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R&D for green technologies in a dynamic oligopoly: Schumpeter, Arrow and inverted-U's[1]

Gustav Feichtinger,[2] Luca Lambertini,[3] George Leitmann[4] and Stefan Wrzaczek[5]

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[2]Institute of Statistics and Mathematical Methods in Economics, Vienna University of Technology

[3]Corresponding author: Department of Economics, University of Bologna, Strada Maggiore 45, 40125 Bologna, Italy; phone: +39-0512092623; fax: +39-0512092664; luca.lambertini@unibo.it

[4]College of Engineering, University of California at Berkeley

[5]Department of Business Administration, University of Vienna

Abstract

We extend a well known differential oligopoly game to encompass the possibility for production to generate a negative environmental externality, regulated through Pigouvian taxation and price caps. We show that, if the price cap is set so as to fix the tolerable maximum amount of emissions, the resulting equilibrium investment in green R&D is indeed concave in the structure of the industry. Our analysis appears to indicate that inverted-U-shaped investment curves are generated by regulatory measures instead of being a 'natural' feature of firms' decisions.

JEL Codes: C73, L13, O31

Keywords: dynamic games, oligopoly, environmental externality, R&D

1 Introduction

The departure point of the analysis illustrated in this paper lies at the intersection between two different debates, one being centered upon the relation between competition and innovation, whose most recent development is known as the *Schumpeterian growth theory* initiated by Aghion and Howitt (1998), the other belonging to environmental economics and focussing on the optimal design of policy instruments, such as environmental standards, pollution rights and Pigouvian taxation, to stimulate firms' investments in abatement and/or replacement technologies (for an updated survey, see Lambertini, 2013).

The acquired industrial organization approach to the bearings of market power on the size and pace of technical progress can be traced back to the indirect debate between Schumpeter (1934, 1942) and Arrow (1962) on the so-called Schumpeterian hypothesis, which, in a nutshell, says that one should expect to see an inverse relationship between innovation and the intensity of competition or market structure. Irrespective of the nature of innovation (either for cost reductions or for the introduction of new products), a large theoretical literature attains either Schumpeterian or Arrovian conclusion (for exhaustive accounts, see Tirole, 1988; and Reinganum, 1989).[1] That is, partial equilibrium theoretical IO models systematically predict a *monotone* relationship, in either direction.

The picture drastically changes as soon as one takes instead the standpoint of modern growth theory. In particular, Aghion *et al.* (2005) stress

[1] See also Gilbert (2006), Vives (2008) and Schmutzler (2010) for add-on's on this discussion, where still the Schumpeter vs Arrow argument is unresolved.

that empirical evidence shows a *non-monotone* relationship between industry concentration (or, the intensity of market competition) and aggregate R&D efforts: this takes the form of an *inverted-U curve*, at odds with all existing theoretical IO models; in the same paper, the authors provide a model yielding indeed such a concave result, and fitting the data. A thorough discussion, accompanied by an exhaustive review of the related lively debate, can be found in Aghion *et al.* (2013).

One could say that the inverted-U emerging from data says that Arrow is right for small numbers, while Schumpeter is right thereafter. Alternatively, on the same basis one could also say that neither Arrow nor Schumpeter can match reality, if our interpretation of their respective views is that "competition (resp., monopoly) outperforms monopoly (resp., competition) along the R&D dimension". Be that as it may, there arises the need of constructing models delivering a non-monotone relationship between some form of R&D (for process, product or environmental-friendly innovations) and the number of firms in the industry.

With this purpose in mind, here we extend a noncooperative differential game model dating back to Leitmann and Schmitendorf (1978) and Feichtinger (1983) to describe an industry in which firms sell a homogeneous good and accumulate capacity over time through costly investments; firms' activities entail polluting emissions hindering welfare, and the government adopts a Pigouvian taxation policy aimed at providing them with an incentive to internalise the environmental externality and therefore undertake R&D projects for pollution abatement. As in the original model, the mark-up is exogenously fixed, and here is though of as an additional regulatory tool in the hands of the public authority.

Our main results can be outlined as follows. First, we show that there exists a unique open-loop equilibrium which is subgame perfect and saddle-point stable, for any pair of policy instruments. Then, taking again the mark-up and tax rate as given, we prove that the aggregate green R&D effort is monotonically increasing in the number of firms, which is a definitely Arrovian result. Subsequently, we endogenise the regulatory toolkit, allowing first the policy-maker to set that Pigouvian tax rate so as to maximise steady state social welfare; in such a case, the aggregate R&D effort is strictly convex in the number of firms. If optimal taxation is accompanied by a mark-up tailored on industry structure so as to limit the overall volume of emissions, then there emerges a general condition on the shape of the price regulation scheme whereby the industry investment is indeed concave w.r.t. the number of firms.

The remainder of the paper is organised as follows. The setup is laid out in section 2, while the equilibrium analysis is in section 3. Section 4 illustrate the design of policy tools and its consequences on aggregate R&D efforts. Concluding remarks are in section 5.

2 The game

As anticipated in the introduction, here we extend the model introduced by Leitmann and Schmitendorf (1978) and further investigated by Feichtinger (1983), to allow for the presence of an environmental externality and green R&D investments. In the remainder, we will label this framework as the 'LSF model' for brevity. The market exists over $t \in [0, \infty)$, and, as in Dragone *et al.* (2010), it is served by $N \geq 1$ *a priori* symmetric firms with individual capacity $x_i(t) \geq 0$.[2] Given a fixed profit margin $p \geq 0$, the instantaneous profit of firm i is

$$\pi_i(t) = px_i(t) - \frac{u_i^2(t)}{2} - \gamma k_i(t) - \frac{k_i^2(t)}{2} - \tau s_i, \tag{1}$$

where $\gamma > 0$ is a parameter. Capacity $x_i(t)$ changes according to

$$\dot{x}_i(t) = u_i(t) - \delta x_i(t), \tag{2}$$

where $u_i(t)$ is the investment of firm i at time t and $\delta > 0$ is the decay rate of individual capacity. $s_i(t)$ and $k_i(t)$ denote the firm's polluting emissions and R&D effort respectively, and τ is the tax rate.

The emissions of a firm follow the dynamics

$$\dot{s}_i(t) = x_i(t) - zk_i - h\sum_{j \neq i} k_j - \eta s_i(t) \tag{3}$$

where z is a positive parameter, $\eta > 0$ is the natural decay rate of emissions, and parameter $h \in [0, z)$ measures the spillover effect received from rivals' R&D activity.

The total instantaneous volume of emissions at the industry level is $S(t) = \sum_{i=1}^{N} s_i(t)$. Therefore the social welfare function at any time can be defined as

$$SW(t) = \sum_{i=1}^{N} \pi_i(t) + CS(t) - S(t) + \tau \sum_{i=1}^{N} s_i(t). \tag{4}$$

[2]In the original formulation of the model, $x_i(t)$ is firm i's sales volume, and $u_i(t)$ its advertising investment. However, one can think of these variables as representing, respectively, installed capacity (with each firm selling at full capacity at any time) and the instantaneous investment to increase it.

Each firm has two control variables, investment in capacity $u_i(t)$ and investment in green R&D $k_i(t)$. The policy maker has two instruments, the Pigouvian tax rate τ (which may be usefully thought of as incorporating the price of emission rights) and the regulated mark-up p. To avoid time inconsistency issues, we consider the policy menu applied onto the steady state only. The structure of the model identifies a linear state game (it wouldn't be so if either the policy were function of the state or the demand function were endogenously determined). Therefore, the open-loop solution is subgame perfect, respecting the original LSF formulation.

3 Equilibrium analysis

Firm i's ($i = 1, \ldots, N$) current-value Hamiltonian (from now on we suppress the time argument)[3]

$$H_i(\mathbf{s}, \mathbf{x}, \mathbf{k}, \mathbf{u}) = \pi_i + \lambda_{ii}\dot{x}_i + \sum_{j \neq i} \lambda_{ij}\dot{x}_j + \mu_{ii}\dot{s}_i + \sum_{j \neq i} \mu_{ij}\dot{s}_j \tag{5}$$

generates the following first order conditions (inner solution) for firm i's ($i = 1, \ldots, N$) controls

$$\frac{\partial H_i}{\partial u_i} = \lambda_{ii} - u_i = 0 \tag{6}$$

$$\frac{\partial H_i}{\partial k_i} = -\gamma - k_i - z\mu_{ii} - h\sum_{j \neq i} \mu_{ij} = 0 \tag{7}$$

[3] In this respect, a remark is in order: note that, in general, the objective functional π_i has to be multiplied by the general multiplier λ_0 to allow for the *abnormal case* (see e.g. Leitmann (1981)). However, in the current model that abnormal case can be ruled out as can be readily shown.

Thus we obtain the following optimal controls of firm i $(i = 1, \ldots, N)$

$$\begin{aligned} u_i^*(t) &= \lambda_{ii}(t) \\ k_i^*(t) &= -\gamma - z\mu_{ii}(t) - h \sum_{j \neq i} \mu_{ij}(t). \end{aligned} \tag{8}$$

Furthermore, each firm i obtains the following dynamic equations for the costates $(i, j = 1, \ldots, N,\ i \neq j)$

$$\begin{aligned} \dot{\lambda}_{ii} &= (\rho + \delta)\lambda_{ii} - p - \mu_{ii} - \sum_{j \neq i} \mu_{ij} v \\ \dot{\lambda}_{ij} &= (\rho + \delta)\lambda_{ij} - \sum_{j \neq i} \mu_{ij} \\ \dot{\mu}_{ii} &= (\rho + \eta)\mu_{ii} + \tau \\ \dot{\mu}_{ij} &= (\rho + \eta)\mu_{ij}. \end{aligned} \tag{9}$$

In order to characterize the optimal long run solution of the system we have to derive the equilibria of the above defined system of differential equations (i.e. state and costate equations of all firms). In this model the equilibrium is unique. For the adjoint variables we obtain $(i, j = 1, \ldots, N,\ i \neq j)$

$$\begin{aligned} \hat{\lambda}_{ii} &= \frac{1}{\rho + \delta}\Big(p - \frac{\tau}{\rho + \eta}\Big) \\ \hat{\lambda}_{ij} &= 0 \\ \hat{\mu}_{ii} &= \frac{-\tau}{\rho + \eta} \\ \hat{\mu}_{ij} &= 0. \end{aligned} \tag{10}$$

Inserting into (8) yields the following equilibrium controls $(i = 1, \ldots, N)$

$$\begin{aligned} \hat{u}_i^* &= \frac{1}{\rho + \delta}\Big(p - \frac{\tau}{\rho + \eta}\Big) \\ \hat{k}_i^* &= \frac{z\tau}{\rho + \eta} - \gamma. \end{aligned} \tag{11}$$

Using these expressions for the state equations we obtain ($i = 1, \ldots, N$)

$$\begin{aligned}
\hat{x}_i &= \frac{1}{\delta(\rho+\delta)}\Big(p - \frac{\tau}{\rho+\eta}\Big) \\
\hat{s}_i &= \frac{1}{\eta}\Big[\frac{1}{\delta(\rho+\delta)}\Big(p - \frac{\tau}{\rho+\eta}\Big) - \Big(\frac{z\tau}{\rho+\eta} - \gamma\Big)(z + h(N-1))\Big].
\end{aligned} \tag{12}$$

Since all firms are assumed to be *a priori* symmetric, we define the steady state values as $\hat{u} := \hat{u}_i$, $\hat{k} := \hat{k}_i$, $\hat{x} := \hat{x}_i$ and $\hat{s} := \hat{s}_i$. Due to the economic meaning of the model, we have to assume that the controls and the states are non-negative for all $t \in [0, \infty)$. The following Lemma provides assumptions such that the non-negativity is fulfilled in equilibrium.

Lemma 1 *The steady state variables of the state and control variables of every player i ($i = 1, \ldots, N$) are non-negative if the following assumptions on the parameters are fulfilled*

$$p(\rho+\eta) \geq \tau \geq \frac{\gamma(\rho+\eta)}{z} \tag{A1}$$

$$\frac{1}{h}\Big[\frac{1}{\delta(\rho+\delta)}\Big(p - \frac{1\tau}{\rho+\eta}\Big)\Big(\frac{z\tau}{\rho+\eta} - \gamma\Big)^{-1} - z\Big] + 1 \geq N \tag{A2}$$

Condition (A1) guarantees non-negativity of the controls (see (11)). Non-negativity of $\hat{x}_i$ is implied by (A1) and that of $\hat{s}_i$ by (A2). The analysis of the Jacobian matrix of the system shows that

Proposition 2 *The unique equilibrium $(\hat{x}, \hat{s}, \hat{u}^*, \hat{k}^*)$ is a saddle point.*

From the adjoint equations it is easy to show that $\mu_{ij}(t) = \lambda_{ij}(t) = 0$. Due to the structure of the system it is possible to derive an analytical expression of the stable path, i.e.

$$\begin{aligned}
x_i(t) &= \hat{x} + (x_{i0} - \hat{x})e^{-\delta t} \\
s_i(t) &= \hat{s} - (x_{i0} - \hat{x})\frac{1}{\eta + \delta}(e^{-\delta t} - e^{-\eta t}) + (s_{i0} - \hat{s})e^{-\eta t} \\
\mu_{ii}(t) &= \hat{\mu}_{ii} \\
\lambda_{ii}(t) &= \hat{\lambda}_{ii}
\end{aligned} \tag{13}$$

Now we are able to define a sufficient assumption that the controls and the states are positive for all $t \in [0, \infty)$.

Lemma 3 *Let (A1) and (A2) hold. Then the following assumption is sufficient to ensure that all controls and states are positive over the whole planning horizon.*

$$x_{i0} \leq \hat{x}, \qquad s_{i0} \leq \hat{s}, \qquad \delta \leq \eta.$$

Proof: $u_i^*(t) > 0$ and $k_i^*(t) > 0$ are trivial by the signs of the adjoint variables in (13). $x_i(t) > 0$ is implied by $x_{i0} \leq \hat{x}$ (see (13)). For $s_i(t)$ we obtain from (13)

$$s_i(t) = \hat{s}(1 - e^{-\eta t}) + s_{i0}e^{-\eta t} + (\hat{x} - x_{i0})\frac{1}{\eta + \delta}(e^{-\delta t} - e^{-\eta t}) \tag{14}$$

Since $\eta > 0$ the first and the second term are trivially (strictly) positive. $\delta \leq \eta$ implies $e^{-\delta t} - e^{-\eta t} \geq 0$ for all $t \geq 0$ (equality is the case only for $t = 0$). Thus also the third term of the above expression for $s_i(t)$ is non-negative.

The foregoing analysis has a seemingly not-so-intriguing ancillary implication:

Corollary 4 *Since $K^* = Nk^*$ is everywhere increasing in N for all $\tau > (\eta + \rho)\gamma/z$, the behaviour of aggregate R&D is Arrovian for any given Pigouvian policy allowing for a positive investment.*

However, there is more to it, which can be shown to emerge as soon as one admits the reasonable possibility for regulation to enter the picture along two dimensions: one is obviously τ, as is usually the case in environmental economics, the other is p, which is a specific feature of the present model. Here, the mark-up is fixed, and this fact can be interpreted as a consequence of a price cap imposed by a public authority. The research question we are about to assess in the following section is the following: is the portfolio of policy instruments $\{p, \tau\}$ going to modify the apparently monotone behaviour of aggregate R&D efforts K^* outlined in Corollary 4? And, if so, in what direction?

4 Environmental policy and aggregate investment

The bearings of p and τ on aggregate R&D incentives can be appreciated by addressing the issue in the following terms. It is already known (see Benchekroun and Long, 1998, 2002, *inter alia*) that there exists a level of Pigouvian taxation driving the industry to the first best which would be obtained under social planning. Call this tax rate $\tau^{SP}(p, N)$. This tax rate must maximise the steady state level of the social welfare function, defined as

$$SW^*(\tau) = N\pi^*(\tau) + CS^*(\tau) - N(1-\tau)s^*(\tau) \tag{15}$$

where

$$CS^*(\tau) = \frac{(a-p)Nx^*(\tau)}{2} \tag{16}$$

is consumer surplus, calculated postulating the existence of a linear and decreasing market demand function $\widehat{p} = a - Nx^*$ in which $a > 0$ is consumers' reservation price; indeed, $\widehat{p}$ is the price that would prevail if the mark-up were unregulated. Moreover, (15) accounts for the additional fact that the revenue produced by Pigouvian taxation, $N\tau s^*(\tau)$, is redistributed to consumers as a windfall.

Then, $\tau^{SP}(p,N)$ can be easily calculated by solving the necessary condition $\partial SW^*(\tau)/\partial\tau = 0$, satisfied by the unique tax rate:[4]

$$\tau^{SP}(p,N) = \frac{(\eta+\rho)\left[p(\delta-\rho)\eta - (\delta+\rho)(a\eta-2) + 2\delta(\delta+\rho)^2(z+h(N-1))z\right]}{2\delta\eta\left[1+(\delta+\rho)^2 z^2\right]}. \tag{17}$$

Now observe that

$$\frac{\partial K^*\left(\tau^{SP}(p,N)\right)}{\partial N} = 0 \text{ in } N = \max\left\{1, \widehat{N}\right\}, \tag{18}$$

$$\widehat{N} = \frac{2\delta\gamma\eta^2 + 4c\delta(\delta+\rho)^2(\gamma\eta+h-z) + p(\rho-\delta)\eta + (\delta+\rho)(a\eta-2)}{4h\delta(\delta+\rho)^2 z},$$

and

$$\frac{\partial^2 K^*\left(\tau^{SP}(p,N)\right)}{\partial N^2} = \frac{2h(\delta+\rho)^2 z^2}{\eta\left[1+(\delta+\rho)^2 z^2\right]} > 0, \tag{19}$$

showing that, if $\widehat{N} \geq 2$, then in correspondence of $\widehat{N}$ the aggregate R&D level $K^*\left(\tau^{SP}(p,N)\right)$ is indeed being minimised. Hence, in this scenario no inverted-U may arise (at most, if $\widehat{N}$ is admissible, a U-shaped curve obtains), since:

[4] The second order condition is satisfied, as

$$\frac{\partial^2 SW^*(\tau)}{\partial\tau^2} = -\frac{N\left(1+(\delta+\rho)^2 z^2\right)}{(\delta+\rho)^2(\eta+\rho)^2} < 0$$

always.

Lemma 5 *If p is given, the equilibrium aggregate R&D effort is convex in N.*

On the basis of the above Lemma, it seems that Pigouvian taxation is in itself insufficient to deliver an inverted-U R&D curve: the opposite shape does in fact appear if $\widehat{N} \geq 2$ (if so, then for a limited number of firms the Schumpeterian hypothesis is confirmed, while for sufficiently large number of firms the Arrovian position prevails).

What if p is set by the government for some purpose? Suppose first that a public agency is in charge of regulating the mark-up of this industry having in mind objectives such as the entry process, consumer surplus or the volume of industry emissions. Be that as it may, the resulting regulatory measure can be defined as $p = p(N)$, so that the mark-up is a function of industry structure. Substituting $p(N)$ into $\tau^{SP}(p, N)$, the optimal tax rate is then defined in terms of industry structure (as well as the parameters of the model), and can be relabelled as $\tau^{SP}(N)$. Then, the aggregate R&D effort at the steady state equilibrium writes as follows:

$$K^*\left(\tau^{SP}(N)\right) = \frac{N\left[z\tau^{SP}(N) - \gamma(\rho+\eta)\right]}{\rho+\eta}$$

with

$$\frac{\partial K^*\left(\tau^{SP}(N)\right)}{\partial N} = \frac{z\tau^{SP}(N) - \gamma(\rho+\eta) + Nz \cdot \partial\tau^{SP}(N)/\partial N}{\rho+\eta} \tag{20}$$

and

$$\frac{\partial^2 K^*\left(\tau^{SP}(N)\right)}{\partial N^2} = \frac{z\left[2 \cdot \partial\tau^{SP}(N)/\partial N + N \cdot \partial^2\tau^{SP}(N)/\partial N^2\right]}{\rho+\eta} \tag{21}$$

If there exists a value of N at which (20) is nil, this is implicitly identified by

$$\frac{\partial \tau^{SP}(N)}{\partial N} = \frac{\gamma(\rho+\eta) - z\tau^{SP}(N)}{Nz} < 0 \tag{22}$$

as $z\tau^{SP}(N) > \gamma(\rho+\eta)$ in order for the equilibrium R&D effort to be positive. Looking back at (17), it appears that (i) as long as p is not a function of industry structure, $\partial \tau^{SP}(p, N)/\partial N > 0$; and (ii) the derivative of the optimal tax w.r.t. N may become negative only if p is indeed a decreasing function of N.

Then, using (22), (21) becomes:

$$\frac{\partial^2 K^*\left(\tau^{SP}(N)\right)}{\partial N^2} = \frac{2\left[\gamma(\rho+\eta) - z\tau^{SP}(N)\right] + zN^2 \cdot \partial^2 \tau^{SP}(N)/\partial N^2}{N(\rho+\eta)} \tag{23}$$

whose sign determines whether the solution to $\partial K^*\left(\tau^{SP}(N)\right)/\partial N = 0$ is a maximum or a minimum.

A sensible way of modelling the role of price regulation rests on considering that, in general, $\tau^{SP}(p, N)$ - although maximising steady state social welfare - does not ensure the minimization of the externality or the attainment of any given cap $\bar{S}$ targeted by the public agency in charge of the environmental policy.

If indeed the government wants to reduce emissions to a given level $\bar{S}$, it must set the regulated price at the level solving $Ns^* = \bar{S}$, which is

$$p^{SP}\left(\bar{S}\right) = \frac{2\bar{S}\delta^2\eta^2\left(1+\varsigma^2\right) + N\left[2 + 2\delta\left(h(N-1)+z\right)(2\varsigma - \delta\Phi) - a\eta\Psi\right]}{\eta N\left[1 - \delta z\left((\delta+3\rho)z + h(N-1)(\rho-\delta)z\right)\right]} \tag{24}$$

where $\varsigma \equiv z(\delta+\rho)$, and

$$\begin{gathered}\Phi \equiv \gamma\eta + \left[\gamma\eta - h(N-1) - z\right]\varsigma^2; \\ \Psi \equiv 1 + \delta\varsigma\left[h(N-1)+z\right].\end{gathered} \tag{25}$$

Price (24) is in fact a function of N and we may further investigate the bearings of $p^{SP}\left(\bar{S}\right)$ on the equilibrium R&D effort of the industry.

The adoption of such a regulated price delivers

$$K^{*}\left(\tau^{SP}\left(p^{SP}\left(\bar{S}\right),N\right),p^{SP}\left(\bar{S}\right)\right)=$$
$$\left[\delta\eta^{2}\left(\delta-\rho\right)\bar{S}-\eta\left(az+\gamma+2z\delta\gamma\varsigma\right)N+\right.$$
$$\left.2z\left(1+\delta\varsigma\left(z+h\left(N-1\right)\right)\right)N\right]/ \tag{26}$$
$$\eta\left[1+z\delta\left(z\left(\delta+3\rho\right)-h\left(N-1\right)\left(\delta-\rho\right)\right)\right].$$

Now we can differentiate $K^{*}\left(\cdot\right)$ w.r.t. N, obtaining:

$$\frac{\partial K^{*}\left(\cdot\right)}{\partial N}=\frac{\left(\Upsilon_{1}+\Upsilon_{2}\right)\Upsilon_{3}+\Upsilon_{4}-\Upsilon_{5}}{\eta\Upsilon_{3}^{2}} \tag{27}$$

where

$$\Upsilon_{1}\equiv z\left(2-a\eta\right)-\gamma\eta$$
$$\Upsilon_{2}\equiv 2z\delta\varsigma\left[z+h\left(N-1\right)-\gamma N\right]$$
$$\Upsilon_{3}\equiv\left[1+z\delta\left(z\left(\delta+3\rho\right)-h\left(N-1\right)\left(\delta-\rho\right)\right)\right] \tag{28}$$
$$\Upsilon_{4}\equiv hz\delta\left(\delta-\rho\right)\left[\bar{S}\delta\eta^{2}\left(\delta-\rho\right)+2Nz\left(1+z\left(h\left(N-1\right)+z\right)\delta\left(\delta+\rho\right)\right)\right]$$
$$\Upsilon_{5}\equiv Nhz\delta\eta\left(\delta-\rho\right)\left[\gamma+z\left(a+2\delta\gamma\varsigma\right)\right]$$

Then, differentiating (27) w.r.t. N, we have the following:

$$\frac{\partial^{2}K^{*}\left(\cdot\right)}{\partial N^{2}}=\frac{\Upsilon_{3}^{2}\Upsilon_{2}'-2\Upsilon_{3}'\left(\Upsilon_{4}-\Upsilon_{5}\right)-\Upsilon_{3}\left[\left(\Upsilon_{1}+\Upsilon_{2}\right)\Upsilon_{3}'-\Upsilon_{4}'+\Upsilon_{5}'\right]}{\eta\Upsilon_{3}^{3}} \tag{29}$$

in which $\Upsilon_{j}'\equiv\partial\Upsilon_{j}'/\partial N$, $j=2,3,4,5$.

The equation $\partial K^{*}\left(\cdot\right)/\partial N=0$ has two roots:

$$N_{\pm}=1+\frac{1+z^{2}\delta\left(\delta+3\rho\right)}{hz\delta\left(\delta-\rho\right)}\pm\frac{\sqrt{\varsigma\Omega}}{hz^{2}\delta\left(\delta^{2}-\rho^{2}\right)\sqrt{2}} \tag{30}$$

where

$$N_+ - N_- = \frac{\sqrt{2\varsigma\Omega}}{hz^2\delta\left(\delta^2 - \rho^2\right)} > 0 \tag{31}$$

for all $\delta > \rho$, provided $\Omega \geq 0$ in such a way that $N_\pm \in \mathbb{R}$, with

$$\Omega \equiv \overline{S}hz\delta^2\eta^2 (\delta - \rho)^2 + \tag{32}$$

$$[1 + z\delta (h (\delta - \rho) + z (\delta + 3\rho))] \times$$

$$\left[4z\delta\varsigma^2 - \gamma\eta (\delta - \rho) + z (a\eta\rho - \delta (a\eta - 4)) - 2z\gamma\delta\eta\varsigma \left(\delta^2 - \rho^2\right)\right]$$

For future reference, define $\Xi \equiv 1 + z\delta (h (\delta - \rho) + z (\delta + 3\rho))$ and

$$\Lambda \equiv 4z\delta\varsigma^2 - \gamma\eta (\delta - \rho) + z (a\eta\rho - \delta (a\eta - 4)) - 2z\gamma\delta\eta\varsigma \left(\delta^2 - \rho^2\right), \tag{33}$$

which allw us to formulate the following

Lemma 6 *If* $\delta > \rho$, $\Omega \geq 0$ *for all*

$$\overline{S} \geq \max\left\{0, -\frac{\Xi \cdot \Lambda}{hz\delta^2\eta^2 (\delta - \rho)^2}\right\}.$$

If instead $\delta \in (0, \rho)$, $\Omega \geq 0$ *for all*

$$\overline{S} \in \left[0, \frac{\Xi \cdot \Lambda}{hz\delta^2\eta^2 (\delta - \rho)^2}\right].$$

Lemma 6 says that (i) if the efficiency of natural carbon sinks is higher than the discount rate, the solutions $N_\pm$ to $\partial K^* (\cdot) / \partial N = 0$ are real if $\overline{S}$ is large enough; (ii) if instead the opposite applies, $\overline{S}$ must be low enough in order for $N_\pm \in \mathbb{R}$.

The expressions $N_\pm$ can be substituted into (29) to verify that

$$\left.\frac{\partial^2 K^* (\cdot)}{\partial N^2}\right|_{N=N_+} = -\frac{4\sqrt{2}hz\delta\varsigma^2}{\eta\sqrt{\varsigma\Omega}} < 0 \tag{34}$$

and

$$\left.\frac{\partial^2 K^*(\cdot)}{\partial N^2}\right|_{N=N_-} = \frac{4\sqrt{2}hz\delta\varsigma^2}{\eta\sqrt{\varsigma\Omega}} > 0 \tag{35}$$

The foregoing analysis produces the following:

Proposition 7 *If $\delta \in (0, \rho)$ and $\overline{S} \in \left[0, \Xi\Lambda / \left(hz\delta^2\eta^2 (\delta - \rho)^2\right)\right]$, then $\Omega \geq 0$ and $N_+ < 0 < N_-$. In this parameter range, $K^*(\cdot)$ is convex in N, taking its unique minimum at $N = N_-$.*

If instead $\delta > \rho$ and $\overline{S} \geq \max\left\{0, -\Xi\Lambda / \left(hz\delta^2\eta^2 (\delta - \rho)^2\right)\right\}$, then $\Omega \geq 0$ and $N_- < 0 < N_+$. In this parameter range, $K^*(\cdot)$ is concave in N and takes its unique maximum at $N = N_+$. In the remainder of the parameter space, $N_\pm \notin \mathbb{R}$.

The above Proposition illustrate the existence of parameter constellations wherein the aggregate advertising effort in steady state is non-monotone in the number of firms, taking the form of either a U-shaped curve or an inverted U-shaped curve w.r.t. the number of firms in the industry. However, it is also interesting to single out the regions in which the curve in question is indeed monotone. These are identified in the following:

Corollary 8 *In the parameter regions where $\Omega < 0$, $N_\pm \notin \mathbb{R}$, and therefore $\partial K^*(\cdot)/\partial N$ has the same sign as $\rho - \delta$. This entails that $K^*(\cdot)$ is monotone in N.*

Proof. This result can be easily proved noting that the coefficient of N^2 the numerator of $\partial K^*(\cdot)/\partial N$ in (27) is indeed $2h^2z^2\delta^2\varsigma(\rho - \delta)$, while the denominator of $\partial K^*(\cdot)/\partial N$, i.e., expression $\eta\Upsilon_3^2$, is positive. Hence, when $\Omega < 0$,

- $K^*(\cdot)$ decreases monotonically in N for all $\delta > \rho$;

- $K^*(\cdot)$ decreases monotonically in N for all $\delta \in (0, \rho)$.

This concludes the proof. ■

This amounts to saying that, when $\Omega < 0$, aggregate green R&D has a Schumpeterian (resp., Arrovian) flavour when discounting is low (resp., high) enough.

A special case where $K^*(\cdot)$ is monotone in N is the following. From (26), we see that if $h = 0$, then

$$K^*\left(\tau^{SP}\left(p^{SP}\left(\bar{S}\right), N\right), p^{SP}\left(\bar{S}\right)\right)\big|_{h=0} =$$

$$\left[\bar{S}z\delta\eta^2(\delta-\rho) + 2Nz\left(1+\delta(\delta+\rho)z^2\right) - \right. \tag{36}$$

$$\left. N\eta\left(az+\gamma\left(1+2z^2\delta(\delta+\rho)\right)\right)\right] / \left[\eta\left(1+\delta z^2(\delta+3\rho)\right)\right],$$

which is necessarily monotone in N. In particular:

Corollary 9 *In the special case in which technological spillovers are absent, take*

$$\bar{S} > \max\left\{0, \frac{N\left[\eta\left(az+\gamma\left(1+2z^2\delta(\delta+\rho)\right)\right) - 2z\left(1+z^2\delta(\delta+\rho)\right)\right]}{z\delta(\delta-\rho)\eta^2}\right\}$$

to ensure $K^*(\cdot)|_{h=0} > 0$. *Then,*

$$\frac{\partial\, K^*(\cdot)|_{h=0}}{\partial N} = \frac{2z\left(1+z^2\delta(\delta+\rho)\right) - \eta\left(az+\gamma\left(1+2z^2\delta(\delta+\rho)\right)\right)}{\eta\left(1+z^2\delta(\delta+3\rho)\right)} \gtreqless 0$$

for all

$$\eta \lesseqgtr \widehat{\eta} \equiv \frac{2z\left(1+z^2\delta(\delta+\rho)\right)}{az+\gamma\left(1+2z^2\delta(\delta+\rho)\right)}.$$

Corollary 9 entails that, if the individual firm's abatement capability is unaffected by the rivals', the behaviour of aggregate green R&D as N changes is Arrovian (resp., Schumpeterian) if the environment's recycling rate is sufficiently low (resp., high). I.e., it is *as if* the industry were complementing the natural absorption activities if the latter are not particularly effective (which corresponds to the Arrovian case), and conversely (which instead corresponds to the Schumpeterian case).

Now it is appropriate to provide a numerical example illustrating the arising of an inverted U-shaped curve. Fixing parameter values

$$a = 150;\ h = 1/10;\ z = 2/5;\ \bar{S} = 12 \times 10^3;$$

$$\delta = 2/3;\ \gamma = 3/2;\ \eta = 2;\ \rho = 1/10, \tag{37}$$

aggregate R&D steady state investment $K^*\left(\tau^{SP}\left(p^{SP}\left(\bar{S}\right), N\right), p^{SP}\left(\bar{S}\right)\right)$ can be drawn as in Figure 1, where the concavity of industry effort emerges clearly and $K^*\left(\tau^{SP}\left(p^{SP}\left(\bar{S}\right), N\right), ^{SP}\left(\bar{S}\right)\right)$ is maximised at $N = N_+ \simeq 822$ (while $N_- < 0$). In correspondence of these numerical values, $\partial p\left(\overline{S}\right)/\partial N \simeq -0.0097$ and from (22-23) we have $\partial \tau^{SP}(N)/\partial N \simeq -0.0059$ and $\partial^2 K^*(\cdot)/\partial N^2 \simeq -0.0938$.

Something more can be said about the effects of the size of the population of firms. Concerning the supply side, we have

$$\begin{array}{l} k^*\left(\tau^{SP}\left(p^{SP}\left(\bar{S}\right), N\right), ^{SP}\left(\bar{S}\right)\right) = 0 \text{ at } N \simeq 7348 \\ x^*\left(\tau^{SP}\left(p^{SP}\left(\bar{S}\right), N\right), ^{SP}\left(\bar{S}\right)\right) = 0 \text{ at } N \simeq 7497 \end{array} \tag{38}$$

which implies that there exists a non-negligible range of N, namely, $(7349, 7389)$ in which the individual R&D effort drops to zero but firms still produce and sell to consumers. As instead to the welfare performance of this industry, one

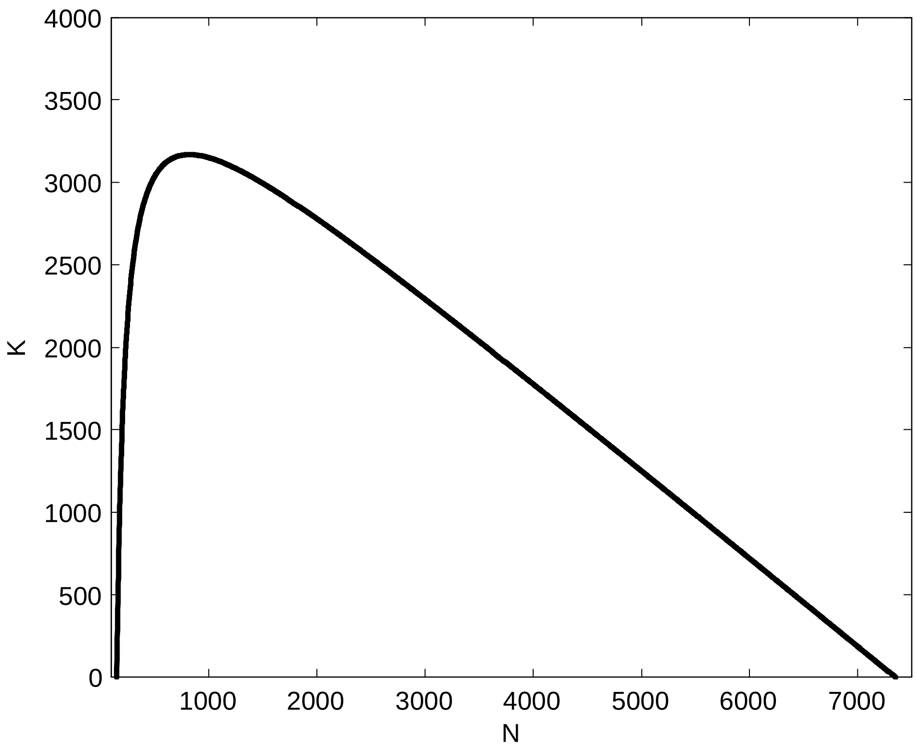


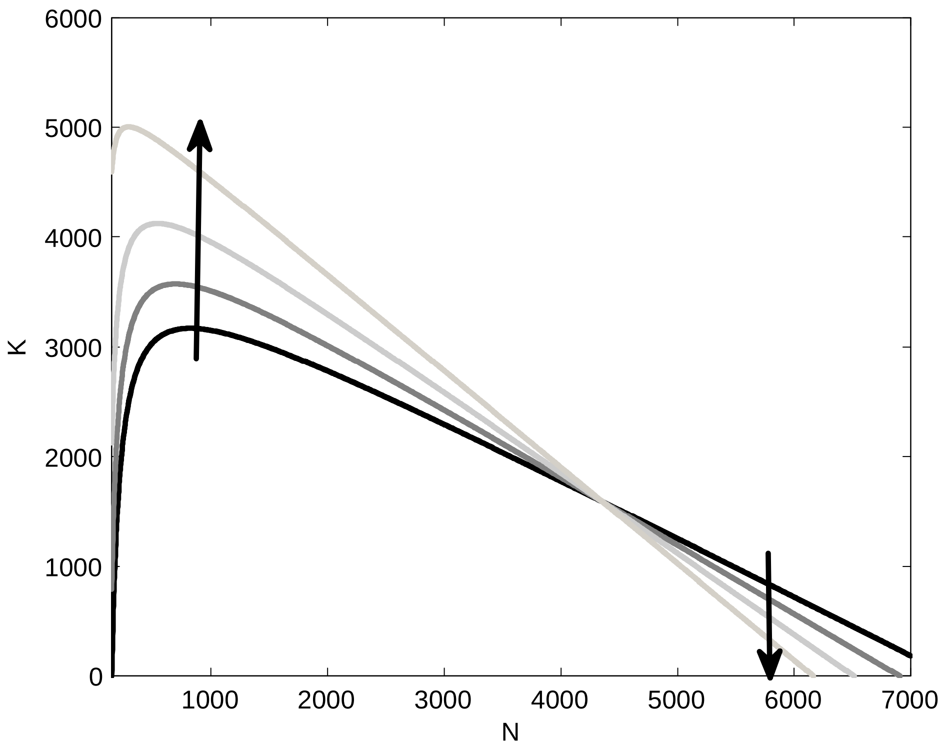
Figure 1: The inverted-U curve

can check that $SW^*\left(\tau^{SP}\left(p^{SP}\left(\bar{S}\right),N\right),^{SP}\left(\bar{S}\right)\right)$ is maximised at $N \simeq 3522$. This result (at least in the numerical example based on the above values) illustrates a situation in which consumer surplus matters more than the environmental externality, so that the industry structure that maximises welfare is a lot more fragmented than that maximising the aggregate volume of green R&D.

The inverted-U is relatively stable with respect to the model parameters. However, the level and the position of the curve differs. Table 1 summarizes the dependence. The first column includes the effect on the peak, the second collumn the effect on the level of the curve and the third one links the whole effect to each of the five graphs in Figure 2, where the qualitative effect on the inverted-U shape is illustrated.

	effect on the peak	**effect on curve level**	
ρ	move to the right	ambiguous: increase for low N, decrease for high N	top left panel
η	move to the right	ambiguous: decrease for low N, increase for high N	top right panel
γ	no/marginal	increase	middle left panel
a	no/marginal	increase	middle left panel
z	move to the left	decrease	middle right panel
$\overline{S}$	move to the right	decrease	low panel
δ	move to the right	decrease	low panel
h	move to the left	decrease	middle right panel

Table 1: Dependence of the inverted-U curve on the model parameters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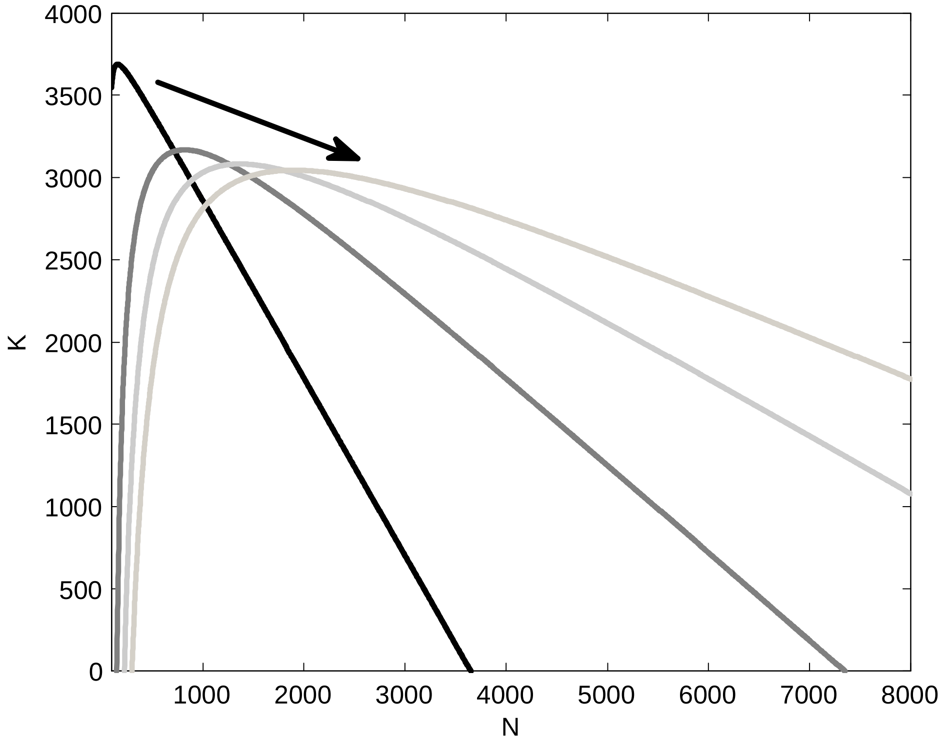
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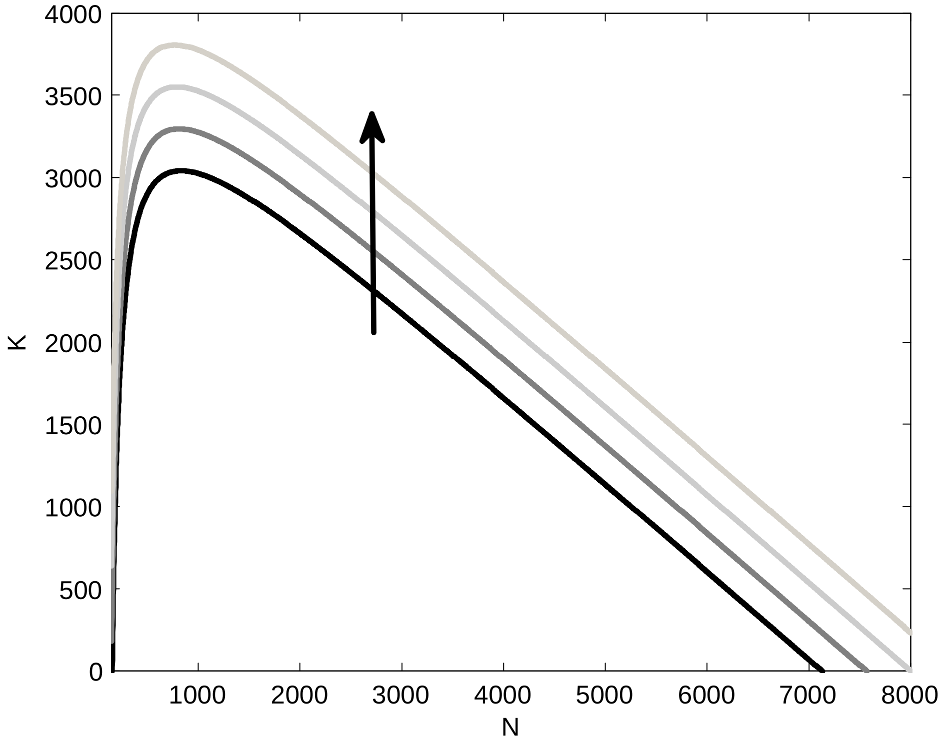

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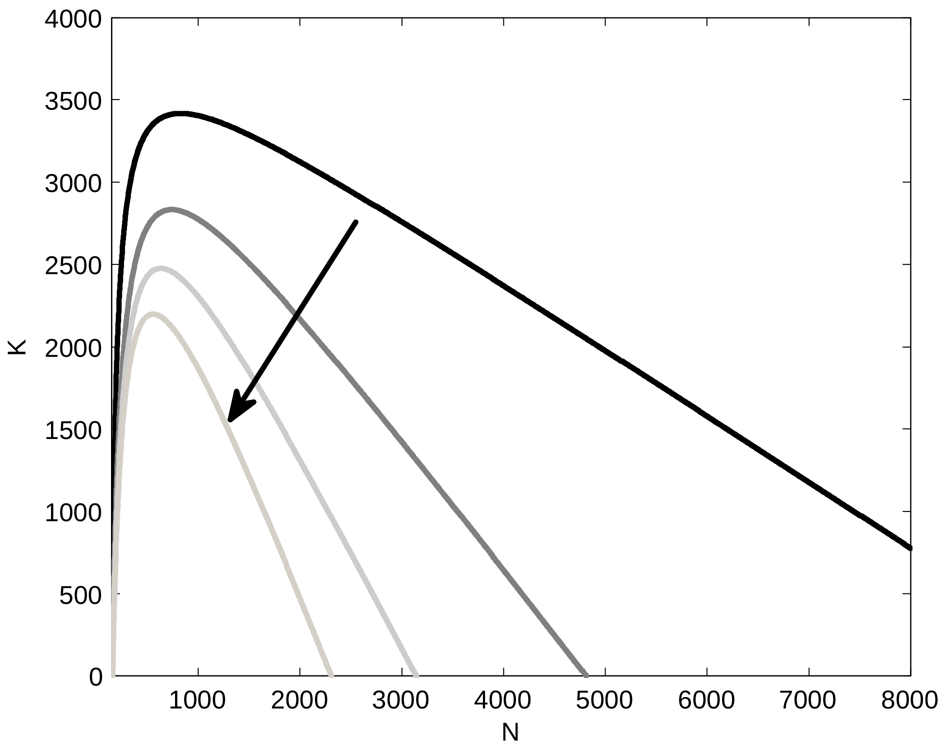

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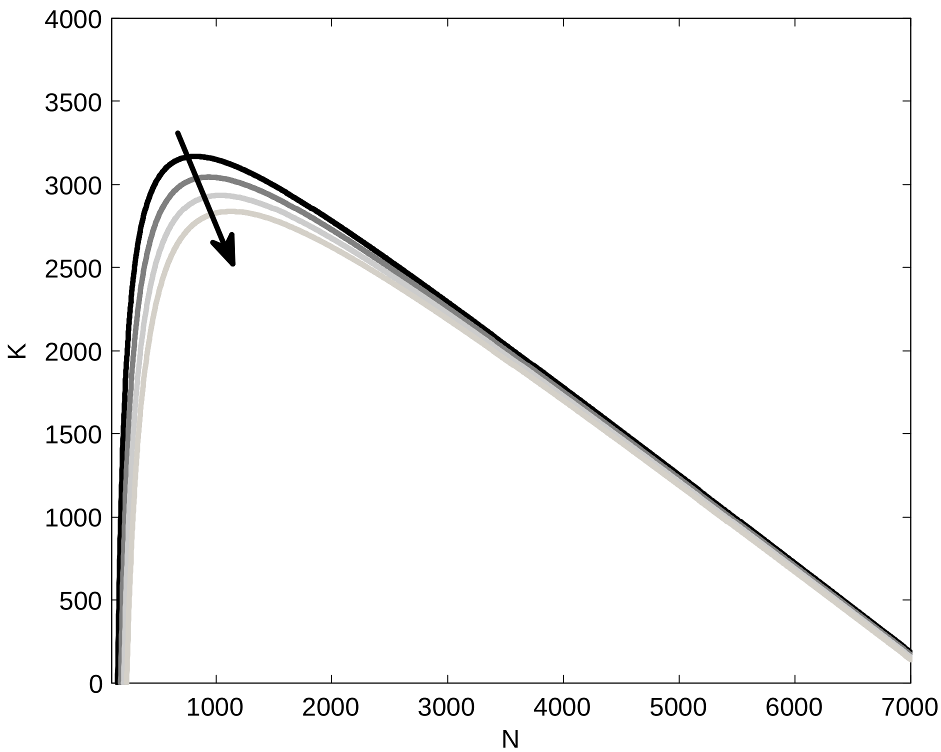

Figure 2: Sensitivity of the inverted-U curve (curve shifts along the arrow for the corresponding increasing parameter)

4.1 Discussion

Concerning the concavity of $K^*\left(\tau^{SP}\left(p^{SP}\left(\bar{S}\right),N\right),^{SP}\left(\bar{S}\right)\right)$ with respect to N, the foregoing analysis seems to imply that the arising of inverted-U curves is the consequence of the pressure of regulatory policy (possibly, as is the case in our model, of the adoption of multiple tools at the same time, to pursue different although - in some way - related objectives). This could be a plausible explanation for the lack of analogous outcomes in the vast literature discussing the bearings of industry structure on aggregate R&D, that has been produced so far in IO.

The acquired wisdom on the matter, delivering monotone predictions in one way or the other, can be quickly summarised as follows. The Schumpeterian hypothesis claims that market power is the driver of innovation, and therefore monopoly should be expected to stand out as the market form producing the highest R&D incentives. This argument rests on the so-called *efficiency effect*, whereby a monopolist can at least replicate the behaviour of any oligopolistic or perfectly competitive industry. Adhering to this view, one should expect to observe aggregate R&D to decrease monotonically in the number of firms. The opposite perspective is based on Arrow's *replacement effect*, whereby a monopolist has a lower incentive to innovate than a competitive industry (or any oligopoly in between) because, even if the innovation is patentable, the monopolist's benefit reduces to replace itself by acquiring the patent, while a smaller firm operating initially under much less favourable conditions might gain monopoly power by getting to the patent office before any of its rivals does.[5] The large subsequent literature has alternatively confirmed one view or the other, with the exception of Aghion *et al.* (2005), where the only available model showing a non-monotone result accompanies an empirical evidence with analogous properties. It is worth stressing that most, if not all of this literature relies on theoretical models where policy instruments are either absent ot taken as exogenously given.

How can we justify or interpret the arising of a concave aggregate R&D effort in the presented model? The source of this effect must be found in some

[5] A full account of this discussion is in Tirole (1988, ch. 10) and Reinganum (1989).

aspect that the previous literature has overlooked, such that the outcome is a non-monotone mixture of Arrow's replacement effect appearing first, to be replaced by Schumpeter's efficiency effect. The present model has several special features. First of all, a patent system is left out of the picture. Additionally (i) individual efforts spill over to rivals; and (ii) innovation is green, which amounts to saying that R&D is spurred by emission taxation. That is, we are treating a particular type of investment which would be altogether nil without an equally specific policy. Yet, Pigouvian taxation *per se* is not an explanation of the arising of an inverted-U curve, as we know from Corollary 4 and Lemma 5. In particular, the latter would imply a U-shaped curve, not the opposite. Hence, the responsibility of our result must be imputed to the remaining policy instrument, the regulated price $p^{SP}\left(\bar{S}\right)$. From (11), we have that the aggregate effort is

$$K^{*}=N\left(\frac{z\tau}{\rho+\eta}-\gamma\right) \tag{39}$$

In (39) we can plug $\tau=\tau^{SP}\left(p,N\right)$ from (17); however, $\tau^{SP}\left(p,N\right)$ being linear and increasing in N, this yields a convex relationship between K^{*} and N. Therefore, the source of the inverted-U curve is *not* Pigouvian taxation. What creates it is the additional policy measure regulating price, i.e., $p^{SP}\left(\bar{S}\right)$ from (24), using which we can rewrite (39) as follows:

$$K^{*}=N\left(\frac{z\tau\left(p\left(N\right),N\right)}{\rho+\eta}-\gamma\right) \tag{40}$$

Now observe that

$$\frac{\partial K^{*}}{\partial N}=\frac{z\left[\tau+N\left(\partial\tau/\partial n+\partial\tau/\partial p\cdot\partial p/\partial n\right)\right]}{\rho+\eta}-\gamma \tag{41}$$

and

$$\frac{\partial^2 K^*}{\partial N^2} = \frac{z\left[2\left(\frac{\partial \tau}{\partial n}+\frac{\partial \tau}{\partial p}\frac{\partial p}{\partial n}\right)+N\left(\frac{\partial^2 \tau}{\partial n^2}+2\frac{\partial^2 \tau}{\partial n \partial p}\frac{\partial p}{\partial n}+\frac{\partial^2 \tau}{\partial p^2}\left(\frac{\partial p}{\partial n}\right)^2+\frac{\partial^2 p}{\partial n^2}\frac{\partial \tau}{\partial p}\right)\right]}{\rho+\eta} \tag{42}$$

Setting (41) equal to zero, we obtain

$$\frac{\partial \tau}{\partial N} = \frac{\gamma(\eta+\rho) - z(\tau + N \cdot \partial\tau/\partial p \cdot \partial p/\partial n)}{Nz} \tag{43}$$

This can be substituted into (42), which can also be further simplified using additional pieces of information that we can draw from expression (17), whereby

$$\frac{\partial^2 \tau}{\partial N^2} = \frac{\partial^2 \tau}{\partial N \partial p} = \frac{\partial^2 \tau}{\partial p^2} = 0. \tag{44}$$

Hence, (42) simplifies as follows:

$$\frac{\partial^2 K^*}{\partial N^2} = \frac{2\left[\gamma(\eta+\rho) - z\tau\right] + zN^2 \cdot \partial\tau/\partial p \cdot \partial^2 p/\partial n^2}{N(\rho+\eta)} \tag{45}$$

Observing (45), we may note that

$$\gamma(\eta+\rho) - z\tau = -(\eta+\rho)\,k^* < 0 \tag{46}$$

and $\partial\tau/\partial p \gtreqless 0$ for all $\delta \gtreqless \rho$ - which again can be easily deduced from (17). Accordingly, we may take a final step and rewrite (45) in a more intuitive form:

$$\frac{\partial^2 K^*}{\partial N^2} = \frac{zN^2 \cdot \partial\tau/\partial p \cdot \partial^2 p/\partial n^2 - 2(\eta+\rho)\,k^*}{N(\rho+\eta)} \tag{47}$$

which is negative for all

$$k^* > \max\left\{\frac{zN^2 \cdot \partial\tau/\partial p \cdot \partial^2 p/\partial n^2}{2(\eta+\rho)}, 0\right\}. \tag{48}$$

If we confine our attention to the parameter region defined by $\delta > \rho$, which is what we have done to generate the inverted-U curve appearing in Figure 1 - then $\partial\tau/\partial p > 0$; therefore, in this range $\partial^2 p/\partial N^2 < 0$ suffices to ensure that K^* is indeed concave w.r.t. N for all $k^* > 0$.

Having said that, two natural questions arises, namely, (i) should we conclude that it is altogether impossible to reproduce the same result if regulation is assumed away in differential games investigating some form of R&D for either process or product innovation? The few existing examples (see Cellini and Lambertini, 2002, 2009, for instance)[6] indeed yield monotone outcomes, but are by no means general; (ii) shall we deem the usual assumption of a linear market demand responsible for monotone outcomes? In fact, empirical research (Hausman, 1981; Varian, 1982, 1990, inter alia) has shown that most markets are characterised by non linear demand functions, which are best approximated by isoelastic curves. These extensions are left for future research.

5 Concluding remarks

We have characterised green R&D incentives for firms operating in an industry where production pollutes the environment the government regulates the mark up and adopts a Pigouvian tax policy to decrease emissions and stimulate the introduction of clean technologies. The model delivers a thus far rare result, in the form of an inverted-U aggregate R&D expenditure at

[6] One could also address in the same spirit other dynamic models whose focus is on the investment in advertising to expand the demand level or goodwill stock, as in Cellini and Lambertini (2003a,b) and the references therein.

equilibrium. The implication of our analysis seems to be that the empirical evidence concerning the emergence of inverted-U curves is a consequence of some form of regulation that modifies the aggregate behaviour of the industry as compared to the predictions of theoretical models where regulation is either totally exogenous or just assumed away. Whether ours is a special (and fortunate) case or instead an indication of some general rule previously overlooked, is a question left for future research.

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