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Indeterminate forecast accuracy under indeterminacy*

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Abstract

This paper studies whether the observed time variation in the forecast accuracy of macro-econometric models can be reconciled with the monetary policy stance that induces (in)determinacy in stylized DSGE models. Using a small-scale New Keynesian monetary framework as laboratory and structural parameters calibrated to the estimates obtained on U.S. data from different macroeconomics regimes, we exploit reduced-form econometric models—such as Vector Autoregressions—to assess their regime-specific forecastability. We show that conducting (pseudo) out-of-sample forecast comparisons in the presence of indeterminacy is a non-trivial exercise, even when sunspot shocks play no role in generating the data. Overall, our simulation experiment suggests that equilibrium indeterminacy need not lead to superior (absolute or relative) forecast accuracy. This finding challenges the view that the deteriorating performance of forecast models over the Great Moderation relative to the Great Inflation was entirely due to changes in the U.S. monetary policy.

Keywords: DSGE, Forecasting, Indeterminacy, VAR system.

J.E.L.: C53, C62, E17

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1 Introduction

A recent debate in the (inflation) forecasting literature revolves around the inability of elaborate macro-econometric models to improve on simple univariate predictors, since the onset of the so-called Great Moderation. Contributions in the field include, but are not limited to, Atkeson and Ohanian (2001), Orphanides and van Norden (2005), Faust and Wright (2009), Rossi and Sekhposyan (2010), Christoffel *et al.* (2010), Edge and Gurkaynak (2010). While shown to be robust across a large variety of models, e.g. activity-based Phillips curves (Stock and Watson, 2007) and factor-augmented autoregressions (D'Agostino *et al.*, 2007), this finding has been largely associated with the emergence of weakly persistent inflation dynamics, as mostly dominated by transitory rather than permanent components (e.g. Stock and Watson, 2007).[1]

The present paper re-examines and qualifies the 'indeterminacy-persistence-forecastability' nexus which has been advocated by some scholars as a possible explanation of the superior forecast accuracy of macro-econometric models prior to the Great Moderation (e.g. Benati and Surico, 2008; Fujiwara and Hirose, 2014). Specifically, we investigate whether the observed time variation in the forecast accuracy of macro-econometric models can be reconciled with the monetary policy stance that induces (in)determinacy in stylized dynamic stochastic general equilibrium (DSGE) models. It is well known that such models may admit locally non-unique (indeterminate) stable equilibria under reasonable parameterizations (e.g. Lubik and Schorfheide, 2004). Equilibrium indeterminacy is generically associated with a richer autocorrelation structure, i.e. a stronger degree of endogenous persistence of resulting equilibrium representations, see e.g. Broze and Szafarz (1991), Lubik and Schorfheide (2003), Fanelli (2012); it also allows nonfundamental shocks (sunspot noise) to affect the model's reduced form dynamics. In principle, these peculiar time-series properties may result in superior predictive power of the indeterminate version of a given model, provided the degree of sunspot uncertainty is not too large. This observation naturally raises two distinct though intimately related research questions: first, is indeterminacy *per se* bound to favor data predictability *in absolute terms*? Second, can the declining *relative* predictive accuracy of macro-econometric models across two historical periods of U.S. business cycle be unambiguously framed in the context of equilibrium indeterminacy?

We conduct a simulation experiment to address both these questions. Using the small-scale New Keynesian model investigated in Benati and Surico (2008, 2009) as laboratory, we generate artificial datasets under determinacy and indeterminacy and exploit Vector Autoregressions (VARs) and other conventional time series models to assess both absolute and relative

[1] See also D'Agostino and Surico (2012) for a thorough analysis of inflation predictability in the U.S. across monetary regimes of the XXth century.

forecasting performances in either equilibrium regime. Structural parameters are calibrated to the estimates obtained by Benati and Surico (2009) on U.S. data on different macroeconomic regimes. When we generate data under determinacy, structural parameters are fixed to the estimates obtained by Benati and Surico (2009) on the Great Moderation period and reflect a setup characterized by an 'active' monetary policy able to prevent self-fulling inflation expectations. On the other hand, when we generate data under indeterminacy, structural parameters are fixed to the estimates obtained by Benati and Surico (2009) on the Great Inflation period and reflect a setup characterized by a 'passive' monetary policy that does not react aggressively enough to inflation shocks inducing multiple (stable) equilibria (e.g., Clarida *et al.*, 2000; Lubik and Schorfheide, 2004; Boivin and Giannoni, 2006; Castelnuovo and Fanelli, 2015*a*). Simulations show that, irrespective of whether the model's solution features sunspot noise or slightly deviates from the Minimum State Variable (MSV) solution, for certain values of the arbitrary parameters that govern solution multiplicity the (pseudo) out-of-sample forecasts of inflation obtained with VARs exhibit smaller average root-mean-square errors (RMSEs) than forecasts generated by univariate autoregressive (AR) or random walk (RW) predictors. For other values of these parameters, by contrast, the opposite occurs. Hence, a stronger (absolute and relative) predictive ability of VARs depends quite heavily on the selected equilibrium in the indeterminate set which is assumed to generate the data.

The first lesson we learn from our experiment is that establishing forecast accuracy on the basis of quadratic loss functions proves highly challenging in the presence of indeterminacy, notwithstanding the richer correlation structure and stronger degree of endogenous persistence featured by indeterminate equilibria. Specifically, it is argued that appropriate forecasting under indeterminacy based on reduced form time series models requires analyzing the relevant features of the underlying data generating process (DGP), and how they might contribute to forecast error. At least two peculiar features of equilibrium indeterminacy in DSGE models should be accounted for. One is the occurrence of arbitrary parameters that index solution multiplicity under indeterminacy, which affect the number of predictors and the parametric restrictions in the prediction framework. Remarkably, forecast errors might reflect the possible time-varying nature of these parameters. The other is the impact of sunspot (e.g. nonstructural) shocks that may drive the dynamics of the variables in addition to structural ones under indeterminacy, and hence may add to the intrinsic risk of prediction. We show that, regardless of whether sunspot noise occurs or not, the (pseudo) out-of-sample performance of a reduced form forecast model, in comparison to rival predictors, critically depends on how the arbitrary parameters interact with the structural ones in the moving average part of the solutions. We make this point clear at the very outset of our investigation through an example based on a simple linear rational expectations model.

The second lesson is that 'more stable' environments (determinacy) may enhance forecastability, while the situation is mixed under indeterminacy due to highly involved dynamics of the variables to forecast. Maintaining that the New Keynesian monetary framework represents a reasonable *prima facie* approximation of the post-WWII U.S. business cycle, and that a change in the stance of monetary policy contributed to the reduction of inflation persistence, the superior forecast performance of econometric models documented prior to the Great Moderation can be interpreted as a contingency characterizing the indeterminate equilibrium featured by U.S. business cycle during the Great Inflation period.

Our analysis enriches the literature on the decline in inflation predictability during the Great Moderation based on small-scale DSGE models with new insights. Benati and Surico (2008) exploit a small-scale monetary New Keynesian model to explore the extent to which the persistence and predictability of inflation vary with the parameters of the monetary rule, and conclude that a more aggressive policy stance towards inflation caused the decline in inflation predictability. These authors refer to a DSGE-based measure of inflation predictability, whereas we appeal to reduced form forecasting models and employ a standard measure of forecast uncertainty (the RMSEs). Remarkably, Benati and Surico's (2008) conclusions are based on the idea that more (DSGE-based) persistence leads to superior (DSGE-based) predictability. Our results provide a qualification to this argument, as they help clarify the subtle link between indeterminacy (determinacy) of DSGE equilibria and their dynamic (regime-specific) properties on the one hand, and the predictive ability of reduced form VARs, on the other hand. In the same vein, Fujiwara and Hirose (2014) suggest that forecast difficulties in the Great Moderation period can be potentially associated with the occurrence of equilibrium determinacy. More specifically, they argue that the documented active monetary policy behavior during the Great Moderation episode insulated the economy from nonfundamental shocks and hence prevented excessive business cycle fluctuations. The resulting reduction of the persistence and volatility of inflation and output turned out to penalize the forecastability of macro-econometric models. Conversely, the superior forecastability documented on the time span preceding the Great Moderation, conventionally denoted 'Great Inflation' period (1954-1984) would be, according to this argument, a by-product of equilibrium indeterminacy induced by the 'passive' monetary policy conduct of the Fed. From a technical point of view, our paper extends the analysis in Fujiwara and Hirose (2014) to a multivariate framework and helps to qualify some of their arguments. From a macroeconometric point of view, we follow Fujiwara and Hirose (2014) in not restricting attention to unique equilibrium models, but our analysis is broad in scope as we are interested in exploring the role of indeterminacy in favoring VARs relative to univariate predictors of macroeconomic time series.

The remainder of this paper is organized as follows. Section 2 provides the main idea of the paper by discussing connections between equilibrium (in)determinacy and forecastability in the context of a simple linear rational expectations model. Section 3 introduces the reference small-scale New Keynesian structural model and reports its state-space representation under either equilibrium regime. Section 4 presents and interprets the results of our Monte Carlo experiments in light of the empirical stylized facts that characterize the forecasting literature on U.S. macroeconomic variables. Section 5 offers a few concluding remarks. Appendix A complements the analytical results obtained with the univariate linear rational expectations model with a detailed Monte Carlo experiment, while Appendix B provides details on the derivation of the equilibria associated with the New Keynesian structural framework.

2 Background

To set ideas, we consider a simple univariate illustrative example, already analyzed in Lubik and Schorfheide (2004), among others, and used in Fujiwara and Hirose (2014) to introduce forecastability under indeterminacy.

Let X_t be a (scalar) random variable defined on a properly filtered probability space, whose dynamics are governed by the following expectational difference equation:

$$X_t = \frac{1}{\theta} E_t X_{t+1} + \omega_t, \quad \omega_t \sim WN(0, \sigma_\omega^2) \tag{1}$$

where $E_t X_{t+1} := E(X_{t+1} \mid \mathcal{F}_t)$, $\mathcal{F}_t := \sigma(X_t, ..., X_1)$ represents the conditioning information set at time t, ω_t is a structural shock, and $\theta \in \Re$ is a structural parameter. X_0 and ω_0 are considered fixed at time $t = 1$. As is known, any solution to (1) satisfies the recursive equation:

$$X_t = \theta X_{t-1} - \theta \omega_{t-1} + \eta_t \tag{2}$$

where $\eta_t := X_t - E_{t-1} X_t$ is the endogenous expectation error. When $\theta > 1$ (determinacy), the (locally) unique non-explosive solution is given by

$$X_t = \omega_t \tag{3}$$

implying that X_t follows white noise dynamics. When $\theta < 1$ (indeterminacy), by contrast, the endogenous expectation error is not constrained by stability requirements, hence any covariance-stationary martingale difference sequence (MDS) η_t will deliver a rational expectations stationary equilibrium of the form in Eq. (2).[2]

[2] Without loss of generality, we abstract from the random walk case, $\theta = 1$, for our focus is on stable (asymptotically stationary) solutions.

The expectation error η_t can be expressed as a linear combination of the model's structural disturbance and a conditionally mean-zero sunspot shock, i.e. $\eta_t = \tilde{m}\omega_t + s_t$ (Lubik and Schorfheide, 2003), where $\tilde{m}$ is an arbitrary parameter unrelated to θ and σ^2_ω, and s_t is an $\mathcal{F}_t$-measurable MDS ($E_t s_{t+1} = 0$) with (assumed) constant variance σ^2_s. For simplicity, we assume $\tilde{m}$ to be non-stochastic and time-invariant. The full set of solutions under indeterminacy is described by the ARMA(1,1)-type process

$$X_t = \theta X_{t-1} + \tilde{m}\omega_t - \theta\omega_{t-1} + s_t. \tag{4}$$

Simple inspection of Eqs. (3) and (4) reveals that, at the most basic level, the content of indeterminacy for the forecaster is essentially twofold. First, dynamic properties of the model's equilibrium are richer under indeterminacy than the determinate (pure noise) case. For $\tilde{m} \neq 1$ and non-zero σ^2_s, Eq. (4) gives rise to a large variety of equilibria, while for $\tilde{m} = 1$ and $\sigma^2_s = 0$, and despite $\theta < 1$, Eq. (4) collapses to a MSV solution which is observationally equivalent to the determinate equilibrium in Eq. (3). While inducing a larger lag structure and hence persistence in X_t, indeterminacy also implies serial correlation in the composite error term $v_t{:=}\tilde{m}\omega_t - \theta\omega_{t-1} + s_t$ forcing Eq. (4). As a result, the presence of pure sunspot noise as well as of an arbitrary response of the endogenous variable to the structural shock have the potential to induce higher volatility in the data generated under Eq. (4) relative to the data generated under Eq. (3).[3] Second, indeterminacy induces richly parameterized time series representations of equilibria, in which the occurrence of indeterminate dynamics induced by both the structural shock ω_t (via the impact coefficient $\tilde{m}$) and the sunspot shock s_t crucially affects any employed measure of forecast accuracy based on quadratic loss functions.

Overall, the relevant question is how significant the contribution of the above mentioned features to the forecast error associated to a given model is when the data are generated according to Eq. (4). In this respect, the common intuition, also reported in Fujiwara and Hirose (2014), is that since X_t in Eq. (4) exhibits richer dynamics relative to Eq. (3), the endogenous persistence implied by Eq. (4) can help forecasters predict the future path of X_t using a wide range of time series models. We explore this conjecture by considering a forecaster endowed with data $X_1, X_2, ..., X_T$ who wishes to predict the future path of X_t using a simple AR(1) model. To simplify the analysis, we focus on a sunspot-free environment, meaning that sunspot shocks are absent from the set of indeterminate equilibria, i.e. $\sigma^2_s = 0$ (implying $s_t = 0$ a.s. for each t) in Eq. (4), so that multiplicity of solutions is solely governed by the indeterminacy parameter $\tilde{m}$.

[3] In principle, volatility in x_t may be further amplified by endogenous expectations revisions which are arbitrarily related to fundamentals, whereas the converse might occur for a suitable choice of the parameter $\tilde{m}$. Moreover, different dynamic structures of the underlying model, e.g. those featuring lagged expectations, may allow for serially correlate sunspots to arise in equilibrium (e.g. Sorge, 2012).

Furthermore, we focus on one-step ahead forecasts $X^f_{T+1|T}$:=$E(X_{T+1} \mid \mathcal{F}_T)$ only, for any given period T.

As mentioned, the forecaster predicts the future path of X_t by using the AR(1) model:

$$X_t = \beta X_{t-1} + u_t \ , \ u_t \sim WN(0, \sigma_u^2) \ , \ \ |\beta| < 1 \ , \ \ t = 1, ..., T. \tag{5}$$

That is, the forecaster is not aware of the regime-dependent nature of the underlying DGP. If Eq. (3) is the true DGP, this amounts to over-specifying the response dynamics including an irrelevant predictor, whereas an omitted-variable bias arises when Eq. (4) generates the data. Hence, our naive forecaster is either failing to impose relevant restrictions on the lag structure of the underlying (forecasting) model, or rather forcing the moving average part of the model's solution into the error process. In either case, $X^f_{T+1|T} := \beta X_T$ and the one-step ahead forecast error is given by

$$e_{T+1} = \begin{cases} \omega_{T+1} - \beta\omega_T & \text{under determinacy} \\ (\theta - \beta)X_T + \ \tilde{m}\omega_{T+1} - \theta\omega_T & \text{under indeterminacy } (\tilde{m} \neq 1) \end{cases}$$

so that, after some manipulations we obtain the mean squared forecast error:

$$E(e^2_{T+1}) = \begin{cases} (1+\beta^2)\sigma^2_\omega & \text{under determinacy} \\ g(\beta, \theta, \tilde{m})\sigma^2_\omega & \text{under indeterminacy } (\tilde{m} \neq 1) \end{cases} \tag{6}$$

where $g(\beta, \theta, \tilde{m})$ is a nonlinear function in the arguments β, θ and $\tilde{m}$, defined by the expression

$$g(\beta, \theta, \tilde{m}) = \frac{(\theta - \beta)^2}{1 - \theta^2} \left[\tilde{m}^2 + \theta^2 - 2\theta^2\tilde{m}\right] - 2(\theta - \beta)\theta\tilde{m} + \tilde{m}^2 + \theta^2. \tag{7}$$

Note that for $\tilde{m} = 1$ (MSV solution), $g(\beta, \theta, 1) = (1 + \beta^2)$. Eqs. (6)-(7) suggest that despite the AR(1) model omits an important predictor under indeterminacy, its forecast performance under indeterminacy may be superior (inferior) than the one under determinacy depending on whether it holds the condition $g(\beta, \theta, \tilde{m}) < (1+\beta^2)$ $(g(\beta, \theta, \tilde{m}) > (1+\beta^2))$. Let $\hat{\beta}_T$ be the OLS estimator of β obtained from Eq. (5) with observations $X_1, X_2, ..., X_T$. In principle, for fixed values of the structural parameters θ and σ^2_ω, and given $\hat{\beta}_T$, it is possible to find values of $\tilde{m}$ such that $g(\hat{\beta}_T, \theta, \tilde{m}) < (1 + \hat{\beta}^2_T)$ or $g(\hat{\beta}_T, \theta, \tilde{m}) > (1 + \hat{\beta}^2_T)$. To see this, observe that the estimator $\hat{\beta}_T$ is consistent under determinacy and is asymptotically biased under indeterminacy, namely:

$$\hat{\beta}_T \xrightarrow{p} \begin{cases} \beta = 0 & \text{under determinacy} \\ \beta = \theta - \theta\frac{(1-\theta^2)}{(\tilde{m}^2+\theta^2-2\theta^2\tilde{m})} & \text{under indeterminacy } (\tilde{m} \neq 1) \end{cases} \tag{8}$$

where '$\xrightarrow{p}$' means convergence in probability for $T \longrightarrow \infty$. Assume that T is large. Replacing β with $\hat{\beta}_T$ in Eq. (6) and using the convergences in Eq (8), the function $g(\hat{\beta}_T, \theta, \tilde{m})$ in Eq.

(7) depends on θ and $\tilde{m}$ alone. For instance, for θ:=0.95 - which is the calibrated value of θ used in the Monte Carlo experiment of Appendix A under indeterminacy - we have that e.g. the value of the indeterminacy parameter $\tilde{m}$:=0.85 leads to $g(\hat{\beta}_T, \theta, \tilde{m}) < (1 + \hat{\beta}_T^2) \approx 1$, which implies that forecast accuracy is superior under indeterminacy; conversely, e.g. the values $\tilde{m}$:=0.80 and $\tilde{m}$:=1.01 lead to $g(\hat{\beta}_T, \theta, \tilde{m}) > (1 + \hat{\beta}_T^2) \approx 1$, which implies that forecast accuracy is superior under determinacy. In Appendix A we confirm and supplement this analysis with a detailed Monte Carlo investigation which explores relative forecast performance at different forecast horizons.

This highly stylized example appears to suggest that any forecasting exercise based on RMSEs as measure of forecast accuracy is destined to become more challenging and undecipherable a-priori under indeterminacy relative to the case of determinacy. This is a natural consequence of the arbitrariness which characterized the moving average part of the model's solution under indeterminacy. Overall, results obtained from the simple linear rational expectations model in Eq. (1) show that (i) it is not necessarily true that using typical forecast models forecastability improves if the DGP belongs to the set of indeterminate equilibria; (ii) the endogenous persistence implied by the indeterminate equilibria in Eq. (4) needs not necessarily help forecasters predict the future path of X_t, regardless of whether the forecast exercise is conducted with time series models that omit relevant propagation mechanisms or not; (iii) a very high degree of uncertainty stemming from nonfundamental (sunspot) shocks can weaken the predictive ability of macro-econometric models, yet their forecast accuracy under indeterminacy relative to determinacy can prove to be inferior even in sunspot-free economies.

There is no reason to think that this simple example is special. In the following sections, we show that the argument can be generalized to more realistic model-based forecasting environments.

3 Structural model and equilibria

Our simulation experiment is based upon Benati and Surico's (2009) New Keynesian monetary business cycle model, given by the three equations:

$$\tilde{x}_t = \gamma E_t \tilde{x}_{t+1} + (1-\gamma)\tilde{x}_{t-1} - \delta(R_t - E_t \pi_{t+1}) + \omega_{\tilde{x},t} \tag{9}$$

$$\pi_t = \frac{\beta}{1+\beta\alpha} E_t \pi_{t+1} + \frac{\alpha}{1+\beta\alpha}\pi_{t-1} + \kappa \tilde{x}_t + \omega_{\pi,t} \tag{10}$$

$$R_t = \rho R_{t-1} + (1-\rho)(\varphi_\pi \pi_t + \varphi_{\tilde{x}} \tilde{x}_t) + \omega_{R,t} \tag{11}$$

where

$$\omega_{i,t} = \rho_i \omega_{i,t-1} + \varepsilon_{i,t}, \quad -1 < \rho_i < 1, \quad \varepsilon_{i,t} \sim \text{WN}(0, \sigma_i^2) \;\; , \;\; i = \tilde{x}, \pi, R \tag{12}$$

and expectations are conditional on the information set $\mathcal{F}_t$, i.e. $E_t\cdot := E(\cdot \mid \mathcal{F}_t)$. The variables $\tilde{x}_t$, π_t, and R_t stand for the output gap, inflation, and the nominal interest rate, respectively; γ is the weight of the forward-looking component in the dynamic IS curve; α is price setters' extent of indexation to past inflation; δ is households' intertemporal elasticity of substitution; κ is the slope of the Phillips curve; ρ, φ_π, and $\varphi_{\tilde{x}}$ are the interest rate smoothing coefficient, the long-run coefficient on inflation, and that on the output gap in the monetary policy rule, respectively; finally, $\omega_{\tilde{x},t}$, $\omega_{\pi,t}$ and $\omega_{R,t}$ in eq. (12) are the mutually independent, autoregressive of order one disturbances and $\varepsilon_{\tilde{x},t}$, $\varepsilon_{\pi,t}$ and $\varepsilon_{R,t}$ are the structural (fundamental) shocks.

This or similar small-scale models have successfully been employed to conduct empirical analysis concerning the U.S. economy. Clarida *et al.* (2000), Lubik and Schorfheide (2004) and Castelnuovo and Fanelli (2015*a*) have investigated the influence of systematic monetary policy over the U.S. macroeconomic dynamics; Boivin and Giannoni (2006) and Benati and Surico (2009) have replicated the U.S. Great Moderation, Benati (2008) and Benati and Surico (2009) have investigated the drivers of the U.S. inflation persistence; Castelnuovo and Surico (2010) have replicated the VAR dynamics conditional on a monetary policy shock in different sub-samples.

The output gap in Eq. (9) is defined by $\tilde{x}_t := x_t - x_t^n$, where x_t is output and x_t^n is the natural rate of output. We complete the structural model specification by postulating that x_t^n is driven by a technology shock and follows the Random Walk process

$$x_t^n = x_{t-1}^n + \varepsilon_{x^n,t} \quad , \quad \varepsilon_{x^n,t} \sim \mathrm{WN}(0, \sigma_{x^n}^2). \tag{13}$$

Eq. (13) will be used to define the measurement equation associated with the state-space equilibrium representation of our DSGE model, see Appendix B.

It is worth emphasizing that our choice of the small-scale system (9)-(13) as laboratory need not be associated with a comparatively inferior forecast performance vis-à-vis larger scale models like e.g. the Smets and Wouters's (2007) framework. As argued in Herbst and Schorfheide (2012), the additional features incorporated into larger models do not involve a uniform improvement in the quality of density forecasts and prediction of co-movements of output, inflation, and interest rates, relative to three-equations systems.

We compact the structural system composed by Eqs. (9)-(12) in the general representation

$$\Gamma_0 X_t = \Gamma_f E_t X_{t+1} + \Gamma_b X_{t-1} + \omega_t \tag{14}$$

$$\omega_t = \Xi \omega_{t-1} + \varepsilon_t \ , \ \ \varepsilon_t \sim \mathrm{WN}(0, \Sigma_\varepsilon) \tag{15}$$

$$\Xi := dg(\rho_{\tilde{x}}, \rho_\pi, \rho_R) \quad , \quad \Sigma_\varepsilon := dg(\sigma_{\tilde{x}}^2, \sigma_\pi^2, \sigma_R^2)$$

where $X_t{:=}(\tilde{x}_t, \pi_t, R_t)'$, $\omega_t{:=}(\omega_{\tilde{x},t}, \omega_{\pi,t}, \omega_{R,t})'$, $\varepsilon_t{:=}(\varepsilon_{\tilde{x},t}, \varepsilon_{\pi,t}, \varepsilon_{R,t})'$ and

$$\Gamma_0{:=}\begin{pmatrix} 1 & 0 & \delta \\ -\kappa & 1 & 0 \\ -(1-\rho)\varphi_{\tilde{y}} & -(1-\rho)\varphi_{\pi} & 1 \end{pmatrix}, \Gamma_f{:=}\begin{pmatrix} \gamma & \delta & 0 \\ 0 & \frac{\beta}{1+\beta\alpha} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_b{:=}\begin{pmatrix} 1-\gamma & 0 & 0 \\ 0 & \frac{\alpha}{1+\beta\alpha} & 0 \\ 0 & 0 & \rho \end{pmatrix}.$$

Let $\theta{:=}(\gamma, \delta, \beta, \alpha, \kappa, \rho, \varphi_{\tilde{x}}, \varphi_\pi, \rho_{\tilde{x}}, \rho_\pi, \rho_R, \sigma^2_{\tilde{x}}, \sigma^2_\pi, \sigma^2_R)'$ be the vector of structural parameters. The elements of the matrices Γ_0, Γ_f, Γ_b and Ξ depend nonlinearly on θ and, without loss of generality, the matrix $\Gamma_0^{\Xi}{:=}(\Gamma_0 + \Xi\Gamma_f)$ is assumed to be non-singular. The space of all theoretically admissible values of θ is denoted by $\mathcal{P}$ and X_0 and X_{-1} are fixed initial conditions.

A solution to system (14)-(15) is any stochastic process $\{X_t^*\}_{t=0}^{\infty}$ such that, for $\theta \in \mathcal{P}$, $E_t X^*_{t+1} = E(X^*_{t+1} \mid \mathcal{F}_t)$ exists and it solves the system (14)-(15) at any t, for fixed initial conditions. A reduced form solution is a member of the solution set whose time series representation is such that X_t can be expressed as a function of ε_t, lags of X_t and ε_t and, possibly, other arbitrary MDSs with respect to $\mathcal{F}_t$, possibly independent of ε_t, called 'sunspot shocks' (e.g. Broze and Szafarz, 1991).

As is known, determinacy/indeterminacy is a system property that depends on a subset of elements in θ, see Lubik and Schorfheide (2004) and Fanelli (2012). More specifically, the solution uniqueness properties of the system of Euler equations (14)-(15) depend on whether θ lies in the determinacy or indeterminacy region of the parameter space. Thus, the theoretically admissible parameter space $\mathcal{P}_\theta$ is decomposed into two disjoint subspaces, the determinacy region, $\mathcal{P}_\theta^D$, and its complement $\mathcal{P}_\theta^I{:=}\mathcal{P}_\theta \backslash \mathcal{P}_\theta^D$. We assume that for each $\theta \in \mathcal{P}_\theta$, an asymptotically stationary (stable) reduced form solution to system (14)-(15) exists, hence the case of non-stationary possibly 'explosive' (unstable) indeterminacy is automatically ruled out. Since we consider only stationary solutions, the set $\mathcal{P}_\theta^I$ contains only values of θ for which multiple stable solutions arise.

A technical discussion of the equilibria associated with the DSGE system (14)-(15) is reported in Appendix B, where the interested reader is referred to. Next we summarize the state-space representation of the DSGE model in the two scenarios, which are the tools through which the data are generated in our experiments.

Under determinacy, the so-called ABCD form (Fernández-Villaverde *et al.* 2007) of the determinate equilibrium is represented by the system

$$\begin{aligned} \underset{2n\times 1}{z_t^d} &= \underset{2n\times 2n}{A^d(\theta)}\ \underset{2n\times 1}{z_{t-1}^d} + \underset{2n\times(n+b)}{B(\theta)}\ \underset{(n+b)\times 1}{u_t^d} \\ \underset{p\times 1}{y_t} &= \underset{p\times 2n}{C(\theta)}\ \underset{2n\times 1}{z_{t-1}^d} + \underset{p\times(n+b)}{D(\theta)}\ \underset{(n+b)\times 1}{u_t^d} \end{aligned} \tag{16}$$

where $z_t^d{:=}(X_t', X_{t-1}')'$ is the state vector, n is the dimension of the state vector X_t in Eq. (14) ($n = 3$ in our specific case), y_t is the vector of observable variables, which in our specific case is given by $y_t{:=}(\Delta x_t, \pi_t, R_t)'$, Δx_t being output growth, $A^d(\theta)$, $B(\theta)$, $C(\theta)$ and $D(\theta)$ are conformable matrices whose elements depend nonlinearly on θ, and $u_t^d{:=}(\varepsilon_t', v_t')'$ is the vector containing all system innovations, i.e. the fundamental shocks ε_t and the innovations associated with the measurement system, v_t, if any. The superscript 'd' stands for 'determinacy'.

Under indeterminacy, instead, the ABCD form associated with equilibria is given by

$$\begin{aligned} \underset{3n\times 1}{z_t^{in}} &= \underset{3n\times 3n}{A^{in}(\theta)}\ \underset{3n\times 1}{z_{t-1}^{in}} + \underset{3n\times(4n+b)}{B(\theta,\tilde{m})}\ \underset{(4n+b)\times 1}{u_t^{in}} \\ \underset{p\times 1}{y_t} &= \underset{p\times 3n}{C(\theta)}\ \underset{3n\times 1}{z_{t-1}^{in}} + \underset{p\times(4n+b)}{D(\theta,\tilde{m})}\ \underset{(4n+b)\times 1}{u_t^{in}} \end{aligned} \tag{17}$$

where the superscript 'in' stands for 'indeterminacy'. Here, the state vector $z_t^{in}{:=}(X_t', X_{t-1}', X_{t-2}')'$ involves an additional lag compared to the case of determinacy, while $y_t{:=}(\Delta x_t, \pi_t, R_t)'$ is the same as before. Notably, the 'B' and 'D' matrices in Eq. (17) depend not only on the structural parameters θ, but also on a vector of auxiliary parameters, unrelated to θ, that we collect in the vector $\tilde{m}$. $u_t^{in}{:=}(e_t', v_t')'$ is the vector containing all system innovations, i.e. the shocks e_t and the innovations associated with the measurement system, v_t, if any. Notably, e_t in u_t^{in} contains not only the fundamental shocks ε_t, but also additional stochastic terms, collected in the vector ζ_t, $e_t{:=}(\varepsilon_t', \zeta_t')'$, where ζ_t is a vector that has the same dimesion as ε_t and features a number $n_2 \leq \dim(\varepsilon_t)$ of possibly non-zero stochastic terms independent on ε_t; the remaining $\dim(\varepsilon_t)$-n_2 elements of ζ_t are equal to zero, see Appendix B.

Thus, while the determinate equilibrium in system (16) depends only on the state variables and the structural parameters θ, the class of indeterminate equilibria summarized by system (17) features (i) higher lag order, (ii) a set of auxiliary parameters ($\tilde{m}$) in addition to the structural parameters, and (iii) possible additional shocks unrelated to the fundamental shocks (the non-zero elements of ζ_t). As shown in Appendix B, the 'parametric indeterminacy' that springs from $\tilde{m}$ characterizes the moving average part of the VARMA-type reduced form solution for X_t. Such parameters index solution multiplicity and may arbitrarily amplify or dampen the fluctuations of the variables in y_t other those implied by the fundamental shocks. The 'stochastic indeterminacy' stems from the non-zero sunspot shocks which enter the vector ζ_t. These shocks may arbitrarily alter the dynamics and volatility of the system, see Lubik and Schorfheide (2003, 2004) for discussions. A special case of interest is obtained when $\tilde{m}{=}vec(I_{n_2})$ and no sunspot shocks drive the reduced form; in this case, despite θ lies in the indeterminacy region of the parameter space, the equilibrium collapses to a MSV solution with the same lag order as the determinate solution.[4]

[4]Notice however that the autoregressive parts $A^{in}(\theta)$ and $A^d(\theta)$ will generically be different, since θ lies in different subsets of the parameter space given the equilibrium regime in place.

The representation in Eq. (17) can be fruitfully used to simulate indeterminate (linearized) DSGE models. One obstacle so far to the full investigation of indeterminate equilibria in the multivariate context has been the difficulty in generating the data under the multiple solution hypothesis, without constraining too heavily the representation of solutions. Our analysis helps to fill this gap. Notably, we are not restricted to the selection of a particular solution—the orthogonal one (Lubik and Schorfheide, 2003)—from the indeterminate set.[5] In the next section we use the representation of the DSGE model in Eq. (16) to generate data from the DSGE model under determinacy, and the representation in Eq. (17) to generate data under indeterminacy.

4 Simulation experiment

In this section we explore VAR forecast performance on data simulated from the New Keynesian monetary business cycle model under different scenarios. To do so, system (14)-(15) is used to generate artificial data using the representation in Eq. (16) under determinacy, and the representation in Eq. (17) under indeterminacy. Structural parameters are calibrated to the estimates obtained by Benati and Surico (2009) on the Great Inflation period and Great Moderation period, respectively. For either of the two equilibrium regimes, we generate N=1000 synthetic datasets under determinacy and indeterminacy, assuming Gaussian fundamental shocks, and exploit the following forecast models:

(a) A VAR system for $y_t := (\Delta x_t, \pi_t, R_t)'$;

(b) univariate AR(1) models for π_t and Δx_{tt};

(c) univariate RW models for π_t and Δx_{tt}.

where Δx_t is the growth rate of output. While the VAR in (a) is our reference forecast model, the models in (b) and (c) serve as conventional benchmarks (e.g., Atkeson and Ohanian, 2001). Owing to their flexibility, reduced form VARs have naturally lent themselves for forecasting since their inception. Moreover, VARs are the more parsimonious models that capture interdependences across the endogenous variables whose co-movements the New Keynesian model attempts to predict. Notice that for our purposes it does not matter whether the employed VAR is misspecified or not relative to the actual (regime specific) DGP; it indeed serves as a reference models to assess absolute and relative predictive accuracy.[6]

Consistent with Benati and Surico (2008, 2009)'s empirical analysis, we consider periods of

[5] Recently, Farmer *et al.* (2015) have provided a method to solve and estimate indeterminate linear rational expectations models using standard software packages, which might be also used as an alternative.

[6] We are aware that also (structural) VAR(MA) models have been extensively used to identify, estimate and forecast the driving force(s) behind business cycles, see e.g. Stock and Watson (2002), Benati and Surico (2009), Kascha and Mertens (2009). However, VARs allow us to keep our arguments as simple as possible.

unequal lengths in the two cases: T=94 observations under determinacy and T=119 observations under the various indeterminacy scenarios we consider. Specifically, data under determinacy are generated from system (16), where the structural parameters θ are calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'After the Volcker Stabilization'. Hence, we mimic the scenario documented by Benati and Surico (2009) for the Great Moderation period, namely a structural system characterized by an 'active' monetary policy able to prevent self-fulling inflation expectations. We label such a scenario the 'Great Moderation-type' DGP. For ease of exposition, the calibrated θ used in the simulation experiment under determinacy is reported in the right column of Table 1. Data under indeterminacy are generated from system (17) with the structural parameters θ calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2001), column 'Before October 1979'. In this case, we mimic the scenario documented by Benati and Surico (2009) for the Great Inflation period, namely a structural system characterized by a 'passive' monetary policy that does not react aggressively enough to inflation shocks, inducing multiple (stable) equilibria. In this framework, the degree of indeterminacy of the system is one (i.e. n_2=1 in terms of the notation used in Section 3), hence we also need to calibrate the indeterminacy (scalar) parameter $\tilde{m}$ and the variance of the (univariate) sunspot shock $\zeta_t = s_t$ (which is also assumed Gaussian). We label such a scenario the 'Great Inflation-type' DGP. The calibrated parameters used in the simulation experiment under indeterminacy is again reported in the left column of Table 1, along with the indeterminacy parameters, summarized at the bottom. The differences in the values assumed by the structural parameters θ in the two scenarios are highlighted in bold in Table 1. It can be noticed that the main divergence across the two DGPs essentially lies in the conduct of monetary policy, namely in the long run response of the policy rate to output gap and inflation shocks.

On each dataset simulated as detailed above, we apply the models (a), (b) and (c). The first $T - P$ observations are used to estimate the models in (a) and (b) by OLS, and the last P observations are used to compute forecasts for π_t and Δx_t and the associated RMSEs. The VAR lag order is selected using Schwarz's criterion, considering 1 up to 3 lags. The *absolute* VAR forecast performance is measured by the average (across simulations) RMSEs, while the *relative* VAR forecast performance with respect to the benchmarks in (b)-(c) is computed by the ratio of the corresponding average (across simulations) RMSEs. As a measure of persistence, we employ the (absolute value of the) estimated largest root of the VAR companion matrix for the model in (a), and the estimated autoregressive coefficients for the univariate model in (b). Alternative measures of persistence in the multivariate framework have been recently proposed by Cogley and Sargent (2005) and Cogley *et al.* (2010). While Cogley and Sargent's (2005) measure, based on the normalized spectrum at frequency zero, frames naturally in the frequency-domain approach, Cogley's *et al.* (2010) R^2-like measure of persistence seems particularly suited for the case of VAR systems with drifting-parameters. Given our time-domain approach and the idea of using fixed-parameters VARs on the two subsamples, the largest root of the VAR companion matrix appears a tenable summary measure of the overall persistence of the variables in $y_t{:=}(w_t, \pi_t, R_t)'$.[7]

Results from the simulation experiment are summarized in Table 2 for the evaluation windows $P{=}8$ (eight quarters). First, in *absolute* terms, inflation forecast uncertainty of VAR-based forecasts is generally lower under determinacy compared to the case of indeterminacy.[8] This finding lines up with our a-priori conjecture: the forecastability of time series models is superior in relatively more stable environments in which only fundamental shocks drive fluctuations, and deteriorates in environments in which additional (intrinsic and extrinsic) not accounted sources of business cycle fluctuations other than fundamental shocks are at work.

Second, persistence tends to increase as the approximating VAR is 'far' from the MSV solution. However, for particular values of the indeterminacy parameters, the largest root of the approximating VAR can be smaller under indeterminacy relative to the case of determinacy. Hence, the richer correlation structure and stronger degree of endogenous persistence featured by indeterminate equilibria need not be associated with superior forecast performance *relative* to the case of determinacy. In fact, the failure of the forecast model to capture the essential features of the true DGP – which typically involves quite complex cross-equation restrictions under indeterminacy – may adversely affect the associated forecast error, which is a conditional property, even when the variables of interest feature higher persistence. The forecast performance of VARs under the Great Inflation-type scenario tends to deteriorate, for fixed indeterminacy parameter, as the uncertainty resulting from sunspot shocks (σ_s^2) increases.[9]

[7] See also Koop *et al.* (1996) and Fanelli and Paruolo (2010) for a comprehensive treatment of measures of shock persistence in multivariate models like VARs. Instead, a detailed analysis of the persistence of U.S. inflation may be found in e.g. Pivetta and Reis (2007), where alternative measures of persistence are discussed for univariate models. Fuhrer (2010) also explores the notion of inflation persistence in macroeconomic theory.

[8] As expected, the (average) RMSEs associated with the VAR-based forecasts obtained under a MSV indeterminate equilibrium is very close to the (average) RMSE obtained under Great Moderation-type scenario. It is worth remarking that differently from the simple univariate case discussed in Section 2, the MSV solution in this case has the same time-series representation as the determinate solution but is characterized by different values for the structural parameters θ, in the sense that θ lies in different regions of the parameter space in the two cases. Hence, the average RMSEs associated with the VAR model under the MSV DGP, need not coincide numerically with the average RMSEs associated with the VAR under the determinate DGP.

[9] Along the same lines, Canova and Gambetti (2010) emphasize that, conditional on lags of the endogenous variables, past expectations Granger-cause current values of the latter under indeterminacy but not under determinacy, and hence – irrespective of higher persistence – the omission of (proxies for) such expectations from the

Third, when considering *relative* forecast accuracy, we observe that the VAR-based forecasts are substantially similar to that of the AR(1)-based forecasts under determinacy, but may be inferior or superior to the AR(1)-based forecasts under indeterminacy, depending on the values taken by the indeterminacy parameters $\tilde{m}$ and σ_s^2. For some values of $\tilde{m}$ and σ_s^2, the VAR forecasts are inferior to that of AR(1) models. For other values, the converse occurs. A similar property characterizes the VAR-based forecasts relative to the RWs. In general, it is not possible to claim that the relative performance of the VAR model is systematically superior or inferior to that of our univariate benchmarks under indeterminacy.

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As mentioned, Benati and Surico (2008) observe that the evolution of the U.S. monetary policy stance might have caused a change in both the persistence of inflation and its predictability. While we share with Benati and Surico (2008) the view that the evolution of the U.S. monetary policy stance might have caused a reduction in inflation persistence, our study calls into question the 'indeterminacy-persistence-forecastability' nexus, at least when reduced form time series models are used. More specifically, our results point out that as long as we forecast with VARs, the richer and more persistent inflation dynamics generated by stylized monetary DSGE models under indeterminacy need not be associated with superior predictive accuracy.

forecast model may result in larger prediction errors in the former regime.

5 Concluding remarks

This paper has investigated the consequences of indeterminacy for quantitative forecasting through popular macro-econometric models. Our simulation-based analysis suggests that establishing superior forecast accuracy based on macro-econometric reduced form models and quadratic loss functions becomes challenging in the presence of equilibrium indeterminacy, even when extrinsic uncertainty (i.e. sunspot noise) plays no role in generating the data. Although indeterminacy need not imply superior forecastability - however measured - it may well be the case that a particular multivariate model estimated on data generated under indeterminate equilibria produces better forecasts than univariate predictors, regardless of the level of persistence and volatility that characterizes the observed time series.

An increasing literature has explored the forecast performance of monetary DSGE models (possibly featuring financial frictions) vis-à-vis conventional forecasting tools such as univariate and multivariate time series models or naive forecasts, see e.g. Gürkaynak *et al.* (2013) or Giacomini (2015) for a critical review. According to our analysis, the richer time series representation of the variables which emerges under indeterminacy should be properly identified and incorporated in the econometric model to enhance predictive accuracy. However, as shown in Lubik and Schorfheide (2003, 2004), Fanelli (2012) and Castelnuovo and Fanelli (2015a), making inference on the regime-specific features of indeterminate equilibrium models is a complicate task even when the analyst specifies the correct statistical model for the data.

Appendix A: Monte Carlo results from a simple linear rational expectations model

In this Appendix we consider a Monte Carlo experiment based on the simple linear rational expectations model discussed in Section 2, Eq. (1). We investigate the relative empirical performance of the AR(1) model in Eq. (5) under determinacy and indeterminacy, respectively. N =1000 artificial samples of length $T = 500$ are generated from Eq. (1) under the two scenarios. Under determinacy, for each of the 1000 simulations we generate 600 synthetic observations from Eq. (3), setting the variance of the structural shock to σ_ω^2:=0.5; the first 100 observations are then discarded. Under indeterminacy, for each of the 1000 simulations we generate 600 synthetic observations from the ARMA(1,1)-type process in Eq. (4) by calibrating the structural parameters to θ:=0.95 and σ_ω^2:=0.5, and selecting the indeterminacy parameter $\tilde{m}$ from the set $\mathcal{M}$:={1, 1.01, 0.98, 0.85, 0.80, 0.015}; with no loss of generality with respect to our argument, the sunspot shock is set to zero (σ_s^2:=0). Also in this case, the first 100 observations are discarded.

The choice of using a relatively large sample of T=500 observations is essentially motivated by the 'large T' argument used in Section 2, with the possibility of comparing part of the results. Furthermore, since the indeterminate equilibrium involves more (or an equal number of) parameters than its determinate counterpart, downplaying the role of parameter estimation error should be relatively more relevant for the former relative to the latter. The values X_0 and ω_0 are fixed to zero at time $t = 1$. The set $\mathcal{M}$ contains the point $\tilde{m} = 1$ which generates a MSV solution (indistinguishable from the determinate solution) and other points which are relatively 'close' to the MSV solution. Recall that we have discussed analytically in Section 2, for large T, the impact of the choices $\tilde{m}$=0.85, $\tilde{m}$=0.80 and $\tilde{m}$=1.01 on one step-ahead forecast accuracy of the AR(1) model.

On each simulated dataset, we use the first $T - P$ observations to estimate the parameters β and σ_u^2 of the AR(1) model through OLS, and the last P observations to evaluate its forecast accuracy. Absolute forecastability is measured by the average (across simulations) RMSEs. The forecast performance of the AR(1) model under indeterminacy *relative* to determinacy is assessed by the ratio of the average (across simulations) RMSEs obtained in the two cases. But as is known, absolute measures of predictive accuracy is not likely to be informative, and the problem of assessing the forecastability of a given model is best answered by using relative evaluation methods that use a benchmark. We therefore consider a 'theory-based' optimal benchmark, represented by a forecaster who perfectly knows the DGP, i.e. whether she is forecasting future paths for X_t under determinacy or indeterminacy, as well as the true values of the parameters, namely σ_ω^2 if the data are generated under determinacy, and θ, σ_ω^2 and $\tilde{m}$ if the data are generated under indeterminacy.[10] In our simple example, the indeterminate (stable) solution has purely backward dynamics, and the variance of (rational) forecast errors grows with the forecasting horizon at any given date t in which predictions are made. Hence, this benchmark provides a theoretical lower-bound on the forecasting performance of less-than-rational forecasting models, e.g. those that are not in the form of the model's indeterminate solution. The forecast performance of the AR(1) model *relative* to this 'theory-based' benchmark is assessed by the ratio of the average (across simulations) RMSEs obtained in the two regimes.

The results of our numerical experiment are reported in Table A1. The first column of Table A1 collects the absolute (average) RMSEs obtained with the AR(1) model under determinacy and in the five indeterminacy cases. The second column reports the ratio between the average (across simulations) RMSEs obtained under the indeterminacy scenarios on the average (across simulations) RMSEs obtained under determinacy. The third column reports the forecast performance of the AR(1) model *relative* to the 'theory-based' optimal benchmark.

First, we observe that for any considered forecast evaluation window, indeterminacy does not necessarily imply superior forecastability. The average RMSEs obtained under indeterminacy may be lower or higher than the average RMSEs obtained under determinacy, depending on the values of $\tilde{m}$. As expected, the forecast performance under the MSV equilibrium is the same as under the determinate solution. But for $\tilde{m}$=1.01 and $\tilde{m}$=0.98, the performance of the AR(1) model may change (albeit slightly) across the two regimes. It is worth remarking that for $P = 1$ (which corresponds to the case of one step-ahead forecasts), results in Table A1 are consistent with the considerations developed in Section 2.

Secondly, the third column of Table A1 shows that, under determinacy, the misspecification of the AR(1) model bears no consequences on the property of the OLS estimators of β and σ_u^2, and therefore does not affect its forecast performance *relative* to the benchmark. Indeed, the chosen univariate predictor has the same average (across simulation) RMSEs as the 'theory-based' benchmark predictor. The same happens under the MSV indeterminate solution. The picture changes, by contrast, for the other type of indeterminate equilibria, for which we observe a faltering forecast performance for the AR(1) predictor.

It may be argued that these findings mainly stem from our choice of a first-order autoregressive predictor as benchmark. To understand whether our argument is weakened in the presence of higher-order autoregressive forecasting models, we replace the AR(1) benchmark with an

[10] This benchmark forecasting model coincides with the actual law of motion of the economy, and empirical forecasts of future endogenous variables will necessarily coincide with model-consistent ones. In fact, the h-step ahead 'theory-based' optimal forecasts will be $E_{T-P}X_{T-P+h} = 0$ under determinacy, and $E_{T-P}X_{T-P+h} = \theta(X_{T-P+h-1} - \omega_{T-P+h-1})$ for $h = 1$ and $E_{T-P}X_{T-P+h} = \theta E_{T-P}X_{T-P+h-1}$ for $h \geq 2$ under indeterminacy.

AR(4) predictor, and explore whether this richer process may deliver better forecasts under indeterminacy relative to determinacy exploiting the higher persistence associated with the former regime. From a pure time-series perspective, when the DGP belongs to the class of ARMA processes for which a stable AR(∞) representation for X_t exists, the AR(4) model indeed provides a better (truncated) approximation to the stable AR(∞) process than the AR(1). In our simulation exercise, the ARMA processes for which the moving average part is invertible and thus X_t has a stable AR(∞) representation are those for which the indeterminacy parameter $\tilde{m}$ belongs to the subset $\mathcal{M}^f$:={0.85, 0.80, 0.015} $\subset \mathcal{M}$.[11] The ratio of the (average) RMSEs computed with the AR(1) and AR(4) predictors (across simulations) is reported in the last column of Table A1. Two main findings stand out: first, under indeterminacy and for large values of P, the AR(4) tends to outperform (albeit not markedly) the AR(1) benchmark provided $\tilde{m} \in \mathcal{M}^f$; second, there seems to be no clear indication of a higher AR(4)-based forecast accuracy (relative to a first-order autoregression) under indeterminacy vis-à-vis determinacy across all evaluation windows and indeterminate equilibria.

Appendix B: DSGE equilibria and their representation

In this Appendix, we discuss the solutions associated with the DSGE model compacted in Eqs. (14)-(15). To keep exposition as general as possible, throughout this Appendix we denote with n the dimension of the state vector X_t in Eq. (14) (notice that $n = 3$ in our specific case). Moreover, we use the notations '$A(\theta)$' and '$A{:=}A(\theta)$' to indicate that the elements of the matrix A depend nonlinearly on the structural parameters θ, hence in our setup $\Gamma_0{:=}\Gamma_0(\theta)$, $\Gamma_f{:=}\Gamma_f(\theta)$, $\Gamma_b{:=}\Gamma_b(\theta)$, $\Xi{:=}\Xi(\theta)$ and $\Gamma_0^{\Xi}{:=}\Gamma_0^{\Xi}(\theta)$. We call 'stable' a matrix that has all eigenvalues inside the unit disk and 'unstable' a matrix that has at least one eigenvalue outside the unit disk. Thus, denoted with $\lambda_{\max}(\cdot)$ the absolute value of the largest eigenvalue of the matrix in the argument, we have $\lambda_{\max}(A(\theta)) < 1$ for stable matrices and $\lambda_{\max}(A(\theta)) > 1$ for unstable ones. We also consider the partition $\theta{:=}(\theta_s', \theta_\varepsilon')'$, where θ_ε contains the non-zero elements of $vech(\Sigma_\varepsilon)$ and θ_s all remaining elements. The 'true' value of θ, $\theta_0{:=}(\theta_{0,s}', \theta_{0,\varepsilon}')'$, is assumed to be an interior point of $\mathcal{P}$. The corresponding partition of the parameter space is $\mathcal{P}{:=}\mathcal{P}_{\theta_s} \times \mathcal{P}_{\theta_\varepsilon}$. This partition is

[11] While determinacy in the context of Eq. (1) involves a one-to-one mapping between the endogenous variable x_t and the structural shock ω_t, indeterminacy may generate non-invertibility, i.e. the reduced form (4) might not be inverted to a (possibly infinite-order) autoregressive representation with one-sided lag polynomial (invertibility in the past). More generally, both determinacy and indeterminacy may be associated with non-invertibility even when equilibrium reduced forms are only driven by structural (fundamental) shocks. While non-invertibility may hinder the possibility of fully recovering the shock ω_t from an estimated causal AR model for the process X_t, this issue is immaterial for forecasting purposes, as the MA representation in (4) is naturally chosen to be invertible. This argument fully generalizes to VAR-based forecasting (e.g. Alessi *et al.*, 2011).

introduced because the determinacy/indeterminacy of the system depends only on the values taken by θ_s.

A detailed derivation of the time series representation of the reduced form solutions associated with the New-Keynesian DSGE system (14)-(15) is reported in Castelnuovo and Fanelli (2015*b*). Using the Binder and Pesaran's (1995) solution method, they show that uniqueness/multiplicity of solutions is governed by the stability/instability of the matrix $G(\theta_s){:=}(\Gamma_0^{\Xi}-\Gamma_f\Phi_1)^{-1}\Gamma_f$, where Φ_1 stems from the solution of a quadratic matrix equation.

Determinacy

For $\theta_s \in \mathcal{P}_{\theta_s}^{D}$, the matrix $G(\theta_s){:=}(\Gamma_0^{\Xi}-\Gamma_f\Phi_1)$ is stable, i.e. $\lambda_{\max}(G(\theta_s)){<}1$, and the reduced form solution to system (14)-(15) can be represented in the form

$$(I_n-\Phi_1(\theta_s)L-\Phi_2(\theta_s)L^2)X_t=u_t \quad , \quad u_t{:=}\Upsilon(\theta_s)^{-1}\varepsilon_t \tag{18}$$

where L is the lag operator ($L^jX_t=X_{t-j}$), $\Phi_1(\theta_s)$, $\Phi_2(\theta_s)$ and $\Upsilon(\theta_s)$ are 3×3 matrices whose elements depend nonlinearly on θ_s and embody the cross-equation restrictions implied by the small New-Keynesian model. The matrices $\Phi_1(\theta_s)$ and $\Phi_2(\theta_s)$ in Eq. (18) are obtained as the unique solution to the second-order quadratic matrix equation

$$\mathring{\Phi}{=}(\mathring{\Gamma}_0-\mathring{\Gamma}_f\mathring{\Phi})^{-1}\mathring{\Gamma}_b \tag{19}$$

where $\mathring{\Gamma}_f$, $\mathring{\Gamma}_0$, $\mathring{\Gamma}_b$ and the stable matrix $\mathring{\Phi}$ are respectively given by

$$\mathring{\Gamma}_0{:=}\begin{pmatrix}\Gamma_0^{\Xi} & 0_{n\times n}\\ 0_{n\times n} & I_n\end{pmatrix} \ , \ \mathring{\Gamma}_f{:=}\begin{pmatrix}\Gamma_f & 0_{n\times n}\\ 0_{n\times n} & 0_{n\times n}\end{pmatrix} \ , \ \mathring{\Gamma}_b{:=}\begin{pmatrix}\Gamma_{b,1}^{\Xi} & \Gamma_{b,2}^{\Xi}\\ I_n & 0_{n\times n}\end{pmatrix} \ , \ \mathring{\Phi}{:=}\begin{pmatrix}\Phi_1 & \Phi_2\\ I_3 & 0_{3\times 3}\end{pmatrix},$$

and $\Gamma_{b,1}^{\Xi}{:=}(\Gamma_b+\Xi\Gamma_0)$, $\Gamma_{b,2}^{\Xi}{:=}-\Xi\Gamma_b$ and $\Upsilon(\theta_s){:=}(\Gamma_0(\theta_s)-\Gamma_f(\theta_s)\Phi_1(\theta_s))$. The matrix $\Phi_1{:=}\Phi_1(\theta_s)$ is the one that enters the definition of $G(\theta_s)$.

A convenient representation of the equilibrium in Eq. (18) is given by

$$\underset{z_t^d}{\begin{pmatrix}X_t\\ X_{t-1}\end{pmatrix}}=\underset{A^d(\theta_s)}{\begin{pmatrix}\check{\Phi}_1 & \check{\Phi}_2\\ I_n & 0_{n\times n}\end{pmatrix}}\underset{z_{t-1}^d}{\begin{pmatrix}X_{t-1}\\ X_{t-2}\end{pmatrix}}+\underset{G^d(\theta_s)}{\begin{pmatrix}\check{\Upsilon}^{-1}\\ 0_{n\times n}\end{pmatrix}}\varepsilon_t \tag{20}$$

where $\check{\Phi}_1=\Phi_1(\theta_s)$, $\check{\Phi}_2=\Phi_2(\theta_s)$, $\check{\Upsilon}=\Upsilon(\theta_s)$, the matrices $A^d(\theta_s)$ and $G^d(\theta_s)$ are $2n\times2n$ and $2n\times n$, respectively, and the superscript 'd' stands for 'determinacy'.

Let $y_t{:=}(y_{1,t},y_{2,t},\cdots,y_{p,t})'$ be the $p\times1$ vector of observable variables. When all variables in X_t are observed, $y_t=X_t$, the state system (20) along with the measurement system: $y_t=Hz_t^d$, $H{:=}(I_n:0_{n\times n})$, give rise to a VAR representation for y_t (X_t) with coefficients that depend on

θ through the CER in Eq. (19). In general, however, not all variables in X_t belong to the forecaster's (observable) information set. The measurement system will take the form

$$y_t = Hz_t^d + Qv_t \tag{21}$$

where H is a $p \times 2n$ matrix, v_t a $b \times 1$ vector ($b \leq p$) of measurement errors with covariance matrix Σ_v, and Q is a $p \times b$ selection matrix. For the specific structural model we consider in the paper, the counterpart of the measurement system (21) is given by

$$\underbrace{\begin{pmatrix} \Delta x_t \\ \pi_t \\ R_t \end{pmatrix}}_{y_t} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} \tilde{x}_t \\ \pi_t \\ R_t \\ \tilde{x}_{t-1} \\ \pi_{t-1} \\ R_{t-1} \end{pmatrix}}_{z_t^d} + \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{Q} \underbrace{\varepsilon_{o^n,t}}_{v_t}$$

and is obtained by exploiting the RW assumption in Eq. (13). Let $u_t := (\varepsilon_t', v_t')'$ be the $(n+b)$-dimensional vector containing all system innovations. By substituting Eq. (20) into Eq. (21) and using some algebra, one obtains the ABCD form (Fernández-Villaverde *et al.* 2007):

$$\begin{aligned} \underset{2n\times 1}{z_t^d} &= \underset{2n\times 2n}{A^d(\theta_s)}\ \underset{2n\times 1}{z_{t-1}^d} + \underset{2n\times(n+b)}{B(\theta_s)}\ \underset{(n+b)\times 1}{u_t} \\ \underset{p\times 1}{y_t} &= \underset{p\times 2n}{C(\theta_s)}\ \underset{2n\times 1}{z_{t-1}^d} + \underset{p\times(n+b)}{D(\theta_s)}\ \underset{(n+b)\times 1}{u_t} \end{aligned} \tag{22}$$

where $B(\theta_s) := (G^d(\theta_s) : 0_{2n\times b})$, $C(\theta_s) := HA^d(\theta_s)$ and $D(\theta_s) := (HG^d(\theta_s) : Q)$.

System (22) must be in 'minimal form' and must be identified (locally) as discussed in Komunjer and Ng (2011). In general, it is possible to manipulate the state space representation such that an identified system in minimal form is obtained eventually. If the system passes these checks, Eq. (22) can be used as the DGP implied by our New-Keynesian DSGE model under determinacy. Replacing θ_s with θ gives system (16) in Section 3 of the paper.

Indeterminacy

For $\theta_s \in \mathcal{P}_{\theta_s}^I$, the matrix $G(\theta_s) := (\Gamma_0^{\Xi} - \Gamma_f \Phi_1)^{-1}\Gamma_f$ is unstable, i.e. $\lambda_{\max}(G(\theta_s)) > 1$, and the class of reduced form solutions associated with the New-Keynesian system (14)-(15) becomes more involved from a dynamic standpoint. When $\lambda_{\max}(G(\theta_s)) > 1$, the matrix $G(\theta_s)$ can be decomposed in the form

$$G(\theta_s) = P(\theta_s) \begin{pmatrix} \Lambda_1 & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & \Lambda_2 \end{pmatrix} P^{-1}(\theta_s)$$

where $P(\theta_s)$ is a $n \times n$ non-singular matrix, Λ_1 is the $n_1 \times n_1$ ($n_1 < n$) Jordan normal block that collects the eigenvalues of $G(\theta_s)$ that lie inside the unit disk, and Λ_2 is the $n_2 \times n_2$ ($n_2 \leq n$) Jordan normal block that collects the eigenvalues of $G(\theta_s)$ that lie outside the unit disk. Observe that $n_1 + n_2 = n$, and n_2 determines the 'degree of multiplicity' of solutions, which in our setup coincides with the number of unstable roots of $G(\theta_s)$.

The reduced form solutions can be given the VARMA-type representation for X_t:

$$(I_n - \Pi(\theta_s)L)(I_n - \Phi_1(\theta_s)L - \Phi_2(\theta_s)L^2)X_t = (\Psi(\theta_s, \tilde{m}) - \Pi(\theta_s)L)V(\theta_s, \tilde{m})^{-1}\varepsilon_t + \tau_t \quad (23)$$

$$\tau_t := (\Psi(\theta_s, \tilde{m}) - \Pi(\theta_s)L)V(\theta_s, \tilde{m})^{-1}P(\theta_s)\zeta_t + P(\theta_s)\zeta_t \quad (24)$$

where the matrices $\Phi_1(\theta_s)$ and $\Phi_2(\theta_s)$ are defined as in the case of determinacy, see Eq. (19), while the matrices $\Pi(\theta_s)$, $\Psi(\theta_s, \tilde{m})$ and $V(\theta_s, \tilde{m})$ are respectively given by

$$\Pi(\theta_s) := P(\theta_s) \begin{pmatrix} 0_{n_1 \times n_1} & 0_{n_1 \times n_2} \\ 0_{n_2 \times n_1} & \Lambda_2^{-1} \end{pmatrix} P^{-1}(\theta_s) \quad , \quad \Psi(\theta_s, \tilde{m}) := P(\theta_s) \begin{pmatrix} I_{n_1} & 0_{n_1 \times n_2} \\ 0_{n_2 \times n_1} & \tilde{M} \end{pmatrix} P^{-1}(\theta_s)$$

$$V(\theta_s, \tilde{m}) := (\Gamma_0(\theta_s) - \Gamma_f(\theta_s)\Phi_1(\theta_s)) - \Xi(\theta_s)\Gamma_f(\theta_s)(I_n - \Psi(\theta_s, \tilde{m})).$$

(See Castelnuovo and Fanelli (2015*b*) for a detailed derivation of the representation in Eqs. (23)-(24)). The $n_2 \times n_2$ sub-matrix $\tilde{M}$ in $\Psi(\theta_s, \tilde{m})$ contains a set of arbitrary auxiliary parameters that do not depend on θ_s; for ease of reference, throughout we collect these parameters in the vector $\tilde{m} := vec(\tilde{M})$. The unstable eigenvalue of $G(\theta_s)$ are 'flipped' in this representation and enter the $\Pi(\theta_s)$ matrix (which is stable) in the autoregressive part of sytem (23). The 'additional' vector moving average term τ_t in sytem (23) depends on an extra source of random fluctuations potentially independent on the fundamental disturbances ε_t, i.e. on the $n \times 1$ vector $\zeta_t := (0'_{n_1 \times 1}, s'_t)'$, where s_t is a $n_2 \times 1$ MDS which collects the 'sunspot shocks' featured by the system. We assume, without any loss of generality, that s_t has a time-invariant covariance matrix Σ_s. The sunspot shocks might be also absent form the reduced form solution, i.e. $\Sigma_s = 0_{n_2 \times n_2}$ implying $\zeta_t = 0_{n \times 1}$ a.s. (and $\tau_t = 0_{n \times 1}$ a.s.). This situation is typically denoted 'indeterminacy without sunspots'. It can be noticed that in the special case in which jointly $\tilde{M} = I_{n_2}$ and $\zeta_t = 0_{n \times 1}$ a.s., $\Psi(\theta_s, vec(I_{n_2})) = I_n$, $V(\theta_s, vec(I_{n_2})) = \Upsilon(\theta_s) := (\Gamma_0(\theta_s) - \Gamma_f(\theta_s)\Phi_1(\theta_s))$ so that system (23) collapses to a MSV solution and X_t has the same representation as in the determinate case, see Eq. (18).

A convenient summary of the class of indeterminate equilibria described by Eqs. (23)-(24)

is given by the system

$$\underbrace{\begin{pmatrix} X_t \\ X_{t-1} \\ X_{t-2} \end{pmatrix}}_{z_t^{in}} = \underbrace{\begin{pmatrix} -(\check{\Phi}_1+\check{\Pi}) & (\check{\Phi}_1\check{\Pi}-\check{\Phi}_2) & \check{\Phi}_2\check{\Pi} \\ I_n & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & I_n & 0_{n\times n} \end{pmatrix}}_{A^{in}(\theta_s)} \underbrace{\begin{pmatrix} X_{t-1} \\ X_{t-2} \\ X_{t-3} \end{pmatrix}}_{z_{t-1}^{in}} + \underbrace{\begin{pmatrix} \check{K}_1 & \check{K}_2 \\ 0_{n\times 2n} & 0_{n\times 2n} \\ 0_{n\times 2n} & 0_{n\times 2n} \end{pmatrix}}_{G^{in}(\theta_s,\tilde{m})} \underbrace{\begin{pmatrix} e_t \\ e_{t-1} \end{pmatrix}}_{\varepsilon_t^0} \tag{25}$$

where $\check{\Phi}_1 = \Phi_1(\theta_s)$, $\check{\Phi}_2 = \Phi_2(\theta_s)$, $\check{\Pi} = \Pi(\theta_s)$, $\check{\Psi} = \Psi(\theta_s,\tilde{m})$, $\check{V} = V(\theta_s,\tilde{m})$, $\check{K}_1 := [\check{\Psi}\check{V}^{-1} : (\check{\Psi}\check{V}^{-1}+I_n)\check{P}]$, $K_2 := [\check{\Pi}\check{V}^{-1} : \check{\Pi}\check{V}^{-1}\check{P}]$ are $n\times 2n$ matrices, $e_t := (\varepsilon_t', \zeta_t')'$ is a $2n\times 1$ vector that collects the fundamental and sunspot shocks of the system, the matrices $A^{in}(\theta_s)$ and $G^{in}(\theta_s,\tilde{m})$ are $3n\times 3n$ and $3n\times 4n$, respectively, and the superscript 'in' stands for 'indeterminacy'.

Given the $p\times 1$ vector of observables y_t, the associated measurement system is given by

$$y_t = Hz_t^{in} + Qv_t \tag{26}$$

and, a part from their dimensions, the matrices H, Q and the vector v_t have the same role as in Eq. (21). For the specific structural model we consider in the paper, the counterpart of the measurement system (26) is given by

$$\underbrace{\begin{pmatrix} \Delta x_t \\ \pi_t \\ R_t \end{pmatrix}}_{y_t} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} \tilde{x}_t \\ \pi_t \\ R_t \\ \tilde{x}_{t-1} \\ \pi_{t-1} \\ R_{t-1} \\ \tilde{x}_{t-2} \\ \pi_{t-2} \\ R_{t-2} \end{pmatrix}}_{z_t^{in}} + \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{Q} \underbrace{\varepsilon_{o^n,t}}_{v_t}.$$

Upon defining the $(4n+b)$-dimensional vector $u_t := (e_t, v_t')'$ which contains the complete set of innovations, the ABCD form of the indeterminate equilibria reads

$$\begin{aligned} \underset{3n\times 1}{z_t^{in}} &= \underset{3n\times 3n}{A^{in}(\theta_s)} \underset{3n\times 1}{z_{t-1}^{in}} + \underset{3n\times(4n+b)}{B(\theta_s,\tilde{m})} \underset{(4n+b)\times 1}{u_t} \\ \underset{p\times 1}{y_t} &= \underset{p\times 3n}{C(\theta_s)} \underset{3n\times 1}{z_{t-1}^{in}} + \underset{p\times(4n+b)}{D(\theta_s,\tilde{m})} \underset{(4n+b)\times 1}{u_t} \end{aligned} \tag{27}$$

where $B(\theta_s,\tilde{m}) := (G^{in}(\theta_s,\tilde{m}) : 0_{2n\times b})$, $C(\theta_s) := HA^{in}(\theta_s)$ and $D(\theta_s,\tilde{m}) := (HG^{in}(\theta_s,\tilde{m}) : Q)$.

Also in this case, provided system (27) is in 'minimal form' and is identified (locally), it can be used as the DGP implied by our New-Keynesian DSGE model under indeterminacy. Replacing θ_s with θ it corresponds to system (17) in Section 3 of the paper.

References

[1] Alessi, L., Barigozzi, M. and M. Capasso (2011), Nonfundamentalness in Structural Econometric Models: A Review, *International Statistical Review* 79, 16-47.

[2] Atkeson A. and Ohanian, L.E. (2001), Are Phillips Curves Useful for Forecasting Inflation? *Quarterly Review* 25, 2-11.

[3] Benati L. (2008), Investigating Inflation Persistence Across Monetary Regimes, *Quarterly Journal of Economics* 123, 1005-1060.

[4] Benati L. and Surico, P. (2008), Evolving U.S. Monetary Policy and the Decline of Inflation Predictability, *Journal of the European Economic Association* 6, 634-646.

[5] Benati, L. and Surico, P. (2009), VAR Analysis and the Great Moderation, *American Economic Review* 99, 1636-1652.

[6] Binder M. and Pesaran, M.H. (1995), Multivariate Rational Expectations Models and Macroeconomic Modelling: a Review and Some New Results. In *Handbook of Applied Econometrics*, Pesaran MH, Wickens M. (eds.). Blackwell: Oxford; 139-187.

[7] Boivin J. and Giannoni, M.P. (2006), Has Monetary Policy become More Effective? *Review of Economics and Statistics* 88, 445-462.

[8] Broze L. and Szafarz, A. (1991), *The Econometric Analysis of Non-Uniqueness in Rational Expectations Models*, North-Holland, Amsterdam.

[9] Canova, F. and Gambetti, L. (2010), Do Expectations Matter? The Great Moderation Revisited, *American Economic Journal: Macroeconomics* 2, 183-205.

[10] Castelnuovo E. and Surico, P. (2010), Monetary Policy, Inflation Expectations and the Price Puzzle, *Economic Journal* 120, 1262-1283.

[11] Castelnuovo, E. and Fanelli, L. (2015*a*), Monetary Policy Indeterminacy and Identification Failures in the U.S.: Results from a robust test, *Journal of Applied Econometrics* 30, 924–947.

[12] Castelnuovo, E. and Fanelli, L. (2015*b*), Monetary Policy Indeterminacy and Identification Failures in the U.S.: Results from a robust test, *Journal of Applied Econometrics* 30, 924–947, Supporting Information.

[13] Christoffel K., Coenen G. and A. Warne. (2010), Forecasting with DSGE Models, in *Oxford Handbook on Economic Forecasting*, eds. M.P. Clements and D.F. Hendry, Oxford University Press, USA.

[14] Clarida R.J., Galì J. and M. Gertler (2000), Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory, *Quarterly Journal of Economics* 115, 147-180.

[15] Cogley, T. and Sargent, T. J. (2005), Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US, *Review of Econonic Dynamics* 8, 262-302.

[16] Cogley, T., Primiceri, G. and T.J. Sargent (2010), Inflation-Gap Persistence in the US, *American Economic Journal: Macroeconomics* 2, 43-69.

[17] D'Agostino A., Giannone, D. and Surico, P. (2007), (Un)Predictability and Macroeconomic Stability, CEPR Discussion Papers 6594.

[18] D'Agostino A. and Surico, P (2012)., A Century of Inflation Forecast, *Review of Economics and Statistics* 94, 1097-1106.

[19] Edge R.M. and Gurkaynak, R.S. (2010), How Useful Are Estimated DSGE Model Forecasts for Central Bankers? *Brookings Papers on Economic Activity* 41, 209-259.

[20] Fanelli L. (2012), Determinacy, Indeterminacy and Dynamic Misspecification in Linear Rational Expectations models, *Journal of Econometrics* 170, 153-163.

[21] Fanelli, L. and Paruolo, P. (2010), Speed of Adjustment in Cointegrated Systems, *Journal of Econometrics*, 158, 130-141.

[22] Farmer, R.E.A., Khramov, V. and Nicolò, G. (2015), Solving and estimating indeterminate DSGE Models, *Journal of Economic Dynamics and Control* 54, 17-36.

[23] Faust J. and Wright, J.H. (2009), Comparing Greenbook and reduced form Forecasts Using a Large Realtime Dataset, *Journal of Business & Economic Statistics* 27, 468-479.

[24] Fernández-Villaverde, J., Rubio-Ramírez, J., Sargent, T. and M. Watson (2007), ABCs (and Ds) of Understanding VARs, *American Economic Review* 97, 1021-1026.

[25] Fuhrer, J.C. (2010), Inflation Persistence, In *Handbook of Monetary Economics* 3, Friedman, B.M. and M. Woodford (eds.), North Holland.

[26] Fujiwara I. and Hirose, Y. (2014), Indeterminacy and Forecastability, *Journal of Money, Credit and Banking* 46, 243-251.

[27] Giacomini, R. (2015), Economic Theory and Forecasting: Lessons from the Literature, *Econometrics Journal* 18, C22-C41.

[28] Gürkaynak, R., Kısacıkoğlu, B. and B. Rossi (2013), Do DSGE Models Forecast More Accurately Out-of-Sample than Reduced Form Models? In: T. Fomby, L. Kilian and A. Murphy (eds.), *VAR Models in Macroeconomics - New Developments and Applications: Essays in Honor of Christopher A. Sims*, Advances in Econometrics vol. 32, 27-80.

[29] Herbst, E. and Schorfheide, F. (2012), Evaluating DSGE Model Forecasts of Comovements, *Journal of Econometrics* 171, 152–166

[30] Komunjer, I. and Ng, S. (2011), Dynamic Identification of Dynamic Stochastic General Equilibrium Models, *Econometrica* 79, 1995-2032.

[31] Koop, G., Pesaran, H.M. and S. Potter, (1996), Impulse Response Analysis in Nonlinear Multivariate Models, *Journal of Econometrics* 74, 119–147

[32] Lubik, T.A. and Schorfheide, F. (2003), Computing Sunspot Equilibria in Linear Rational Expectations Models, *Journal of Economic Dynamics and Control* 28, 273-285.

[33] Lubik, T.A. and Schorfheide, F. (2004), Testing for Indeterminacy: An Application to U.S. Monetary Policy, *American Economic Review* 94, 190-217.

[34] Orphanides, A. and van Norden, S. (2005), The Reliability of Inflation Forecasts Based on Output Gap Estimates in Real Time, *Journal of Money, Credit and Banking* 37, 583-601.

[35] Pivetta, F. and Reis, R. (2007), The Persistence of Inflation in the United States, *Journal of Economic Dynamics and Control* 31, 1326-1358.

[36] Rossi, B. and Sekhposyan, T. (2010), Have Models' Forecasting Performance Changed Over Time, and When? *International Journal of Forecasting* 26, 808-835.

[37] Smets, F. and Wouters, R. (2007), Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach, *American Economic Review*, 97 586–606.

[38] Sorge M.M. (2012), News Shocks or Parametric Indeterminacy? An Observational Equivalence Result in Linear Rational Expectations Models, *Economics Letters* 114, 198-200.

[39] Stock, J.H., and Watson, M.W. (2002), Has the Business Cycle Changed and Why? in M. Gertler and K. Rogoff, eds., *NBER Macroeconomics Annuals* 2002.

[40] Stock J.H., and Watson, M.W. (2007), Why Has U.S. Inflation Become Harder to Forecast? *Journal of Money, Credit and Banking* 39, 3-33.

TABLES

Table 1. Values of the structural parameters of the New Keynesian model in Eqs. (9)-(12) used in the simulation experiments of Section 4.

		Great Inflation DGP	Great Moderation DGP
Parameter	Interpretation	θ^{det}	θ^{ind}
γ	IS: forward looking term	0.744	0.744
δ	IS: inter. elast. of substitution	0.124	0.124
α	NKPC: indexation past inflation	0.059	0.059
κ	NKPC: slope	0.044	0.044
ρ	Rule, smoothing term	**0.595**	**0.834**
$\varphi_{\tilde{x}}$	Rule, reaction to output gap	**0.527**	**1.146**
φ_{π}	Rule, reaction to inflation	**0.821**	**1.749**
$\rho_{\tilde{o}}$	Output gap shock, persistence	0.796	0.796
ρ_{π}	Inflation shock, persistence	0.418	0.418
ρ_R	Policy rate shock, persistence	0.404	0.404
$\sigma^2_{\tilde{o}}$	IS: shock variance	0.055	0.055
σ^2_{π}	NKPC: shock variance	0.391	0.391
σ^2_R	Policy rule: shock variance	0.492	0.492
$\sigma^2_{o^n}$	Natural rate of output: shock variance	0.25	0.25
$\tilde{m}$,	Indeterminacy parameters	1; 1.01; 0.98	-
σ^2_s	Variance of sunspot shock	0; 2; 5	-

NOTES: θ under determinacy (θ^{det}) is calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'After the Volcker Stabilization'. θ under indeterminacy (θ^{ind}) is calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'Before October 1979'. In bold the parameter values that change across the two regimes.

Table 2. Absolute and relative (average) RMSE for inflation and output growth computed from data simulated from the New-Keynesian DSGE model in Eqs. (9)-(13) under determinacy and indeterminacy on different sample lenghts.

	Evaluation window: $P = 8$			
	Absolute (a)	Relative to RW (a/c)	Relative to AR (a/b)	VAR persistence
		$T = 94$		
DETERMINACY				
inflation	1.1714	0.7655	1.0094	0.8496
output growth	1.3580	0.7778	1.0048	
		$T = 119$		
INDETERMINACY: MSV solution				
inflation	1.2793	0.7413	1.0029	0.8214
output growth	1.3373	0.7498	1.0001	
INDETERMINACY: $\tilde{m}$=1.01, σ_s^2=0				
inflation	2.6674	0.8118	0.9647	0.8454
output growth	5.5618	0.8018	0.9942	
INDETERMINACY: $\tilde{m}$=0.98, σ_s^2=0				
inflation	2.4867	0.8467	0.9835	0.8238
output growth	5.5395	0.8014	0.9954	
INDETERMINACY: $\tilde{m}$=1.01, σ_s^2=2				
inflation	5.5149	1.0265	1.0064	0.9710
output growth	6.2177	0.8284	0.9995	
INDETERMINACY: $\tilde{m}$=1.01, σ_s^2=5				
inflation	7.7233	1.0369	1.0170	0.9729
output growth	7.0334	0.8538	0.9998	

NOTES: Results are based on N=1000 simulations. Data under determinacy are generated by simulating system (16) for $\theta = \theta^{\det} \in P_\theta^D$, where $\theta^{\det}$ is calibrated as in the third column of Table 1. Data under indeterminacy are generating by simulating system (17) for $\theta = \theta^{ind} \in P_\theta^I$, where θ^{ind} and the indeterminacy parameters $\tilde{m}$ and σ_s^2 are calibrated as in the fourth column of Table 1. Forecasts are computed using $T - P$ observations to estimate the model and the last P observations to evaluate forecasts (fixed scheme) and the corresponding RMSEs. (a) denotes the three-variate VAR system for y_t:=$(\Delta o_t, \pi_t, R_t)'$, whose lag order is selected using Schwarz's (SC) information criterion considering 1 up to 4 lags; 'RW' stands for univariate random walk, i.e. model (c); 'AR(1)' stands for univariate autoregressive model of order one, i.e. model (b). 'VAR persistence reportes the absolute value of the largest estimated root of the VAR companion matrix'.

Table A1. RMSEs for X_t computed from Eq. (1) under determinacy and indeterminacy.

Equilibrium regime	AR(1)	Relative to DET.	Relative to BENCH.	Relative to AR(4)
T=500	Evaluation window: $P = 1$			
DETERMINACY	0.5561	-	1	0.9958
INDETERMINACY. MSV	0.5561	1	1	0.9958
INDETERMINACY, $\tilde{M} = 1.01$	0.5627	1.0120	1.0114	0.9960
INDETERMINACY, $\tilde{M} = 0.98$	0.5469	0.9830	0.9829	0.9947
INDETERMINACY, $\tilde{M} = 0.85$	0.5393	0.9698	1.1046	0.999
INDETERMINACY, $\tilde{M} = 0.80$	0.5564	1.0005	1.249	1.0157
INDETERMINACY, $\tilde{M} = 0.015$	0.5273	0.948	63.18	0.9932
T=500	Evaluation window: $P = 8$			
DETERMINACY	0.6791	-	1	0.9979
INDETERMINACY, MSV	0.6791	1	1	0.9979
INDETERMINACY, $\tilde{M} = 1.01$	0.6866	1.0110	1.0009	0.9972
INDETERMINACY, $\tilde{M} = 0.98$	0.6662	0.9810	1.0008	0.9976
INDETERMINACY, $\tilde{M} = 0.85$	0.6545	0.9638	1.0932	1
INDETERMINACY, $\tilde{M} = 0.80$	0.6799	1.0011	1.1639	1.0144
INDETERMINACY, $\tilde{M} = 0.015$	1.1526	1.6972	1.1360	1.0146
T=500	Evaluation window: $P = 16$			
DETERMINACY	0.6919	-	1	0.9996
INDETERMINACY, MSV	0.6919	1	1	0.9996
INDETERMINACY, $\tilde{M} = 1.01$	0.6995	1.011	1.0008	0.9993
INDETERMINACY, $\tilde{M} = 0.98$	0.6785	0.9810	1.0002	0.9993
INDETERMINACY, $\tilde{M} = 0.85$	0.6637	0.9592	1.0604	1.001
INDETERMINACY, $\tilde{M} = 0.80$	0.6890	0.9958	1.1046	1.009
INDETERMINACY, $\tilde{M} = 0.015$	1.4076	2.0344	1.0554	1.0003

NOTES: Results are based on N=1000 simulations. Data under determinacy are generated from Eq. (3) with σ_{ω}^{2}=0.5. Data under indeterminacy are generated from Eq. (4) with θ:=0.95, σ_{ω}^{2}:=0.5 and $\sigma_s = 0$, for different values of $\tilde{M}$ and fixed initial conditions. 'AR(1)': average (across simulations) absolute RMSEs obtained with the model in Eq. (5). The first $T - P$ observations are used to estimate the model and the remaining P observations to compute the RMSEs. 'Relative to DET.': ratio between the average RMSE obtained with the AR(1) model under indeterminacy and the average RMSEs obtained with the AR(1) model under determinacy. 'Relative to BENCH.': ratio between the average AR(1)-based RMSEs and the average RMSEs obtained under a 'theory-based' benchmark. 'Relative to AR(4)': ratio between the average AR(1)-based RMSEs and the average AR(4)-based RMSEs.