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Measuring the output gap using large datasets

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Abstract

We propose a new measure of the output gap based on a dynamic factor model that is estimated on a large number of U.S. macroeconomic indicators and which incorporates relevant stylized facts about macroeconomic data (co-movements, non-stationarity, and the slow drift in long-run output growth over time). We find that, (1) from the mid-1990s to 2008, the U.S. economy operated above its potential; and, (2) in 2018:Q4, the labor market was tighter than the market for goods and services. Because it is mainly data-driven, our measure is a natural complementary tool to the theoretical models used at policy institutions.

JEL classification: C32, C38, C55, E0.

Keywords: Output Gap; Non-stationary Approximate Dynamic Factor Model; Trend-Cycle Decomposition.

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1 Introduction

A fundamental issue in policymaking and macroeconomics is the decomposition of aggregate output into potential output and the output gap. Indeed, the former is a crucial input for long-term projections; the latter is important to measure the cyclical position of the economy. However, both of these quantities are unobservable, and there is disagreement in the economics profession on how to estimate them.

Nowadays, economists use three main approaches to estimate the output gap, each of which derives from a different definition of potential output (Kiley, 2013)—the output gap being simply the deviation of aggregate output from its potential: the statistical approach, the production-function approach, and the New Keynesian approach. According to the statistical approach (e.g., Beveridge and Nelson, 1981), potential output is the long-run stochastic trend of output. According to the production-function approach (e.g., Congressional Budget Office, 2001), potential output is the level of output consistent with current technologies and normal use of capital and labor. And, according to the New Keynesian approach (e.g., Justiniano et al., 2013), potential output is the level of output obtained in the absence of any nominal (and financial) friction and of inefficient shocks. Not only are the concepts of potential output and the output gaps stemming from these definitions different, but these different definitions oftentimes yield different conclusions as well.

This paper proposes a new measure of potential output and the output gap that is closely related to the statistical approach. The novelty of our measure is that it is based on a model

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that is estimated on a large number of U.S. macroeconomic indicators and which incorporates relevant stylized facts about macroeconomic data, such as that macroeconomic time series tend to co-move and be non-stationary, and that long-run output growth has drifted slowly over time.

Our methodology works in two steps. First, we estimate a large-dimensional non-stationary dynamic factor model using a dataset of about one hundred U.S. quarterly macroeconomic indicators (Quah and Sargent, 1993; Barigozzi and Luciani, 2019). Intuitively, in this step, we isolate the factors driving the common dynamics (i.e., those dynamics generated by macroeconomic shocks), thus backing-out idiosyncratic dynamics and measurement error. For this first step to be successful, using a large number of variables is crucial, since aggregating a large number of variables allows us to disentangle macroeconomic fluctuations from idiosyncratic dynamics (Stock and Watson, 2002; Bai and Ng, 2002).

Second, we decompose the estimated common factors into common trends and common cycles using principal component analysis on the estimated factors (Peña and Poncela, 2006; Zhang et al., 2019). Intuitively, in this step, we separate permanent dynamics from transitory dynamics. In our framework, potential output is that part of aggregate output explained by the common trends, while the output gap is that part of aggregate output explained by the common cycles.

In the past 30 years, many papers have suggested different econometric approaches to obtaining a trend-cycle decomposition of aggregate output. Some of these papers have used univariate models (e.g., Watson, 1986; Morley et al., 2003; Kamber et al., 2018). Some other have used multivariate methods like large-dimensional (but stationary) models (Aastveit and Trovik, 2014; Morley and Wong, 2020) or non-stationary (but low dimensional) factor models (e.g., Fleischman and Roberts, 2011; Jarociński and Lenza, 2018; Hasenzagl et al., 2020). Our factor model is both non-stationary and large-dimensional, thus accounting for both stochastic and secular trends and capturing information coming from a wide spectrum of the economy. Therefore, our output gap estimate is driven by all of the forces driving the co-movements in the U.S. economy, including real, nominal, and financial shocks.

A synthetic view of our main findings is in Figure 1, where in the left panel we show our estimate of the output gap and the one produced by the Congressional Budget Office (CBO), which plays an important role in the policy discussions in the United States. Our approach differs from that of the CBO because our estimate of the output gap is primarily data-driven, whereas the CBO's estimate is based mainly on theoretical macroeconomic models. Indeed, we make no theoretical macroeconomic assumptions, e.g., we forgo imposing a Phillips curve or Okun's law. Instead, we make just a few "empirically based" economic assumptions, such as that average output growth drifts slowly over time (Antolin-Diaz et al., 2017), and we try to let the data speak as much as possible. This methodological difference makes our measure a natural complementary tool to the theoretical models used by the CBO and at other policy institutions.

As shown in Figure 1, our output gap estimate looks very similar to the CBO estimate in that the dating of the turning points perfectly coincides. However, the two estimated measures started to diverge somewhat significantly in the mid-1990s. Whereas the CBO estimates that the output gap closed just in 2006, our model suggests a persistent overheating of the US economy from the mid-1990s to the Great Recession. This result conforms particularly well with Borio et al. (2017)'s narrative, who point out that credit growth was a key factor in overheating the economy from the late nineties to the Great Recession, and it is compatible with the "productivity view" of Fernald et al. (2017).

Our model also allows us to estimate the unemployment gap (in our framework, the part of the unemployment rate driven by the common cycles), which is often used as an alternative indicator of the cyclical position of the economy (see Figure 1, right panel). In general, we find that the unemployment gap moves together with the output gap, a feature that the CBO imposes via an Okun's law; however, departures from this co-movement can be quite large. In particular, in 2018:Q4, our estimate of the unemployment gap signals that the economy was operating above its potential, while our estimate of the output gap suggests that the U.S. economy was operating at its potential. In other words, our model suggests that the labor market was tighter than the market for goods and services at the end of 2018.

Finally, it is well known that filtering and data revision often lead to highly unreliable and misleading estimates of the output gap in real time (Orphanides and van Norden, 2002). In this respect, our output gap measure revises very little when considering samples no more than five years shorter than our benchmark sample. However, the size of the revisions becomes more relevant when considering samples that are ten or more years shorter than our benchmark sample.

The rest of this paper is structured as follows. In Section 2, we present our methodology. In Section 3, we describe the data used, and in Section 4, we discuss the model set-up. Then, in Section 5, we present and discuss our estimates of potential output and the output gap. Section 6 studies the real-time properties of our output gap measure, and Section 7 concludes.

2 Methodology

2.1 Non-stationary dynamic factor model

Let us assume to observe n time series $\mathbf{y}_t = \{y_{it} : i = 1, \ldots, n\}$ over T periods, i.e., $t = 1, \ldots, T$. The dynamic factor model we propose in this paper is based on the idea that each variable (y_{it}) evolves over time around a secular (deterministic or slow-moving) trend ($\mathcal{D}_{it}$), and that the dynamics around this trend are generated by (i) a few common factors ($\boldsymbol{f}_t$) capturing co-movements across series and across time, which are the result of macroeconomic dynamics, and (ii) an idiosyncratic component (ξ_{it}) capturing sectoral and local dynamics as well as measurement errors. Formally, we consider the following model

$$y_{it} = \mathcal{D}_{it} + \boldsymbol{\lambda}_i'(L)\boldsymbol{f}_t + \xi_{it}, \qquad i = 1, \ldots, n, \quad t = 1, \ldots, T, \tag{1}$$

$$\mathcal{D}_{it} = b_{it-1} + \mathcal{D}_{it-1} + \epsilon_{it}, \qquad \epsilon_{it} \sim \mathcal{N}(0, \sigma^2_{\epsilon_i}), \tag{2}$$

$$b_{it} = b_{it-1} + \eta_{it}, \qquad \eta_{it} \sim \mathcal{N}(0, \sigma^2_{\eta_i}), \tag{3}$$

$$\boldsymbol{f}_t = \boldsymbol{A}(L)\boldsymbol{f}_{t-1} + \mathbf{u}_t, \qquad \mathbf{u}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_u), \tag{4}$$

$$\xi_{it} = \rho_i \xi_{it-1} + e_{it}, \qquad \boldsymbol{e}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_e), \tag{5}$$

where L is the lag-operator, $\boldsymbol{\lambda}_i(L) = \sum_{\ell=0}^{s} \boldsymbol{\lambda}_{i\ell} L^\ell$, $\boldsymbol{A}(L) = \sum_{\ell=0}^{p-1} \boldsymbol{A}_\ell L^\ell$, $s \geq 0$ and $p \geq 0$ are finite integers, and $\boldsymbol{e}_t = (e_{1t} \dots e_{nt})'$. Furthermore, we assume the existence of $q \ll n$ common latent factors, hence $\boldsymbol{f}_t = (f_{1t} \dots f_{qt})'$, $\mathbf{u}_t = (u_{1t} \dots u_{qt})'$, and $\boldsymbol{\lambda}_{i\ell} = (\lambda_{i1\ell} \dots \lambda_{iq\ell})'$ are q-dimensional vectors, and $\boldsymbol{A}_\ell$ are $q \times q$ matrices.

In what follows, we summarize the main assumptions and features of model (1)-(5), and its estimation. We refer the interested reader to Barigozzi and Luciani (2019, 2020) for a more formal treatment.

2.1.1 Assumptions

Non-stationarity in the data is modeled by means of three main assumptions.

Assumption 1. *The process $\{\mathbf{u}_t\}$ is a zero-mean weak white noise with covariance $\boldsymbol{\Sigma}_u$ positive definite. Moreover, there exists two positive integers r and d such that $r + d = q$ and $\det(\mathbf{I}_q - \boldsymbol{A}(z)z) = 0$ has r roots in $z = 1$, while the other d roots lie outside the unit circle.*

Assumption 1 is equivalent to saying that (i) the common factors $\boldsymbol{f}_t$ are cointegrated with cointegration rank d; and (ii) $\boldsymbol{f}_t$ is driven by $r = (q - d)$ common trends.

Assumption 2. *Let n_I be the number of variables among $y_{1t}, \dots, y_{nt}$ for which $\xi_{it} \sim I(1)$, i.e. $\rho_i = 1$. Then, $0 < n_I < n$.*

Assumption 2 implies that some (but not all) idiosyncratic components might be $I(1)$, and it is motivated by the fact that assuming $n_I = 0$ (i.e., all idiosyncratic components are stationary) implies that any $(r+1)$-dimensional vector is cointegrated, an assumption which in general is not supported by the data—see also the theory and empirical results presented in Barigozzi et al. (2020a,b). However, since some macroeconomic variables are known to be cointegrated (e.g., income and consumption) it is reasonable to assume $n_I < n$. This intuition is confirmed on our dataset, as about half of the $n = 103$ variables considered have a non-stationary idiosyncratic component, i.e., $n_I = 44$ (see Section 3 and Section A of the Complementary Appendix for details).

Assumption 3. *(I) Let n_b be the number of variables among $y_{1t}, \dots, y_{nt}$ for which $b_{it} = b_i \neq 0$ for all t. Then, $0 < n_b < n$ and, for these n_b variables, we set $\epsilon_{it} = 0$ and $\eta_{it} = 0$, so that we can write $\mathcal{D}_{it} = a_i + b_i t$, where a_i and b_i are two finite real numbers, with the exception of GDP and GDI for which only $\epsilon_{it} = 0$, so that $\mathcal{D}_{it} = b_{it-1} + \mathcal{D}_{it-1}$ and $b_{it} = b_{it-1} + \eta_{it}$, with $\{\eta_{it}\}$ being a zero mean weak white noise process with variance $\sigma^2_{\eta_i} > 0$.*
(II) For all other $(n - n_b)$ variables, for which $b_{it} = 0$ for all t, we set $\epsilon_{it} = 0$ and $\eta_{it} = 0$, so that we can write $\mathcal{D}_{it} = a_i$, where a_i is a finite real number, with the exception of the unemployment rate, for which only $\eta_{it} = 0$, so that $\mathcal{D}_{it} = \mathcal{D}_{it-1} + \epsilon_{it}$, with $\{\epsilon_{it}\}$ being a zero mean weak white noise process with variance $\sigma^2_{\epsilon_i} > 0$.

In Part (I), we assume that the linear trend is common to at most *some of* the variables in the dataset but not to *all* the variables in the dataset. This is a realistic assumptions for a typical macroeconomic dataset. Indeed, series belonging to the real side of the economy, e.g., GDP, exhibit a clear upward trend, hence b_i is likely to be strongly significant; by contrast, for nominal series, e.g., inflation, which do not exhibit any trending behavior, b_i is likely to be not significantly different from zero. This intuition is confirmed on our dataset, as only about half of the $n = 103$ variables considered display a significant linear trend, i.e., $n_b = 61$ (see Section 3 and Section A of the Complementary Appendix for details).

Moreover, Assumption 3 allows for time variation in the secular component of GDP, GDI, and the unemployment rate. These assumptions are crucial to correctly estimate potential output and the natural rate of unemployment, hence the output gap and the unemployment gap.[1] Specifically, Part (I) allows the mean of GDP (GDI) growth to drift slowly over time; hence we model its secular component as a local linear trend. The goal here is to capture

[1]Section J.4 of the Complementary Appendix shows results when we do not allow for time variation in the secular component of GDP, GDI, and the unemployment rate.

the secular decline in long-run U.S. growth documented by several authors (see Antolin-Diaz et al., 2017, for example, and references therein). The literature has pointed to several factors that can explain this secular slowdown citing, among others, slower productivity growth and demographic factors, such as the decline of population growth and the slower pace of increase in educational attainment (Gordon, 2018). Indeed, as pointed out by Ng (2018), a linear deterministic trend cannot adapt to those changes, thus leaving too much predictable variations unexplained, which would, in turn, inflate our output gap at the end of our sample.

Part (II) allows the mean of the unemployment rate to drift gradually over time, hence it evolves over time as a local level model. The idea here is that several idiosyncratic labor market and demographic characteristics might generate an idiosyncratic trend in the unemployment rate (see e.g., Daly et al., 2011). For example, labor supply factors such as the entry and exit from the labor market of the baby boom generation, the increased participation rate among women, and changes in youth labor force share. Or labor demand factors such as, for example, possible skill or location mismatch between job openings and the characteristics of the unemployed. Finally, the emergency and extended benefits, which are a standard policy response to elevated cyclical unemployment that expire once the labor market recovers, might have transitory effects on the natural rate of unemployment.[2]

Summing up, Assumption 3 restricts the mean of all the variables but GDP, GDI, and the unemployment rate to be to be either a linear trend (if $b_i \neq 0$) or a constant (if $b_i = 0$). These are meant to be an approximation for a slowly moving trend or mean. Indeed, as we will explain in Section 2.1.2, although it is computationally feasible, it is complicated to estimate

[2]In the U.S. standard unemployment benefits lasts for 26 weeks. After 26 weeks, there are two programs that allow to extend further the unemployment benefits (the Emergency Unemployment Compensation and Extended Benefits), which extend unemployment benefits eligibility up to 99 weeks. These two measures were introduced during the Great Recession, the previous maximum eligibility was 65 weeks.

a model in which all variables have a time-varying trend. We stress that Assumption 3 is perfectly in line with the literature on large stationary dynamic factor models. Indeed, in this literature, the common practice is to transform variables to stationarity and then to center them (i.e. subtracting the sample mean) before estimating the model, an operation equivalent to detrending the variables in levels. The novelty with respect to the literature, is the fact that we allow for some time variation in $\mathcal{D}_{it}$ for GDP, GDI, and the unemployment rate. To the best of our knowledge, in the large factor model literature, the only paper that allows for a similar time variation is Antolin-Diaz et al. (2017), who allow the mean of GDP growth to drift slowly over time.[3]

Remark. Given Assumptions 1–3, it is important to note that the vector $\mathbf{y}_t$ can include both stationary and non-stationary variables. Indeed, whenever $y_{it} \sim I(0)$, we will have $b_i = 0$, $\xi_{it} \sim I(0)$, and the loadings polynomial $\boldsymbol{\lambda}_i(L)$ will contain the factor $(1-L)^r$ thus canceling the r unit roots in $\boldsymbol{f}_t$. We note here that this happens in practice when estimating the parameters on our dataset without the need of forcing it (see the first two columns of Table 2).

Finally, the model is identified by means of the three following assumptions.

Assumption 4. *Let* $\boldsymbol{\Lambda} = ((\boldsymbol{\lambda}'_{10}\cdots\boldsymbol{\lambda}'_{1s})'\cdots(\boldsymbol{\lambda}'_{n0}\cdots\boldsymbol{\lambda}'_{ns})')'$ *be the* $n \times q(s+1)$ *matrix of all factor loadings, then there exists a* $q(s+1) \times q(s+1)$ *matrix* $\mathbf{H}$ *such that* $\mathrm{rk}(\mathbf{H}) \geq q$ *and* $\lim_{n\to\infty} n^{-1}\boldsymbol{\Lambda}'\boldsymbol{\Lambda} = \mathbf{H}$.

Assumption 5. *The process* $\{\boldsymbol{e}_t\}$ *has zero-mean and covariance* $\boldsymbol{\Sigma}_e$ *positive definite. Moreover, there exist a finite constant* $C > 0$, *independent of* n *and* T, *such that* $(nT)^{-1}\sum_{i,j=1}^{n}\sum_{t,s=1}^{T}|\mathsf{E}[e_{it}e_{js}]| \leq C$.

[3]In line with Antolin-Diaz et al. (2017) in Section J.6 of the Complementary Appendix, we show how our results change if we impose that output and consumption grow at the same rate in the long run. Moreover, in Section J.7 of the Complementary Appendix, we show how our results change if we allow for a time-varying mean also in consumer price inflation.

Assumption 6. $\mathsf{E}[u_{jt}e_{is}] = 0$ *for any* i, j, t, s.

Assumption 4 is equivalent to saying that the factors $\boldsymbol{f}_t$ are pervasive, i.e., they have non-negligible effects on all variables at least at one lag, while Assumption 5 implies that the idiosyncratic innovations ($\boldsymbol{e}_t$) are weakly cross-sectionally correlated and therefore do not have a pervasive effect. Moreover, they are allowed to be weakly serially correlated. These assumptions are similar to the typical assumptions made in the factor model literature (e.g., Bai and Ng, 2002, 2004; Barigozzi et al., 2020b). From Assumptions 4–6, it can be shown that in the limit $n, T \to \infty$ the space spanned by the factors can be recovered by aggregating the data, and this justifies from a methodological point of view our choice of using large datasets.

Finally, we add a constraint derived from the National Account concept of "output."

Assumption 7. $\boldsymbol{\lambda}_{\text{GDP}}(L) = \boldsymbol{\lambda}_{\text{GDI}}(L)$ *and* $\mathcal{D}_{\text{GDP},t} = \mathcal{D}_{\text{GDI},t}$.

Assumption 7 implies that GDP and GDI respond to the common factors and to the secular trend in the same way. When combined with the fact that the idiosyncratic component of GDP and GDI is $I(0)$, this assumption implies that the discrepancy between GDP and GDI is only temporary and driven by stationary idiosyncratic factors and measurement errors.[4]

The rationale behind Assumption 7 is that GDP and GDI are both measures of U.S. aggregate output—GDP tracks all expenditures on final goods and services produced, while GDI tracks all income received by those who produced the output. Therefore, the difference between GDP and GDI is exclusively the result of measurement error—using the NIPA table definition "statistical discrepancy." Based on this rationale, in recent years, there has been interest in combining GDP and GDI to come up with a better estimate of aggregate output

[4]This restriction is indeed corroborated by the data, as $\widehat{\boldsymbol{\lambda}}_{\text{GDP}}(L)$ and $\widehat{\boldsymbol{\lambda}}_{\text{GDI}}(L)$ are nearly identical even when we do not impose the restriction. In numbers, the mean of $|\Delta x_{\text{GDP},t} - \Delta x_{\text{GDI},t}|$ is 0.4 with a standard deviation of 0.5, while the mean $|\widehat{\boldsymbol{\lambda}}_{\text{GDP}}(L)\Delta\widehat{\boldsymbol{f}}_t - \widehat{\boldsymbol{\lambda}}_{\text{GDI}}(L)\Delta\widehat{\boldsymbol{f}}_t|$ is just 0.1 with a standard deviation of 0.1. Section J.3 of the Complementary Appendix shows results when this assumption is removed.

(e.g., Aruoba et al., 2016; Council of Economic Advisers, 2015) to which the Council of Economic Advisers refers to as Gross Domestic Output (GDO). Therefore, in our setting the (log of) GDO is defined as

$$\mathrm{GDO}_t = \mathcal{D}_{\mathrm{GDP},t} + \boldsymbol{\lambda}'_{\mathrm{GDP}}(L)\boldsymbol{f}_t = \mathcal{D}_{\mathrm{GDO},t} + \boldsymbol{\lambda}'_{\mathrm{GDO}}(L)\boldsymbol{f}_t.$$

2.1.2 Estimation

The model specified in (1)-(5) has a state-space form, and we estimate it by Quasi-Maximum Likelihood, implemented through the Expectation Maximization (EM) algorithm, where in the E-step the factors $\boldsymbol{f}_t$ are estimated with the Kalman Smoother. This approach was previously considered, e.g., by Watson and Engle (1983) and Doz et al. (2012), in the stationary setting, and by Quah and Sargent (1993) in the non-stationary setting. The theoretical properties of the estimators we use here are studied in Barigozzi and Luciani (2019), where consistency of the estimated loadings and factors is proved as $n, T \to \infty$.[5]

The EM algorithm is an iterative procedure that produces estimates of all parameters of the model, that is of the loadings $\boldsymbol{\lambda}_i(L)$, the VAR parameters $\boldsymbol{A}(L)$ and $\boldsymbol{\Sigma}_u$, the diagonal elements of $\boldsymbol{\Sigma}_e$, the variances $\sigma^2_{\epsilon_i}$, $\sigma^2_{\eta_i}$, and the parameters a_i and b_i, defined in Assumption 3. Once the parameters are estimated, we estimate the latent factors $\boldsymbol{f}_t$ via a last run of the Kalman smoother. While the technical details on the implementation of the EM are in Section C of the Complementary Appendix, here we just discuss three main aspects: initialization of the algorithm, the model for the idiosyncratic components, and the model for linear trends.

We initialize the EM algorithm by first estimating the loadings via Principal Component (PC) analysis on differenced data. In this way we avoid to incur in spurious effects due to

[5]For simplicity of presentation, we assumed Gaussianity of all the shocks in the model. However, Gaussianity is not needed to prove consistency as long as the innovations have finite fourth moments (see also the arguments made in Barigozzi and Luciani (2020), in the stationary case).

the presence of idiosyncratic unit roots (Onatski and Wang, 2019) or deterministic linear trends (Ng, 2018). The factors are then obtained by projecting the data onto the loadings and then the VAR parameters in (4) are estimated by least squares (see Bai and Ng, 2004 and Barigozzi et al., 2020b for details).

We then choose as initial value $\sigma^{2(0)}_{\eta_{\text{GDO}}} = 10^{-3}$, which implies that over ten years, the time-varying mean of GDO growth varies with a reasonable standard deviation of around 0.25 percentage points at an annual rate.[6] And, we set the initial value $\sigma^{2(0)}_{\epsilon_{\text{UR}}} = 10^{-2}$, implying that on average, over ten years, the time-varying mean of the unemployment rate varies with a standard deviation of around eight basis points. In Section J.5 of the Complementary Appendix, we report robustness results for different starting values of these two variances.

As for the idiosyncratic component, we forgo estimating (5), rather we impose $\rho_i = 0$ when ξ_{it} is stationary, and $\rho_i = 1$ when ξ_{it} is $I(1)$, meaning that we treat n_I idiosyncratic components as additional latent states. In both cases, in the EM algorithm we treat $\boldsymbol{e}_t$ as if it were a weak white noise process, i.e., serially uncorrelated. Furthermore, we estimate the model without accounting explicitly for the cross-sectional dependence across idiosyncratic shocks. That is, we estimate only the diagonal terms of the idiosyncratic covariance matrix $\boldsymbol{\Sigma}_e$ (its inverse being needed for the Kalman smoother), while setting to zero all off-diagonal entries. Although by imposing such a simplified structure we are in fact estimating a mis-specified model, this approach does not affect consistency of the estimates, provided that Assumption 5 holds.

In order to choose which idiosyncratic component to model as a random walk, we run the test proposed by Bai and Ng (2004) for the null hypothesis of a unit root, setting $\rho_i = 0$ if we reject the null, and $\rho_i = 1$ otherwise.[7] Detailed results of the test are available in Section

[6]To give a metric, Antolin-Diaz et al. (2017) calibrate their time-varying mean of GDP growth so that over a period of 10 years it has a standard deviation of 0.25 percentage points at an annual rate.

[7]An alternative strategy to determine which idiosyncratic component to model as a random walk consists in applying some judgment and overriding the test to guarantee consis-

A of the Complementary Appendix.

Finally, as per Assumption 3, before estimating the model we need to choose for which variable to include a linear trend. To this end, we test for significance of the sample mean of Δy_{it}, and we set $b_i = 0$ only when we fail to reject the null, while otherwise b_i is estimated in the EM algorithm. This is done for all variables with the exception of GDP, GDI, and unemployment rate for which we impose Assumption 3, thus adding two more latent states, $\mathcal{D}_{\mathrm{GDO},t}$ and $\mathcal{D}_{\mathrm{UR},t}$, to the model. More details on the test, and detailed results of it, are available in Section A of the Complementary Appendix.

2.2 Trend-cycle decomposition

Assumption 1 implies that the vector $\boldsymbol{f}_t$ is cointegrated with d cointegration relations, i.e., there exists a $q \times d$ vector $\boldsymbol{\beta}$ such that $\boldsymbol{\beta}'\boldsymbol{f}_t \sim I(0)$. It can be proven—see, e.g., Escribano and Peña (1994)—that because the common factors are cointegrated, they admit a factor representation:

$$\boldsymbol{f}_t = \boldsymbol{\Psi}_1 \boldsymbol{\tau}_t + \boldsymbol{\gamma}_t, \tag{6}$$

where $\boldsymbol{\tau}_t = (\tau_{1t} \dots \tau_{rt})'$ is the vector of r common trends, such that $\tau_{jt} \sim I(1)$ for all j and $\boldsymbol{\Psi}_1$ is $q \times r$ with $\mathrm{rk}(\boldsymbol{\Psi}_1'\boldsymbol{\Psi}_1) = r$, while $\boldsymbol{\gamma}_t$ is a q-dimensional stationary vector.

Representation 6 is not unique, and the literature has proposed different representations based on different sets of assumptions—e.g., Stock and Watson (1988); Kasa (1992); Gonzalo and Granger (1995). In particular, the literature has focused on two main identification issues. The first is about the way common trends are defined—are trends restricted to be pure random walks (e.g., Stock and Watson, 1988), or are they allowed to contain transitory elements as well (e.g., Lippi and Reichlin, 1994)? The second is about how to allocate the common shocks between the permanent and transitory components: for example, they can be independent (e.g., Kasa, 1992; Harvey, 1985) or they can generate independent fluctuations

tency in the treatment of variables belonging to the same group. Robustness results to this strategy are available in Section J.9 of the Complementary Appendix.

only in the long run (e.g., Gonzalo and Granger, 1995). Here we adopt an agnostic, non-parametric approach, which is close to the trend-cycle decomposition proposed by Kasa (1992), and that has also been considered by Bai (2004), Peña and Poncela (2006), and Zhang et al. (2019). The next assumption characterizes the common trends.

Assumption 8. $\boldsymbol{\tau}_t = \mathbf{B}'\boldsymbol{f}_t$, *where* $\mathbf{B}$ *is a* $q \times r$ *matrix such that* $\mathrm{rk}(\mathbf{B}) = r$.

Assumption 8 requires the common trends to be linear combinations of the common factors, a common assumption in many trend-cycle decompositions (e.g., Kasa, 1992; Gonzalo and Granger, 1995). By combining Assumption 8 with equation (6), we have that $\boldsymbol{\gamma}_t = (\mathbf{I}_q - \boldsymbol{\Psi}_1\mathbf{B}')\boldsymbol{f}_t$, which shows that also the stationary component $\boldsymbol{\gamma}_t$ is a linear combination of the factors. Now, because the only linear combination of the factors that yields a stationary process is given by $\boldsymbol{\beta}'\boldsymbol{f}_t$, it must be that $(\mathbf{I}_q - \boldsymbol{\Psi}_1\mathbf{B}') = \boldsymbol{\Psi}_2\boldsymbol{\beta}'$, for some $\boldsymbol{\Psi}_2$ of dimension $q \times d$ and such that $\mathrm{rk}(\boldsymbol{\Psi}_2) = d$. Therefore, letting $\boldsymbol{c}_t = \boldsymbol{\beta}'\boldsymbol{f}_t$, we can rewrite (6) as

$$\boldsymbol{f}_t = \boldsymbol{\Psi}_1\boldsymbol{\tau}_t + \boldsymbol{\Psi}_2\boldsymbol{c}_t. \tag{7}$$

Therefore, the stationary component $\boldsymbol{\gamma}_t$ is driven by d processes $\boldsymbol{c}_t = (c_1 \dots c_d)'$, which are indeed common cycles for $\boldsymbol{f}_t$ (Vahid and Engle, 1993).

Finally, let us assume

Assumption 9. $\mathsf{E}[\tau_{it}c_{jt}] \neq 0$ *for any* i, j, t.

Assumption 9 imposes that the common trends and the common cycles are contemporaneously uncorrelated, but it is not ruling out lagged dependences, so that, in general, $\mathsf{E}[\tau_{it}c_{js}] \neq 0$ for $t \neq s$, hence $\mathrm{E}[\Delta\tau_{it}\Delta c_{jt}] = \mathrm{E}[(\tau_{it} - \tau_{it-1})(c_{jt} - c_{jt-1})] \neq 0$. By combining Assumptions 8 and 9 we see that it must be the case that $\mathbf{B} = \boldsymbol{\beta}_\perp$, such that $\boldsymbol{\beta}_\perp'\boldsymbol{\beta} = \mathbf{0}$. Hence, from (7), $\boldsymbol{\Psi}_1 = \boldsymbol{\beta}_\perp(\boldsymbol{\beta}_\perp'\boldsymbol{\beta}_\perp)^{-1}$ and $\boldsymbol{\Psi}_2 = \boldsymbol{\beta}(\boldsymbol{\beta}'\boldsymbol{\beta})^{-1}$, and, by imposing the normalizations $\boldsymbol{\beta}_\perp'\boldsymbol{\beta}_\perp = \mathbf{I}_r$ and $\boldsymbol{\beta}'\boldsymbol{\beta} = \mathbf{I}_d$:

$$\boldsymbol{f}_t = \boldsymbol{\beta}_\perp\boldsymbol{\beta}_\perp'\boldsymbol{f}_t + \boldsymbol{\beta}\boldsymbol{\beta}'\boldsymbol{f}_t = \boldsymbol{\beta}_\perp\boldsymbol{\tau}_t + \boldsymbol{\beta}\boldsymbol{c}_t. \tag{8}$$

In our decomposition, the cycles are the Error Correction (EC) terms (a feature also of the

Gonzalo and Granger, 1995, decomposition), and the trends are what lies in the orthogonal space to the EC terms. In other words, the common cycles represent fluctuations around the long-run equilibria given by the common trends (see also the definition of output gap given by the CBO and discussed in Section 5).

Moreover, by letting $\boldsymbol{Z}_t = (\boldsymbol{\tau}_t' \;\; \boldsymbol{c}_t')'$, and $\boldsymbol{\Gamma} = (\boldsymbol{\beta}_\perp \;\; \boldsymbol{\beta})$, and by combining (4) and (8) we have (note that under the chosen normalization $\boldsymbol{\Gamma}^{-1} = \boldsymbol{\Gamma}'$):

$$\boldsymbol{Z}_t = \boldsymbol{\Gamma}' \boldsymbol{A}(L) \boldsymbol{\Gamma} \boldsymbol{Z}_{t-1} + \boldsymbol{\Gamma}' \mathbf{u}_t. \tag{9}$$

In particular, noting that, because of cointegration, we can write $[\mathbf{I}_q - \boldsymbol{A}(1)] = \boldsymbol{\alpha}\boldsymbol{\beta}'$, with $\boldsymbol{\alpha}$ of dimension $q \times d$ it can be shown that (9) is equivalent to,

$$\boldsymbol{\tau}_t = \boldsymbol{\tau}_{t-1} - \boldsymbol{\beta}_\perp' \boldsymbol{\alpha} \boldsymbol{c}_{t-1} + \boldsymbol{\beta}_\perp' \boldsymbol{A}^*(L) \boldsymbol{\beta}_\perp \Delta \boldsymbol{\tau}_{t-1} + \boldsymbol{\beta}_\perp' \boldsymbol{A}^*(L) \boldsymbol{\beta} \Delta \boldsymbol{c}_{t-1} + \boldsymbol{\beta}_\perp' \mathbf{u}_t, \tag{10}$$

$$\boldsymbol{c}_t = (\mathbf{I}_d - \boldsymbol{\beta}' \boldsymbol{\alpha}) \boldsymbol{c}_{t-1} + \boldsymbol{\beta}' \boldsymbol{A}^*(L) \boldsymbol{\beta}_\perp \Delta \boldsymbol{\tau}_{t-1} + \boldsymbol{\beta}' \boldsymbol{A}^*(L) \boldsymbol{\beta} \Delta \boldsymbol{c}_{t-1} + \boldsymbol{\beta}' \mathbf{u}_t, \tag{11}$$

where $\boldsymbol{A}^*(L) = \sum_{j=0}^{p-2} \boldsymbol{A}_j^* L^j$, with $\boldsymbol{A}_j^* = -\sum_{\ell=j+1}^{p-1} \boldsymbol{A}_\ell$. This gives the law of motion of the common trends and the common cycles.

From (10), we can see that our trend-cycle decomposition does not impose that the common trends evolve as random walks but as a general multivariate ARIMA process, which specific form depends on the law of motion of the common factors. A possible rationale for deviating from the random walk assumption is given by Lippi and Reichlin (1994, p. 21), who point out that, if we interpret the trend as productivity, the random walk assumption is "overly restrictive[, as] it rules out well-known features of technological adoption: once a new method has been introduced at the firm level, productivity increases follow a learning process; moreover, arrivals of new knowledge are not absorbed by all firms at the same time, but spread slowly throughout the economy." That said, in Section 5 and Section G of the Complementary Appendix, we show that our results are little affected if we assume that the common trends evolve as random walks.

Likewise, from (11), we can see that the common cycles follow a process which can be more general than the usual AR(2) often imposed in the literature (e.g., Jarociński and Lenza, 2018).

Finally, by combining (1) and (8), we have the trend-cycle decomposition of the data:

$$y_{it} = \mathcal{D}_{it} + \boldsymbol{\lambda}_i'(L)\boldsymbol{\beta}_\perp\boldsymbol{\tau}_t + \boldsymbol{\lambda}_i'(L)\boldsymbol{\beta}\boldsymbol{c}_t + \xi_{it} \tag{12}$$

from which we can define potential output PO_t, the output gap OG_t, the natural rate of unemployment NR_t, and the unemployment gap UG_t as follows:

$$PO_t = \mathcal{D}_{\text{GDO},t} + \boldsymbol{\lambda}_{\text{GDO}}'(L)\boldsymbol{\beta}_\perp\boldsymbol{\tau}_t, \tag{13}$$

$$OG_t = \boldsymbol{\lambda}_{\text{GDO}}'(L)\boldsymbol{\beta}\boldsymbol{c}_t, \tag{14}$$

$$NR_t = \mathcal{D}_{\text{UR},t} + \boldsymbol{\lambda}_{\text{UR}}'(L)\boldsymbol{\beta}_\perp\boldsymbol{\tau}_t, \tag{15}$$

$$UG_t = \boldsymbol{\lambda}_{\text{UR}}'(L)\boldsymbol{\beta}\boldsymbol{c}_t. \tag{16}$$

In other words, in our framework potential output is the sum of the local linear trend of GDO, $\mathcal{D}_{\text{GDO},t}$, which captures the secular decline in long-run U.S. output growth, and that part of GDO which is driven by the common trends, $\boldsymbol{\tau}_t$. The output gap is that part of GDO that is driven by the common cycles $\boldsymbol{c}_t$. The natural rate is the sum of the time-varying mean, $\mathcal{D}_{\text{UR},t}$, meant to capture slow-moving idiosyncratic labor market and demographic characteristics, and that part of fluctuations of the unemployment rate that are caused by the common trends. Lastly, the unemployment gap is that part of the unemployment rate driven by the common cycles.

To estimate trends and cycles we need to face two problems: (i) the factors $\boldsymbol{f}_t$ are not observed, and (ii) the cointegrating vector $\boldsymbol{\beta}$ and its orthogonal complement $\boldsymbol{\beta}_\perp$ are unknown. In practice, we do have a consistent estimator of the factors from Section 2.1.2, and, based on the factor representation of the common factors (6), we can estimate $\boldsymbol{\beta}$ and $\boldsymbol{\beta}_\perp$ by means of PC analysis, as shown by Zhang et al. (2019). Finally, to quantify the uncertainty attached to the estimated trends and cycles in equation (12) we use a bootstrap procedure that is presented in Section D of the Complementary Appendix.

3 Data

Our analysis is carried out on a large macroeconomic dataset comprising $n = 103$ quarterly series from 1960:Q1 to 2018:Q4 describing the U.S. economy for a total of $T = 236$ observations.[8] Specifically, our dataset includes national account statistics, industrial production indexes, different price indexes, various labor market indicators including indicators from both the household survey and the establishment survey, as well as labor cost and compensation indexes, monetary aggregates, credit and loans indicators, housing market indicators, interest rates, the oil price, and the S&P500 index. While which broad categories of data to include in the dataset is standard in the large dimensional factor model literature, which and how many series from each category to include is not standard.

When constructing a dataset for estimating a large dimensional dynamic factor model, we face a trade-off. On the one hand, the model is consistently estimated for n growing to infinity. On the other hand, Boivin and Ng (2006) show that an excess amount of cross-sectional correlation among idiosyncratic components worsens the performance of the model. Therefore, we have to balance between the need for adding variables to describe the U.S. economy better and to have a larger n, and the risk to add only idiosyncratic correlation.

In practice, we first estimated the model on a dataset comprising more than 130 indicators. Then, we eliminated all the redundant variables, defined as those variables with very high idiosyncratic cross-correlation.[9] We made some exceptions to this process, though, to

[8]Although we estimate the model starting in 1960, we decided to start all the plots in 1970. This helped us improve graphs readability, particularly because the most interesting part of our estimates is about what happened in the last 30 years or so.

[9]For example, we eliminated the 6-Month T-Bill interest rate whose idiosyncratic component has a correlation greater than 0.9 with both the 3-Month and the 1-Year T-Bill interest rate. Likewise, we eliminated the 5-Year T-Note rate and the 3-Year T-Note rate that have correlation higher than 0.9 with the 10-Year T-Note rate and the 1-year T-Bill rate, respectively.

retain some important variables that would have otherwise be eliminated (e.g., we kept consumption even though we have its three components in the dataset). As a result of this procedure, some broad categories might look under-represented, and some other over-represented. For example, we have just four interest rates, while payroll employment is disaggregated in many sectors. This is the case because including more interest rates would have added nothing but cross-correlation among the idiosyncratic components. By contrast, disaggregated payroll employment indicators do not exhibit much idiosyncratic cross-correlation, hence each of these indicators is adding potential information.

Moving to the treatment of the variables, broadly speaking, we take logs of all variables in levels that are not already expressed in percentage points. Furthermore, we keep all variables in levels, except price indicators and monetary aggregates, which are differenced once—in other words, we include price inflation and money growth in our model, rather than the price level and money stock. Indeed, as we discuss in Section 2.1.1, our model can handle both $I(1)$ and $I(0)$ variables. Therefore, the default strategy is not to transform any variable unless it exhibits some $I(2)$ dynamics, in which case the best approach is to take first differences of that specific variable. In our dataset, the only variables that exhibit some $I(2)$ dynamics are price indicators (deflators, CPI, PCE prices, and PPIs) and monetary aggregates, which is why we took first difference of them (similar transformation have also been used, for example, by Stock and Watson, 2016).[10]

The complete list of variables and transformations is reported in Section A of the Complementary Appendix.

[10] In Section J.8 of the Complementary Appendix, we show how our results change if we do not transform any variable in our dataset, i.e., if we include price indicators and monetary aggregates in levels.

4 Model setup

Before estimating the model, we need to choose the number of common factors q and of common trends r, the number of lags s in the factor loadings, and the number of lags p in the VAR for the common factors in (4). While the number of lags p in the VAR for the common factors can be consistently selected applying the BIC on the estimated factors (see, Barigozzi and Luciani, 2020), how to determine q, r, and s, deserve a proper discussion.

Because of Assumptions 4 and 5, the covariance matrix of the differenced data vector $\Delta \mathbf{y}_t = (\Delta y_{1t} \dots \Delta y_{nt})'$ has at most $q(s+1)$ eigenvalues diverging with n, all the others being bounded for all n. Furthermore, the q largest eigenvalues of the spectral density of $\Delta \mathbf{y}_t$ diverge with n at all frequencies but at zero-frequency: due to the presence of common trends, at zero-frequency, only r eigenvalues diverge, while all the others are bounded for all n. These three conditions allow us to recover the number of common factors q and of common trends r, and of lags s—see Hallin and Liška (2007), D'Agostino and Giannone (2012), and Barigozzi et al. (2020b) for further details.

To choose the number of lags s in the factor loadings, for a given choice of q, we pick s such that the share of variance explained by the first $k = q(s+1)$ eigenvalues of the covariance matrix of $\Delta \mathbf{y}_t$ coincides with the share of variance explained by the first q eigenvalues of the spectral density matrix of $\Delta \mathbf{y}_t$ (averaged over all frequencies)—see also D'Agostino and Giannone (2012). According to this criteria, it is clear that $k \simeq 2q$, meaning that $s \simeq 1$ (result not shown).

As for the number of factors and trends, let us start with the consideration that a large number of papers estimate the output gap by imposing one trend and one cycle (e.g., Fleischman and Roberts, 2011; Jarociński and Lenza, 2018; Hasenzagl et al., 2020). In light of this, a sensible strategy is to estimate a model with two common factors (one trend-one cycle), i.e., with $q = 2$.

The upper-left plot in Figure 2 shows the spectral density of the first difference of the common trend and of the common cycle when the model is estimated with two common factors. The estimated cycle captures frequencies corresponding to a period shorter than two years, and its spectral density has approximately the same shape of that of the PCE price inflation rate, more specifically it has a shape closer to that of PCE energy price inflation than to that of core PCE price inflation (see the lower-right plot in Figure 2). In some sense, this result is not surprising, as several papers have documented that the first two factors of the U.S. economy appear to be one "real" and one "nominal" factor (e.g., Stock and Watson, 2016). What our decomposition is adding to what the literature has documented is that these two common factors can be seen as a common "real" trend (in first differences the shape of its spectral density is similar to that of GDP growth, lower-left plot Figure 2), and as a "nominal" cycle.

To summarize, it is clear that with the first cycle dominated by high frequencies, it is impossible to get any meaningful estimate of the output gap or the unemployment gap. Therefore, we look for a richer parametrization with more cycles. This choice is supported by the test by Onatski (2009) and the information criterion by Hallin and Liška (2007) that suggest a value of q between three and four (not shown). Finally, the information criterion by Barigozzi et al. (2020b) clearly supports the presence of one common trend, i.e., $r = 1$ (not shown). Therefore, the above statistical procedures suggest the presence of one common trend, which is in line with many theoretical models assuming a common productivity trend as the sole driver of long-run dynamics (e.g., Del Negro et al., 2007), and either two or three common cycles.

The upper-middle plot in Figure 2 shows the spectral density of the first difference of the common trend and of the two common cycles obtained when the model is estimated with three common factors. The additional cycle captures mainly frequencies with period of about five years, and its spectral density has a shape somewhat similar to that of the unemployment rate. In other words, under this parametrization we have a representation of the economy with one common "real" cycle, and one common "nominal" cycle.

Is there any rationale to include a fourth factor, i.e., a third cycle? Table 1 shows the variance of the idiosyncratic component of selected variables—note that the idiosyncratic components of all the variables in the table are stationary. Most of the variance of GDP is accounted for by the common factors even with $q = 2$, and, moreover, the idiosyncratic variance of GDP decreases just modestly if additional factors are included in the model.[11] By contrast, the idiosyncratic variance of the other variables is quite sensitive to the number of factors. Starting with inflation as measured by the quarter-on-quarter percent change in the PCE price index, the model with three factors has an idiosyncratic variance that is more than 90% lower than the one with two common factors. Moving to the unemployment rate, the model with three common factors explains only part of its total variation, whereas once the fourth factor is included the idiosyncratic variance is more that 90% lower than the idiosyncratic variance of the model with three factors. The same is true for other job market indicators shown in Table 1, such as the labor force participation rate (LFPR), the employment to population ratio (EPR), and private payroll employment (EMP). For all these variables, the idiosyncratic variance of the model with four common factors is more than 80% lower than the model with three common factors.

Why do we need a third cycle to explain the unemployment rate? A possible explanation is that, as documented by Barnichon (2010) among others, prior to the mid 1980s, productivity was procyclical; afterwards, it was acyclical or countercyclical. As a result, the cyclical relationship between unemployment and output changed. This change cannot be captured by the model with just one "real" cycle; hence the need for an additional cycle.

Figure 3, which shows the estimated common trend and the three estimated common cycles, supports this assumption: the second and third cycle seem to be positively correlated until around the early 1990s and then negative correlated after that. This timing roughly matches the change in the cyclical behavior of productivity described in Barnichon (2010).

In summary, the results shown in this Section show how the model with three common

[11]Note that Table 1 shows the variance of the level of the estimated idiosyncratic component of log GDP. Since log GDP is 983.98 in 2018:Q4, a variance of 0.28 is negligible.

factors is a possible candidate, but at the same time it has weak explanatory power for the unemployment rate.[12] Therefore, since the unemployment rate is a crucial indicator of the cyclical position of the economy, our benchmark specification includes four common factors ($q = 4$) and one common trend ($r = 1$), implying three common cycles ($d = 3$).[13]

Under this parametrization, as shown in Figure 3, the first cycle looks like an inflation cycle, the second cycle seems to be related with GDP, and the third cycle seems to be related with the unemployment rate; hence, we have a representation of the economy with two common "real" cycles and one common "nominal" cycle (see also the upper-right plot in Figure 2). This interpretation is corroborated by the results in Table 2, which shows the estimated loadings of the common trend, $\widehat{\boldsymbol{\lambda}}_\ell \widehat{\boldsymbol{\beta}}_\perp$, $\ell = 0, 1$, and of the three common cycles, $\widehat{\boldsymbol{\lambda}}_\ell \widehat{\boldsymbol{\beta}}_i$, $i = 1, 2, 3$, $\ell = 0, 1$. As can be seen, (i) the common trend is mainly loaded by GDP and payroll employment (EMP), while many other variables, among others PCE price inflation, have loadings such that $\widehat{\boldsymbol{\lambda}}_0 \widehat{\boldsymbol{\beta}}_\perp \simeq -\widehat{\boldsymbol{\lambda}}_1 \widehat{\boldsymbol{\beta}}_\perp$; (ii) the first cycle is heavily loaded by inflation, while it is weakly loaded by GDP; finally, (iii) the second and third cycle are mainly loaded by real variables, while nominal variables loads these cycles very little (for example the oil price essentially does not load the third cycle).

5 Potential output and the output gap

In this section, we present estimates of potential output and the output gap. In contrast, in Section E of the Complementary Appendix, we present estimates of the natural rate of

[12]In Section J.1 of the Complementary Appendix we show that, while the model with three common factors produces a meaningful estimate of the output gap, it fails to provide a sensible estimate of the cyclical component of the unemployment rate.

[13]In Section F.2 of the Complementary Appendix, we evaluate the forecasting performance of our model. Results are inconclusive, and a more parsimonious specification might do better. That said, the focus of this paper is not forecasting; hence, our benchmark specification is not optimized for forecasting purposes.

unemployment and the unemployment gap.

In what follows, we compare our estimates with those published by the Congressional Budget Office (CBO). The CBO estimates potential output and the output gap by using the so-called "production-function approach" (Kiley, 2013) according to which potential output is that level of output consistent with current technologies and normal utilization of capital and labor, and the output gap is the deviation of output from potential output. To be more precise, the CBO model is based upon a textbook Solow growth model, with a neoclassical production function, and estimates trends in the components of GDP based on a variant of the Okun's law. In other words, in the CBO framework, the actual output is above its potential (the output gap is positive) when the unemployment rate is below the natural rate of unemployment, which is in turn defined as the non-accelerating inflation rate of unemployment (NAIRU), i.e., that level of unemployment consistent with stable inflation—for further details see Congressional Budget Office (2001).

Figure 4 shows our measure of potential output (solid line), where the shaded areas around our estimate are 68% and 84% confidence bands, respectively, together with the estimate produced by the CBO (dashed line), and GDP (dotted line). The plots on the top row and the bottom-left plot show the estimate of the level of potential output in three different periods, while the bottom-right plot shows the estimate of year-over-year (*yoy*) potential output growth.

As we can see, the two trend estimates start to diverge at the end of the 1990s, with the CBO estimating higher potential growth (i.e., a steeper increase in the trend) than our model (4.2% vs. 3.7% in 2000:Q1). However, at the beginning of the 2000s, both trend estimates slowdown, and they reunite at the end of the sample.

Our results contrast with those of Coibion et al. (2018), who estimate that potential output fell during the financial crisis, but then since 2010 has started to grow at approximately the same pace as before the crisis. However, as pointed out by Ng (2018) the path of potential output estimated by Coibion et al. (2018) is heavily influenced by their choice to include a deterministic linear trend in the model for GDP, which is responsible for the large output gap they estimate at the end of their sample in 2017:Q4 (see also Bullard, 2012). This argument is supported by the results in Section J.4: when imposing a trend with a constant slope for GDO, our estimate of potential output at the end of the sample is higher, and the output gap is wider than when allowing for a time-varying slope. Those results support the argument of Ng (2018).

Moving to the output gap, the left plot of Figure 5 shows our estimate together with the one produced by the CBO. Overall, our estimate is remarkably similar to that of the CBO. In particular, the peaks and troughs of the two measures align, thus indicating that the dating of the turning points perfectly coincides.

However, an important difference between the two estimates is visible during the period 1997-2008. The two output gap measures started to diverge somewhat significantly at the end of 1997, which approximately dovetails with the divergence in the estimate of potential output. Then, starting in 2000:Q3 the two measures roughly co-move; however, according to the CBO the level of the output gap was negative between 2002:Q1 and 2005:Q4, whereas according to our measure in that same period the output gap was positive. In other words, while the CBO estimates that the U.S. economy reached its potential just a couple of years before the financial crisis, our model signals a persistent overheating of the economy from the mid-1990s to 2008.

What does explain the wedge between our estimate and the CBO's in 1997–2008? Before answering this question, we shall point out that, as shown in Figure 4, our estimate of potential output growth fluctuates over time substantially more than the estimate of the CBO; in particular, it declines with recessions and increases in expansions.[14] Therefore, we

[14]From a technical point of view, as we discussed in Section 2.2, in our decomposition, the common trends evolve as a general ARIMA process; hence $\Delta\boldsymbol{\tau}_t$ evolves as an ARMA process. Therefore, our estimate of potential output growth tends to fluctuate by construction. Actually, unless trend growth is modeled as a random walk, and unless the variance of the

first want to rule out that this excess volatility in our potential output growth estimate is responsible for the wedge, and to this end, we consider different trends' specifications.

Figure 6 shows the estimate of potential output and the output gap obtained with a different version of our model. In this alternative version of the model, the common stochastic trend, call it $\tau_t^\star$, follows a random walk process, which is estimated by applying the Kalman Filter to τ_t. The common cycles, $\boldsymbol{c}_t^\star$, are then estimated as $\boldsymbol{c}_t^\star = \boldsymbol{f}_t - \boldsymbol{\beta}_\perp \tau_t^\star$ (additional details on estimation of this model and results can be found in Section G of the Complementary Appendix).

As shown in the left plot of Figure 6, the new estimate of *yoy* potential output growth is just a little bit more volatile than that of the CBO and less than our benchmark specification. Nonetheless, our model still estimates that potential output growth in the second half of the 1990s was slower than estimated by the CBO and that it slowed down just before the 2001 recession. Moreover, the new estimate of the output gap is nearly identical to our benchmark estimate.

Having cleared out that it is not excess volatility in our potential output growth estimate that drives our results, we can now come back to the wedge between our output gap estimate and the CBO's in 1997–2008? Figure 7 shows the decomposition of *yoy* GDP growth into the contribution of potential output and the output gap. Compared to our model, the CBO attributes a larger portion of GDP growth in the second half of the 1990s to potential growth. In particular, the CBO attributes most of the growth in 1999 and 2000 to potential output, and it estimates that potential growth started decreasing a few years after the 2001 recession. By contrast, our model attributes a large part of GDP growth in 1999 and 2000 to potential

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shock driving the trend is kept small (for example by imposing a tight a prior), any statistical model will yield an estimate of potential output growth that exhibits some degree of short-run fluctuations (see also the comparison with the estimate of potential output obtain by other authors with similar statistical methods shown in Section H of the Complementary Appendix).

output, but then it estimates that potential growth slows down with the 2001 recession. This difference in potential growth generates a wedge between the level of our output gap and the CBO output gap, which persists until the Great Recession.

Our results conform particularly well with Borio et al. (2017)'s narrative, who point out that credit growth was a key factor in overheating the economy from the late nineties to the Great Recession. This is a sensible economic interpretation of our results, as in the years before the financial crisis, a historically high share of sub-prime loan origination (Haughwout and Okah, 2009) and a large amount of equity withdrawal from housing (Fuster et al., 2017) were key factors in fueling the housing boom and in boosting consumption. A similar interpretation of the US economy comes from Furlanetto et al. (2021), who estimate that the output gap was positive from the mid-1990s until the Great Recession and attribute this result to the presence of financial frictions and financial shocks in their model. Another narrative that is compatible with our results is the one suggested by Fernald (2015) and Fernald et al. (2017), who argue that labor force participation and total factor productivity growth slowed before the Great Recession.[15]

Our model, being a reduced form statistical model, is not equipped to discriminate between these hypotheses and to isolate which force is actually driving our results. In Section L of the Complementary Appendix, we study the effect of removing specific groups of variables, but none seems to be the sole responsible for our results. Likewise, even when we consider smaller datasets (as long as the datasets are not too small), we get similar results. This is because the signal we are extracting is common to all variables and not specific to

[15]Incidentally, this "productivity view" is also compatible with our estimate of the output gap after the Great Recession, which is narrower than the CBO. This result is also in line with the "hysteresis view" according to which recessions reduce capital accumulation, have a long-term impact on workers who lose their jobs, and might disrupt the economic activities that produce technological progress (see, e.g., the discussion in Ball, 2014, and Blanchard et al., 2015).

any of them. Therefore, as long as we have enough variables to extract this signal, removing some variables does not alter our estimates.

To conclude, our reading and the CBO reading of the economic developments between the second half of the 1990s and the Great Recession are based on different and untestable assumptions about the law of motion of potential output and the output gap. Therefore, there is no obvious way to select which one of these two interpretations is best.

In Section F.1 of the Complementary Appendix, we compare the forecasting performance of our output gap measure in predicting output and inflation with those of the CBO measure. This might be a potential criterion to discriminate between the two measures, but the results are pretty much inconclusive—our output gap measure performs as good as, or marginally better than, the CBO measure. Furthermore, in Section 6, we compare the two measures for their real-time reliability, and in the last five years, our estimate of the output gap tends to revise less than that of the CBO. That said, in no way are we trying to say that our measure is better than the CBO, and our narrative should be considered "the truth." Rather, we are saying that our narrative is legit, and as we show in Section H of the Complementary Appendix, other authors using statistical models somewhat close to our models find that the economy was tighter from the early 2000s to the Great Recession than what the CBO estimate asserts.

6 Real-time estimation of the output gap

Since the seminal work of Orphanides and van Norden (2002), who show that end-of-sample revisions of GDP are of the same order of magnitude as the output gap, there has been much debate on the reliability of such estimates in real time. There are two reasons why a model-based estimate of the output gap revises in real-time: first, because the data upon which the model is estimated get revised over time. Second, because as new information comes in, both the estimates of the model parameters and of the latent factors, might change.

In this Section, we study the real-time properties of our output gap estimate through both a real-time and a *quasi*-real-time exercise on expanding windows. In the real-time exercise, which was limited by our ability to retrieve real-time data vintages for all 103 variables in our dataset—details on the real-time dataset are in Section B of the Complementary Appendix—we compare the revision to our output gap estimate with the revision to the CBO measure. In the *quasi*-real-time exercise, which is run over a much longer period, we dig deeper into the reliability of our output gap estimate. Finally, in Section I of the Complementary Appendix, we run a *quasi*-real-time exercise on contracting windows, which will be useful to assess the robustness of our model to the changes in the US business cycle that happened with the Great Moderation (see e.g., Gadea et al., 2020).[16]

The upper plots in Figure 8 show real-time estimates of the output gap for both our measure and the CBO measure. Two main conclusions can be drawn: first, our estimate of the output gap revises less than that of the CBO. Second, by comparing the estimate obtained on the first vintage of data (dashed black line) with the one obtained on the last vintage (solid black line), it is interesting to notice that both our model and the CBO model revised in the same direction, i.e., both models interpreted incoming data as signaling that there was less slack in the economy.

The lower-left plot in Figure 8 shows the *quasi*-real-time estimates of the output gapOur output gap measure revises very little when considering samples no more than five years shorter than our benchmark sample. However, the size of the revisions become relevant when considering samples that are ten or more years shorter than our benchmark sample. That said, our *quasi*-real-time estimates still indicate a constant overheating of the economy from the mid-1990s to the Great Recession.

[16] In an expanding windows exercise, the model is estimated on different samples, each starting in 1960 and ending in a different year (and quarter). In a contracting windows exercise, the model is estimated on different samples, each starting in a different year (and quarter) and ending in 2018:Q4.

Finally, the lower-right plot in Figure 8 shows that the real-time and *quasi*-real-time estimates of the output gap are very similar—the Mean Absolute Revision computed on comparable samples is 17 basis points in the *quasi*-real-time exercise and 21 basis points in the real-time exercise, with a Root Mean Squared Revision of 24 and 28 basis points, respectively. Therefore, since the difference between the real-time estimates and the *quasi*-real-time estimates are solely the results of data revisions, we can confirm the results in Orphanides and van Norden (2002) according to which output gap revisions are mainly caused by parameter estimation, rather than data revision.

7 Conclusions

This paper proposes a new measure of the output gap based on a dynamic factor model estimated on a large number of U.S. macroeconomic indicators. Our model incorporates relevant stylized facts about macroeconomic data, including that macroeconomic time series tend to co-move and be non-stationary, and that long-run output growth has drifted slowly over time.

Our estimate of the output gap tracks very well the one produced by the Congressional Budget Office (CBO), but from the late 1990s to the Great Recession. Whereas the CBO estimates that the output gap was negative between 2001 and 2006, our model suggests a persistent overheating of the U.S. economy from the mid-1990s to the Great Recession.

Our reading and the CBO reading of the economic developments between the second half of the 1990s and the Great Recession are based on different and untestable assumptions about the law of motion of potential output and the output gap. That said our model is offering a plausible narrative which is consistent with the data. Hence, we believe that our model, which is bringing to the table the ability to incorporate a large dataset, provides an additional measure that can be assessed in conjunction with others to gauge economic developments in real time.

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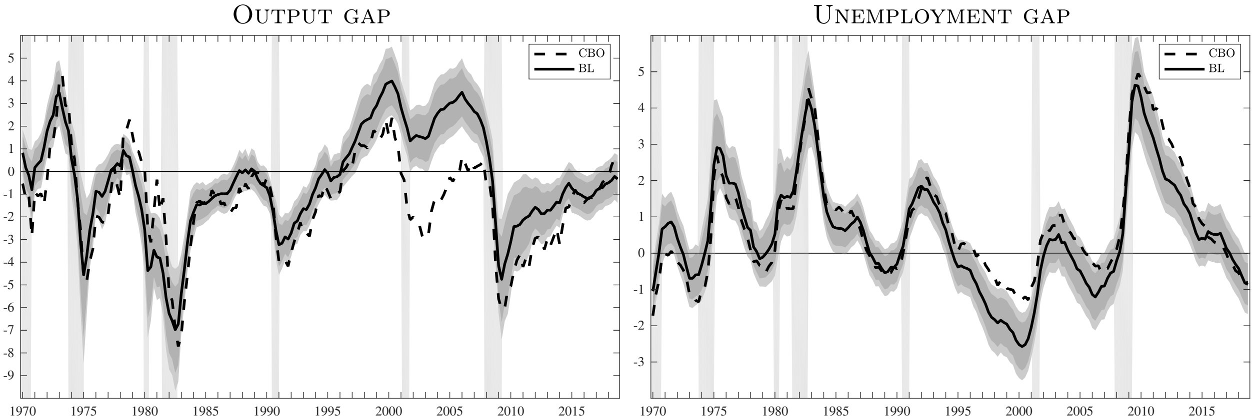
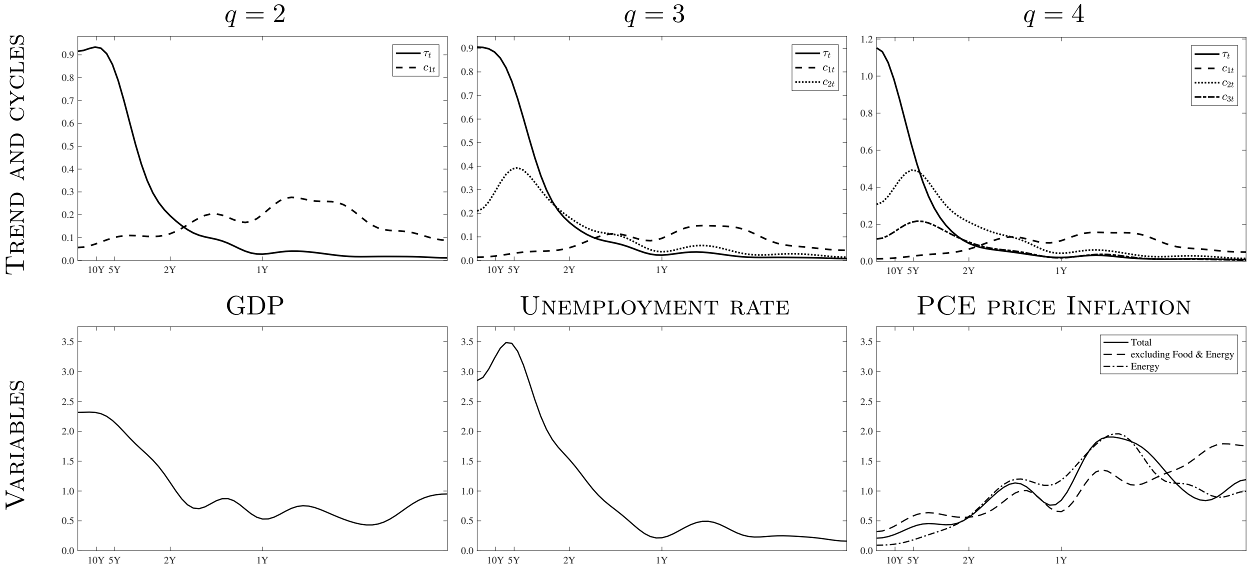
Figure 1: Synthetic view of our main findings



The dashed line is the CBO estimate, while the solid line is our estimate. The shaded areas around our estimate are 68% and 84% confidence bands, respectively.

Figure 2: Spectral densities

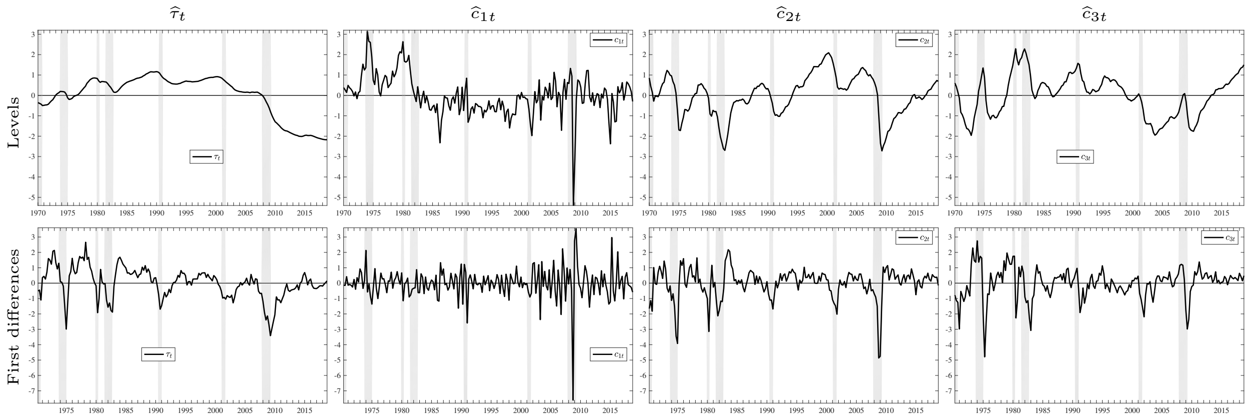


In the top row, the solid line is the normalized spectral densities of the common trend $\Delta\widehat{\boldsymbol{\tau}}_t$, while the other lines are the spectral densities of the common cycles $\Delta\widehat{\boldsymbol{c}}_t$. In the left plot, $q = 2$. In the middle plot, $q = 3$. In the right plot, $q = 4$.

The bottom row reports the normalized spectral densities of Δy_{it}.

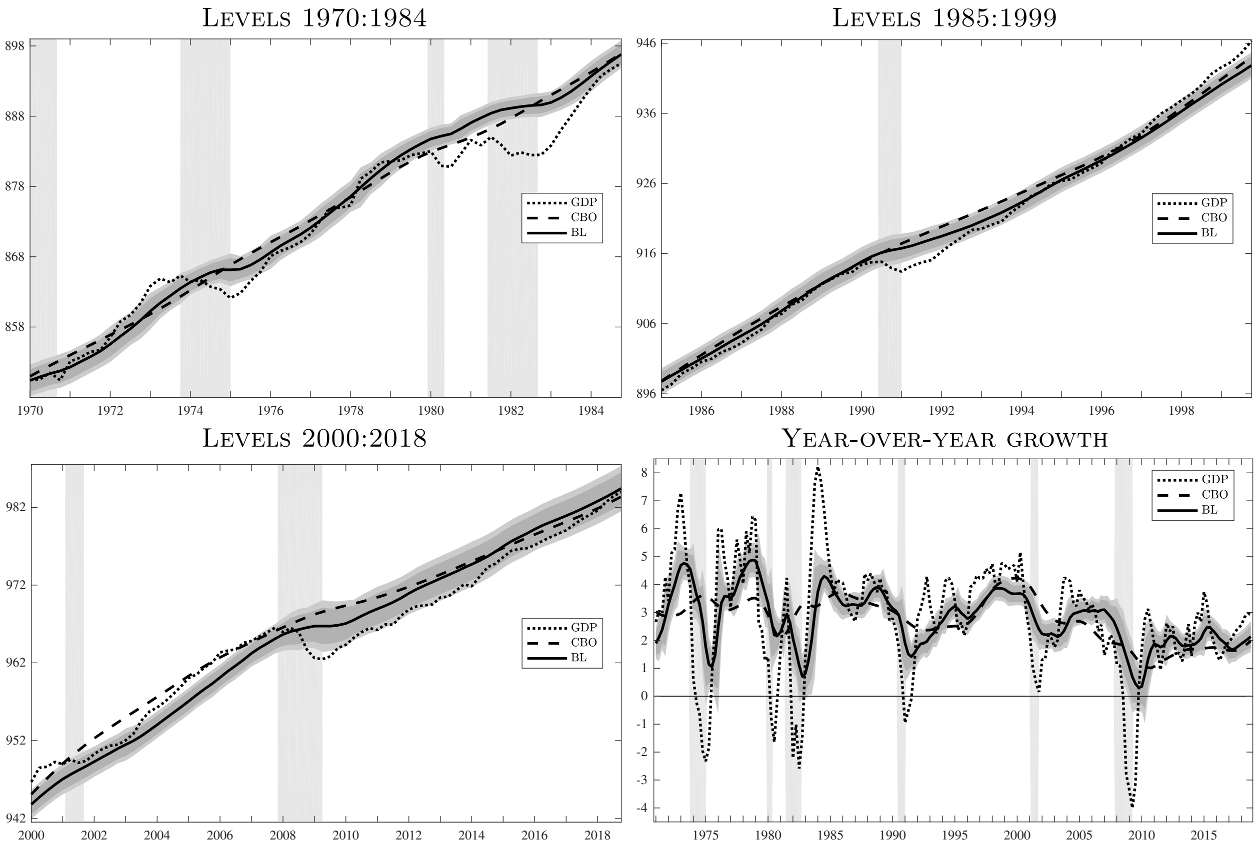
In all plots, the x-axis are periods $\mathcal{T}$ measured in years such that the corresponding frequencies are given by $\theta = 2\pi/4\mathcal{T}$.

Figure 3: Trend and Cycles



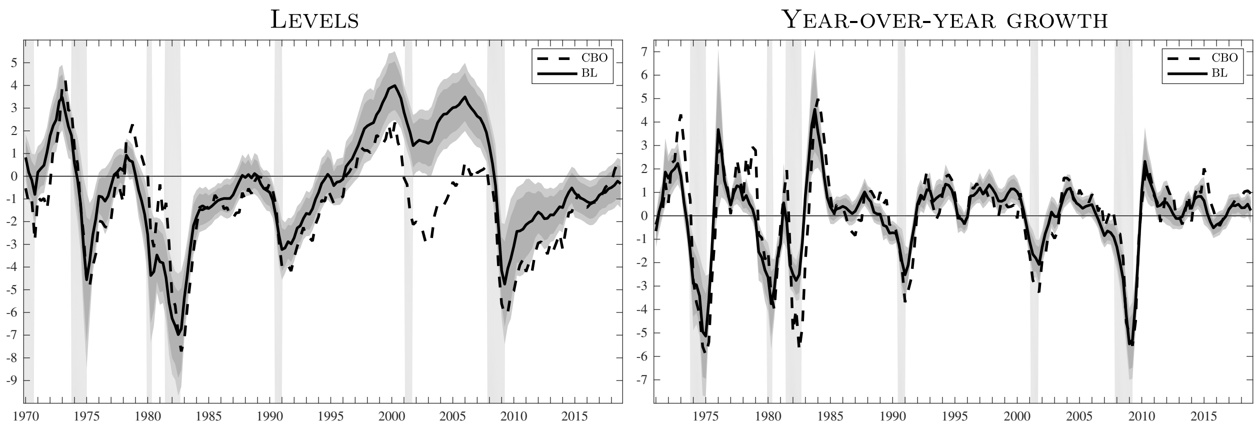
Estimated common trend and common cycles when $q = 4$.

Figure 4: Potential Output



The dashed line is the CBO estimate. The solid line is our estimate and the shaded areas around our estimate are 68% and 84% confidence bands, respectively. The dotted line is log GDP. All plots shows levels, but the lower-right plot which shows four-quarter differences.

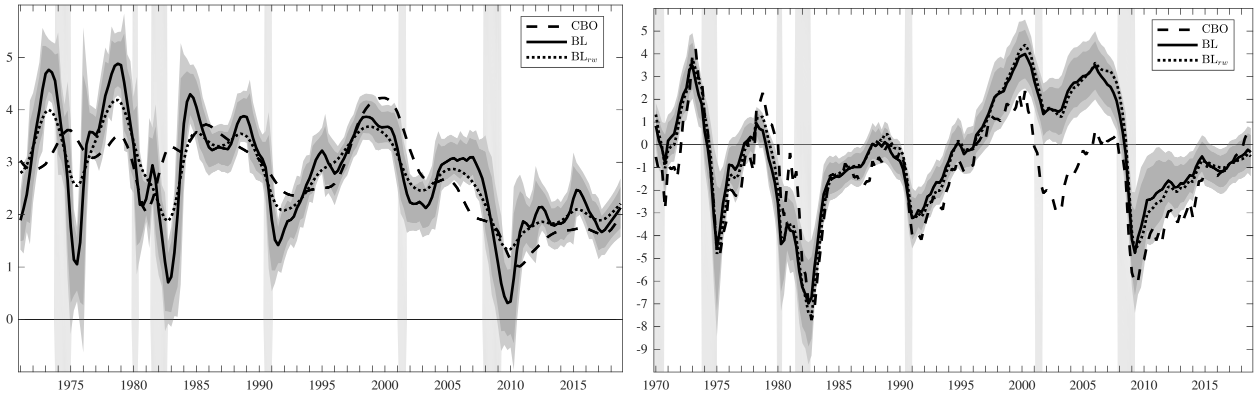
Figure 5: Output gap



The dashed line is the CBO estimate. The solid line is our estimate, and the shaded areas around our estimate are 68% and 84% confidence bands, respectively. The right plot shows log level, the left plot shows four-quarter differences.

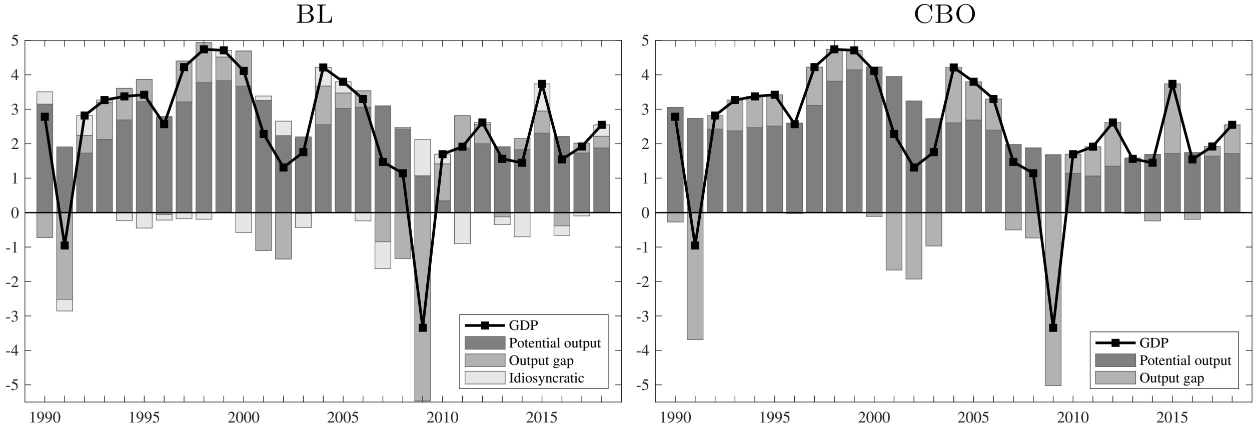
Figure 6: Alternative trend-cycle decomposition

Potential output growth (year-over-year) | Output gap



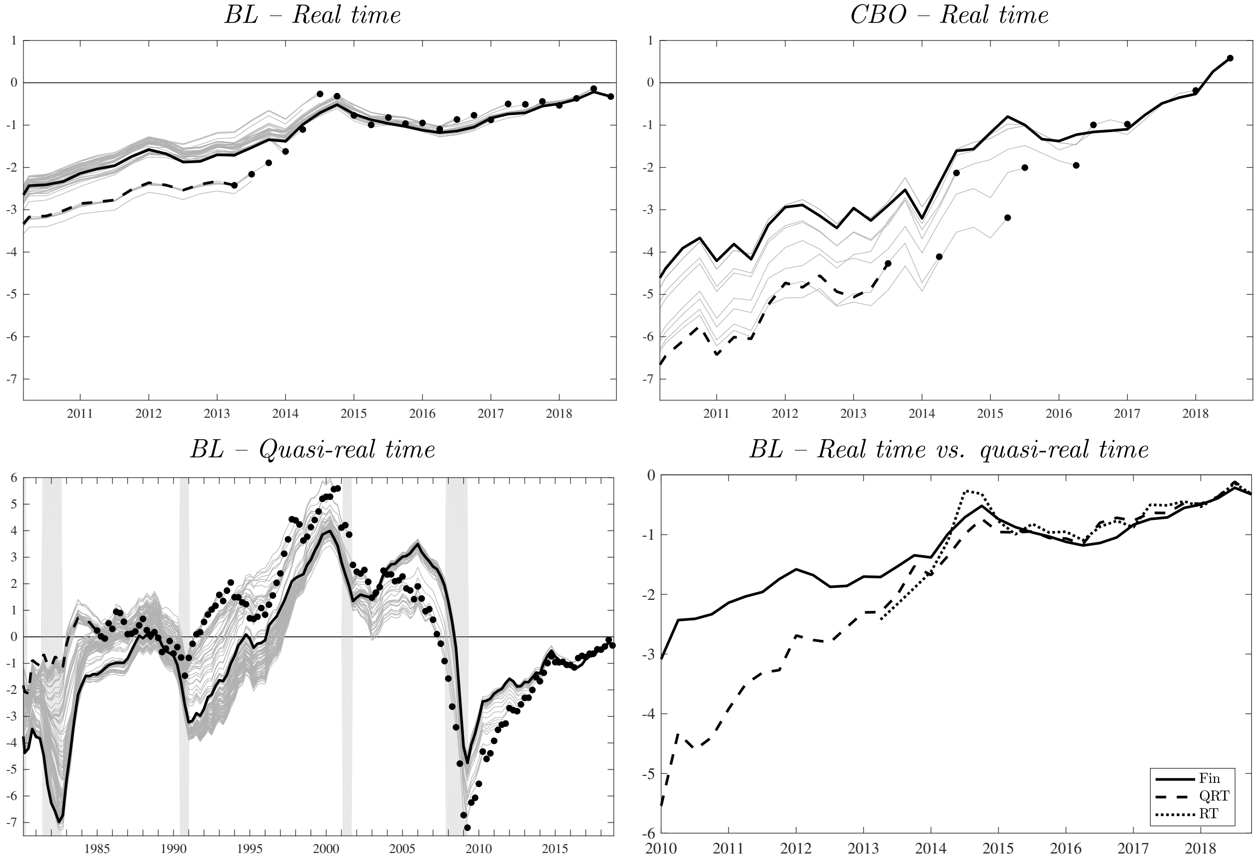
The dashed line is the CBO estimate. The solid line is our benchmark estimate and the shaded areas around our estimate are 68% and 84% confidence bands, respectively. The dotted line is our estimate obtained with the new specification of the model in which the common trend evolve as a random walk (detailed information can be found in Section G of the Complementary Appendix). The left plot shows four-quarter differences, the right plot shows levels.

Figure 7: Year-over-year GDP growth decomposition



The black line with dot markers is *yoy* GDP growth. The colored bars are the contribution to GDP growth of the different components. To make the chart more readable, for each year, we show just the data point in Q4.

Figure 8: Output gap: Real-time and quasi-real-time estimation



Upper plots: the dashed black line is the estimate of the output gap obtained with the vintage of data from August 30 2013, the solid black line is the estimate obtained with the vintage of data from on March 29 2019, and the thin grey lines are the estimate obtained with all the other vintages. Each black dots represent the estimate of the output gap for quarter Q and year Y obtained with the vintage of data ending at quarter Q and year Y. For CBO the dashed black line represents the estimate available as of April 11 2014.

Lower-left plot: the dashed black line is the estimate of the output gap obtained on the sample ending in 1985:Q1, the solid black line is the estimate obtained on the benchmark sample ending in 2018:Q4, and the thin grey lines are the estimate obtained on the other samples.

Lower-right plot: the solid line is the estimate of the output gap obtained on our benchmark sample (the "final" estimate), the dashed line is the *quasi*-real-time estimate, and the dotted line is the real-time estimate.

Table 1: Idiosyncratic variances

	GDP	PCE	PCEx	FFR	UR	LFPR	EPR	EMP
$q = 2$	0.325	0.232	0.196	7.297	0.940	2.504	1.045	5.161
$q = 3$	0.308	0.016	0.042	2.087	0.673	1.813	1.949	5.536
$q = 4$	0.267	0.009	0.017	2.041	0.020	0.118	0.053	1.046
$q = 5$	0.255	0.016	0.028	0.665	0.013	0.061	0.026	0.773

This table shows the variance of the idiosyncratic component of selected variables. "GDP" stands for Gross Domestic Product, "PCE" stands for Personal Consumption Expenditure price inflation, "PCEx" is PCE price inflation excluding food and energy, "FFR" is Fed Funds Rate, "UR" stands for Unemployment Rate, "LFPR" stands for Labor Force Participation Rate, "EPR" stands for Employment to Population Ratio, "EMP" is total private payroll Employment.

Table 2: ESTIMATED FACTOR LOADINGS

	$\widehat{\boldsymbol{\lambda}}_0\widehat{\boldsymbol{\beta}}_\perp$	$\widehat{\boldsymbol{\lambda}}_1\widehat{\boldsymbol{\beta}}_\perp$	$\widehat{\boldsymbol{\lambda}}_0\widehat{\boldsymbol{\beta}}_1$	$\widehat{\boldsymbol{\lambda}}_1\widehat{\boldsymbol{\beta}}_1$	$\widehat{\boldsymbol{\lambda}}_0\widehat{\boldsymbol{\beta}}_2$	$\widehat{\boldsymbol{\lambda}}_1\widehat{\boldsymbol{\beta}}_2$	$\widehat{\boldsymbol{\lambda}}_0\widehat{\boldsymbol{\beta}}_3$	$\widehat{\boldsymbol{\lambda}}_1\widehat{\boldsymbol{\beta}}_3$
GDP	0.55	0.40	-0.21	-0.16	1.43	0.16	0.99	0.01
PCE	1.92	-1.84	3.96	0.52	-1.74	1.44	0.50	-0.61
PCEx	4.51	-4.40	3.84	2.10	-4.10	3.34	1.95	-2.22
PCEe	-0.86	0.88	2.27	-0.71	0.87	-0.74	-0.28	0.25
FFR	5.61	-5.26	3.42	1.97	-3.66	3.08	0.48	-1.97
UR	0.40	-0.56	0.02	0.88	-1.92	-1.04	-0.29	0.96
LFPR	-9.09	10.83	-6.26	-3.15	9.51	-8.87	-5.65	4.47
EPR	-5.43	6.45	-3.04	-2.26	6.18	-3.57	-3.20	2.19
EMP	1.58	-0.07	-0.04	0.01	1.06	0.10	-0.88	-0.01
OIL	-0.62	0.63	1.93	-1.05	0.51	-0.41	-0.34	0.34
SP500	-7.66	7.66	-3.12	-1.62	7.84	-4.82	1.36	-1.30

"GDP" stands for Gross Domestic Product, "PCE" stands for Personal Consumption Expenditure price inflation, "PCEx" is PCE price inflation excluding food and energy (aka core PCE price inflation), "PCEe" is Energy PCE price inflation, "FFR" is Fed Funds Rate, "UR" stands for Unemployment Rate, "LFPR" stands for Labor Force Participation Rate, "EPR" stands for Employment to Population Ratio, "EMP" is total private payroll Employment, "OIL" is the WTI Oil price, and "SP500" is the S&P stock price index.