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Abstract

We propose a non-recursive identification scheme for uncertainty shocks which exploits breaks in the volatility of macroeconomic variables and is novel in the literature on uncertainty. This approach allows us to simultaneously address two major questions in the empirical literature: Is uncertainty a cause or effect of decline in economic activity? Does the relationship between uncertainty and economic activity change across macroeconomic regimes? Results based on a small-scale VAR with U.S. monthly data suggest that (i) uncertainty is an exogenous source of decline of economic activity, (ii) the effects of uncertainty shocks amplify in periods of economic and financial turmoil.

Keywords: Heteroskedasticity, Identification, Non-recursive SVAR, Uncertainty shocks, Volatility regime.

J.E.L.: C32, C51, E44, G01.

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1 Introduction

Since the aftermath of the recent Global Financial Crisis (GFC), there has been revamped attention on the role played by uncertainty as a driver of the business cycle. Three main findings have emerged from the extant literature: first, heightened uncertainty triggers a contraction in real activity; second, uncertainty tends to be higher during economic recessions; third, the effects of uncertainty shocks are not constant over time. The first finding is consistent with the theoretical literature that shows why uncertainty can have negative macroeconomic effects. The prevailing view is that uncertainty is recessionary in presence of real options effects (e.g. Bloom, 2009) or financial frictions (e.g. Christiano *et al.*, 2014). However, uncertainty appears also to endogenously increase during recessions, as lower economic growth induces greater dispersion at the micro level and higher aggregate volatility. This second finding is consistent with the theoretical literature on 'endogenous uncertainty', which contends that uncertainty is rather a consequence, not a cause, of declining economic activity, as in e.g. Van Nieuwerburgh and Veldkamp (2006), Bachmann and Moscarini (2012), Fajgelbaum *et al.* (2017), Gourio (2014), Navarro (2014) and Plante *et al.* (2018). The fact that the relationship between uncertainty and real activity may not be constant over time is consistent with theoretical models that show how the effects of heightened uncertainty can be amplified in extreme conditions like high financial stress (e.g. Gilchrist *et al.*, 2014; Alfaro *et al.*, 2018; Arellano *et al.*, 2018) or when monetary policy is constrained by the zero lower bound (Basu and Bundick, 2017).

Whether causality runs from uncertainty to real activity, or from real activity to uncertainty, or in both directions, and whether this relationship changes under different macroeconomic conditions are issues which can be investigated empirically within a Structural VAR (SVAR) framework. The first issue requires moving away from recursive identification schemes, which are by construction ill suited to shed light on the reverse causality issue. This topic has been explicitly analyzed in Ludvigson *et al.* (2018a) and Carriero *et al.* (2018b), reporting mixed evidence. The second issue requires moving away from linear SVARs which would not allow to uncover possibly regime-dependent effects of uncertainty shocks. This concern has been addressed in the recent literature, and evidence that uncertainty shocks have time-varying effects has been provided by, among others, Alessandri and Mumtaz (2018), and Caggiano *et al.* (2014, 2017a). These early attempts of examining causality and time variation of uncertainty shocks have looked at the two issues in isolation. In light of the findings in the literature, however, this seems to be a strong limitation: if the relationship between uncertainty and real activity is indeed time-varying (or regime-dependent), it may very well be the case that also the direction of causality might change over time, something which a time-invariant SVAR would be unable to uncover.

This paper fills this gap by proposing a non-recursively identified SVAR model which exploits breaks in the (unconditional) volatility of post-WW2 U.S. macroeconomic variables. Within this framework, we can allow both for on-impact effects of uncertainty on real activity, and vice versa, and for regime-dependence in these effects. As discussed in Magnusson and Mavroeidis (2014), structural breaks induced by policy shifts and/or the occurrence of financial crises, provide exogenous identifying information which can be fruitfully used for inference. The identification strategy we apply extends the standard 'identification-through-heteroskedasticity' approach, popularized in the empirical macroeconomic literature by Rigobon (2003), Rigobon and Sack (2003) and Lanne and Lütkepohl (2008), to the case where the structural parameters (on-impact coefficients), and hence the associated impulse response functions (IRFs), may vary across volatility regimes, see Bacchiocchi and Fanelli (2015) and Bacchiocchi *et al.* (2018). In this setup, changes in the VAR covariance matrix can be also ascribed to variations in the structural parameters and identification is achieved by imposing restrictions on the changes that characterize these parameters across volatility regimes. This opens up interesting possibilities for practitioners relative to 'standard' SVARs. In general, there are more moment conditions which can be used to identify the shocks jointly with theory-based restrictions, and the method is flexible enough to jointly allow for recursive and non-recursive structures across volatility regimes, provided a necessary and sufficient rank condition is respected. This is particularly important when addressing the issue of exogeneity/endogeneity of uncertainty, since it endows us with a formal test for exogeneity with the highly desirable property of accounting for potential dependence to macroeconomic (volatility) regimes.

We estimate, as in Ludvigson *et al.* (2018a), a small-scale SVAR with three variables: a measure of real activity, Y_t; an index of macroeconomic uncertainty, U_{Mt}; and an index of financial uncertainty, U_{Ft}. Real activity is proxied by either industrial production or employment, and the indices of macroeconomic and financial uncertainty are taken from Jurado *et al.* (2015) and Ludvigson *et al.* (2018a), respectively.[1] As argued in Ludvigson *et al.* (2018a), the joint use of macroeconomic and financial uncertainty indices is crucial to correctly uncover the relationship between uncertainty and real activity, since they can display substantially different properties. Data are monthly, and span the 1960-2015 sample. Using recursive and rolling-windows estimates of the VAR covariance matrix, we show that two main volatility breaks are consistent with the pattern of data, and can be associated with two important episodes of the U.S. history: one is the onset of the Great Moderation, and the other is the GFC of 2007-2008. This leads to the

[1] Other measures of macro uncertainty available in the literature have been proposed by Rossi *et al.* (2016) and Scotti (2016). We use the measure proposed by Jurado *et al.* (2015) to be consistent with the VAR specification in Ludvigson *et al.* (2018a), see below. In Carriero *et al.* (2018a) uncertainty and its effects are instead estimated in a single step within the same model.

identification of three broad volatility regimes in the data, which correspond to three well-known macroeconomic regimes: the 'Great Inflation' period (1960M8-1984M3), the 'Great Moderation' period (1984M4-2007M12) and the 'Great Recession+Slow Recovery' period (2008M1-2015M4).[2] We then identify shocks by specifying a non-recursive structural model which exploits the differences in the average level of volatility displayed by macroeconomic variables in the three different sub-samples.

Our main findings can be summarized as follows. First, macroeconomic uncertainty can be better described as an exogenous driver of the U.S. business cycle. Macroeconomic uncertainty shocks trigger a decline of U.S. real economic activity, whose magnitude and persistence is estimated to be larger during Great Recession+Slow Recovery period, while the opposite is not supported by the empirical evidence. This finding holds true in all three macroeconomic regimes and is robust to several perturbations of the baseline model, such as the use of alternative measures of real activity and macroeconomic uncertainty, and also controlling for financial stress. Second, from the Great Moderation onwards, the pass-through of financial uncertainty to real economic activity is found to be indirect: financial uncertainty shocks trigger macroeconomic uncertainty and, via this channel, a contraction in real activity, with effects which amplify after the GFC. Financial uncertainty does not respond to real economic activity shocks nor to macroeconomic uncertainty shocks. Third, the estimated impulse responses differ substantially from those coming from a benchmark represented by a SVAR identified with heteroskedasticity along the lines of Lanne and Lütkepohl's (2008) method, i.e. by imposing that the autoregressive and the structural parameters are fixed across volatility regimes.

Overall, our findings support the claim that uncertainty, both macro and financial, is an exogenous driver of the business cycle, with contractionary effects on real activity that change over time. While we share the exogeneity of financial uncertainty with other contributions (e.g., Ludvigson et al., 2018a), one key finding of our paper is the exogeneity of macroeconomic uncertainty. We explicitly test this assumption in our structural model, and do not reject it. To this end, we consider two overidentified non-recursive SVARs, one featuring 'endogenous' macroeconomic uncertainty in the three volatility regimes, and a restricted (nested) version in which macroeconomic uncertainty does not respond contemporaneously to real activity shocks in the three volatility regimes. The SVAR with 'endogenous' macroeconomic uncertainty (and exogenous financial uncertainty) is rejected at the 5% significant level, while the SVAR featuring 'exogenous' macroeconomic uncertainty (and exogenous financial uncertainty) is supported by the data. Both specifications implicitly assume that financial uncertainty does not respond on

[2]Given the strong and well established association between the (average) volatility of most macroeconomic variables and specific macroeconomic regimes of U.S. economic history (e.g. McConnel and Perez-Quiros, 2000), throughout the paper we use the terms 'volatility regime' and 'macroeconomic regime' interchangeably.

impact to negative economic shocks. It is important to stress that this assumption, which is required to jointly identify economic activity and macro uncertainty shocks, is not arbitrary but is supported by the reduced form evidence associated with the estimated SVAR, which suggests that financial uncertainty is poorly correlated with real economic activity until the beginning of the Great Recession+Slow Recovery period, and is correlated with macroeconomic uncertainty only starting from the beginning of the 1980s.

The closest papers to ours are Ludvigson *et al.* (2018a) and Carriero *et al* (2018b). Both papers deal with the issue of exogeneity/endogeneity of uncertainty. Similarly to Ludvigson *et al.* (2018a), our results are consistent with the view that financial uncertainty is exogenous to the business cycle. However, in stark contrast with their findings and in line with Carriero *et al.* (2018b), we find strong evidence that macroeconomic uncertainty is an exogenous driver of the business cycle.[3]

Ludvigson *et al.* (2018a) propose a novel set-identification strategy in a time-invariant framework, that allows the joint identification of uncertainty and real activity shocks, without imposing any restrictions on the contemporaneous relations (see also Ludvigson *et al.*, 2018b). Their identification strategy uses two types of *shock-based* restrictions. The first is what they label 'event constraints', which require that the identified financial uncertainty shocks must be large enough during two major financial disruptions, e.g. the 1987 stock market crash and the 2007-09 financial crisis. The second set of constraints are 'correlation constraints', which require that (i) macroeconomic and financial uncertainty shocks must be negatively correlated with aggregate stock market returns, and (ii) financial uncertainty shocks must be more highly correlated with stock market returns than macroeconomic uncertainty shocks. Using the same VAR specification as ours, they find that only financial uncertainty can be considered exogenous to the business cycle, while macroeconomic uncertainty should be treated as an endogenous response to business cycle fluctuations. They also find that while financial uncertainty shocks are contractionary shocks, macro uncertainty shocks have positive effects on real activity, in line with 'growth-options' theories. This major difference on the role of macroeconomic uncertainty can be explained by considering the different identification methods. In Ludvigson *et al.* (2018a), identification is based on external information, which is used asymmetrically between the two types of shocks: event constraints are imposed only on financial uncertainty shocks, and it is therefore unclear what is the actual identification information behind macroeconomic uncertainty. Moreover, relative to their analysis, the flexibility of our SVAR allows us to uncover relevant regime-dependent effects: financial uncertainty becomes a crucial factor for business

[3]To save space, a more comprehensive discussion of how our paper is connected to the large empirical literature on the identification of uncertainty shocks can be found in the Technical Supplement.

cycle developments only after the 1980s. This result lines up with Ng and Wright (2013)'s argument that financial factors have played a crucial role in driving the U.S. business cycle after the mid 1980s, and is consistent with Caldara *et al.* (2016) and Caldara and Scotti (2018). Interestingly, our non-recursive SVAR shows that financial uncertainty affects real economic activity mostly indirectly, by fostering greater macroeconomic uncertainty.

Carriero *et al.* (2018b) jointly identify real activity and uncertainty shocks by using a novel stochastic volatility approach in the context of bivariate VARs which feature measures of macroeconomic and financial uncertainty (one at a time), along with measures of real economic activity. Accordingly, they do not separately identify the effects of macroeconomic and financial sources of uncertainty on economic fluctuations. Their empirical evidence is partly consistent with ours: they also document that macroeconomic uncertainty is broadly exogenous to business cycle fluctuations, while they find that financial uncertainty might, at least in part, arise as an endogenous response to some macroeconomic developments. The identification approach in Carriero *et al.* (2018b) is based on a stochastic volatility mechanism, hence it is inherently different from our heteroskedasticity-based approach to identification. Our method requires the occurrence of separate variance regimes which must be either known or inferred from the data, and this may possibly affect the inference and identification results if the volatility breaks are misspecified. The stochastic volatility approach in Carriero *et al.* (2018b) hinges on the specification of an independent stochastic process which governs the changes of the variances over time. This adds flexibility to the model and facilitates identification issues, but also raises computational issues. For instance, the extension of Carriero *et al.* (2018b)'s approach to the case of three-variate SVARs, which would allow to separately identify the effects of macroeconomic and financial uncertainty shocks, may become computationally demanding. Moreover, our approach allows, without imposing, regime-specific effects of uncertainty shocks, which may uncover important changes in the transmission mechanism over time, as we find for the effects of financial uncertainty.

The paper is organized as follows. Section 2 introduces the identification problem and presents our non-recursive identification approach. Section 3 discusses the data and the empirical results obtained from the estimated SVAR. Section 4 provides some concluding remarks. Additional technical details and empirical results and robustness checks are confined in an on-line Technical Supplement.[4]

[4]Available online at https://drive.google.com/file/d/1cK3HPPWPEc7fG7J_VNafNalDYy0n66_g/view

2 Econometric framework

In this Section, we outline our econometric methodology to deal with both regime-dependence and the joint identification of uncertainty and real activity shocks. Subsection 2.1 presents the general setup and discusses the nature of the problem one faces in 'standard' SVARs, while Subsection 2.2 extends the analysis to the 'identification-through-heteroskedasticity' method exploited in the paper.

2.1 Identifying uncertainty and real economic activity shocks under homoskedasticity

Consider the following SVAR:

$$X_t = c + \Phi_1 X_{t-1} + ... + \Phi_p X_{t-p} + Be_t = \Pi W_t + Be_t \quad , \quad e_t \sim \text{WN}(0_{n\times 1},\, I_n) \ , \ t = 1, ..., T \quad (1)$$

where T is the sample length, p is the system lag order, X_t is the $n \times 1$ vector of endogenous variables, c is a $n\times 1$ constant, Φ_i, $i = 1, ..., p$ are $n\times n$ matrices of parameters, $\Pi := (\Phi_1, ..., \Phi_p, c)$, $W_t := (X'_{t-1}, ...X'_{t-p}, 1)'$, B is a $n \times n$ non-singular matrix containing what we call 'structural parameters', and e_t is the vector of mean zero, (normalized) unit variance and uncorrelated structural shocks. It is assumed that the autoregressive polynomial $\Phi(L)$:$=I_n - \Phi_1 L - ... - \Phi_p L^p$ is such that the solutions to $\det(\Phi(z)) = 0$ satisfy $|z| > 1$. Let

$$\eta_t = Be_t \quad (2)$$

be the $n \times 1$ vector of reduced form innovations, with (unconditional) covariance matrix $\Sigma_\eta = BB'$.

Suppose we are interested in the dynamic effects of the structural shocks in e_t. Let A be the VAR companion matrix, X^c_t:$=(X'_t, X'_{t-1}, ..., X'_{t-p+1})'$ the state vector associated with the VAR companion form and R:$=(I_n, 0_{n\times n}, ..., 0_{n\times n})$ a selection matrix such that X_t=RX^c_t, $RR' = I_n$. As is known, the dynamic response of X_{t+h} to shock e_{jt} to the variable X_{jt} is summarized by the (population) IRF:

$$IRF_j(h) := R\,(A)^h\, R' b_j \quad , \quad h = 0, 1, 2, ..., \quad j = 1, ..., n \quad (3)$$

where b_j is the j-th column of B, i.e. B:$=(b_{\bullet j} : b_j : b_{j\bullet})$, and $b_{\bullet j}$ and $b_{j\bullet}$ are the sub-matrices that contain the columns that precede (if any) and follow (if any) the column b_j, respectively. Absent further restrictions on the coefficients, the IRF in eq. (3) requires that b_j is identified in the sense that it contains independent information relative to the columns in $b_{\bullet j}$ and/or in $b_{j\bullet}$. For $h = 0$, the IRF in eq. (3) is such that, up to possible normalizations of the shocks, the

element b_{lj} of the B matrix in eq. (2) captures the instantaneous (on-impact) effect of the j-th structural shock on the l-th variable of the system.

Consider now our specific case, where $n = 3$. Let Y_t denote a (scalar) measure of real activity, and let U_{Mt} and U_{Ft} be two (scalar) measures of macro and financial uncertainty, respectively, so that $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$. In the absence of further restrictions, the structural relationship in eq. (2) is given by the following system of equations

$$\underbrace{\begin{pmatrix} \eta_{Mt} \\ \eta_{Yt} \\ \eta_{Ft} \end{pmatrix}}_{\eta_t} = \underbrace{\begin{pmatrix} b_{MM} & b_{MY} & b_{MF} \\ b_{YM} & b_{YY} & b_{YF} \\ b_{FM} & b_{FY} & b_{FF} \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} e_{Mt} \\ e_{Yt} \\ e_{Ft} \end{pmatrix}}_{e_t} \tag{4}$$

where we conventionally call e_{Mt} 'macroeconomic uncertainty shock', e_{Ft} 'financial uncertainty shock' and e_{Yt} 'real economic activity shock'. As is known, at least three restrictions are needed in eq. (4) to identify the shocks in a 'Gaussian setup'.[5] The covariance matrix $\Sigma_\eta = BB'$ provides $n(n+1)/2 = 6$ symmetry restrictions to identify the 9 elements of B, leaving 3 element unidentified. A common solution to this problem is to specify B as a triangular matrix, which provides the 3 zero (identifying) restrictions. The empirical literature on the identification of uncertainty shocks largely relies on the use of recursive SVARs because the interest typically lies on the effect of uncertainty shocks on Y_t, while it is presumed that U_{Mt} (U_{Ft}) responds to shocks to Y_t only with lags. If one imposes an upper (lower) triangular structure on B, or 'conventional' zero restrictions, it is not possible to identify simultaneously the parameters of interest b_{YM}, b_{YF}, b_{MY} and b_{FY}, meaning that 'reverse causality' cannot be addressed.

The reverse causality issue and the related identification problem can in principle be tackled by using valid external instruments that permit to increase the number of useful moment conditions other than $\Sigma_\eta = BB'$, without further restricting B; see e.g. Stock and Watson (2012, 2018) and Mertens and Ravn (2013); see also Carriero *et al.* (2015). Ludvigson *et al.* (2018a) discuss the peril of such an approach in the uncertainty framework, and improve upon this methodology by arguing that if U_{Mt} and U_{Ft} are potentially endogenous (i.e. they may respond to e_{Yt}), then it is difficult to find credible observable exogenous external instruments for the uncertainty shocks.

While the combined use of external instruments and set-identification methods allow to address the reverse causality issue, it does not help dealing with the problem of possibly regime-

[5] It is worth stressing that regardless of the type of identifying restrictions we impose on B, we do not have enough information in this stylized small-scale model to claim that e_{Yt} is a demand or supply shock. In general, e_{Yt} could be a combination of technology, monetary policy, preferences and government expenditures. For this reason, and in line with Ludvigson *et al.* (2018a), we refer to e_{Yt} as 'real activity shock'. Likewise, we do not have enough information to disentangle whether uncertainty shocks originate from economic policies and/or technology.

dependent effects of uncertainty shocks, which is the issue analyzed next.

2.2 Using heteroskedasticity to identify uncertainty and real economic activity shocks

In order to jointly address the reverse causality and possible regime-dependent effects of uncertainty shocks, one needs to combine a non-recursive structure for B with the case where the elements in B may change across macroeconomic regimes with the changes in the unconditional covariance matrix Σ_η, generating regime-dependent IRFs. We solve this problem by exploiting the heteroskedasticity displayed by the reduced form errors η_t across different macroeconomic regimes that characterize the U.S. business cycle. Our identification methodology is based on the existence of different volatility regimes in the post-WW2 U.S. business cycle, i.e. different values that Σ_η may take across sub-samples. Allowing for changes in the structural parameters B represents a major generalization relative to the 'standard' identification approach based on heteroskedasticity developed in Rigobon (2003), Lanne and Lütkepohl (2008) and Lanne *et al.* (2010); see also Lewis (2018).[6]

Going back to the SVAR for $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$ defined in eq. (1), consider the unconditional covariance matrix Σ_η:

$$\Sigma_\eta = E(\eta_t\eta_t'){:=}\begin{pmatrix} \sigma_M^2 & \sigma_{M,Y} & \sigma_{M,F} \\ & \sigma_Y^2 & \sigma_{Y,F} \\ & & \sigma_F^2 \end{pmatrix}, \tag{5}$$

where, $\sigma_{M,Y} = E(\eta_{Mt}\eta_{Yt})$, $\sigma_{M,F} = E(\eta_{Mt}\eta_{Ft})$ and $\sigma_{Y,F} = E(\eta_{Yt}\eta_{Ft})$. For ease of exposition, assume that there are two structural changes in this unconditional error covariance matrix, which correspond to the existence of three distinct volatility regimes.[7] If $t{=}T_{B_1}$ and $t{=}T_{B_2}$ denote the dates of the two structural breaks, with $1 < T_{B_1} < T_{B_2} < T$, then the reduced form VAR in eq. (1) can be generalized to:

$$X_t = \Pi(t)W_t + \eta_t \quad , \quad \Sigma_\eta(t){:=}E(\eta_t\eta_t') \quad , \quad t = 1, ..., T \tag{6}$$

[6] We refer to Lütkepohl (2013), Lütkepohl and Netšunajev (2017) and Kilian and Lütkepohl (2017, Chap. 14) for a review of this literature. Chen and Netšunajev (2018) provide an application of such methodology in the context of uncertainty shocks.

[7] This is the case we will deal with in our empirical section. Our analysis, however, can be easily generalized to the case in which there are m structural breaks in the unconditional error covariance matrix, corresponding to $m + 1$ volatility regimes in the data. The inferential issues that arise when the break dates are misspecified is a topic which has not been yet explicitly analyzed in the identification-through-heteroskedasticity literature, and is the subject of future research. Podstawki and Velinov (2018) have extended the identification approach we present and apply in this paper to the case in which the VAR parameters switch endogenously across volatility regimes.

where $W_t := (X'_{t-1}, ..., X'_{t-p}, 1)'$ contains lagged regressors and a constant, $\Pi(t)$ is the matrix of associated slope (autoregressive) coefficients given by

$$\Pi(t) := \Pi_1 \times \mathbb{1}(t \leq T_{B_1}) + \Pi_2 \times \mathbb{1}(T_{B_1} < t \leq T_{B_2}) + \Pi_3 \times \mathbb{1}(t > T_{B_2}) \tag{7}$$

and, finally, the error covariance matrix $\Sigma_\eta(t)$ is given by

$$\Sigma_\eta(t) := \Sigma_{\eta,1} \times \mathbb{1}(t \leq T_{B_1}) + \Sigma_{\eta,2} \times \mathbb{1}(T_{B_1} < t \leq T_{B_2}) + \Sigma_{\eta,3} \times \mathbb{1}(t > T_{B_2}) \tag{8}$$

where $\mathbb{1}(\cdot)$ is the indicator function. Key to our identification approach is that $\Sigma_{\eta,1} \neq \Sigma_{\eta,2} \neq \Sigma_{\eta,3}$. Important for our analysis, notice that the specification in eq.s (6)-(8) covers the case in which also the slope (autoregressive) parameters vary across volatility regimes ($\Pi_1 \neq \Pi_2 \neq \Pi_3$).

We assume that the system described by eq.s (6)-(8) is subject to a set of regularity assumptions (Assumptions 1-3 in the Technical Supplement) which allow standard inference. Given the existence of three volatility regimes, the SVAR is defined by the structural specification:

$$\begin{aligned} \eta_t &= B e_t && 1 \leq t \leq T_{B_1} \\ \eta_t &= (B + Q_2)\, e_t && T_{B_1} < t \leq T_{B_2} \\ \eta_t &= (B + Q_2 + Q_3)\, e_t && T_{B_2} < t \leq T \end{aligned} \tag{9}$$

where B, Q_2 and Q_3 are 3×3 matrices containing structural parameters and $e_t := (e_{Mt}, e_{Yt}, e_{Ft})'$ is the vector of structural shocks such that $E(e_t) = 0_{3\times 1}$ and with normalized covariance matrix $E(e_t e'_t) := I_3$.[8] As before, we call e_{Mt} 'macroeconomic uncertainty shock', e_{Ft} 'financial uncertainty shock' and e_{Yt} 'shock to real activity'. In eq. (9), B is the non-singular matrix which governs the structural contemporaneous relationships (on-impact responses) between the variables and the shocks in the first volatility regime. The matrix Q_2 captures the changes in the structural parameters, if any, from the first to the second volatility regime, hence the non-singular matrix $(B + Q_2)$ captures the structural contemporaneous relationship (on-impact responses) between the variables and the shocks in the second volatility regime. The matrix Q_3 captures the change in the structural parameters, if any, from the second to the third volatility regime, hence the non-singular matrix $(B + Q_2 + Q_3)$ captures the structural relationship (on-impact responses) between the variables and the shocks in the third volatility regime.

[8] An alternative and equivalent parametrization of the SVAR in eq. (9) is discussed in the Technical Supplement, and is based on the assumptions that the structural shocks have a diagonal matrix covariance matrix which changes across volatility regimes, i.e. $E(e_{i,t} e'_{i,t}) := \Lambda_i := diag(\lambda_{i,1}, ..., \lambda_{i,n})$, where $e_{i,t}$ is the vector of structural shocks at time t in the regime volatility i, and $\lambda_{i,j}$ is the variance of the structural shock to variable j in the volatility regime i. The IRFs presented and discussed in eq. (15) below can be 'scaled' accordingly. To keep exposition as simple as possible, in the paper we refer, without loss of generality, to the parametrization of the SVAR in eq. (9).

Eq. (9) leads to the system of second-order moment conditions

$$\Sigma_{\eta,1} = BB' \tag{10}$$

$$\Sigma_{\eta,2} = (B+Q_2)(B+Q_2)' \tag{11}$$

$$\Sigma_{\eta,3} = (B+Q_2+Q_3)(B+Q_2+Q_3)' \tag{12}$$

which link the reduced form to the structural parameters. Equations (10)-(12) provide $r = \frac{3}{2}n(n+1)$ identifying restrictions on B, Q_2 and Q_3 induced by symmetry. The total number of elements in B, Q_2 and Q_3 is $3n^2$, hence it is necessary to impose at least $3n^2 - r$ additional constraints to achieve identification. These $3n^2 - r$ identifying constraints are provided by economic reasoning about the way the on-impact coefficients may change across regimes, which means that the suggested identification approach combines both data properties (i.e. the heteroskedasticity provided by the data) and theoretical considerations reflected in the specification of the structure of the matrices B, $(B+Q_2)$ and $(B+Q_2+Q_3)$. Let ψ be the vector defined as $\psi{:=}(vec(B)', vec(Q_2)', vec(Q_3)')'$. The set of theory-based linear identifying restrictions on B, Q_2 and Q_3 can be represented compactly in explicit form by:

$$\psi = G\theta + d \tag{13}$$

where θ is the vector containing the 'free' elements in B, Q_2 and Q_3, G is a known $3n^2 \times \dim(\theta)$ selection matrix of full column rank, $d{:=}(d_B', d_{Q_2}', d_{Q_3}')'$ is a $3n^2 \times 1$ vector containing known elements.[9] The moment conditions in eq.s (10)-(12) along with the constraints in eq. (13) can be conveniently summarized in the expression

$$\sigma^+ = g(\theta) \tag{14}$$

where $\sigma^+{:=}(vech(\Sigma_{\eta,1})', vech(\Sigma_{\eta,2})', vech(\Sigma_{\eta,3})')'$ is $r \times 1$, and $g(\cdot)$ is a nonlinear (differentiable) vector function (see Bacchiocchi and Fanelli, 2015 for details). It turns out that the necessary and sufficient rank condition for identification is that the Jacobian matrix $J(\theta) := \frac{\partial g(\theta)}{\partial \theta'}$ be regular and of full column rank when evaluated in a neighborhood of the true parameter value θ_0. The necessary order condition is $\dim(\theta) \leq r$. The Jacobian $J(\theta)$ can be derived analytically or evaluated numerically. Thus, in order to identify the shocks it is necessary that the restrictions in eq.s (10)-(13) satisfy also the necessary and sufficient rank condition.

We denote with $\tilde{B} = B(\theta)$, $\tilde{Q}_2 = Q_2(\theta)$ and $\tilde{Q}_3 = Q_3(\theta)$ the counterparts of B, Q_2 and Q_3 which fulfill the identification conditions. Interestingly, $\tilde{B}$ (first regime), $(\tilde{B}+\tilde{Q}_2)$ (second

[9] Other than accounting for (possibly) non-homogeneous restrictions (meaning that the vector d can be non-zero), eq. (13) allows for cross-regime constraints, i.e. simultaneous restrictions which involve the elements of the matrices B, Q_2 and Q_3 like, for example, $b_{12} + q_{2,12} = 0$ or $b_{12} + q_{2,12} + q_{2,12} = 1$, where b_{12}, $q_{2,12}$ and $q_{3,12}$ are the (1,2) elements of B, Q_2 and Q_3, respectively.

regime) and $(\tilde{B}+\tilde{Q}_2+\tilde{Q}_3)$ (third regime), may be either triangular or 'full' depending on the specification at hand, therefore reverse causality phenomena can in principle be modeled. Notably, in this setup overidentified SVARs, i.e. those for which $\dim(\theta) < r$, can be tested against the data.

The so-identified SVAR generates regime-dependent IRFs. Let A_i, $i = 1, 2, 3$ be the reduced form companion matrices associated with the system in eq. (6). The dynamic response of X_{t+h} to a one-standard deviation shock in variable j at time t is summarized by the (population) IRFs:

$$IRF_j(h) := \begin{cases} R'(A_1)^h R\tilde{b}_j & t \leq T_{B_1} \\ R'(A_2)^h R(\tilde{b}_j + \tilde{q}_{2j}) & T_{B_1} < t \leq T_{B_2} \\ R'(A_3)^h R(\tilde{b}_j + \tilde{q}_{2j} + \tilde{q}_{3j}) & t > T_{B_2} \end{cases} \qquad \begin{matrix} h = 0, 1, ..., h_{\max} \\ j = M, Y, F \end{matrix} \tag{15}$$

where R is the selection matrix introduced in Section 2.1, $\tilde{b}_j$ is the j-th column of the matrix $\tilde{B}$, $\tilde{b}_j + \tilde{q}_{2j}$ is the j-th column of the matrix $\tilde{B} + \tilde{Q}_2$, $\tilde{b}_j + \tilde{q}_{2j} + \tilde{q}_{3j}$ is the j-th column of the matrix $\tilde{B} + \tilde{Q}_2 + \tilde{Q}_3$, respectively, and $h_{\max}$ is the largest horizon considered. Even in the special case in which the slope (autoregressive) coefficients do not vary across volatility regimes, i.e. when $A_1 = A_2 = A_3$ (meaning that $\Pi_1 = \Pi_2 = \Pi_3$ in eq. (7)), the IRFs in eq. (15) change across volatility regimes because of the changes in the on-impact response coefficients.

3 Model specification and empirical results

In this section, we apply the SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$ presented in eq. (1) and discussed in the previous section to address our two main research questions: (i) Does the response of Y_t to shocks to (U_{Mt}, U_{Ft}) vary across macroeconomic regimes? (ii) Are U_{Mt} and U_{Ft} exogenous sources of fluctuations in Y_t, or do U_{Mt} and U_{Ft} respond endogenously to shocks in Y_t? In Section 3.1 we present the data and in Section 3.2 we provide evidence for the existence of three broad volatility regimes. In Section 3.3 we specify and discuss the baseline non-recursive SVAR and in Section 3.4 we test for exogenous uncertainty and analyze the resultant IRFs.

3.1 Data

Our VAR includes three variables; $U_{Mt}(f), U_{Ft}(f)$ and Y_t, where Y_t is a measure of real economic activity, $U_{Mt}(f)$ is a measure of f-period-ahead macroeconomic uncertainty and $U_{Ft}(f)$ is a measure of f-period-ahead financial uncertainty, where $f = 1$ (one-month) or $f = 12$ (one-year). Our measure of real economic activity is the growth rate of the log of real industrial production, denoted Δip_t. The real industrial production index is taken from the FRED database. The measure of financial uncertainty is taken from Ludvigson *et al.* (2018a), while the index of

macroeconomic uncertainty is taken from Jurado *et al.* (2015).[10] The data are monthly and cover the period 1960M8-2015M4 for a total of $T = 653$ observations. As discussed in Ludvigson *et al.* (2018a), jointly modeling financial and macroeconomic uncertainty is key to obtain a correct understanding of the relationship between uncertainty and the business cycle.

3.2 Volatility breaks

Two crucial features of our VAR are that the identification approach requires breaks in the unconditional volatility of the data, and that a small-scale system like ours is not affected by nonfundamentalness, which implicitly amounts to claim that it does not omit important variables. Our major hypothesis is that the relationship between uncertainty and real activity vary across the main macroeconomic regimes of post-WW2 U.S. business cycle because of changes in the unconditional variance of Y_t. To provide evidence in favour of volatility breaks, we proceed in two steps. First, we provide suggestive evidence of time variation by looking at recursive and rolling windows estimates of the residual variances and covariances in our baseline VAR. Second, we formally test for the existence of two structural breaks using Chow-type tests, with possible break dates identified in the previous step. Next, we deal with potential nonfundamentalness of our VAR by testing for its 'informational sufficiency' using the procedure by Forni and Gambetti (2014) and factors extracted from the McCracken and Ng (2015)'s large set of macroeconomic and financial variables. The detailed investigation of this last issue, sketched in the Technical Supplement, is important in light of the small dimension of $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$ because nonfundamentalness is best seen as an informational deficiency problem. The empirical analysis shows that we do not reject the informational sufficiency of $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$, meaning that we can correctly estimate the effects of uncertainty shocks through IRFs.

We start by estimating our baseline VAR for $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$ with four lags ($p = 4$) both recursively and over 10- and 15-years rolling-windows. The estimates of the six elements of the unconditional VAR error covariance matrix Σ_η are plotted in Figure 1. The graphs on the diagonal report the estimated variances while the off-diagonal terms report the estimated covariances for the recursive (blue line), the 10-years (red line) and the 15-years (yellow line) rolling windows VARs. The graph in the position (2,2) reports the unconditional variance of the residuals of the second equation of our VAR, the one associated with Y_t, i.e. σ_Y^2 in Σ_η in eq. (5). The graph clearly shows that the average volatility level is time-varying, being higher during the seventies and eighties, declining from the mid-eighties until the end of 2007, and then increasing again after the financial crisis of 2007–08 before stabilizing. All the remaining graphs in Figure 1 broadly confirm the presence of three volatility regimes. As expected, the two main changes of

[10] The Technical Supplement discusses at length how the two proxies of uncertainty have been constructed.

volatility occur in correspondence of the beginning of the Great Moderation and Great Recession periods, respectively. The two dashed vertical lines correspond to the possible break dates, i.e. T_{B_1} = 1984M3 and T_{B_2} =2007M12. These two break dates would partition the whole sample period 1960M8-2015M4 into three different sub-samples: the Great Inflation period (1960M8-1984M3, $T = 280$), the Great Moderation period (1984M4-2007M12, $T = 285$), and the Great Recession+Slow Recovery period (2008M1-2015M4, $T = 88$).[11] It is worth noting, however, that while the unconditional variance associated with the proxy of macroeconomic uncertainty roughly follows the same volatility pattern as the unconditional volatility of Y_t (position (1,1) in Figure 1), the unconditional variance associated with the proxy of financial uncertainty increases until the beginning of the nineties, probably because of the process of financial innovation which characterizes U.S. financial markets (position (3,3) in Figure 1). Interestingly, these differences in volatility patterns provide identification information in our approach.

The evidence reported in Figure 1 is broadly consistent with the information conveyed in Table 1.[12] The second column of Table 1 summarizes the OLS-based estimates of the VAR covariance matrix Σ_η on the whole sample, i.e. under the null hypothesis that there are no volatility regimes in the data ($H_0' : \Sigma_{\eta,1} = \Sigma_{\eta,2} = \Sigma_{\eta,3}$), and then separately on the three volatility sub-periods.[13] As already shown in Figure 1, these results confirm that unconditional variances and covariances have changed over time. Table 1 also summarizes some diagnostic statistics associated with the estimated models, which suggest that VAR residuals tend to be not Gaussian but not serially correlated within regimes. The non-normality of VAR disturbances is detected, as expected, on the overall sample period but also within macroeconomic regimes and is fully consistent with the analysis in e.g. Cúrdia *et al.* (2014). We remark that the possible

[11] As concerns the third volatility regime, according to the U.S. National Bureau of Economic Research the Great Recession began in December 2007 and ended in June 2009, thus extending over 19 months. Thus, we treat T_{B_2} =2007M12 as the date in which the Great Moderation ends. Considering three distinct volatility regimes does not necessarily rule out the possibility that the VAR for X_t:=$(U_{Mt}, Y_t, U_{Ft})'$ might display unconditional (or possibly conditional) heteroskedastic disturbances within regimes, other than across them. This is clearly seen from the graphs in Figure 1 but, as discussed below, does not represent a major obstacle to the implementation of our identification approach.

[12] Admittedly, the evidence in Figure 1 could also support a time-varying specification. We refer to Mumtaz and Theodoridis (2018) and Carriero *et al.* (2018a, 2018b) for different views on how time-varying specifications can be fruitfully exploited to address empirically the role of uncertainty. As already stressed, crucial to our identification approach is the existence of broad volatility regimes in the data.

[13] The OLS estimates in Table 1 correspond to maximum likelihood estimates generated by maximizing Gaussian densities within each of the considered samples. In the Technical Supplement, we also discuss a classical minimum distance (CMD) estimation approach which does not require any distributional assumption. We prefer to stick to Gaussian maximum likelihod estimation of our SVAR to be as close as possible to the more familar identification-through-heteroskedasticiy approach put forth by Lanne and Lütkepohl (2008) in the context of SVARs.

presence of within-regimes heteroskedasticity (conditional or unconditional), while affecting the full efficiency of our estimates, does not represent a major obstacle to the identification strategy presented below.

To verify formally the hypothesis that there are two main structural breaks in the VAR error covariance matrix at the dates T_{B_1} =1984M3 and T_{B_2} = 2007M12, we compute a set of Chow-type tests and misspecification-type tests. We first test whether the joint null hypothesis of absence of structural breaks in all VAR coefficients:

$$H_0 : \begin{pmatrix} \Pi_1 \\ \Sigma_{\eta,1} \end{pmatrix} = \begin{pmatrix} \Pi_2 \\ \Sigma_{\eta,2} \end{pmatrix} = \begin{pmatrix} \Pi_3 \\ \Sigma_{\eta,3} \end{pmatrix} = \begin{pmatrix} \Pi \\ \Sigma_{\eta} \end{pmatrix} \tag{16}$$

is rejected and, conditional on the rejection of H_0, we test the null hypothesis of absence of volatility regimes

$$H_0' : \Sigma_{\eta,1} = \Sigma_{\eta,2} = \Sigma_{\eta,3} \tag{17}$$

under the maintained restriction: $\Pi_1 = \Pi_2 = \Pi_3 = \Pi$ on slope coefficients. Results are summarized in the bottom panel of Table 1 which reports the LR tests for the hypotheses H_0 and H_0', respectively. Both H_0 and H_0' are strongly rejected by the data. As a final check, we investigate to what extent the detected regime-dependence in the residual covariance matrix can be ascribed to the regime-dependence that characterizes the autoregressive parameters. To do so, we estimate the VAR in eq. (6) by allowing the autoregressive parameters to change as in eq. (7) with T_{B_1} =1984M3 and T_{B_2} = 2007M12, keeping the covariance matrix Σ_η constant. In the so-estimated model, we perform a test for the null hypothesis of (unconditional) homoskedasticity in the residuals (H_0''), which is reported in the lower panel of Table 1. Results show that the hypothesis of homoskedasticity is strongly rejected by the data. This evidence confirms that the changes that characterize the unconditional covariance matrix Σ_η can not be solely ascribed to the changes in the autoregressive parameters. Overall, our results are consistent with Aastveit *et al.* (2017) who, using a wide range of econometric techniques, provide substantial evidence against the stability of common VARs in the period since the Great Recession.

Other than documenting the existence of three broad volatility regimes in the data, Table 1 provides some rough evidence about the changing nature of the relationships between our proxies of uncertainty, U_{Mt} and U_{Ft}, and real economic activity, Y_t. Although it is not possible to infer any causality direction from the correlations in Table 1, the data clearly point towards changing relationships. The information provided by the correlations in Table 1 will be used to inform the structural specification in the next section.

Overall, the estimated reduced form system for X_t:=$(U_{Mt}, Y_t, U_{Ft})'$ provides a reasonable fit to the data and is 'informational sufficient'. We consider it a statistically satisfactory reduced form representation of the non-recursive SVAR specified next.

3.3 Non-recursive SVAR specification

In this section we discuss the specification of the matrices of structural parameters $\tilde{B} = B(\theta)$, $\tilde{Q}_2 = Q_2(\theta)$ and $\tilde{Q}_3 = Q_3(\theta)$ in eq. (9). The vector of structural shocks is $e_t{:=}(e_{Mt}, e_{Yt}, e_{Ft})'$, and we call conventionally e_{Mt} 'macroeconomic uncertainty shock', e_{Ft} 'financial uncertainty shock' and e_{Yt} 'real activity shock', see the discussion in Section 2.

To inform the structural specification, valuable indications may be inferred from the VAR residuals correlation matrices sketched in Table 1. Three main empirical facts emerge from this table. First, the negative correlation between (the residuals associated with) macroeconomic uncertainty and industrial production growth increases by about 50%, from -14% to -21%, when moving from the Great Moderation to the Great Recession+Slow Recovery period. This reduced form evidence is consistent with the structural analysis in e.g. Caggiano *et al.* (2017a) and Plante *et al.* (2018). Second, the correlation between (the residuals associated with) financial uncertainty and industrial production growth turns negative only in the Great Recession+Slow Recovery period (3.8%, 3.2% and -8.9%, respectively) and is not significant. Third, the correlation between (the residuals associated with) macroeconomic and financial uncertainty increases substantially across the three volatility regimes (12%, 32% and almost 40%, respectively), suggesting that the two sources of uncertainty developed in a relatively independent way during the Great Inflation period, and started to be much more correlated thereafter, when periods of financial turmoil have become more prominent. These three empirical facts suggest that the (negative) relationship between macroeconomic uncertainty and real economic activity is likely regime-dependent and intensifies after the GFC. On the other hand, the channel which connects financial uncertainty and real economic activity appears to be indirect: since financial uncertainty is virtually uncorrelated with real activity, its effects on the business cycle, if any, might work only via its correlation with macroeconomic uncertainty. This latter correlation is almost irrelevant in the first subsample, and increases only after the mid-1980s.

Based on these considerations, we formulate our hypotheses on the structural parameters. The three volatility regimes detected in the previous section provide us with $r = 3/2(n)(n+1) = 18$ moment conditions. In the absence of restrictions, B, Q_2 and Q_3 contain $3n^2{=}27$ elements, hence it is necessary to place at least $3n^2 - r = 9$ parameter constraints on these matrices in order to achieve identification. These restrictions, which can be represented compactly as in eq. (13), must satisfy the necessary and sufficient identification rank condition discussed in Section 2.2, i.e. the Jacobian matrix associated with the function in eq. (14) must be regular and full column rank. We consider a total of 11 identifying restrictions (which lead to 11-9=2

overidentification restrictions) and the following matrices $\tilde{B}$, $\tilde{B}+\tilde{Q}_2$ and $\tilde{B}+\tilde{Q}_2+\tilde{Q}_3$:

Great Inflation:

$$\tilde{B}:=\begin{pmatrix} b_{MM} & b_{MY} & 0 \\ b_{YM} & b_{YY} & 0 \\ 0 & 0 & b_{FF} \end{pmatrix}$$

Great Moderation:

$$\tilde{B}+\tilde{Q}_2:=\begin{pmatrix} b_{MM}+q_{2,MM} & b_{MY} & q_{2,MF} \\ b_{YM}+q_{2,YM} & b_{YY}+q_{2,YY} & 0 \\ q_{2,FM} & 0 & b_{FF}+q_{2,FF} \end{pmatrix}$$

Great Recession + Slow Recovery:

$$\tilde{B}+\tilde{Q}_2+\tilde{Q}_3:=\begin{pmatrix} b_{MM}+q_{2,MM} & b_{MY} & q_{2,MF}+q_{3,MF} \\ b_{YM}+q_{2,YM}+q_{3,YM} & b_{YY}+q_{2,YY}+q_{3,YY} & q_{3,YF} \\ q_{2,FM} & 0 & b_{FF}+q_{2,FF}+q_{3,FF} \end{pmatrix}, \tag{18}$$

so that the vector of structural parameters θ contains 16 non-zero elements ($\dim(\theta)$=16).

The specification of the matrix $\tilde{B}$ (Great Inflation) in eq. (18) is based on one crucial hypothesis. Inspired by Ng and Wright (2013) and the already commented reduced-form evidence in Table 1, we maintain that heavily regulated financial markets before the 1980s slowed down the response of financial markets to non-financial dynamics on the one hand, and the response of macroeconomic variables to the uncertainty generated by financial markets on the other hand. Thus, financial uncertainty is assumed not to respond on-impact to real activity shocks ($b_{FY}=0$) nor to macro uncertainty shocks ($b_{FM}=0$), and real activity is assumed not to respond on-impact to financial uncertainty shocks ($b_{YF}=0$), though lagged responses are not ruled out and depend on the estimated dynamics. Likewise, it is also assumed that financial uncertainty does not exert contemporaneous effects on macroeconomic uncertainty ($b_{MF}=0$). Overall, according to the $\tilde{B}$ matrix in eq. (18), macroeconomic uncertainty can be potentially endogenous, depending on the significance of the parameter b_{MY}, while financial uncertainty is treated as a variable that can react only with lags.

Moving to the second volatility regime (Great Moderation), the non-recursive structure of the matrix $\tilde{B}+\tilde{Q}_2$ in eq. (18) is still consistent with the idea that macroeconomic uncertainty shocks may affect real economic activity instantaneously through the parameter $b_{YM}+q_{2,YM}$ ($q_{2,YM}$ captures the change of impact relative to the Great Inflation) and, in turn, real activity shocks may affect macro uncertainty through the parameter $b_{MY}+q_{2,MY}$ (again, $q_{2,MY}$ captures the change relative to the Great Inflation). Differently from the Great Inflation period, however, we now admit that causation among the two sources of uncertainty may run both ways through the

parameters $q_{2,FM}$ (position (3,1)) and $q_{2,MF}$ (position (1,3)), respectively. This is done to infer whether the increased correlation between U_{Mt} and U_{Ft} observed during the Great Moderation relative to the Great Inflation can be ascribed to financial or macro uncertainty shocks, or to both types of shocks. For instance, with $q_{2,FM} = 0$ and $q_{2,MF} \neq 0$ ($q_{2,MF} > 0$) in eq. (18) we might conclude that causality runs from financial uncertainty shocks to macroeconomic uncertainty alone.

Finally, the causality relationships entailed by the structure of the matrix $\tilde{B}+\tilde{Q}_2+\tilde{Q}_3$ (Great Recession+Slow Recovery) in eq. (18) is similar to that of the Great Moderation, the main difference being that we now allow financial uncertainty shocks to affect real economic activity both directly (through the parameter $q_{3,YF}$) and indirectly through its effect on macroeconomic uncertainty (through the parameter $q_{2,MF} + q_{3,MF}$ where, recall, $q_{3,MF}$ captures the possible change of effect relative to the Great Moderation).

Overall, the SVAR based on the specification in eq. (18) is identified in the sense that it satisfies the rank condition discussed in Section 2.2, and gives rise to $r - \dim(\theta) = 2$ (testable) overidentification restrictions. Financial uncertainty is given the 'passive' role of merely amplifying the shocks before the 1980s, while the role of financial markets and the uncertainty stemming from them are brought back to the center-stage of business cycle after the mid-1980s.[14] Notably, the specified structural model features possibly endogenous uncertainty, since it allows the structural parameter b_{MY} to be non-zero. Hence, testing $b_{MY} = 0$ amounts to testing for exogeneity of macroeconomic uncertainty.

Our testing procedure compares the specification in eq. (18) with a restricted version which features two additional hypotheses about the pass-through from uncertainty to real economic activity: one is the hypothesis of 'exogenous' macroeconomic uncertainty, $b_{MY} = 0$, and the other is the hypothesis that macroeconomic uncertainty shocks do not trigger financial uncertainty, $q_{2,FM} = 0$, so that their structural relationship is unidirectional. Jointly, the two restrictions

$$\begin{array}{ll} b_{MY} = 0 & \text{'exogenous' macro uncertainty} \\ q_{2,FM} = 0 & \text{'one-way' causality from financial to macro uncertainty} \end{array} \tag{19}$$

imply, when imposed in eq. (18), an overidentified system which features $r-\dim(\theta) = 4$ (testable) overidentification restrictions.

[14] Our choice is also supported by institutional facts, in particular the changes in the norms regulating financial markets which occurred in the early 1980s, like the Depository Institutions Deregulation and Monetary Control Act in 1980, particularly the termination of regulation Q, and the Garn-St. Germain Act of 1982, which granted easier access to financial liquidity to households and firms only from the mid-eighties onwards.

3.4 On-impact and dynamic causal effects

The non-recursive SVAR specified in eq. (18) is based on the idea that the changes in the covariance matrices $\Sigma_{\eta,1} \neq \Sigma_{\eta,2} \neq \Sigma_{\eta,3}$ associated with the break dates $T_{B_1} = 1984M3$ and $T_{B_2} = 2007M12$, are explained by the occurrence of breaks in the structural parameters. As already observed in Section 2.2, the main difference between our approach and Lanne and Lütkepohl (2008)'s approach is that in the latter the autoregressive parameters in eq. (7) are kept constant, and the changes in the unconditional covariance matrix in the three volatility regimes are modelled by the simultaneous diagonalization:

$$\Sigma_{\eta,1} = BB' \quad , \quad \Sigma_{\eta,2} = B\Lambda_2 B' \quad , \quad \Sigma_{\eta,3} = B\Lambda_3 B \tag{20}$$

where the elements of B are fixed, $\Lambda_2 \neq \Lambda_3 \neq I_n$ are two diagonal matrices with positive elements on the diagonal which satisfy a set of identification conditions discussed in detail in Lanne *et al.* (2010). Eq. (20) gives rise to dynamic causal effects which are invariant to volatility regimes. Hence, before moving to the estimation of our structural model, it seems natural to test to what extent the specification in eq. (20) is supported/rejected by the data. The model in eq. (20) entails an overidentified system which incorporates three (testable) restrictions (indeed there are $r = 18$ reduced form covariance parameters and 9+6=15 distinct elements in B, Λ_2 and Λ_3). The likelihood ratio test for the overidentification restrictions implied by the specification in eq. (20) is equal to 10.11 with associated p-value of 0.0176, hence the model is rejected at the 5% level of significance. We interpret this result as supportive of the fact that the on-impact coefficients of SVARs in the uncertainty framework can possibly change across major macroeconomic regimes of the U.S. economic history. In the analysis that follows, in order to highlight the importance of regime-dependent coefficients, the IRFs implied by the SVAR in eq. (20) will serve as comparative benchmark against our model.

The non-recursive SVAR specified in eq. (18) is estimated on the period 1960M8-2015M4 by imposing the three volatility regimes associated with the two break dates $T_{B_1} = 1984M3$ and $T_{B_2} = 2007M12$. The (quasi-)maximum likelihood estimates of the structural parameters θ that enter the matrices $\tilde{B}$, $\tilde{B} + \tilde{Q}_2$ and $\tilde{B} + \tilde{Q}_2 + \tilde{Q}_3$ are reported, for $f = 1$ (one-month uncertainty), in Table 2, along with analytic and bootstrap standard errors.[15] The upper panel

[15] Bootstrap standard errors are computed using Kilian's (1998) bootstrap-after-bootstrap method, keeping the break dates $T_{B_1} = 1984M3$ and $T_{B_2} = 2007M12$ fixed and resampling (non-parametrically) separately within each volatility regime. This method is also used to compute 90% bootstrap confidence bands for the IRFs that follow. Brüggemann *et al.* (2016) have shown that estimation uncertainty in IRFs produced by SVARs may increase dramatically in the presence of conditional heteroskedasticity compared to an i.i.d setup, depending especially on the persistence characterizing the underlying conditional heteroskedasticity processes. In our setup, the occurrence of conditional heteroskedasticity within the three volatility regimes is an issue which can not be

of Table 2 refers to the the specification in eq. (18), while the lower panel refers to the same model estimated under the two additional restrictions in eq. (19). The estimated structural parameters $\hat{\theta}$ in Table 2 correspond to the on-impact responses featured by our IRFs.

We first discuss the reverse causality/exogeneity issue, then we analyze the dynamic causal effects implied by the estimated IRFs.

Reverse causality/exogeneity. The estimates in the two panels of Table 2 deliver an answer to our first research question, i.e. whether macroeconomic uncertainty is an exogenous source of economic fluctuations or an endogenous response to it, or both. We first analyze the model in the upper panel of Table 2, which allows for 'endogenous' macroeconomic uncertainty ($b_{MY} \neq 0$) and bidirectional causality between macroeconomic and financial uncertainty from the Great Moderation onwards ($q_{2,FM} \neq 0$, $q_{2,MF} \neq 0$, $q_{3,MF} \neq 0$). The LR test for the two overidentification restrictions featured by this model is equal to 7.35 and has a p-value of 0.025, hence the model is not supported by the data at the 5% level of significance.

The parameter b_{MY}, which captures the on-impact response of macroeconomic uncertainty to real economic activity shocks in the three volatility regimes, is not statistically significant. The hypothesis of 'exogenous' macroeconomic uncertainty is largely supported by the data as the LR test for $b_{MY} = 0$ is equal to 0.056 and has a p-value of 0.94. The estimated parameter $q_{2,FM}$ proves to be not strongly significant, confirming our intuition that since the 1980s the pass-through between the two sources of uncertainty is unidirectional: from financial uncertainty shocks to macroeconomic uncertainty. The estimated structural model in the lower panel of Table 2 incorporates these two additional restrictions, see eq. (19). In this case, the LR test for the four overidentification restrictions featured by the SVAR is equal to 8.03 with associated p-value of 0.091, which does not lead us to reject the model at the 5% significance level. A LR test for the structural model in the lower panel against the one in the upper panel of Table 2 is equal to 0.672 and has p-value equal to 0.713. Overall, our empirical evidence supports the specification in eq.s (18)-(19).[16]

ruled out a priori, given the diagnostic tests in Table 1. Brüggemann *et al.* (2016) have also shown that the residual-based moving block bootstrap results in asymptotically valid inference, see also Kilian and Lütkepohl (2017, Ch. 12). Since available simulation results suggest that the performance of different bootstrap methods is often hardly distinguishable in finite samples, Brüggemann *et al.* (2016) recommend that practitioners should be aware of the fact that reported impulse response intervals may understate the actual estimation uncertainty in the presence of conditional heteroskedasticity. With this in mind, and in the absence of a detailed quantification of the extent of conditional heteroskedasticity in our SVAR (which is beyond the scopes of this paper), we interpret all reported bootstrap confidence bands for IRFs with caution.

[16] As a Referee has pointed out, the increase of the p-value that characterizes the two LR tests can also be explained by the fact that the latter can be less powerful than the former due to the increased number

It could be argued that macroeconomic and financial uncertainty are treated asymmetrically in our model. Indeed, although the reduced form evidence speaks loudly about the role of financial uncertainty, the non-response of financial uncertainty to real economic activity shocks has been imposed in the structural specification. To address this issue, we re-estimate the SVARs in eq.s (18)-(19) by inverting the positions of $U_{M,t}$ and $U_{F,t}$ in the vector X_t. This leads to a radical change in the role played by the two sources of uncertainty in the system and the way they transmit to the business cycle. In this case, the LR test for the overidentification restrictions is equal to 28.47 with a p-value of 0.00, which strongly rejects the model. We interpret this evidence as fully consistent with the pass-through of financial and macroeconomic uncertainty to real economic activity hypothesized in our baseline model. In particular, the data seem to support the fact that in the Great Moderation and Great Recession+Slow Recovery periods financial uncertainty shocks foster greater uncertainty about future economic growth.

These findings on reverse causality allow us to make direct contact with Ludvigson *et al.* (2018a), the closest paper to ours in this respect. In line with their results, our analysis is consistent with the view that financial uncertainty is a driver of the business cycle, not a reaction to it. According to our identification scheme, however, financial uncertainty affects the business cycle indirectly by triggering greater macroeconomic uncertainty on-impact. Instead we find remarkable differences with Ludvigson *et al.* (2018a) when we look at the behavior of macroeconomic uncertainty: while they report that macroeconomic uncertainty shocks could be characterized as an endogenous response to business cycle fluctuations and have positive effects on real activity, we find that macroeconomic uncertainty is exogenous to the business cycle and has a negative on-impact effect on real activity. Ludvigson *et al.* (2018a) base their conclusions on a novel methodology which combines the external instruments approach with the mechanics of set-identification (see also Ludvigson *et al.*, 2018b). The endogeneity of macroeconomic uncertainty they document might reflect the 'asymmetric' characterization of financial and macroeconomic uncertainty shocks implicit in their approach, i.e. the fact that the 'event constraints' are imposed on financial uncertainty only, and that the 'correlation constraints' employ aggregate stock market returns as the only external variable with informational content about uncertainty shocks. Our analysis unveils important time-variation (regime-dependency) in the dynamic responses to uncertainty shocks which could further explain the differences between

of restrictions being tested. In the robustness section of the Technical Supplement, we show that all p-values associated with the LR tests discussed in this section increase dramatically once we replace, *ceteris paribus*, the measure of macroeconomic uncertainty, U_{Mt}, with an alternative one obtained by 'purging' U_{Mt} from a subset of financial variables, denoted U^p_{Mt}. Our choice of using U_{Mt} and not directly U^p_{Mt} in the estimation of our baseline SVAR is motivated by the idea of using the same information set as Ludvigson *et al.* (2018a) to facilitate comparison.

their results and ours.

Our empirical evidence fully lines up with Carriero *et al.* (2018b) as concerns the exogeneity of macroeconomic uncertainty. Carriero *et al.* (2018b) identify the shocks by a novel stochastic volatility approach based on non-recursive SVARs which include measures of macroeconomic and financial uncertainty, one at a time. In their model the on-impact coefficients are constant but an independent stochastic process drives the volatility of the system and facilitates the identification of the shocks compared to our regime-dependent method. The extension of their approach to the case of three-equations systems is computationally demanding, and this fact probably explains why they do not separately identify the effects of macroeconomic and financial sources of uncertainty on economic fluctuations, and why in their analysis financial uncertainty is not 'as exogenous' as we find in our setup.

IRFs. The implied IRFs are computed as in eq. (15) by replacing A_1, A_2 and A_3 and $\tilde{B}$, $\tilde{B}+\tilde{Q}_2$ and $\tilde{B}+\tilde{Q}_2+\tilde{Q}_3$ with their estimates, and are plotted in Figures 2-5 over an horizon of $h_{\max}$ =60 periods (5 years). Figure 2 plots the IRFs obtained on the three volatility regimes for $f = 1$ (one-month uncertainty). Figures 3-5 plot the IRFs separately for each regime, disentangling the case $f = 1$ (one-month uncertainty) from the case $f = 12$ (one-year uncertainty).[17] All plots show responses to one standard deviation changes in e_{jt}, $j = M, Y, F$ in the direction that leads to an increase in its own variable X_{it}, $i = M, Y, F$, where $X_{Mt} = U_{Mt}$, $X_{Yt} = Y_t$ and $X_{Ft} = U_{Ft}$, respectively. This normalization allows us to directly compare the responses of real economic activity in the three volatility regimes.

In Figure 2 (which can be fully appreciated in color), the blue IRFs refer to the Great Inflation period, the red IRFs to the Great Moderation period and the yellow IRFs to the Great Recession+Slow Recovery period. The first row reports the response of macroeconomic uncertainty to the three structural shocks, the second row reports the response of industrial production, and the third row reports the response of financial uncertainty. In this case, confidence bands have not been reported to ease reading.[18] In order to compare results with a benchmark, Figure 2 also plots the IRFs generated by the SVAR identified by the Lanne and Lütkepohl's (2008) method, discussed at the beginning of this section, i.e. the structural model based on the specification in eq. (20) and regime-invariant autoregressive coefficients.

The graphs in Figure 2 suggest four main comments. First, there is evidence of substantial time variation in the impulse responses: the estimated IRFs differ quantitatively and quali-

[17] To save space, a detailed comment of the IRFs in Figures 3-5 can be found in the Technical Supplement

[18] Recall that the reduced form analysis in Section 3.2 shows that there are significant differences between all VAR coefficients (autoregressive parameters and covariance matrices) across the three volatility regimes. Accordingly, the three IRFs in each graph of Figure 2 read as transformations of parameters which have been established to be statistically different in their population values.

tatively across the three volatility regimes. Although uncertainty shocks curb industrial production growth in all three macroeconomic regimes, the persistence of the response and the number of periods after which the negative peak is reached vary across regimes. Second, the effects of macroeconomic uncertainty shocks on all variables are larger and more persistent in the Great Recession+Slow Recovery period. In particular, macroeconomic uncertainty seems to have played a sizable role in driving persistently down economic activity during this period. Third, while real economic activity reacts negatively and persistently to uncertainty shocks, uncertainty reacts only mildly to real activity shocks, if anything. Fourth, there exist differences, as expected, between the IRFs estimated with our non-recursive SVAR and the IRFs produced by keeping the structural parameters fixed.

Overall, combined with the reduced form evidence in Section 3.2, Figures 2-5 provide a positive answer to our second research question: the short-run relationship between uncertainty and real economic activity changes qualitatively and quantitatively across macroeconomic regimes. A researcher who ignores the regime-dependent nature of uncertainty shocks is likely to estimate compounded effects, which hide the different dynamics displayed in the data.

The estimated IRFs can also be framed in a recent debate on the role of uncertainty during the zero lower bound. According to Plante *et al.* (2018), during the zero lower bound, which roughly coincides with the Great Recession+Slow Recovery period, macroeconomic variables were more responsive to negative shocks hitting the economy because of the inability of the Fed to use conventional instruments to stabilize the economy, inducing a general increase in the uncertainty surrounding future growth. The IRFs in Figures 4 and 5 show that there is indeed a difference in the response of uncertainty to real economic shocks when moving from the Great Moderation to the Great Recession+Slow Recovery period. However, while this effect helps to explain why the correlation between uncertainty and real economic activity increases after the GFC (see the correlations in Table 1), it is not sufficient to claim that uncertainty is an endogenous (causal) response to real economic activity shocks because according to our analysis the response is at most lagged of one period, but is not instantaneous.

Finally, the estimated IRFs also line up with several contributions in the literature which highlight how uncertainty shocks have had larger effects after the GFC. This can be due to large financial frictions, as in Alfaro *et al.* (2018), Caggiano *et al.* (2017b), and Gilchrist *et al.* (2014), or to the presence of the zero lower bound, as in Caggiano *et al.* (2017a) and Basu and Bundick (2017). They also support theoretical and empirical research that highlights how uncertainty shocks might have time-varying effects which depend on different macroeconomic conditions like, e.g. the level of financial frictions (Alfaro *et al.*, 2018; Gilchrist *et al.*, 2014, Alessandri and Mumtaz, 2014), the stance of the business cycle (Cacciatore and Ravenna, 2016;

Caggiano *et al.*, 2014), or the stance of monetary policy (Basu and Bundick, 2017, Caggiano *et al.*, 2017a).

4 Concluding remarks

This paper has addressed two controversial issues that characterize the empirical literature on uncertainty: whether time-variation in uncertainty should be considered as an exogenous driver of the business cycle or, rather, an endogenous response to it, and whether the real effects of uncertainty shocks have changed over time with the changes in macroeconomic conditions. The two issues have been analyzed simultaneously with a small-scale non-recursive SVAR estimated on U.S. post-WW2 data, by resorting to an 'identification-through-heteroskedasticity' approach which is novel in the literature on uncertainty. Unlike other existing identification approaches, our framework allows us to jointly estimate regime-dependent effects of uncertainty shocks, and is general enough to account for reverse causality, i.e. to allow for a contemporaneous response of both real activity to uncertainty shocks and of uncertainty to real activity shocks.

Empirical results suggest that there are important differences in the impact and propagation mechanism of uncertainty shocks across the three main macroeconomic regimes that characterize the U.S. business cycle, and that uncertainty, both macro and financial, is better approximated as an exogenous source of economic decline rather than an endogenous response to it. We find that macroeconomic uncertainty shocks have always had a contractionary impact on real activity, but that these effects have become larger since the GFC. In turn, after the 1980s, financial uncertainty shocks affect real economic activity by fostering greater macroeconomic uncertainty.

Overall, our findings support the theoretical models where uncertainty is treated as an exogenous driver of economic fluctuations, as in e.g. Bloom (2009) and Basu and Bundick (2017), and the empirical specifications where uncertainty enters recursive SVARs. In this respect, our analysis is partially consistent with the evidence reported in Ludvigson *et al.* (2018a) and is not at odds with Carriero *et al.* (2018b).

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TABLE 1. Estimated reduced form VAR covariance and correlation matrices and Chow-type tests for structural breaks.

Overall period: 1960M8-2015M4 (T=653)		
log-Likelihood = 2871.6	$\hat{\Sigma}_\eta = \begin{pmatrix} 1.05e\text{-}04^* & -0.011^* & 6.94e\text{-}05^* \\ & 0.433^* & 6.80e\text{-}04 \\ & & 7.39e\text{-}04^* \end{pmatrix}$,	$\hat{\rho}_\eta = \begin{pmatrix} 1 & -0.167^* & 0.249^* \\ & 1 & 0.038 \\ & & 1 \end{pmatrix}$
$N_{DH} = 645.281[0.000]$		
$AR_5 = 19.216[0.997]$		
GI: 1960M8-1984M3 (T=280)		
log-Likelihood = 1170.5	$\hat{\Sigma}_{\eta,1} = \begin{pmatrix} 1.25e\text{-}04^* & -0.001^* & -3.49e\text{-}05^* \\ & 0.587^* & 0.002 \\ & & 6.72e\text{-}04^* \end{pmatrix}$,	$\hat{\rho}_{\eta,1} = \begin{pmatrix} 1 & -0.157^* & 0.120^* \\ & 1 & 0.086 \\ & & 1 \end{pmatrix}$
$N_{DH} = 111.242[0.000]$		
$AR_5 = 35.521[0.843]$		
GM: 1984M4-2007M12 (T=285)		
log-Likelihood = 1399.9	$\hat{\Sigma}_{\eta,2} = \begin{pmatrix} 7.18\text{-}05^* & -5.58e\text{-}04^* & 7.62e\text{-}05^* \\ & 0.221^* & 4.24e\text{-}04 \\ & & 7.84e\text{-}05^* \end{pmatrix}$,	$\hat{\rho}_{\eta,2} = \begin{pmatrix} 1 & -0.140^* & 0.321^* \\ & 1 & 0.032 \\ & & 1 \end{pmatrix}$
$N_{DH} = 317.978[0.000]$		
$AR_5 = 51.729[0.228]$		
GR+SR: 2008M1-2015M4 (T=88)		
log-Likelihood = 428.1	$\hat{\Sigma}_{\eta,3} = \begin{pmatrix} 8.51e\text{-}05^* & -0.001^* & 7.90e\text{-}05^* \\ & 0.378^* & -0.001 \\ & & 4.62e\text{-}04^* \end{pmatrix}$,	$\hat{\rho}_{\eta,3} = \begin{pmatrix} 1 & -0.214^* & 0.398^* \\ & 1 & -0.089 \\ & & 1 \end{pmatrix}$
$N_{DH} = 108.785[0.000]$		
$AR_5 = 79.523[0.001]$		
H_0 :	$\text{LR}_T = 253.99[0.000]$ (no breaks in all VAR coefficients)	
H_0' :	$\text{LR}_T = 96.061[0.000]$ (no breaks in the VAR covariance matrix)	
H_0'' :	$\text{F}_T = 4.221[0.000]$ (homoskedasticity in the VAR residuals with regime-dependent autoregressive parameters)	

Notes: Results are based on a VAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$ (with four lags) with $Y_t = \Delta ip_t$ (industrial production growth). Top panel: estimates on the whole sample. Second, third and fourth panels: estimates on partial samples. Bottom panel: LR Chow-type tests for the null H_0 in eq. (16) and the null H_0' in eq. (17), and F-type test for the null H_0'' of vector homoskedasticity in the VAR residuals with regime-dependent autoregressive parameters. Asterisks (*) denote statistical significance at the 10% confidence level. N_{DH} is the Doornik-Hansen multivariate test for Gaussian disturbances. AR_5 is an LM-type test for the absence of residual autocorrelation against the alternative of autocorrelated VAR disturbance up to 5 lags. P-values in brackets. 'GI'=Great Inflation, 'GM'=Great Moderation and 'GR+SR'=Great Recession + Slow Recovery.

TABLE 2. Estimated structural parameters.

GI: 1960M8-1984M3	GM: 1984M4-2007M12	GR+SR: 2008M1-2015M4

SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$

$$\hat{B} := \begin{pmatrix} \underset{(0.0005)}{\underset{(0.0007)^*}{0.0112}} & \underset{(0.0029)}{\underset{(0.0055)^*}{0.0003}} & 0 \\ \underset{(0.1969)}{\underset{(0.1989)^*}{-0.1377}} & \underset{(0.0466)}{\underset{(0.0633)^*}{0.7540}} & 0 \\ 0 & 0 & \underset{(0.0011)}{\underset{(0.0018)^*}{0.0259}} \end{pmatrix}, \quad \hat{B}+\hat{Q}_2 := \begin{pmatrix} \underset{(0.0009)}{\underset{(0.0010)^*}{0.0075}} & \underset{(0.0029)}{\underset{(0.0055)^*}{0.0003}} & \underset{(0.0016)}{\underset{(0.0015)^*}{0.0040}} \\ \underset{(0.1634)}{\underset{(0.1767)^*}{-0.0907}} & \underset{(0.0360)}{\underset{(0.0616)^*}{0.4613}} & 0 \\ \underset{(0.0056)}{\underset{(0.0029)^*}{-0.0047}} & 0 & \underset{(0.0015)}{\underset{(0.0040)^*}{0.0276}} \end{pmatrix}, \quad \hat{B}+\hat{Q}_2+\hat{Q}_3 := \begin{pmatrix} \underset{(0.0009)}{\underset{(0.0010)^*}{0.0075}} & \underset{(0.0029)}{\underset{(0.0055)^*}{0.0003}} & \underset{(0.0021)}{\underset{(0.0023)^*}{0.0054}} \\ \underset{(0.2130)}{\underset{(0.1967)^*}{-0.1225}} & \underset{(0.0637)}{\underset{(0.1100)^*}{0.5967}} & \underset{(0.0766)}{\underset{(0.1286)^*}{-0.0839}} \\ \underset{(0.0056)}{\underset{(0.0029)^*}{-0.0047}} & 0 & \underset{(0.0020)}{\underset{(0.0030)^*}{0.0210}} \end{pmatrix}$$

Model with 'endogenous' macroeconomic uncertainty: 2 overidentification restrictions: $LR_T = 7.3506_{\chi^2(df=2)}[0.0253]$

SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$

$$\hat{B} := \begin{pmatrix} \underset{(0.0005)}{\underset{(0.0006)^*}{0.0112}} & 0 & 0 \\ \underset{(0.0455)}{\underset{(0.0595)^*}{-0.1203}} & \underset{(0.0320)}{\underset{(0.0438)^*}{0.7569}} & 0 \\ 0 & 0 & \underset{(0.0011)}{\underset{(0.0017)^*}{0.0259}} \end{pmatrix}, \quad \hat{B}+\hat{Q}_2 := \begin{pmatrix} \underset{(0.0003)}{\underset{(0.0004)^*}{0.0081}} & 0 & \underset{(0.0005)}{\underset{(0.0005)^*}{0.0028}} \\ \underset{(0.0280)}{\underset{(0.0408)^*}{-0.0757}} & \underset{(0.0194)}{\underset{(0.0244)^*}{0.4641}} & 0 \\ 0 & 0 & \underset{(0.0012)}{\underset{(0.0036)^*}{0.0280}} \end{pmatrix}, \quad \hat{B}+\hat{Q}_2+\hat{Q}_3 := \begin{pmatrix} \underset{(0.0003)}{\underset{(0.0004)^*}{0.0081}} & 0 & \underset{(0.0014)}{\underset{(0.0031)^*}{0.0037}} \\ \underset{(0.0617)}{\underset{(0.0810)^*}{-0.1150}} & \underset{(0.0453)}{\underset{(0.0844)^*}{0.6006}} & \underset{(0.0653)}{\underset{(0.1123)^*}{-0.0552}} \\ 0 & 0 & \underset{(0.0016)}{\underset{(0.0024)^*}{0.0215}} \end{pmatrix}$$

Model with 'exogenous' macroeconomic uncertainty: 4 overidentification restrictions specified in eq. (19): $LR_T = 8.0268_{\chi^2(df=4)}[0.0906]$

Notes: Estimated structural parameters based on the non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta ip_t$ (industrial production growth), specified in eq. (18). The estimates in the lower panel are based on the additional restrictions in eq. (19). 'LR_T' are likelihood-ratio tests for the overidentification restrictions. Hessian-based standard errors in parenthesis; bootstrap standard errors are denoted with asterisks '*', see footnote 15 for details; 'GI'=Great Inflation; 'GM'=Great Moderation; 'GR+SR'=Great Recession + Slow Recovery.

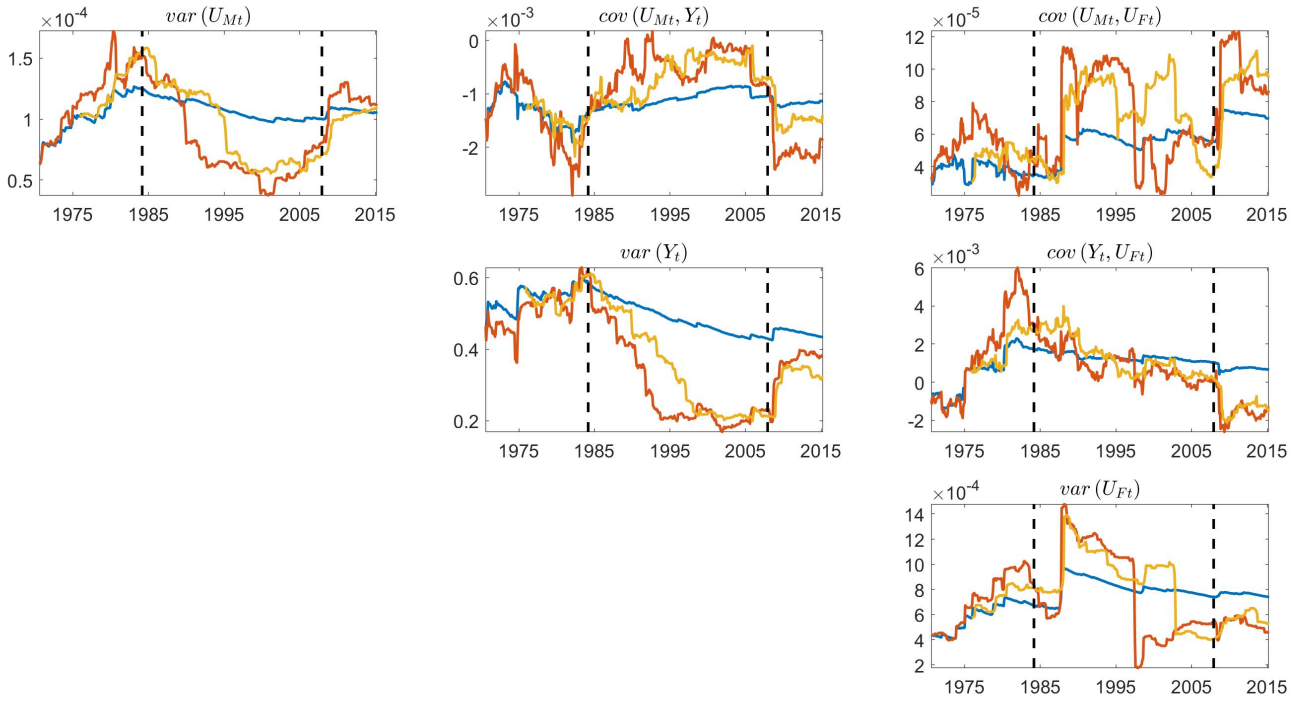

Figure 1: Recursive (blue line), 10-years (red line) and 15-years (yellow line) rolling windows estimates of the error covariance matrix of the VAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta ip_t$ (industrial production growth). Dashed black lines denote the two break dates T_{B_1}=1984M3 and T_{B_2}=2007M12 which separate the three volatility regimes used to identify the shocks. Overall sample: 1960M8-2015M4.

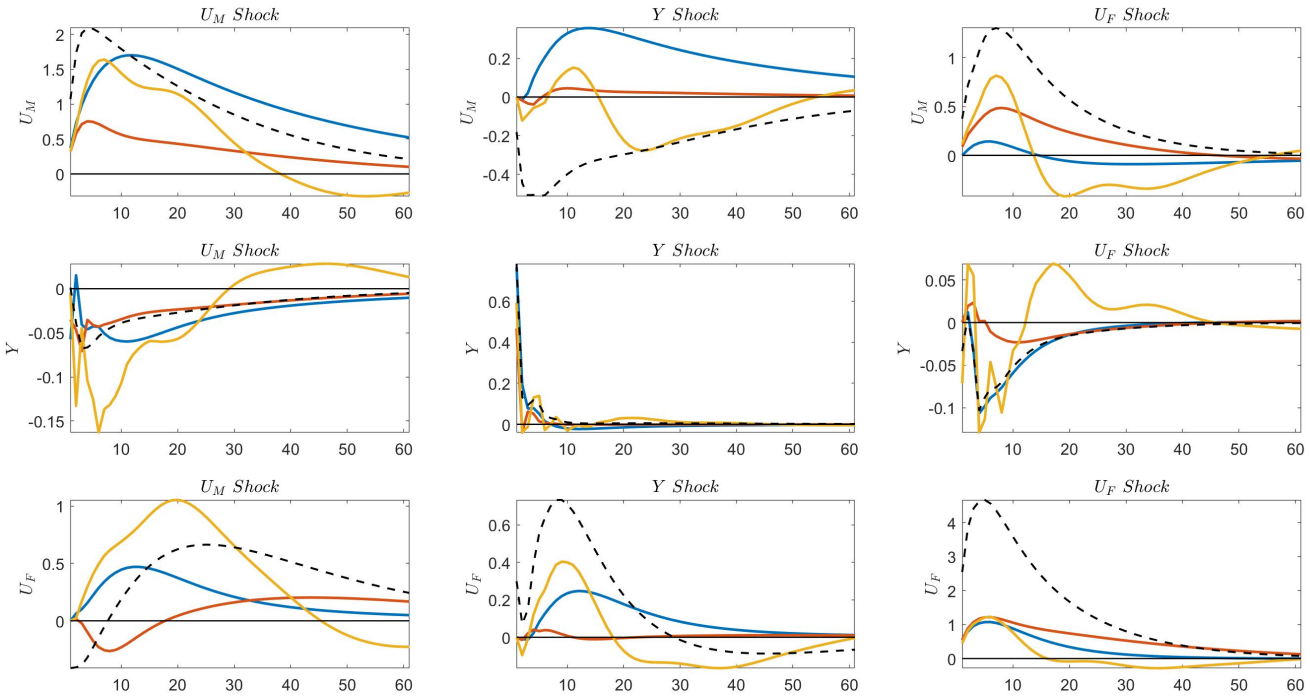

Figure 2: IRFs obtained from the beseline non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta ip_t$ (industrial production growth) specified in eq.s (18)-(19). U_{Mt} and U_{Ft} refer to the one-month ($f = 1$) uncertainty horizon. The blue line refers to the first volatility regime (Great Inflation, 1960M8-1984M3); the red line refers to the second volatility regime (Great Moderation, 1984M4-2007M12); the yellow line refers to the third volatility regime (Great Recession + Slow Recovery, 2008M1-2015M4). Dashed black lines plot the IRFs obtained with the SVAR in eq. (20), estimated on the whole sample, 1960M8-2015M4. Responses are measured with respect to one standard deviation changes in structural shocks.

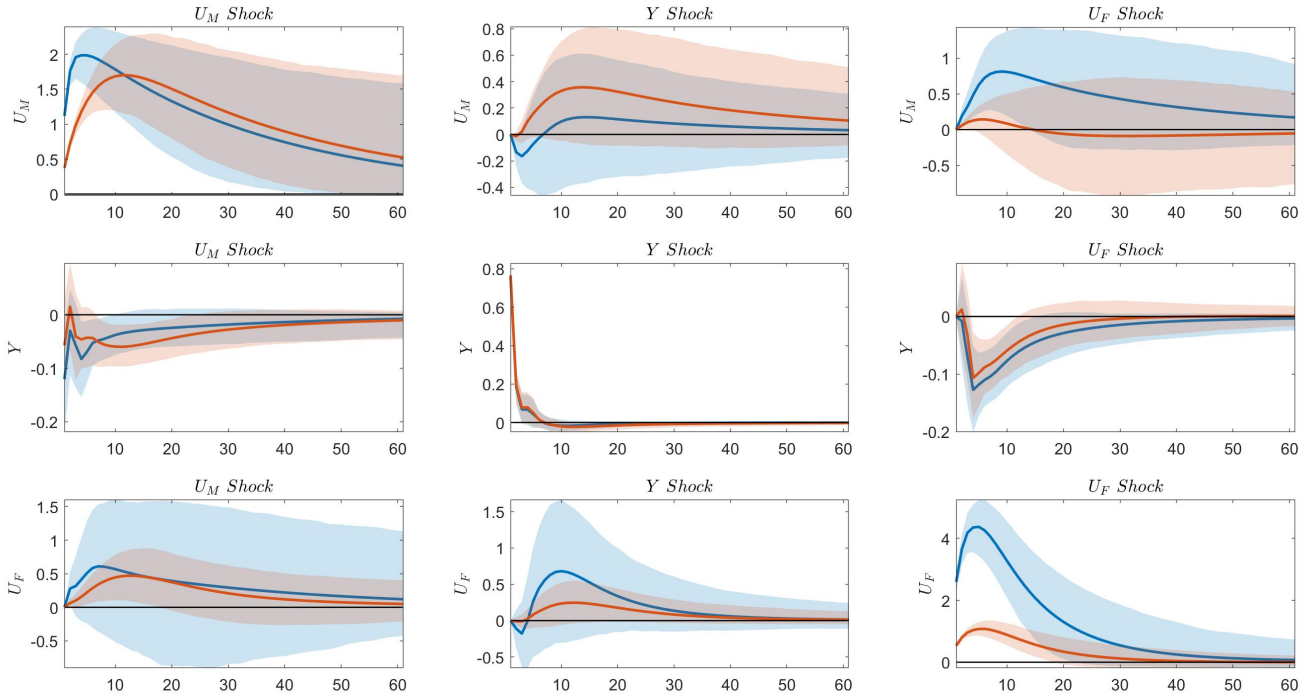

Figure 3: IRFs obtained in the first volatility regime (Great Inflation, 1960M8-1984M3) from the baseline non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta ip_t$ (industrial production growth) specified in eq.s (18)-(19). The blue lines refer to the one-month ($f = 1$) uncertainty horizon and blue shaded areas denote the associated 90% bootstrap confidence bands; the red lines refer to the one-year ($f = 12$) uncertainty horizon and red shaded areas denote the associated 90% bootstrap confidence bands. Responses are measured with respect to one standard deviation changes in structural shocks.

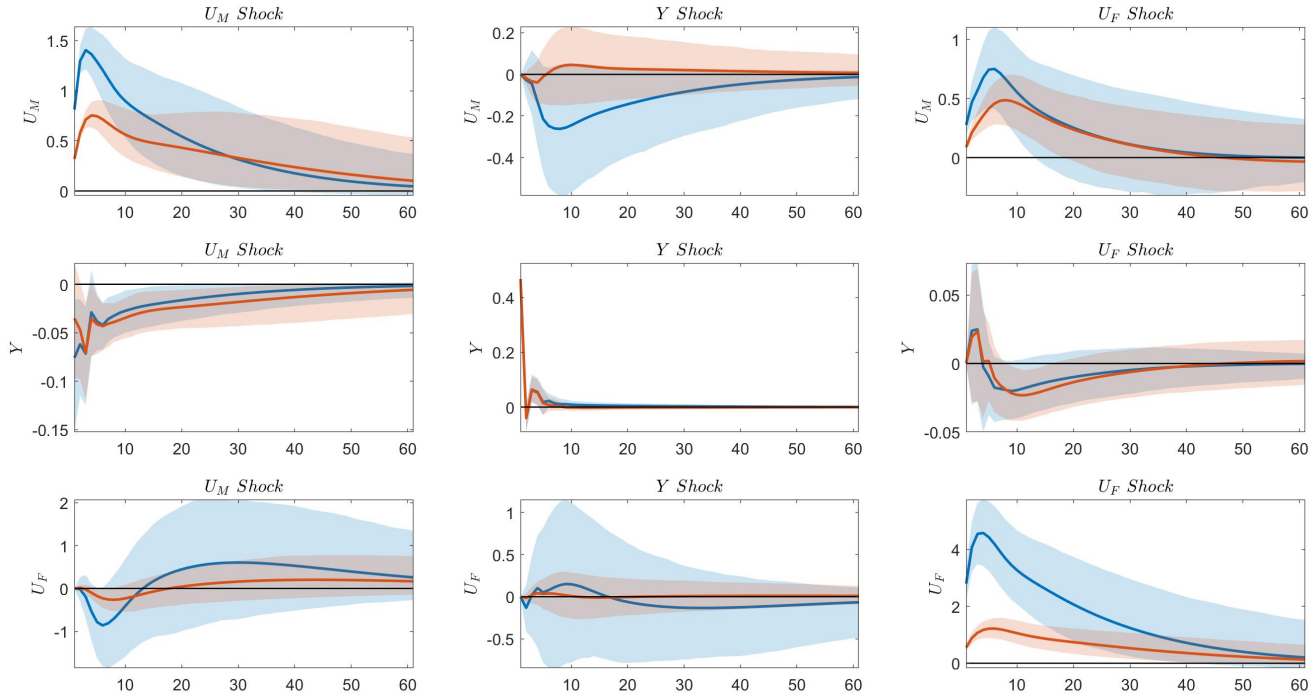

Figure 4: IRFs obtained in the second volatility regime (Great Moderation, 1984M4-2007M12) from the baseline non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta ip_t$ (industrial production growth) specified in eq.s (18)-(19). The blue lines refer to the one-month ($f = 1$) uncertainty horizon and blue shaded areas denote the associated 90% bootstrap confidence bands; the red lines refer to the one-year ($f = 12$) uncertainty horizon and red shaded areas denote the associated 90% bootstrap confidence bands. Responses are measured with respect to one standard deviation changes in structural shocks.

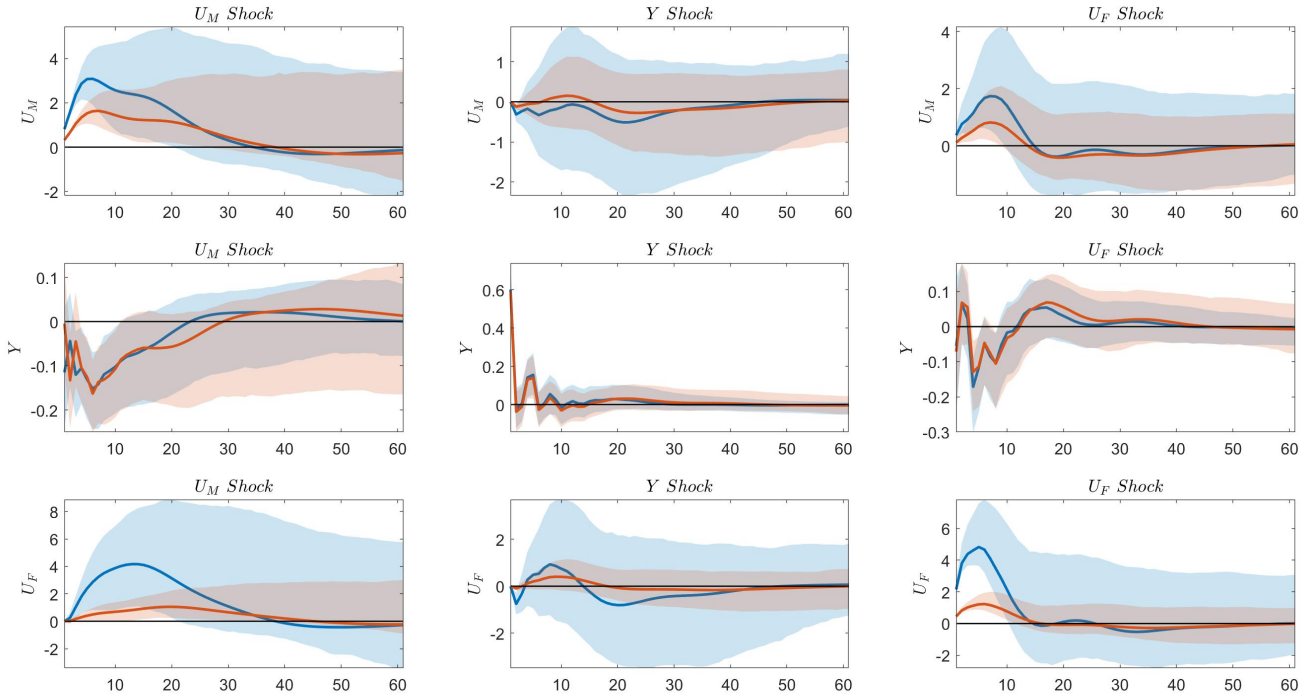

Figure 5: IRFs obtained in the third volatility regime (Great Recession + Slow recovery, 2008M1-2015M4) from the baseline non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta ip_t$ (industrial production growth) specified in eq.s (18)-(19). The blue lines refer to the one-month ($f = 1$) uncertainty horizon and blue shaded areas denote the associated 90% bootstrap confidence bands; the red lines refer to the one-year ($f = 12$) uncertainty horizon and red shaded areas denote the associated 90% bootstrap confidence bands. Responses are measured with respect to one standard deviation changes in structural shocks.

Technical Supplement to

Uncertainty Across Volatility Regimes

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TS.1 Introduction

This Technical Supplement develops/expands a number of topics only partly discussed in the paper and provides additional empirical results.

Section TS.2 formalizes the assumptions and regularity conditions which permit standard asymptotic inference in the VAR system with breaks in the covariance matrix, which is at the basis of our non-recursive SVAR specification and identification approach. Section TS.3 presents an alternative equivalent parametrization of the non-recursive SVAR and compares our approach with the one in Lanne and Lütkepohl (2008). Section TS.4 presents an alternative estimation approach to the likelihood-based method discussed in the paper, which re-interprets the estimation of our non-recursive SVAR with volatility regimes as a classic minimum distance (CMD) problem. Section TS.5 summarizes how the proxies of uncertainty U_{Mt} and U_{Ft} used in the paper and taken from Jurado *et al.* (2015) and Ludvigson *et al.* (2018), respectively, have been constructed. Section TS.6 investigates whether the VAR systems based on $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$, $X_t^*{:=}(CS_t, Y_t, U_{Ft})'$ and $X_t^*{:=}(U_{Mt}, Y_t, CS_t)'$, where CS_t is a measure of credit spreads, are 'informational sufficient' in the sense of Forni and Gambetti (2014). Section TS.7 provides a regime-by-regime comment of the IRFs plotted in Figures 3-5 of the paper. Section TS.8 investigates whether the estimated dynamic causal effects also support significant long-run impacts as measured by cumulated long-run multipliers. Finally, Section TS.9 provides the graphs and detailed comments for three main robustness checks which are only briefly mentioned in the paper. Finally, Section TS.10 frames our paper in the empirical literature on the identification of uncertainty shocks.

TS.2 Model assumptions

In this section we formalize the set of assumptions behind the maximum likelihood estimation of our non-recursive SVAR with breaks in the error covariance matrix. Since maximum likelihood estimation is based on a Gaussian likelihood function, hereafter we denote our estimator with the acronym QML, where 'Q' stands for quasi-maximum likelihood. For simplicity, we focus on the case of $m = 2$ breaks and $m + 1 = 3$ volatility regimes in the data, which is the situation we face in the empirical section of the paper.

Our reference model is the SVAR discussed in Section 2 of the paper. The reduced form belongs to the class of 'VAR models with structural changes in regression coefficients and in covariance matrices' considered in Bai (2000), the only difference being that throughout the paper we treat, without limiting the scopes of our analysis, the two volatility change points (break dates) T_{B_1} and T_{B_2} as known. Our identification and estimation approach, however, can also be applied by relaxing the assumption that T_{B_1} and T_{B_2} are known because it is in principle possible to infer these dates directly from the data along the lines suggested by e.g. Qu and Perron (2007) (see also references therein).

Let $\mathcal{F}_t{:=}\sigma(X_t, X_{t-1}, ..., X_1)$ be the sigma-field generated by the sequence $X_t, X_{t-1}, ..., X_1$ and let $\|\cdot\|$ be the Euclidean norm. Let $(\Pi_i^0, \Sigma_{\eta,i}^0)$ be the true values of the VAR parameters $(\Pi_i, \Sigma_{\eta,i})$, $i = 1, 2, 3$ in eq.s (7)-(9) of the paper. $T_{B_i}^0 = [T\tau_i^0]$, $0 < \tau_i^0 < 1$, $i = 1, 2$ are the true break dates.

Assumption 1 *The sequence* $\{\eta_t, \mathcal{F}_t\}$ *is a MDS* ($E(\eta_t \mid \mathcal{F}_{t-1}) = 0_{n\times 1}$) *which, in addition, satisfies the condition* $\sup_t E(\|\eta_t\|^{4+\delta}) < \infty$.

Assumption 2 $\Sigma_{\eta,i}^0 \neq \Sigma_{\eta,i+1}^0$, $i = 1, 2, 3$. *In addition, each entry in* $\Sigma_{\eta,i}^0$ *is different from the corresponding entry in* $\Sigma_{\eta,i+1}^0$. *Each true regime parameter* $(\Pi_i^0, \Sigma_{\eta,i}^0)$ *corresponds to that of a stationary process so that unit roots and explosive roots are ruled out.*

Assumption 3 $T_{B_i}^0 = [T\tau_i^0]$, *where* τ_i^0 *is the true fraction of the sample,* $i = 1, 2$, *are known.*

Assumption 1 is a relatively standard regularity condition which models the VAR disturbances as MDS (conditional on past information) and requires the existence of up to fourth moments. The first part of Assumption 2 requires that the differences of unconditional covariance matrices across regimes involve all elements of the covariance matrix. Actually, for the purposes of inference on the reduced form parameters, Assumption 2 could be relaxed by simply requiring that there exists an entry in $\Sigma_{\eta,i}^0$ which is different from the corresponding entry in $\Sigma_{\eta,i+1}^0$. We impose the stronger condition that all elements of the covariance matrices differ across volatility regimes to guarantee the identifiability of the non-recursive SVAR.

More precisely, Assumption 2 posits that the covariance matrices $\Sigma^0_{\eta,1}$, $\Sigma^0_{\eta,2}$ and $\Sigma^0_{\eta,3}$ provide enough information to identify shocks in a non-recursive framework. We refer to Magnusson and Mavroeidis (2014) for a thorough discussion of the inferential issues, including weak identification issues, that may arise when possible instabilities in the moments and certain heterogeneity in the data generating process is assumed; see also Lewis (2017). The second part of Assumption 2 establishes that each volatility regime is characterized by 'asymptotically stable' VAR processes. Assumption 3 posits that the break dates are known to the econometrician but, as already observed, could be relaxed.

Under Assumptions 1-3, the inference on the parameters $(\Pi_i, \Sigma_{\eta,i})$, $i = 1, 2, 3$ in the SVAR is standard, see e.g. Bai (2000) and Qu and Perron (2007). Therefore, also the inference on the IRFs stemming from the associated SVAR is of standard type.

TS.3 More on the identification through heteroskedasticity

In this Section, we provide examples of SVARs identified along the lines discussed in Section 2.2 of the paper (Subsection TS.3.1), and then discuss an equivalent reparameterization of our non-recursive SVAR (Subsection TS.3.2). Here we also show that SVARs identified through heteroskedasticity along the lines originally proposed by Lanne and Lütkepohl (2008) and then generalized in Lanne *et al.* (2010), i.e. assuming that the elements of the matrix B do not change across volatility regimes, reads as a very special case of our approach under a particular set of conditions. As in the paper, we focus on the case of $m = 2$ breaks and $m+1 = 3$ volatility regimes.

TS.3.1 Examples

Consider, for illustration, the SVAR for $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$ ($n = 3$) estimated in the paper. The three volatility regimes are associated with the three covariance matrices $\Sigma_{\eta,1}$, $\Sigma_{\eta,2}$ and $\Sigma_{\eta,3}$, respectively. Let '$\times$' denote a generic parameter different from zero. Next we provide some cases where the specified matrices B, Q_2 and Q_3, denoted $\tilde{B}$, $\tilde{Q}_2$ and $\tilde{Q}_3$, respectively, lead to identified SVARs.

Case 1

$$\tilde{B} := \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \ , \ \tilde{Q}_2 := \begin{pmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix} \ , \tilde{Q}_3 := \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}.$$

The specified matrices $\tilde{B}$ (first regime) $\tilde{B}+ \tilde{Q}_2$ (second regime) and $\tilde{B}+ \tilde{Q}_2 + \tilde{Q}_3$ (third regime) are non-recursive, and it is seen that in the move from the first to the second

volatility regime only the structural parameters in the lower triangle of $\tilde{B}$ change, while in the move from the second to the third volatility regime only the diagonal elements of $\tilde{B}+\tilde{Q}_2$ change. In this case, $\dim(\theta) = 18 = r = 3/2n(n+1)$, hence the necessary order condition is satisfied. Obviously, the necessary and sufficient rank condition must be checked by evaluating the full column rank of the Jacobian $J(\theta) := \frac{\partial g(\theta)}{\partial \theta'}$, see Bacchiocchi and Fanelli (2015) for details.

Case 2

$$\tilde{B} := \begin{pmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix} \ , \ \tilde{Q}_2 := \begin{pmatrix} \times & \times & \times \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix} \ , \tilde{Q}_3 := \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}.$$

The specified matrix $\tilde{B}$ (first regime) is recursive, while $\tilde{B}+\tilde{Q}_2$ (second regime) and $\tilde{B}+\tilde{Q}_2+\tilde{Q}_3$ (third regime) are not. In particular, only the diagonal element change in the move from the second to the third volatility regime. In this case, $\dim(\theta) = 15 < r = 3/2n(n+1) = 18$, hence the necessary order condition is satisfied and the SVAR is overidentified (with $r - \dim(\theta) = 3$ overidentification restrictions), provided the '$\times$' that enter the Jacobian $J(\theta) := \frac{\partial g(\theta)}{\partial \theta'}$ are such that $J(\theta)$ has full column rank rank 15.

Case 3

$$\tilde{B} := \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix} \ , \ \tilde{Q}_2 := \begin{pmatrix} \times & 0 & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix} \ , \tilde{Q}_3 := \begin{pmatrix} 0 & 0 & \times \\ \times & \times & \times \\ 0 & 0 & \times \end{pmatrix}.$$

This is the structure estimated in the paper (see the upper panel of Table 2). The specified matrices $\tilde{B}$ (first regime), $\tilde{B}+\tilde{Q}_2$ (second regime) and $\tilde{B}+\tilde{Q}_2+\tilde{Q}_3$ (third regime) are non-recursive and feature 'endogenous uncertainty'. In this case, $\dim(\theta) = 16 < r = 3/2n(n+1) = 18$, hence the necessary order condition is satisfied and the SVAR is overidentified (with $r - \dim(\theta) = 2$ overidentification restrictions) provided the '$\times$' that enter the Jacobian $J(\theta) := \frac{\partial g(\theta)}{\partial \theta'}$ are such that $J(\theta)$ has full column rank 16.

TS.3.2 Alternative parameterization

We discuss an alternative but equivalent parametrization of the non-recursive SVAR presented in Section 2.2 of the paper. In Section 2.2 of the paper, the structural shocks have been normalized such that their variance is the identity matrix in all three volatility regimes i.e. $E(e_{i,t}e'_{i,t}) = I_n$, for $i = 1, 2, 3$, where $e_{i,t}$ is the vector of structural shock at time t in regime i. Actually, it is

possible to consider an equivalent specification of our SVAR in which the structural shocks $e_{i,t}$ are allowed to have covariance matrices given by

$$E(e_{i,t}e_{i,t}') = \Lambda_i := \begin{pmatrix} \lambda_{i,1} & 0 & 0 \\ 0 & \lambda_{i,2} & 0 \\ 0 & 0 & \lambda_{i,3} \end{pmatrix} \quad , i = 1, 2, 3$$

where $\lambda_{i,j}$ is the variance of the structural shock to variable j in volatility regime i. In this case, eq. (10) in the paper becomes:

$$\begin{aligned} \eta_t &= B^\circ \Lambda_1^{1/2} e_{1t}^\circ & 1 \leq t \leq T_{B_1} \\ \eta_t &= (B^\circ + Q_2^\circ)\Lambda_2^{1/2} e_{2t}^\circ & T_{B_1} < t \leq T_{B_2} \\ \eta_t &= (B^\circ + Q_2^\circ + Q_3^\circ)\, \Lambda_3^{1/2} e_{3t}^\circ & T_{B_2} < t \leq T \end{aligned}$$

where $e_{it}^\circ := \Lambda_i^{-1/2} e_{i,t}$, $i = 1, 2, 3$ and with $B^\circ, (B^\circ + Q_2^\circ)$ and $(B^\circ + Q_2^\circ + Q_3^\circ)$ we denote the analogs of the matrices $B, (B + Q_2)$ and $(B + Q_2 + Q_3)$ in eq. (10). These matrices are subject to the set of identifying restrictions:

$$\psi^\circ = G^\circ \theta + d^\circ$$

which represent the counterpart of the linear restrictions in eq. (13) of the paper. In this case, however, $\psi^\circ := (vec(B^\circ)', vec(Q_2^\circ)', vec(Q_3^\circ)', vecd(\Lambda_1)', vecd(\Lambda_2)', vecd(\Lambda_3)')'$ is $3(n^2+n)$, where $vecd(\cdot)$ is the vec operator which selects only the diagonal elements of a matrix, G° is a known selection matrix of dimensions $3(n^2+n) \times \dim(\theta)$, θ is the vector of free (unrestricted) elements that enters the matrices B°, Q_2°, Q_3°, Λ_1, Λ_2 and Λ_3, and d° is a known vector $3(n^2+n)$. In this setup, a typical identification restriction is the normalization of the diagonal elements of the matrices $B^\circ, (B^\circ + Q_2^\circ)$ and $(B^\circ + Q_2^\circ + Q_3^\circ)$ to '1'. The vector ψ° will be $3n^2 \times 1$ vector when this normalization is incorporated in the analysis.[1]

With this alternative parameterization, the moment conditions in eq.s (10)-(12) of the paper become

$$\Sigma_{\eta,1} = B^\circ \Lambda_1 B^{\circ\prime} \quad \text{(TS.1)}$$

$$\Sigma_{\eta,2} = (B^\circ + Q_2^\circ)\, \Lambda_2\, (B^\circ + Q_2^\circ)' \quad \text{(TS.2)}$$

$$\Sigma_{\eta,3} = (B^\circ + Q_2^\circ + Q_3^\circ)\, \Lambda_3\, (B^\circ + Q_2^\circ + Q_3^\circ)' \quad \text{(TS.3)}$$

[1] Bacchiocchi (2017) proposes a different parametrization that allows both changes in the structural parameters and in the variances of the shocks across different volatility regimes, without normalizing the diagonal elements to '1', but he considers an additional matrix of structural parameters (denoted A matrix) in the parameterization.

and the necessary order conditions and the necessary and sufficient rank conditions for identification are similar to the one discussed in the paper.[2]

Defined ι_j to be the j-th column of the identify matrix, the dynamic response of X_{t+h} to a shock in variable j at time t of size $e_{j,t} = \delta_j$ is in this case summarized by the (population) IRFs:

$$IRF_j(h) := \begin{cases} R'(A_1)^h R B^\circ \frac{1}{\lambda_{1,j}} \Lambda_1 \iota_j \delta_j & t \leq T_{B_1} \\ R'(A_2)^h R \left(B^\circ + Q_2^\circ\right) \frac{1}{\lambda_{2,j}} \Lambda_2 \iota_j \delta_j & T_{B_1} < t \leq T_{B_2} \\ R'(A_3)^h R (B^\circ + Q_2^\circ + Q_3^\circ) \frac{1}{\lambda_{3,j}} \Lambda_3 \iota_j \delta_j & t > T_{B_2} \end{cases} \qquad \begin{matrix} h = 0, 1, ..., h_{\max} \\ j = M, Y, F \end{matrix} \tag{TS.4}$$

which, apart from the scaling, mimic the ones in eq. (15) of the paper. Here, R is a selection matrix, A_i, $i = 1, 2, 3$ are the VAR companion matrices in the three volatility regimes and $h_{\max}$ is the largest horizon considered.

Apparently, this alternative parameterization is more general than the one used in the paper because in this case the difference $\Sigma_{\eta,1} \neq \Sigma_{\eta,2} \neq \Sigma_{\eta,3}$ is (apparently) explained either by the changes in the variance of the structural shocks e_{it} in the three volatility regimes, or by the different impact of the shocks across the volatility regimes if $Q_2^\circ \neq 0_{n\times n}$ and $Q_3^\circ \neq 0_{n\times n}$ or, possibly, by a combination of these two factors. Actually, given B°, Q_2°, Q_3°, Λ_1, Λ_2 and Λ_3 that satisfy the moment conditions in eq.s (TS.1)-(TS.3), it is always possible to find matrices B, Q_2 and Q_3 such that it holds

$$\begin{aligned} BB' &= B^\circ \Lambda_1 B^{\circ\prime} \\ (B + Q_2)(B + Q_2)' &= (B^\circ + Q_2^\circ) \Lambda_2 (B^\circ + Q_2^\circ)' \\ (B + Q_2 + Q_3)(B + Q_2 + Q_3)' &= (B^\circ + Q_2^\circ + Q_3^\circ) \Lambda_3 (B^\circ + Q_2^\circ + Q_3^\circ)' \end{aligned} \tag{TS.5}$$

meaning that it exists an equivalent parameterization of the SVAR which can be expressed in the form of eq.s (10)-(12) of the paper. This is indeed obtained by choosing B, Q_2 and Q_3 as

$$B = B^\circ \Lambda_1^{1/2}$$

$$Q_2 = (B^\circ + Q_2^\circ) \Lambda_2^{1/2} - B^\circ \Lambda_1^{1/2} = B^\circ (\Lambda_2^{1/2} - \Lambda_1^{1/2}) + Q_2^\circ \Lambda_2^{1/2}$$

$$Q_3 = (B^\circ + Q_2^\circ + Q_3^\circ) \Lambda_3^{1/2} - (B^\circ + Q_2^\circ) \Lambda_2^{1/2} = (B^\circ + Q_2^\circ)(\Lambda_3^{1/2} - \Lambda_2^{1/2}) + Q_3^\circ \Lambda_3^{1/2}$$

and means, for instance, that the on-impact effects of the shocks in the third period, which are captured by the elements of the matrix $(B^\circ + Q_2^\circ + Q_3^\circ) \Lambda_3^{1/2}$, can be equally captured by the

[2] It means that also in this case we have a nonlinear relationship of the form $\sigma^+ = g^\circ(\theta)$ in which $g^\circ(\cdot)$ is a nonlinear differentiable vector function such that the Jacobian matrix $J^\circ(\theta) := \frac{\partial g^\circ(\theta)}{\partial \theta'}$ must be regular and of full column rank when evaluated in a neighborhood of the true parameter value θ_0.

matrix $(B + Q_2 + Q_3)$.[3]

Finally, consider the Lanne and Lütkepohl (2008)'s approach to the identification of SVARs through heteroskedasticity, see also Lanne *et al.* (2010). As is known, in the case of three volatility regimes the changes $\Sigma_{\eta,1} \neq \Sigma_{\eta,2} \neq \Sigma_{\eta,3}$ do not depend in this setup on the changes in the on-impact coefficients. It is therefor possible to refer to the simultaneous diagonalization:

$$\Sigma_{\eta,1} = BB' \tag{TS.6}$$

$$\Sigma_{\eta,2} = B\Lambda_2 B' \tag{TS.7}$$

$$\Sigma_{\eta,3} = B\Lambda_3 B \tag{TS.8}$$

in which $\Lambda_2 \neq \Lambda_3 \neq I_n$ are two diagonal matrices with positive elements on the diagonal which must satisfy a set of identification conditions discussed in detail in Lanne *et al.* (2010) and Kilian and Lütkepohl (2017, Ch. 14). Since the matrix B in eq.s (TS.6)-(TS.8) is kept constant, the IRFs produced by the so-identified SVAR do not change across volatility regimes. It is further seen that in this case, there are $r = 3/2(n)(n+1)$ reduced form elements in the matrices $\Sigma_{\eta,1}, \Sigma_{\eta,2}$ and $\Sigma_{\eta,3}$, and n^2+2n elements in the matrices B, Λ_2 and Λ_3, hence the system features $3/2(n)(n+1) - n^2 + 2n = 1/2(n)(n-1) = 3$ (testable) overidentification restrictions. It is then possible to test the empirical validity of the SVAR by testing the overidentification restrictions implied by the specification in eq.s (TS.6)-(TS.8).

Assume now that the matrix B is specified 'full' with its n^2 elements unrestricted. We denote this situation with $B := B^f$. Conditional on $B := B^f$, the representation in eq.s (TS.6)-(TS.8) is equivalent to our representation in eq.s (10)-(12) of the paper if it holds the equality

$$\begin{aligned} \left(B^f + Q_2\right)\left(B^f + Q_2\right)' &= B^f \Lambda_2 B^{f\prime} \\ \left(B^f + Q_2 + Q_3\right)\left(B^f + Q_2 + Q_3\right)' &= B^f \Lambda_3 B^{f\prime} \end{aligned}$$

which is valid if

$$\begin{aligned} Q_2 &:= B^f(\Lambda_2^{1/2} - I_n) \\ Q_3 &:= B^f(\Lambda_3^{1/2} - \Lambda_2^{1/2}). \end{aligned}$$

This result shows that also in our approach, for $B := B^f$, Q_2 and Q_3 can be chosen such that one obtains the same parameterization as in Lanne and Lütkepohl (2008)'s which leads to regime-invariant IRFs.

[3] However, it is worth stressing that the zero identification restrictions one imposes on the parameterization based on B°, Q_2° and Q_3° (given Λ_1, Λ_2 and Λ_3), do not necessarily have zero couterparts in the parameterization based on B, Q_2 and Q_3.

TS.4 Alternative estimation approach

In this section we sketch an alternative estimation approach for our baseline non-recursive SVAR. The alternative estimation method reads as classical minimum-distance (CMD) approach and does not require the Gaussian density (within regimes) assumption which characterizes the Gaussian maximum likelihood used in the paper.

The idea is that the relationship in eq. (14) of the paper, written here as

$$\sigma^+ - g(\theta) = 0_{r\times 1} \tag{TS.9}$$

can be interpreted as a measure of distance between the reduced form parameters (error variances and covariances) in σ^+ and the structural parameters θ. Moreover, under Assumptions 1-3 we can estimate σ^+ consistently so that eq. (TS.9) forms the basis for the CMD estimation of θ.

Our starting point is the condition

$$T^{1/2}(\hat{\sigma}_T^+ - \sigma_0^+) \xrightarrow{d} N(0_{r\times 1}, V_{\sigma^+})$$

which holds under Assumptions 1-3 of Section TS.2. Here $\hat{\sigma}_T^+ := (vech(\hat{\Sigma}_{\eta,1})', vech(\hat{\Sigma}_{\eta,2})', vech(\hat{\Sigma}_{\eta,3})')$ is a consistent (say, OLS) estimate of σ^+, σ_0^+ is the true value of σ^+, the symbol '$\xrightarrow{d}$' denotes converge in distribution as $T \to \infty$, and V_{σ^+} is a block-diagonal asymptotic covariance matrix with form

$$V_{\sigma^+} := \begin{pmatrix} V_{\sigma_1^+} & & \\ & V_{\sigma_2^+} & \\ & & V_{\sigma_3^+} \end{pmatrix}, \quad V_{\sigma_i^+} := 2D_3^+(\Sigma_{\eta,i} \otimes \Sigma_{\eta,i})(D_3^+)' \ , \ i = 1,2,3 \tag{TS.10}$$

where $D_3^+ := (D_3'D_3)^{-1}D_3'$ is the Moore-Penrose inverse of the duplication matrix D_3 (Magnus and Neudecker, 2007); empty spaces denote zeros. Notice that V_{σ^+} can be estimated consistently by $\hat{V}_{\sigma^+}$ by simply replacing $\Sigma_{\eta,i}$ with their consistent estimates $\hat{\Sigma}_{\eta,i} := \frac{1}{T_i}\sum_{t=1}^{T_i}(X_t - \hat{\Pi}_i W_t)(X_t - \hat{\Pi}_i W_t)'$, $i = 1,2,3$ in eq. (TS.10).

We have all the ingredients to define the CMD estimation problem:

$$\min_{\theta} \ \left(\hat{\sigma}_T^+ - g(\theta)\right)' \left(\hat{V}_{\sigma^+}\right)^{-1} \left(\hat{\sigma}_T^+ - g(\theta)\right) \tag{TS.11}$$

which provides a CMD estimate $\hat{\theta}_T$. When $r >$dim(θ), a test of overidentification restrictions implied by the SVAR specification is immediately available after estimation, because under the null hypothesis $\sigma_0^+ = g(\theta_0)$ it holds:

$$T\left(\hat{\sigma}_T^+ - g(\hat{\theta}_T)\right)' \left(\hat{V}_{\sigma^+}\right)^{-1} \left(\hat{\sigma}_T^+ - g(\hat{\theta}_T)\right) \xrightarrow{d} \chi^2(r - \dim(\theta)). \tag{TS.12}$$

As an example, we have reported in the two panels of Table TS.1 the CMD estimates of the parameters of the baseline SVAR specified in eq.s (18)-(19) of the paper (with asymptotic standard errors only), considering both the standard measure U_{Mt} of macroeconomic uncertainty that characterizes the model presented in the paper and its 'purged' counterpart, U^p_{Mt}, discussed in Section TS.9.2. Comparing the estimates in the upper panel of Table TS.1 with the Gaussian maximum likelihood estimates reported in the lower panel of Table 2 of the paper, we notice that results are numerically similar. Accordingly, the IRFs implied by the CMD estimation will be substantially similar to the ones reported in the paper. The analysis of the finite sample performances of the CMD and the Gaussian maximum likelihood methods in SVARs identified through heteroskedasticity, and their comparison in the presence of non-normal distributions are important topics which deserve a thorough investigation which goes beyond the scopes of the paper and of this supplementary material.

TS.5 Measures of uncertainty

In this section we briefly review how the two proxies of uncertainty used in the paper have been built.

Following Jurado *et al.* (2015) and Ludvigson *et al.* (2018), the time series that proxy the uncertainty indexes $U_{it}(f)$, $i = M, F$, where f denotes the uncertainty horizon ($f = 1$ one-month uncertainty and $f = 12$ one-year uncertainty in the paper), are estimated as the average of the time-varying volatility, as produced by stochastic volatility models, of the forecast error of each series in a large panel of macroeconomic ($U_{Mt}(f)$) and financial variables ($U_{Ft}(f)$), conditional on information available.

To keep presentation as simple as possible, consider the quantity:

$$U_{it}(f) := \lim_{N_i \to \infty} \frac{1}{N_i} \sum_{j=1}^{N_i} U^i_{jt}(f) \quad , \quad i = M, F \tag{TS.13}$$

where N_i is the number of time series in category $i = M, F$ for which individual indices of the type

$$U^i_{jt}(f) := \left(E\left[\left(e^i_{jt+f} \right)^2 \mid \mathcal{I}_t \right] \right)^{1/2} \quad , \quad i = M, F \tag{TS.14}$$

are computed. In eq. (TS.14), $e^i_{j,t+f} := v^i_{j,t+f} - E(v^i_{j,t+f} \mid \mathcal{I}_t)$; $v^i_{j,t+f}$ is the individual time series at time $t + f$ that belongs to the category $i = M, F$; $\mathcal{I}_t$ is the information set available at time t; $E(v^i_{j,t+f} \mid \mathcal{I}_t)$ is the conditional forecast of $v^i_{j,t+f}$ based on information $\mathcal{I}_t$; $e^i_{j,t+f}$ is the associated conditional forecast error. Eq. (TS.14) defines the uncertainty associated with the jth variable in the category $i = M, F$ as the square root of the conditional volatility generated by the (unpredictable) forecast error associated with that variable. Eq. (TS.13)

aggregates all the individual uncertainties in the category $i = M, F$. In particular, $N_M = 134$ 'monthly macroeconomic time series' covering the sample 1960M7-2015M4 are used for $U_{Mt}(f)$ and $N_F = 147$ 'monthly financial indicators' are used for $U_{Ft}(f)$; see Ludvigson *et al.* (2018) and references therein for details.

We now focus on how the individual measures of uncertainty that enter eq. (TS.14) are estimated in practice. Jurado *et al.* (2015) use factor augmented autoregressive models to estimate the conditional forecasts $E(v^i_{j,t+f} \mid \mathcal{I}_t)$ and the decomposition $e^i_{j,t+f} = \gamma^i_{j,t+f}\varepsilon^i_{j,t+f}$, where $\varepsilon^i_{j,t+f}$ is iidN(0,1) and $\gamma^i_{j,t+f}$ is driven by stochastic volatility models of the form

$$\log\left(\gamma^i_{j,t+f}\right)^2 = \alpha^i_j + \delta^i_j \log\left(\gamma^i_{j,t+f-1}\right)^2 + \tau^i_j \xi_{j.t+f} \ , \ \ \xi_{j,t+f} \sim iidN(0,1) \ , \quad i = M, F$$

where the parameters $(\alpha^i_j,\ \delta^i_j,\ \tau^i_j)$ are subject to standard regularity conditions. Given the estimates of $(\alpha^i_j,\ \delta^i_j,\ \tau^i_j)$ for the jth variable in the category $i = M, F$, one gets the dynamics of $U^i_{jt}(f)$ in eq. (TS.14) and from these the measures of uncertainty in eq. (TS.13) are obtained by aggregation.

The measures of macroeconomic and financial uncertainty U_{Mt} and U_{Ft} are plotted in Figure TS.1 for $f = 1$, and are the variables which enter our baseline VAR.

TS.6 Information sufficiency and omitted variables analysis

In this section we check the 'informational sufficiency' of the small-scale VAR estimated in the paper by using the testing procedure of Forni and Gambetti (2014). For each macroeconomic regime, we consider an augmented VAR system (a FAVAR) comprising X_t plus a vector of factors, included in the vector V_t, extracted from the McCracken and Ng (2015)'s large set of macroeconomic and financial variables. We then test whether V_t Granger-causes X_t. We also investigate the properties of the structural shocks estimated from our baseline structural specification with respect to some historical events. To do this, we divide this section into two parts. In the first part, we analyze the information sufficiency of the reduced form VARs for $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$, $X^*_t{:=}(CS_t, Y_t, U_{Ft})'$, and $X^{*\circ}_t{:=}(U_{Mt}, Y_t, CS_t)'$. In the second part, we describe some properties of the estimated structural shocks $\hat{e}_t$ obtained from the non-recursive SVAR for $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$, see the specification in eq.s (18)-(19) of the paper and Table 2 in the paper, and from the estimated SVARs for $X^*_t{:=}(CS_t, Y_t, U_{Ft})'$, and $X^{*\circ}_t{:=}(U_{Mt}, Y_t, CS_t)'$.

Informational sufficiency and omitted information

Given our small-scale system, a natural concern is whether the VAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$ satisfies the necessary and sufficient conditions which permit to correctly recover the structural shocks of interest. To do so we test the 'informational sufficiency' of the specified VAR.

Indeed, in light of the small dimension of X_t, not rejecting the informational sufficiency of $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$ allows us to rule out problems of nonfundamentalness, so that we can correctly estimate the effects of uncertainty shocks through IRFs. In practice, we estimate a FAVAR model for the vector $W_t{:=}(X_t', V_t')'$ where $V_t{:=}(v_1, v_{2t}, v_{3t}, v_{4t})'$ contains orthogonal factors extracted from a large set of macroeconomic and financial variables which jointly account for almost 90% of the entire variability, see McCracken and Ng (2015), and then run Granger-causality tests of V_t on X_t. This allows to check whether there exists a substantial discrepancy between the econometrician's information set and the agent's information set which, if present, would compromise the recovering of the shocks. We estimate the FAVAR for W_t on the Great Inflation, Great Moderation and Great Recession+Slow recovery periods, respectively and on each macroeconomic regime test whether V_t Granger-causes X_t. The upper panel of Table TS.2 reports bootstrapped p-values associated with the test for the null of absence of Granger-causality, equation-wise and at the system level.[4] It can be noticed that the null hypothesis is not generally rejected at the 5% level of significance.

The mid and lower panels of Table TS.2 repeat the same exercise for the two alternative VAR specifications based on $X_t^* := (CS_t, Y_t, U_{Ft})'$ and $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$, respectively, CS_t being a measure of financial frictions proxied with the spread between yields on Baa- and Aaa-rated long-term industrial corporate bonds (source FRED). As for the baseline specification, also in this case there are no rejections of the null hypotheses, at least at the system level, at the standard 5% (equation-wise we do not reject only at the 1% critical level).

The results reported in Table TS.2 refer to a FAVAR model with four lags for the dependent variables and eight lags for the factors. The results, however, are robust to more parsimonious specifications with respect to the number of factors included in the analysis.[5]

In addition to informational sufficiency, a further simple check can be directly based on the structural shocks $\hat{e}_{jt}$, $j = M, Y, F$ estimated from the baseline non-recursive SVAR for $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$ (see eq.s (18)-(19) and Table 2 of the paper). The shocks are obtained through $\hat{e}_t = \hat{B}_i^{-1}\hat{\eta}_t$, $i = 1, 2, 3$, where $\hat{B}_1 = \hat{B}, \hat{B}_2 = (\hat{B} + \hat{Q}_2)$ and $\hat{B}_3 = (\hat{B} + \hat{Q}_2 + \hat{Q}_3)$ are fixed at the estimates reported in Table 2 of the paper. It is natural to analyze whether $\hat{e}_t$ still contains predictable information with respect to the inflation rate (π_t) and the federal fund rate (i_t) which are variables excluded from the baseline three-equation VAR. To do so, we regress $\hat{e}_t := (\hat{e}_{Mt}, \hat{e}_{Yt}, \hat{e}_{Ft})'$ on two lags of π_t and i_t and then test whether the associated regression coefficients are jointly significant equation-wise and at the system level. The results of the tests are reported in the upper panel of Table TS.3 in the form of bootstrapped p-values. It can

[4] All bootstrap exercises are carried out by adapting Kilian's (1998) method. See footnote 15 in the paper for details.

[5] The complete set of results is available from the authors upon request.

be noticed that we do not reject the null hypothesis of irrelevant regressors at the 5% nominal significance level. We repeat the same exercise considering the structural shocks estimated from the non-recursive SVARs for $X_t^* := (CS_t, Y_t, U_{Ft})'$ and $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$, respectively, see Section TS.9.1 below. Results are summarized in the mid and lower panels of Table TS.3 and, again, suggest that the structural shocks produced by our small-scale non-recursive SVARs are not seriously affected by the omission of the inflation rate and the federal funds rate.

As a final check, we come back on the factors $V_t := (v_1, v_{2t}, v_{3t}, v_{4t})'$ considered before in the informational sufficiency analysis. We first perform a regression of $\hat{e}_t$ on the first two lags of the first factor (i.e. v_{1t-1} and v_{1t-2}), which account for more than 55% of the total variability. Then, we repeat the analysis by regressing $\hat{e}_t$ on the first two lags of all four factors which jointly account for almost 90% of the entire variability. The non rejection of the null hypothesis that V_t does not bear relevant information on $\hat{e}_t$ is substantially confirmed and is also valid for the two alternative SVARs containing the credit spread indicator. The complete set of results is reported in Table TS.4.

Estimated structural shocks and important historical events

Having verified the 'statistical' properties of the estimated structural shocks $\hat{e}_{jt}$, $j = M, Y, F$, plotted in Figure TS.2 for the baseline specification, we analyze qualitatively whether they reproduce important historical events characterizing the U.S. and global economy. The upper panel of Figure TS.2 plots the estimated macroeconomic uncertainty shock, $\hat{e}_{Mt}$, the mid panel plots the estimated real activity shock, $\hat{e}_{Yt}$, and the lower panel plots the estimated financial uncertainty shocks $\hat{e}_{Ft}$. The horizontal dotted black lines in the graphs correspond to 2 standard deviations above/below the unconditional mean of each series, while the shaded areas summarize NBER official recession dates.

From the graphs, it clearly emerges that the estimated shocks are systematically higher in coincidence of the NBER recession dates (the shaded areas in the graphs). An interesting exception refers to the so-called Black Monday (October 19, 1987), when stock markets crashed around the world. The crash, originating from Hong Kong, almost immediately spreads to Europe, hitting also the U.S. The Dow Jones Industrial Average (DJIA) fell more than 22%. Other interesting events are the International Monetary Fund (IMF) Crisis in the United Kingdom in 1976 which forced the government to borrow $3.9 billion from the IMF, generating instabilities in the U.S. financial market as well, and the reactions of U.S. policy authorities in late 1978 that, facing with a collapse in confidence in the dollar, announced the mobilization of more than $20 billion to defend the currency's value in foreign exchange markets.

The graphs in Figure TS.2 interestingly also show that the identified macro and financial uncertainty shocks are substantially different. Prominent macro uncertainty shocks are those

corresponding to the two oil price shocks of the 1970s and the fiscal battles in the 2010s, while the stock market crash in 1987 and the Asian crisis in the late 1990s are examples of two major financial uncertainty shocks which did not cause any increase in macroeconomic uncertainty. The GFC in 2007-2009 is an example of a shock that has increased both macro and financial uncertainty. It is also interesting to notice that the identified financial uncertainty shocks is consistent with the 'event constraints' in Ludvigson *et al.* (2018).

TS.7 Dynamic causal effects: regime-by-regime analysis

In this section we examine in detail, and separately for the three volatility regimes, the dynamic causal effects estimated in the paper. In particular we focus on the IRFs reported in Figures 3-5 of the paper.

The differences between these IRFs in the three macroeconomic regimes can be appreciated by looking at the numbers in Table TS.4 which extrapolate the significant peaks of the IRFs in the three regimes, along with the number of months necessary to achieve these peaks. Table TS.4 indicates that within each macroeconomic regime, both the magnitude and persistence of the effects of uncertainty shocks increase with the length of the uncertainty horizon f. Moreover, the negative effects of uncertainty shocks tend to be higher on the Great Recession+Slow Recovery sample, which is the period characterized by higher financial frictions after the GFC of 2007-2008. Finally, it is seen that real economic activity shocks have a significant effect on macroeconomic and financial uncertainty one month after the shock.

We now turn more specifically on the IRFs sketched in Figures 3-5 of the paper. Recall that shaded areas represent 90% bootstrap confidence bands.

IRFs: Great Inflation. Figure 3 in the paper plots the dynamic responses of the variables in $X_t{:=}(U_{Mt}, Y_t, U_{Ft})'$ to each structural shock during the Great Inflation period (1960M8-1984M3) considering both $f = 1$ (one-month uncertainty, blue line) and $f = 12$ (one-year uncertainty, red line). The graphs show that positive shocks to macroeconomic uncertainty lead to a decline in industrial production growth, which is statistically significant for a large number of months. IRFs are shorter-lived and less persistent in the case $f = 1$. For $f = 1$, the largest effect is on impact and is equal to -0.11 percentage points, while for $f = 12$ the negative significant peak is obtained 8 months after the shock and is equal to -0.072 percentage points.

Positive shocks to financial uncertainty have lagged (recall that there is no instantaneous impact according to the specification in eq.s (18)-(19) of the paper) and slightly less persistent negative effects on industrial production growth relative to the case of macroeconomic uncertainty shocks. The effect of financial uncertainty shocks lasts for roughly 12 months after the

shock, reaches its maximum significant negative effect 3 months after the shocks and is equal to -0.121 ($f = 1$) and -0.101 ($f = 12$) percentage points, respectively.

Notably, regardless of whether one considers macroeconomic or financial uncertainty shocks, industrial production growth does not overshoot its trend after recovering, suggesting that the decline in industrial production might be permanent.[6] Conversely, for both $f = 1$ and $f = 12$, macroeconomic and financial uncertainty do not respond significantly at any lag to shocks to real economic activity.

Overall, for the Great Inflation period, the IRFs in Figure 3 of the paper corroborate the hypothesis that both macroeconomic and financial uncertainty shocks trigger recessionary effects (the latter with lags only). They also support the view that uncertainty acts as a driver, rather than a consequence, of business cycle fluctuations.

IRFs: Great Moderation. Figure 4 of the paper plots the dynamic responses of the variables in X_t:=$(U_{Mt}, Y_t, U_{Ft})'$ to each structural shock on the Great Moderation period (1984M4-2007M12). The graphs show that positive shocks to macroeconomic uncertainty lead to a decline in industrial production growth with a slowdown which remains statistically significant for about 12 months after the shock for $f = 1$, and for more than 20 months after the shocks for $f = 12$. The peak effect of macroeconomic uncertainty shocks is on-impact and is equal to -0.076 percentage points for $f = 1$, while it occurs 4 months after the shock and is equal to -0.041 percentage points for $f = 12$.

Positive shocks to financial uncertainty lead to a delayed decline in industrial production growth. Indeed, for both $f = 1$ and $f = 12$, the highest significant negative effect are achieved 8 and 9 months after the shocks, respectively. Also in this case, both macroeconomic and financial uncertainty shocks seem to lead to a permanent drop in industrial growth because the dynamics of Y_t does not overshoot its trend significantly after recovering. Interestingly, positive shocks to financial uncertainty trigger a prolonged (more than 12 months) significant response of macroeconomic uncertainty, while financial uncertainty does not respond significantly to macroeconomic uncertainty shocks, confirming the indirect pass-through discussed in the paper.

The estimated IRFs further confirm that the two measures of uncertainty do not respond significantly to real economic activity shocks at any lag, other than on-impact.[7]

[6] This phenomenon is further scrutinized in the next section in which we report the associated cumulated long-run multipliers which quantify the final effect of uncertainty shocks on the level of industrial production once all dynamic adjustments have been taken into account.

[7] Bekaert *et al.* (2013) find a positive response of their measure of financial uncertainty to positive shocks to industrial production growth: they consider the sample 1990M1-2007M7, which is compatible with our Great Moderation period, but also their IRFs are not statistically significant. Popescu and Smets (2010) show that real

Overall, the IRFs in Figure 4 in the paper show that both macroeconomic and financial uncertainty curb real economic activity during the Great Moderation period. However, while macroeconomic uncertainty exerts a direct impact on industrial production growth, financial uncertainty fosters macroeconomic uncertainty on-impact and produces more delayed effects on real economic activity.

IRFs: Great Recession+Slow Recovery. Figure 5 of the paper plots the dynamic responses of the variables in X_t:=$(U_{Mt}, Y_t, U_{Ft})'$ to each structural shock on the Great Inflation+Slow Recovery period (2008M1-2015M4). In this case, positive shocks to macroeconomic uncertainty lead to a sharp decline in industrial production growth with a slowdown which remains statistically significant for about 10 months after the shock for both $f = 1$ and $f = 12$. The highest negative impact of macroeconomic uncertainty shocks is reached 5 months after the shock and is equal to -0.144 ($f = 1$) and -0.153 ($f = 12$) percentage points, respectively. Compared to the previous subsamples, the response of industrial production is more persistent and larger in magnitude.

Positive shocks to financial uncertainty produce comparatively more jagged responses of industrial production growth. The highest negative significant peak is obtained 3 months after the shock and is equal to -0.190 ($f = 1$) percentage points and -0.151 ($f = 12$) percentage points, respectively. Again, positive shocks to financial uncertainty trigger an instantaneous increase of macroeconomic uncertainty which lasts for more than 6 months, while financial uncertainty does not respond significantly to macroeconomic uncertainty shocks, confirming the mechanism detected also on the Great Moderation period.

Overall, the IRFs in Figure 5 in the paper show that the real effects of uncertainty shocks have become larger during the Great Recession+Slow Recovery period. While macroeconomic uncertainty has a direct impact on real economic activity, financial uncertainty fosters macroeconomic uncertainty on-impact and produces delayed effects on real economic activity.

As a final check on the relieability of the estimated dynamic causal effects, we have carried out an exercise in which we have forced the reduced-form autoregressive parameters of the estimated VAR to remain constant across volatility regimes (i.e. $\Pi_1 = \Pi_2 = \Pi_3 = \Pi$ in our notation), and have re-estimated the structural parameters on the so-obtained covariance matrices. This means that we have re-estimated the structural parameters based on the moment conditions $\Sigma_{\eta 1} = BB'$, $\Sigma_{\eta 2} = (B + Q_2)(B + Q_2)'$ and $\Sigma_{\eta 3} = (B + Q_2 + Q_3)(B + Q_2 + Q_3)'$, subject to the necessary and sufficient identification rank constraint. Thus, we have 'frozen-out' the

industrial production shocks in Germany have significant, yet non-monotonic, effects on perceived uncertainty. In their case, both uncertainty and risk premia initially fall in response to positive output shocks, but eventually increase.

effect induced by the changes in the autoregressive parameters across volatility regimes. This exercise has produced a new set of IRFs whose differences across regimes can solely be ascribed to the changes in the structural parameters captured by the matrices Q_2 and Q_3. The so-computed new IRFs are shown, in dashed lines, in Figure TS.3 and are contrasted with the just commented IRFs computed in the paper (solid lines) under the (statistically verified) condition $\Pi_1 \neq \Pi_2 \neq \Pi_3$. We have not reported confidence bands to improve readibility.

As expected, given the rejection of the test of the constant autoregressive parameters in Table 1 of the paper, the differences in the IRFs in Figure TS.3 are much more accentuated across regimes relative to those with constant autoregressive parameters. This is, of course, an expected outcome. However, it can be noticed that important differences across regimes do emerge also when the only source of variability are the changes in the structural parameters. This can be appreciated by looking, for instance, at the dynamic response of macroeconomic uncertainty to financial uncertainty shocks (graph in the position (1,3)). One crucial feature of our structural model is the pass-through of financial uncertainty: in our setup, financial uncertainty shocks affect the economy indirectly by triggering macroeconomic uncertainty. The graph shows that even forcing the autoregressive parameters to hold constant across regimes (against the empirical and statistical evidence) the impact is larger on the Great Recession+Slow Recovery.

However, the main message from Figure TS.3 is that ignoring variations in the autoregressive parameters may lead one to wrongly estimate the dynamic causal effects of uncertainty shocks. The graphs suggest that by falsely imposing the condition $\Pi_1 = \Pi_2 = \Pi_3 = \Pi$ may lead one to underestimate the effects of the uncertainty shocks.

TS.8 Long-run multipliers

As is known, IRFs provide short-run (transitory) dynamic causal effects. In addition, IRFs are explicitly aimed at identifying 'structural shocks' rather than measuring causal links between time series, see e.g. Dufour and Renault (1998), Bruneau and Jondeau (1999), Yamamoto and Kurozumi (2006) and Dufour *et al.* (2006) for a thorough discussion. For instance, it can be easily shown that zero on-impact responses may become non-zero after a certain number of periods. In this section we complement the analysis based on the IRFs reported in the paper with long-run total multipliers (or long-run cumulative impulse response matrix). These multipliers capture the (cumulative) limit impact of the structural shocks on the variables, if statistically significant, by taking into account all dynamic adjustments at work in the system.[8]

[8]Interestingly, while Granger-noncausality at all horizons implies a long-run multiplier equal to zero, the converse does generally not hold, hence the condition of zero long-run effect is less stringent than the one of absence of Granger-causality at all forecasting horizons, see e.g. Fanelli and Paruolo (2010).

In our setup, long-run multipliers are given by

$$CIRF_j = \sum_{h=0}^{\infty} IRF_j(h) := \begin{cases} R'(I_3 - A_1)^{-1} R\tilde{b}_j & t \leq T_{B_1} \\ R'(I_3 - A_2)^{-1} R(\tilde{b}_j + \tilde{q}_{2j}) & T_{B_1} < t \leq T_{B_2} \\ R'(I_3 - A_3)^{-1} R(\tilde{b}_j + \tilde{q}_{2j} + \tilde{q}_{3j}) & t > T_{B_2} \end{cases} \qquad j = M, Y, F$$

where we have used the same notation as in the paper. The structural specification of the SVAR is that in eq.s (18)-(19) of the paper. Estimates are summarized in Table TS.6 for $f = 1$ (one-month uncertainty) and $f = 12$ (one-year uncertainty), respectively. The upper panel of Table TS.6 refers to the baseline non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, the middle panel refers to the non-recursive SVAR based on $X_t^* := (CS_t, Y_t, U_{Ft})'$, while the lower panel refers to the non-recursive SVAR based on $X_t^* := (U_{Mt}, Y_t, CS_t)'$, where CS_t is proxied by considering the spread between yields on Baa- and Aaa-rated long-term industrial corporate bonds. In both cases the structural specification is summarized in eq.s (18)-(19) of the paper. Each estimated long-run multiplier is associated with a bootstrap-based standard error. Recall that since we consider the long-run cumulative impulse response matrix, the estimates obtained in correspondence of '$e_{Mt} \to Y_t$', '$e_{Ft} \to Y_t$' and '$e_{CSt} \to Y_t$' capture the long-run effect of uncertainty shocks and credit shocks on the industrial production level.

The multipliers in Table TS.6 confirm that regardless of macroeconomic regimes and the length of the uncertainty horizon f, macroeconomic uncertainty shocks cause a permanent decline in real economic activity. The long-run total multiplier associated with macroeconomic uncertainty shocks is negative and strongly significant. Instead, the long-run multipliers associated with the impact of financial uncertainty shocks are not statistically significant. Overall, Table TS.6 leads one to rule out the hypothesis that a rebound takes place after uncertainty shocks curb economic activity. Indeed, the long-run (permanent) effect of these shocks is either negative and significant (macroeconomic uncertainty shocks), or not significant (financial uncertainty shocks).

Focusing instead on the reverse causality issue, the long-run multipliers in Table TS.6 show that there are no significant long-run effects of real economic activity shocks on macroeconomic and financial uncertainty.

Finally, results in the lower panel of Table TS.6 confirm that while credit spreads shocks do not have permanent long-run effects on financial uncertainty, financial uncertainty shocks trigger a strong deterioration of credit conditions. This result is particularly evident on the Great Recession+Slow Recovery period.

TS.9 Robustness checks

In this section we report a set of robustness checks that complement the results discussed in the paper and above. In Section TS.9.1 we analyze the role of financial frictions. In Section TS.9.2 we replace the measure of macroeconomic uncertainty used in the paper, U_{Mt}, with a measure of 'real uncertainty', denoted U^p_{Mt} , which is obtained by purging U_{Mt} from variables associated with financial markets. Finally, in Section TS.9.3 we check whether results are robust to the use of a different proxy of real economic activity, i.e. employment (growth).

Overall, the robustness checks discussed throughout this section show that the main findings of the paper hold true after changing the baseline specification in different directions, i.e. (i) uncertainty, both macroeconomic and financial, is better characterized as an exogenous driver of the business cycle rather than an endogenous response to it, (ii) financial uncertainty shocks affect real economic activity mostly indirectly by triggering macroeconomic uncertainty on-impact; (iii) the effects of (macroeconomic) uncertainty shocks are time-varying and depend on the macroeconomic regime.

TS.9.1 Financial frictions

The baseline empirical analysis reported in the paper considers macroeconomic and financial uncertainty jointly but ignores financial frictions. As argued in Bachmann *et al.* (2013), the prolonged negative response of production to a surprise increase in uncertainty might indicate that channels other than 'wait and see' may be relatively more important in the United States. A number of recent papers have brought attention to such alternative channels. Arellano *et al.* (2012) build a quantitative general equilibrium model in which an increase in uncertainty, in the presence of imperfect financial markets leads firms to downsize projects to avoid default; this impact is exacerbated through an endogenous tightening of credit conditions and leads to a persistent reduction in output. Similarly, Christiano *et al.* (2014) develop a large-scale New Keynesian model with financial frictions in which risk shocks have persistent effects on output. The role of financial frictions as amplifiers of the effects of uncertainty shocks and cause of possible permanent decline in economic activity is also rationalized in Gilchrist *et al.* (2014), Alessandri and Mumatz (2018) and Alfaro *et al.* (2018). Caldara *et al.* (2016) and Caggiano *et al.* (2017b) analyze the interaction between financial conditions and economic uncertainty and find that uncertainty shocks have an especially negative impact in situations where they trigger a tightening of financial conditions. Furlanetto *et al.* (2017) disentangle the role of credit and uncertainty shocks and find that shocks originating in the credit markets have larger and longer-lived effects than uncertainty shocks. A common element in these contributions is

that uncertainty interacts with financial frictions to generate sizable and persistent reductions in production.

We estimate non-recursive SVARs for two different vectors of endogenous variables: $X_t^* := (CS_t, Y_t, U_{Ft})'$ and $X_t^{*\circ}:=(U_{Mt}, Y_t, CS_t)'$, respectively, where CS_t is the spread between yields on Baa- and Aaa-rated long-term industrial corporate bonds (source FRED). The reduced form analyses of these systems, not reported here but available upon request to the authors, confirm the existence of three broad volatility regimes in the data which can be associated with the Great Inflation, Great Moderation and Great Recession+Slow Recovery, respectively.

The identification schemes in these two cases are slightly different with respect to those specified in the paper and can be summarized as follow:

$X_t^* := (CS_t, Y_t, U_{Ft})'$**:**

$$\tilde{B} := \begin{pmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix} \quad , \quad \tilde{Q}_2 := \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix} \quad , \tilde{Q}_3 := \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ \times & 0 & \times \end{pmatrix}; \qquad \text{(TS.15)}$$

$X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$**:**

$$\tilde{B} := \begin{pmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix} \quad , \quad \tilde{Q}_2 := \begin{pmatrix} \times & 0 & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix} \quad , \tilde{Q}_3 := \begin{pmatrix} 0 & 0 & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}. \qquad \text{(TS.16)}$$

The former (eq. (TS.15)) has the same logic as the specification in eq.s (18)-(19) of the paper, the main difference being that macroeconomic uncertainty is replaced by the chosen proxy of financial frictions. The SVAR is overidentified (14 structural parameters are estimated using 18 moment conditions coming from the reduced-form residuals) and the LR test for the overidentification restrictions has a p-value equal of 0.15. Thus, the model is not rejected by the data. The latter (eq. (TS.16)) is based on the idea that macroeconomic uncertainty shocks can immediately affect real economic activity since the Great Inflation, with potential different effects across volatility regimes. Also in this case the SVAR is overidentified (with 3 degrees of freedom) and the LR test for the overidentification restrictions has a p-value equal to 0.07, hence we do not reject the null hypothesis at the 5% nominal level of significance. Both specifications in eq.s (TS.15)-(TS.16) incorporate the hypothesis that financial and macroeconomic uncertainty do not respond on-impact to real economic activity shocks, i.e. the exogeneity of the two sources of uncertainty. This assumption is in line with the findings discussed in the paper for the main specification.

The estimated matrices $\tilde{B}$, $\tilde{Q}_2$ and $\tilde{Q}_3$ are not reported to save space but can be obtained from the authors upon request. The implied IRFs are plotted in Figures TS.4-TS.6 ($X_t^* := (CS_t, Y_t, U_{Ft})'$) and Figures TS.7-TS.9 ($X_t^{*\circ}$:=$(U_{Mt}, Y_t, CS_t)'$), respectively. The baseline findings discussed in the paper on the contractionary effect of uncertainty shocks are confirmed also controlling for first-moment financial shocks: the effect is more pronounced during the Great Recession+Slow Recovery period.

TS.9.2 Real uncertainty

For $f := 1$, the proxies of uncertainty $U_{Ft}(1)$ and $U_{Mt}(1)$ plotted in Figure TS.1 display comovement but also have independent variations, as the correlation between them is 'only' 0.58. Part of this correlation might be simply due, however, to the fact that $U_{Mt}(f)$ includes by construction also the uncertainty from a category of financial variables which potentially overlap with the variables used to build the index $U_{Ft}(f)$. For this reason, we proceed by employing a measure of macroeconomic uncertainty which is extracted from a smaller dataset including only real activity indicators ('real uncertainty', U_{Mt}^p). We re-estimate our non-recursive SVAR by simply replacing $U_{Mt}(f)$ with $U_{M,t}^p(f)$, adopting the same structural specification discussed in the paper (see eq.s (18)-(19)). Results are summarized in Table TS.7, which reproduces exactly Table 2 in the paper

The interesting fact that emerges from the results in Table TS.7 is that the empirical evidence on reverse causality/exogeneity discussed in Section 3.4 of the paper is strengthened. Now the LR test for the two overidentification restrictions featured by this model is equal to 1.36 and has a p-value of 0.44, hence the model is supported by the data at the 5% level of significance.

Focusing more specifically on the parameter b_{MY}, which captures the on-impact response of macroeconomic uncertainty to real economic activity shocks in the three volatility regimes, it is seen that this is not statistically significant. The hypothesis of 'exogenous' macroeconomic uncertainty is largely supported by the data as the LR test for $b_{MY} = 0$ is equal to 0.0012 and has a p-value of 0.97. The estimated parameter $q_{2,FM}$ proves to be not strongly significant, confirming our intuition that since the eighties the pass-through between the two sources of uncertainty is one-way: from financial uncertainty shocks to macroeconomic uncertainty.

The estimated structural model in the lower panel of Table 2 incorporates the two restrictions in eq. (19) of the paper. In this case, the LR test for the four overidentification restrictions featured by the SVAR is equal to 2.36 with associated p-value of 0.50. A LR test for the structural model in the lower panel against the one in the upper panel of Table 2 is equal to 1.01 and has p-value equal to 0.60. Overall, the empirical evidence based on $X_t := (U_{Mt}^p, Y_t, U_{Ft})'$ supports the specification in eq.s (18)-(19).

The dynamic causal effects estimated in this case are qualitatively and quantitatively comparable to those reported in the paper with the 'extended' indicator for macroeconomic uncertainty U_{Mt}. Figures TS.10-TS.12 plot the implied IRFs for both $f = 1$ (yellow line) and $f = 12$ (green line) and $Y_t = \Delta ip_t$ (industrial production growth).

TS.9.3 Real economic activity: employment growth

In this section we reproduce the analysis presented in the paper for the baseline case $X_t := (U_{Mt}, Y_t, U_{Ft})'$ by measuring real economic activity by $Y_t = \Delta emp_t$, where emp_t is the log of the employment level (source: FRED database).

The reduced form analysis confirms the existence of three broad volatility regimes in the data which can be associated with the Great Inflation, Great Moderation and Great Recession+Slow Recovery periods, respectively. The structural specification is the same as the one in Section 3.3 of the paper (see eq.s (18)-(19)). Also in this case, the overidentification restrictions are not rejected by the data when using the one-month uncertainty horizon $f = 1$, and is only marginally rejected for the one-year uncertainty horizon $f = 12$. The complete set of results can be obtained from the authors upon request.

Figures TS.13-TS.15 plot the dynamic responses of the variables in $X_t := (U_{Mt}, Y_t, U_{Ft})'$ to each structural shock in the Great Inflation, Great Moderation and Great Recession + Slow Recovery periods, respectively. Albeit there are quantitative differences relative to the baseline case, overall, the analysis confirms qualitatively the results discussed in the paper using industrial production growth for Y_t.

TS.10 Related literature

The two main research questions discussed in the paper and the empirical findings connect our analysis to different strands of the literature. Since our approach allows for time variation, our paper relates to the contributions that have shown that uncertainty shocks have had effects that are not constant over time, e.g. Beetsma and Giuliodori (2012), Choi (2013), Bontempi *et al.* (2018), Mumtaz and Theodoridis (2018) and Caggiano *et al.* (2017b). The main message from this strand of the literature is that uncertainty shocks are more powerful if the economy is in extreme conditions, such as an economic recession and high financial strain. In line with them, we also find that uncertainty shocks have possibly time-varying effects which can be associated with macroeconomic (volatility) regimes. All these contributions, however, either identify and estimate recursive SVARs separately on different sub-samples, or estimate time-varying recursive SVARs which cannot account for the reverse causality issue.

Our findings on the larger effects of uncertainty shocks in the aftermath of the GFC are in line with Basu and Bundick (2017) and Caggiano *et al.* (2017a), who highlight the role played by the stance of monetary policy in magnifying the effects of uncertainty shocks. Differently from their papers, we do not investigate the causes of why real activity reacts more in the GFC, but we confirm their findings with a more general identification approach that, crucially in a period of high economic and financial turmoil, does not require a recursive structure. We can also relate to the findings by Plante *et al.* (2018). These authors contend that, since during the zero lower bound the central bank was unable to offset negative shocks by applying 'conventional' methods, macroeconomic variables were more responsive to the negative shocks hitting the economy and, accordingly, the uncertainty surrounding future growth increased. According to this 'endogenous uncertainty' mechanism, the response of macroeconomic uncertainty to real economic activity shocks in the Great Moderation period should differ substantially from the response in the Great Recession+Slow Recovery regime, which roughly coincides with the zero lower bound period. Our empirical results support only partially this mechanism: in line with Plante *et al.* (2018), our estimated IRFs show that in the Great Recession+Slow Recovery regime real economic activity shocks trigger a significant response of uncertainty. Unlike them, however, we find that this happens only with a lag, and not on-impact, a result that does not lend support to the 'endogenous uncertainty' mechanism. More generally, in our setup macroeconomic uncertainty does not respond contemporaneously to real economic shocks in all volatility regimes.

On the methodological side, our paper is related to works that have identified uncertainty shocks using non-recursive schemes. Recent examples are methods based on the combination of external instruments with other restrictions (Ludvigson *et al.* 2017; Piffer and Podstawski, 2017; Carriero *et al.*, 2015), methods based on the penalty function approach (Caldara *et al.* 2016) and methods based on sign restrictions (e.g. Furlanetto *et al.* 2017). None of these contributions, however, has examined the joint issue of reverse causality and time dependence.

Finally, the links to Carriero *et al.* (2018) and Ludvigson *et al.* (2018) have been examined exhaustively in the paper.

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TABLE TS.1. Estimated structural parameters, CMD method.

GI: 1960M8-1984M3	GM: 1984M4-2007M12	GR+SR: 2008M1-2015M4
SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$		
$\hat{B} := \begin{pmatrix} 0.0112 \\ (0.0005) & 0 & 0 \\ -0.1205 & 0.7569 & 0 \\ (0.0450) & (0.0316) & \\ 0 & 0 & 0.0257 \\ & & (0.0011) \end{pmatrix}$	$\hat{B}+\hat{Q}_2 := \begin{pmatrix} 0.0080 & 0 & 0.0030 \\ (0.0003) & & (0.0005) \\ -0.0755 & 0.4640 & 0 \\ (0.0279) & (0.0194) & \\ 0 & 0 & 0.0280 \\ & & (0.0012) \end{pmatrix}$	$\hat{B}+\hat{Q}_2+\hat{Q}_3 := \begin{pmatrix} 0.0080 & 0 & 0.0047 \\ (0.0003) & & (0.0009) \\ -0.1150 & 0.6015 & -0.0553 \\ (0.0687) & (0.0452) & (0.0655) \\ 0 & 0 & 0.0215 \\ & & (0.0016) \end{pmatrix}$
SVAR for $X_t := \left(U^p_{Mt}, Y_t, U_{Ft}\right)'$		
$\hat{B} := \begin{pmatrix} 0.0164 \\ (0.0007) & 0 & 0 \\ -0.1434 & 0.7550 & 0 \\ (0.0454) & (0.0317) & \\ 0 & 0 & 0.0259 \\ & & (0.0011) \end{pmatrix}$	$\hat{B}+\hat{Q}_2 := \begin{pmatrix} 0.0091 & 0 & 0.0024 \\ (0.0003) & & (0.0005) \\ -0.0948 & 0.4687 & 0 \\ (0.0276) & (0.0196) & \\ 0 & 0 & 0.0280 \\ & & (0.0012) \end{pmatrix}$	$\hat{B}+\hat{Q}_2+\hat{Q}_3 := \begin{pmatrix} 0.0091 & 0 & 0.0041 \\ (0.0003) & & (0.0010) \\ -0.2047 & 0.5956 & -0.0948 \\ (0.0658) & (0.0446) & (0.0675) \\ 0 & 0 & 0.0222 \\ & & (0.0017) \end{pmatrix}$

Notes: CMD estimates of the parameters of the baseline SVAR specification in eq.s (18)-(19) of the paper, with asymptotic standard errors. The estimates in the upper panel are based on the measure of macroeconomic uncertainty used in the paper, while the estimates in the lower panel refer to the 'purged' measure of macroeconomic uncertainty discussed in Section TS.9.2.

TABLE TS.2. Information sufficiency: Bootstrap p-values of the Granger causality tests for the first four factors in the FAVAR model.

	GI: 1960M8-1984M3	GM: 1984M4-2007M12	GR+SR: 2008M1-2015M4
	VAR for $X_t^F := (U_{Mt}, Y_t, U_{Ft}, v_1, v_2, v_3, v_4)'$		
U_{Mt}	0.20	0.06	0.99
Y_t	0.05	0.02	0.88
U_{Ft}	0.20	0.61	0.88
System	0.08	0.32	0.87
	VAR for $X_t^{*F} := (CS_t, Y_t, U_{Ft}, v_1, v_2, v_3, v_4)'$		
CS_t	0.01	0.17	0.13
Y_t	0.01	0.02	0.05
U_{Ft}	0.12	0.49	0.20
System	0.07	0.26	0.08
	VAR for $X_t^{*\circ\circ F} := (U_{Mt}, Y_t, CS_t, v_1, v_2, v_3, v_4)'$		
U_{Mt}	0.03	0.17	0.75
Y_t	0.01	0.02	0.70
CS_t	0.01	0.30	0.32
System	0.02	0.12	0.43

Notes: Upper panel: the FAVAR model contains the variables $X_t^F := (U_{Mt}, Y_t, U_{Ft}, v_1, v_2, v_3, v_4)'$. Mid panel: the FAVAR contains the variables $X_t^{*F} := (CS_t, Y_t, U_{Ft}, v_1, v_2, v_3, v_4)'$. Lower panel: the FAVAR model contains the variables $X_t^{*\circ\circ F} := (U_{Mt}, Y_t, CS_t, v_1, v_2, v_3, v_4)'$. In all specifications, v_{it}, $i = 1, \ldots, 4$ are the first four factors described in Section TS.6, $Y_t = \Delta ip_t$ (industrial production growth) and CS_t is the proxy of financial frictions. 'GI'=Great Inflation, 'GM'=Great Moderation and 'GR+SR'=Great Recession + Slow Recovery.

TABLE TS.3. Structural shocks and monetary policy stance: Bootstrap p-values of the Granger causality tests for interest rate (i_t) and inflation rate (π_t) on the estimated structural shocks.

	two lags for i_t and π_t	eight lags i_t and π_t
SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$		
$\hat{e}_{Mt}$	0.05	0.04
$\hat{e}_{Yt}$	0.18	0.89
$\hat{e}_{Ft}$	0.73	0.88
System	0.28	0.39
SVAR for $X_t^* := (CS_t, Y_t, U_{Ft})'$		
$\hat{e}_{CSt}$	0.62	0.12
$\hat{e}_{Yt}$	0.24	0.61
$\hat{e}_{Ft}$	0.39	0.47
System	0.44	0.22
SVAR for $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$		
$\hat{e}_{Mt}$	0.07	0.10
$\hat{e}_{Yt}$	0.43	0.81
$\hat{e}_{CSt}$	0.75	0.20
System	0.35	0.25

Notes: Upper panel: the structural shocks are estimated through a SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$. Mid panel: the structural shocks are estimated through a SVAR for $X_t^* := (CS_t, Y_t, U_{Ft})'$. Lower panel: the structural shocks are estimated through a SVAR form $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$. $Y_t = \Delta ip_t$ (industrial production growth) and CS_t is the proxy of financial frictions. Overall sample: 1960M8-2015M4.

TABLE TS.4. Structural shocks and factors: Bootstrap p-values of the Granger causality test for the factors $(v_1, v_2, v_3, v_4)'$ on the estimated structural shocks.

Shock	two lags for v_{1t}	two lags for $(v_1, v_2, v_3, v_4)'$	eight lags for $(v_1, v_2, v_3, v_4)'$
SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$			
$\hat{e}_{Mt}$	0.15	0.06	0.66
$\hat{e}_{Yt}$	0.25	0.14	0.50
$\hat{e}_{Ft}$	0.34	0.85	0.86
System	0.29	0.45	0.51
SVAR for $X_t^* := (CS_t, Y_t, U_{Ft})'$			
$\hat{e}_{CSt}$	0.85	0.55	0.43
$\hat{e}_{Yt}$	0.03	0.17	0.43
$\hat{e}_{Ft}$	0.19	0.74	0.69
System	0.08	0.22	0.45
SVAR for $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$			
$\hat{e}_{Mt}$	0.12	0.17	0.18
$\hat{e}_{Yt}$	0.25	0.18	0.32
$\hat{e}_{CSt}$	0.52	0.56	0.76
System	0.18	0.25	0.50

Notes: Upper panel: the structural shocks are estimated through a SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$. Mid panel: the structural shocks are estimated through a SVAR for $X_t^* := (CS_t, Y_t, U_{Ft})'$. Lower panel: the structural shocks are estimated through a SVAR form $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$. In all specifications, v_{it}, $i = 1, \ldots, 4$ are the first four factors described in Section TS.6. $Y_t = \Delta ip_t$ (industrial production growth) and CS_t is the proxy of financial frictions. Overall sample: 1960M8-2015M4.

TABLE TS.5. IRFs estimated from the non-recursive SVAR, negative peaks (percentage points).

	GI: 1960M8-1984M3		GM: 1984M4-2007M12		GR+SR: 2008M1-2015M4	
	$f=1$	$f=12$	$f=1$	$f=12$	$f=1$	$f=12$
	SVAR for $X_t := (U_{Mt}, Y_t, U_{Mt})'$					
$e_{Mt} \to Y_t$	$-0.120(0)$	$-0.060(10)$	$-0.076(0)$	$-0.071(2)$	$-0.152(5)$	$-0.163(5)$
$e_{Ft} \to Y_t$	$-0.127(3)$	$-0.107(3)$	$-0.020(8)$	$-0.023(10)$	$-0.172(3)$	$-0.129(3)$
$e_{Yt} \to U_{Mt}$	–	–	–	–	$-0.263(1)$	$-0.014(1)$
$e_{Yt} \to U_{Ft}$	–	–	–	–	$-0.749(1)$	–

Notes: Highest negative (significant) responses of $Y_t = \Delta ip_t$ (industrial production growth) to one standard deviation change in macroeconomic (e_{Mt}) and financial (e_{Ft}) uncertainty shocks and highest negative (significant) responses of macroeconomic (U_{Mt}) and financial (U_{Ft}) uncertainties to one standard deviation change in real economic activity shocks (e_{Yt}), at the one-month ($f=1$) and one-year ($f=12$) uncertainty horizons, obtained from the non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$ specified in eq.s (18)-(19) of the paper. In parenthesis the number of months after the shock at which the highest negative peak is reached.

TABLE TS.6. Long-run total multipliers.

	GI: 1960M8-1984M3		GM: 1984M4-2007M12		GR+SR: 2008M1-2015M4	
	$f=1$	$f=12$	$f=1$	$f=12$	$f=1$	$f=12$
SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$						
$e_{Mt} \to Y_t$	−1.7019 (0.7348)	−2.2196 (0.7967)	−0.9397 (0.3241)	−1.3127 (0.4036)	−1.3018 (0.558)	−1.3486 (0.5708)
$e_{Mt} \to U_{Ft}$	0.2245 (0.3248)	0.1443 (0.1366)	0.2121 (0.2997)	0.0973 (0.175)	0.7513 (0.4044)	0.2146 (0.1711)
$e_{Yt} \to U_{Mt}$	0.0461 (0.127)	0.169 (0.157)	−0.0593 (0.0423)	0.0112 (0.034)	−0.0919 (0.1579)	−0.0427 (0.1013)
$e_{Yt} \to U_{Ft}$	0.1197 (0.1335)	0.0583 (0.0489)	−0.0475 (0.1496)	0.0081 (0.0634)	−0.0747 (0.2769)	−0.0011 (0.0992)
$e_{Ft} \to U_{Mt}$	0.3168 (0.2428)	−0.0435 (0.2598)	0.1209 (0.0911)	0.07 (0.0782)	0.0851 (0.158)	−0.0197 (0.1254)
$e_{Ft} \to Y_t$	−1.7768 (0.6393)	−0.9368 (0.6885)	−0.3133 (0.3502)	−0.2419 (0.4122)	−0.0525 (0.4943)	0.1828 (0.5672)
SVAR for $X_t^* := (CS_t, Y_t, U_{Ft})'$						
$e_{CS_t} \to Y_t$	0.2413 (0.4589)	0.3448 (0.4717)	−0.1688 (0.2913)	−0.1487 (0.3127)	−0.5377 (0.4641)	−0.4029 (0.6328)
$e_{CS_t} \to U_{Ft}$	−0.4068 (0.2485)	−0.1225 (0.0852)	−0.1285 (0.3389)	−0.0631 (0.1575)	−0.2388 (0.3179)	−0.152 (0.2585)
$e_{Yt} \to CS_t$	−0.3419 (0.4268)	−0.2923 (0.4424)	−0.201 (0.268)	−0.2194 (0.2635)	−0.7212 (0.4572)	−0.4945 (0.8146)
$e_{Yt} \to U_{Ft}$	0.0773 (0.1049)	0.0222 (0.0363)	0.0154 (0.1619)	0.004 (0.0732)	−0.0531 (0.2962)	0.0391 (0.1789)
$e_{Ft} \to CS_t$	2.9376 (0.8616)	3.1684 (0.9147)	1.0665 (0.4584)	1.0964 (0.5196)	1.5886 (0.5726)	1.9799 (1.3223)
$e_{Ft} \to Y_t$	−1.2581 (0.4496)	−1.3002 (0.5684)	−0.3622 (0.3079)	−0.382 (0.3394)	−0.6925 (0.6284)	−0.7833 (0.8836)
SVAR for $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$						
$e_{Mt} \to Y_t$	−1.0867 (0.3997)	−1.1889 (0.4896)	−0.8748 (0.3328)	−1.1436 (0.4141)	−1.7607 (1.0005)	−2.0008 (2.1123)
$e_{Mt} \to CS_t$	2.7605 (0.575)	3.0773 (0.7324)	1.1034 (0.4959)	1.698 (0.6011)	2.1507 (1.2396)	2.3079 (2.341)
$e_{Yt} \to U_{Mt}$	0.0425 (0.0768)	0.1667 (0.0925)	−0.04 (0.0534)	0.0246 (0.044)	−0.0276 (0.3262)	0.0402 (0.2895)
$e_{Yt} \to CS_t$	0.0349 (0.3629)	0.6716 (0.4498)	−0.2027 (0.3469)	0.2476 (0.3949)	−0.4283 (1.0714)	−0.104 (1.546)
$e_{CS_t} \to U_{Mt}$	−0.3033 (0.1141)	−0.3115 (0.1259)	0.0764 (0.0916)	0.0063 (0.0717)	−0.0289 (0.3176)	−0.2374 (0.3108)
$e_{CS_t} \to Y_t$	0.6763 (0.316)	0.8896 (0.3524)	−0.225 (0.3409)	0.1253 (0.3919)	−0.1502 (0.8412)	0.8391 (1.5468)

Notes: Estimated long-run total multipliers produced by the non-recursive SVAR (see eq.s (18)-(19) of the paper), for $X_t := (U_{Mt}, Y_t, U_{Ft})'$ (top panel), $X_t^* := (CS_t, Y_t, U_{Ft})'$ (middle panel), and $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$ (bottom panel), at one-month ($f=1$) and one-year ($f=12$) uncertainty horizons, $Y_t = \Delta ip_t$ (industrial production growth) and CS_t is the proxy of financial frictions. Bootstrap standard errors in parenthesis. 'GI'=Great Inflation, 'GM'=Great Moderation and 'GR+SR'=Great Recession + Slow Recovery.

TABLE TS.7. Estimated structural parameters.

GI: 1960M8-1984M3	GM: 1984M4-2007M12	GR+SR: 2008M1-2015M4

SVAR for $X_t := \left(U_{Mt}^p, Y_t, U_{Ft}\right)'$

$$\hat{B} := \begin{pmatrix} \underset{(0.0007)}{\underset{(0.0010)^*}{0.0164}} & \underset{(0.0035)}{\underset{(0.0089)^*}{0.0001}} & 0 \\ \underset{(0.1659)}{\underset{(0.1631)^*}{-0.1365}} & \underset{(0.0434)}{\underset{(0.0492)^*}{0.7562}} & 0 \\ 0 & 0 & \underset{(0.0011)}{\underset{(0.0018)^*}{0.0260}} \end{pmatrix}, \quad \hat{B}+\hat{Q}_2 := \begin{pmatrix} \underset{(0.0013)}{\underset{(0.0016)^*}{0.0083}} & \underset{(0.0035)}{\underset{(0.0089)^*}{0.0001}} & \underset{(0.0021)}{\underset{(0.0027)^*}{0.0044}} \\ \underset{(0.1790)}{\underset{(0.1753)^*}{-0.1024}} & \underset{(0.0421)}{\underset{(0.0627)^*}{0.4671}} & 0 \\ \underset{(0.0067)}{\underset{(0.0032)^*}{-0.0053}} & 0 & \underset{(0.0017)}{\underset{(0.0044)^*}{0.0276}} \end{pmatrix}, \quad \hat{B}+\hat{Q}_2+\hat{Q}_3 := \begin{pmatrix} \underset{(0.0013)}{\underset{(0.0016)^*}{0.0083}} & \underset{(0.0035)}{\underset{(0.0089)^*}{0.0001}} & \underset{(0.0027)}{\underset{(0.0033)^*}{0.0055}} \\ \underset{(0.2412)}{\underset{(0.2064)^*}{-0.1905}} & \underset{(0.0905)}{\underset{(0.1234)^*}{0.5905}} & \underset{(0.0812)}{\underset{(0.1409)^*}{-0.1439}} \\ \underset{(0.0067)}{\underset{(0.0032)^*}{-0.0053}} & 0 & \underset{(0.0023)}{\underset{(0.0038)^*}{0.0216}} \end{pmatrix}$$

Model with 'endogenous' macroeconomic uncertainty: 2 overidentification restrictions: $LR_T = 1.36302_{\chi^2(df=2)}[0.4422]$

SVAR for $X_t := \left(U_{Mt}^p, Y_t, U_{Ft}\right)'$

$$\hat{B} := \begin{pmatrix} \underset{(0.0007)}{\underset{(0.0010)^*}{0.0164}} & 0 & 0 \\ \underset{(0.0455)}{\underset{(0.0674)^*}{-0.1430}} & \underset{(0.0319)}{\underset{(0.0422)^*}{0.7550}} & 0 \\ 0 & 0 & \underset{(0.0011)}{\underset{(0.0018)^*}{0.0260}} \end{pmatrix}, \quad \hat{B}+\hat{Q}_2 := \begin{pmatrix} \underset{(0.0003)}{\underset{(0.0004)^*}{0.0091}} & 0 & \underset{(0.0005)}{\underset{(0.0007)^*}{0.0028}} \\ \underset{(0.0283)}{\underset{(0.0417)^*}{-0.0943}} & \underset{(0.0196)}{\underset{(0.0260)^*}{0.4690}} & 0 \\ 0 & 0 & \underset{(0.0012)}{\underset{(0.0037)^*}{0.0281}} \end{pmatrix}, \quad \hat{B}+\hat{Q}_2+\hat{Q}_3 := \begin{pmatrix} \underset{(0.0003)}{\underset{(0.0004)^*}{0.0091}} & 0 & \underset{(0.0010)}{\underset{(0.0014)^*}{0.0034}} \\ \underset{(0.0617)}{\underset{(0.1042)^*}{-0.2034}} & \underset{(0.0448)}{\underset{(0.0704)^*}{0.5937}} & \underset{(0.0673)}{\underset{(0.1208)^*}{-0.0944}} \\ 0 & 0 & \underset{(0.0017)}{\underset{(0.0025)^*}{0.0223}} \end{pmatrix}$$

Model with 'exogenous' macroeconomic uncertainty: 4 overidentification restrictions in eq. (19) of the paper: $LR_T = 2.3590_{\chi^2(df=4)}[0.5013]$

Notes: Estimated structural parameters based on the non-recursive SVAR for $X_t := \left(U_{Mt}^p, Y_t, U_{Ft}\right)'$, $Y_t = \Delta ip_t$ (industrial production growth), specified in eq. (18) of the paper. The estimates in the lower panel are based on the additional restrictions in eq. (19). Hessian-based standard errors in parenthesis; bootstrap standard errors are denoted with asterisks '*', see footnote 16 for details; 'GI'=Great Inflation; 'GM'=Great Moderation; 'GR+SR'=Great Recession + Slow Recovery.

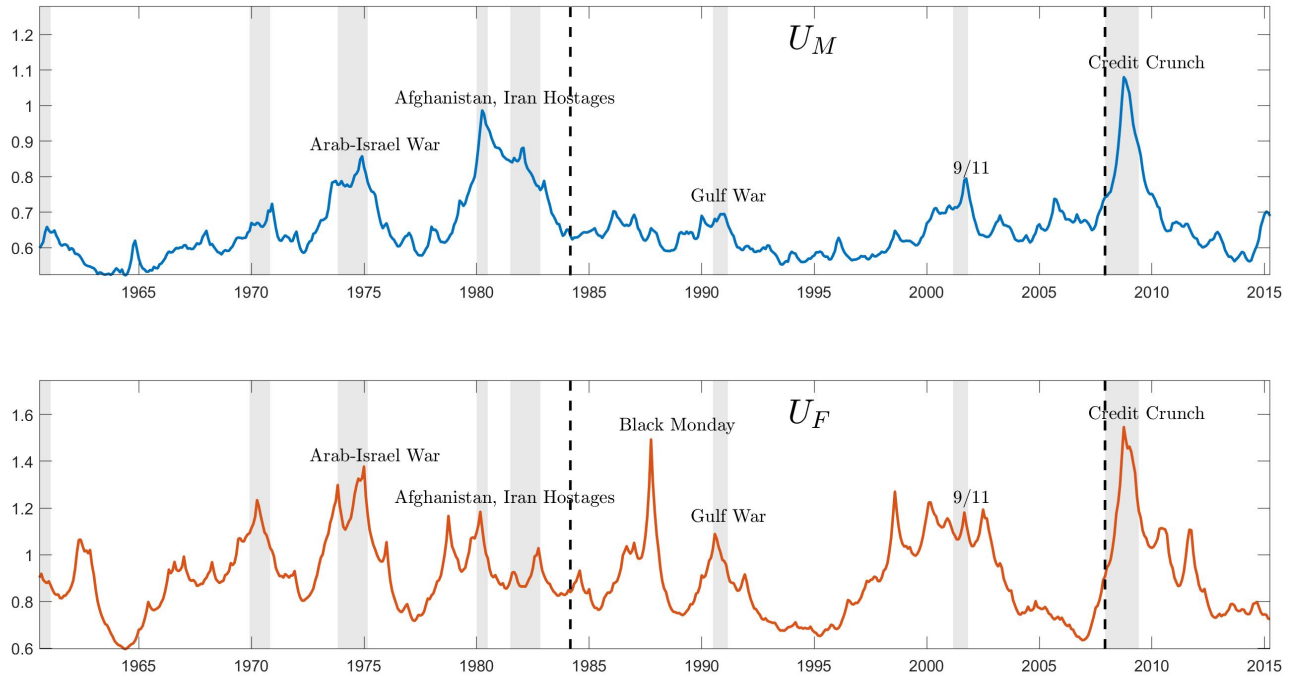

FIGURE TS.1: Measures of one-month ($f = 1$) macroeconomic uncertainty U_{Mt} (top-panel) and financial uncertainty U_{Ft} (bottom-panel). U_{Mt} is taken from Jurado *et al.* (2015); U_{Ft} is taken from Ludvigson *et al.* (2017). Dashed black lines denote the two break dates T_{B_1}=1984M3 and T_{B_2}=2007M12 which separate the three volatility regimes used to identify the shocks. The shaded areas correspond to the NBER recession dates. Overall sample: 1960M8-2015M4.

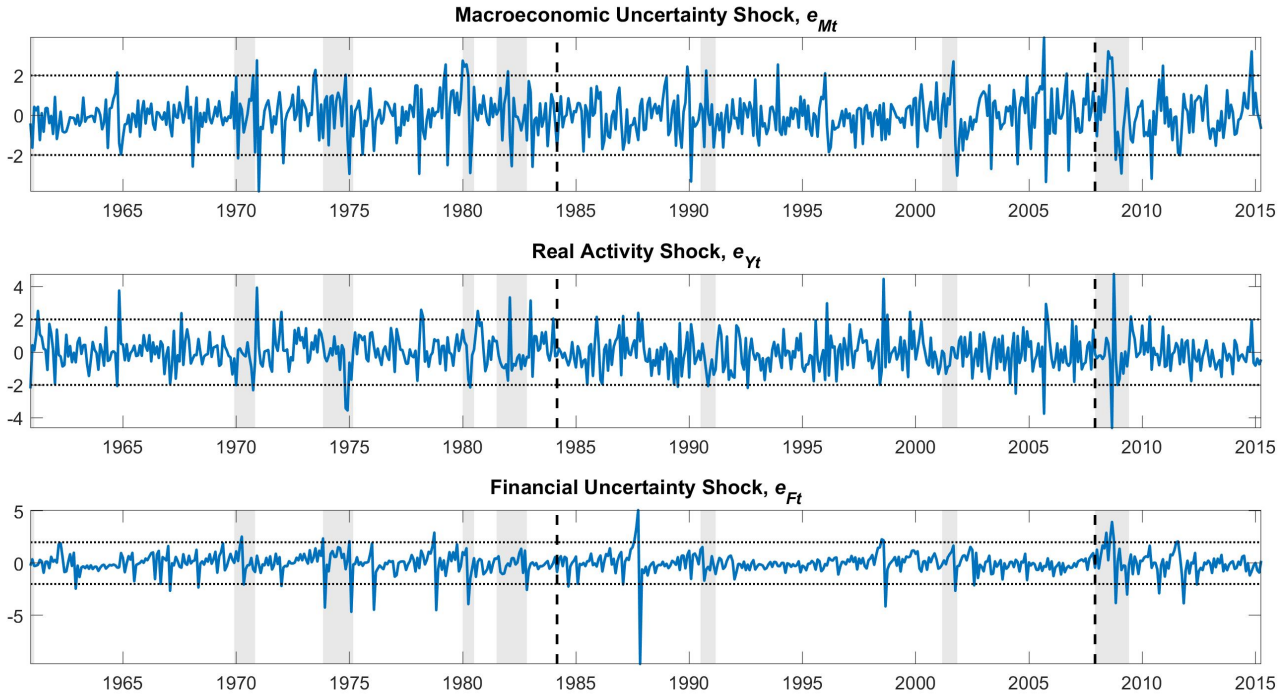

FIGURE TS.2: Estimated structural shocks $\hat{e}_{jt}$, $j = M, Y, F$ where $\hat{e}_t = \hat{B}_i^{-1}\eta_t$, $i = 1, 2, 3$, $\hat{B}_1 = \hat{B}, \hat{B}_2 = (\hat{B} + \hat{Q}_2)$ and $\hat{B}_3 = (\hat{B} + \hat{Q}_2 + \hat{Q}_3)$ from the non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta ip_t$ (industrial production growth), specified in eq.s (18)-(19) of the paper. The first panel plots the estimated macroeconomic uncertainty shock $\hat{e}_{Mt}$, the second panel the estimated real activity shock $\hat{e}_{Yt}$ and the last panel the estimated financial uncertainty shock $\hat{e}_{Ft}$. Vertical dashed black lines are the two break dates T_{B_1}=1984M3 and T_{B_2}=2007M12. Horizontal dotted black lines correspond to 2 standard deviations above/below the unconditional mean of each series. The shaded areas correspond to the NBER recession dates. Overall sample: 1960M8-2015M4.

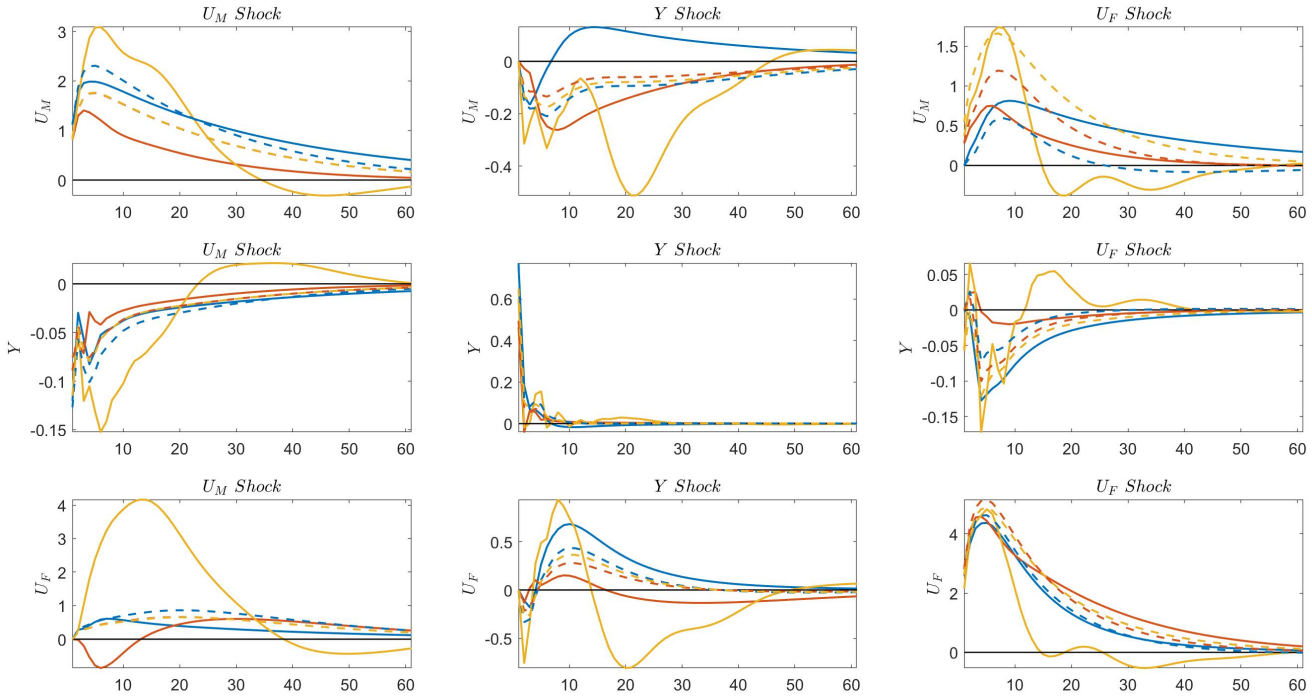

FIGURE TS.3: IRFs obtained from the beseline non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta ip_t$ (industrial production growth) specified in eq.s (18)-(19) of the paper. U_{Mt} and U_{Ft} refer to the one-month ($f = 1$) uncertainty horizon. The blue line refers to the first volatility regime (Great Inflation, 1960M8-1984M3); the red line refers to the second volatility regime (Great Moderation, 1984M4-2007M12); the yellow line refers to the third volatility regime (Great Recession + Slow Recovery, 2008M1-2015M4). Dashed lines plot the IRFs obtained with constant autoregressive parameters and the structural specification in eq.s (18)-(19) of the paper. Responses are measured with respect to one standard deviation changes in structural shocks.

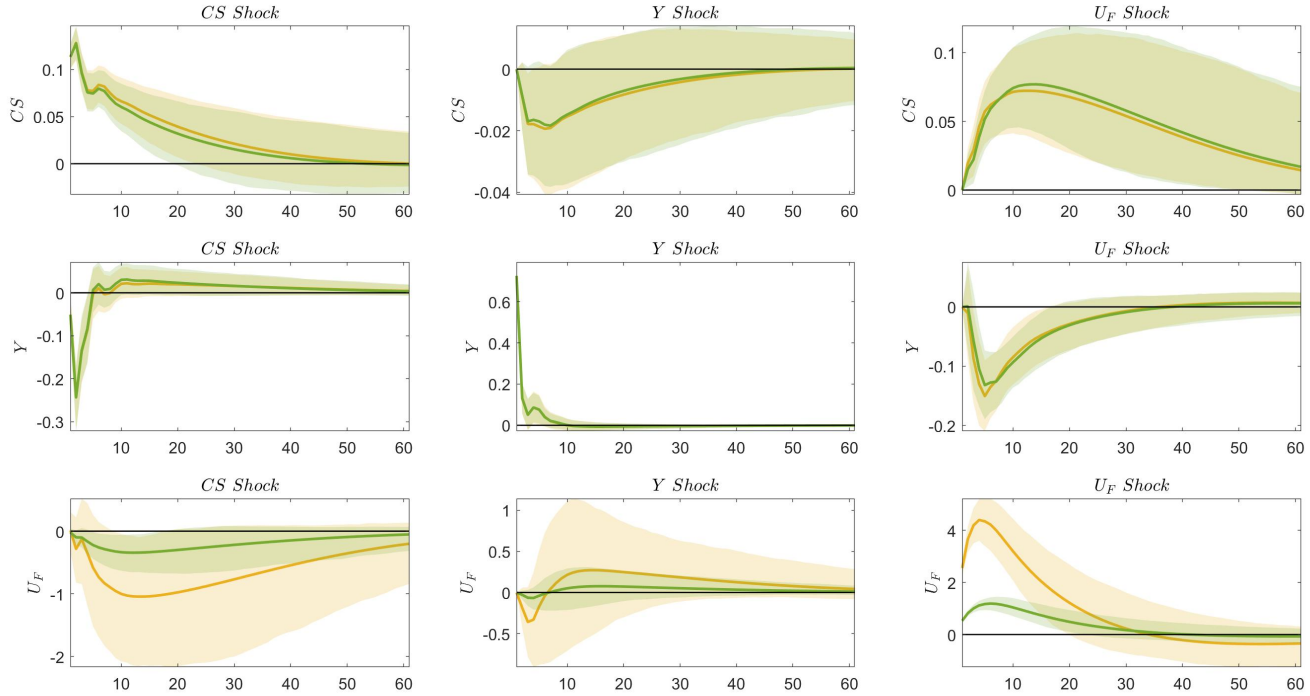

FIGURE TS.4: **Robustness check:** IRFs obtained in the first volatility regime (Great Inflation, 1960M8-1984M3) from the non-recursive SVARs for $X_t^* := (CS_t, Y_t, UF_t)'$, specified in eq. (TS.15). $Y_t = \Delta ip_t$ (industrial production growth); CS_t is the proxy of financial frictions; the yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

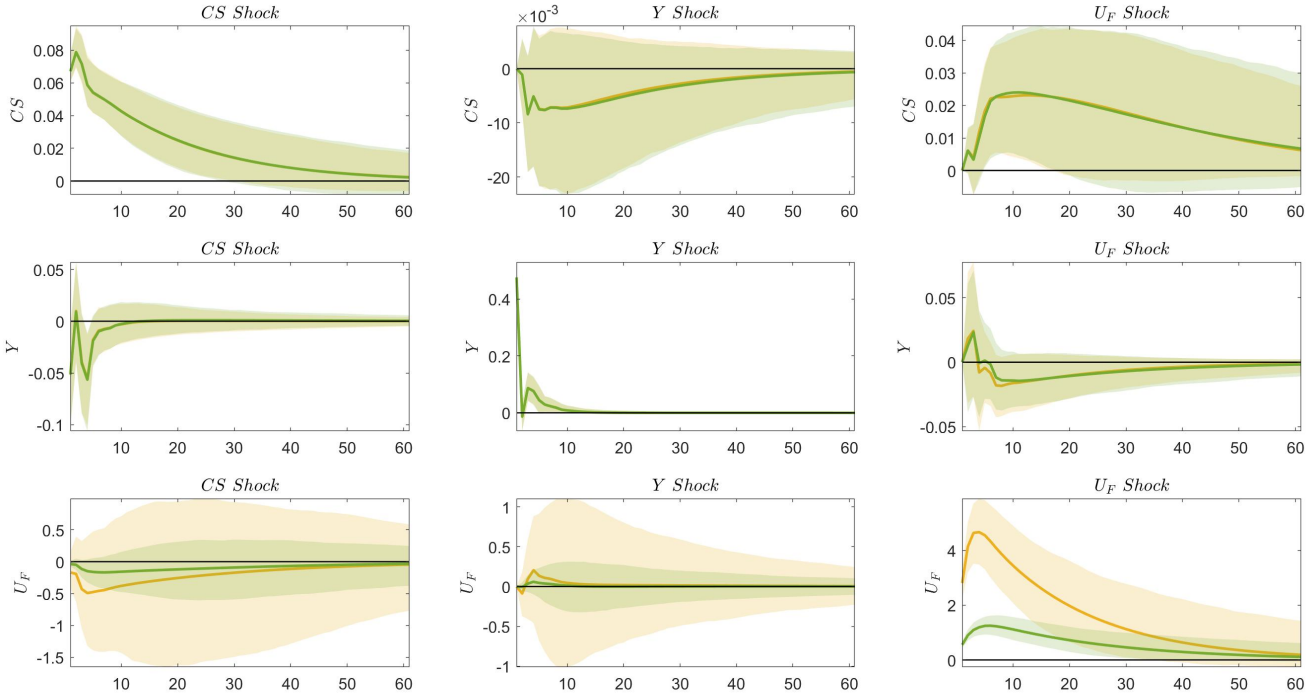

FIGURE TS.5: **Robustness check:** IRFs obtained in the second volatility regime (Great Moderation, 1984M4-2007M12) from the non-recursive SVARs for $X_t^* := (CS_t, Y_t, UF_t)'$, specified in eq. (TS.15). $Y_t = \Delta ip_t$ (industrial production growth); CS_t is the proxy of financial frictions; the yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

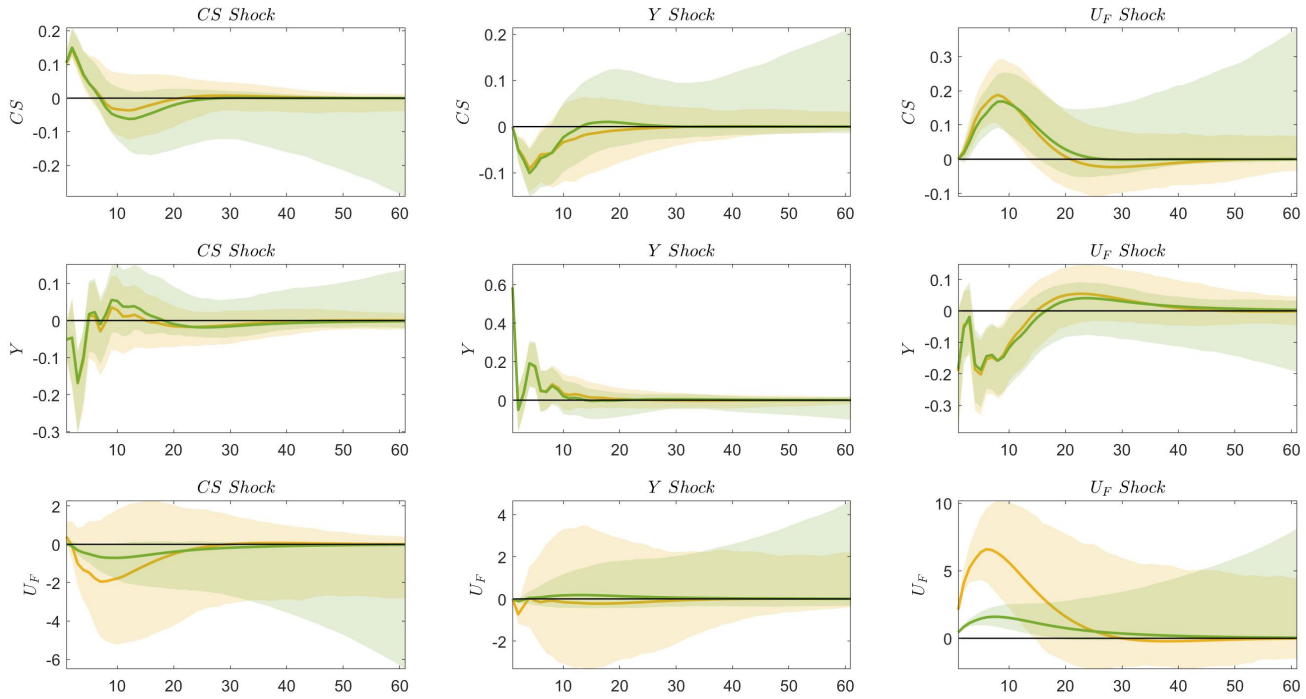

FIGURE TS.6: **Robustness check:** IRFs obtained in the third volatility regime (Great Recession + Slow recovery, 2008M1-2015M4) from the non-recursive SVARs for $X_t^* := (CS_t, Y_t, UF_t)'$, specified in eq. (TS.15). $Y_t = \Delta ip_t$ (industrial production growth); CS_t is the proxy of financial frictions; the yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

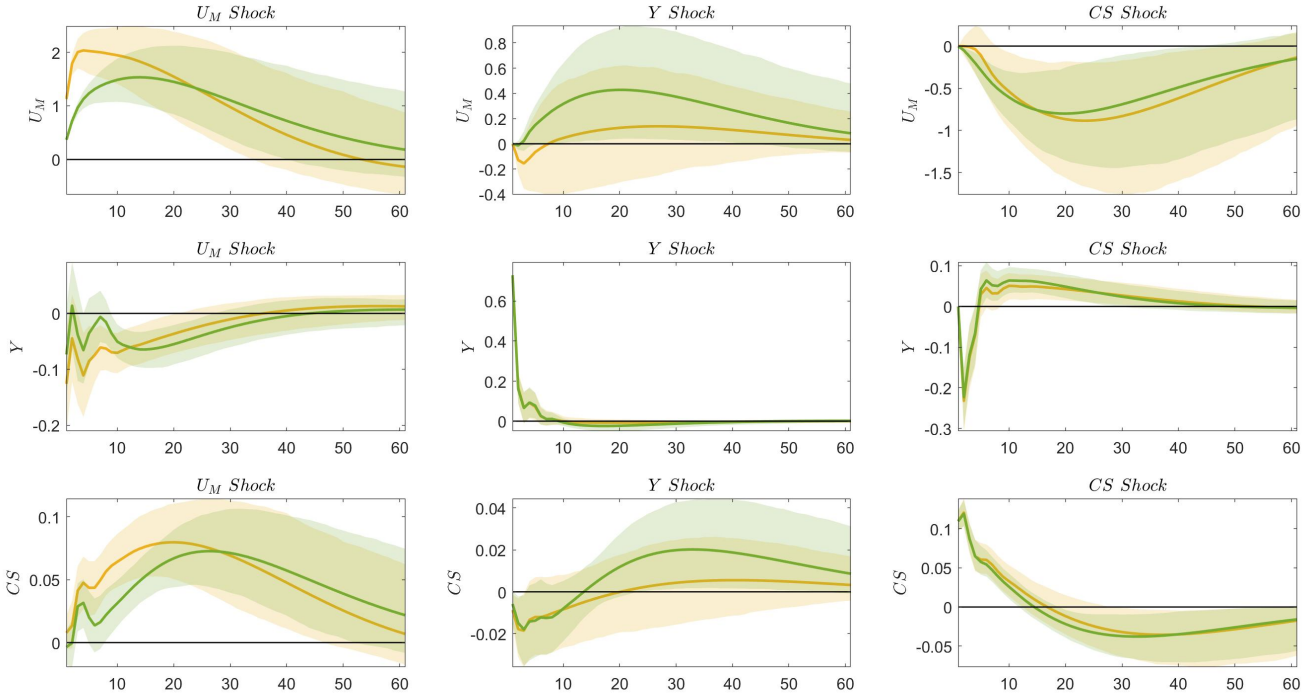

FIGURE TS.7: **Robustness check:** IRFs obtained in the first volatility regime (Great Inflation, 1960M8-1984M3) from the non-recursive SVARs for $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$, specified in eq. (TS.16). $Y_t = \Delta ip_t$ (industrial production growth); CS_t is the proxy of financial frictions; the yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

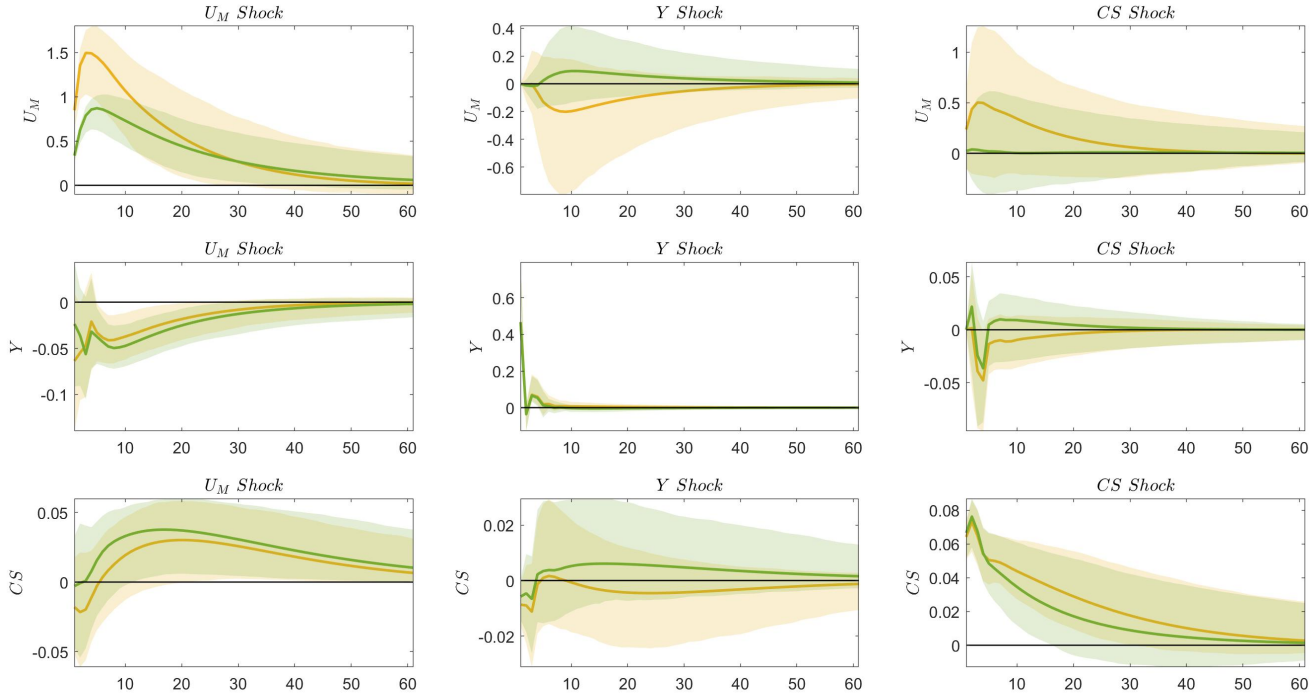

FIGURE TS.8: **Robustness check:** IRFs obtained in the second volatility regime (Great Moderation, 1984M4-2007M12) from the non-recursive SVARs for $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$, specified in eq. (TS.16). $Y_t = \Delta ip_t$ (industrial production growth); CS_t is the proxy of financial frictions; the yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

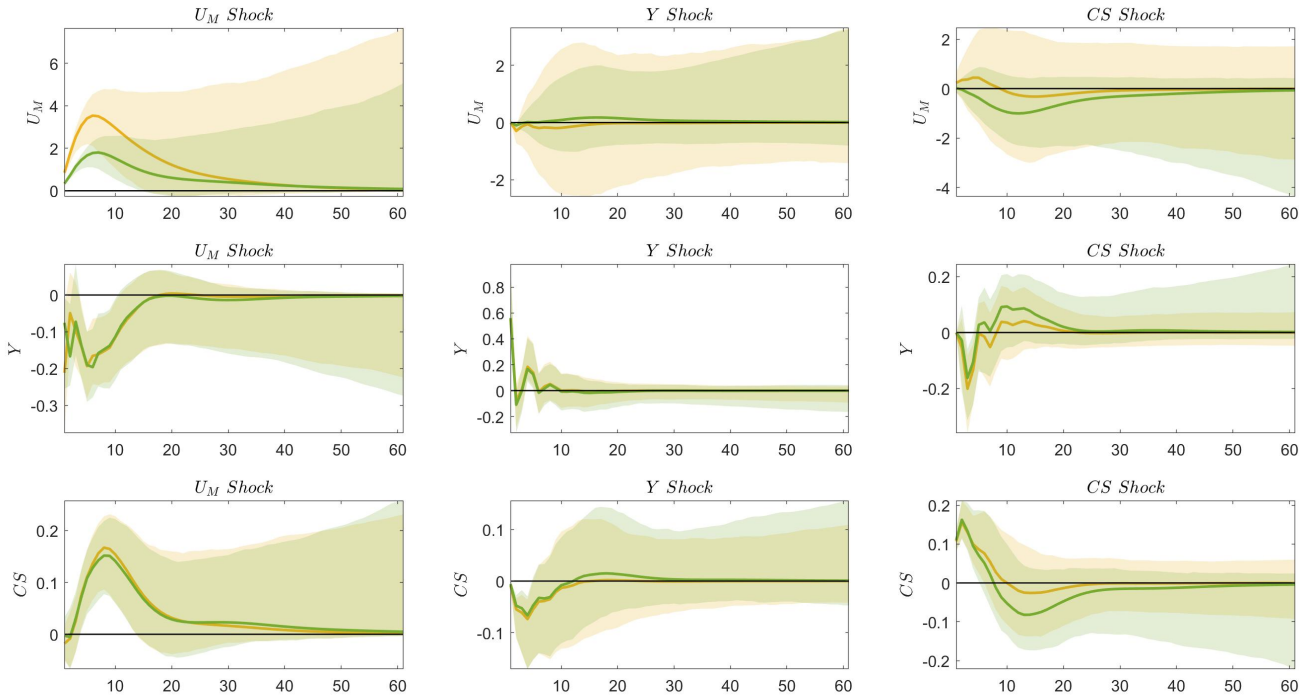

FIGURE TS.9: **Robustness check:** IRFs obtained in the third volatility regime (Great Recession + Slow recovery, 2008M1-2015M4) from the non-recursive SVARs for $X_t^{*\circ} := (U_{Mt}, Y_t, CS_t)'$, specified in eq. (TS.16). $Y_t = \Delta ip_t$ (industrial production growth); CS_t is the proxy of financial frictions; the yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

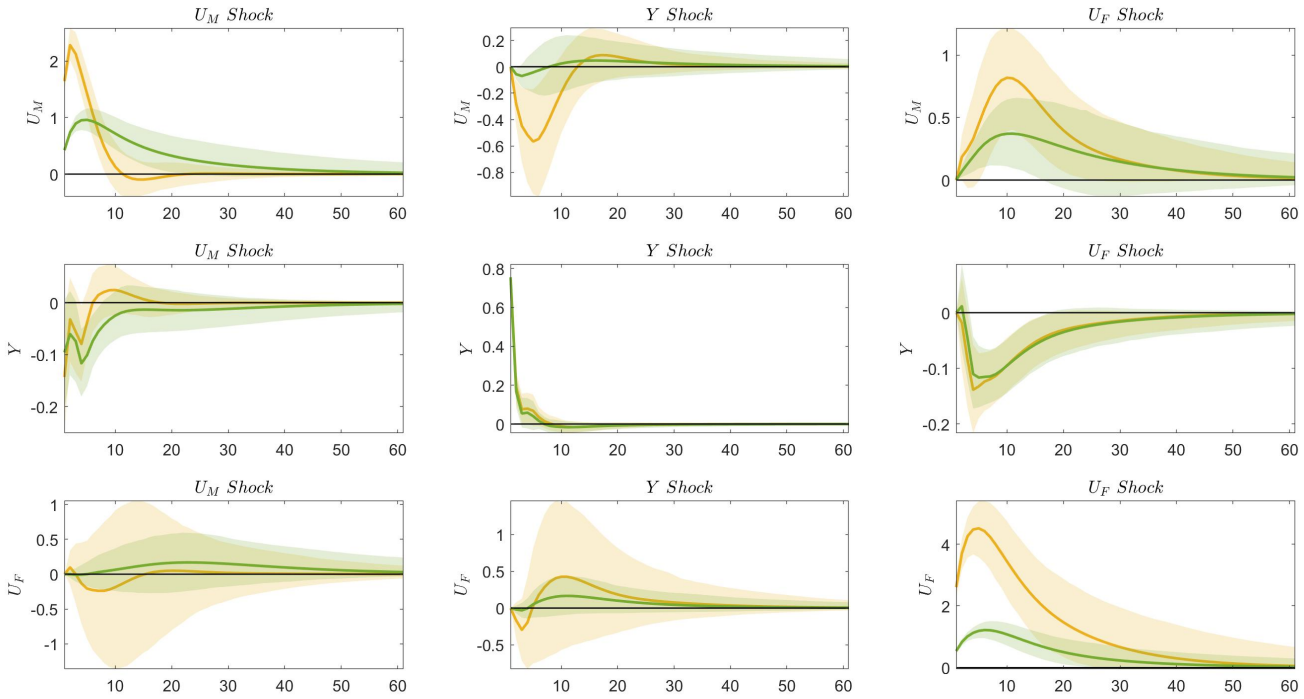

FIGURE TS.10: **Robustness check:** IRFs obtained in the first volatility regime (Great Inflation, 1960M8-1984M3) from the non-recursive SVAR for $X_t^p := \left(U_{Mt}^p, Y_t, U_{Ft}\right)'$, $Y_t = \Delta ip_t$ (industrial production growth), specified in eq.s (18)-(19) of the paper. U_{Mt}^p is the measure of real uncertainty of Ludvigson *et al.* (2017). The yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

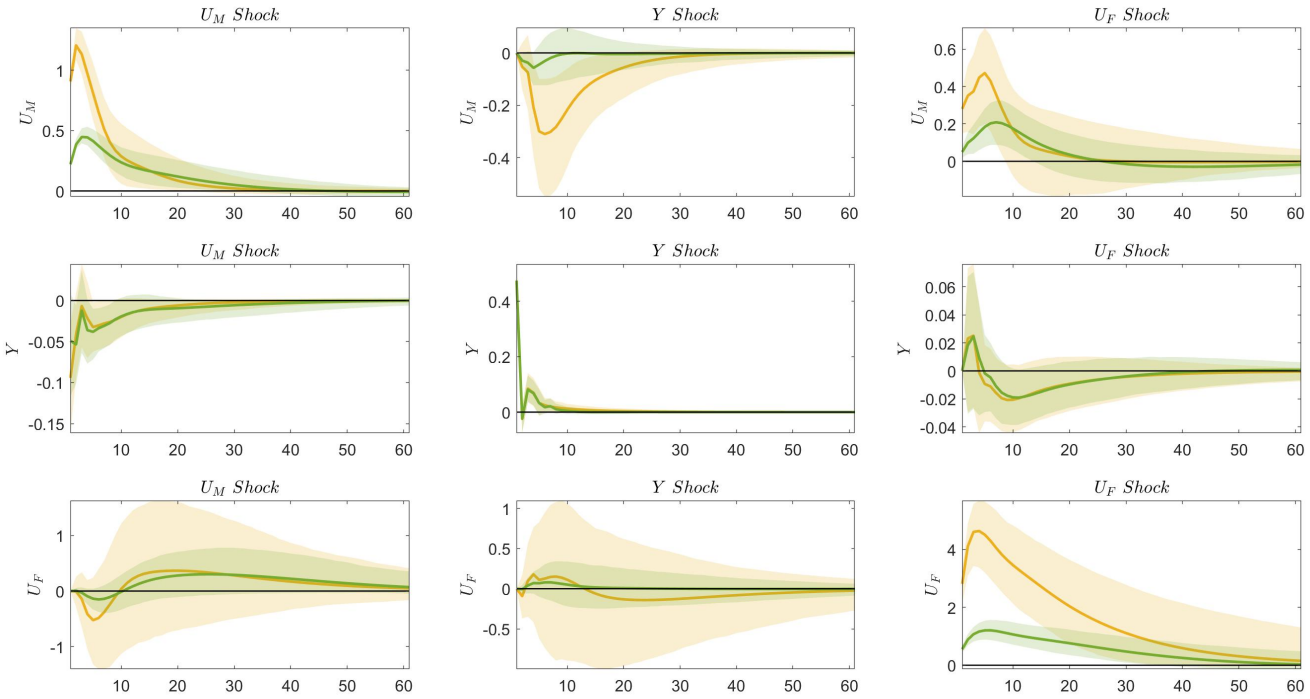

FIGURE TS.11: **Robustness check:** IRFs obtained in the second volatility regime (Great Moderation, 1984M4-2007M12) from the non-recursive SVAR for $X_t^p := \left(U_{Mt}^p, Y_t, U_{Ft}\right)'$, $Y_t = \Delta ip_t$ (industrial production growth), specified in eq.s (18)-(19) of the paper. U_{Mt}^p is the measure of real uncertainty of Ludvigson *et al.* (2017). The yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

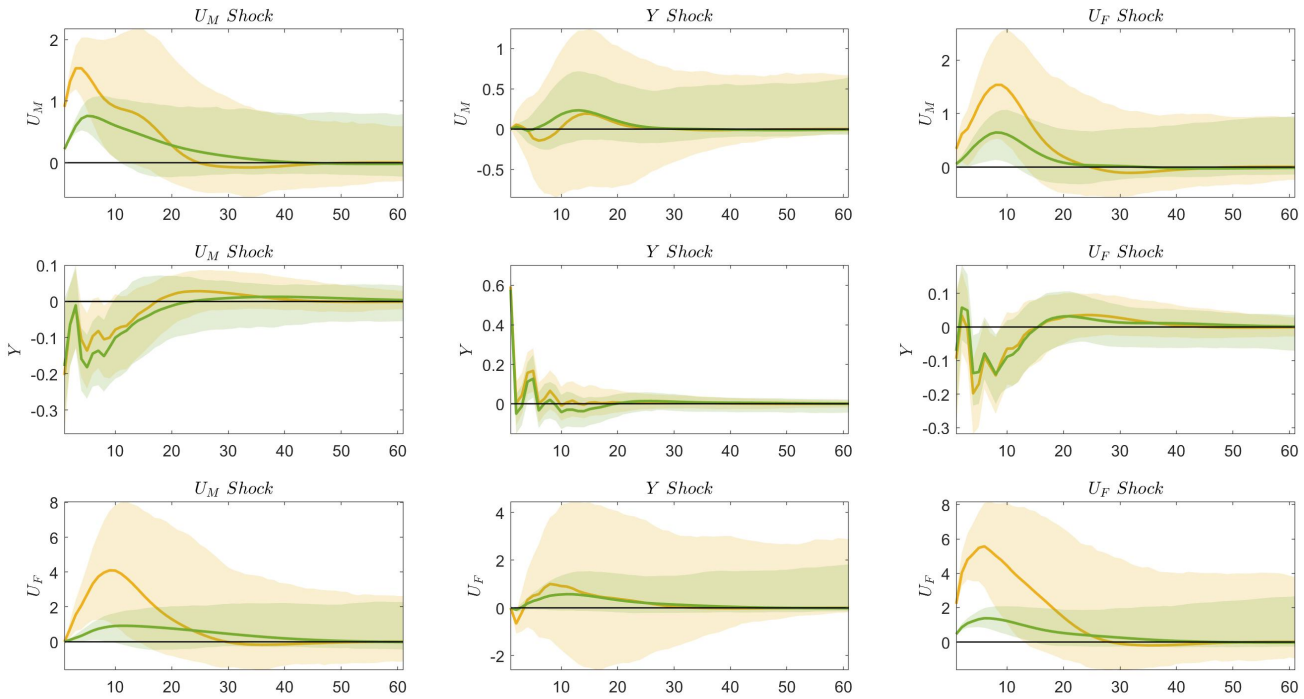

FIGURE TS.12: **Robustness check:** IRFs obtained in the third volatility regime (Great Recession + Slow Recovery, 2008M1-2015M4) from the non-recursive SVAR for $X_t^p := \left(U_{Mt}^p, Y_t, U_{Ft}\right)'$, $Y_t = \Delta ip_t$ (industrial production growth), specified in eq.s (18)-(19) of the paper. U_{Mt}^p is the measure of real uncertainty of Ludvigson *et al.* (2017). The yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

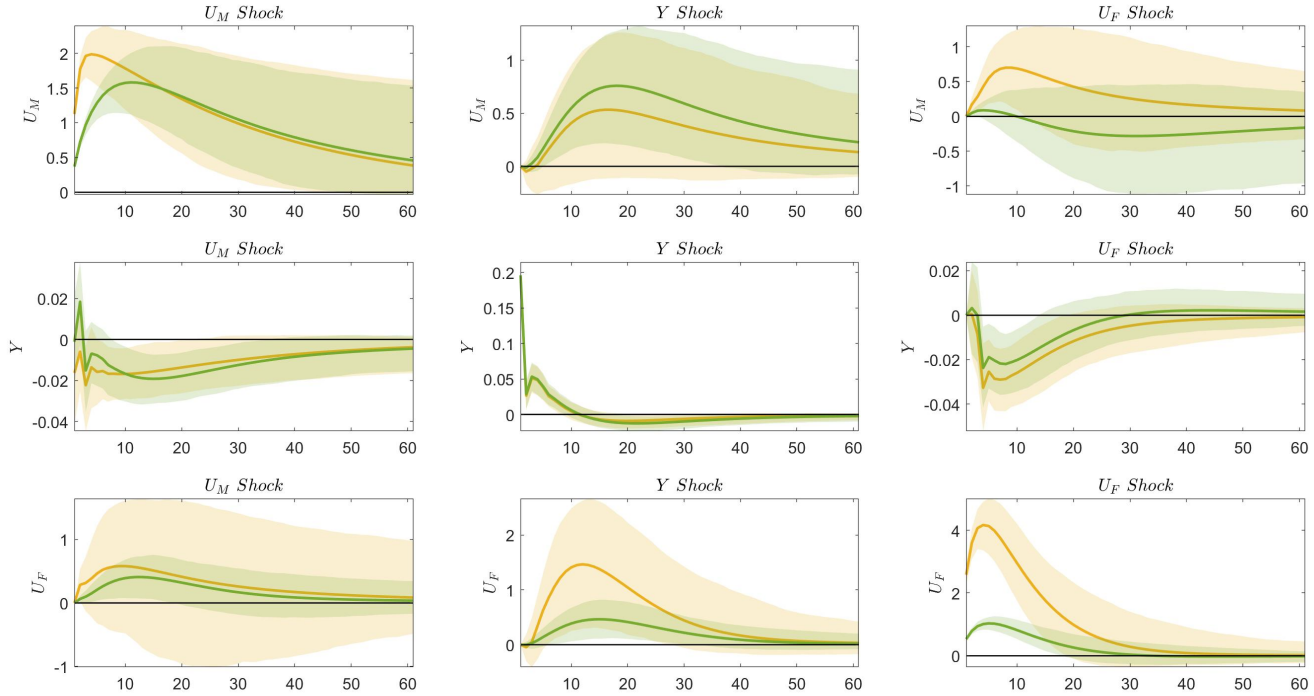

FIGURE TS.13: **Robustness check:** IRFs obtained in the first volatility regime (Great Inflation, 1960M8-1984M3) from the non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta emp_t$ (employment growth) specified in eq.s (18)-(19) of the paper. The yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

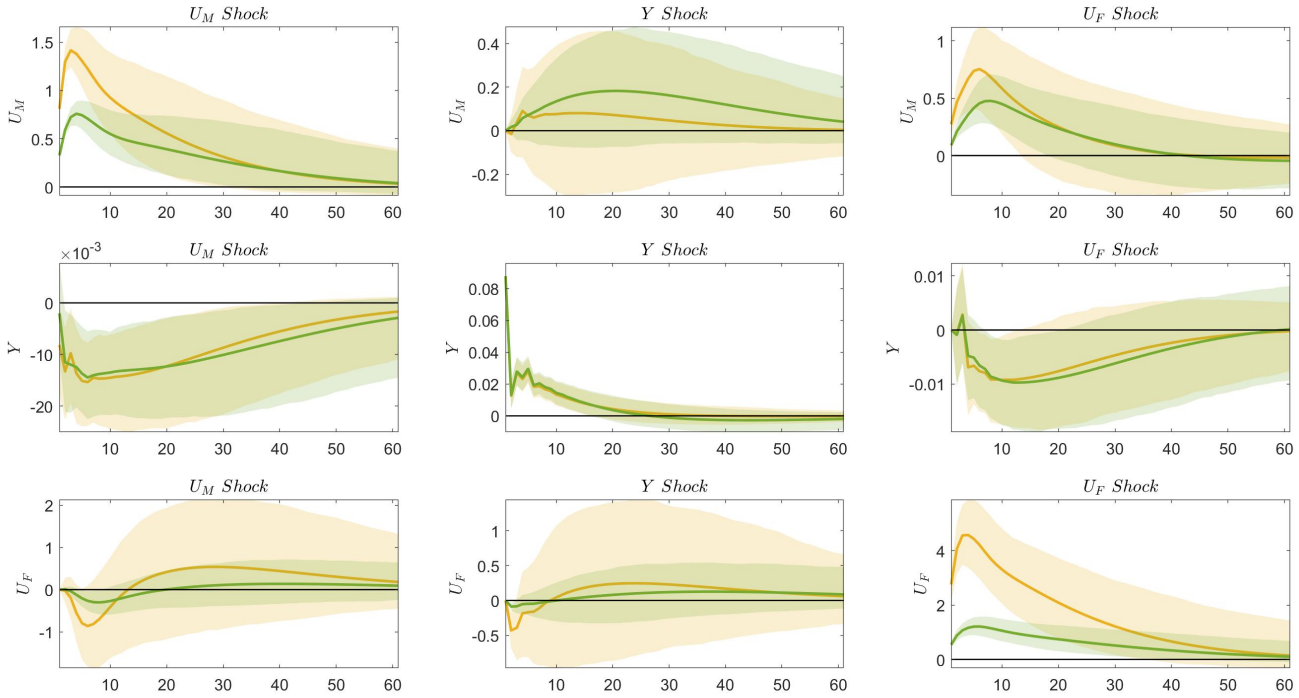

FIGURE TS.14: **Robustness check:** IRFs obtained in the second volatility regime (Great Moderation, 1984M4-2007M12) from the non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta emp_t$ (employment growth) specified in eq.s (18)-(19) of the paper. The yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.

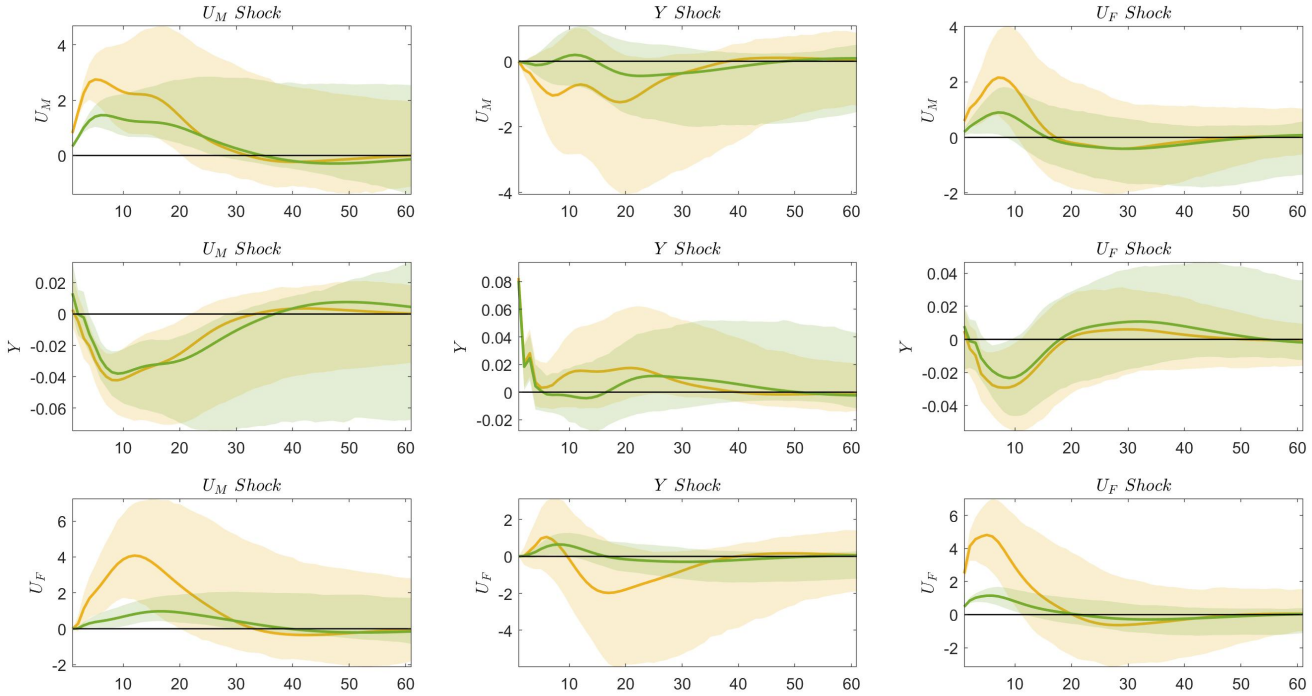

FIGURE TS.15: **Robustness check:** IRFs obtained in the third volatility regime (Great Recession + Slow Recovery, 2008M1-2015M4) from the non-recursive SVAR for $X_t := (U_{Mt}, Y_t, U_{Ft})'$, $Y_t = \Delta emp_t$ (employment growth) specified in eq.s (18)-(19) of the paper. The yellow lines refer to the one-month ($f = 1$) uncertainty horizon and yellow shaded areas denote the associated 90% bootstrap confidence bands; the green lines refer to the one-year ($f = 12$) uncertainty horizon and green shaded areas denote the associated 90% bootstrap confidence bands; bootstrap confidence bands are computed using Kilian's (1998) method. Responses are measured with respect to one standard deviation changes in structural shocks.