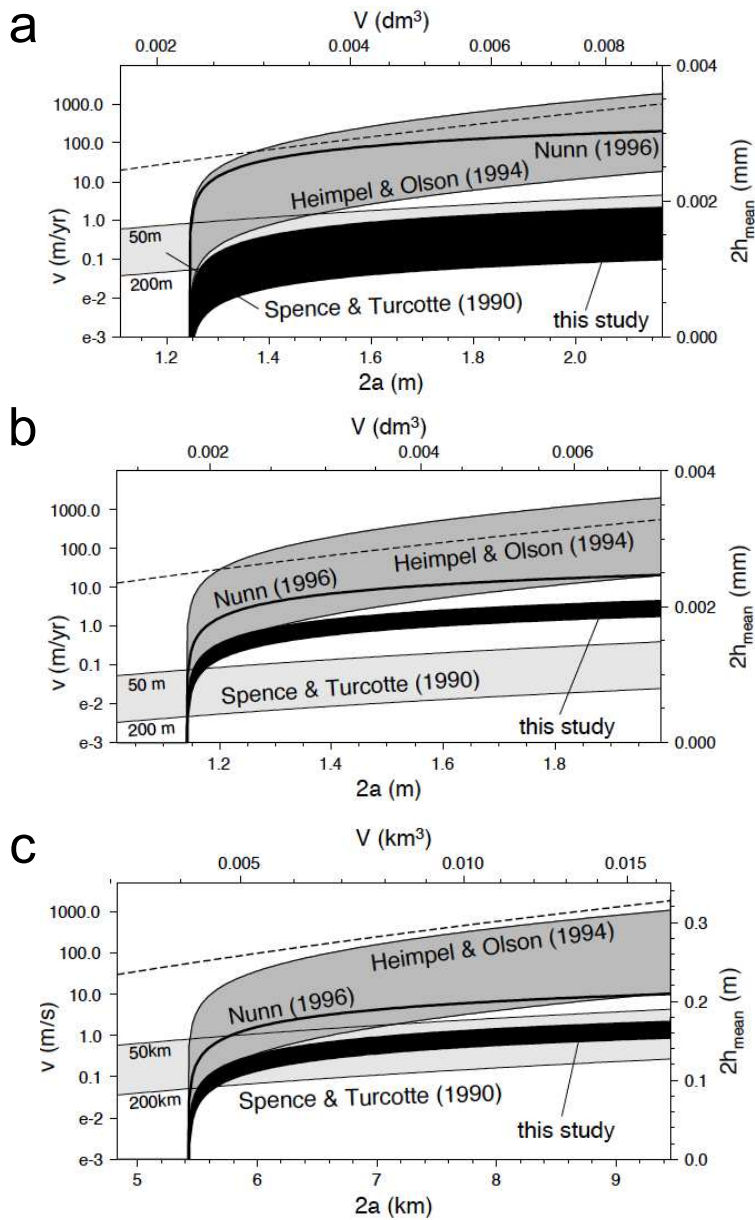


Figure 13: a) Predicted propagation velocities of water-filled fractures in pressurized sediments by Dahm (2000b) (see paper for details). Model estimates are compared with results by Nunn (1996), Spence and Turcotte (1990) (using propagation distances between 50 and 200 m) and Heimpel and Olson (1994) (assuming a yield strength between 100 and 1000 MPa). The average thickness is plotted as a dashed line. Fig. 8 from Dahm (2000b), with permission. b) Predicted propagation velocities for oil-filled fractures in pressurized sediments. Fig. 9 from Dahm (2000b), with permission. c) Predicted propagation velocities for magma-filled fractures in upper mantle rocks. Fig. 10 from Dahm (2000b), with permission.

1303 sulting from the interaction of dikes with heterogeneous stress fields. Under
1304 this method, several interacting tensile and dip-slip dislocations are combined
1305 to model an inclined dike; boundary conditions on the dike plane are pre-
1306 scribed for the dike overpressure and for a total release of shear stress. The
1307 models differ in how the dike trajectories are selected: Dahm (2000a) uses an
1308 energy-release criterion in which the energy release is calculated for virtual
1309 elongations in several directions and the direction leading to the maximum
1310 release is chosen (Nuismer, 1975); in contrast, Muller et al. (2001) select
1311 the direction minimising the shear stress, because this will be perpendic-
1312 ular to the direction of maximum hoop stress around the tip of the dike
1313 (Erdogan and Sih, 1963). Only if the dike tip is highly resolved on the nu-
1314 merical mesh, the minimum-shear stress criterion gives results very similar
1315 to the maximum strain energy release (this because the former is based on
1316 the shape of the dike tip, which must be calculated very precisely). The
1317 energy release criterion works for coarser discretizations because it takes into
1318 account the shape of the entire fracture and is less sensitive to details in the
1319 tip. In both models, the dike aperture at the trailing dislocation is checked
1320 for negative values, which can occur due to closing associated with propaga-
1321 tion of a confined pocket of fluid. If the aperture is negative, the dislocation
1322 at the tail is deleted or, alternatively, the aperture is set to zero and the
1323 linear system for the set of dislocations is re-solved to find the equilibrium
1324 configuration. This approach allows for a quasi-static model of dike propa-
1325 gation. External stress fields can be easily introduced in such a numerical
1326 scheme (see Sec. 5).

1327 The numerical models provide a framework for other generalizations. For
1328 example, the model by Dahm (2000a) includes compressibility of the magma
1329 and imposes a conservation of mass rather than of volume. Maccaferri et al.
1330 (2010) and Maccaferri et al. (2011) improve on models of Dahm (2000a) by
1331 including gravitational potential energy in the energy balance equation (see
1332 also Sec. 5 below). However, current models do not yet provide the ability to
1333 consider dike curving and growth with a finite fracture toughness and fluid
1334 viscosity at the same time. Recent models by Maccaferri et al. (2010) and
1335 Maccaferri et al. (2011) do not include fluid viscosity, while the semi-implicit
1336 models by Taisne and Tait (2009) and do not include dike curving.

1337 In a field that is related to modeling of dike propagation, the hydraulic
1338 fracturing techniques that have advanced unconventional oil and gas produc-
1339 tion have led to a proliferation of novel numerical models (see Sec. 6). These
1340 consider the growth of fluid-filled, pressure-driven cracks and, up to the point

1341 where buoyancy forces or the cooling/solidification of the magma are invoked,
1342 they are essentially identical to elasto-hydrodynamic dike propagation mod-
1343 els. Advances in modeling of hydrofracture represent a resource for new
1344 and more powerful approaches to modeling dike propagation. The power
1345 of Boundary Element and Boundary Integral Methods have been harnessed
1346 to simulate hydraulic fracture growth since the 1980s (e.g. A. H.-D. Cheng,
1347 1984; Vandamme and Curran, 1989). However, there are some significant
1348 challenges. These include devising efficient computational schemes so that so-
1349 lutions can be obtained in practically-relevant time frames, bridging the gap
1350 between the mostly 2D simulations and the mostly 3D physical phenomena,
1351 and accounting for complexity that is ubiquitous in geological environments.
1352 These are partially overcome by:

- 1353 • Design-motivated modeling of network growth of hydraulic fractures
1354 by integrating so-called pseudo-three-dimensional (P3D) modeling with
1355 Discrete Fracture Networks (Meyer and Bazan, 2011; Kresse et al., 2013).
1356 This approach uses a drastically-simplified, local elasticity relationship
1357 that is valid when the hydraulic fracture is very long relative to its
1358 height, i.e. blade-like in shape. This simplification induces orders of
1359 magnitude reduction in computational time but the applicability is
1360 limited to relatively blade-like hydraulic fractures.
- 1361 • Boundary Element Models that consider hydraulic fracture interaction
1362 with pre-existing fractures (Zhang et al., 2009; Dahi-Taleghani and Olson,
1363 2011). These allow consideration of one of the most important sources
1364 of complexity, but are limited to relatively few natural fracture inter-
1365 actions in a two-dimensional framework.
- 1366 • Simulators that overcome the limitations of P3D and 2D models by ac-
1367 counting for planar hydraulic fracture growth in a 3D medium (Peirce and Detournay,
1368 2008). The computational cost is partially offset by implicit time step-
1369 ping, thus allowing coarser discretization of time, and embedding ap-
1370 propriate asymptotic behavior of the near-tip opening, thus allowing
1371 coarser discretization of space.
- 1372 • The eXtended Finite Element Method (XFEM), that overcomes the
1373 need to re-mesh a traditional FEM model as the hydraulic fracture
1374 grows. These models are capable of simulating growth in complex ge-
1375 ological settings and are, in principle, extensible to three dimensions

1376 (Lecampion, 2009; Dahi-Taleghani and Olson, 2011; Gordeliy and Peirce,
1377 2013; Chen, 2013; Weber et al., 2013; Ru et al., 2013). In some cases,
1378 XFEM includes specific functions to enrich the finite element basis that
1379 ensure accurate and efficient computation when under strong fluid-solid
1380 coupling (Lecampion, 2009; Gordeliy and Peirce, 2013; Chen, 2013).

- 1381 • Damage mechanics-FEM based models for three-dimensional growth of
1382 hydraulic fractures including the impact of stochastically heterogeneous
1383 rocks (Wangen, 2011; Li et al., 2012; Guest and Settari, 2012).
- 1384 • Distinct Element Models (DEM) capable of modeling highly complex,
1385 three-dimensional growth patterns including interaction with Discrete
1386 Fracture Networks (Damjanac et al., 2010; Nagel et al., 2013). The
1387 main limitation of these models is the need for very fine discretization
1388 in order to accurately match benchmarks (an A. P. Peirce et al., 2013).

1389 While all of these approaches provide steps toward 3D modeling, the
1390 overall challenge is that the most thoroughly benchmarked models are 2D
1391 or 3D models that constrain growth to one or more planes. This is very
1392 limiting because dikes have been inferred to bend and twist at some volca-
1393 noes (Bagnardi et al., 2013; Xu and Jónsson, 2014) while adjusting from one
1394 stress domain to the next, so that a fully 3D approach would be required to
1395 predict the dynamics of such dikes. Currently, it is out of reach to model
1396 this behavior with workable computational times and with benchmarking
1397 that demonstrates suitable confidence that the model is providing a correct
1398 solution to the basic, underlying mechanical problem.

1399 **5. Results including interaction of dikes with the surroundings**

1400 The challenges associated with modelling dike propagation go beyond the
1401 issue of the relative importance of magma viscosity and rock fracture tough-
1402 ness. Dikes almost invariably grow in environments that draw into question
1403 the simplifications that make the modelling problems tractable. Here we re-
1404 view a range of complications associated with the interaction between dikes
1405 and their surroundings, how they have been addressed, and the associated
1406 progress in understanding the dynamics of dikes.

1407 *5.1. External stress field*

1408 As described above (Sect. 2), the trajectory of a dike is controlled by the
1409 orientation of the principal stresses. Spatial variation of that orientation will

1410 force a propagating dike to change its direction. However, drastic turns of
1411 dikes during their propagation (e.g. from dike to sill orientation) are not in-
1412 stantaneous but occur over a finite distance. This arises because dikes do not
1413 propagate in perfect alignment with the stress field, but rather are continu-
1414 ously adjusting toward alignment. This is particularly true when the stress
1415 field is heterogeneous or the dike driving pressure is very high (Dahm, 2000a;
1416 Watanabe et al., 2002). Menand et al. (2010) study experimentally the char-
1417 acteristic length scales over which a horizontal compressive stress field exerts
1418 a steering effect on ascending, air-filled cracks in gelatin. Their dimensional
1419 analysis shows that this distance varies exponentially with the ratio of crack
1420 effective buoyancy to horizontal compressive stress. Up-scaled to natural
1421 systems, these results imply a spatial scale of a few hundreds meters to a few
1422 kilometers for a dike-to-sill rotation to occur, so that this mechanism should
1423 be important for crustal-scale processes. Dike bending and twisting has been
1424 inferred to occur also at the volcano edifice scale Bonaccorso et al. (2010);
1425 Bagnardi et al. (2013); Xu and Jónsson (2014); material heterogeneities may
1426 however also play a dominant role (see Sec. 5.5 below).

1427 Numerical models (Dahm, 2000a; Maccaferri et al., 2010, 2011) return
1428 the general result that the rotation of a dike in a heterogeneous stress field
1429 occurs over spatial scales that are of the same order of magnitude as the
1430 dimension of the crack. Maccaferri et al. (2011) analyze different scenar-
1431 ios for an external stress field: compression, extension and the load of a
1432 volcanic edifice (Sec. 5.2). They find for example that a kilometric spa-
1433 tial scale is needed to turn a dike into a sill or a sill into a vertical dike.
1434 The spatial scale is influenced by the ratio between overpressure within the
1435 dike (which can be estimated with the equation $\Delta p = \rho g L/4$) and com-
1436 pressive/extensional/loading/unloading stresses, as shown experimentally by
1437 Watanabe et al. (2002) (Sections 5.2 and 5.3).

1438 *5.2. Load of a volcanic edifice*

1439 The load of a volcanic edifice modifies the local stress field and therefore
1440 exerts a control on the trajectory of dikes ascending nearby. Gudmundsson
1441 (2002), for example, reports observations of inclined sheets and dikes dipping
1442 toward central volcanoes. The theoretical problem of how ascending dikes
1443 are influenced by the stress field associated with a volcanic edifice has been
1444 studied with several approaches.

1445 Dahm (2000a) used a boundary element approach (Sec. 4.5) to model
1446 expected trajectories in such a stress field. He finds that gravitational loads

1447 attract ascending dikes, which tend to focus at the base of the volcano, erupt
1448 and reinforce the process by piling up more material in the same location,
1449 adding to the overburden. Also, he highlights how dikes are not driven ex-
1450 clusively by the principal stresses, but also by the gradient of tectonic/local
1451 stresses and magma buoyancy, and the trajectories might be more compli-
1452 cated than what is simply suggested by the principal stresses.

1453 Maccaferri et al. (2011) extended the results of Dahm (2000a) by calculat-
1454 ing the trajectories of dikes ascending at various initial angles, directly below
1455 the volcanic edifice or offset from it. No external stress field was added in
1456 this case. Most of the trajectories stream into the base of the volcanic edifice,
1457 but occasionally a dike may escape from the trap and erupt at a consider-
1458 able distance from it (Fig. 14F1–4). If the buoyancy is not sufficient (if the
1459 dikes are not very large), the dikes will stop just below the base of the load
1460 (Fig. 14F1), extending laterally and erupting, or creating/feeding a shallow
1461 crustal magma reservoir.

1462 Muller et al. (2001) carried out laboratory experiments to investigate the
1463 effect on analog dikes of a mass lying on top of a brittle gelatin block
1464 (Fig. 14D). Watanabe et al. (2002) performed similar analog experiments
1465 with gelatin. They calculated how much dike trajectories are deflected as a
1466 function of the ratio between dike driving pressure and gravitational loading.
1467 They also noticed that ascending, fluid-filled cracks decelerate in proximity
1468 of the load and eventually stop, with the deceleration proportional to the size
1469 of the load. Bonaccorso et al. (2010) observed that the 2001 dike at Etna
1470 tilted towards the volcano summit during ascent, consistent with the theoret-
1471 ical and experimental predictions just outlined (Fig. 14E). They used results
1472 from analog modeling by Watanabe et al. (2002) to infer the overpressure of
1473 the dike and the overpressure at the magma chamber at breakout.

1474 Kervyn et al. (2009) used air-filled cracks in gelatin and golden syrup in
1475 sand-plaster to explore how volcano load controls magma ascent and vent
1476 locations. They found rising dikes approaching the conic stress field are
1477 arrested by the compressive stress of the load and begin extending laterally.
1478 Pinel and Jaupart (2004) studied the influence of volcano loads on the lateral
1479 extension of shallow dikes and considered how this influences the location of
1480 eruptive vents (Sec. 5.12).

1481 Taken together, these results also indicate that the stress field caused by
1482 gravitational loading may be the reasons why large volcanic edifices such as
1483 Etna, for example, develop a stable magma reservoir (which can be detected
1484 by measurements of crustal deformation).

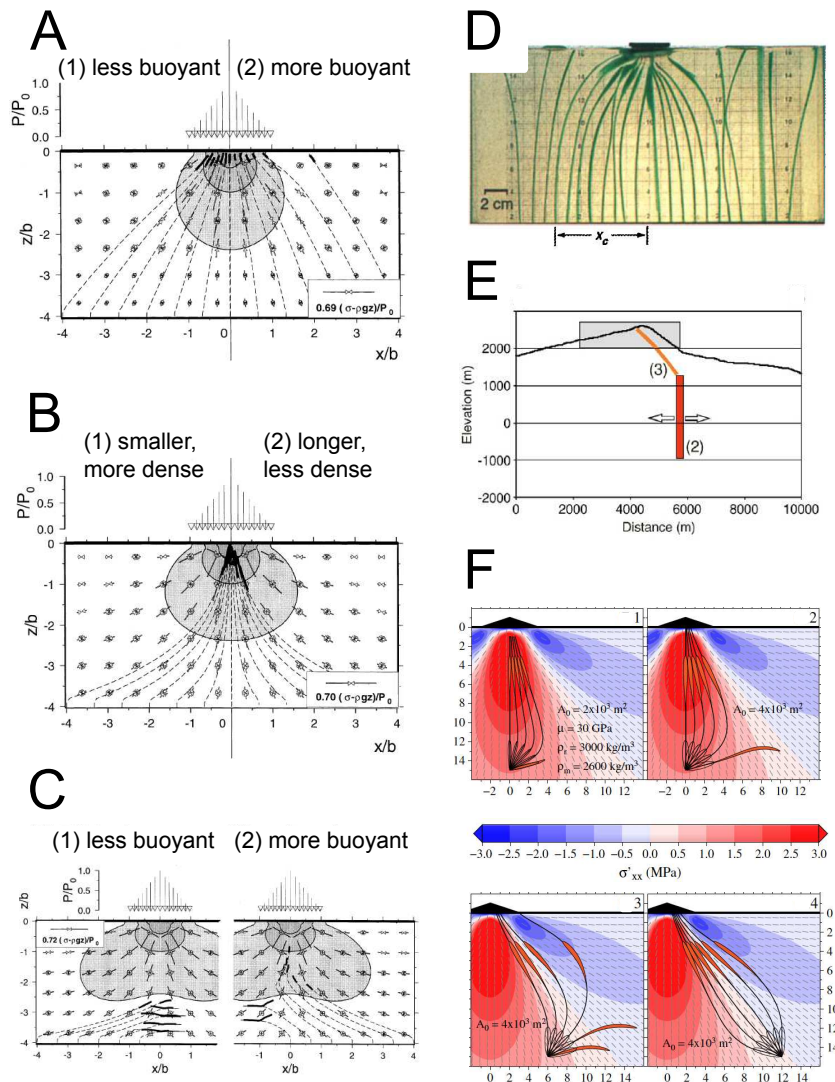


Figure 14: Experimental and numerical models of dike ascent in a stress field modified by loading of volcanic edifices. A) Boundary element model of dike trajectories under the influence of a triangle-shaped volcanic load. The principal stresses rotate and promote ascent towards the volcano. The individual dikes are not interacting with each other. Less buoyant dikes follow the trajectories closely (1), more buoyant dikes need more space to rotate (2). Fig. 7 from Dahm (2000a), with permission. B) Same as A), but with an additional tectonic compressive stress equal to $P_L/12$, where P_L is the pressure caused by the volcanic edifice. Fig. 8 from Dahm (2000a), with permission. C) If the compressive tectonic stress is higher ($P_L/6$) some of the trajectories turn horizontal, and sill emplacement is favoured. Fig. 9 from Dahm (2000a), with permission. D) Trajectories of analog dikes in gelatin under the influence of a load applied to the surface. Fig. 1a from Muller et al. (2001), with permission. E) Dike rotation observed for the 2001 dike intrusion at Etna. The rotation was interpreted as originating from the load of the volcano edifice and summit. From Bonaccorso et al. (2010), with permission. F) Influence of a triangle-shaped volcano load on the trajectories of dikes departing with different orientations from an axial (top) or a off-axis (bottom) magma chamber. The compressive stress induces dike arrest at depth, promoting the creation of a magma chamber. Fig. 5 from Maccaferri et al. (2011), with permission.

1485 5.3. Unloading

1486 Seasonal loading/unloading conditions may occur at high latitudes or
1487 altitudes due to cyclic icecap melting and formation. Albino et al. (2010)
1488 studies how this causes variations in magma pressure and therefore influ-
1489 ences the likelihood of dike initiation. They find that unloading favors dike
1490 nucleation and the occurrence of seismicity.

1491 Loading due to a topographic weight (Sec. 5.2) causes a rotation of the
1492 principal stresses in the crust, steering ascending dikes into the base of a vol-
1493 cano. Similarly, unloading due to mass removal influences dike propagation,
1494 and has the opposite effect of defocusing ascending dikes to the side of a re-
1495 gion which has been unloaded. Significant unloading may occur over various
1496 time scales during volcano flank collapse, deglaciation, or crustal thinning.
1497 Hooper et al. (2011) calculated the unloading effect caused by icecap melt-
1498 ing at Kverkfjöll, Iceland. They found that the orientation of the ascending
1499 dike, inferred from inversion of InSAR data, is not consistent with the tec-
1500 tonic stress state modified by the current icecap melting. However, it would
1501 would fit with the more intense modification induced by icecap melting after
1502 deglaciation (Fig. 15A). They inferred that deglaciation modifies the capacity
1503 to store magma in the crust.

1504 Corbi et al. (2014) developed an axially symmetric finite element model
1505 for the unloading effect due to a massive withdrawal of magma from a volcano
1506 reservoir and the associated caldera formation. The unloading was modeled
1507 as a decompression over the caldera area, amounting to the missing topog-
1508 raphy from the most recent caldera collapse, superposed to an isotropically
1509 stresses volcano. The latter assumption is justified by observing that volcanic
1510 edifices form over long time scales; multiple dike intrusions and anelastic re-
1511 lease of deviatoric stresses compensate over such long time scales any devi-
1512 ations from an isotropic state of stress. Therefore, only recent sub-surface
1513 mass changes (caldera collapses) would be uncompensated for. The model
1514 was applied to Fernandina volcano, Galàpagos: under those assumptions, σ_3
1515 is oriented vertically below the caldera, favoring the creation of horizontal
1516 sill-shaped magma chambers. On the flanks, close to the surface, σ_3 is out of
1517 plane: the pattern of the principal stresses is consistent with the bending and
1518 twisting of the recent dikes inferred from crustal deformation data and from
1519 the pattern of the surface fissures at Fernandina (Bagnardi et al., 2013).

1520 Maccaferri et al. (2014) studied the trajectories of dikes in rift environ-
1521 nments by coupling the gravitational unloading due to crustal thinning and
1522 the creation of a topographic depression with an extensional stress field (see

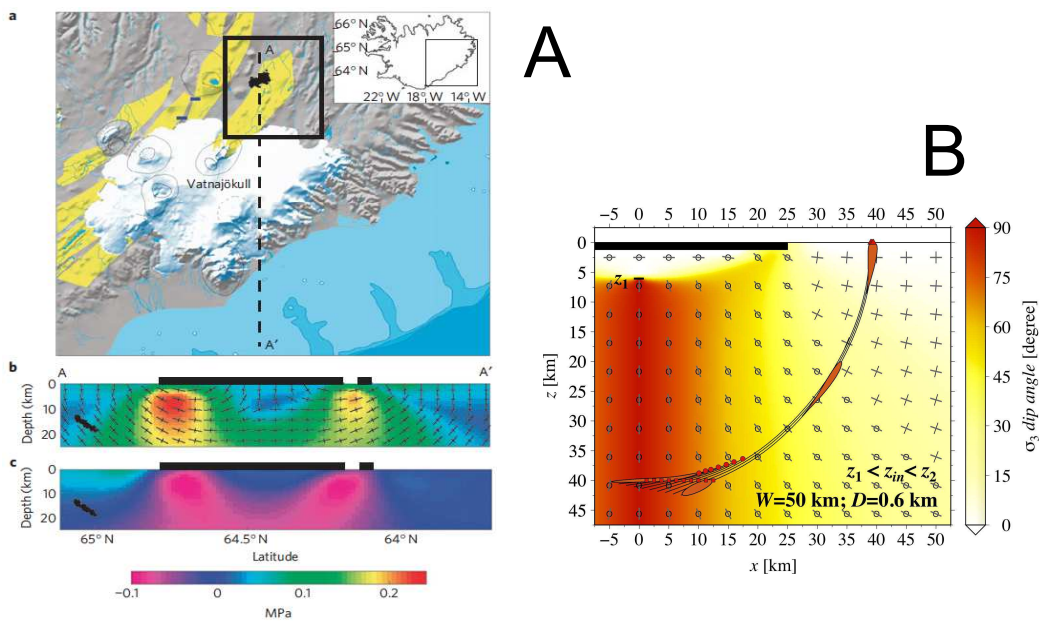


Figure 15: Numerical models of diagonal dike ascent in a stress field caused by unloading. A) The unloading at is caused by icecap melting during the last deglaciation. From Hooper et al. (2011), with permission. B) The stress field is the sum of tectonic extension and unloading due to crustal thinning. From Maccaferri et al. (2014), with permission.

1523 also Sect. 5.4). The unloading resulting from crustal thinning induces decom-
1524 pression melting in the mantle, so that magma will pond at the crust-mantle
1525 boundary and release magma-filled dikes. Consistent with the focusing effect
1526 that is obtained by studies on gravitational loading, Maccaferri et al. (2014)
1527 found that the principal stresses in the crust are rotated by the effect of the
1528 unloading forces, and that ascending dikes will follow diagonal trajectories
1529 that steer them away from the rift axis towards the shoulders of the rift.
1530 This model is applied to explain the distribution of volcanism in rifts and
1531 the existence of off-rift volcanoes, offset of tens of kilometers with respect to
1532 the source of volcanism below the rift (Fig. 15B).

1533 *5.4. Extensional and compressional tectonics*

1534 Great potential lies in applications of dike modeling to magmatic tectonic
1535 environments. These models open the possibility to reveal the mechanisms
1536 of formation of large scale volcanotectonic features, such as the morphology
1537 of slow or fast spreading ridges, rifts, volcano chains in subduction zones.

1538 Kühn and Dahm (2004) employ a viscoelastic version of the model by
1539 Dahm (2000a) to study dike ascent at mid-ocean ridges. Their model in-
1540 cludes the passive motion of the mantle through a 2D isothermal corner
1541 flow. They conclude that the observed focussing of melt beneath mid-ocean
1542 spreading axes cannot be explained by corner flow models and additional
1543 mechanisms are needed, such as large magma reservoirs or permeability bar-
1544 riers. Kühn and Dahm (2008) includes dike-dike interaction (Sec. 5.7) to
1545 study the formation of shallow magma reservoirs at fast or slow spreading
1546 MOR.

1547 Choi and Buck (2010) discuss the influence of upper mantle viscosity on
1548 the topography profile of fast-spreading mid ocean ridges. They develop a nu-
1549 merical model based on Qin and Buck (2008) including mechanical coupling
1550 between tectonic extension and diking. The model has two layers, a crust
1551 with viscosity evolving with time overlaying a mantle with constant viscos-
1552 ity. Amagmatic periods, where extension is loading the system, are modeled
1553 through a finite differences scheme. They are punctuated by sudden dike
1554 intrusions, modeled by means of a BEM code, where the vertical extent of
1555 the dikes is optimized to compensate for the residual tectonic stretching (dif-
1556 ference between residual stress from the last diking period cumulated with
1557 the extension added during the amagmatic period). Choi and Buck (2010)
1558 find that the topography profile has a strong dependence on the viscosity of
1559 the mantle, with an axis high or a valley forming for low- or high-viscosity

1560 mantle rocks, respectively. Moreover, very high viscosity below mid-ocean
1561 ridges could lead to dikes that intrude into the mantle.

1562 Parsons et al. (1992) focuses on the paradox of magma ponding and hor-
1563 izontal intrusion of basaltic magma at various depths, common in various
1564 tectonic environments, including extensional ones where vertical ascent is
1565 theoretically expected to dominate. The paradox is that the stress conditions
1566 favoring horizontal intrusions (σ_3 vertical) are expected to block the opening
1567 of vertical feeder conduits necessary for their formation. Parsons et al. (1992)
1568 discuss a number of mechanisms, mainly of rheological nature (alternation
1569 between very rigid to viscoelastic layers, density layering) but also the effect
1570 of previous intrusions on the next: many vertical intrusions compensate ex-
1571 tension and may cause stress rotations. This may be behind dikes turning
1572 into sills.

1573 Maccaferri et al. (2014) offers an alternative explanation for the deep hor-
1574 izontal sheet intrusions found in extensional settings. The decompression
1575 caused by the decrease of weight on the lower crustal sheets and mantle due
1576 to crustal thinning may be responsible for a vertical σ_3 and therefore favor
1577 horizontal intrusions. Vertical feeder dikes may then be driven, as suggested
1578 by Parsons et al. (1992), by rheological differences between layers or by the
1579 local stress field due to the pressurization of the horizontal sills: locally σ_3
1580 may become horizontal again, leading to nucleation of a vertical dike, that
1581 would turn as a sill as soon as its tip reaches an area outside of the influence
1582 of the sill-induced stresses.

1583 Some studies consider the effect of extensional or compressional tectonics
1584 coupled with loading/unloading due to modifications of the mass distribution
1585 on the surface, for example crustal thickening or thinning. Dahm (2000a)
1586 includes a compressive tectonic stress in addition to the stress caused by the
1587 load of a volcanic edifice. The dike trajectories become closer to each other,
1588 and for a particularly intense compressive stress, the dikes turn into sills and
1589 build a system of stacked intrusions that may generate a stratified magma
1590 reservoir (Fig. 14B and C). Muller et al. (2001) also study the trajectories of
1591 dikes in the stress field of a volcanic load, with application to intervolcanic
1592 spacing in the Cascade volcano province.

1593 *5.5. Rigidity layering*

1594 Dikes hosted in layered rocks that have associated variations in material
1595 parameters can be strongly affected by those variations. As predicted by
1596 numerical models (Bonafede and Rivalta, 1999a; Gudmundsson, 2002, 2003;

1597 Rivalta and Dahm, 2004), host-rock anisotropy and heterogeneity is recog-
1598 nised as the main control on the observed local (fine scale) variations in the
1599 thickness of a dike. Dikes are often observed to be arrested at or to intersect
1600 several layers with strong contrasts in the elastic parameters. Geshi et al.
1601 (2012) predicted with finite element models the geometries of dikes from the
1602 caldera walls of Miyakejima and Piton de la Fournaise.

1603 Maccaferri et al. (2010) investigated the effect of layering on the travel
1604 path of ascending magma-filled dikes. Propagation across the layer inter-
1605 face is modelled using published analytical solutions for tensile and dip-slip
1606 dislocations in a medium made up of two welded half-spaces with differ-
1607 ent elastic parameters (Bonafede and Rivalta, 1999b; Rivalta et al., 2002).
1608 Maccaferri et al. (2010) find that the rigidity change at the interface causes a
1609 deviation of fluid-filled fractures crossing it: if the fractures pass from a high-
1610 rigidity layer to a low-rigidity one, they will be deflected towards the vertical
1611 direction, while the opposite holds if the fractures pass from a low-rigidity
1612 to a high-rigidity medium (see Fig. 16a and b, respectively). Above some
1613 critical incidence angle that depends on the rigidity ratio at the interface,
1614 the ascending dikes may deviate to become horizontal sills. Maccaferri et al.
1615 (2010) validated their numerical model with gelatin experiments. An inclined
1616 air-filled crack was created at the bottom of the container by injecting air
1617 through an inclined hole. The empirical angle of 'refraction' at the interface
1618 was compared with the results of the numerical model run with experimental
1619 parameters; the two were found to be in perfect agreement, within uncertain-
1620 ties.

1621 An analysis of energy release during dike propagation can be used to
1622 predict the velocity of the dike and where it might be stopped by material
1623 variations. Drawing a horizontal line in Fig. 16c and d to represent the en-
1624 ergy needed to create new crack surface (or, in other words, to break the
1625 medium), and observing when this line crosses the curve of the energy re-
1626 lease, we can deduce that a dike moving from a high-rigidity medium will
1627 accelerate until it reaches the interface and then suddenly decelerate; how-
1628 ever, it will maintain a larger velocity in the weaker medium than in the
1629 stiffer medium. Dike stopping is not predicted in this case. These theoretical
1630 deductions are consistent with observations of gelatin experiments (Sec. 3.2)
1631 (Rivalta et al., 2005). On the other hand, for a crack traveling from a low-
1632 rigidity medium to a high-rigidity one, it is plausible that for the crack tip at
1633 the interface between the media the energy released by propagation is lower
1634 than the energy needed to create new crack surface. In such a situation a

1635 dike is predicted to stop before reaching the interface or at the interface (for
1636 example, for $\Delta E = 15$ MPa m, Fig. 16c and d). If the crack does man-
1637 age to cross the interface (for example for $\Delta E = 10$ MPa m), depending on
1638 the rigidity ratio it may continue propagating ($r < 1.66$) or not ($r > 2.5$). One
1639 must, however, bear in mind that in natural systems (and more complicated
1640 models), the third dimension may also accomodate dike extension, and that
1641 this may be energetically preferred over ascending in unfavored conditions.
1642 Vertical ascent may become favored after lateral extension; see movie at
1643 <http://youtu.be/MJHslWoMXoI>, Rivalta et al. (2013b,c).

1644 5.6. Density layering

1645 The level of neutral buoyancy (LNB), where the density of the material
1646 within a dike becomes equal to the density of the wall rock, affects the dy-
1647 namics of propagation of a dike. The LNB is not a physical barrier to the
1648 propagation of a rising magma. The total hydrostatic pressure, which is ob-
1649 tained by integrating the hydrostatic pressure gradient $\Delta\rho g$ from the level
1650 of melt production or from the lower tip of the dike (Takada, 1989; Lister,
1651 1991, Fig. 3), is maximum at a LNB. Therefore the magma will preferen-
1652 tially spread at this level, but may still penetrate into layers of rocks of lesser
1653 density than the magma. This will modify the upward velocity of the dike,
1654 as explained below.

1655 Taisne and Jaupart (2009) solved for the time-dependent propagation of
1656 a crack through a medium with vertical stratification of density. Previous
1657 work using a similar methodology (see Sec. 4.3) obtained only stationary
1658 solutions for constant physical conditions. For example, Lister (1990a) con-
1659 sidered a viscous fluid propagating in a homogeneous elastic medium under
1660 constant source conditions. The results provided a basic state solution used
1661 to validate the formulation and code of Taisne and Jaupart (2009). They
1662 then went further, quantifying the effect of spatially variable material prop-
1663 erties on the speed of propagation of a dike. For decreasing but positive
1664 buoyancy, the dimension of the dike adjust to the new properties of the sur-
1665 rounding medium; dike width increases while upward speed decreases. If
1666 the dike penetrates a layer where it has a negative buoyancy, pressure will
1667 build up at the interface and a sharp deceleration occurs Taisne and Jaupart
1668 (2009). Furthermore, Taisne and Jaupart (2009) show this pressure increase
1669 may, in turn, initiate a sill, especially at a discontinuous decrease in density
1670 (Fig. 17A).

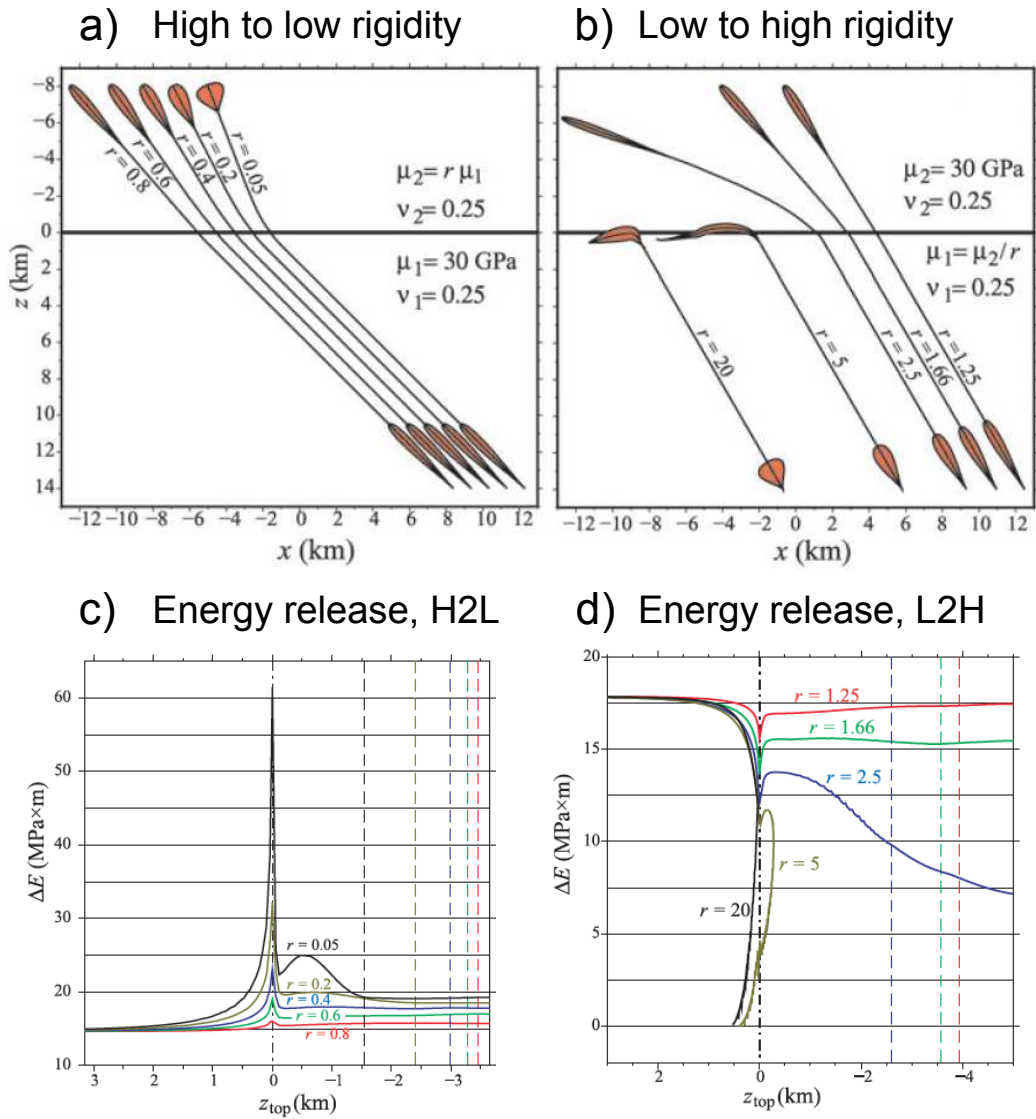


Figure 16: Dike trajectories deviate when dikes cross interfaces separating material with different rigidity. The angle of deviation depends on the rigidity ratio and on the angle of incidence, similarly to light in optics, but also on buoyancy and driving pressure. Maccaferri et al. (2010) calculated the trajectories with a boundary element code (Sec. 4.5). a) Trajectories for dikes transiting from high to low rigidity, paths are relative to varying rigidity ratios, as declared in figure. b) Same as (a), but the dikes transit from a low to a high rigidity layer. c) Total (elastic + gravitational) energy released during propagation from high to low rigidity rock (panel a). d) Same as (c), but relative to panel b). Figure modified from Figs. 7, 8, 10, 12 in Maccaferri et al. (2010), with permission

1671 Maccaferri et al. (2011) studies the trajectories of inclined dikes under
1672 different stratification scenarios by allowing for density layering that is inde-
1673 pendent of rigidity layering. Deflections similar to what described in Par. 5.5
1674 are obtained only if there is a discontinuity in the rigidity. If only the density
1675 is discontinuous across a horizontal interface, the dike will continue in the
1676 same direction as it crosses the interface between the two layers of different
1677 density (Fig. 17B). The dike may stop if it experiences a state of negative
1678 buoyancy, but its leading tip will still penetrate significantly into the medium
1679 of low density (Fig. 17B, C.3). In this case, the typical inverse tadpole shape
1680 assumed during vertical ascent is replaced by a pointy profile.

1681 It should be noted that the dikes are not allowed to extend in the third
1682 dimension in either of the models described above. The increasing pres-
1683 sure in the upper part of the dike will favor lateral, rather than verti-
1684 cal, propagation and thus instead of leading to sill inception the results by
1685 Taisne and Jaupart (2009) and Maccaferri et al. (2011) may feed results ob-
1686 tain by Pinel and Jaupart (2004) or Lister (1990a) dealing with horizontal
1687 migration of dikes.

1688 *5.7. Dike–dike interaction*

1689 Focusing of ascending, magma-filled dikes by dike–dike interaction was
1690 modelled numerically and experimentally by Ito and Martel (2002). In par-
1691 ticular, Ito and Martel (2002) consider the deviation expected when an as-
1692 cending dike feels the stress field due to a pre-existing, stalled dike. They
1693 find dikes will, in general, tend to interstect or merge to previously ascended
1694 dikes. In their numerical model, the contribution of external stresses and
1695 other parameters such as buoyancy and driving pressure is also included and
1696 is related to how effectively dikes focus and merge.

1697 Building on this research, Kühn and Dahm (2008) combines the simula-
1698 tion of fracture propagation with dike interaction (Sec. 5.1). Dikes interact
1699 by adapting to the stress field caused by preceding dikes that arrested in the
1700 crust, which leads to focussing and crossing of dykes. The method is applied
1701 to study how magma chambers and sheeted dyke complexes may form at
1702 mid-ocean ridges. They find that interaction between dykes can have signif-
1703 icant consequences under certain conditions: the interaction is small under
1704 lateral tension that is large compared to the pressure in the dike head; oth-
1705 erwise, dikes tend to attract each other and form large, magma-filled bodies
1706 or sill layers.

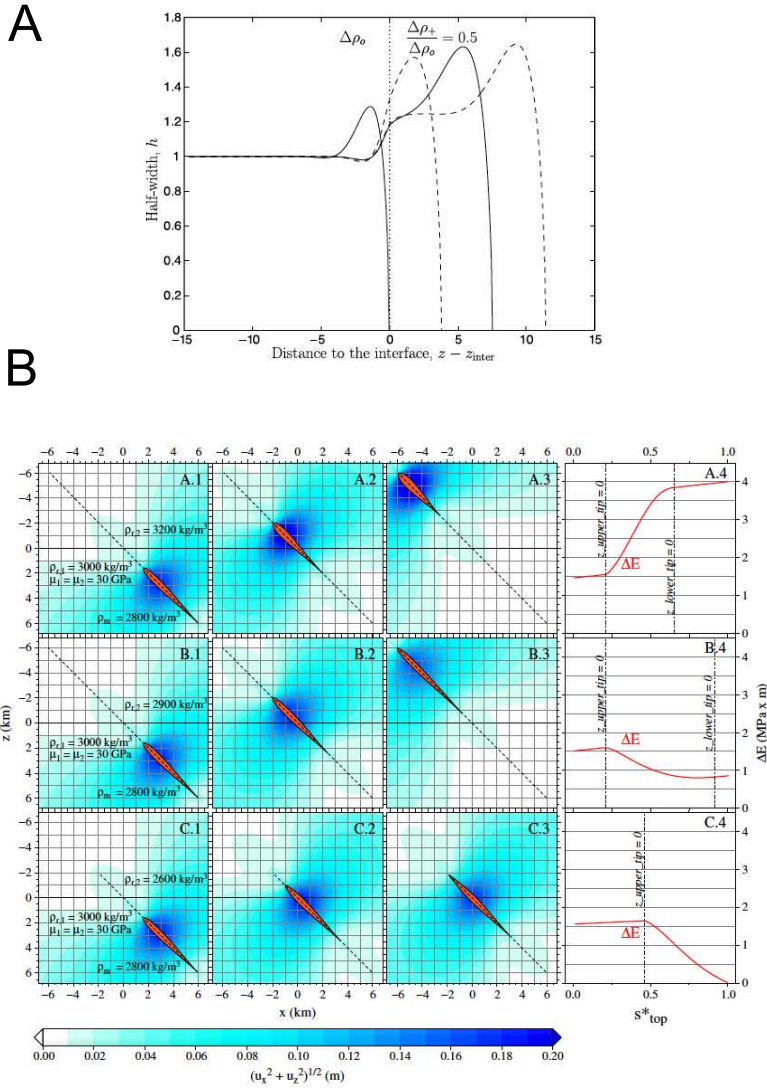


Figure 17: A) Results from a lubrication theory approach on dike propagation in a density-layered medium. The shape of a dike penetrating into a layer with reduced buoyancy is shown. The interface lies at $z = z_{inter}$ (vertical dotted line). Results are shown for various snapshots at fixed time interval. The nose region rapidly adjusts to new dimensions in the upper layer. Calculations are made for a dimensionless stress-intensity factor equal to 1, i.e., $Kc/K^* = 1$. From Taisne and Jaupart (2009), with permission. B) In each row, successive snapshots are shown of a dike propagating in a medium made up of two welded half-spaces with different density. From Maccaferri et al. (2011), with permission. The viscous flow of magma within the dike is neglected, as in Weertman theory. Density of rock and magma are declared in the images, where ρ_{r1} and ρ_{r2} are the densities of the lower and upper half-space, respectively, μ_1 and μ_2 are the rigidities, ρ_m is magma density. The modulus of the displacement vector induced in the medium is shaded in the background. The dashed line represents the energetically preferred path and the opening of the dike is exaggerated by a factor 1400. The final column shows the specific total energy release, plotted as function of a spatial coordinate along the dike path.

1707 Dike–dike interaction has also been studied in relation to rifting episodes,
1708 where sequences of dike intrusions compensate the strain accumulated over
1709 centuries at divergent plate boundaries (see Section 5.8). For example, em-
1710 placement location of the dikes in the Manda-Hararo segment (Afar, Ethiopia)
1711 is found to be influenced by the location of previously emplaced dikes. In
1712 particular, dikes tend to emplace in locations where tectonic tension has ei-
1713 ther not yet been compensated by previous dikes (local opening minima) or
1714 has accumulated at their tips (Hamling, 2010; Grandin et al., 2010b). In
1715 this respect, dikes behave much like earthquakes: they occur in fault areas
1716 where stress has not been relieved by previous, recent earthquakes, or occur
1717 as aftershocks where stress has increased due to inhomogeneous slip or at the
1718 edge of faults (see also Sect. 5.8 below).

1719 *5.8. Scaling and dike sequences in rifting episodes*

1720 The scaling laws of earthquakes and main shock–aftershocks sequences
1721 have been the subject of great interest in literature. Dike sequences in rift-
1722 ing episodes offer a chance to compare earthquake sequences with a process
1723 similar in that it relieves accumulated tectonic stresses, but different in that
1724 a source of magma needs to be available to compensate for the volume of
1725 tectonic extension.

1726 Buck et al. (2006) developed a numerical model to study main-dikes/after-
1727 dikes sequences in rifting episodes. The model includes the following features:
1728 (a) the relative tension associated with tectonic extension (the tensile stress
1729 gradient drives the dikes away from the magma chamber); (b) a compressive
1730 stress compensation produced by each emplaced dike (this has the role/effect
1731 of reducing the tectonic stress difference after each dike has been emplaced);
1732 (c) a magma chamber that undergoes a pressure drop during diking (this is
1733 responsible, together with the level of pre-existing tensile stress, for stopping
1734 the dikes); and (d) the existence of some threshold values of driving pressure
1735 required to initiate diking from the magma chamber (i.e. for the pressure in
1736 incipient dikes to be sufficient to overcome the fracture toughness of rock
1737 and continue propagation). Under these assumptions, the model predicts
1738 sequences of dikes that mimic many of the characteristics observed during
1739 rifting episodes, as observed during the 1975–1984 Krafla sequence, and the
1740 sequence in the Manda-Harraro segment of the East African Rift that started
1741 in 2005 (Wright et al., 2012).

1742 Passarelli et al. (2014b) analyzed the statistics of rifting episodes at diver-
1743 gent plate boundaries and found that they have scaling characteristics that

1744 are similar to mainshock–aftershocks sequences. The volumes of the dikes
1745 from rifting episodes are distributed according to a power law that mimics
1746 the Gutenberg-Richter magnitude–frequency relation found for earthquakes
1747 worldwide (Fig. 18), and the seismic moment released by the dikes as a func-
1748 tion of time is consistent with the release rate of seismic moment through
1749 aftershocks (Omori law). The strong control from tectonic extension (for
1750 divergent plate boundaries this is likely dominant with respect to magma
1751 overpressure) and dike–dike interactions (see also Sec. 5.7) are inferred to be
1752 an important process controlling these scaling laws.

1753 *5.9. Fracturing, faulting, induced seismicity*

1754 Dikes and earthquakes interact in two main ways: (i) dikes induce fault-
1755 ing and earthquakes during propagation, and (ii) earthquakes on pre-existing
1756 faults or fractures influence propagating dikes. Here we review the main
1757 studies that link propagating dikes with faulting and fracturing. We discuss
1758 results specific to hydraulic fracturing in Sec. 6. In Sec. 3.1.3 we presented
1759 an overview of the main observations on dike-induced seismicity. From a
1760 modelling perspective, the mechanisms behind the generation of seismicity
1761 by propagating dikes has been investigated using a range of analytical, nu-
1762 merical, statistical and experimental approaches.

1763 Buck et al. (2005) model numerically the generation of faulting by diking
1764 at mid-ocean ridges with the purpose of explaining the formation of abyssal-
1765 hill-bounding faults pervading the surface of the oceanic crust. As Buck et al.
1766 (2005) observe, given that dike intrusions occur at lower stress than is needed
1767 for faulting, many authors assume that faults at mid-ocean ridges form only
1768 during the time intervals between diking events when no magma is available
1769 for dike intrusion (Carbotte and Macdonald, 1990). However, this simple
1770 model does not fit more detailed observations related to the dip direction of
1771 the faults, their development as a function of distance from the axis and the
1772 presence of complex structures at the intersection of ridges and transforms.
1773 In the model by Buck et al. (2005) faults develop where the stress overcomes
1774 brittle yielding. Elastic, viscous and brittle-plastic deformation accompany-
1775 ing isostatic balancing due to density changes is taken into account. The role
1776 played by dikes in the models is central but it can be simplified to columns of
1777 magma intruding at the axis at regular time intervals. The models examines
1778 end-members (buoyancy- vs. stretching-dominated ridges, leading to axial
1779 highs and valleys, respectively) and the proportion of extension accommo-
1780 dated by diking.

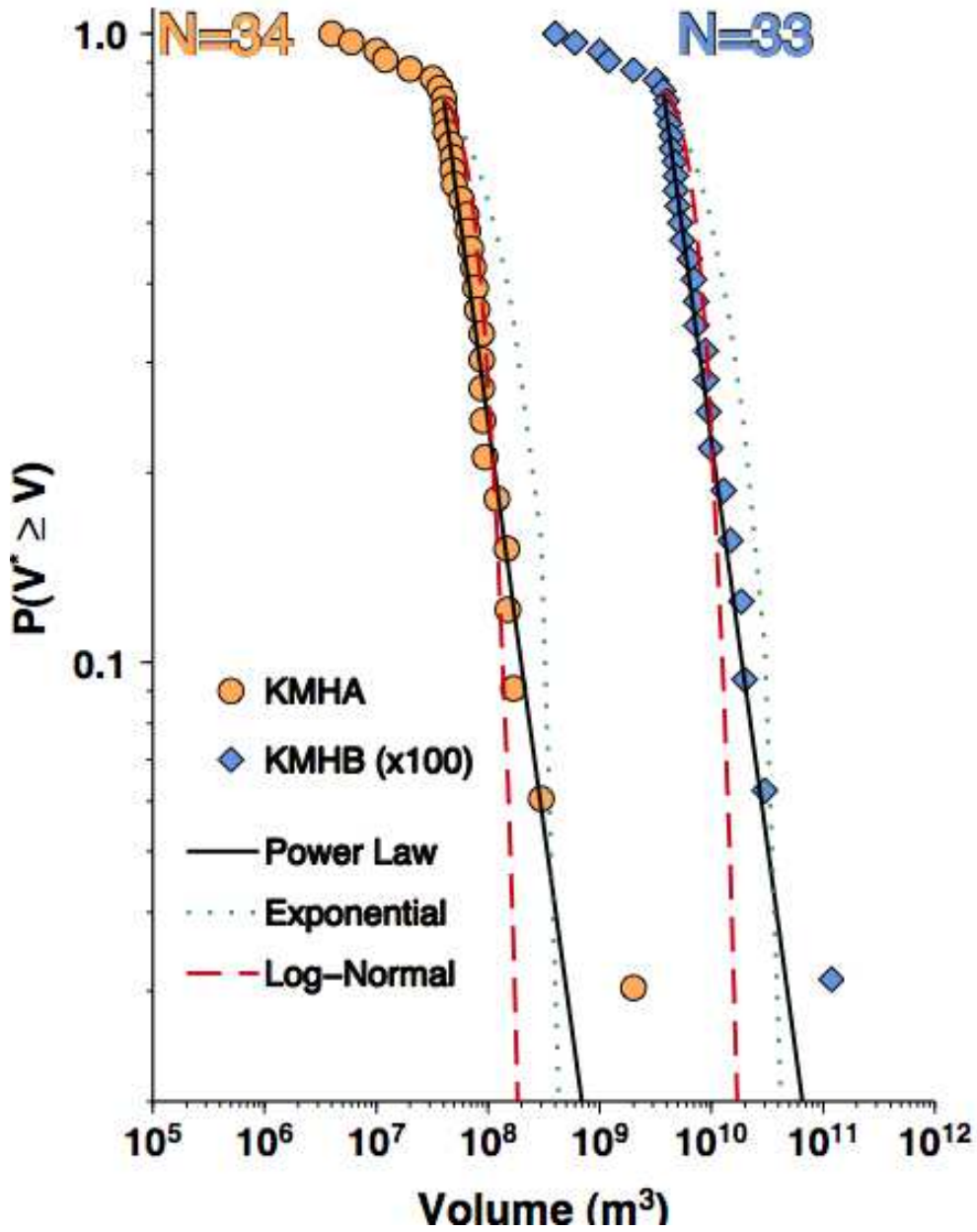


Figure 18: The frequency–total volume distribution of dikes from the Krafla and Mandahararo rifting episodes follow a power law, analogous to the Gutenberg-Richter relation for aftershock sequences. From Passarelli et al. (2014b), with permission.

1781 The seismicity linked to dike intrusions is known to be related to advance
1782 of the crack tip, but it is also known to mirror the response of the surround-
1783 ing rock to the deformation and stresses induced by the magma intrusion
1784 (see e.g. Rubin and Gillard, 1998; Traversa et al., 2010). Rubin and Gillard
1785 (1998) examine the stress field induced by a propagating dike in a bid to ex-
1786 plain the origin of dike-induced seismicity. They include a dike-tip cavity in
1787 their model and study the stress field in its proximity. The tip cavity is a low
1788 pressure zone where magma cannot penetrate quickly and may attract flu-
1789 ids from rock pores. Seismicity is found to be most likely around this area,
1790 provided that pre-existing, favorably oriented fractures exist. Rubin et al.
1791 (1998) studied the seismicity induced by the 1978 and 1984 dikes at Kilauea.
1792 They observe that the spatial distribution of dike-induced earthquakes re-
1793 flects areas where the differential stress is high and does not necessarily re-
1794 flect the real extent of the dike. They also observe repeating seismic events
1795 coming from the same fault patch. This observation has recently been con-
1796 firmed by Jakobsdóttir et al. (2008) and White et al. (2011), who observed
1797 earthquakes of inverse polarity coming from the same patch, meaning that
1798 the same fault patch (or two patches very close spatially) experienced slip in
1799 opposite directions within short time intervals (seconds or minutes).

1800 Rivalta and Dahm (2004) develop a boundary element model for a dike
1801 embedded in a fractured medium. In this model a pressurized dike inter-
1802 acts with randomly distributed fractures that are also interacting with each
1803 other. The dike opening is calculated as a function of the density of frac-
1804 tures. The weakening and inhomogeneities represented by the fractures cause
1805 an increased, irregular opening profile in the dike, as described qualitatively
1806 by Ida (1992) (Sec. 5.12). The average dike opening from the numerical
1807 model agrees with predictions from effective media theory, which assumes a
1808 fractured medium to be homogeneous but with modified, effective elastic pa-
1809 rameters (Davis and Knopoff, 1995; Dahm and Becker, 1998). These can be
1810 obtained as a function of fracture density by solving a differential equation.
1811 The model is applied to explain post-intrusion seismicity and deformation for
1812 the 2000 dike intrusion at Miyakejima. The deformation and the seismicity
1813 are found to be linked through an exponential law during the post-intrusion
1814 phase. Rivalta and Dahm (2004) conclude that progressive weakening of a
1815 medium due to diffuse seismicity may induce additional opening in the dike,
1816 if more magma is available. This, in turn, feeds back to the generation of
1817 more seismicity. A different relation was found for the aftershocks and defor-
1818 mation following earthquakes. Savage and Yu (2007) found a linear relation

1819 between the number of aftershocks and amount of post-seismic relaxation for
1820 the first 160 days following the Chengkung earthquake and for the first 560
1821 days following the Parkfield earthquake. The difference between the response
1822 of rock to diking and faulting may be due to the presence of friction on faults.
1823 On the contrary, dikes are frictionless fractures and the dike walls are free to
1824 deform for some time after the intrusion. The model by Rivalta and Dahm
1825 (2004) is static; no attempt has yet been made to model numerically dike
1826 propagation in a fractured medium (but see Le Corvec et al. (2013) for an
1827 experimental study of fluid-filled fracture propagation in a faulted medium).

1828 Our understanding of the source-physics of earthquakes has informed the
1829 development of theories for how dikes induce seismicity. The rate-and-state
1830 theory of friction on a fault (Dieterich, 1994) links a local Coulomb stress
1831 change (defined as the shear stress diminished by the friction times the nor-
1832 mal compressive stress) to a change in the seismicity rate. This theory pro-
1833 vides a link between models of dike shape evolution during propagation and
1834 observables related to seismicity. It has been used to make inferences on
1835 the correlation between the amount of seismic energy release during dike-
1836 induced earthquake swarms and the rate and volume of propagating magma:
1837 (Pedersen et al., 2007) show how the relationship between volume change and
1838 earthquake rate varies greatly between different intrusions and is strongly
1839 linked to the background stress state or background seismicity rate. The
1840 equations of the theory are solved for time-dependent stressing (which is
1841 what occurs during dike injections) by many authors (see e.g. Dieterich et al.,
1842 2000; Segall et al., 2006; Hainzl et al., 2010; Maccaferri et al., 2014). The
1843 calculation of Coulomb Stress changes and of changes in the seismicity rate
1844 with the rate-state theory are rich with details that are beyond the scope
1845 of the present review (see instead Harris (1998); Stein (1999); Cocco et al.
1846 (2000); Hainzl et al. (2010); Toda et al. (2012) for Coulomb stress studies
1847 and tools and Dieterich et al. (2000); Toda and Stein (2002); Segall et al.
1848 (2006); Maccaferri et al. (2013); Segall (2013) for dike-related applications of
1849 the rate-state theory). The 2D nature of most numerical models of propa-
1850 gating dikes restricts applications of Coulomb stress studies and rate-state
1851 theory to 2D, with the resulting uncertainty that adding a third dimen-
1852 sion may change the results in a significant way. A promising application is
1853 described by Segall et al. (2013), who shows how inversions of crustal defor-
1854 mation data can be combined with rate-state earthquake nucleation theory
1855 (Dieterich, 1994) to get an improved picture of the shape of a propagating
1856 dike.

1857 Using data from the 2000 intrusion at Miyakejima, Passarelli et al. (2014a)
1858 studied the relation between double-couple fault plane solutions of induced
1859 earthquakes, the Coulomb stress induced by the dike and the statistics of
1860 the earthquakes. They find that the focal mechanisms match well with the
1861 optimally-oriented planes for the 3D Coulomb stress change around an el-
1862 liptical penny-shaped dike, resulting in a strong correlation between rake,
1863 strike and dip of the earthquakes, smoothly varying around the dike edges.
1864 They also find that earthquakes with a predominantly strike-slip mechanism
1865 follow the usual Gutenberg-Richter statistics for tectonic earthquakes, while
1866 predominantly normal faulting mechanisms lack in proportion large magni-
1867 tude events. According to the pattern of the Coulomb stresses, such normal
1868 faulting earthquakes are expected to occur mainly in the crustal layer above
1869 the dike. A lack of large magnitude events may occur because of a limited
1870 thickness of such a layer, limiting the physical dimension of the faults, or,
1871 alternatively, because of the decreased strength of the rock in shallow layers,
1872 so that the ability to sustain large stress accumulation is limited.

1873 A new seismological approach has been applied at Piton de la Fournaise
1874 volcano to track magma motion during an intense seismic swarm. It is based
1875 on seismic wave attenuation, and on the theory that ratio of radiated energy
1876 between 2 stations is a function of the location of the source. Any time
1877 evolution of this ratio can be confidently interpreted in terms of migration of
1878 the source, namely the propagating dike. Application of this method to Piton
1879 de la Fournaise shows that the dynamics of propagation is complex, with
1880 variation in upward velocities punctuated by phases of arrest (Taisne et al.,
1881 2011a).

1882 *5.10. Coupling of magma chambers and dikes, connectivity, induced defor-* 1883 *mation*

1884 The behaviour of dikes is controlled by their hydraulic connectivity to a
1885 magma reservoir. This control is so important it was the basis for a clas-
1886 sification of dikes into two types Nakashima (1993): dikes fed by a magma
1887 chamber versus propagating, isolated magma pockets. To understand why
1888 these two categories exist, we must ask what are the conditions under which
1889 a dike nucleated from a magma reservoir may hydraulically separate from it?
1890 Key factors include the rheology of the host medium, the temperature dif-
1891 ference between magma and host rock, the viscosity of the magma, and the
1892 distance between the dike tip and the chamber. We know that shallow dikes
1893 nucleated from magma reservoirs in the elastic crust will be coupled to the

1894 magma reservoir for at least some of the propagation distance. For example,
1895 the 2000 intrusion at Miyakejima continued to thicken for weeks after being
1896 arrested at several km distance from the magma chamber (Nishimura et al.,
1897 2001), implying that connectivity may be large even in cases where magma
1898 chamber and dike tip are widely separated. Quantitative evidence of the
1899 coupling between magma reservoir and dike was presented by Tryggvason
1900 (1984), who made a careful comparison of dike widening-related volume in-
1901 crease and magma chamber volume loss for the first dikes from the 19875–
1902 1984 Krafla rifting episode. An early model about chamber-dike coupling
1903 is by Mériaux and Jaupart (1995). The model includes a pressurised reser-
1904 voir shaped as a funnel with elliptical opening connected to a fissure. Rock
1905 fracturing is neglected in the model and the governing equations are solved
1906 for the pressure and the fluid flux as functions of reservoir size and magma
1907 supply rate to the reservoir. High supply rates and small chamber sizes lead
1908 to small amounts of magma flowing into the fissure and therefore to larger
1909 reservoir pressurization and magma volume stored. The delay time between
1910 the onset of reservoir inflation and the opening of the fissure decreases with
1911 increasing reservoir size. Rapid deflation of the reservoir occurs if the supply
1912 rate decreases with time.

1913 Inverting measurements of surface deformation associated with dike intru-
1914 sion has provided useful insight into the characteristics of the dike. Mostly,
1915 such studies have offered only a static picture of the emplacement process.
1916 Crustal deformation data can sometimes be used to constrain the dynamics
1917 of dike propagation, but models attempting to invert time-dependent defor-
1918 mation are rare. Even less common are inversions that employ physics-based
1919 models, which can be used for a better understanding of dike dynamics and of
1920 the interaction with other deformation-inducing sources such as major faults
1921 and magma chambers.

1922 Segall et al. (2001) develops a time-dependent dike growth model to ex-
1923 plain the decreasing rate of volume change of the dike during the 1997 intru-
1924 sion at Kilauea. Simple models of dikes fed at constant pressure or constant
1925 inflow from magma chambers do not predict a decreasing volume rate and
1926 could not be applied to the intrusion. Segall et al. (2001) observe that the
1927 rate of volume decrease at the magma chamber mirrored the volume in-
1928 crease at the dike, suggesting that the growth was limited by the ability of
1929 the chamber to sustain pressure during the intrusion. Their model suggests
1930 that eruptions during intrusion are favored by compressive stress regimes,
1931 large, compressible reservoirs, and high connectivity between chamber and

1932 dikes. Magma level changes at Pu'u O'o lava lake, hydraulically connected
1933 to the magma chamber, were also used to give an estimate for its total vol-
1934 ume, around 20 km³. Rivalta and Segall (2008) collected evidence of large
1935 mismatches between observed volume changes at magma chambers and dike
1936 volumes during intrusion events. Dikes are often observed as having a much
1937 larger volume than the volume change at the feeding reservoir. They sug-
1938 gested that magma compressibility and the different elastic compression of
1939 the host medium for different magma chamber shapes are responsible for this
1940 discrepancy. Blake (1981), Sigmundsson et al. (1992), Johnson et al. (2000)
1941 had already recognized that volume changes at spherical magma chambers
1942 do not correspond to the true volume of magma injected or extracted, be-
1943 cause the compression/decompression of the much larger volume of magma
1944 residing in the chamber compensates some of the volume injected/extracted.

1945 Further studies focused on solving the flow of magma between coupled
1946 magma-filled structures. Rivalta (2010) developed a model of the pressure
1947 drop during mass transfer from a chamber to a dike and found evidence for
1948 such a coupling for several lateral dike propagations, as demonstrated by the
1949 pressure drop at the magma chamber and the synchronous volume gain by
1950 the dike. The coupling was probably active for the total propagation time
1951 for two of the after-dikes of the sequence at the Manda Harraro segment,
1952 Afar, Ethiopia, (propagation time in the range of about 3–16 hours), for the
1953 1997 dike intrusion at Kilauea volcano, Hawaii, (about two days) and for
1954 the 1978 dike intrusion at Krafla volcano, Iceland. As for the 2000 intrusion
1955 at Miyakejima, Japan, the dike was pressurised by the chamber at least for
1956 the first 12 hours of propagation but it is unclear whether it continued for
1957 the total propagation time of about a week. It is possible that other driving
1958 forces, such as the topographic load, assumed a greater role in the later
1959 stages of the propagation so that the coupling, although still present, lost its
1960 importance.

1961 Tarasewicz et al. (2012) develop a model for how the connectivity be-
1962 tween magma storage zones at different depth may evolve during volcanic
1963 activity. During the 2010 Eyjafjallajökull eruption in Iceland, seismic activ-
1964 ity was observed to counterintuitively propagate downwards over about 30
1965 km depth and ten weeks time of volcanic unrest. Systematic changes in the
1966 petrology of the magma suggest a multilayered plumbing system. Tapping
1967 of each of the individual storage zones preceded an explosive surge in erup-
1968 tion rate. Tarasewicz et al. (2012) explain these systematics in terms of a
1969 decompression wave that triggers magma release from progressively deeper

1970 sills in the crust.

1971 Segall et al. (2013) present a method to combine the information from
1972 crustal deformation and from the seismicity linked to a propagating dike in
1973 order to obtain the time-dependent distributed opening of the dike. The
1974 method is based on linking deformation (and thus stress changes) to the
1975 induced seismicity rate through the rate-state earthquake nucleation model
1976 (Dieterich, 1994). The dike is modeled as a horizontally extending rectan-
1977 gular crack in an elastic half-space, subject to spatially uniform but time-
1978 dependent overpressure. Some forward models are presented with the ex-
1979 pected hypocenters of induced earthquakes (Fig. 19). These forward models
1980 show that time-dependent seismicity rate behaves differently for rock vol-
1981 umes above or below the dike or at intermediate depth (adjacent to the
1982 dike). Similar space-time patterns have been observed in nature (Sect. 3.1.3)
1983 and in industrial operations, see also model by Dahm et al. (2010). The
1984 method is then applied to the 2007 Fathers Day lateral intrusion in Kilauea
1985 Volcano. Segall et al. (2013) find that it is difficult to fit both deformation
1986 and seismicity with a simple dike model with a horizontal propagation and a
1987 vertical tip line. In particular, the location of the dike's bottom edge relative
1988 to the deep part of the migrating seismicity cloud is difficult to constrain.
1989 The approach is very promising in that it integrates physics-based knowledge
1990 with observations.

1991 5.11. *Dike arrest*

1992 Why and how a dike ceases to propagate is a central question in volcanol-
1993 ogy. In this manuscript we have already explored the case of arrest due to a
1994 stress barrier caused by a volcanic load (Sec. 5.2) or by layering (Sec. 5.5).
1995 Magma solidification within the dike (Sec. 5.14) and faulting on pre-existing
1996 or dike-created structures (Sec. 5.9) are also thought to be potential causes of
1997 dike arrest. Furthermore, pressure decrease at the magma chamber has been
1998 shown to cause dike arrest (Rivalta, 2010). This issue merits some atten-
1999 tion from numerical modeling in the future, as there are many unanswered
2000 questions.

2001 Qin and Buck (2008) discuss partial stress release during diking. This
2002 can originate from limited magma supply when the dike is coupled to a
2003 magma chamber and results in an upper limit on the width of dikes. Multiple
2004 dike intrusions may then be required to release the entire accumulated stress
2005 (Sec. 5.8).

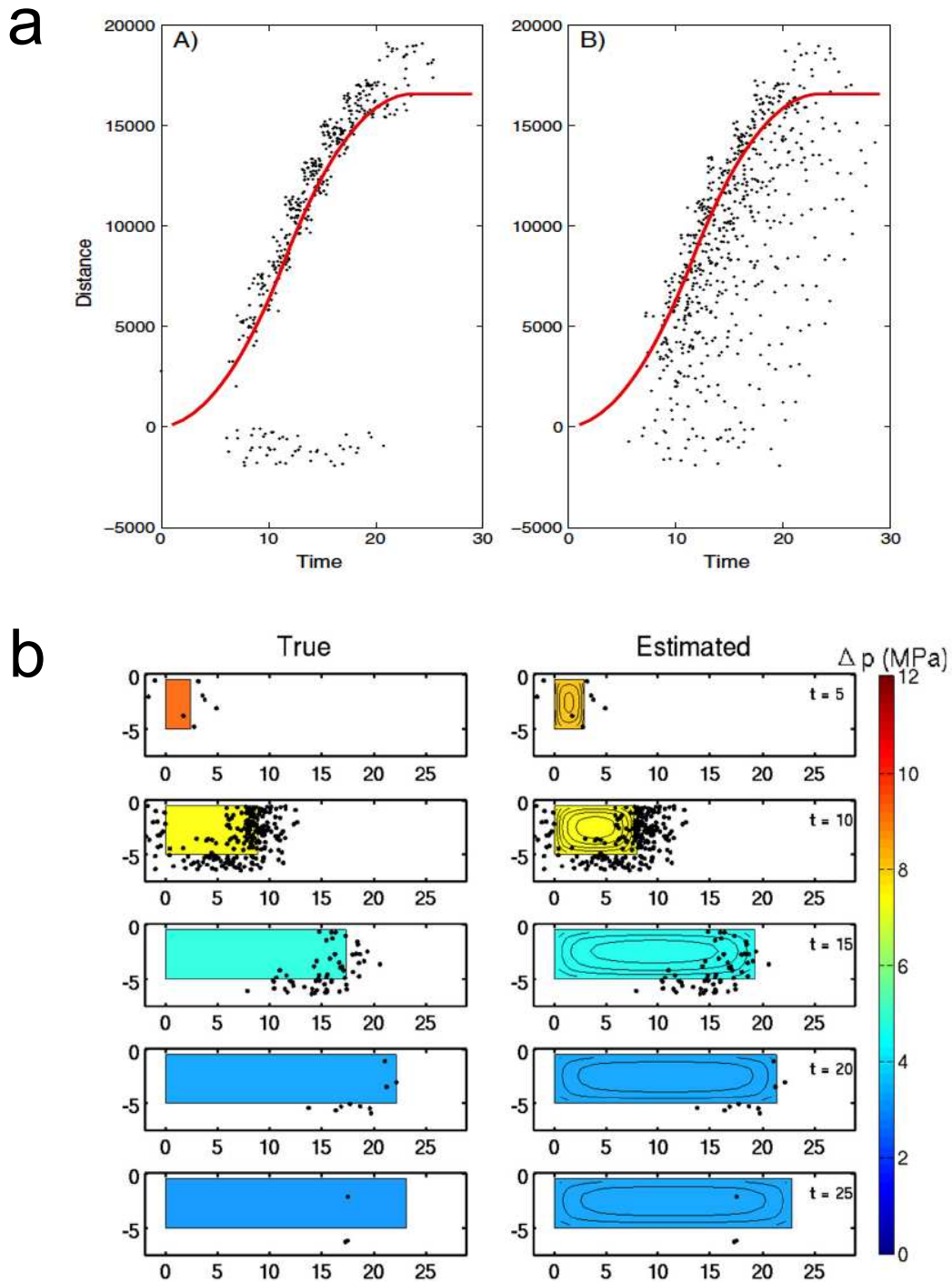


Figure 19: a) Predicted space-time hypocenter distribution (A) adjacent to dike and (b) below the dike. Red curve represents position of propagating dike tip. Fig. 5 from Segall et al. (2013), with permission. b) Side view of simulation results: comparison between (left) input dike model and (right) that estimated from inversion. Color maps dike overpressure; contours in right column are curves of constant dike opening. Fig. 8 from Segall et al. (2013), with permission.

2006 Baer (1991) examines dike exposures in the Ramon area of Israel and
2007 calculates the stress intensity factor for an extension crack approaching a
2008 mechanical interface. He concludes that dike segmentation and arrest are
2009 controlled mainly by local stresses, shear moduli differences between adja-
2010 cent layers, and, partly, by bedding plane slippage. Taisne et al. (2011b)
2011 explore 3-dimensional effects using laboratory experiments on a constant-
2012 volume, vertically propagating dike and consider the conditions under which
2013 to expect dike arrest and/or horizontal extension. They also investigate the
2014 behavior of a dike penetrating into a low density layer using numerical anal-
2015 ysis. From both analyses they predict the minimum volume of magma that
2016 must be injected into the dike for an eruption to take place. A more com-
2017 plicated situation occurs in the intrusion of dikes into sedimentary, porous
2018 rocks, where fluidization of rock is induced by pore pressure increase, causing
2019 fingering in the shape of the dike (Sec. 5.13).

2020 *5.12. Predicting vent locations and times of intrusions and eruptions*

2021 Even if volcanoes often give notice of unrest through seismic activity, de-
2022 formation or unusual degassing, knowing when and where an eruption could
2023 occur has not been an easy task in volcanic hazard. Significant effort has
2024 been devoted in identifying the temporal and spatial pattern of eruptions and
2025 understanding the physical mechanisms controlling the creation of eruptive
2026 vents. One of the topics currently under investigations is monogenetic vol-
2027 canism, where volcanic fields are composed of tens of volcanic vents created
2028 and occupied by only one eruption.

2029 Ida (1992) presents a model for the variation in opening width along a
2030 magma-filled fracture, assuming laminar flow of an incompressible fluid. He
2031 correlates the formation of fissures, polygenetic volcanic edifices or mono-
2032 genetic vents to the changes induced in the width of the dike by the specific
2033 tectonic settings. A variable width along dike length leads to preferred loca-
2034 tions for magma erupting at the surface. He also observes that various degrees
2035 of inhomogeneity in the crust lead to a non-uniform increase in width that
2036 may evolve in separated vents. Ida (1999) models dike growth by assuming a
2037 fissure with opening constant over length but not constant in time. The dike
2038 is coupled to a magma chamber (see Sec. 5.10) and the opening is affected
2039 by elasticity and the external tectonic stress. Tip processes are neglected
2040 completely but discussed thoroughly. The model is used to evaluate the ef-
2041 fect of the external stress field on whether the dike will erupt or form and
2042 intrusion. He finds that extensional stress conditions favor growth only until

2043 a limited length, while compressive or moderately extensional stresses favor
2044 'unlimited' growth until the dike erupts. The model is applied to explain
2045 trends in deformation data from Izu-Tobu and Izu-Oshima, Japan.

2046 Pinel and Jaupart (2004) study the influence of volcano loads on the ex-
2047 tension of shallow dikes, distinguishing three regimes: (1) eruption through
2048 the summit, occurring when the load is not very significant and magma is
2049 buoyant, (2) storage beneath the edifice, occurring when the load is large
2050 and magma buoyancy is not very high, (3) horizontal propagation to feed
2051 a distal eruptive vent, which can only be achieved if the load by an edifice
2052 prevents eruption in the dike nucleation area and if magma is negatively
2053 buoyant at shallow depth. The model explains the distribution of erupted
2054 products which show decreasing magma evolution with increasing distance
2055 from the focal area.

2056 Segall (2013) reviews the conditions under which dike intrusions may
2057 be predicted from inflation–deflation patterns at the magma chamber. He
2058 concludes that in general, it is not possible to define a threshold in the
2059 amount of inflation that would immediately lead to the rupture of the magma
2060 chamber's walls and the initiation of an intrusion. The main reason for this
2061 is that the stress conditions in the host medium are likely to be changed by
2062 each intrusion, which will relieve some of the tectonic tensile stress (see also
2063 Sec. 5.8). Indeed, the level of inflation related to dike initiation differed for
2064 each intrusion during the 1975–1984 Krafla rifting episode (Sturkell et al.,
2065 2006).

2066 *5.13. Segmentation*

2067 While models typically assume stable propagation of a dike's leading edge,
2068 segmentation and fingering instabilities are predicted by theory and observed
2069 in the field and laboratory. Segmentation, which here refers to out-of-plane
2070 bifurcation of the dike front and which can lead to observed en-echelon dikes,
2071 is commonly interpreted to result from rotation of the stresses or, equiva-
2072 lently, from a mixed mode of fracture, opening (Mode I) and tearing (Mode
2073 III) of the dike tip (Pollard et al., 1975, 1982). This interpretation draws
2074 its original inspiration from the laboratory experiments performed in glass
2075 by Sommer (1969), who showed that under mixed-mode mechanical loading
2076 the segmentation occurred when the stress rotation angle was greater than
2077 a threshold value of about 3.3 degrees which, according to the formulae pre-
2078 sented by Pollard et al. (1982), corresponds to a ratio of the Mode III stress

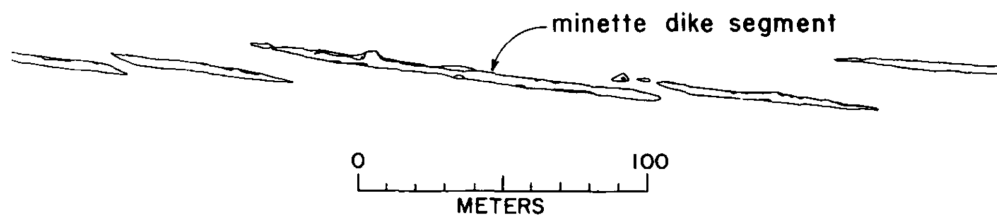
2079 intensity factor K_{III} to the Mode I stress intensity factor K_I of ≈ 0.05 –
2080 0.10, depending on the ratio of the extensional to the shear stresses. How-
2081 ever, more recent experiments wherein fluid-driven cracks were subjected to
2082 mixed Modes I–III loading have shown that segmentation can occur at even
2083 lower ratios, i.e. with $K_{III}/K_I \approx 0.01$ (Wu et al., 2009). Furthermore, it
2084 can be shown that the angle of the segments relative to the overall strike
2085 of the array of en echelon dikes is related to the stress rotation angle and
2086 extensional to shear stress ratio (Pollard et al., 1982). As a consequence, the
2087 morphology of en echelon dikes can be used to aid in the interpretation of
2088 paleostresses; however, experiments have demonstrated that the twist angle
2089 of the segments is overestimated by theory (Cooke and Pollard, 1996). Well
2090 known examples of segmented dikes include the petal-like morphology of the
2091 dikes near Ship Rock, New Mexico, USA (Figure 20a) (Delaney and Pollard,
2092 1981; Delaney et al., 1986).

2093 Another mechanism, which is potentially applicable to a similarly wide
2094 range of settings, has been identified through recent analysis and experi-
2095 mentation (Figure 20b). That work shows that an emergent finger-like mor-
2096 phology can also be related to an instability to transverse perturbations of
2097 a buoyancy-driven ascending dike front (Touvet et al., 2011). Such pertur-
2098 bations can naturally arise from heterogeneity in the rock properties at a
2099 scale comparable with the dike thickness, and therefore should be ubiquitous
2100 in natural host rocks. For California’s Inyo dike, it has been argued that
2101 segmentation is driven by variations in the host-rock rheology and fracturing
2102 style (Reches and Fink, 1988).

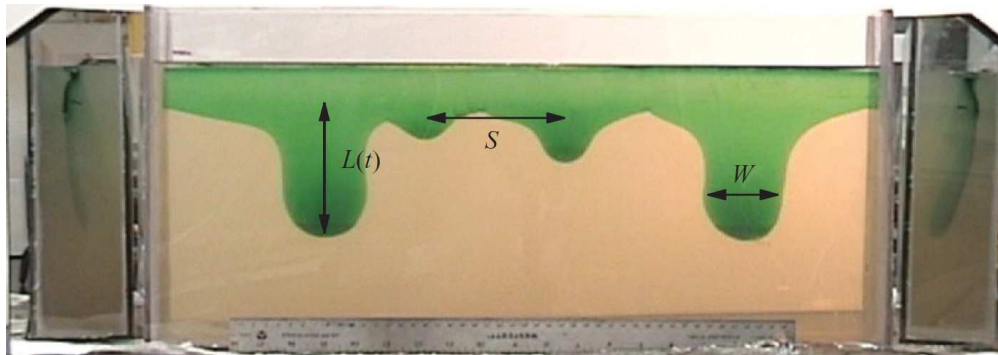
2103 *5.14. Heat exchange, cooling*

2104 Heat transfer between magma in dykes and the host rock is interest-
2105 ing for a number of reasons. First, cooling of the magma in propagating
2106 dykes is thought to be one of the factors inducing deceleration and arrest
2107 (Fialko and Rubin, 1998). Second, dyke intrusions heat and weaken the crust
2108 or lithosphere (Havlin et al., 2013), influencing processes ranging from the
2109 accumulation of cooled intrusions below volcanic edifices to crustal thinning
2110 in continental rifts.

2111 Solidification of magma flowing in an open conduit has been studied
2112 by Delaney and Pollard (1982); Bruce and Huppert (1990); Lister (1994a,b).
2113 Lister and Kerr (1991) state that solidification, or a step in viscosity, of the
2114 magma would have comparable effect on the propagation as an increase of
2115 fracture toughness. Rubin (1993b) and later Bolchover and Lister (1999)



a)



b)

Figure 20: Two types of segmentation. a) Map of part of minette dike near Ship Rock, New Mexico, from Delaney and Pollard (1981), b) Fingering instability of negatively buoyant analogue dike in gelatine, from Touvet et al. (2011).

2116 study the early stage of a propagating magma subject to solidification and de-
2117 rive a critical length below which solidification prevents propagation. Taisne and Tait
2118 (2011) conducted experiments involving magma solidification during migra-
2119 tion. They found that despite a constant flux of injection, the propagation
2120 occurred in successive steps alternating with a phase of arrest associated
2121 with inflation. In those experiments the average behavior of the propagation
2122 could be captured through the surface-creation rate, rather than the upward
2123 propagation rate. The authors compare the surface-creation rate to the rate
2124 of seismicity associated with magmatic intrusion and show how this approach
2125 may lead to early estimation of the physical conditions driving the injection,
2126 such as the volumetric flow of magma injected.

2127 Heat transfer is an important process during the evolution of continental
2128 rifts. Bialas et al. (2010) developed a 2D numerical approach to investi-
2129 gate the relation between the volume of magma intruded into dikes and the
2130 amount of lithospheric weakening. The simulations consider the available rift-
2131 ing force and the lithospheric structure (including a depth- and temperature-
2132 dependent rheology) and a boundary element model for the dikes based on
2133 Crouch and Starfield (1983) and Weertman (1971a). The amount of magma
2134 (and total dike thickness) needed to weaken the lithosphere is discussed in
2135 terms of model parameters as well as the conditions under which magmatic
2136 or amagmatic rifts develop on Earth.

2137 Daniels et al. (2014) obtained numerical solutions to the conservation of
2138 energy equation to quantify the transfer of heat from dikes to the continental
2139 crust during rifting. The thermal models are benchmarked against a priori
2140 constraints on crustal structure and dike intrusion episodes in Ethiopia. The
2141 study finds that through sequences of dike intrusions, the crust heats and
2142 weakens rapidly (in less than 1 Ma). The model is applied to the Main
2143 Ethiopian Rift (MER) and to the Red Sea rift (RSR) in Afar, which show
2144 a different elastic thickness and seismogenic depths. By applying the heat
2145 transfer models to these constraints, converted into constraints for crustal
2146 temperatures, Daniels et al. (2014) calculate that no more than half of the
2147 MER extension since ≈ 2 Ma has been accommodated by dike intrusion.

2148 *5.15. Volatile exsolution and fragmentation in dikes*

2149 In recent years there has been significant progress in modelling the evo-
2150 lution of the physical properties of magma with changing pressure in a reser-
2151 voir Mastin and Ghiorso (2000); Dobran (2001); Huppert and Woods (2002);
2152 Sahagian (2005). However, models of dike propagation that include such

2153 systematics are rare and, in particular, the effect of gas exsolution on a
 2154 propagating dike remains poorly understood. In general, three directions
 2155 have been taken: modeling the bulk effects on compressibility of bubble for-
 2156 mation, without entering in the details of phase transitions; modeling the
 2157 dynamics or the effects of a gas pocket at the top of the dike; coupling gas
 2158 exsolution laws with the equations of flow for the magma.

2159 Pioneering work by Lister (1990b) shows how the accumulation of gas
 2160 at the tip of the dike affects its shape. These results were obtained by
 2161 assuming a stationary solution and therefore do not account for the effect of
 2162 decompression as the dike ascends. Following the experimental work done
 2163 by Menand and Tait (2001), Maimon et al. (2012) model a gas pocket at the
 2164 top of a propagating dike. They find that any gas pocket forming by ascent
 2165 and accumulation of bubbles within the dike will propagate at a faster speed
 2166 than the magma below, and will escape the dike by fracturing the rock ahead.

2167 For viscous magma, and neglecting the motion of the bubbles into the
 2168 magma, Taisne and Jaupart (2011) shows that compressibility and fragmen-
 2169 tation processes lead to counterintuitive results. The mass conservation in
 2170 the system can be written as $\rho_m W \Sigma = Q$, where ρ_m represents magma den-
 2171 sity, W the mean upward velocity, Σ the cross section of the dike and Q
 2172 the mass flux. This could be re-written as a volume conservation, such as
 2173 $Q/\rho_m = W \Sigma$. This way, it is easy to see that a decreasing density will in-
 2174 duce either an increasing velocity with thinning of the dike, or a decreasing
 2175 velocity with fattening of the dike. In order to understand this behavior we
 2176 can write the modified Navier-Stokes equation as follows:

$$\frac{\partial P_e}{\partial z} = -\frac{3\mu}{2h^3}\phi + (\rho_s - \rho_m)g, \quad (36)$$

2177 where the two terms on the right-hand side represent the viscous pressure
 2178 drop and the buoyancy. This equation shows that the viscous pressure drop
 2179 tends to decrease the elastic pressure gradient and that this gradient is max-
 2180 imized when viscosity is negligible, while buoyancy tends to increase it. As-
 2181 suming a viscous mixture of magma and gas, the continuous increase of the
 2182 buoyancy induced by the decompression of the rising magma will induce a
 2183 thinning and increasing velocity of the propagating dike. While approach-
 2184 ing the surface, the gas volume fraction will increase and fragmentation may
 2185 occur within the propagating dike. In this case a sudden drop in viscos-
 2186 ity occurs because the mixture will change from magma bearing bubbles to
 2187 gas bearing droplet of magma, inducing an increase of the elastic pressure

2188 gradient.

2189 This gradient is maximized when viscosity is negligible, thus the elastic
2190 pressure gradient in the fragmented region is greater than the one in the vis-
2191 cous region. This explains why we have an inflation of the fragmented region
2192 with respect to the viscous region, and why deceleration in the propaga-
2193 tion is induced by the fragmentation. In other words that increase of elastic
2194 pressure within the fragmented region induce inflation that allow magma
2195 accumulation without involving thinning and acceleration.

2196 *5.16. Coupling to the asthenosphere*

2197 In many settings where magma traverses the lithosphere and crust in
2198 dikes, the magmatic source is located in the asthenosphere. Magma transport
2199 within the asthenosphere is thought to be by flow through the permeable net-
2200 work of pores between mantle grains (e.g. McKenzie, 1984; Scott and Stevenson,
2201 1986; Spiegelman and McKenzie, 1987); under this theory, magma ascends
2202 by porous flow to the base of the lithospheric cold thermal boundary layer.
2203 The behaviour of the magma at the base of the lithosphere remains an open
2204 question. Sparks and Parmentier (1991) considered magmatic interaction
2205 with the bottom of the lithosphere, where the conductive geotherm above
2206 intersects the geotherm of the approximately adiabatic mantle below. Here
2207 the temperature of the magma/mantle system drops below the melting tem-
2208 perature, creating a barrier to porous flow that has typically been consid-
2209 ered as impermeable. This barrier is thought to stall the ascent of melts,
2210 creating a boundary layer where magma accumulates with an overpressure
2211 due to its buoyancy. If the boundary is sloping, the gradient of this over-
2212 pressure has a horizontal component that drives magma to migrate later-
2213 ally (Sparks and Parmentier, 1991). A thermo-mechanical instability (Katz,
2214 2008) or mantle heterogeneity (Katz and Weatherley, 2012) can interrupt
2215 this lateral transport and cause melt to pool at the base of the lithosphere,
2216 where magmatic overpressure may become large.

2217 Magmatic overpressure in this setting should lead to dike propagation
2218 into the lithosphere above. Until recently, however, models of diking and
2219 brittle failure were distinct from models of porous melt migration. Work
2220 by Havlin et al. (2013) and Keller et al. (2013), taking two different ap-
2221 proaches, addresses this disconnect and seeks to quantify the magmatic inter-
2222 action between the asthenosphere and the lithosphere. Havlin et al. (2013)
2223 consider a model in which a one-dimensional asthenospheric column feeds

2224 magma upward by porous flow to an impermeable boundary, where a de-
2225 compacting boundary layer develops with increasing porosity and overpres-
2226 sure. Dike initiation and propagation from this boundary layer are modelled
2227 by theory simplified from Lister (1990b), Meriaux and Jaupart (1998), and
2228 Roper and Lister (2005). Magma accumulates until the overpressure reaches
2229 a critical value given by $P_{\text{crit}} = K_c/\sqrt{h} - \sigma$, where $K_c = 1$ MPa is the critical
2230 stress intensity factor, h is the height of an incipient dike, and $\sigma = 5$ or
2231 10 MPa is the ambient tectonic stress normal to the dike (tension positive).
2232 Havlin et al. (2013) assume the pressure gradient within a propagating dike
2233 is balanced by buoyancy forces and viscous resistance to flow; they then com-
2234 pute the magmatic flux Q that satisfies this balance and also matches the
2235 reservoir overpressure P_d and dike-tip criterion for propagation. The over-
2236 pressure in the decompaction layer evolves with time according to a pressure
2237 diffusion equation for a compressible, poroelastic material (e.g. Brace et al.,
2238 1968; Wong et al., 1997). The dike and the decompaction layer are coupled
2239 by matching Q and P_d , giving a consistent model that predicts the growth of
2240 dike height based on magmatic flux and volume conservation. Dike growth
2241 ceases when the dike freezes across or when the dike-magma flux goes to
2242 zero. The reservoir overpressure then grows steadily until it again reaches
2243 the critical value; the duration of the complete cycle is used to define a dike-
2244 recurrence time interval as a function of system parameters. Havlin et al.
2245 (2013) then use the recurrence interval, predicted dike spacing, and total
2246 dike volume to compute a volumetric flow rate out of the asthenosphere and
2247 into the lithosphere. This, in turn, implies a heat flux that they use to predict
2248 the thermal erosion of the lithosphere.

2249 The strength of the approach of Havlin et al. (2013) is that it uses sym-
2250 metry assumptions, analytical methods, and approximations to bridge scales
2251 of magmatic accumulation in the asthenosphere (10^3 – 10^4 m, 10^4 – 10^5 yr)
2252 to scales of transport through dikes in the lithosphere (10^{-2} – 10^2 m, 10^{-3} –
2253 10^{-2} yr). However these idealisations also give rise to an important weak-
2254 ness: an inability to model the heterogeneous conditions present in a more
2255 realistic view of the lithosphere/asthenosphere system. These strengths and
2256 weaknesses are reversed in the approach taken by Keller et al. (2013), who
2257 extend the formulation of McKenzie (1984) and Bercovici et al. (2001) to
2258 incorporate a visco-elasto-plastic rheology with an effective stress principle
2259 (Terzaghi, 1943; Skempton, 1960). Here the idea is to capture a broad range
2260 of magma/mantle interaction, from viscous deformation to brittle/elastic dik-
2261 ing and faulting, within a single, continuum formulation. Solutions to the

2262 nonlinear system of governing equations are obtained numerically. Model
2263 behaviour is studied through a series of test-cases where a pool of magma
2264 is injected at the bottom boundary of a $\sim 4 \times 6$ km domain filled with ho-
2265 mogeneous solid that is subject to an imposed extensional strain rate of
2266 $10^{-15} \text{ sec}^{-1}$. These calculations show three categories of roughly distinct
2267 behaviour depending on the imposed viscosity of the host rock. The first
2268 of these corresponds to distributed, viscous deformation of low-viscosity as-
2269 thenospheric mantle (10^{18} Pa s), which has been previously modelled. At
2270 moderate host-rock viscosity (10^{21} Pa s), magmatic transport becomes lo-
2271 calised into channels caused by large magma overpressure at small shear
2272 stress. This may be considered an intermediate regime before dikes emerge
2273 in the case of the high host-rock viscosity representing mantle lithosphere
2274 (10^{22} Pa s , resulting in extensional stress of $\sim 10 \text{ MPa}$). In this latter case,
2275 elasto-plastic deformation associated with tensile failure is dominant, and
2276 a sub-vertical, melt-rich crack propagates upward at low magmatic over-
2277 pressure. At the highest imposed viscosity (10^{23} Pa s), tensile and shear
2278 failure combine to localise deformation onto a system of dikes and normal
2279 faults. Keller et al. (2013) go on to consider lithospheric models with depth-
2280 dependent strength profiles of the crust and lithosphere that produce complex
2281 patterns of deformation and melt transport from the asthenosphere into dikes
2282 and faults.

2283 Due to limitations of model resolution, dikes in the models of Keller et al.
2284 (2013) actually have a porosity of $\sim 25\%$ and a width that is one or two grid
2285 cells ($\sim 40 \text{ m}$); the viscous resistance to flow within them arises from the
2286 Darcy drag term in the governing equations. Hence these dikes behave dif-
2287 ferently from classical models of dikes as fluid-filled cracks. In particular, they
2288 are much wider and grow much slower than those modelled semi-analytically
2289 by Havlin et al. (2013), despite the fact that both studies consider the same
2290 range of extensional tectonic stress. And because Keller et al. (2013) do not
2291 consider thermal evolution in their formulation, their dikes cannot freeze, and
2292 hence do not have a recurrence interval, making it difficult to compare with
2293 the long-term behaviour predicted by Havlin et al. (2013). However, numer-
2294 ical simulations provide the flexibility to model complex lithospheric archi-
2295 tecture and spatially variable magmatic sources. Finally, both approaches
2296 neglect the (thermo)dynamics of the narrow, transitional zone between pore-
2297 hosted and dike-hosted melt transport, where temperatures remain on or
2298 near the solidus; Havlin et al. (2013) formally neglect this transition while
2299 Keller et al. (2013) do not resolve it. The consequences of this deficiency

2300 are unknown because there is little (if any) understanding of the dynamics
2301 localised in this zone.

2302 Despite their shortcomings, these two studies represent complementary
2303 approaches to coupling magma transport through the asthenosphere and
2304 lithosphere. They seem to initiate two avenues of research leading toward
2305 models of (for example) off-axis volcanism at mid-ocean ridges, batholith em-
2306 placement, continental rifting, and the seismic character of the lithosphere-
2307 asthenosphere boundary.

2308 **6. Investigation along with industrial hydraulic fractures**

2309 Although hydraulic fracture can provide a useful analogue for diking, it
2310 is worth emphasising that dikes and industrial hydraulic fractures differ in a
2311 variety of ways. Magma can solidify during dike propagation so that a com-
2312 plete model should consider the coupling between these processes. Buoyancy
2313 forces typically drive dike propagation and but not hydraulic fractures. In-
2314 dustrial hydraulic fractures are driven by fluids that are orders of magnitude
2315 less viscous; leak-off of fluid into the surrounding formation is typically im-
2316 portant for industrial hydraulic fractures and expected to be negligible for
2317 dikes; industrial hydraulic fractures attain maximum volumes that are orders
2318 of magnitude smaller than dikes.

2319 Still, there are many similarities between the mechanisms driving dike
2320 propagation and industrial hydraulic fractures. Both processes are funda-
2321 mentally fluid-driven cracks and are typically modeled by the coupled equa-
2322 tions of elasto-hydro-dynamic crack propagation, described in Section 4.3.
2323 Both processes result in induced seismicity and microseismicity. Both pro-
2324 cesses can result in the formation of essentially single features, i.e. a localized
2325 dike or hydraulic fracture, or they can result in a swarm or network of fluid-
2326 driven cracks. Both dikes and hydraulic fractures can be altered in their
2327 propagation when they encounter barrier layers or faults. These issues are
2328 explored in more detail in the subsections that follow.

2329 There is a broad synergy between the study of dikes and hydraulic frac-
2330 tures. Not only are the mechanisms similar, these studies are naturally com-
2331 plementary because dikes provide detailed, mappable data regarding the fi-
2332 nal configuration attained by fluid-driven cracks, albeit with relatively little
2333 known about the boundary conditions associated with the system that re-
2334 sulted in the observed geometry. And while the boundary conditions are

2335 well known for industrial hydraulic fractures, the final geometry is never di-
2336 rectly observable (with the exception of experiments in which hydraulic frac-
2337 tures are placed ahead of the advance of a mine and then directly mapped
2338 (Elder, 1977; Lambert et al., 1980; Warpinski et al., 1982; Warpinski, 1985;
2339 Warpinski and Teufel, 1987; Diamond and Oyler, 1987; Steidl, 1991; Jeffrey et al.,
2340 1992, 1995; van As and Jeffrey, 2002; Jeffrey et al., 2009)). Hence, dike and
2341 hydraulic fracture studies are complementary because the quality of data and
2342 data gaps for each case are the complement of the other.

2343 *6.1. Growth barriers*

2344 Growth barriers are of great interest both to dike propagation and hy-
2345 draulic fracturing. In dike propagation, layers that serve as growth bar-
2346 riers control the potential for arrest of ascending dikes and therefore are
2347 among the most important considerations in predicting volcanic hazards
2348 (Gudmundsson et al., 1999; Gudmundsson, 2005). Furthermore, growth bar-
2349 riers are essential for understanding the transition of dikes to sills (Rivalta et al.,
2350 2005; Kavanagh et al., 2006; Maccaferri et al., 2011; Taisne et al., 2011b)
2351 and the concomitant potential for magma-chamber formation (Gudmundsson,
2352 2011a).

2353 In industry, the issue of vertical propagation of hydraulic fractures is re-
2354 ferred to as “height growth” because it deals with the limitations to the
2355 vertical extent, or height, of vertically oriented but laterally propagating
2356 blade-like hydraulic fractures in the presence of bounding layers that serve
2357 as partial impediments to vertical propagation. In this context, ascertain-
2358 ing the presence, or lack thereof, of barriers to vertical hydraulic fracture
2359 growth is paramount for design of hydraulic fractures that grow in produc-
2360 tive strata and avoid growth into layers that will not positively contribute to
2361 the economics of the well (“growth out of zone”). Microseismic mapping of
2362 thousands of hydraulic fracturing stages from shale reservoirs in North Amer-
2363 ica has shown that the separation between underground water sources and
2364 shale reservoirs is so great relative to the height growth that it is probably of
2365 little relevance to the overall environmental risk profile of shale gas/oil pro-
2366 duction (Fisher and Warpinski, 2012; Warpinski, 2013). However, the ability
2367 to predict height growth could be useful for understanding the potential for
2368 contamination of ground water associated with stimulating much shallower
2369 coal-seam gas wells (EPA, 2004).

2370 The geometrical similarities between observed dikes and hydraulic frac-
2371 tures associated with a growth barrier are striking. Dikes have been observed

2372 to cross, arrest, or deflect when they encounter potential barriers (Fig. 21)
2373 (Gudmundsson et al., 1999; Gudmundsson and Loetveit, 2005; Gudmundsson,
2374 2011a). Inspired by these contrasting behaviors, experiments in gelatine
2375 have obtained analogues to each case based on variation of the relative ma-
2376 terial properties, especially the stiffness, in a two-layered system (Fig. 22)
2377 (Rivalta et al., 2005; Kavanagh et al., 2006). Similarly, mine-through map-
2378 ping of hydraulic fractures in coal seams shows both arrest and deflection
2379 to the horizontal when the hydraulic fracture encounters a relatively stiffer
2380 and stronger rock layer (Figure 23a-b) (Elder, 1977; Lambert et al., 1980;
2381 Diamond and Oyler, 1987; Jeffrey et al., 1992). Furthermore, mine-through
2382 experiments in coal demonstrate meters to tens-of-meter scale examples of
2383 more complex geometries that can arise in the presence of multiple layers.
2384 Notably there is a tendency for hydraulic fractures to cross stiff/strong layers
2385 if they are thin and bounded by relatively thick soft/weak coal layers and,
2386 intriguingly, to grow on the contact above a thin stiff layer that is separated
2387 from a thick stiff layer by a thin soft layer (Fig. 23c) (Lambert et al., 1980;
2388 Diamond and Oyler, 1987; Jeffrey et al., 1992, 1995).

2389 Although the observations are similar, there are nuanced but impor-
2390 tant differences in the interpretations made by research communities study-
2391 ing dike and hydraulic fracturing. The starting point is actually similar,
2392 with both communities recognizing the fundamental importance of vari-
2393 ations in the in-situ stresses and the mechanical properties of the layers
2394 (Simonson et al., 1978; Teufel and Clark, 1984; Warpinski and Teufel, 1987;
2395 Gudmundsson et al., 1999; Economides and Nolte, 2000; Rivalta et al., 2005;
2396 Kavanagh et al., 2006; Gudmundsson, 2011a; Maccaferri et al., 2011). How-
2397 ever, the dike research community has tended to consider the role of stresses
2398 in terms of the impact of rotation of the orientation of the least compressive
2399 stress direction in the vicinity of a magma chamber, the surface, or a volcanic
2400 edifice (Gudmundsson et al., 1999; Gudmundsson, 2011a; Maccaferri et al.,
2401 2011); it has placed a relatively strong emphasis on the importance of the
2402 contrast in mechanical properties between the layers, most notably the re-
2403 lative stiffness (Rivalta et al., 2005; Kavanagh et al., 2006). In contrast, the
2404 hydraulic fracturing research community considers height growth to be pri-
2405 marily driven by the relative magnitude of the minimum in-situ compres-
2406 sive stress among the layers; contrasts in strength are considered to play
2407 a relatively minor role and the stiffness contrasts among layers are consid-
2408 ered to be of key importance — but this is primarily via their impact on
2409 stress variation among the layers when they are subjected to remote tectonic

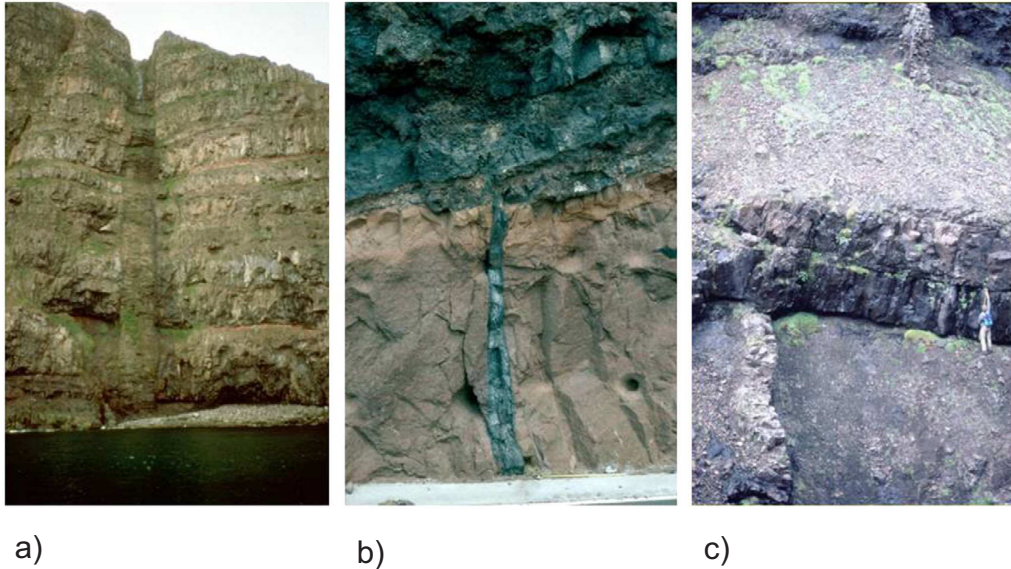


Figure 21: Dykes meeting contacts. a) A vertical, 6-m-thick dyke penetrates all the basaltic lava flows in North Iceland. b) A vertical, 0.3-m-thick dyke becomes arrested at the contact between a pyroclastic layer and a basaltic lava flow in Tenerife, Canary Islands. c) The dyke becomes singly deflected along the contact to form a sill between a basaltic sheet and a lava flow, Southwest Iceland. From Gudmundsson (2011a), with permission.

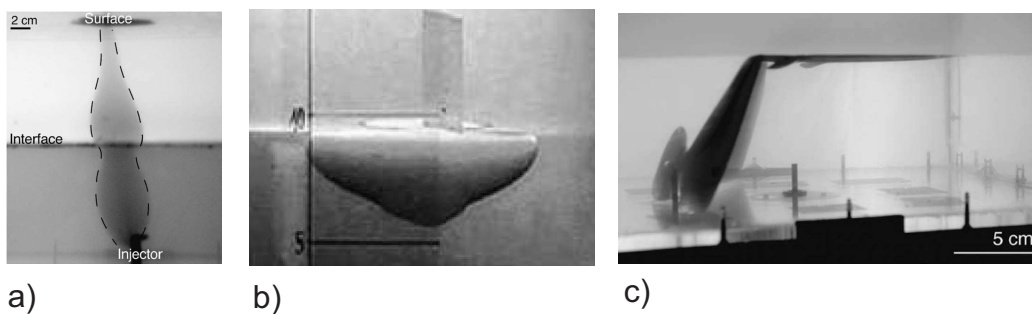


Figure 22: Experiments in gelatine showing: a) Analogue dike crossing into a less dense and less rigid upper layer, from Kavanagh et al. (2006), with permission, b) Arrest when an ascending analogue dike encounters a relatively stiff layer ($G_U/G_L = 5.5$ for shear modulus G) and lacks sufficient buoyant driving force to propagate in the upper layer, after Rivalta et al. (2005), with permission Fig 5a, c) Formation of an analogue sill along the contact between a lower soft layer and an upper stiff layer in a case with $E_U/E_L = 6.6$ for Young's modulus E , from Kavanagh et al. (2006), with permission.

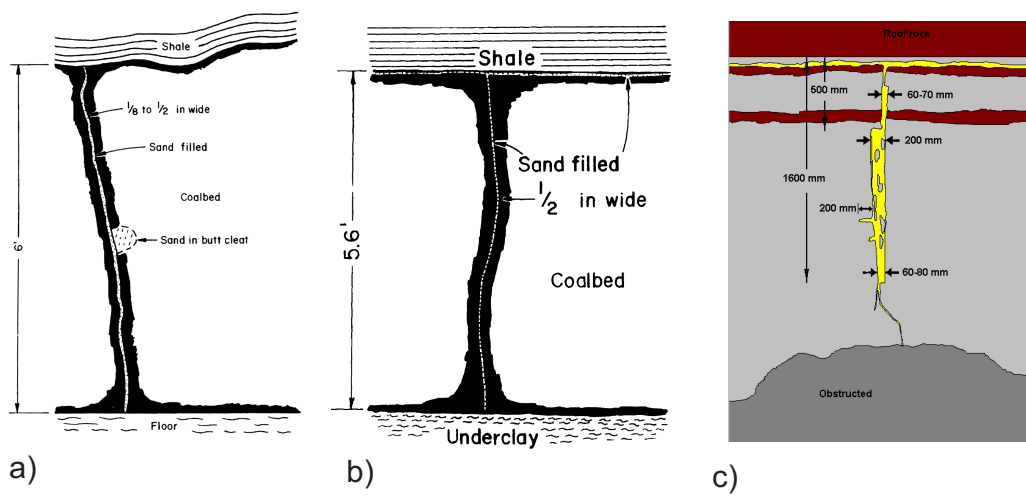


Figure 23: Examples from mine through mapping of hydraulic fractures in coal showing: a) Arrest of height growth at relatively stiff/strong roof and floor layers, from Diamond and Oylar (1987), with permission, b) Formation of horizontal growth component along the contact between the coal seam and a relatively stiff/strong roof rock, from Diamond and Oylar (1987), with permission, c) Direct crossing of the stiff/strong but thin layer between two coal layers and growth of a horizontal component above a relatively stiff/strong layer that is separated from the stiff/strong roof rock by a relatively thin layer of soft/weak coal, redrawn after Jeffrey et al. (1992), with permission.

2410 loading (Teufel and Clark, 1984; Warpinski and Teufel, 1987; Naceur et al.,
2411 1990; Gu and Siebrits, 2008). Put another way, both communities agree that
2412 stiff/strong layers serve as barriers to growth, however the hydraulic fractur-
2413 ing community typically understands this to be because stiff layers tend to
2414 have higher stresses than neighboring lower stiffness layers, while the dike
2415 research community tends to interpret the contrasts in material properties
2416 themselves to be more directly responsible for dike arrest.

2417 *6.2. Induced seismicity*

2418 Seismic energy is released during both hydraulic fracturing and dike prop-
2419 agation. The timing and location of seismic events provide valuable in-
2420 formation on fracture growth. For dike propagation, monitoring typically
2421 focuses on events with magnitudes larger than 1; stress transfer associ-
2422 ated with dike intrusion has been suggested to be associated with magni-
2423 tude 6 and larger earthquakes (Julian, 1983; Savage and Cockerham, 1984;
2424 Toda et al., 2002). In contrast, hydraulic fractures for the petroleum industry
2425 produce microseismicity of magnitude lower than 1, with the vast majority
2426 of events smaller than magnitude zero (Warpinski et al., 2012a; Warpinski,
2427 2013). Note that here we limit our discussion to microseismicity associated
2428 with the hydraulic fracturing itself and do not consider the regional increase
2429 in seismicity with observed magnitudes up to 5.7 that have been inferred
2430 to result from or be exacerbated by deep wastewater injection (e.g. Council,
2431 2012; van der Elst et al., 2013). Deep wastewater injection, although it is as-
2432 sociated with disposal of a waste stream generated by unconventional oil and
2433 gas production including hydraulic fracturing, takes place at greater depth,
2434 larger time scales, and lower pressures, which is to say that it is a substan-
2435 tively different process. Note also that hydraulic stimulation of geothermal
2436 wells located in deep, hot, low permeability crystalline rocks in some cases
2437 are associated with microseismicity in a similar range to what is observed
2438 for oil and gas hydraulic fracturing (Pearson, 1981; Evans et al., 2005), but
2439 has been observed to exceed magnitude 3 in both Basel, Switzerland and the
2440 Cooper Basin, Australia (Deichmann and Giardini, 2009; Mukuhira et al.,
2441 2010).

2442 The difference in magnitude range attests to the difference in scale be-
2443 tween the two processes. Still, in both dike propagation and hydraulic frac-
2444 turing there is an observed prevalence of shear (strike-slip) events that is
2445 taken to indicate that the most important mechanism for generating seismic-
2446 ity is slippage of nearby faults (Hill, 1977; Pearson, 1981; Savage and Cockerham,

2447 1984; Shapiro et al., 1997; Toda et al., 2002; Rutledge and Phillips, 2003;
2448 Shapiro et al., 2006; Warpinski et al., 2012b). For dikes, the activation of
2449 nearby faults is thought to be directly induced by the stress surrounding
2450 the intrusion (Toda et al., 2002) or from fault pressurization due to stress
2451 or thermally-induced fluid migration (Hill, 1977; Sibson, 1996). For hy-
2452 draulic fractures, the main source mechanism is commonly taken to be slip-
2453 page of critically-oriented natural fractures and faults that are pressurized by
2454 fluid which diffuses out of the hydraulic fracture and through the formation
2455 (Pearson, 1981; Rutledge and Phillips, 2003; Warpinski et al., 2012b). The
2456 parabolic shape of the seismic leading and trailing fronts associated with
2457 hydraulic fractures is taken as support for this mechanism (Parotidis et al.,
2458 2004; Shapiro et al., 2006) and, conversely, it has been proposed that the
2459 spatial evolution of microseismicity can be used to deduce the reservoir per-
2460 meability (Shapiro et al., 1997, 2002). This parabolic shape of the micro-
2461 seismic leading and trailing fronts (Fig. 24) stands in contrast to the shape
2462 of the seismic fronts observed for dikes (Fig. 5) and attests to the relative
2463 importance of fluid diffusion in the generation of microseismicity associated
2464 with hydraulic fracturing. This difference may also be associated with the
2465 relative importance of direct fault activation from deformation-induced stress
2466 changes propagating at the rate of propagation for dikes.

2467 *6.3. Networks and swarms*

2468 While models often limit consideration to simple propagation geometries,
2469 there is field evidence from the petroleum industry that certain conditions
2470 lead to complex, network-like growth. The generation of complex hydraulic
2471 fracture networks in the Barnett shale in Texas, USA, was first inferred
2472 from microseismic data in the early 2000s (Maxwell et al., 2002; Fisher et al.,
2473 2004). It is one of the modern petroleum industry’s most influential discover-
2474 ies because it is believed that this network growth is an important contributor
2475 to the economical production of gas from the Barnett formation and it there-
2476 fore significantly shaped the stimulation strategies that have been employed
2477 as shale gas production has expanded to commercial scale to several other
2478 formations in North America (King, 2010).

2479 Meanwhile, numerical analysis has demonstrated that the relative dif-
2480 ference between the least principal stress, which is invariably horizontally-
2481 directed in shale reservoirs, and the other horizontal principal stress compo-
2482 nent is a determining factor in whether hydraulic fractures grow as networks
2483 or localized features (Olson and Dahi-Taleghani, 2009; Kresse et al., 2013).

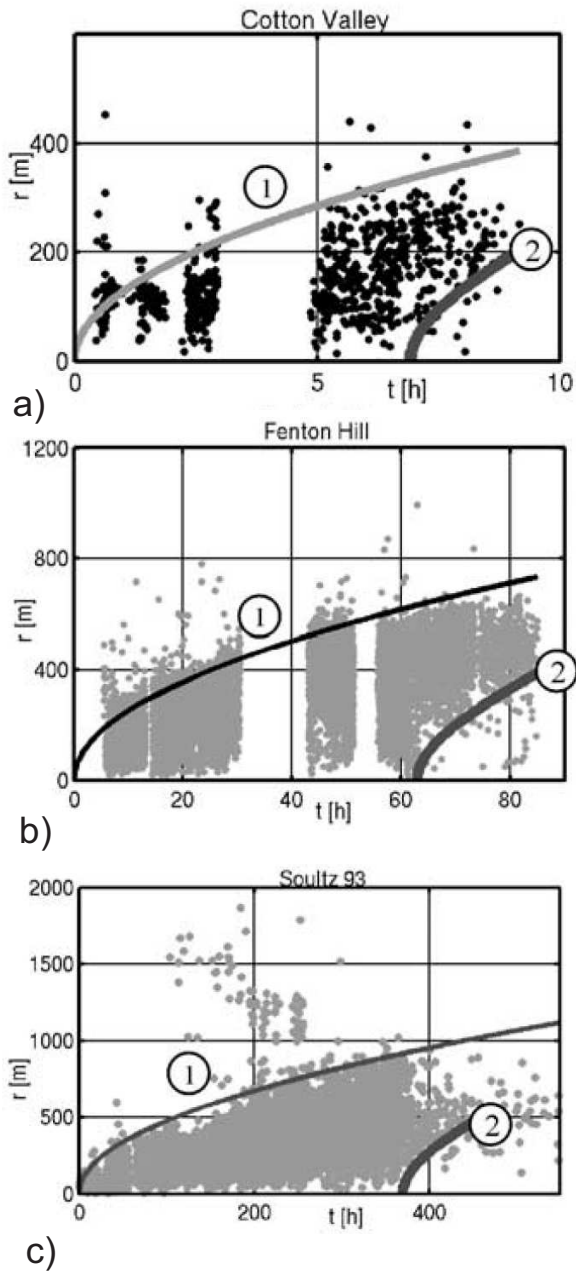


Figure 24: Distance r between the microseismic event and the injection point as a function of time, where the seismic back fronts (curve 2) and parabolic envelopes (curve 1) are computed using the same diffusivity for each case study according to the methods in Parotidis et al. (2004). Examples are: a) Cotton Valley gas field, Texas, USA; b) Fenton Hill hot rock geothermal site, New Mexico, USA; c) Soultz-sous-Forêts hot rock geothermal site, France. From Parotidis et al. (2004), with permission.

2484 For the Barnett shale, the two horizontal stresses are thought to be very
2485 nearly equal (e.g. Abbas et al., 2013), and according to numerical models this
2486 scenario is ideally suited to network-like growth. Additionally, an analytical
2487 model (Bunger et al., In Press) has shown that injection from multiple entry
2488 points will tend to result in a fracture spacing that is about 1.5 times the
2489 vertical extent of hydraulic fractures which are limited in height by barrier
2490 layers (see Section 6.1); this explains the observed tendency of hydraulic
2491 fractures to attain a spacing of the main well-transverse network branches of
2492 around 150 m in the Barnett where their vertical extent is limited to about
2493 100 m (Fisher et al., 2004).

2494 In summary, the experience so far of the petroleum industry, when com-
2495 bined with numerical and analytical modeling, has identified at least two
2496 ingredients for network growth: 1) limited height growth and 2) relatively
2497 similar horizontal principal stresses. While there are almost certainly other
2498 important factors in network growth of hydraulic fractures, the two that
2499 have been identified so far can also give useful insights for other applica-
2500 tions. Firstly, these factors explain why Engineered Geothermal System
2501 (EGS) trials have rarely, if ever, achieved a complex, interconnected growth
2502 of hydraulic fractures (Jung, 2013) in spite of a stimulation strategy that
2503 is designed with that goal (Tester et al., 2006). EGS sites have typically
2504 had relatively substantial differences between the minimum and intermed-
2505 iate principal stress magnitudes (e.g. Barton et al., 1988; Valley and Evans,
2506 2007) and the massive nature of the formations means they do not provide
2507 height-limiting growth barriers (or similarly, one of the lateral extents when
2508 hydraulic fractures are oriented in a horizontal plane). Without these ingre-
2509 dients it is not surprising that observations over the past decades point to a
2510 strong tendency to generate localized growth.

2511 Beyond engineering applications, the ingredients for network growth can
2512 also give insights into the mechanisms leading to growth of giant radiat-
2513 ing dike swarms, of which there are 199 documented on Earth, 163 doc-
2514 umented on Venus, and which are also abundant on Mars (Ernst et al.,
2515 2001). The abundance of these giant swarms, as well as smaller-scale vol-
2516 canic swarms (e.g. Odé, 1957; Gudmundsson, 1983; Walker, 1986; Sigurdsson,
2517 1987; Paquet et al., 2007), indicates that the conditions for their formation
2518 are not pathological. Still, the formation of closely-spaced arrays of dikes ap-
2519 pears to be specific to laterally-propagating, blade-like dikes that are limited
2520 in height. Drawing on a modification of the analytical hydraulic fracturing
2521 model developed by Bunger (2013), Bunger et al. (2013) have shown that the

2522 characteristic spacing that naturally seems to arise in dike swarms is similar
2523 to the dike height, consistent with model predictions. This discovery suggests
2524 that limited height, i.e. blade-like growth, is not only a fundamental ingredi-
2525 ent for growth of arrays of hydraulic fractures but also for the formation of
2526 dike swarms.

2527 **7. Outlook: Perspectives and challenges**

2528 Despite the many advances reviewed above, we remain far from having an
2529 ideal dike model. The single most important development that could drasti-
2530 cally change the way we model dikes would be to move into three dimensions.
2531 While many problems are well approximated by 2D models, there are many
2532 others where a 3D model would be instrumental for the understanding of the
2533 physical process. In particular, 2D models of ascending dikes do not allow the
2534 possibility of lateral extension, while this is how dikes are observed to behave
2535 in the field (Gudmundsson, 2011b) and in experiments. The transition from
2536 vertical to horizontal propagation is very important for volcanic applications,
2537 when magma contained in dikes becomes neutrally or negatively buoyant and
2538 propagates laterally (if pressurized or driven by external gradients).

2539 A possible strategy to solve this problem is to explore methods used in
2540 the industry to model hydraulic fractures and adapt those numerical models
2541 to dikes. It is also necessary that experimental work focuses on providing
2542 observations to constrain those models. However, large dike injections are
2543 often in areas that are infrequently monitored; for example, mid-ocean ridges,
2544 where most large dike injection are probably occurring, are deep below the
2545 ocean and are hence difficult and expensive to access. Future studies on the
2546 currently ongoing dike intrusion event at Bárðunga, Iceland, will certainly
2547 lead to a step in our understanding of the dynamics of dikes.

2548 Dike models must be based on reliable estimates of large-scale crustal
2549 properties such as rock toughness and elastic moduli measured at low fre-
2550 quencies. On the other hand, constrains from models may provide unique
2551 insight into rock parameters that will likely be a key to progress in some
2552 far-reaching debates in geomechanics.

2553 While waiting for progress in 3D modeling of dikes, we can focus on
2554 some outstanding questions that can be at least partially addressed with 2D
2555 models:

- 2556 • The different phases of nucleation, propagation and stopping of dikes;

- 2557 • Interaction between dikes and their feeding sources;
- 2558 • The rifting cycle on Mid Ocean Ridges: Rifting episodes, post-diking
2559 and inter-diking deformation (in parallel with earthquakes occurring
2560 on plate boundaries);
- 2561 • Interaction between faulting and diking;
- 2562 • Dike-faulting/tectonics interactions (e.g. Kilauea and Etna, where large
2563 tectonic faults interact with rifts);
- 2564 • Incorporation of realistic magma properties including volatile exsolu-
2565 tion, realistic rheology (sudden changes in viscosity are critical), and
2566 magma compressibility;
- 2567 • Development of volcanic conduits and vents from dikes,
- 2568 • Development of magma reservoirs from dikes,
- 2569 • Improved forecasts of the time, size and location of potential eruptions;
- 2570 • Understanding the effect of dikes on local or large-scale crustal proper-
2571 ties (thermal effects, permeability changes, mechanical response of the
2572 medium, seismic properties),
- 2573 • Understanding the role played by magma in the development of major
2574 tectonic features such as rifts, subduction zones, volcanic arcs.

2575 Each individual application may be best solved with a tailored approach.
2576 The details of tip processes or of the velocity of dikes may be studied most
2577 effectively by including only the fluid flow within the dike. On the other
2578 hand, the trajectory of dikes or other geometrically complex processes may
2579 be studied by simplifying or neglecting any viscous flow within the crack.

2580 The complementarity of methods (laboratory and numerical modeling)
2581 has proven very useful in the past and will remain so in the future. It would
2582 be very helpful also to increase communication between disciplines, in partic-
2583 ular between geophysics, petrology and field geology; more communication
2584 with industry would be beneficial in order to take advantage of the comple-
2585 mentarity of diking and hydraulic fracturing.

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