

RICERCA - FORMAZIONE

Formative Assessment *for mathematics* teaching and learning

Teacher Professional
Development Research
by Videoanalysis
Methodologies

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R-F

Ricerca-Formazione

collana diretta da

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La Collana accoglie studi teorici ed empirico-sperimentali che indagano il rapporto tra ricerca e formazione degli insegnanti; essa nasce dalla comune volontà di un gruppo di studiosi e ricercatori di diverse università italiane interessati a questa tematica e con specifiche competenze di ricerca in ambito educativo.

I continui cambiamenti che attraversano il mondo della scuola e che coinvolgono direttamente coloro che operano al suo interno in qualità di insegnanti, dirigenti, educatori, necessitano di professionalità altamente specializzate e allo stesso tempo flessibili, in grado di interpretare le trasformazioni in atto e di gestire la complessità che oggi è presente nei contesti scolastici. Per questo è importante promuovere un rapporto sempre più stretto e sinergico tra la ricerca accademica e la scuola, affinché questa relazione possa essere letta in modo biunivoco e paritario.

La formazione iniziale e in servizio del personale scolastico, e degli insegnanti in particolare, rappresenta una leva decisiva per il miglioramento della qualità dell'offerta formativa, l'innalzamento dei risultati di apprendimento degli alunni e il funzionamento delle istituzioni scolastiche, in un'ottica di equità e di democrazia del sistema di istruzione. La ricerca educativa, con i suoi molteplici approcci teorici e metodologici, deve poter offrire nuovi ambiti di riflessione e strumenti d'intervento per formare competenze e sostenere lo sviluppo professionale degli insegnanti. La possibilità di progettare, realizzare e monitorare interventi e strategie efficaci, sul fronte sia della ricerca sia dell'educazione e dell'istruzione, nasce dalla capacità di far interagire competenze diverse e attivare processi didattici e organizzativi rispondenti ai bisogni di bambini, giovani e adulti. In tale prospettiva, si può parlare di metodologie orientate alla ricerca-formazione, da considerare soprattutto come una scelta metodologica per fare ricerca con gli insegnanti e per il loro sviluppo professionale e il miglioramento della scuola. Una scelta che caratterizza, accompagna e sostanzia (nelle sue finalità e procedure applicative) le specificità e il rigore dei vari approcci metodologici della ricerca empirica, nelle loro declinazioni di volta in volta quantitative, sperimentali, fenomenologiche e qualitative.

La ricerca-formazione pertanto, oltre a rappresentare un settore di studio interdisciplinare, che comprende molteplici apporti teorici ed epistemologici, viene considerata, all'interno della presente collana, soprattutto come un modo di fare ricerca insieme ai professionisti dell'insegnamento, inaugurando nuovi campi d'azione verso cui convogliare risorse e interessi comuni. In questo senso, la collana valorizzerà contributi capaci di evidenziare la contiguità tra insegnamento e ricerca, prestando particolare attenzione alle modalità di coinvolgimento degli insegnanti, al rigore procedurale, alla ricaduta formativa dei risultati raggiunti.

In particolare, gli aspetti presentati di seguito delineano l'idea di Ricerca-Formazione cui la collana si ispira; essi possono pertanto costituire un orientamento per gli autori.

Una Ricerca-Formazione, per essere tale, richiede:

1. una esplicitazione chiara della finalità della ricerca in termini di crescita e sviluppo della professionalità degli insegnanti direttamente coinvolti e un'attenzione a documentare e analizzare le ricadute in termini di cambiamento;
2. la creazione di un gruppo di R-F di cui facciano parte ricercatore/i e insegnanti, nel quale vengano chiariti i diversi ruoli dei partecipanti e in cui vengano negoziati e chiariti obiettivi e oggetti, scelte valoriali e metodologiche della R-F;

3. la centratura sulle specificità dei contesti - istituzionali e non - in cui si svolge la R-F, che si concretizza in tutte le fasi della ricerca attraverso un'analisi dei vincoli e delle risorse in essi presenti;
4. un confronto continuo e sistematico fra i partecipanti alla ricerca sulla documentazione dei risultati e dei processi messi in atto nei contesti scolastici e in quelli della formazione;
5. l'attenzione alla effettiva ricaduta degli esiti nella scuola, sia per l'innovazione educativa e didattica, sia per la formazione degli insegnanti.

La collana intende accogliere contributi di studiosi italiani e di altri paesi, sotto forma di monografie, volumi collettanei, rapporti di ricerca e traduzioni relativi a studi e ricerche che realizzino una sinergia tra università e scuola, compresi volumi che documentino percorsi di Ricerca-Formazione realizzati nelle scuole.

Una particolare sezione della Collana accoglierà inoltre volumi relativi a risultati di ricerche empiriche che affrontino specificamente le questioni della formazione alla/della professionalità docente.

La collana è diretta da un gruppo di quattro studiosi di diverse università italiane che condividono finalità e scelte metodologiche del progetto editoriale e che mantengono un rapporto di confronto e di scambio costante con il Comitato scientifico.

Attraverso la collana, la Direzione e il Comitato scientifico intendono promuovere un ampio confronto tra ricercatori, studiosi, insegnanti, educatori e tutti coloro che a diverso titolo sono coinvolti nei processi di istruzione e formazione.

Comitato scientifico

Lucia Balduzzi (Bologna); Anna Bondioli (Pavia); Cristina Coggi (Torino); Martin Dodman (Bolzano); Giuliano Franceschini (Firenze); Antonio Gariboldi (Modena e Reggio Emilia); Laurent Jeannine (Cergy Pontoise, Paris); Patrizia Magnoler (Macerata); Elisabetta Nigris (Milano-Bicocca); Loredana Perla (Bari); Graziella Pozzo (Torino); Raúl Ruiz-Cecilia (Granada).

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CENTRO DI RICERCA EDUCATIVA
SULLA PROFESSIONALITÀ DELL'INSEGNANTE



Disegno di Aldo Spizzichino

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2. Why formative assessment in Mathematics?

by *Andrea Ciani**, *Ira Vannini**, *Federica Ferretti***

1. Introduction

Assessment in classroom has always been a key tool in order to promote, or to hinder, democratic values at school. An education system that does promote quality and equity for the learning achievements of its students, uses assessment as a key element to qualify the action of teaching in a democratic way, both at the beginning and during the process of teaching-learning; moreover it will consider the differences among the students and their possible learning difficulties as opportunities to make the teaching actions flexible in order to reach goals of quality for all (Vertecchi, 1976; Grandi, 1977; Weeden, Winter, Broadfoot, 2002).

As we can read in Crahay (2013), it has to be a kind of assessment which adheres to a principle of equality of achievements (Bloom, 1968; Black, William, 1998; Guskey, 2005), hence to an idea of “fairness” in teaching, by offering more to whom possesses less.

This need of fairness in achieving the competences for citizenship (OECD, 2015; Eurydice, 2012) is more evident in every education system when considering basic competences and at high and junior high school level, before the completion of the compulsory cycle of studies. In particular, relevant problems appear in the field of math teaching, with important gaps in the conduct of the specific teaching-learning processes.

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2. Formative Assessment: assessment for learning for all the students

Since its origins (Scriven, 1967; Vertecchi, 1976), the main function attributed to formative assessment (FA) is to be a regulator tool for teaching and learning.

Referring to the current international scientific debate on this issue, we can say that formative assessment is characterized specifically as an assessment for learning (Weeden *et al.*, 2002; Allal, Laveault, 2009). This means that it has to be an assessment which is functional to backing up and promoting learning; it is embedded in the teaching-learning process in a dynamic way, modifying the teaching actions by following the needs of the students. The aim will never be to just attribute marks, or to make a résumé on the abilities of a student; formative assessment helps a teacher to gather information to improve and to get her/his teaching action more effective.

Thus, when a teacher uses formative assessment, s/he is implementing two fundamental actions (Vertecchi, 1976):

- a diagnostic analysis of the achievements (knowledge, abilities) that the student is acquiring and of which meta-cognitive strategies the student is following;
- a reconstruction of the teaching path by following the student's needs and differentiating times and methods of the didactic process.

Thanks to this diagnostic function, formative assessment analyzes the learning situations and can give information in order to take coherent and effective decisions. It focuses on the “errors” of the student and of the teacher, by considering them as resources for designing and re-designing interventions in view of the teaching goals.

This kind of assessment requires a high professionalization of the teacher, which has to implement a continuous assessment attention during the teaching process, as a coach in the training of an athlete or a team (Bennet, 2010; 2015) who proposes activities and tasks to the trainees (as a trial for their abilities), detects and immediately corrects their errors (by discussing with the trainees about them), understands the specific needs and gives formative feedback.

As every assessment procedure (Gattullo, 1967), also FA is characterized by three steps (Gitomer, Zisk, 2015, p. 3):

- an initial step of cognitive representation of which data we want to collect (“what we are trying to measure”);
- a step of specific gathering of data, by empiric observation (“how we collect evidence”);
- the interpretation of the data (“how we make sense of the evidence”).

Collecting evidence is an unavoidable phase (Ruiz-Primo, Furtak, 2004) inasmuch as it characterizes FA as a specifically evaluative action, both when it is formally or informally done (informally as in the course of a teacher's day-to-day activities) (Bell, Cowie, 2001; Duschl, 2003; Shavelson *et al.*, 2003). Without a willing gathering of evidence we would not be doing FA, but just a teaching activity.

The next step, data interpretation, is equally important. Doing summative assessment this step would end in the attribution of marks or of a judgment, doing FA any judgment is suspended. It is formative feedback that must take place in this moment, instead: the teacher's answer to the needs/requirements of the student. Researches show that feedback – together with FA – is the crucial element, which contributes, in a statistically significant way, to improve the results in the students' learning (Hattie, Timperley, 2007; Hattie, 2009; 2012; Huelser, Metcalfe, 2012).

For this reason the teacher's practices in class are particularly important, both in the moment when data about the students' achievements are gathered and analyzed, and when interpreting the data, elaborating hypotheses about the kind of mistakes the students do and implementing feedback actions to help them in the critical steps in their apprehension. All this is really fundamental in the teaching of mathematics.

The feedback activity is a complex set of actions by the teacher, not easily described by a set of rules or given operations; for this reason researches in this field are particularly relevant and compelling: it is necessary, in fact, to clear up, in detail, which are the most effective conducts that the teacher has to implement in classes activities when facing a “stumbling” student.

Via the feedback, the teacher should manage to make the student's errors explicit, and make them valuable as an asset in the learning process; in the meanwhile the teacher has to sustain the students' motivation to learn and to mobilize all their meta-cognitive strategies in order to overcome the obstacles. Here the didactic mediation is substantial; the teacher must use several and differentiated didactic tools, give additional explanations, sustain the students' aloud reasoning (Weeden, Winter, Broadfoot, 2002; Bennet, 2010; Doabler *et al.*, 2014).

Several researches about teachers' behavior highlight that they agree about the fundamental role of FA for the quality of teaching, nevertheless – in their practice – they follow more often summative assessment praxes. Also, in spite of the fact that they use FA in their ongoing activities, they may use superficial tests, propose mechanical answers, or give a feedback that is too generic (Looney, 2011, p. 10). In fact, they seem not to be prepared to interpret the evidence they have about their students' learning and often they attribute to external reasons the impossibility to

implement FA (too many students in their classes, too large curricula to teach, organizational difficulties in their schools) (OECD, 2005).

3. International debate: different views on the formative assessment

The formative assessment, as it has been outlined since its inception, has been conceived, imagined, designed to increase students' learning. Black & Wiliam studies and researches' have allowed to spread and share in the world an idea of FA and to support the efficacy of its practice. Although Black & Wiliam works' was referring to local contexts, data, considerations, it became a fundamental basis for the comparison and development of the international debate on the FA.

The international debate on the FA is based on additional interpretations or new approaches, without contradicting its initial conception even if the educational perspectives of application could be radically different (e.g. FA in behaviourist or in social-constructivist view).

In general, as Torrance (2012, p. 324) has documented, over the years the FA has defined more its meaning and the spectrum of related practices:

from mastery learning and criterion-referenced graded assessment programmes in the 1970s and 1980s (Bloom, 1974; Popham, 1978; Pennycuick & Murphy, 1988); through systems approaches to feedback (Ramprasad, 1983; Sadler, 1989); to the variety of approaches to recording, reporting and profiling achievement which emerged in the 1980s and 1990s in the UK (Broadfoot, 1986; Torrance, 1991; Pole, 1993). Currents of formative assessment, also known as 'assessment for learning', are often linked to Black and Wiliam's (1998a) review of the literature, subsequent developments by the UK Assessment Reform Group (2002) and associated work by Black and colleagues.

If initially, in the U.S., early interest tended to revolve around mastery learning e and formative assessments' practices to develop programmes of 'measurement-driven instruction' (Popham, 1987; Airasian, 1988), over time a wider vision has been established, associating it with the assessment for learning.

Stiggins (2005), one the first promoters in North America of this vision, explains how assessment for learning is an approach to the formative assessment and defines its differences with a traditional view. In its traditional form, formative assessment has been thought of as providing teachers with more frequent evidence of students' mastery of standards to

help teachers make useful instructional decisions. In this way, formative assessment is intended to enhance student learning.

The purpose is using a lot of different evaluations methods to provide students teachers and parents with a continuous flow of tests of the student progress in mastering competences that support or guide to set the standards: during the learning, students are inside the assessment process, watching them grow, and feeling in control of their success. The most important difference between the traditional approach and assessment for learning is that the former intend to inform the teachers about student achievement, while the latter also wants to inform students about their own learning.

Assessment FOR learning rests on the understanding that students are data-based instructional decision makers too, a perspective all but ignored in our assessment legacy and in previous approaches to school improvement. Another difference is that traditional formative thinking tends to want more frequent assessment of student mastery of the standards themselves, while assessment FOR learning focuses on day-to-day progress in learning as students climb the curricular scaffolding leading up to state standards (Stiggins, 2005, p. 328).

It is precisely on the guiding principles of assessment for learning that different ways of viewing the FA are born. In fact, in a similar way to what happens in the US, the initial conception of FA proposed by Bloom has been enlarged with other elements, in particularly

In the enlarged perspective of formative assessment developed in French-language publications, the idea of remediation of learning difficulties (feedback + correction) is replaced by the broader concept of regulation of learning (feedback + adaptation). In an enlarged conception, external regulation (by the teacher, by the test, by remedial material) is redefined as scaffolding that assists students' development of self-regulation. [...]This means fostering the active involvement of students in formative assessment through procedures of self-assessment, reciprocal peer-assessment, and joint teacher-student assessment (Allal, 2005, p. 245).

In the French FA enlarged perspectives, the learning regulation (interactive, retroactive and proactive) is fundamental. Allal writes that concept of regulation in the French-language literature about FA is linked to:

- the degree of active student involvement in these actions;
- the ways students make use of tools and resources present in the instructional environment to adapt or enrich their learning activity;
- the meaning the students and teachers give to the various aspects of assessment;
- the ways in which teachers and students negotiate assessment.

Thus, vision that emerges in the French pedagogical debate supports a socio-constructivist perspective of FA, close or almost superimposable to the approach of *assessment for learning*.

Although in Germany the FA in a Bloom/Black&William sense is not consolidated yet and there are few FA studies in comparison with UK, US and France, the alternative education has emphasized that teachers should be aware of how they provide feedback to students, as feedback indicating personal growth to students could foster their learning and motivational development (Köller, 2005).

In Italy, as in several other countries, the effort to promote a FA culture in schools following the Bloom perspective is quite substantial, above all in universities and in the world of educational research, but is also increasingly combined with the attention to the teaching, motivational and communicative processes. For this reason, in Italy people talk more and more of FA following the *assessment for learning* approach and the FAMT&L research has adopted this term to indicate a complete and upgraded view of FA.

4. Formative Assessment in Mathematics

The important role of the assessment is underlined in most National Curricula and it is entrusted to teachers, individual schools and ministerial institutions. The assessment is under the responsibility of the teacher in all the phases, both of the planning and of the operational process of teaching; for instance, in Italian National Curricula, the formative function of evaluation is clearly emphasized, as are its crucial role inside the learning process and its function as continuous stimulus for improvement.

Formative assessment for learning should indeed be essential part in all phases of the process of teaching and learning. From this perspective, its main function is regulative and its main objective is to help teachers and students to continuously adapt the teaching/learning process, especially if it is oriented to the individualisation of teaching procedures (Tornar, 2001). Specifically, formative and diagnostic function of evaluation is carried out, referring to a formative path, *ex ante* and *in itinere* with the aim of collecting detailed information both at initial stages and during the process of learning, whenever students are faced with difficulties, so to design educational targeted interventions and to have a constant and reciprocal feedback between teacher and pupil. Hence the purpose of formative assessment is essentially educational and its main aim is to provide feedback and information to the teachers, so that they can then do something to promote their students' learning. Formative assessment

is also characterized by the fact that it does not provide the explanation of a judgment or a vote, but is rather characterized by a significant enhancement of errors, considered as a resource to improve the quality of the educational path (Gagatsis & Christou, 1997; Gagatsis & Kyriakides, 2000, Zan, 2007). Some studies have allowed us to collect empirical evidence that could prove, for example, that the systematic use of the *in itinere* evaluation of the students' progress generates significant increases in their final performance. Moreover, the quality of the feedback, the active participation of the students in the evaluation process, some aspects of verbal interaction in the classroom and the effects of evaluation on self-confidence and motivation, have shown themselves to be crucial for the effectiveness of formative assessment to promote learning (Scallon, 1985; Black & William, 1998). From these studies a discussion at the international level has developed, getting to the conclusion that formative assessment is an "assessment for learning" (Weeden *et al.*, 2002).

5. Why formative assessment in Mathematics?

Teaching and learning processes concerning mathematics and sciences are a fundamental component of school activities, and they are preliminary to many of the skills that are significant in life and necessary for the citizenship formation. Mathematics paves the way for different ways of thinking and for applications that affect daily life, allowing critical interpretation and assessment of the huge amount of information produced in the modern learning society. Moreover, it also determines the practice of citizenship through logical reasoned and motivated decision-making processes about social issues.

Without adequate competence-building in mathematics (thinking, logical reasoning, etc.) it becomes virtually impossible to fully access the contemporary world made by information, communication and technology. This affects the opportunities for all to be involved in social and economic life. This idea is also stressed in the concept of "mathematical literacy", as defined in the OCSE Programme for International Student Assessment – PISA (OECD, 2013) as the capability to identify, understand and engage in mathematics, and to make well-founded judgements about the role that mathematics plays in an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen.

Despite researchers' and teachers' engagement, the crisis in mathematical education & learning is becoming very diffused and profound. Mathematics tends to be seen by young and adults as an uninteresting discipline.

According to recent international research (OECD, 2014; Eurydice, 2012), the main difficulties in mathematics learning are represented by:

- a severe lack of mathematical skills among students aged 15th in many European countries (OECD, 2005) based on problems in mathematical didactics starting from the 11-14/15 years age group;
- teaching methodologies and pedagogic lack in giving a broad “sense of number” and the ability to work with it (with figures, measures, statistics and probability);
- incorrect use of formative assessment and need to introduce strategies of teaching and learning individualization;
- in general, rising de-motivation for school learning, starting especially in the 11-14/15 years age;
- more specifically, de-motivation for learning mathematics.

The 11-14/15 years age group appears to be the one that requires innovative and adequate interventions to improve mathematical and – especially – numeracy skills, reclaiming both to mind for the lost ground in mathematical curiousness amongst youngsters as well to bring all students to obtain satisfactory results with a better and broader commitment to mathematical reasoning in any social-economic application.

It appears to be important to design innovative ways to invest in teacher training for mathematics, in particular through training paths that put teachers in collective planning situations and bring them to build plans, strategies and methodologies for teaching mathematics effectively. It goes without saying that this does not only means to investigate on the subject, but also on the pedagogical-didactical learning strategies, in particular in the field of assessment, stressing the value of formative assessment and individualized teaching strategies, planned to specifically respond to students’ rhythm and learning styles.

For this reason, it was very important to investigate on mathematics teachers’ beliefs and misconceptions about assessment in the classroom and to analyse learning activities in the classroom, investigating on teachers’ rationales behind learning difficulties in mathematics in order to plan adequate interventions for remedial programming.

At the same time, we think that each teacher should acquire the necessary competences for reflecting on the content of mathematical education as well as on the pedagogical-didactical aspects. This implies to have an internal focus on mathematics itself and an external focus on the links of mathematics to other subjects. In both cases, it is important for teachers to learn, assess and diagnose students’ learning needs and to give answer to them with adequate teaching methodologies.

During all project activities and implementations, the mathematics has been seen and validated as a multi-functional and multi-disciplinary

subject in school. One of the fundamental principles on which we are based is that the teachers need to be aware about their crucial position in school and about their need to reflect on difficulties and mistakes, to find the causes of them and to plan the interventions for remedial programming through efficacy strategies and tools of formative assessment.

In this perspective it is therefore necessary to train mathematic teachers not only in mathematics and didactics of mathematics, but also in the application of these disciplines in their own context. From this point of view, the solution lies mostly in the hands of teachers and teacher-trainers, more specifically in designing and giving them the instruments of educational planning and assessment.

6. FAMT&L view on the Formative Assessment

In 2008, Sattler has defined assessment as a way to understand a student, in order to make informed decisions related to classroom practices that involve him. In most theoretical frameworks for evaluation the first step of the evaluation process it is the diagnostic assessment, but it is quite of common knowledge that the evaluation process, and in particular the role of diagnostic and formative assessment, is often based on mechanical procedures influenced by implicit philosophies of each teacher, in particular in mathematics (Speranza, 1997).

Since the '90s there has been a growing awareness about the need for significant changes in assessment practices of learning in mathematics and likely this is related to the fact that more and more mathematics has been identified as a key competence within the frame of life skills (Shepard, 1989; Webb & Coxford, 1993). The evaluation process is considered as an integral part of the learning process (Desforges, 1989) and the importance attached to the assessment in mathematics is reflected both by the impact of increasing disciplinary national and international standardized assessments (see, for example, OECD, 2015) and by the research work about the practices of classroom assessment (DES, 1987; NCTM, 1989).

In detail, as for learning mathematics, usually teachers assess knowledge, skills and abilities of students. In the school practice, assessment in mathematics is often divided and organized on the base of the content of teaching, although it is an increasingly shared notion that to properly assess cross components of learning is needed. In recent times especially there is an increasing interest on what competence in mathematics means and how it can be evaluated. FA is connected with a concept of learning according to which all students are able to acquire, at a adequate level, the basic skills of a discipline. Learning passes through the

use of teaching methodologies which can respond effectively to different learning time for each student, to different learning styles and to zones of proximal development.

Basing on these references and results, we adopt the following definition of *formative assessment*:

FA in the classroom is an assessment FOR teaching and learning.

It is part of the teaching-learning process and regulates it; it identifies, in an analytical way, the strengths and weaknesses of student's learning in order to allow teachers to reflect on it and maybe modify their own practices; it allows a formative feedback to establish a dialogue between teacher and student and to design educational interventions aimed to the recovery; it promote and foster the learning by all students through differentiated teaching that ensures each student different rhythms and different teaching and learning strategies; it *involves the students in the analysis of their own errors/weaknesses and their own ability to promote self- and peer-assessment and active participation in the teaching-learning process.*

It is intended to give information, feedback and feed forward – in and outside of the classroom – related to the development of mathematical life-skills. In particular:

- it addresses the different components and aspects of mathematical learning of the students (conceptual, procedural, communicative, semiotic);
- it is involved in analyzing problem posing and solving strategies, misconceptions, organization of mathematical experience, students' beliefs, students' image of mathematics and of specific segments of mathematics; students' behavior and classroom interaction when involved in different mathematical tasks;
- it is a critical issue to make clear what the outputs of teacher's choices are (transposition of mathematical contents, interface between contents and methods).

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