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SOME RECONSTRUCTION HYPOTHESES OF LEONARDO'S PROJECT FOR THE *TIBURIO* OF THE MILAN CATHEDRAL BY USING 3D DIGITAL MODELS

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Abstract

This paper analyses possible interpretations of the unbuilt Leonardo's project for the *tiburio* of the Milan cathedral, depicted in two sheets of the *Codex Atlanticus*, using digital tools, mainly 3D modelling techniques. Starting from existing studies and hypothetical reconstructions of Leonardo's project, exploiting only analogue methods (i.e. 2D drawing techniques), new in-depth analyses are presented. In detail the use of 3D modelling allows a systematic analysis of the possible solutions for the ability to easily reconstruct the plan of each solution investigated starting from the section represented by Leonardo in the *Codex Atlanticus*. Results lead to multiple interpretative solutions of this complex architecture.

Keywords

Leonardo da Vinci, *Tiburio*, Cathedral, Milan, Unbuilt architecture, Virtual reconstruction, 3D modeling.

1. *Introduction*

The present contribution analyses some possible reconstructions of Leonardo's project for the *tiburio* of the Milan cathedral using digital tools, mainly 3D modelling techniques.

In 1487 the Milan authority for the construction of the cathedral, the *Fabbrica del Duomo*, asked architects and engineers for a proposal of completion for the *tiburio* and the solution of a structural problem concerning the main pillars (Beltrami, 1903; Pedretti, 1978). Leonardo gave a detailed proposal displayed in two sections/projections drawn on two different paper sheets today classified as f. 850r and f. 851r of the *Codex Atlanticus* (CA), and at least in one wooden model, today lost.

Leonardo's drawings are complex plots not allowing an easy reading, both for the lack of an associate plan and for the difficulty to distinguish between section and foreshortened elevation, most likely coexisting, to which is added the presence of pricked marks of the pouncing technique, which therefore indicate a true size of proportions, even of those elements that should be foreshortened (for example the ribs of the vaults).

The fact that the project was never built contributes to the difficulty of understanding how he imagined the solution for the *tiburio* figured in these two sheets. The information that several other sketches provide regarding this project is also limited.

Even if these sketches tell us that Leonardo, during his entire life, carried out in-depth studies upon that matter, always referring to ancient and contemporary architecture and analyzing also structural problems and practical constructive solutions (Firpo, 1963; Maltese, 1975; Frommel, 2019), however no decisive contribution is given to research about his real conception of the *tiburio*.

This ambiguity generated over time different interpretations, both in the project as a whole and in the details of the structure.

The aim of a new study of the subject of these two drawings using digital reconstruction tools is, then, to understand Leonardo's project (Fig. 1), limiting the misunderstandings of the translation of the two-dimensional into 3D, and analyzing only a small group of different, but consistent and valid interpretations of Leonardo's design.

Digital techniques, mainly the use of 3D modelling systems, allow a systematic analysis of Leonardo's possible intentions for their ability to easily reconstruct the plan of each solution investigated starting from the section. More, digital vector-based representations allow to

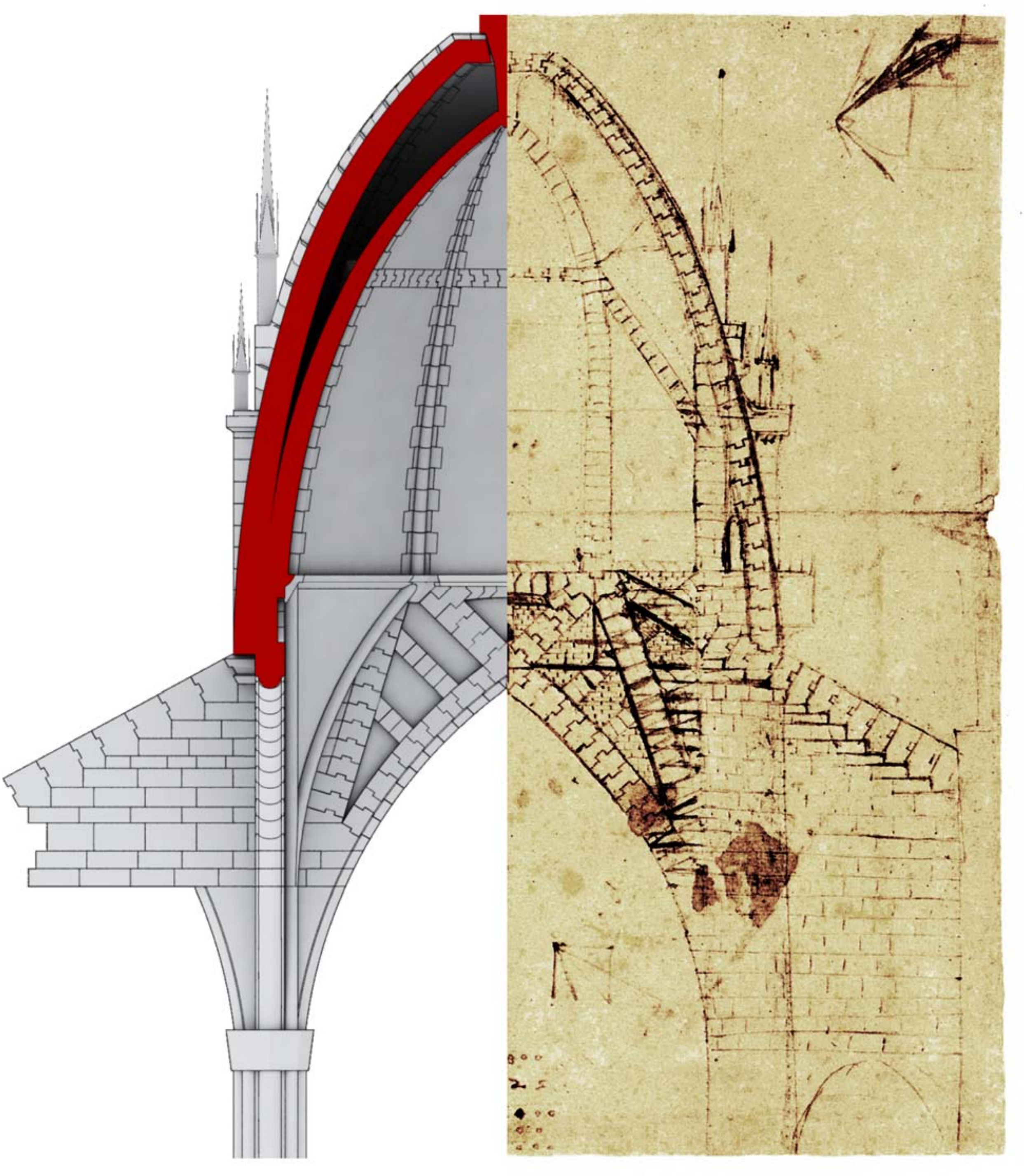

Fig. 1: Comparison between the 3D reconstruction and an edited detail of the f. 850r, Codex Atlanticus.

quickly generate multiple solution variants viewable in 3D and then both in elevation (as Leonardo's *tiburio* drawings) and in plan.

In detail, the 3D-based method used follows a well-defined workflow for the reconstruction of no more existing or unbuilt architectures grounded in shared working hypotheses, and our specific solutions (Steadman, 1989; Apollonio, Gaiani, & Sun, 2013; Apollonio, 2016; Frommel, Gaiani, & Garagnani, 2018): the analysis of different data sources, the formulation of some conjectures by deduction or induction and their consequent iterative verification, led to the definition of two "most probable" solutions.

The first one follows and deepens the analogical reconstruction proposed by Jean

Guillaume and Sabine Frommel in 1987 (Guillaume, 1987). The second solution arises from a new and completely different interpretation of Leonardo's drawing and, while presenting some common parts with the first, determines a very different structural output.

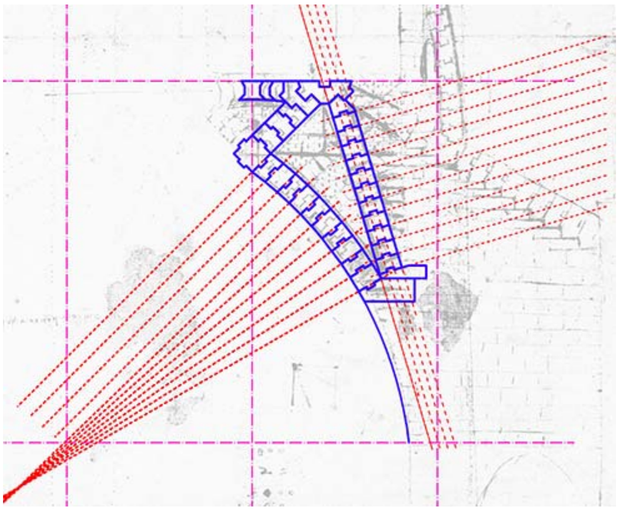

Fig. 2: 2D definition of the pointed arch, that has been regularized by dividing its length into 10 segments in addition to the keystone; each part has been divided transversely into three equal segments to define the tongue and groove joint.

The 3D models were defined following a methodological approach based on a well-defined workflow starting with the transposition of Leonardo's sheet into a 2D CAD reference drawing[1] (Fig. 2), followed by the integration of the information derived from surveys of the Duomo (Beltrami, 1903; Ferrari da Passano, 1973)[2]. Then all these 2D inputs are translated in 3D and, finally, the rough model is completed shaping the smallest elements, like ashlars and voussoirs.

In the specific case of the *tiburio*, the approach allowed to deduce the plans of the two hypotheses, starting from the common main section (the representation of Leonardo). We can notice that this procedure inverts the Vitruvian procedure in which the elevation is derived from the plan (from *ichnographia* to *orthographia*; Bartoli, 1978; Dezzi Bardeschi, 2017, pp. 91-92).

2. *Leonardo & the* tiburio

During his stay in Milan at the court of Lodovico Sforza, from 1487 until 1490, Leonardo explored the topic of centralized churches systematically and in the process classified them in his drawings according to typological and morphological principles (Guillaume, 1987, pp. 224-245; Frommel & Guillaume, 2019, pp. 35-41). These drawings of the Manuscript B (Institut de France, Paris) provide an important contribution to one of the more fascinating themes of the Renaissance. Main axes and diagonal spaces characterize a lot of those ideal projects, but the artist was also highly interested in unified systems. The sketches are represented both as plans and three-dimensional renderings, in the form of perspectives, bird's-eye views or axonometries, which allow to control the whole organism in a highly efficient way. An original approach, that reveals that he looked at architecture as a whole. No other drawings of the sort by any contemporary architect survive, however it's possible that Leonardo conceived these drawings in dialogue with Donato Bramante, who was active in Milan at the same time, designing spectacular churches such as Santa Maria presso San Satiro and Santa Maria delle Grazie (Giordano, 1998, pp. 183-186). In any case, the latter seems to draw on these variations of the theme of central plans when he designed the reconstruction of Saint Peter in 1505 for Julius II. And, vice versa, one cannot exclude that Leonardo, his long-lasting friend, was also involved in the Roman design (Frommel, 2019, pp. 101-102).

In the same Milanese years, Leonardo faces the completion of the *tiburio* of the cathedral of Milan, the tower-like construction that, according to the Lombard tradition, crowns the crossing (Schofield 1989; Frommel 2019, pp. 68-75).

The building-site had started in 1386, and by 1480 the vaults of the crossing, the choir and five bays of the nave had been achieved. Now emerged the tricky problem: how to erect a *tiburio* 50 meters tall on slender pillars and fragile arches. The task was very challenging. Though the *tiburio* was a traditional Lombard solution, it had never

[1] The reference drawing used for the creation of the 2D base is the sheet 850r of the CA, which seems to be a more definitive version of Leonardo's project in terms of detail.

[2] The validation of proportions was carried out simultaneously with the verification of the ancient units of measurement used at the time of the project, i.e. the *braccio milanese,* equal to 0.594936 meters. The interaxle spacing between the main pillars, equal to 19.2 meters, i.e. 32 *braccia milanesi* and 4 *once*, was taken as the principal reference measure to scale the entire drawing.

before taken such breath-taking dimensions! Sebregondi, Gritti, Repishti, & Schofield (2019) formulated some hypotheses which, however, the references considerered in this solution are not those proposed here.

In 1481, after the death of Guiniforte Solari, the architect in charge, the level just below the octagonal drum had been reached and the *operai* asked for advice. To solve the problem, it was necessary to reinforce the construction horizontally, to avoid the pillars collapsing. At first, the famous specialist Johann Nexemperger was called from Straßburg, but shortly after his interventions in 1486, cracks in the walls appeared and the architect preferred to leave immediately (Pedretti, 1978, p. 34). Leonardo's caption "Del Tedesco in Domo", on one of the sheets of the Manuscript B (f. 10v) probably refers to him.

The *Fabbrica del Duomo* turned now to local masters like Amadeo and Dolcebono and to prominent specialists like Luca Fancelli, Francesco di Giorgio, Donato Bramante and Leonardo (Guillaume, 1987, pp. 209-210; Frommel & Guillaume, 2019, p. 29). The masters couldn't hide that it would be hard to find a solution for a "monument without bones and without measure". Following the example of the Florentine cathedral, a competition was promoted in the hope of receiving multiple solutions to compare.

Between 1487 and 1490 Leonardo produced about 20 sketches and drawings, a wooden model and a letter to the *operai* with general considerations on the "doctor-architect" – who, according to Leon Battista Alberti's *topos*, heals a sick building (CA, folio 730r; Firpo 1963, pp. 22-26; Pedretti, 1978, p. 34). It represents one of the more detailed projects of his whole career, founded on a precise survey of the cathedral. Between July and September 1487, the carpenter Bernardino di Abbiate was paid 34 *lire imperiali* for a model conform to Leonardo's drawings, a very detailed one, that required more than a month to be carried out (Pedretti 1978, p. 34).

Leonardo developed his project further verifying his ideas on the model, because in January 1488 he received money from the *operai* to make changes, executed probably by himself (Pedretti 1978, p. 34). Both the CA drawings present pricked marks for the pouncing technique, therefore it is assumed that the 850r was used to make a copy (the 851r), as noticed by Sebregondi et al. (2019, p. 168), and maybe then they were used to report the measurements directly on the wood to construct the model. Soon afterwards the model was ready, but in May 1490 he took it back again in order to improve other details, these additional modifications were also paid by the *operai.*

This is quite astonishing since until this moment the model of Leonardo had not yet been mentioned in the documents in the Duomo archives. On the 27 June 1490, during a meeting with the duke Lodovico in the castle of Milan, the *operai* charged the local masters Amadeo and Dolcebono to take on the building site and asked them to collaborate with Francesco di Giorgio. Finally, it was the two local architects that worked on the *tiburio* from 1490 to 1500.

What happened between May and June 1490? Did Leonardo decide to withdraw from the competition? Was the feasibility of his solution and the stability of the structure questioned? Did the *operai* and the jury disapprove of the Florentine character of the project, which evoked Brunelleschi's cupola of Santa Maria del Fiore?

According to another hypothesis he could have given his model to Francesco di Giorgio, one of the famous specialists of his time, who used and modified it. This would explain that in June 1490, a fortnight after his arrival in Milan in May, Francesco di Giorgio was already able to present a model (Fergusson, 1977, pp. 175-192; Pedretti, 1978, pp. 32-52; Marani, 1982, pp. 81-92).

Finally, it is also possible that Leonardo was absorbed by the new commission of the Equestrian statue of Lodovico Sforza and renounced to the *tiburio* competition. As it is well known, he rarely carried out a work till the end.

3. *The Leonardo's drawings and sketches for the* tiburio

As Leonardo's many drawings confirm, while he was preparing the project for the *tiburio*, he was studying the anatomy of the human body: in a similar way, he analyzed the building as an organism made of flesh and bones, nerves and tendons. He focused his attention on the problem of the active forces in architectural construction, and the distribution of loads and loads and stresses. In contrast of most of his drawings and sketches, for the *tiburio* there is no idealized or utopian vision, but a precise study evolving by consecutive steps. One of the first stages is certainly recorded in a sketch of the Manuscript B[3], that represents the four pillars of the crossing and includes two other ones of the nave, the transepts and the choir (Guillaume, 1987, pp. 211-212; Frommel & Guillaume, 2019, pp. 30-31). To reinforce the structure, big arches connect a pillar of the crossing with one of the adjoining spaces in order to create an octagonal base. This way, the eight keystones, instead of the arches, will support the weight and transmit it to the peripherical areas (Fig. 3). The solution was skillful, but the octagon resulting from this system was irregular and didn't fit with the already executed building.

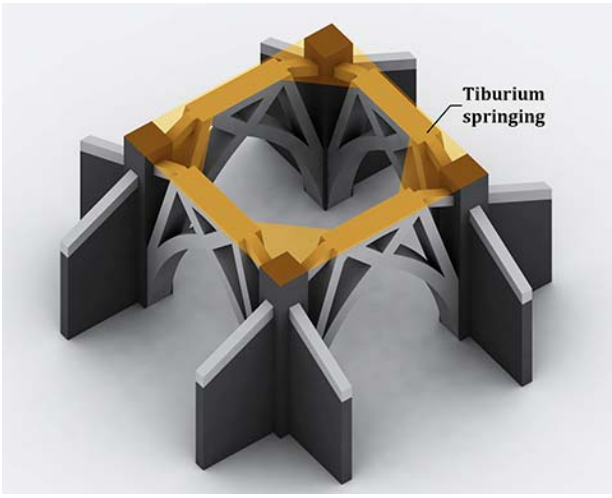

Fig. 3: Simplified reconstruction of the ideal system of intersection of pointed arches

The next solution recorded in the CA's f. 851r illustrates, in a convincing manner, the interplay of forces: the wall does not assume a supporting function, but only elements like arches and beams inside of it (Guillaume, 1987, pp. 212-223; Frommel & Guillaume, 2019, pp. 29-33; Frommel, 2019, pp. 68-75). The joints of the arches' stones are interlocked, reinforcing the stability (Fig. 4). It can be seen as a variation of the *spina di pesce* of Brunelleschi in the cupola of Florence, where Brunelleschi succeeded a self-supporting construction, transmitting the weight from the drum to the pillars and the arches.

The double-shell cupola also ties to the latter. Such a system allowed to build a majestic exterior cupola, that would dominate the urban landscape, and a smaller inner cupola in harmony with the proportions of the inside of the church.

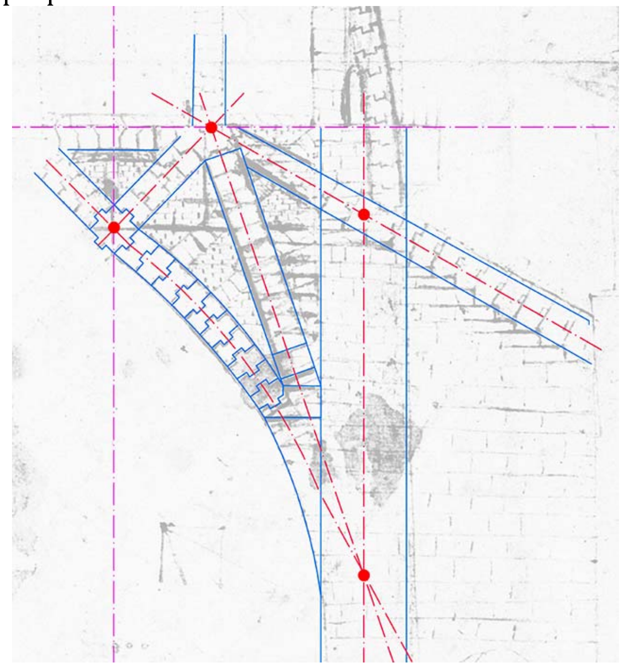

Fig. 4: Axes and intersection points of the load-bearing elements of the lower structure

Because the inner shell rises on a square plan and the exterior one on an octagonal, this allows to insert buttresses with double curvature in the corners. They had to support the crowning lantern and act like gothic pinnacles.

The virtual reconstruction, carried out here, shows that the system is efficient and coherent, but the pattern is not easily compatible with Lombard tradition. Leonardo tried in different designs to find more aesthetically "Lombard" solutions, but finally his project gave rise to a problem of "convenienza".

Even in a period like the Renaissance, marked by a renovation of typologies and patterns, patrons and architects had to respect the traditional technics and features. Examples like Peruzzi's projects for San Petronio in Bologna, which provoked a heated debate in 1521/1522, illustrate this in a very concrete manner (Panofsky, 1978, pp. 216-217). So, it is possible that the *operai* of the cathedral of Milan had the impression that their tradition had not been respected adequately. Such decisions depend on the local climate and the visions and taste of religious orders and patrons.

[3] Manuscript B - 2173 (c1488-90), Bibliothèque de l'Institut de France, Paris (f. 27r)

In the same years a Florentine master like Giuliano da Maiano designed a cupola according to the model of Santa Maria del Fiore for the cathedral of Loreto, later completed by Giuliano da Sangallo (Frommel, 2018, pp. 45-54). Likewise, it seems that the cardinal Ascanio Sforza, the brother of the duke, appreciated the bold propositions drawn by Bramante for the cathedral of Pavia (Giordano, 1998, pp. 187-190). However, at the end of the *Quattrocento,* sometimes completing a church, confronted the architects with a complicated "diplomatic" situation. In the case of the Milanese *tiburio*, it seems tradition and national pride played a significant role and made it difficult for Leonardo to impose his solution.

4. The reconstruction of Leonardo's project

4.1 The first hypothesis: a 3D reconstruction based on Guillaume/Frommel's solution

The two drawings of the CA preserved in the Veneranda Biblioteca Ambrosiana in Milan, the sheet 850r[4] and the 851r[5], represent a cross section of the *tiburio*, the first one in a more detailed version (Marinoni, 2006; Sebregondi et al., 2019, p. 167).

The upper section of the drawing (above the spring of *tiburio*) is open to various interpretations, due to the difficulty to distinguish between sectioned and/or view elements (mainly the sail of the vault), most likely coexisting, being these drawings a tool used by Leonardo to deepen and investigate different solutions.

Guillaume's and Frommel's text published in 1987 presents two slightly different reconstruction hypotheses of Leonardo's project for the *tiburio* (Guillaume, 1987). One of these was taken as a starting point to create a very simplified first model (Fig. 5), with the aim of verifying fairness and congruence of the original 2D drafted reconstruction, and to understand lacks needing further investigations, exploiting a detailed 3D digital model. Therefore, to define completely this first 3D based accurate solution, the observation of sheet 850r and the understanding of its parts went at the same pace as the analysis of the previous interpretation of Guillaume - Frommel.

The solution represented in sheet 850r consists in the intersection of large pointed arches, which both lean on the main pillars and on those of the side naves. These intersections determine

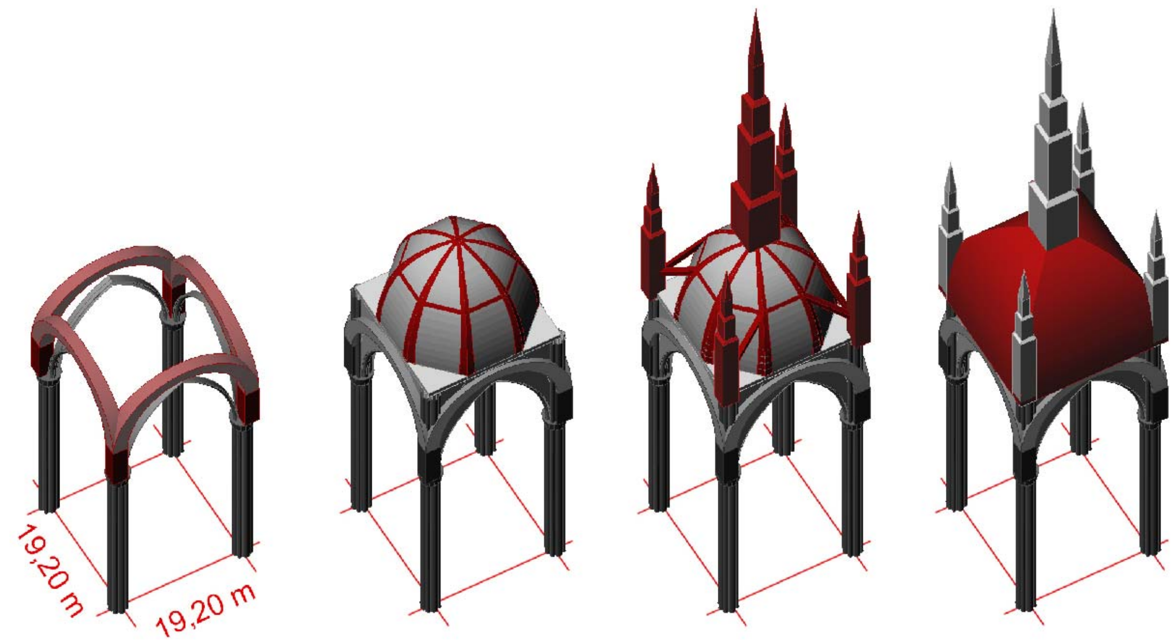

Fig. 5: Simplified 3D model of the Guillaume - Frommel reconstruction.

[4] Folio 850 recto, or previously 310 r.b; 332x293 mm; it shows pricked marks for pouncing technique; in the lower left part there are some arithmetic operations for calculating the weight of the *tiburio*; dated to 1487.

[5] Folio 851 recto, or previously 310 v.b; 282x237 mm; it shows pricked marks for pouncing technique; dated to 1487.

the eight points of support for the octagonal vault, corresponding to the eight keystones of each pointed arch[6] (Fig. 6). According to the proposal of Guillaume - Frommel, the upper part represents a double cloister vault, the internal one on an octagonal base, while the external one on a square base.

This interpretation could be further confirmed by the reference to the dome of Santa Maria del Fiore in Florence, even if it differs in the structural typology, e.g. for the presence of ribs, absent in the Florentine dome. We must consider, in fact, that the young Leonardo worked in Florence as an apprentice in Verrocchio's studio, in the same years in which Verrocchio worked on the installation of the bronze sphere above the lantern of the Florentine cathedral. As reported by Paolo Galluzzi (1987, p. 50), Leonardo stated in the manuscript G f. 84v: "Ricordati delle saldature con che si saldò la palla di Santa Maria del Fiore". He, certainly, had the opportunity to study the construction technique of the double Florentine dome (in some sketches it represents the *spinapesce* technique; see f. 933v, CA).

The 3D model allowed to define and specify the Guillaume – Frommel interpretation, at both the scales of the whole architectural object, and of the constructive and structural specific solutions and details, using furtherly material from the two Leonardo's drawings. Leonardo, in fact, defines in the two CA drawings ashlars and voussoirs of the *tiburio*. In both sheets, the elements that constitute the structural parts (architraves, arches, ribs) have highlighted joggled joints. Many evidences of this stone cut can be found in ancient Roman architecture (Lancaster, 2015, pp.152-176) and in

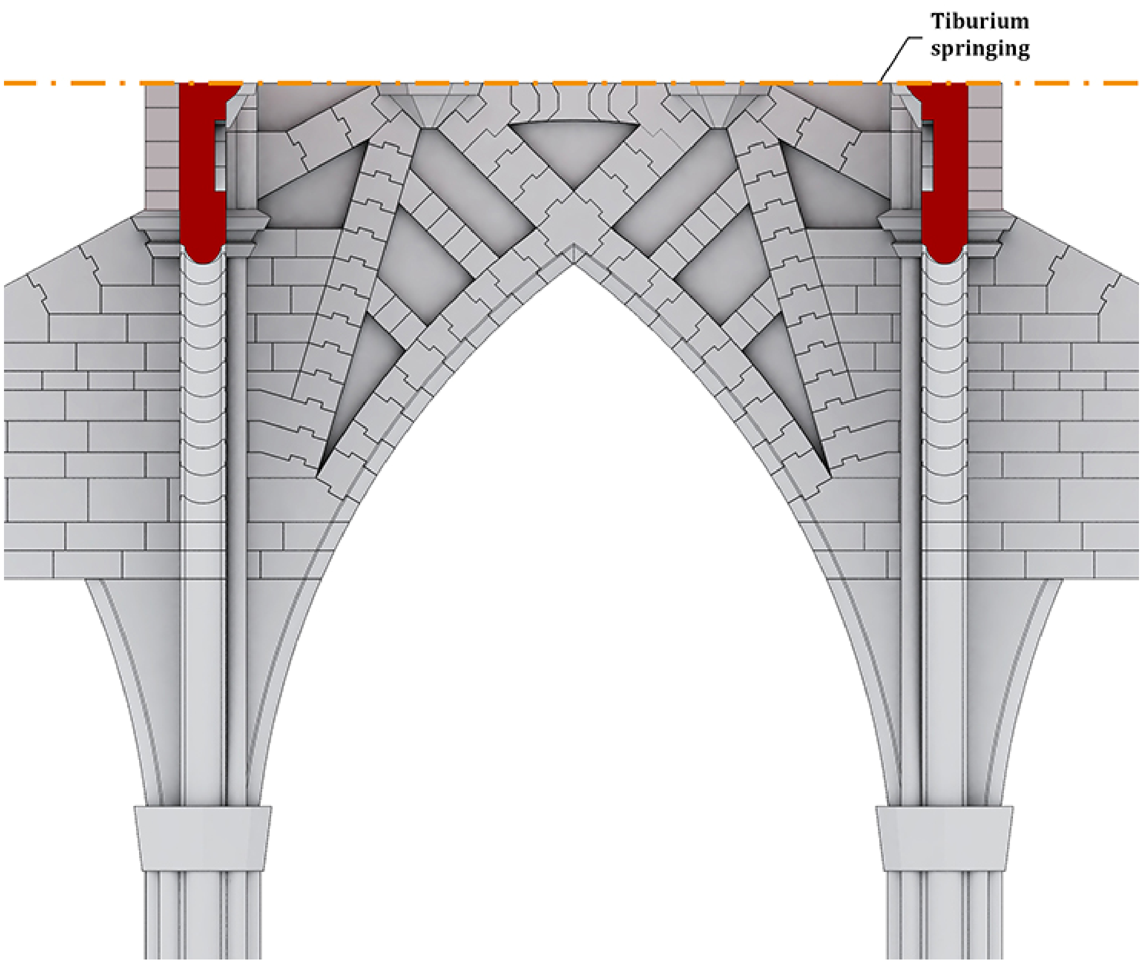

Fig. 6: The 3D model of the lower structure, common in both solutions

[6] The cross arches system can be found in : Manuscript B - 2173 (c1488-90), Bibliothèque de l'Institut de France, Paris (f. 27r); Codex Trivulzianus 2162 (1478-93), Biblioteca Trivulziana, Castello Sforzesco, Milan (f. 27v); Manuscript A – 2172, Bibliothèque de l'Institut de France, Paris (f. 50v, 51r).

some other examples, in which perhaps the joggled joints are used also as a decorative pattern, in addition to the structural reinforcement (see the arches of the mausoleum of Theodoric in Ravenna and the Arch of the Virgin Mary in Jerusalem). However, the practice of cutting stone elements, although already empirically widespread in ancient and Gothic architecture (e.g. in Villard de Honnecourt), will be codified in the stereotomy treatises, starting from the 16th century (see Philibert de l'Orme and later Frézier). It is impossible to be sure of the real experimentation of this technique by Leonardo, as for all his architectural projects (Maltese, 1975). However, there are many sketches representing architraves and structures with joggled ashlars[7] (Fig. 7), that demonstrate the practical know-how of Leonardo (Firpo, 1963, pp. 9-11).

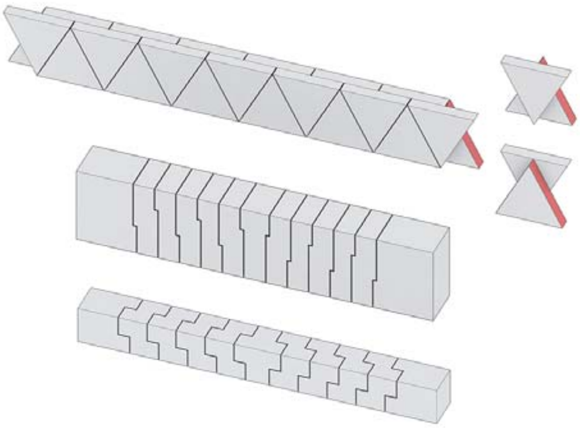

Fig. 7: Some examples of architraves with joggled ashlars. 3D elaboration of the f. 91v and 76r, CA

Considering the suggestions of the drawing and the actual feasibility of the proposed solution, the interaction between ashlars and the different planes on which the structural parts lie has been deeply analyzed and reproduced in the 3D model (Figs. 8 -10).

Other in-depth analyses are related to (a) the intersection of arches of the lower structure, (b) the eight elements in the springing of the octagonal vault, that serve both as a keystone for the pointed arches and as a base for the ribs of the internal vault, (c) the architraves connecting the ribs of the internal vault, (d) the keystone that joins and

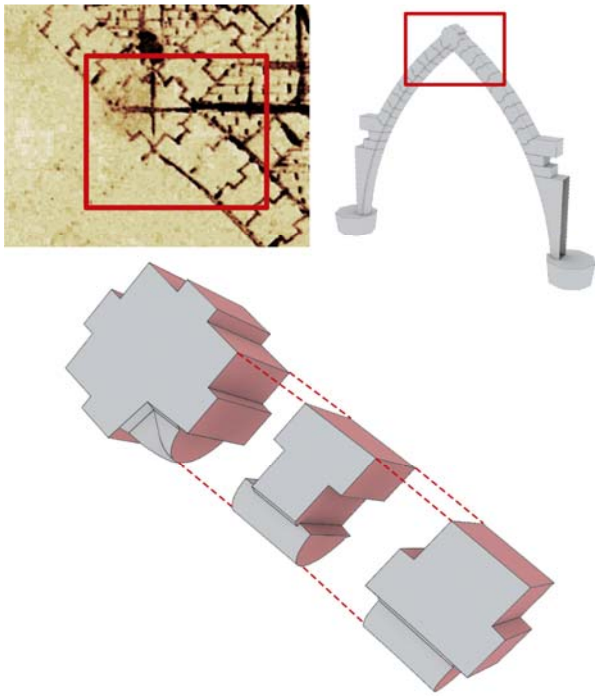

Fig. 8: In each pointed arch, the voussoirs were extruded by two *braccia milanesi* and subsequently, the intrados was moulded to obtain a rounded profile

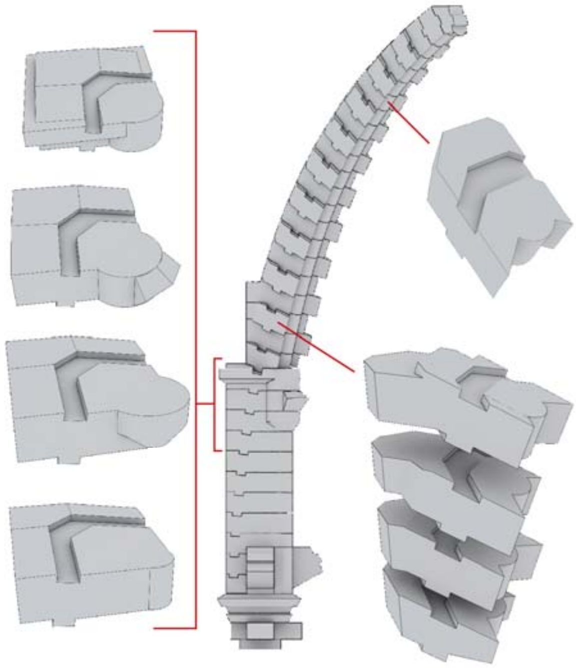

Fig. 9: Each main pillar gathers the structure of two sides and became a rib of the cloister vault and a spire

[7] Sketches representing joggled structures are contained in: Codex Atlanticus (f. 50r, 76r, 91v, 1074r); Manuscript B de Paris (f. 30r, 51v); Codex Trivulzianus (f. 4r).

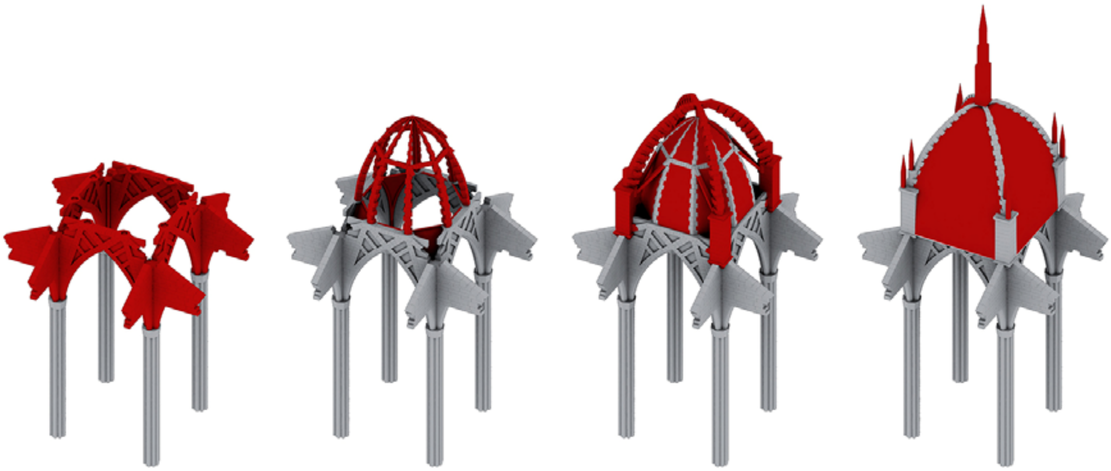

Fig. 10: Phases of the first solution, after Guillaume - Frommel hypothesis

tightens the two kinds of ribs and stabilizes the whole structure.

4.2 The second hypothesis of Leonardo's project

A second hypothetical reconstruction precisely arose by considering alternative constructive typologies of the upper part of the structure, due to the impossibility, in fact, of establishing with certainty whether the lines of the two drawings represent a view or a section.

A Tuscan example (therefore perhaps known by Leonardo), the dome of the baptistery of Saints Giovanni and Reparata in Lucca (Tuscany) (Fig. 11), presents a structural typology similar to the cloister vault on a square base, which, however, gradually becomes circular, as it rises towards the top. Another peculiarity of this vault is the reinforcement elements of the structure, the eight brick ribs placed not only in the corners, but also as a reinforcement in the middle of the four concave surfaces. This reference suggests an alternative hypothesis, in which the section represented by Leonardo refers to a single cloister vault, whose concave surfaces are reinforced by additional ribs (which in the first interpretation corresponded instead to the ribs of the octagonal internal vault).

The purpose of these additional ribs should be to distribute the forces of weight and convey them to the lower parts of the structure (Fig. 12). Some previous studies, such as those carried out by Pedretti (1978, p. 33) on the f. 10v of the Manuscript B of Paris (Fig. 13), reinforce this hypothesis from the point of view of the possible correspondence between the alternative plans proposed by Leonardo for the *tiburio* and the arrangement of the ribs in each concave surface.

So, the Florentine and classical influence would concern not only the design rules, but also the construction techniques, which in this second hypothesis would recall the hyperstatic concrete domes typical of the Roman period (Curcio & Manieri Elia, 1982, p. 139), which Leonardo could have combined with the Gothic ribbed structures,

Fig. 11: Internal view of the dome of the baptistery of Saints Giovanni and Reparata in Lucca (Tuscany). Photo: F. Fantini

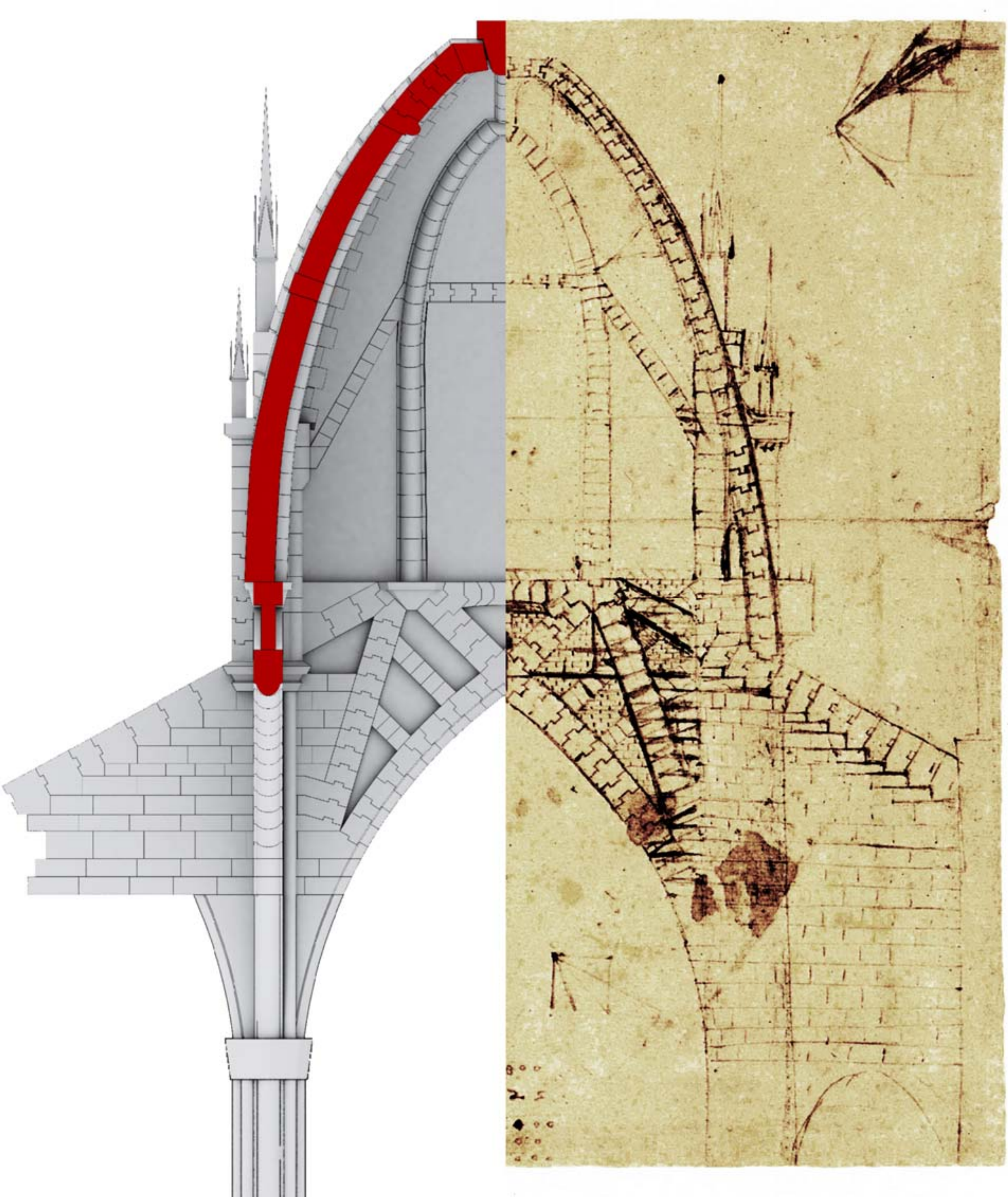

Fig. 12: Comparison between the 3D reconstruction of the second hypothesis and an edited detail the f. 850r, Codex Atlanticus.

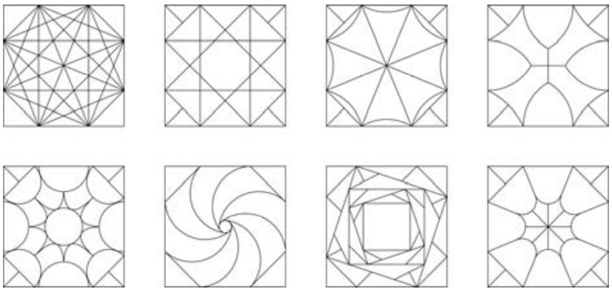

Fig. 13: Geometrical reconstructions of the plans contained in the folio 10v of the Manuscript B of Paris

to convey the loads to follow specific directions in order to reach the eight points of the lower structure, placed over the intersection of large pointed arches, which – as seen above - both lean on the main pillars and on those of the side naves.

5. *Conclusions*

As already stated, the multiple possible readings of the two CA drawings and all the sketches related generate multiple hypothetical reconstructions of Leonardo's project.

The two proposed in this contribution present some critical issues, that were perhaps decisive in inducing the *Fabbrica del Duomo* to reject Leonardo's project.

First, Leonardo's solution influenced by his Florentine heritage should not have well impressed the Milan commission, which certainly focused more on the Lombard and international Gothic style and the typical *tiburio* solution (Brivio, 2005; Sebregondi and Schofield, 2016). Despite the accentuated curvature, Leonardo's vault would have remained much more hidden from the exterior view than a *tiburio*, maybe raised even more by a drum. Furthermore, the upper structure represented by Leonardo seems to necessarily coexist with the lower structure of the pointed arches. Since the springing was already realized in 1487, it would have been too expensive to demolish much of the work done and adopt Leonardo's solution. Moreover, the stone cutting of the ashlars and voussoirs proposed by Leonardo would have been very complex, due to the realization of the joints and exact curvatures.

Perhaps the lack of practical applications has discouraged the *Fabbrica* from adopting such an unusual solution compared to tradition. In addition, in the hypothesis of a double vault without a lantern, the dome would have been completely blind. On this matter, it must be taken into account that many changes were probably made directly on the wooden model and that the topic of light is widely studied by Leonardo in many architectural sketches, referred to alternative solutions for the exterior of the Milan dome (CA, f. 719r; MS B, f 4v; C Trivulzianus, f. 9r).

The last step of this analysis allowed to obtain the plans corresponding to each of the two solutions studied (Fig. 14). In the architectural design, following the indications of Vitruvius, we proceed by first defining the plan (*ichnographia*) and then the elevation (*orthographia*). In this case it was not possible, since there were no planimetric information concerning the proposal defined by Leonardo, but only the section. Therefore, this unusual or infrequent circumstance of hypothetical reconstruction (that is, producing a complete 3D model in order to obtain the planimetric solution), but above all the possibility of considering with a relative level of certainty only the details of vault designed by Leonardo, required an innovative approach and a reverse procedure to the reconstruction process. The formulation of the hypotheses, in fact, was substantiated on the verification of constructive feasibility starting from the details of the individual ashlars, reaching the formulation of the two hypotheses that demonstrated the greatest degree of feasibility.

Summarizing, a first significant result is represented by the three-dimensional verification carried out on the solution hypothesized by Frommel and Guillame in 1987, made at that time with the use of traditional architectural drawing tools, i.e. plans, elevations, sections and axonometric views.A second result is the verification of an alternative hypothesis deriving from the interpretation of the uncertainty data which characterize - as it has been said - the two drawings by Leonardo of Codex Atlanticus concerning the proposal for the *tiburio* of Milan cathedral.

This case study therefore provides some innovative methodological elements to the problem of hypothetical reconstruction of buildings just designed based on archive sources, which contribute to consolidating and enriching the theoretical framework that has been developing over the past few decades.

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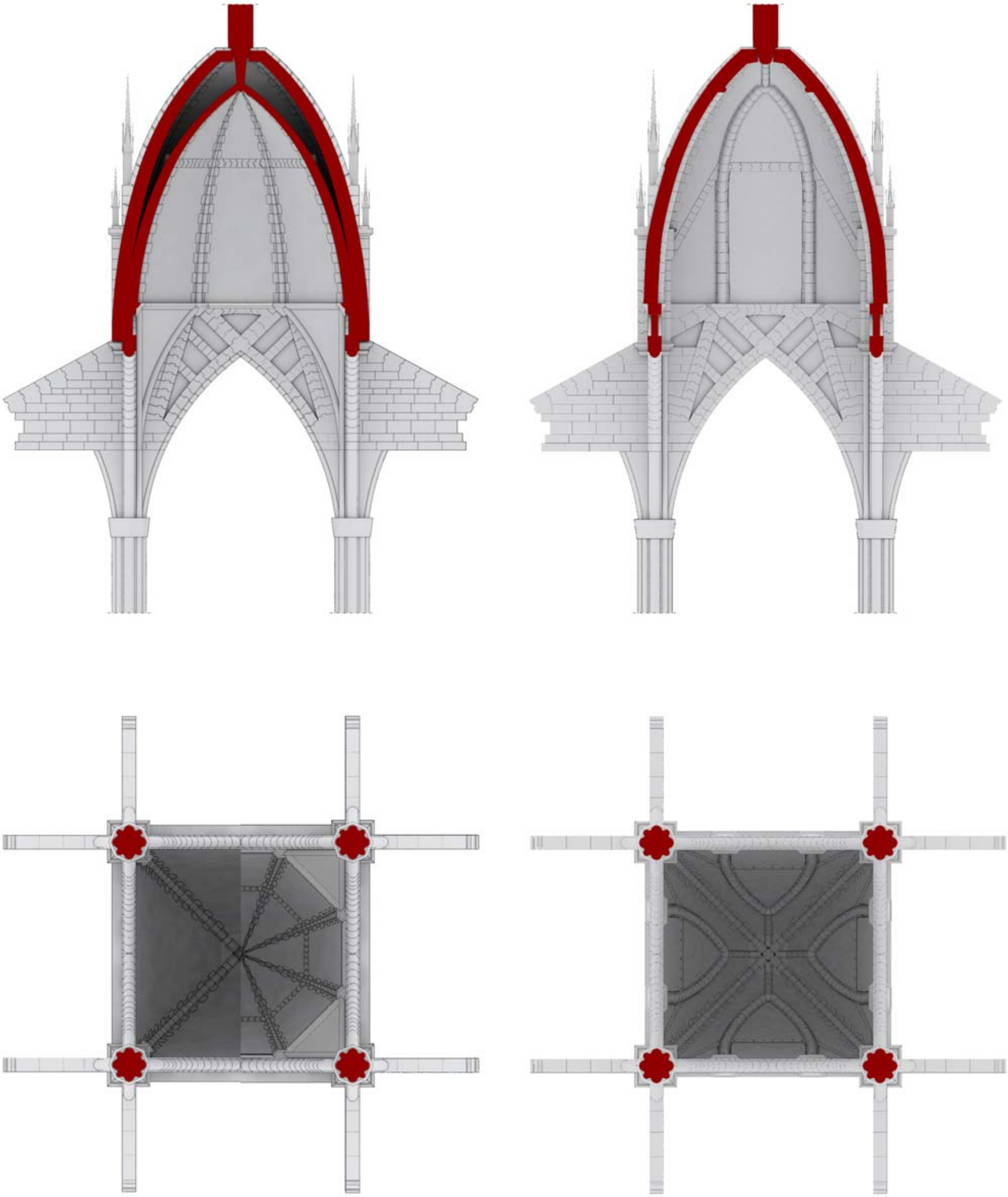

Fig. 14: Plan and section comparison between first hypothesis (left) and second one (right)

References

Apollonio, F.I. (2016). Classification Schemes for Visualization of Uncertainty in Digital Hypothetical Reconstruction. In S. Münster, M. Pfarr-Harfst, P. Kuroczyński, M. Ioannides (Eds.), *3D Research Challenges in Cultural Heritage II* (pp. 173-197). Cham, Switzerland: Springer International Publishing. https://doi.org/10.1007/978-3-319-47647-6_9

Apollonio, F.I., Gaiani, M., & Sun, Z. (2013). 3D modeling and data enrichment in digital reconstruction of architectural heritage. *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, XL-5/W2, 43-48. https://doi.org/10.5194/isprsarchives-XL-5-W2-43-2013

Bartoli, M.T. (1978). Orthographia, Ichnographia, Scaenographia. *Studi e documenti di Architettura*, 8, 197-208.

Beltrami, L. (1903). *Leonardo da Vinci negli studi per il tiburio della cattedrale di Milano*. Milano, Italy.

Brivio, E. (2005). Il Duomo di Milano e la sua fabbrica: ieri, oggi e domani. *La Casana*, 47, 22-31.

Curcio, G., & Manieri Elia, M. (1982). *Storia e uso dei modelli architettonici*. Roma-Bari, Italy: Laterza.

Dezzi Bardeschi, C. (Ed.). (2017). *Abbeceddario Minimo. Cento voci per il restauro*. Firenze, Italy: Altralinea.

Fergusson, F. D. (1977). Leonardo da Vinci and the Tiburio of Milan Cathedral. *Architettura*, VII, 175-192.

Ferrari da Passano, C. (1973). Storia della veneranda fabbrica. In *Il Duomo di Milano*. Milano, Italy: Cassa di risparmio delle provincie lombarde.

Firpo, L. (Ed.). (1963). *Leonardo architetto e urbanista*. Torino, Italy: UTET.

Frommel, C.L. (2018). *L'architettura del santuario e del palazzo apostolico di Loreto da Paolo II a Paolo III*. Loreto, Italy: Edizioni Tecnostampa.

Frommel, S. (2019). *Leonardo da Vinci: Architektur und Erfindungen*. Stuttgart, Germany: Belser.

Frommel, S., & Guillaume, J. (2019). *Leonardo e l'architettura*. Modena, Italy: Panini.

Frommel, S., Gaiani, M., & Garagnani, S. (2018). Progettare e costruire durante il Rinascimento. Un metodo per lo studio di Giuliano da Sangallo. *Disegnare Idee Immagini*, 56, 20-31.

Galluzzi, P. (Ed.). (1987). *Léonard de Vinci ingénieur et architecte*. Montréal, Canada: Musée des beaux-arts de Montréal.

Giordano, L. (1998). Milano e l'Italia nord-occidentale. In F.P. Fiore (Ed.). *Storia dell'architettura italiana. Il Quattrocento* (pp. 166-199). Milano, Italy: Electa.

Guillaume, J. (1987). Le tiburio de la cathédrale de Milan. In P. Galluzzi (Ed.), *Léonard de Vinci ingénieur et architecte* (pp. 209-223). Montréal, Canada: Musée des beaux-arts de Montréal.

Lancaster, L. (2015). *Innovative Vaulting in the Architecture of the Roman Empire: 1st to 4th Centuries CE*. Cambridge, MA, USA: Cambridge University Press.

Maltese, C. (1975). *Gusto e metodo scientifico nel pensiero architettonico di Leonardo*. Firenze, Italy: Giunti Barbèra.

Marani, P. (1982). Leonardo, Francesco di Giorgio e il tiburio del Duomo di Milano. *Arte Lombarda*, 62(2), 81-92.

Marinoni, A. (Ed.) (2006). *Leonardo da Vinci. Il Codice Atlantico*. Firenze, Italy: la Repubblica - Giunti - Il Sole 24 ore.

Panofsky, E. (1978). *Sinn und Deutung in der bildenden Kunst*. Köln, Germany: DuMont. German edition of: Panofsky, E. (1957). *Meaning in the Visual Arts*. New York, NY, USA: Doubleday Anchor Books.

Pedretti, C. (1978). *Leonardo architetto*. Milano, Italy: Electa.

Sebregondi, G.C., & Schofield, R. (2016). First Principles: Gabriele Stornaloco and Milan Cathedral. *Architectural history*, 59, 63–122. https://doi.org/10.1017/arh.2016.3

Sebregondi, G.C., Gritti, J., Repishti, F., & Schofield, R. (2019). *Ad Triangulum. Il Duomo di Milano e il suo tiburio*. Milano, Italy: Il Poligrafo.

Steadman, P. (1989). Modelling Leonardo's Ideas by Computer. In M. Kemp, & J. Roberts (Eds.), *Leonardo da Vinci* (pp. 209-217). New Haven, CT, USA: Yale University Press.