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On the Instability of the R&D Portfolio in a Dynamic Monopoly. Or, One Cannot Get Two Eggs in One Basket

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Abstract

Firms' innovation portfolios include several dimensions ranging from organizational aspects to cost reduction and product characteristics. All of these efforts take place during the product life cycle, and interact with each other in determining the spectrum of features of the product and its performance on the market. This paper contributes to the related theoretical debate, focussing on the possibility of having superior product quality levels at lower marginal production cost over time. To deal with this issue, we investigate the optimal R&D portfolio of a single-product monopolist investing in cost-reducing activities accompanied by efforts improving the quality of its product over an infinite time horizon. It turns out that the firm's relative incentives along the two directions are conditional upon market affluence, measured by consumers' willingness to pay for quality, and R&D efforts are complements in the neighbourhood of the steady state equilibrium. However, the dynamics of the two R&D controls depend on both quality and marginal cost at every instant. Consequently, as the stability analysis reveals, the steady state equilibrium is indeed unstable due to the dynamics of marginal cost, thereby implying that one should not expect the firm to supply an increasing quality level at a decreasing production cost. Hence, the dynamic interplay between R&D controls and the resulting instability affecting production costs also imply that one may not expect to observe product quality to increase and market price to decrease over the product life cycle.

Keywords: Process innovation; Monopoly; Product quality; R&D

1 Introduction

Firms' incentives to innovate in several directions have received a large amount of attention in the existing literature, where, traditionally, innovation may affect the product characteristics or marginal production cost (see, e.g., Tirole, 1988; and Reinganum, 1989, among others). The endogenous connections between multidimensional innovation and demand and industry evolution have been extensively studied in the debate on product life cycle (see Abernathy and Utterback, 1975, 1978; and Adner and Levinthal, 2001, among many others). The resulting view holds it that product innovation necessarily precedes process innovation, this perception being reinforced by empirical evidence (Damanpour and Gopalakrishanan, 2001).

However, the reverse case is also observed (Barras, 1986) as well as the simultaneous presence of product improvement and cost abatement (Pine *et al.*, 1993).

The latter constitutes our point of departure in this paper, where we want to outline some features of the technological evolution over time of a product which is being modified while being already marketed, initial innovations having allowed the firm to supply a product characterised a marginal cost which is sufficiently low to ensure positive demand. Another essential component of the spectrum of product characteristics is quality, as perceived from the consumers' viewpoint. The interplay between market power and the resulting price/quality ratio is an issue that has been lively debated in the extant literature belonging to the theory of industrial organization ever since the pioneering research by Spence (1975) and Mussa and Rosen (1978). In particular, one of the pivotal elements of this discussion has been the monopolist's incentive to distort quality downward to extract as much surplus as possible from consumers.[1]

This has been done taking the efficiency level of the firms' production technology (i.e., marginal cost) as given. More often than not, marginal cost has been entirely left out of the picture for the sake of simplicity. However, doing so, the extant debate on product quality distortions has almost entirely left out of the picture a relevant question which can be formulated in the following terms: may R&D efforts aimed at decreasing production costs be accompanied by similar investment aimed at increasing product quality while the product is being marketed? Put differently, would it be possible to supply a product whose quality gets higher at a lower marginal cost? This would allow firms to increase the mark-up simultaneously in two directions by commanding a higher price via a higher quality level, accompanied by a lower unit production cost. That is, might a firm get two eggs in one basket?

Surprisingly enough, the existing literature in this area of industrial economics offers little material of this kind, featuring a few contributions where product quality is not explicitly taken into account. Indeed, in Lin and Saggi

[1] Further contributions in this vein are those of Itoh (1983), Maskin and Riley (1984), and Champsaur and Rochet (1989). For an overview, see Lambertini (2006). Moreover, there exists a similar but not strictly related discussion in the field of business and management, where product quality is a relevant variable in shaping firms' advertising campaigns. For exhaustive surveys, see Feichtinger *et al.* (1994) and Erickson (2003).

(2002), Rosenkranz (2003) and Lambertini and Mantovani (2009, 2010) product differentiation is modelled in terms of a representative consumer's preference for variety rather than quality, and firms' product innovation efforts modify a degree of product differentiation which has no explicit relationship with quality. Two relevant exceptions are Bonanno and Haworth (1988) and Veldman and Gaalman (2014). Bonanno and Haworth (1988) use a static discrete choice model with vertical differentiation *à la* Mussa and Rosen (1978) to investigate the impact of Bertrand and Cournot competition on process innovation and quality improvements, while Veldman and Gaalman (2014) focus on the role of strategic delegation in shaping bidimensional innovation incentives in a model where product quality enters the utility function of a representative consumer buying a basket of all goods available on the market, to show that the presence of managers endowed with appropriate incentives enhances both cost-reducing and quality-enhancing activities.

It is worth recalling the relevant further contribution by Hayes and Wheelwright (1984), where a detailed description of a 4-steps framework is proposed in order to help manufacturing organizations achieve their strategic goals. In their approach, product innovation and process innovation are viewed as different steps in an optimization path together with a number of distinct characteristics of the production procedures. Subsequent related literature (e.g., the works by Damanpour and Gopalakrishanan, 1994, 2001) treat the relationships between process innovation and product innovation empirically, in oligopoly contexts. These contributions analyse the timing of such innovations, assuming both of them are implemented. To some extent, our analysis is coherent with theirs, in the sense that we are also admitting the presence of both kinds of innovation. However, we focus our attention on the behaviour over time of the R&D efforts exerted by the firm once the product is on the market and its quality and production cost are simultaneously modified *during the product life cycle*.

In line of principle, one could think that it would be desirable to simultaneously have quality increasing and unit production cost decreasing over time, from both the demand and the supply side standpoints. As to whether this may happen, empirical evidence and casual observation are both controversial, and suggest this might not be the case. An example can be found in the car industry. In particular, green hybrid cars are costlier (to producers and consumers alike) than brown ones, all else equal, for two reasons: the first is that market price is set so as to allow firms to abate the large initial R&D investments required to invent, design and put into production electrical power units; the second is that the marginal production costs of the assembled final product is largely affected by the marginal production costs of hybrid propulsion. If indeed green quality is perceived as higher than the old style brown one, then in this case higher product quality goes hand in hand with higher marginal costs and market prices.[2] A similar picture emerges if one considers energy supply. In this case, photovoltaic panels may at most afford to convert about 20% of incoming sunlight into electricity, with the resulting kW/h costing several times more than a kW/h obtained from any combination of fossil sources, which remains true if we compare a combination of renewables and nonrenewable energy sources in general. Windmills might seem an exception in this respect, as they require large upfront costs but very low operating costs and are already supplying energy at a fraction of the unit cost associated with coal plants (cf. Smil, 2010). However, energy from windmills is heavily affected by storage problems and is not available on call.[3] Considering that the ensuing model investigates an industry for nondurables, the case of energy supply appears to fit our setup and reflect its main results.

Now we can illustrate the theoretical setup, addressing the joint presence of process and product innovation through an optimal control model describ-

[2]This theme is receiving a growing amount of attention in models at the intersection between the theory of industrial organization and environmental economics. See Arora and Gangopadhyay (1995), Bansal and Gangopadhyay (2003) and Amacher *et al.* (2004). For a model where green high-quality goods explicitly involve higher marginal production costs than brown low-quality ones, see André *et al.* (2009) and Lambertini and Tampieri (2012).

[3]In computing total and marginal production costs, one should also account for the additional costs associated with a widespread adoption of renewable energy sources, such as those associated with energy density (i.e., the number of kW/h per km^2), which is considerably lower than that of fossil fuels. See Smil (2003, 2008).

ing a profit-maximising monopolist activating a bidimensional R&D portfolio over an infinite horizon. Cost-reducing and quality-enhancing efforts are controls, while marginal production cost and product quality are the state variables. Although, for the sake of simplicity, we assume that the two state dynamics are decoupled, i.e., each state appears in its own dynamic equation only, it turns out that the control dynamics are not decoupled. This adds a desirable pinch of realism to the model and plays a crucial role in shaping the essential features of the outcome of our analysis. Indeed, the stability analysis reveals that, as a direct consequence of this feature, the equilibrium is unstable. In particular, what triggers instability is marginal production cost, whose dynamics offsets the firm's R&D efforts along that dimension. Hence, the bottom line of our analysis is that the monopolist cannot get two eggs in one basket and consequently consumers cannot expect to see quality rising at a progressively lower marginal cost, precisely because of the cross-effects existing between states and controls in the R&D portfolio of the firm.

This is in sharp contrast with Lambertini and Orsini (2015), to the best of our knowledge the single work which has previously investigated the same topic in a similar setup, which differs from ours for a single but essential feature. In Lambertini and Orsini (2015), the monopoly equilibrium is stable because the two dimensions of the firm's R&D portfolio are independent of each other and no cross effects between states and controls appear.[4] This

[4]More precisely, the dynamic equation of the R&D control for process (resp., product) innovation contains only the marginal cost (resp., the quality level). As a result, the Jacobian matrix of the dynamic system is block diagonal and the steady state is a saddle

property, intuitively, is quite demanding and cannot be expected to hold in general.

An explicit aim of the ensuing analysis is in fact to show what happens in a more realistic situation where each R&D effort is affected by both states. In short, we show that if the effectiveness of R&D appearing in the state dynamics is independent of the level of the relevant state variable (either quality or marginal cost), then the model is unstable and this does not allow the firm to increase quality and simultaneously decrease price over the product life cycle because of production cost instability. This reveals that, in order to reach a non-explosive steady state equilibrium, the firm should design its R&D projects by looking for innovation technologies whose returns to scale (either increasing or decreasing) explicitly characterise the evolution of the targeted states.

A last remark is in order. The analysis is carried out in a model where environmental implications of either production or consumption are not explicitly modelled. However, linking our results to the existing debate on environmental quality, from the present setup there emerge interesting implications concerning the possibility of attaining green technologies at marginal costs comparable with those of brown ones.

The remainder of the paper is structured as follows. Section 2 illustrates the setup. The equilibrium analysis is carried out in Section 3, while Section 4 contains the stability analysis. In Section 5, an alternative version of the model, affected by learning-by-doing behaviour in technology development,

point equilibrium.

is presented. Concluding remarks are in Section 6.

2 The setup

Our model describes a market supplied by a single-product monopoly selling a nondurable good of quality $q(t) > 0$ at price $p(t) > 0$ over continuous time $t \in [0, \infty)$. The population of consumers has a constant size $\Theta > 0$, and each consumer is characterised by a marginal willingness to pay for quality $\theta \in [0, \Theta]$. Parameter θ is usually interpreted as a proxy of income or wealth (see Tirole, 1988, ch. 2). The population of consumers is uniformly distributed with density 1 over such interval.[5] At any time $t \in [0, \infty)$, each individual is assumed to buy a single unit of the good or nothing at all.

The utility function we are going to employ dates back to Mussa and Rosen (1978) and has been widely used in the ensuing literature on oligopolistic models with vertically differentiated goods initiated by Gabszewicz and Thisse (1979). It is also the same used in Arora and Gangopadhyay (1995) and Bansal and Gangopadhyay (2003), where quality has an environmental value, whereby polluting emissions decrease in the quality level. In their approach, to which we will come back in Section 4, consumers are environmentally concerned and demand green innovations on the part of firms,

[5] In the literature based on this approach, it is often assumed that density is $1/\Theta$, in such a way that the population of consumers is equal to one. In itself, this is a quite specific assumption. Additionally, as long as the distribution of consumers is rectangular, choosing any specific value of density has no impact on the qualitative properties of the ensuing analysis, and therefore we assume unit density.

consumer awareness acting as a substitute for regulation. The net surplus of an individual indexed by $\theta \in [0, \Theta]$ is

$$U(\cdot) = \theta q(t) - p(t) \geq 0, \tag{1}$$

if the purchase takes place, otherwise it is nil. The consumer indifferent between buying or not is indexed by $\widehat{\theta}(t) = p(t)/q(t)$; accordingly, the instantaneous inverse demand function is

$$p(t) = [\Theta - x(t)]\, q(t), \tag{2}$$

where $x(t)$ is output.[6] Turning to the supply side, some specific hypotheses have to be adopted. We are assuming that the entire R&D activity is carried out in house by the integrated firm.[7] The monopolist is bearing instantaneous costs due to output production and to both process and product innovation. The total cost function borne by the firm at any time t is

$$C(t) = c(t)\, x(t) + bk^2(t) + sy^2(t) \tag{3}$$

where $c(t)$ is marginal production cost; $k(t)$ is the instantaneous R&D effort for quality improvement; $y(t)$ is the effort for process innovation (cost

[6] In Balasubramanian and Bhardwaj (2004) and by Veldman and Gaalman (2014) quantity is linear in the difference between qualities. Their approach models demand in a representative consumer setup with quasi-linear preferences. Here, we adopt instead the discrete choice approach where every consumer buys a single unit of the product characterised by the preferred price-quality ratio.

[7] For an assessment of the bearings of outsourcing on quality improvement, and the related contractual design, see El Ouardighi and Kim (2010) and El Ouardighi and Kogan (2013), *inter alia*.

reduction); and b and s are positive constants. The convexity of R&D costs along both dimensions of innovation account for decreasing returns to R&D activity. Product quality and marginal production cost are state variables, each of them being affected by a specific R&D effort for either product or process innovation. The resulting state equations describing the evolutions of $q(t)$ and $c(t)$ are:

$$\dot{q}(t) = k(t) - \delta q(t) \tag{4}$$

and

$$\dot{c}(t) = -y(t) + \eta c(t). \tag{5}$$

The differential equations (4-5) are linear and feature exogenous obsolescence (or decay) rates of quality and productive efficiency, δ and η, both positive and time-invariant.[8] State dynamics (4-5) are the same as in Li and Ni (2016, p. 105), while in Lambertini and Orsini (2015, p. 371) they are defined as $\dot{q}(t) = [k(t) - \delta] q(t)$ and $\dot{c}(t) = [-y(t) + \eta] c(t)$, respectively. This alternative formulation postulates a proportional impact of R&D, which depends on the current level of either state. This amounts to saying that decreasing (or increasing) returns to R&D activity appear in the state equations. Look for instance at the dynamics of marginal cost, and suppose $c(t)$ is positive but arbitrarily close to zero. If so, then even a very large R&D effort would have a negligible impact, contrary to what happens in (5). Exactly the opposite applies along the quality dimension, as higher quality levels boost the effect

[8] We are supposing that R&D has an immediate impact. This is admittedly a simplifying and unrealistic assumption which, however, is commonly adopted.

of any given R&D effort in this direction. Indeed, all of this is assumed away in (4-5), where the increase in quality and the decrease in marginal cost are both independent of their respective current levels at any time, and one only observes decreasing returns relegated into the convexity of the instantaneous cost function (3) only. As we shall see in the remainder, this feature plays a crucial role in shaping the resulting outcome and its properties.

Given (2-3), the monopolist's instantaneous profits are

$$\pi(\cdot) = p(t)\,x(t) - C(t) = [(\Theta - x(t))\,q(t) - c(t)]\,x(t) - bk^2(t) - sy^2(t)\,. \tag{6}$$

Therefore, the firm must solve the following infinite horizon problem:

$$\max_{x(t),k(t),y(t)} \int_0^\infty e^{-\rho t}\left[[(\Theta - x(t))\,q(t) - c(t)]\,x(t) - bk^2(t) - sy^2(t)\right]dt, \tag{7}$$

The alternative between price-setting and quantity-setting behaviour being of course immaterial in a monopoly model.

The dynamic constraints of problem (7) are described by the state equations (4) and (5), which are endowed with initial conditions $q(0) = q_0 > 0$, $c(0) = c_0 \in (0, \Theta q_0)$, indicating the positivity of both quality and marginal production cost levels at the initial time. It is worth observing that condition $c_0 < \Theta q_0$ requires the initial marginal production cost to be strictly lower than the spending capability of the richest consumer in this market, in order for demand to be positive at $t = 0$. Finally, future profits are discounted at the constant rate $\rho > 0$.

To recap the technical setup, the monopolistic firm has to solve a dynamic optimization problem with three control variables and two state variables. The solution approach we are going to adopt involves the open-loop information structure (or equilibrium).

3 Equilibrium analysis

The firm's current value Hamiltonian is[9]

$$\mathcal{H} = e^{-\rho t}\left(\pi + \lambda\dot{q} + \mu\dot{c}\right) \tag{8}$$

where $\lambda = \zeta e^{\rho t}$ and $\mu = \psi e^{\rho t}$ are the costate variables (evaluated at time t) associated with q and c, respectively. The resulting first order conditions (FOCs) on controls and costate equations are (exponential discounting is omitted for brevity):

$$\frac{\partial\mathcal{H}}{\partial x} = (\Theta - 2x)\,q - c = 0 \tag{9}$$

$$\frac{\partial\mathcal{H}}{\partial k} = -2bk + \lambda = 0 \tag{10}$$

$$\frac{\partial\mathcal{H}}{\partial y} = -2sy - \mu = 0 \tag{11}$$

$$\dot{\lambda} = -\frac{\partial\mathcal{H}}{\partial q} + \rho\lambda \qquad \Longleftrightarrow \qquad \dot{\lambda} = (\delta + \rho)\,\lambda - x\,(\Theta - x) \tag{12}$$

$$\dot{\mu} = -\frac{\partial\mathcal{H}}{\partial c} + \rho\mu \qquad \Longleftrightarrow \qquad \dot{\mu} = (\rho - \eta)\,\mu + x. \tag{13}$$

The accompanying set of transversality conditions is $\lim_{t\to\infty}\lambda q e^{-\rho t} = 0$ and $\lim_{t\to\infty}\mu c e^{-\rho t} = 0$.

[9] Henceforth, we shall omit the explicit indication of the time argument for the sake of brevity.

The FOC (9) can be solved in a quasi-static way to obtain the optimal instantaneous output $x^* = \frac{\Theta q - c}{2q}$, so that monopoly price is $p^* = \frac{\Theta q + c}{2}$. Before proceeding, we may formulate

Lemma 1 *For any given admissible pair of states* (q, c)*, optimal output* $x^* \in (0, \Theta)$.

Proof. To prove the Lemma, it suffices to observe that $x^* \in (0, \Theta)$ iff $p^*/q \in (0, \Theta)$. Now, $\frac{p^*}{q} = \frac{\Theta q + c}{2q}$ is surely positive and $\Theta - \frac{p^*}{q} = \frac{\Theta q - c}{2q} > 0$ due to the fact that $\Theta q > p^* \geq c$ in order to enable the richest consumer to buy, either at the monopoly price or, *a fortiori*, at a competitive price equal to marginal cost. ■

The result stated in Lemma 1 tells that the monopolist will never cover the entire market throughout the infinite time horizon, for any generic product quality and marginal production cost. This, in addition to confirming the obvious restrictive effect of monopoly power on output decisions, implies that we shall not deal with corner solutions in the remainder of the analysis.

Hence, we may proceed with the characterization of optimal R&D efforts. From (10-11), we have the optimal instantaneous controls at any time t:

$$k^* = \max\left\{0, \frac{\lambda}{2b}\right\} ; \qquad y^* = \max\left\{0, -\frac{\mu}{2s}\right\} \tag{14}$$

and the control equations

$$\dot{k} = \frac{\dot{\lambda}}{2b} ; \qquad \dot{y} = -\frac{\dot{\mu}}{2s} \tag{15}$$

which, using (10-11) and (14), can be rewritten as follows:

$$\dot{k} = \frac{c^2 + \left[8bk\left(\delta + \rho\right) - \Theta^2\right] q^2}{8bq^2} \tag{16}$$

$$\dot{y} = \frac{c + [4sy(\rho - \eta) - \Theta]\, q}{4sq} \tag{17}$$

The system composed by (4-5) and (16-17) identifies the state-control system of the dynamic problem at hand. Before proceeding any further, it is important to stress that both state variables simultaneously appear in both control equations. This implies that the two dimensions of the firm's R&D portfolio are not independent of each other. More explicitly, the two dynamic control equations are not separable w.r.t. states, this being a direct consequence of the lack of additive separability in the instantaneous profit function (6). The relevant consequences of this feature of the model will become evident in the remainder, in connection with the stability analysis.

Imposing stationarity on states and controls yields

$$k_{SS} = \frac{(\Theta q_{SS} - c_{SS})(\Theta q_{SS} + c_{SS})}{8bq_{SS}^2(\delta + \rho)} \tag{18}$$

$$y_{SS} = \frac{\Theta q_{SS} - c_{SS}}{4sq_{SS}(\rho - \eta)} \tag{19}$$

Note that, since $\Theta q - c > 0$ necessarily, then $k_{SS} > 0$. As to y^*, this is positive iff $\rho > \eta$, i.e., the process innovation effort is positive provided that the firm's impatience outweighs the rate of depreciation of technology. Therefore, in the remainder, we shall assume $\rho > \eta$, otherwise marginal cost would increase (which would imply that the system cannot reach a steady state). We can now proceed to impose stationarity on (5), which delivers the following steady state value of marginal cost:

$$c_{SS} = \frac{\Theta q_{SS}}{1 + 4sq_{SS}(\rho - \eta)\,\eta} \tag{20}$$

Now we can compare k_{SS} and y_{SS} to see that $k_{SS} > y_{SS}$ for all

$$\Theta > \frac{b(\delta+\rho)[1+4sq_{SS}(\rho-\eta)\eta]}{s(\rho-\eta)[1+2sq_{SS}(\rho-\eta)\eta]} \equiv \Theta_{ky} \tag{21}$$

Relying on the above expression and Lemma 1, we can claim the following:

Proposition 2 *Take $\rho > \eta > 0$. At the steady state, $\Theta > \max\{\Theta_{ky}, c_{SS}/q_{SS}\}$ suffices to ensure $k_{SS} > y_{SS} > 0$.*

Proposition 2 conveys the intuitive message that the equilibrium R&D effort for quality improvement is higher than the effort exerted for process innovation if the marginal willingness to pay for quality of the richest consumer in the market is high enough (or, equivalently, if consumers' affluence is sufficiently high): richer consumers with hedonic tastes are keen on paying higher prices for superior quality levels, which makes marginal cost abatement comparatively less relevant, and the firm is happy to react accordingly along the two R&D dimensions.

A supplementary discussion can be carried out about the presence of complementarity or substitutability between the two forms of innovation, in line with an existing discussion in the literature (see Lambertini, 2003, 2004; and Lin, 2004, *inter alia*). At first sight, judging from (18-19), one would be tempted to conclude that, in the present model, product and process innovation are independent of each other, as $\partial k_{SS}/\partial y_{SS} = \partial y_{SS}/\partial k_{SS} = 0$.

This conclusion, however, can be swept away by observing that both states appear in (18-19), and therefore one can carry out a simple exercise to single out the nature of the influence exerted by one type of innovation on the other. This can be done by relying on (18-20) to assess the effects of a slight variation in q in the neighbourhood of the steady state:

$$\frac{\partial k_{SS}}{\partial y_{SS}} = \frac{\partial k_{SS}/\partial q_{SS}}{\partial y_{SS}/\partial q_{SS}} = \frac{\Theta s\left(\rho - \eta\right)}{b\left(\delta + \rho\right)\left[1 + 4sq_{SS}\left(\rho - \eta\right)\eta\right]} > 0 \qquad (22)$$

for all $\rho > \eta$. This analysis can be summarised in

Lemma 3 *Process and product innovation are complements in the neighbourhood of equilibrium.*

That is, provided both efforts are positive, each one boosts the other in the neighbourhood of the steady state, thereby fostering the global innovative content of the monopolist's product.[10] The intuitive explanation of this result is that any reduction in c and any increase in q increase the profitability of the firm. The first implication is obvious as a lower marginal cost produces a higher profit margin, all else equal; the second can be understood noting that higher quality levels expand the gross spending capability (measured by the product θq) of any consumer, and therefore contribute to expanding sales by attracting additional consumers that would be unwilling to purchase lower qualities. The synergy between the two dimensions of the R&D portfolio highlighted in Lemma 3 is in line with empirical evidence, as we know since Cohen and Klepper (1996) and Damanpour and Gopalakrishanan (2001).

There remains to identify the steady state level of quality. To this aim, we have to impose stationarity on (4), which, using (18) and (20), now writes

[10] The same can be shown to apply at any t, using (10-13). The related calculations are omitted for brevity.

as follows:

$$\dot{q} = \frac{\Theta^2 s(\rho-\eta)\eta q[1+2sq(\rho-\eta)\eta]}{b(\rho+\delta)[1+4sq(\rho-\eta)\eta]^2} - \delta q. \tag{23}$$

For future reference, note that the r.h.s. of the above equation is discontinuous in correspondence of

$$q = -\frac{1}{4s(\rho-\eta)\eta} \equiv \widetilde{q} < 0. \tag{24}$$

Solving $\dot{q} = 0$, we obtain three roots, $q = 0$ and

$$q_{\pm} = \frac{-4b\delta(\delta+\rho) + \Theta\left[\Theta s(\rho-\eta)\eta \pm \sqrt{\Gamma\Psi}\right]}{16bs\delta(\delta+\rho)(\rho-\eta)\eta} \tag{25}$$

with $\Gamma \equiv s(\rho-\eta)\eta$ and $\Psi \equiv \Theta^2 s(\rho-\eta)\eta + 8b\delta(\delta+\rho)$. Note that $q_{\pm} \in \mathbb{R}$ and $q_{-} < -1/[4s(\rho-\eta)\eta] < 0$ in the whole admissible range of parameters, as can be easily ascertained. Since $q \leq 0$ is economically inadmissible, we are left with a single candidate, $q_{SS} = q_{+} > 0$ for all

$$\Theta > \sqrt{\frac{b(\delta+\rho)\delta}{s(\rho-\eta)\eta}} \equiv \Theta_q > 0. \tag{26}$$

Likewise, it can be easily established that k_{SS}, y_{SS}, x_{SS} and π_{SS} are positive iff $\Theta > \Theta_q$. The above expression must be evaluated against (21), which now can be rewritten as follows:

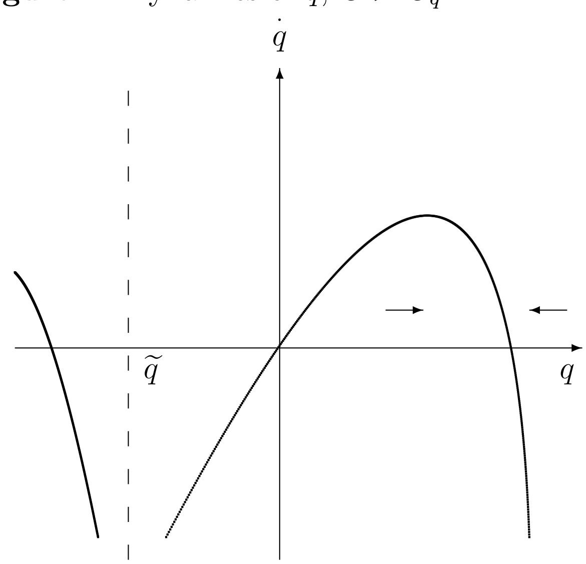
$$\Theta_{ky} = \frac{2b\eta(\delta+\rho) - s\delta(\rho-\eta)}{s(\rho-\eta)\eta} > 0 \tag{27}$$

for all $b > s\delta(\rho-\eta)/[2\eta(\delta+\rho)] \equiv \underline{b}$, with

$$\begin{gathered} \Theta_{ky} > \Theta_q \, \forall \, b > \frac{s\delta(\rho-\eta)}{(\delta+\rho)\eta} \equiv \overline{b} \\ \Theta_{ky} \in (0, \Theta_q) \, \forall \, b \in \left(0, \overline{b}\right). \end{gathered} \tag{28}$$

The foregoing discussion can be summarised in the following Figures 1-2. Figure 1 portrays the case $\Theta > \Theta_q$. In this range, the dynamics of q, illustrated by the horizontal arrows, shows that q_+ is not only positive but also stable. The vertical dashed line indicates the discontinuity at $\widetilde{q}$.

Figure 1: Dynamics of q, $\Theta > \Theta_q$.



The alternative situation occurring in the parameter range identified by $\Theta \in (0, \Theta_q)$ can be disregarded as it is not economically meaningful: in this case, indeed, product quality drops to zero at the steady state equilibrium, which also involves that sales are nil since consumers are unwilling to buy.

Figure 2 offers a partition of the parameter space $\{b, \Theta\}$ in which one can appreciate the bearings of market affluence and the steepness of R&D costs for product innovation on product quality and the relative weights of process and product innovation at the steady state equilibrium.

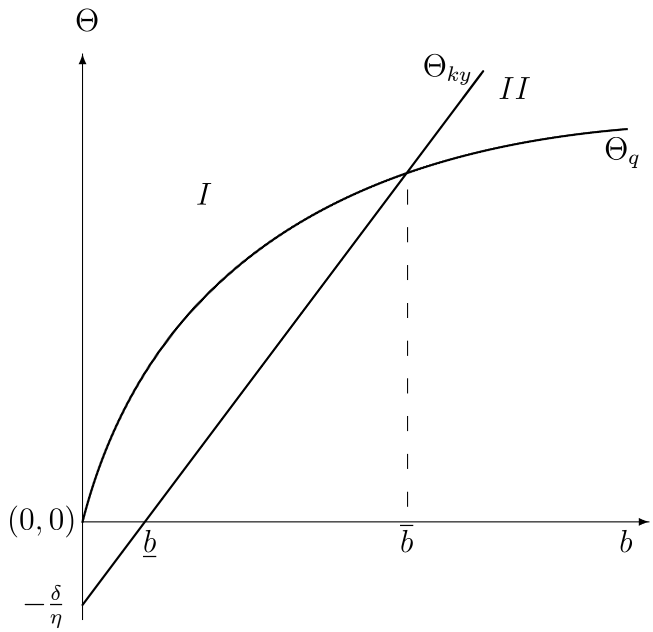
Looking at Figure 2, we must restrict our attention to the region defined by $\Theta > \Theta_q$, since below the curve Θ_q all equilibrium magnitudes are negative. We can formulate the following:

Proposition 4 *Take $\rho > \eta > 0$, and consider the range $\Theta > \Theta_q$:*

- *in area I, $\Theta > \max\{\Theta_q, \Theta_{ky}\}$. Here, $q_{SS} = q_+ > 0$ and $k_{SS} > y_{SS}$;*
- *in area II, $\Theta \in (\Theta_q, \Theta_{ky})$. Here, $q_{SS} = q_+ > 0$ but $k_{SS} < y_{SS}$.*

The second claim appearing in the above Proposition says that, for intermediate levels of Θ, although equilibrium quality is positive, the level of the richest consumer's marginal willingness to pay is low enough to modify the R&D portfolio of the firm in such a way that it finds convenient to put a higher effort in process rather than in product innovation.

Figure 2: Equilibrium analysis in the space $\{b, \Theta\}$.



To complement the analysis, one can look at the welfare consequences of the firm's decision. Define social welfare as $SW = \pi + CS$, with

$$CS = \int_{p/q}^{\Theta} (\theta q - p)\, d\theta \tag{29}$$

measuring consumer surplus. A standard question is whether the monopolist's behaviour, aimed at profit maximization, produces a welfare distortion along the quality dimension, and if so, of what sign. To this aim, use (18-19) and (20) to write

$$CS\left(c_{SS}, y_{SS}, k_{SS}\right) = \frac{2s^2 q^3 \left(\rho - \eta\right)^2 \eta^2 \Theta^2}{\left[1 + 4sq\left(\rho - \eta\right)\eta\right]^2} \tag{30}$$

In correspondence of q_{SS}, profit is maximised and therefore, if (30) is monotone w.r.t. q, then monopoly quality is necessarily socially inefficient in the neighbourhood of the steady state. In particular, it can be easily verified that (30) is monotonically increasing in q, which implies that the firm undersupplies product quality at the steady state: a benevolent social planner would welcome any increase in q to benefit consumers, given the equilibrium price-quantity schedule and the corresponding amount of bidimensional R&D chosen by the firm. This reproduces in a dynamic setup a result dating back to Spence (1975) and Mussa and Rosen (1978).

4 Stability analysis

Here we come to the core of our investigation. The stability properties of the dynamic model can be assessed using Dockner's (1985) method (as illustrated in Dockner and Feichtinger, 1991).

Substituting the instantaneous optimal controls $\{x^*, y^*, k^*\}$ into (8), we can write the maximised Hamiltonian as follows:

$$\mathcal{H}^* = \frac{sq\lambda^2 + b\left[cs\left(c + 2q\Upsilon\right) + q\left(sq\Omega + \mu^2\right)\right]}{4bsq} \tag{31}$$

where $\Upsilon \equiv 2\eta\mu - \Theta$ and $\Omega \equiv \Theta^2 - 4\delta\lambda$. Then, we can construct the following 4×4 Jacobian matrix:

$$J = \begin{bmatrix} \mathcal{H}^*_{\omega z} & \mathcal{H}^*_{\omega\omega} \\ -\mathcal{H}^*_{zz} & \rho I - \mathcal{H}^*_{z\omega} \end{bmatrix} \tag{32}$$

where each $\mathcal{H}^*_{\alpha\beta}$ is a 2×2 matrix of second-order partial derivatives, $z \equiv (q, c)$

is the state vector, $\omega \equiv (\lambda, \mu)$ is the costate vector and I is the 2×2 identity matrix. Hence, (32) rewrites as

$$J = \begin{bmatrix} -\delta & 0 & \dfrac{1}{2b} & 0 \\ 0 & \eta & 0 & \dfrac{1}{2s} \\ -\dfrac{c^2}{2q^3} & \dfrac{c}{2q^2} & \rho+\delta & 0 \\ \dfrac{c}{2q^2} & -\dfrac{1}{2q^3} & 0 & \delta-\eta \end{bmatrix} \tag{33}$$

whose determinant is

$$\Delta_J = \frac{c^2 s\Lambda - bq^2 (\delta+\rho)\,\delta\,[1+4sq(\rho-\eta)\,\eta]}{4bsq^3} \tag{34}$$

where $\Lambda \equiv (\rho-\eta)\,\eta$. At $\{c_{SS}, q_{SS}\}$, (34) is

$$\Delta_J = \frac{\Lambda\left[8b\delta(\delta+\rho) + s\Theta^2\Lambda - \sqrt{\Phi}\right]}{4b} \tag{35}$$

where

$$\Phi \equiv s\Lambda\left[8b(\delta+\rho)\,\delta + s\Theta^2\Lambda\right] \tag{36}$$

and it can be easily shown that the r.h.s. of (35) is strictly negative for all $\rho > \eta$. As we know from Dockner and Feichtinger (1991, Lemma 2, pp. 35-36), this is necessary and sufficient for one eigenvalue of J to be negative and the other three to be positive (or, for one to be positive and two with positive real parts). The four eigenvalues can be calculated using the formula in Dockner and Feichtinger (1991, Lemma 1, p. 35), the single negative one being associated to product quality. Consequently, we may formulate the following result:

Proposition 5 *In the parameter region where $y_{SS} > 0$, the dynamic monopoly model with a bidimensional R&D portfolio produces a one-dimensional stable manifold. The single stable branch is that describing quality improvement.*

This fact has an immediate and clear-cut consequence:

Corollary 6 *The monopolist's R&D portfolio is unstable, due to the dynamic properties of the branch describing the behaviour of marginal production cost.*

This amounts to saying that one should anticipate upward jumps in the marginal cost level, which of course will reverberate onto the other state variable, causing perturbations in the quality level as well. This feature is generated by the fact that the Jacobian matrix (33) is a not a block diagonal one, and suggests that the common observation whereby higher quality products are supplied at higher marginal cost might well be the consequence of the interplay between the two state dynamics - or, as it appears from the control equations (16-17), between the R&D efforts' dynamics - rather than an intrinsic economic property of higher quality goods *per se*.

Additionally, should quality have an environmental interpretation,[11] as in Arora and Gangopadhyay (1995), Bansal and Gangopadhyay (2003) and

[11] This theme is receiving a growing amount of attention in models at the intersection between the theory of industrial organization and environmental economics. For a model where green high-quality goods explicitly involve higher marginal production costs than brown low-quality ones, see André *et al.* (2009) and Lambertini and Tampieri (2012).

Amacher *et al.* (2004), the foregoing analysis would imply that green innovations taking the form of superior quality levels could not be expected to be attained at the same marginal cost as older (brown) goods or technologies. Considering also welfare implications, the downward quality distortion induced by profit incentives highlighted at the end of the previous section would also reduce the greenness of such innovations, curtailing welfare through an increase in polluting emissions and the associated environmental damage adding up to the obvious reduction in consumer surplus.

To interpret the arising of instability, one has to look back at the state-control system made up by (4-5) and (16-17). As noted above, (4-5) illustrate a situation in which the effects of R&D are independent of the levels of states, which, in combination with the simultaneous appearance of both states in the control equations (16-17), clearly implies that the resulting Jacobian matrix is not block diagonal, with instability emerging via the marginal cost dynamics (see the Appendix). This also yields a clearcut indication concerning corporate strategy: in order to avoid instability, the firm - if at all possible - should design the composition of its innovation portfolio to include R&D activities whose nature can be described by state dynamics incorporating decreasing or increasing returns to R&D in the form of multiplicative effects between states and controls. This, as shown in Lambertini and Orsini (2015), yields separated control dynamics and saddle point stability *under the same specification of consumers' preferences and market demand*. Otherwise, if state dynamics exhibit a constant effect of R&D efforts irrespective of the level of the state, the firm is bound to come to terms with the unstable behaviour of its productive efficiency.

5 Learning-by-doing

One key source of technological development adding itself to R&D efforts in both product and process innovation, can be learning-by-doing. In our model, learning-by-doing may be accounted for by adopting the same approach proposed by Thompson (2010) and applied by Li and Ni (2016) to extend the analysis in Lambertini and Orsini (2015). Their main idea amounts to modelling two distinct knowledge accumulation processes, having different growth rates. Namely, if we respectively call $A(t)$ and $B(t)$ the knowledge accumulations of process and product innovation in the interval $[0, t]$, we have:

$$A(t) = A_0 + \alpha \int_0^t k(s)ds, \tag{37}$$

$$B(t) = B_0 + \beta \int_0^t y(s)ds, \tag{38}$$

where A_0 and B_0 respectively indicate initial levels of knowledge accumulations, and α and β are positive growth rates. Inserting the instantaneous costs defined by Thompson (2010) due to learning-by-doing into (3), we obtain a cost function $\widetilde{C}(\cdot)$ which incorporates such further effects:

$$\widetilde{C}(t) = c(t)x(t) + bk^2(t) - \gamma_A(A(t) - A_0) + sy^2(t) - \gamma_B(B(t) - B_0), \tag{39}$$

where γ_A and γ_B are the positive learning rates in the two directions. Consequently, the associated profit function for the monopolistic firm is given by

$$\widetilde{\pi}(\cdot) = [(\Theta - x(t))q(t) - c(t)]x(t) - bk^2(t) + \gamma_A(A(t) - A_0) - sy^2(t) + \gamma_B(B(t) - B_0). \tag{40}$$

Note that the knowledge accumulation functions $A(t)$ and $B(t)$ become additional state variables, because they verify the dynamic constraints

$$\begin{cases} \dot{A}(t) = \alpha k(t) \\ A(0) = A_0 \end{cases}, \qquad \begin{cases} \dot{B}(t) = \beta y(t) \\ B(0) = B_0 \end{cases}. \tag{41}$$

When taking into account the above constraints as well, the maximization problem amounts to

$$\max_{x(t),k(t),y(t)} \int_0^\infty e^{-\rho t} \left[[(\Theta - x(t))\, q(t) - c(t)]\, x(t) \right.$$
$$\left. - bk^2(t) + \gamma_A(A(t) - A_0) - sy^2(t) + \gamma_B(B(t) - B_0) \right] dt \tag{42}$$

subject to

$$\begin{cases} \dot{q}(t) = k(t) - \delta q(t) \\ \dot{c}(t) = -y(t) + \eta c(t) \\ \dot{A}(t) = \alpha k(t) \\ \dot{B}(t) = \beta y(t) \end{cases} \tag{43}$$

with the related initial conditions.

The analysis of such a modified problem is more involved than the previous one, because of the resulting 8×8 Jacobian matrix. This makes it particularly difficult to calculate eigenvalues for the stability analysis. However, we are able to provide some insights.

Reconstructing the optimization procedure yields the following expressions for the derivatives of the control variables:

$$\dot{k}(t) = \frac{\dot{\lambda}_1(t) + \alpha\dot{\lambda}_3(t)}{2b}, \qquad \dot{y}(t) = \frac{-\dot{\lambda}_2(t) + \beta\dot{\lambda}_4(t)}{2s}. \tag{44}$$

Since we do not have sufficient information from the FOCs to reduce them to two ODEs depending on states and controls only, we can stress that a solution comes from the 4-equations system where $\dot{\lambda}_j(t) = 0$ for $j = 1, \ldots, 4$; λ_1 and λ_2 are attached to the dynamics of quality and marginal cost, while λ_3 and λ_4 are attached to the dynamics of knowledge accumulation in the two directions. From the system of necessary conditions, we have[12]

$$\dot{\lambda}_3 = 0 \qquad \Longrightarrow \qquad \lambda_3 = \frac{\gamma_A}{\rho} \tag{45}$$

$$\dot{\lambda}_4 = 0 \qquad \Longrightarrow \qquad \lambda_4 = \frac{\gamma_B}{\rho} \tag{46}$$

Then, by plugging them into the expressions of $k(t)$ and $y(t)$ and solving for λ_1 and λ_2, we obtain

$$\begin{cases} \lambda_1 = 2bk - \dfrac{\alpha\gamma_A}{\rho} \\ \lambda_2 = -2sy + \dfrac{\beta\gamma_B}{\rho} \end{cases} \tag{47}$$

leading to the following control equations:

$$\dot{k} = \frac{2b(\rho+\delta)k - \dfrac{\alpha\gamma_A}{\rho}(\rho+\delta) - \dfrac{1}{4}\left(\theta^2 - \dfrac{c^2}{q^2}\right)}{2b} \tag{48}$$

$$\dot{y} = \frac{-(\rho-\eta)\left(-2sy + \dfrac{\beta\gamma_B}{\rho}\right) - \dfrac{1}{2}\left(\theta - \dfrac{c}{q}\right)}{2s} \tag{49}$$

[12] Once again, we omit time dependence whenever possible to save on notation.

The determination of the steady states of the above equations is analogous to the one carried out in the absence of learning-by-doing effects. If we call k_{LBD} and y_{LBD} the controls in this scenario, we can note that

$$k_{LBD} = k_{SS} + \frac{\alpha\gamma_A}{2\rho b}, \qquad y_{LBD} = y_{SS} + \frac{\beta\gamma_B}{2\rho s}, \tag{50}$$

i.e., the steady state values of R&D controls (and therefore also of states) generated under learning-by-doing can be reached from the previous ones by a simple translation determined by learning-by-doing, discounting and the steepness of the instantaneous R&D costs. When learning-by-doing is nil along both dimensions, the equilibrium structure collapses to the basic one.

To close the analysis carried out in this section, it is worth noting that the inclusion of learning-by-doing does not modify significantly the stability analysis, as the marginal cost dynamics is still responsible for the instability of the entire system. We have omitted the detailed demonstration of this fact, which is a straightforward consequence of the form of (4-5) combined with (48-49), where the nonlinear interplay between states and control persists.

6 Concluding remarks

We have assessed the relationship between the development of different forms of innovation carried out by a monopolist over time. The dynamic scenario we have described provides interesting insights about the properties of the firm's optimal R&D portfolio and the impact of learning-by-doing on the life cycle of a product or service. The main findings can be recapped as follows:

- Product improvement and process innovation can coexist and the former may indeed prevail over the latter, if the market is sufficiently rich, i.e., if the richest consumer's marginal willingness to pay for quality is sufficiently high.

- When both R&D efforts are positive, then they are complements at equilibrium, thus boosting each other. Equilibrium quality falls short of the socially efficient level.

- The R&D portfolio of the firm is affected by an intrinsic instability as its efforts to drive marginal cost downwards are bound to fail because of the dynamics of the latter. Consequently, having a higher quality at lower production cost and market price may well remain a mirage for the firm and for its customers alike.

The third result is the most relevant one, in that it exhibits a peculiar property connecting quality level and market price, which may explain such market phenomena as products which undergo massive innovation without any price reduction. However, it is worth stressing that this conclusion is the outcome of an infinite horizon model in which consumers have hedonic preferences. Under different but equally plausible assumptions, stable equilibria exists, characterised by increasing quality and decreasing price. For instance, see El Ouardighi and Kogan (2013) and El Ouardighi (2014), where product quality exerts a multiplicative effect on a linear market demand function and the time horizon is finite.

Further developments of the present line of research can be envisaged. For instance, it would be relevant to reconstruct the same analysis in an oligopoly model under feedback information, to deduce possible differences with the monopolistic market outlined here. Moreover, another issue remains open to both theoretical and empirical investigation: which production sector is more affected by the above effects? In other words, is it possible to systematically identify sectors in which this intrinsic instability does not emerge? An isolated example is in Lambertini and Orsini (2015), which, however, does not provide a definitive answer. Hence, this task is left for future research.

Appendix

The dynamic properties of the system of state and control equations emerging from the analysis of the Jacobian matrix (33) can be further illustrated by considering each of the two 2×2 matrices along the main diagonal of the 4×4 Jacobian matrix deriving from the state-control system made up by (4-5) and (16-17):

$$J(q,k,c,y) = \begin{bmatrix} \frac{\partial \dot{q}}{\partial q} & \frac{\partial \dot{q}}{\partial k} & \frac{\partial \dot{q}}{\partial c} & \frac{\partial \dot{q}}{\partial y} \\ \frac{\partial \dot{k}}{\partial q} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial c} & \frac{\partial \dot{k}}{\partial y} \\ \frac{\partial \dot{c}}{\partial q} & \frac{\partial \dot{c}}{\partial k} & \frac{\partial \dot{c}}{\partial c} & \frac{\partial \dot{c}}{\partial y} \\ \frac{\partial \dot{y}}{\partial q} & \frac{\partial \dot{y}}{\partial k} & \frac{\partial \dot{y}}{\partial c} & \frac{\partial \dot{y}}{\partial y} \end{bmatrix} \tag{51}$$

This amounts to illustrating the features of the dynamic interplay between (i) q and k for a given level of $c \geq 0$ and (ii) c and y for a given level of $q > 0$. That is, these two exercises serve the purpose of grasping the nature, respectively, of quality improvement (or product innovation) for an exogenous marginal production cost and cost reduction (or process innovation) for an exogenous quality level. The first submatrix is

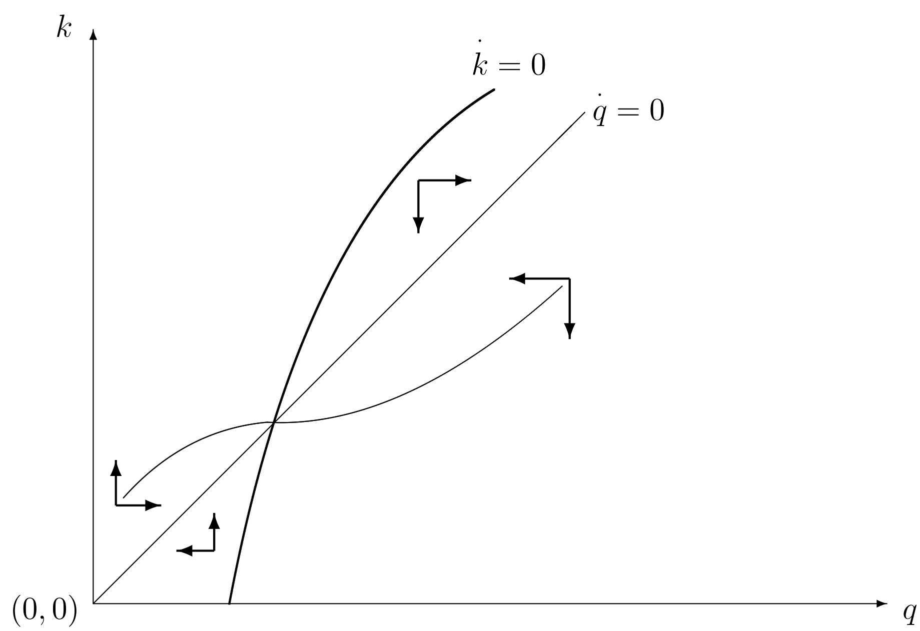
$$J(q,k) = \begin{bmatrix} \frac{\partial \dot{q}}{\partial q} & \frac{\partial \dot{q}}{\partial k} \\ & \\ \frac{\partial \dot{k}}{\partial q} & \frac{\partial \dot{k}}{\partial k} \end{bmatrix} = \begin{bmatrix} -\delta & 1 \\ & \\ \frac{c^2}{4bq^3} & \delta + \rho \end{bmatrix} \tag{52}$$

whose trace and determinant are $T_{J(q,k)} = \rho > 0$ and

$$\Delta_{J(q,k)} = -\delta\left(\delta+\rho\right) - \frac{c^2}{4bq^3} < 0 \tag{53}$$

Since $\Delta_{J(q,k)} < 0$, this side of the firm's dynamic problem is stable in the saddle point sense for any level of the marginal production cost. This fact is portrayed in Figure A.1, where the arrows illustrate the presence of a saddle path to the equilibrium.

Figure A.1: The phase diagram in the space (q,k) for a given c



The second submatrix is

$$J(c,y) = \begin{bmatrix} \dfrac{\partial \dot{c}}{\partial c} & \dfrac{\partial \dot{c}}{\partial y} \\ \\ \dfrac{\partial \dot{y}}{\partial c} & \dfrac{\partial \dot{y}}{\partial y} \end{bmatrix} = \begin{bmatrix} \eta & -1 \\ \\ \dfrac{1}{4sq} & \rho - \eta \end{bmatrix} \tag{54}$$

Its trace is $T_{J(c,y)} = \rho > 0$ while its determinant is

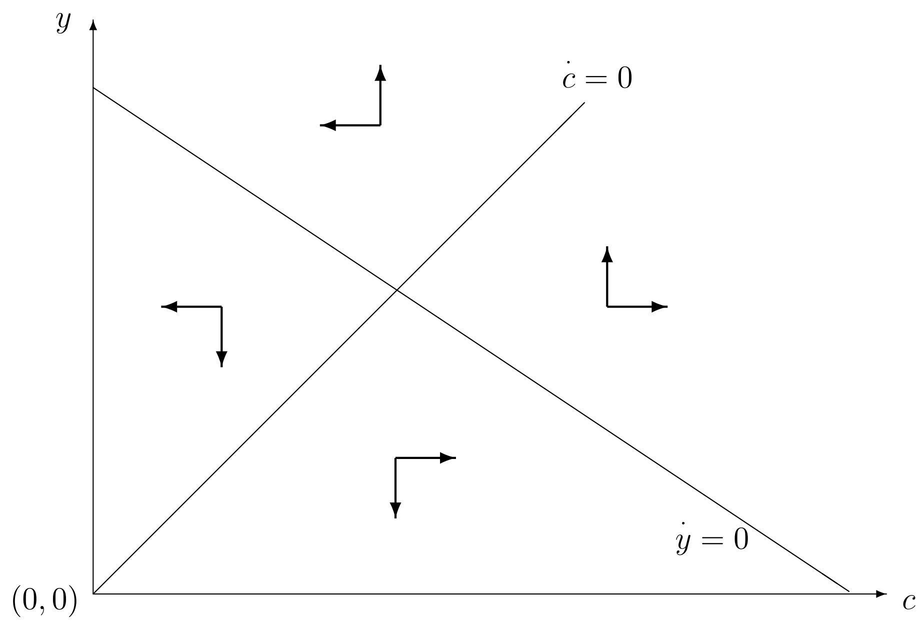
$$\Delta_{J(c,y)} = \eta(\rho - \eta) + \frac{1}{4sq} \tag{55}$$

which, in view of the requirement $\rho > \eta$, is positive everywhere. Whenever the trace and determinant are both positive, the system is necessarily unstable (see Mehlmann, 1988, p. 145). If $\Delta_{J(c,y)} > T^2_{J(c,y)}/4$, we have an unstable focus. Otherwise, if $\Delta_{J(c,y)} \in \left(0, \mathcal{T}^2_{J(c,y)}/4\right]$, we have an unstable node. Here both cases are admissible in line of principle, as the difference $\Delta_{J(c,y)} - T^2_{J(c,y)}/4$ has the sign of

$$sq\left[4\eta(\rho - \eta) - \rho^2\right] + 1 \tag{56}$$

where $4\eta(\rho - \eta) - \rho^2$ is nil at $\eta = \rho/2$, otherwise it is negative. Hence, in the special case $\eta = \rho/2$, $\Delta_{J(c,y)} > T^2_{J(c,y)}/4$ and the solution of the state-control system $\left(\dot{c}, \dot{y}\right)$ is an unstable focus. For any other $\eta < \rho$, it can be either an unstable focus or an unstable node depending on the values of q and s. The arising of instability is illustrated by the arrows in the phase diagram appearing in Figure A.2.

Figure A.2: The phase diagram in the space (c, y) for a given q



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