Proofs of statements and details about the instances

EC.1. Proofs of statements

Proposition 1. There are no dominance relations between inequalities RCII-a and RCII-b.

Proof. Consider a VRPTF instance with $|V_C| = 6$, $|V_F| = 1$, with $V_C = \{1,2,3,4,5,6\}$ and $V_F = \{7\}$. In addition, let $Q = 10$ and $q_1 = 4$, $q_2 = 3$, $q_3 = 5$, $q_4 = q_5 = q_6 = 2$. First select a set $S = \{1,2,4,7\}$ and such that the nodes $\{1,2,4,7\}$ are visited on a route while customers $\{3,5,6\}$ are assigned to facility node 7 of S. The right-hand side of inequality (15) becomes $\lceil 18/10 \rceil - 0 = 2$, while the right-hand side of (17) has value $\lceil 9/10 \rceil - 0 = 1$ and (15) is stronger than (17). Now consider a set $S = \{1,2,3,4,5\}$ containing five customers all visited on a route. Customer 6 is associated with a node in $V' \setminus S$. The right-hand side of (15) takes value $\lceil 18/10 \rceil - 1/4 = 7/4$, while the right-hand side of (17) becomes $\lceil 16/10 \rceil - 0 = 2$ and the second inequality is stronger than the first one. Moreover, note that the FrCC inequalities (4) are dominated by (15), for the first example, and by both (15) and (17) for the second one. □

Lemma 2. *Let $o \in \mathbb{R}$ with $\hat{o} > 0$ and $T = \{m \in \mathbb{R}, n \in \mathbb{Z} : m + n \geq o, m \geq 0\}$. The inequality*

$$m + \hat{o}n \geq \hat{o}\lceil o \rceil \tag{18}$$

is valid for T.

Proof. We have two cases:

(i) $n \geq \lceil o \rceil$. As $m \geq 0$, we have $\dfrac{m}{\hat{o}} \geq 0$, hence $\dfrac{m}{\hat{o}} + n \geq \lceil o \rceil$;

(ii) $n \leq \lfloor o \rfloor$. As $0 < \hat{o} < 1$ we have that

$$\lfloor o \rfloor - n \geq \hat{o}(\lfloor o \rfloor - n). \tag{EC.1}$$

Since $o = \lfloor o \rfloor + \hat{o}$ and using inequality (EC.1), inequality $m + n \geq o$ can be rewritten as:

$$m \geq \hat{o} + \hat{o}(\lfloor o \rfloor - n). \tag{EC.2}$$

The right-hand side of inequality (EC.2), can be rewritten as:

$$\hat{o}(1 + \lfloor o \rfloor) - \hat{o}n = \hat{o}\lceil o \rceil - \hat{o}n \tag{EC.3}$$

thus obtaining $m \geq \hat{o}\lceil o \rceil - \hat{o}n$. □

THEOREM 1. *Let $\alpha_e \geq 0$, $\forall e \in E$, $\beta_i \geq 0$, $\forall i \in V'$ and $\gamma_{ij} \geq 0$, $\forall (i,j) \in A$ and consider the following inequality valid for formulation TI:*

$$\sum_{e\in E} \alpha_e x_e + \sum_{i\in V'} \beta_i y_i + \sum_{(i,j)\in A} \gamma_{ij} z_{ij} \geq o \tag{19}$$

where $o \in \mathbb{R}$ and $\hat{o} > 0$. Then the following inequality:

$$\sum_{e\in E} \varphi^o(\alpha_e) x_e + \sum_{i\in V'} \varphi^o(\beta_i) y_i + \sum_{(i,j)\in A} \varphi^o(\gamma_{ij}) z_{ij} \geq \lceil o \rceil \tag{20}$$

where $\varphi^o(m) = \lfloor m \rfloor + \min\left\{\frac{\hat{m}}{\hat{o}}, 1\right\}$, $m \in \mathbb{R}$, $o \in \mathbb{R}$, $\hat{o} > 0$, is also a valid inequality for formulation TI.

Proof. Let $E^1 \subseteq E$, $E^2 = E \setminus E^1$, $V^1 \subseteq V'$, $V^2 = V' \setminus V^1$ and $A^1 \subseteq A$, $A^2 = A \setminus A^1$. Starting from inequality (19) we can round up the coefficients in E^2, V^2 and A^2 to obtain:

$$\sum_{e\in E^1} \alpha_e x_e + \sum_{i\in V^1} \beta_i y_i + \sum_{(i,j)\in A^1} \gamma_{ij} z_{ij} + \sum_{e\in E^2} \lceil \alpha_e \rceil x_e + \sum_{i\in V^2} \lceil \beta_i \rceil y_i + \sum_{(i,j)\in A^2} \lceil \gamma_{ij} \rceil z_{ij} \geq o. \tag{EC.4}$$

Writing $\alpha_e = \lfloor \alpha_e \rfloor + \hat{\alpha_e}$, $\forall e \in E^1$, $\beta_i = \lfloor \beta_i \rfloor + \hat{\beta_i}$, $\forall i \in V^1$, and $\gamma_{ij} = \lfloor \gamma_{ij} \rfloor + \hat{\gamma_{ij}}$, $\forall (i,j) \in A^1$ and re-arranging terms, we get:

$$\begin{aligned}&(\sum_{e\in E^1} \hat{\alpha_e} x_e + \sum_{i\in V^1} \hat{\beta_i} y_i + \sum_{(i,j)\in A^1} \hat{\gamma_{ij}} z_{ij}) + \\ &(\sum_{e\in E^1} \lfloor \alpha_e \rfloor x_e + \sum_{e\in E^2} \lceil \alpha_e \rceil x_e + \sum_{i\in V^1} \lfloor \beta_i \rfloor y_i + \sum_{i\in V^2} \lceil \beta_i \rceil y_i + \sum_{(i,j)\in A^1} \lfloor \gamma_{ij} \rfloor z_{ij} + \sum_{(i,j)\in A^2} \lceil \gamma_{ij} \rceil z_{ij}) \geq o.\end{aligned} \tag{EC.5}$$

The first part of inequality (EC.5) is non-negative, and the second part is integral for all x, w and z integral. Applying Lemma 2 we get:

$$\begin{aligned}&\frac{1}{\hat{o}}(\sum_{e\in E^1} \hat{\alpha_e} x_e + \sum_{i\in V^1} \hat{\beta_i} y_i + \sum_{(i,j)\in A^1} \hat{\gamma_{ij}} z_{ij}) + \\ &(\sum_{e\in E^1} \lfloor \alpha_e \rfloor x_e + \sum_{e\in E^2} \lceil \alpha_e \rceil x_e + \sum_{i\in V^1} \lfloor \beta_i \rfloor y_i + \\ &\sum_{i\in V^2} \lceil \beta_i \rceil y_i + \sum_{(i,j)\in A^1} \lfloor \gamma_{ij} \rfloor z_{ij} + \sum_{(i,j)\in A^2} \lceil \gamma_{ij} \rceil z_{ij}) \geq \lceil o \rceil.\end{aligned} \tag{EC.6}$$

The coefficients of variables $\{x_e\}$ in (EC.6) are $\lfloor \alpha_e \rfloor + \frac{\hat{\alpha_e}}{\hat{o}}$ if $e \in E^1$ and $\lceil \alpha_e \rceil$ if $e \in E^2$. Similarly, the coefficients of variables $\{y_i\}$ in (EC.6) are $\lfloor \beta_i \rfloor + \frac{\hat{\beta_i}}{\hat{o}}$ if $i \in V^1$ and $\lceil \beta_i \rceil$ if $i \in V^2$ and the coefficients of variables $\{z_{ij}\}$ are $\lfloor \gamma_{ij} \rfloor + \frac{\hat{\gamma_{ij}}}{\hat{o}}$ if $(i,j) \in A^1$ and $\lceil \gamma_{ij} \rceil$ if $(i,j) \in A^2$.

The best choices of coefficients for the sets E^1, V^1 and A^1 are $E^1 = \{e \in E : \hat{\alpha_e} \leq \hat{o}\}$, $V^1 = \{i \in V' : \hat{\beta_i} \leq \hat{o}\}$ and $A^1 = \{(i,j) \in A : \hat{\gamma_{ij}} \leq \hat{o}\}$, respectively.

By defining $\varphi^o(m) = \lfloor m \rfloor + \min\left\{\frac{\hat{m}}{\hat{o}}, 1\right\}$, $m \in \mathbb{R}$, $o \in \mathbb{R}$, $\hat{o} > 0$, inequality (EC.6) becomes inequality (20).□

Theorem 2. *Let us associate penalties $\lambda_i \in \mathbb{R}$, $\forall i \in V_C$, with constraints (27), and $\lambda_i \leq 0$, $\forall i \in V_F$, with constraint (28). For each $i \in V_C$, define $\overline{a}_{i\ell} = a_{i\ell} + \sum_{j\in V_F(R_\ell)} b^j_{i\ell}$, and let $\mathscr{R}_i = \{\ell \in \mathscr{R} : \overline{a}_{i\ell} > 0\}$. For each $i \in V_C$ compute:*

$$\nu_i = q_i \min_{\ell\in\mathscr{R}_i} \left\{ \frac{(c_\ell + p_\ell) - \sum_{j\in V_C} \overline{a}_{j\ell}\lambda_j - \sum_{j\in V_F} a_{j\ell}\lambda_j}{\sum_{j\in V_C} \overline{a}_{j\ell} q_j} \right\} \tag{30}$$

A feasible DSP solution $\mathbf{u}$ *of cost* $z(DSP(\boldsymbol{\lambda}))$ *is given by the following expressions:*

$$u_i = \nu_i + \lambda_i, \forall i \in V_C, \quad \text{and } u_i = \lambda_i, \forall i \in V_F. \tag{31}$$

Proof. Consider a route $\ell \in \mathscr{R}$. Since $\ell \in \mathscr{R}_i$, $\forall i \in V_C(R_\ell)$, from expression (30) we derive:

$$\nu_i \leq q_i \frac{(c_\ell + p_\ell) - \sum_{j\in V_C} \overline{a}_{j\ell}\lambda_j - \sum_{j\in V_F} a_{j\ell}\lambda_j}{\sum_{j\in V_C} \overline{a}_{j\ell} q_j}, \quad \forall i \in V_C(R_\ell). \tag{EC.7}$$

Given a route $\ell \in \mathscr{R}$, from expression (31) we obtain:

$$\begin{aligned}\sum_{i\in V_C} \overline{a}_{i\ell} u_i + \sum_{i\in V_F} a_{i\ell} u_i \leq \sum_{i\in V_C} \overline{a}_{i\ell} q_i \frac{(c_\ell + p_\ell) - \sum_{j\in V_C} \overline{a}_{j\ell}\lambda_j - \sum_{j\in V_F} a_{j\ell}\lambda_j}{\sum_{j\in V_C} \overline{a}_{j\ell} q_j} + \\ \sum_{i\in V_C} \overline{a}_{i\ell}\lambda_i + \sum_{i\in V_F} a_{i\ell}\lambda_i.\end{aligned} \tag{EC.8}$$

Inequality (EC.8) can be written as:

$$\sum_{i\in V_C} \overline{a}_{i\ell} u_i + \sum_{i\in V_F} a_{i\ell} u_i \leq c_\ell + p_\ell, \tag{EC.9}$$

that corresponds to the constraint of problem DSP for route ℓ.□

Let $E(S)$ denote the set of edges in G with both end-nodes in S and, given two disjoint vertex sets S_1, S_2, let $E(S1 : S2)$ denote the set of edges crossing from S_1 to S_2 (i.e., $E(S_1 : S_2) = \delta(S_1) \cap \delta(S_2)$) (if $S_1 = \{i\}$, we simply write $E(i : S_2)$ instead of $E(\{i\} : S_2)$).

Theorem 3. *The LP-relaxation of the SP formulation satisfies both CI and FrCI inequalities, and a weak form of MI inequalities.*

Proof. Consider a set $S \subseteq V'$ with $V_C(S) \neq \emptyset$ and let $T = V_C(S)$ be the set of customers contained in S. Define the surrogate constraint obtained by adding partitioning constraints (27) corresponding to customers in T after having multiplied the equation associated with $i \in T$ by q_i:

$$\sum_{\ell\in\mathscr{R}} q_\ell(T)\xi_\ell = q(T), \tag{EC.10}$$

where $q(T) = \sum_{i \in S} q_i$ and $q_\ell(T) = \sum_{i \in S} q_i \overline{a}_{i\ell}$. Since $q_\ell(T) \leq \min[Q, q(T)]$, we have

$$\sum_{\ell \in \mathscr{R}(T)} \xi_\ell \geq \max[1, q(T)/Q], \tag{EC.11}$$

where $\mathscr{R}(T) = \{\ell \in \mathscr{R} : \overline{a}_{i\ell} = 1 \text{ for some } i \in T\}$. Given a route $\ell \in \mathscr{R}(T)$, define $\overline{q}_\ell(T)$ as the total demand of the customers not in T assigned to the route, i.e. $\overline{q}_\ell(T) = q_\ell(\overline{T})$, where $\overline{T} = (V_C \setminus T) \cap V_C(R_\ell)$. As $q_\ell(T) + \overline{q}_\ell(T) \leq Q$ we have:

$$\sum_{\ell \in \mathscr{R}(T)} Q\xi_\ell \geq \sum_{\ell \in \mathscr{R}} q_\ell(T)\xi_\ell + \sum_{\ell \in \mathscr{R}} \overline{q}_\ell(T)\xi_\ell. \tag{EC.12}$$

From equations (EC.10) and inequalities (EC.12) we derive:

$$\sum_{\ell \in \mathscr{R}(T)} \xi_\ell \geq \max\left\{1, \frac{1}{Q}\left(q(T) + \sum_{\ell \in \mathscr{R}} \overline{q}_\ell(T)\xi_\ell\right)\right\}. \tag{EC.13}$$

Note that any route $\ell \in \mathscr{R}(T)$ contains at least two edges, one having an ending node in S and the other in $\overline{S}$. Therefore, we have:

$$\sum_{\{i,j\} \in \delta(S)} \eta_{ij}^\ell \xi_\ell \geq 2\xi_\ell. \tag{EC.14}$$

Adding inequality (EC.14) for all $\ell \in \mathscr{R}(T)$ we obtain:

$$\sum_{\ell \in \mathscr{R}(T)} \sum_{\{i,j\} \in \delta(S)} \eta_{ij}^\ell \xi_\ell \geq 2 \sum_{\ell \in \mathscr{R}(T)} \xi_\ell. \tag{EC.15}$$

Thus inequalities (EC.13) become

$$\sum_{\ell \in \mathscr{R}(T)} \rho_\ell(S)\xi_\ell \geq 2\max\left\{1, \frac{1}{Q}\left(q(T) + \sum_{\ell \in \mathscr{R}} \overline{q}_\ell(T)\xi_\ell\right)\right\}, \tag{EC.16}$$

where $\rho_\ell(S) = \sum_{\{i,j\} \in \delta(S)} \eta_{ij}^\ell$. Since

$$\overline{q}_\ell(T) \geq \sum_{j \in \overline{T}} \sum_{k \in V_F(S)} b_{j\ell}^k q_j + \frac{1}{2} \sum_{j \in \overline{T}} q_j \sum_{\{i,h\} \in E(S:\{j\})} \eta_{ih}^\ell \tag{EC.17}$$

from (EC.17) we obtain:

$$\sum_{\ell \in \mathscr{R}(T)} \rho_\ell(S)\xi_\ell \geq 2\max\{1, \frac{1}{Q}(q(T) + \sum_{\ell \in \mathscr{R}}(\sum_{j \in \overline{T}} \sum_{k \in V_F(S)} b_{j\ell}^k q_j + \frac{1}{2}\sum_{j \in \overline{T}} q_j \sum_{\{i,h\} \in E(S:\{j\})} \eta_{ih}^\ell)\xi_\ell)\}. \tag{EC.18}$$

We have:

i) $q(T) = \sum_{\ell \in \mathscr{R}} q_\ell(T)\xi_\ell = \sum_{i \in T} q_i(\sum_{\ell \in \mathscr{R}} a_{i\ell}\xi_\ell) + \sum_{i \in T} q_i(\sum_{\ell \in \mathscr{R}} \sum_{k \in V_F(R_\ell)} b_{i\ell}^k \xi_\ell)$;

ii) $\sum_{\ell \in \mathscr{R}} \sum_{j \in \overline{T}} \sum_{k \in V_F(S)} b_{j\ell}^k q_j \xi_\ell = \sum_{j \in \overline{T}} q_j(\sum_{\ell \in \mathscr{R}} \sum_{k \in V_F(S)} b_{j\ell}^k \xi_\ell)$.

Using the equations (32)-(35) linking variables variables ξ with (x, z, w), we derive:

i) $\sum_{i \in T} q_i(\sum_{\ell \in \mathscr{R}} a_{i\ell}\xi_\ell) = \sum_{i \in V_C(S)} q_i y_i$;

ii) $\sum_{i\in T} q_i(\sum_{\ell\in\mathscr{R}}\sum_{k\in V_F(R_\ell)} b^k_{i\ell}\xi_\ell) + \sum_{j\in\overline{T}} q_j(\sum_{\ell\in\mathscr{R}}\sum_{k\in V_F(S)} b^k_{j\ell}\xi_\ell) \geq \sum_{\substack{(i,j)\in A:\\ j\in V_F(S)}} q_i z_{ij}$;

iii) $\frac{1}{2}\sum_{j\in\overline{T}} q_j(\sum_{\ell\in\mathscr{R}})\sum_{\{i,h\}\in E(S:\{j\})}\eta^\ell_{ih}\xi_\ell = \frac{1}{2}\sum_{i\in V_C(\overline{S})}\sum_{j\in S} q_i x_{\{i,j\}}$.

Using the above equations, from (EC.18) we obtain:

$$\sum_{e\in\delta(S)} x_e \geq 2\max\left\{1, \frac{1}{Q}\left(\sum_{i\in V_C(S)} q_i y_i + \sum_{(i,j)\in A: j\in V_F(S)} q_i z_{ij} + \frac{1}{2}\sum_{i\in V_C(\overline{S})}\sum_{j\in S} q_i x_{\{i,j\}}\right)\right\} \tag{EC.19}$$

□

Theorem 4. *Let (x,z,y) be a solution of the LP-relaxation of formulation TI and assume that $q_i \leq Q$, $\forall i\in V_C$, and that $x_e = 0$, $e=\{i,j\}\in E\setminus\{\{0,h\}: h\in V'\}$, if $q_i+q_j > Q$. The separation problem for MI inequalities* (10) *is solvable in polynomial time.*

Proof. Consider the MI inequality for a given set $S\subseteq V'$, $S\neq\emptyset$:

$$\sum_{e\in\delta(S)} x_e \geq \frac{2}{Q}\left(\sum_{i\in V_C(S)} q_i y_i + \sum_{(i,j)\in A: j\in V_F(S)} q_i z_{ij} + \sum_{i\in V_C(\overline{S})}\sum_{j\in S} q_i x_{\{i,j\}}\right). \tag{EC.20}$$

We have:

$$\sum_{e\in\delta(S)} x_e = \sum_{e\in E(0:S)} x_e + \sum_{e\in E(S:\overline{S})} x_e, \tag{EC.21}$$

and for each $i\in V'$ (see equation (2)):

$$\sum_{e\in\delta(i)} x_e = 2y_i = x_{\{0,i\}} + \sum_{e\in E(i:S)} x_e + \sum_{e\in E(i:\overline{S})} x_e. \tag{EC.22}$$

From equation (EC.22), the term $\sum_{i\in V_C(S)} q_i y_i$ of inequality (EC.20) can be rewritten as follows:

$$\sum_{i\in V_C(S)} q_i y_i = \sum_{i\in V_C(S)} \frac{q_i}{2}\left(x_{\{0,i\}} + \sum_{e\in E(i:S)} x_e + \sum_{e\in E(i:\overline{S})} x_e\right). \tag{EC.23}$$

The MI inequality (EC.20) can be rewritten as:

$$\begin{aligned}\sum_{e\in E(0:S)} x_e + \sum_{e\in E(S:\overline{S})} x_e \geq \sum_{i\in V_C(S)} \frac{q_i}{Q} x_{\{0,i\}} + \sum_{i\in V_C(S)}\sum_{e\in E(i:S)} \frac{q_i}{Q} x_e + \\ \sum_{i\in V_C(S)}\sum_{e\in E(i:\overline{S})} \frac{q_i}{Q} x_e + \frac{2}{Q}\sum_{\substack{(i,j)\in A:\\ j\in V_F(S)}} q_i z_{ij} + \sum_{i\in V_C(\overline{S})}\sum_{j\in S}\frac{q_i}{Q} x_{\{i,j\}} + \sum_{i\in V_C(\overline{S})}\sum_{j\in S}\frac{q_i}{Q} x_{\{i,j\}}.\end{aligned} \tag{EC.24}$$

We also have:

$$\sum_{e\in E(0:S)} x_e = \sum_{e\in E(0:V_C(S))} x_e + \sum_{e\in E(0:S\setminus V_C(S))} x_e, \tag{EC.25}$$

and

$$\sum_{e\in E(S:\overline{S})} x_e = \sum_{j\in S} \sum_{\{i,j\}\in E(j:V_C(\overline{S}))} x_{\{i,j\}} + \sum_{j\in S} \sum_{\{i,j\}\in E(j:(V'\setminus V_C(\overline{S}))\setminus S)} x_{\{i,j\}}. \tag{EC.26}$$

Notice that $S\setminus V_C(S) = V_F(S)$ and that $(V'\setminus V_C(\overline{S}))\setminus S = V_F(\overline{S})$. Then, inequality (EC.24) can be rewritten as:

$$\begin{aligned}\sum_{e\in E(0:V_C(S))} x_e + \sum_{e\in E(0:V_F(S))} x_e + \sum_{j\in S} \sum_{\{i,j\}\in E(j:V_C(\overline{S}))} x_{\{i,j\}} + \sum_{j\in S} \sum_{\{i,j\}\in E(j:V_F(\overline{S}))} x_{\{i,j\}} \geq \\ \sum_{i\in V_C(S)} \frac{q_i}{Q} x_{\{0,i\}} + \sum_{i\in V_C(S)} \sum_{e\in E(i:S)} \frac{q_i}{Q} x_e + \sum_{i\in V_C(S)} \sum_{e\in E(i:\overline{S})} \frac{q_i}{Q} x_e + \\ \frac{2}{Q} \sum_{\substack{(i,j)\in A:\\ j\in V_F(S)}} q_i z_{ij} + \sum_{i\in V_C(\overline{S})} \sum_{j\in S} \frac{q_i}{Q} x_{\{i,j\}} + \sum_{i\in V_C(\overline{S})} \sum_{j\in S} \frac{q_i}{Q} x_{\{i,j\}}.\end{aligned} \tag{EC.27}$$

Notice that as $q_i = 0$, $\forall i\in V_F$, we have:

$$\sum_{i\in V_C(S)} \sum_{e\in E(i:\overline{S})} \frac{q_i}{Q} x_e = \sum_{i\in S} \sum_{e\in E(i:\overline{S})} \frac{q_i}{Q} x_e = \sum_{i\in S} \sum_{e\in E(i:V_C(\overline{S}))} \frac{q_i}{Q} x_e + \sum_{i\in S} \sum_{e\in E(i:V_F(\overline{S}))} \frac{q_i}{Q} x_e, \tag{EC.28}$$

$$\sum_{i\in V_C(S)} \sum_{e\in E(i:S)} \frac{q_i}{Q} x_e = \sum_{i\in S} \sum_{e\in E(i:S)} \frac{q_i}{Q} x_e, \tag{EC.29}$$

and

$$\sum_{i\in V_C(\overline{S})} \sum_{j\in S} \frac{q_i}{Q} x_{\{i,j\}} = \sum_{i\in \overline{S}} \sum_{j\in S} \frac{q_i}{Q} x_{\{i,j\}}. \tag{EC.30}$$

Inequality (EC.27) can be rewritten as:

$$\begin{aligned}\sum_{\{0,i\}\in E(0:V_C(S))} (1 - q_i/Q) x_{\{0,i\}} + \sum_{e\in E(0:V_F(S))} x_e + \sum_{j\in S} \sum_{\{i,j\}\in E(j:V_C(\overline{S}))} (1-(q_i+q_j)/Q) x_{\{i,j\}} + \\ \sum_{j\in S} \sum_{\{i,j\}\in E(j:V_F(\overline{S}))} (1-q_j/Q) x_{\{i,j\}} \geq \sum_{i\in S} \sum_{e\in E(i:S)} \frac{q_i}{Q} x_e + \frac{2}{Q} \sum_{\substack{(i,j)\in A:\\ j\in V_F(S)}} q_i z_{ij} + \sum_{i\in \overline{S}} \sum_{j\in S} \frac{q_i}{Q} x_{\{i,j\}}.\end{aligned} \tag{EC.31}$$

Since

$$\begin{aligned}\sum_{i\in S} \sum_{e\in E(i:S)} \frac{q_i}{Q} x_e + \sum_{i\in \overline{S}} \sum_{j\in S} \frac{q_i}{Q} x_{\{i,j\}} = \sum_{j\in S} \left(\sum_{\substack{\{j,i\}\in\delta(j):\\ j<i}} \frac{q_i}{Q} x_{\{j,i\}} + \sum_{\substack{\{i,j\}\in\delta(j):\\ i<j}} \frac{q_i}{Q} x_{\{i,j\}} \right) = \\ \sum_{j\in V'} \left(\sum_{\substack{\{j,i\}\in\delta(j):\\ j<i}} \frac{q_i}{Q} x_{\{j,i\}} + \sum_{\substack{\{i,j\}\in\delta(j):\\ i<j}} \frac{q_i}{Q} x_{\{i,j\}} \right) - \sum_{j\in \overline{S}} \left(\sum_{\substack{\{j,i\}\in\delta(j):\\ j<i}} \frac{q_i}{Q} x_{\{j,i\}} + \sum_{\substack{\{i,j\}\in\delta(j):\\ i<j}} \frac{q_i}{Q} x_{\{i,j\}} \right),\end{aligned} \tag{EC.32}$$

and

$$\begin{aligned}\sum_{\substack{(i,j)\in A:\\ j\in V_F(S)}} q_i z_{ij} = q(V_C) - \sum_{i\in V_C(S)} q_i y_i - \sum_{i\in V_C(\overline{S})} q_i y_i - \sum_{\substack{(i,j)\in A:\\ j\in V_F(\overline{S})}} q_i z_{ij} = \\ q(V_C) - \sum_{i\in V_C} q_i y_i - \sum_{\substack{(i,j)\in A:\\ j\in V_F(\overline{S})}} q_i z_{ij},\end{aligned} \tag{EC.33}$$

inequality (EC.31) can be rewritten as:

$$\sum_{\{0,i\}\in E(0:V_C(S))}(1-q_i/Q)x_{\{0,i\}}+\sum_{e\in E(0:V_F(S))}x_e+\sum_{j\in S}\sum_{\{i,j\}\in E(j:V_C(\overline{S}))}(1-(q_i+q_j)/Q)x_{\{i,j\}}+$$
$$\sum_{j\in\overline{S}}\left(\sum_{\substack{\{j,i\}\in\delta(j):\\ j<i}}\frac{q_i}{Q}x_{\{j,i\}}+\sum_{\substack{\{i,j\}\in\delta(j):\\ i<j}}\frac{q_i}{Q}x_{\{i,j\}}\right)+\sum_{j\in S}\sum_{\{i,j\}\in E(\{j\}:V_F(\overline{S}))}(1-q_j/Q)x_{\{i,j\}}+ \tag{EC.34}$$
$$\frac{2}{Q}\sum_{\substack{(i,j)\in A:\\ j\in V_F(\overline{S})}}q_iz_{ij}\geq\sum_{j\in V'}\left(\sum_{\substack{\{j,i\}\in\delta(j):\\ j<i}}\frac{q_i}{Q}x_{\{j,i\}}+\sum_{\substack{\{i,j\}\in\delta(j):\\ i<j}}\frac{q_i}{Q}x_{\{i,j\}}\right)+\frac{2}{Q}(q(V_C)-\sum_{i\in V_C}q_iy_i).$$

Notice that, as $q_i\leq Q$, $\forall i\in V_C$, $x_e=0$, $e=\{i,j\}\in E\setminus\{\{0,h\}:h\in V'\}$, if $q_i+q_j>Q$, all the variable coefficients of the above inequality are nonnegative whereas the right-hand-side of the inequality does not depend on the set S.

The most violated constraint (EC.34) can now be found by computing a minimum s-t cut on an directed capacitated graph $\overline{G}=(\overline{V},\overline{A})$ with $\overline{V}=V'\cup\{s,t\}$ and $\overline{A}=\{(i,j),(j,i):\forall\{i,j\}\in E\setminus\{\{0,j\}:j\in V'\}\}\cup\{(s,i):\{0,i\}\in E\}\cup\{(i,t):i\in V'\}$. The additional nodes s and t represent source and sink node, respectively. The arcs capacities are defined as follows:

- Every arc (s,i), $i\in V_C$ is associated with a capacity $(1-q_i/Q)x_{\{0,i\}}$;
- Every arc (s,i), $i\in V_F$ is associated with a capacity $x_{\{0,i\}}$;
- Every arc (i,j), $i\in V_C$, $j\in V'$ is associated with a capacity $(1-(q_i+q_j)/Q)x_{\{i,j\}}$;
- Every arc (i,j), $i\in V_F$, $j\in V'$ is associated with a capacity $(1-q_j)x_{\{i,j\}}$;
- Every arc (j,t), $j\in V_C$, is associated with a capacity $\sum_{\substack{\{j,i\}\in\delta(j):\\ j<i}}\frac{q_i}{Q}x_{\{j,i\}}+\sum_{\substack{\{i,j\}\in\delta(j):\\ i<j}}\frac{q_i}{Q}x_{\{i,j\}}$;
- Every arc (j,t), $j\in V_F$, is associated with a capacity $\frac{2}{Q}\sum_{\substack{(i,j)\in A:\\ j\in V_F(\overline{S})}}q_iz_{ij}+(\sum_{\substack{\{j,i\}\in\delta(j):\\ j<i}}\frac{q_i}{Q}x_{\{j,i\}}+\sum_{\substack{\{i,j\}\in\delta(j):\\ i<j}}\frac{q_i}{Q}x_{\{i,j\}})$.

Let $(\overline{S},\overline{V}\setminus\overline{S})$ be the minimum s-t cut of $\overline{G}$ and assume that $t\in\overline{S}$. One can see that if the cut capacity is strictly smaller than right-hand-side of inequality (EC.34) then node set $S=\overline{S}\setminus\{t\}$ defines the most violated inequality (EC.34). No violated inequality exists if the cut has a capacity greater than or equal to the value of the right-hand-side of (EC.34). □

EC.2. Details of the heuristic algorithms

EC.2.1. A constructive heuristic

Given an instance of VRPTF, we define a complete graph $\overline{G}=(\overline{V},\overline{E})$ where the node set $\overline{V}=\{0\}\cup V_C$ contains the depot and the customer nodes. Each edge $e\in\overline{E}$ has a cost given by r_e. Each customer $i\in V_C$ has a demand equal to q_i and the capacity of the vehicles is set to Q. Let $m=\lceil\sum_{i\in V_c}q_i/Q\rceil$ be a lower bound on the minimum number of routes required. The details of our implementation of the three phases are as following.

(i) m "seed" customers are randomly selected to initialize the m routes of the emerging CVRP solution. The remaining customers are partitioned into m subsets by heuristically solving a Generalized Assignment Problem (GAP) where each bin k is associated with the k-th customer selected to initialize a route. The assignment cost a_{ik} for allocating customer i to bin k is $r_{0i} + \alpha r_{\{i,k\}} - \beta r_{0k}$, where α, β are nonnegative parameters. The GAP is solved heuristically. If, for a given m, the GAP solution is infeasible, we set $m = m + 1$ and we repeat the above procedure.

(ii) The route for a subset of customers is determined by solving a TSP on the subgraph induced by the subset. We apply a 3-opt procedure to a starting tour obtained by generating a random sequence of the customers.

(iii) The solution obtained at step (ii) is locally optimized using a classical multiroute improvement procedure consisting of two types of operations: (a) movement of a customer from one route to another; (b) exchange of two customers belonging to different routes. We try all possible such operations until no improvement can be obtained. Each route of the modified solution is then re-optimized with the 3-opt procedure.

Since the initial partitioning and the TSP solutions are based on random choices, we can obtain different solutions executing the three phases several times. In our implementation we run them 2000 times: for the first 1000 runs we set $\alpha = 1.1$ and $\beta = 0.7$ (see step (i)), while in the last 1000 runs we set $\beta = 0$, leaving α unchanged.

The CVRP solution so far obtained, is optimized by iteratively applying two re-optimization procedures: procedure Squeeze used by Baldacci et al. (2007) for the CmRSP, and procedure LS-multiple. The two procedures are repeated in sequence until the current solution can be improved. Procedure Squeeze tries to re-optimize the routing (i.e. the set $\overline{E}$) by allowing a few changes in the customer connections (set $\overline{A}$). Procedure LS-multiple is a multiroute improvement procedure based on customer exchanges among the routes of the current solution.

EC.2.2. A Lagrangean heuristic

Procedure CG is interwoven with a heuristic algorithm that produces a feasible VRPTF solution of cost $\hat{z}$ using the route sets $\tilde{\mathscr{R}}$ (see Step 2 of procedure CG). Given the current $DSP(\boldsymbol{\lambda})$ solution, define vector $\tilde{\boldsymbol{\xi}}$ as follows:

$$\tilde{\xi}_\ell = \sum_{i \in V_C} \overline{a}_{i\ell} \frac{q_i}{q(R_\ell)} \zeta^i_\ell, \quad \ell \in \tilde{\mathscr{R}}, \tag{EC.35}$$

by setting $\zeta^i_{\ell(i)} = 1$ and $\zeta^i_\ell = 0$, $\forall \ell \in \tilde{\mathscr{R}} \setminus \{\ell(i)\}$, $\forall i \in V_C$. Define $C(\ell) = V_C(R_\ell) \cup V_A(R_\ell)$, i.e. $C(\ell)$ is the set of customers either visited on the route or assigned to facilities in $V_F(R_\ell)$. The heuristic algorithm performs the following steps.

1. *Initialization.* Initialize $\hat{z} = 0$, $SOL = \emptyset$ and $\delta(i) = 0$, and $\forall i \in V'$.

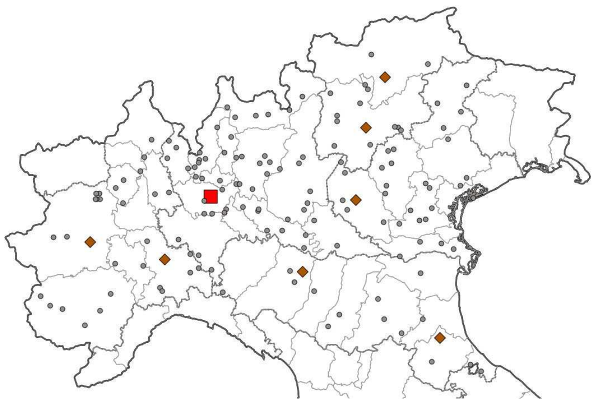
2. *Extract a subset of routes* $SOL \subseteq \tilde{\mathscr{R}}$. Let ℓ^* be the route of $\tilde{\mathscr{R}}$ where $\tilde{\xi}_{\ell^*} = \max\{\tilde{\xi}_\ell : \ell \in \tilde{\mathscr{R}}\}$. Remove ℓ^* from $\tilde{\mathscr{R}}$. If $\delta(i) = 0$, for some $i \in C(\ell^*)$, then update $SOL = SOL \cup \{\ell^*\}$, $\delta(i) = \delta(i) + \overline{a}_{i\ell}$, $\forall i \in C(\ell^*)$, and $\delta(i) = \delta(i) + a_{i\ell}$, $\forall i \in V_F(R^*_\ell)$. Repeat step 2 until $\tilde{\mathscr{R}} = \emptyset$.
3. *Modify the route set* SOL *so that* $\delta(i) \le 1$, $\forall i \in V'$.
   a) Remove from SOL any route $\ell \in SOL$ such that $\delta(i) > 1, \forall i \in C(\ell^*)$, and update $\delta(i)$, $\forall i \in V'$, accordingly. For each $\ell \in SOL$, compute the savings that can be achieved by removing from route ℓ every customer $i \in C(\ell^*)$ having $\delta(i) > 1$. Let $\ell^* \in SOL$ be the route of maximum saving. Remove from route ℓ^* every customer $i \in C(\ell^*)$ with $\delta(i) > 1$, and update $\delta(i)$. Repeat step 3.a until $\delta(i) \le 1$, for each $i \in V_C$.
   b) For each $\ell \in SOL$, compute the total number $\alpha(\ell)$ of customers assigned to every facility $i \in V_F(R^*_\ell)$ having $\delta(i) > 1$. Let ℓ^* be the route having the minimum $\alpha(\ell)$ value. Remove from route ℓ^* every facility $i \in V_F(R_{\ell^*})$ with $\delta(i) > 1$, update $\delta(i)$, and $\delta(j)$, $\forall j \in V_C$, accordingly. Repeat step 3.b until $\delta(i) \le 1$, for each $i \in V_F$.
   c) For each $\ell \in SOL$, remove any facility $i \in V_F(R_\ell)$ with $\delta(i) = 1$ and without customers assigned to it, and update $\delta(i) = \delta(i) - 1$.
4. *Insert unrouted customers.* For each unrouted customer i (i.e., $\delta(i) = 0$) perform the following operations. Compute the minimum extra-cost $exc(i,\ell)$ for inserting i in route $\ell \in SOL$ without considering assignment of i to facilities in $V_F(R_\ell)$. We set $exc(i,\ell) = \infty$ if the total load of the resulting route ℓ exceeds the vehicle capacity Q. Let ℓ^* be such that $exc(i,\ell^*) = \min_{\ell \in SOL}[exc(i,\ell)]$. If $exc(i,\ell^*) = \infty$, then set $\hat{z} = \infty$ and stop; otherwise, insert customer i in route ℓ^* in the position of cost $exc(i,\ell^*)$ and set $\delta(i) = 1$.
5. *Define the* VRPTF *solution* $\boldsymbol{\xi}$. Define $\xi_\ell = 1$, for each $\ell \in SOL$, and $\xi_\ell = 0$, for each $\ell \in \mathscr{R} \setminus SOL$.
6. *Local optimization.* Locally optimize solution $\boldsymbol{\xi}$ by iteratively applying the two re-optimization procedures Squeeze and LS-multiple used also for the constructive heuristic.

EC.3. Details about the instances
EC.3.1. Real-world based instances

A set of six test instances for each area (*North*, *Centre* and *South*) were generated by the company based on the following settings.

- The set of customers is selected from the customers that are currently served on a daily basis by using different criteria. The number of customers varies from a minimum of 54 up top a maximum of 164. The customer demands were computed based on historical data;
- The set of facilities corresponds to the existing set of facilities and can also include new facilities that the company want evaluate in order to revise the current distribution network. Instances with 4, 6, 7, 9, 12, 13, and 18 facilities were generated;

Figure EC.1 Real-world instances: North area



- Two types of fleet of vehicles were considered: vehicles with capacity equal to 24 pallets (*single-unit 3 axes* type of trucks) and vehicles with capacity equal to 36 pallets (*single-trailer 3 axes* type of trucks), respectively;
- The routing cost of a pair of nodes i and j of the network were computed as $r_{\{i,j\}} = c\,dist_{ij}$ where $dist_{ij}$ represents the distance in kilometer between nodes i and j computed using a digital map of the Italian territory, and c is the routing cost per kilometers (currency in expressed in Euro (€)) associated with the type of vehicle (either *single-unit* or *single-trailer*);
- Set $\{F_i\}$ of facilities to which the customers can be assigned are defined directly by the company using different criteria. These criteria take into account the customer demand, required level of service, a priori agreements between the customers and the company, and the distance matrix $[d_{ij}]$ used to compute the routing costs;
- The distribution from the facilities to the customers is performed by means of a fleet of *single-unit 2 axes* type of trucks with a vehicle capacity ranging from 6 to 8 pallets. The distribution cost from the facilities depends on the type of contract that has been defined between the company and the third-party contractor and vary from facility to facility. The distribution cost is a function of the number of pallets associated with the order and the distance between the customer location and the facility. Therefore the assignment cost matrix is defined by the company using the current distribution tariff agreed with the third-party companies.

A total number of 18 instances were generated, 6 instances per areas or depots. Figures EC.3.1, EC.3.1, and EC.3.1 illustrate the layout of the three distributions ares. In the figure, the three depots are represented with squares, and rhombus and circles represent facilities and customers, respectively.

EC.3.2. LRP based instances

From each LRP instance we derived a VRPTF instance as follows.

Figure EC.2 Real-world instances: Centre area

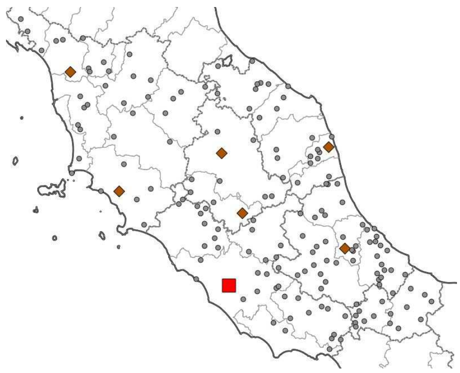


Figure EC.3 Real-world instances: South area

i) The set V_F of facilities, the set V_C of customers (and the associated demands), correspond to the set of depots and customers of the original LRP instance;
ii) The vehicle capacity Q is equal to the vehicle capacity of the original LRP instance;
iii) The depot coordinates were defined as follows. Let x_{min} and x_{max} be the minimum and maximum x-coordinates among the customers and the facilities x-coordinate, respectively; similarly define y_{min} and y_{max}. The coordinate (x, y) of the central depot are defined as follows:

$$x = x_{min} + \lfloor (x_{max} - x_{min})/2 \rfloor \quad \text{and} \quad y = y_{min} + \lfloor (y_{max} - y_{min})/2 \rfloor. \tag{EC.36}$$

The routing and connection costs were generated as follows.

Class A. Routing and assignment costs of a pair of nodes i, j are equal to the Euclidean distance e_{ij}, computed according to the TSPLIB EUC_2D standard.

Class B. For each pair of nodes i, j, the routing cost is $r_{\{i,j\}} = \lfloor \alpha e_{ij} \rfloor$, while the assignment cost is $d_{ij} = \lfloor (10 - \alpha) e_{ij} \rfloor$, where $\alpha = 7.0$.

For all the instances, every customer can be assigned to every facility, i.e. $F_i = V_F$, $\forall i \in V_C$.

We generated a total number of 150 instances, 75 instances per class. The dimensions of the instances vary from very small instances with 12 customers and two facilities up to very large instances with 150 customers and 20 facilities.

EC.4. Details about the computational results on LRP based instances

This section reports the complete details about the computational results on LRP based instances.

Table EC.1 Results on Class A: Akca et al. (2009) LRP based instances

Name	z^*	$\#r$	$\#f$	$\#c$	$\%UB_1$	$\%UB_2$	$\%LB_1$	$\%LB_2$	t_{DA}	$\%LB_C$	t_C	$\%LB$	t_{LB}	$\#cuts$	$\#cols$	#N	$\%Opt$	t_{TOT}
cr30x5a-1	621	5	0	0	100.0	100.0	99.4	100.0	2.6	98.8	2.7	100.0	0.1	0	303	1	100.0	8
cr30x5a-2	665	5	1	2	100.2	102.1	90.6	94.0	2.8	97.5	2.5	99.1	3.2	275	2185	23	100.0	20
cr30x5a-3	575	5	1	2	100.7	100.0	90.8	96.5	2.5	98.5	2.2	98.5	1.3	252	1160	93	100.0	29
cr30x5b-1	727	5	1	1	100.0	100.0	96.4	98.5	2.9	99.7	2.9	100.0	1.3	479	959	1	100.0	10
cr30x5b-2	826	6	0	0	100.0	100.6	92.5	93.6	1.5	93.0	3.7	97.6	2.1	78	1609	367	100.0	79
cr30x5b-3	788	7	1	1	100.1	100.0	94.6	96.1	32.5	94.2	2.8	97.6	1.9	94	1096	1335	100.0	1061
cr40x5a-1	738	7	3	8	100.0	100.0	95.5	97.5	19.5	94.9	4.2	97.9	2.5	98	1530	151	100.0	82
cr40x5a-2	786	6	1	1	100.5	101.0	92.7	96.9	5.2	95.8	8.3	98.2	3.8	357	1627	986	99.3	3615
cr40x5a-3	807	6	4	4	101.1	101.5	94.8	98.0	21.4	95.2	5.3	98.5	2.8	405	1212	562	98.8	3631
cr40x5b-1	964	8	0	0	100.0	101.0	95.9	98.1	11.1	95.6	6.0	98.9	1.3	238	646	187	100.0	49
cr40x5b-2	901	8	2	3	100.0	100.1	93.1	95.7	3.3	95.5	7.9	98.1	3.4	372	910	41	100.0	34
cr40x5b-3	887	8	2	5	100.5	100.5	95.2	96.7	13.5	95.0	6.4	98.1	1.5	295	876	378	100.0	81

Table EC.2 Results on Class A: Prins et al. (2004) LRP based instances

Name	z^*	$\#r$	$\#f$	$\#c$	$\%UB_1$	$\%UB_2$	$\%LB_1$	$\%LB_2$	t_{DA}	$\%LB_C$	t_C	$\%LB$	t_{LB}	$\#cuts$	$\#cols$	#N	$\%Opt$	t_{TOT}
ppw-20-5-0-a	253	5	1	1	100.0	100.0	95.2	96.7	1.5	98.5	1.2	100.0	0.2	11	394	1	100.0	5
ppw-20-5-0-b	211	3	1	2	100.0	100.0	85.3	94.4	7.8	100.0	0.5	97.8	0.6	120	803	17	100.0	12
ppw-20-5-2-a	247	5	1	3	100.0	100.0	90.8	94.5	3.2	95.4	1.3	97.4	0.4	157	711	197	100.0	14
ppw-20-5-2-b	189	3	1	2	100.0	100.0	85.7	93.1	1.6	100.0	0.3	100.0	1.1	181	2212	1	100.0	5
ppw-50-5-0-a	616	12	1	1	100.2	100.2	97.0	98.9	1.6	93.9	14.8	98.9	2.3	224	794	332	100.0	87
ppw-50-5-0-b	400	6	1	2	100.0	102.0	91.8	95.6	6.3	96.8	9.8	96.7	11.9	362	2964	15	96.9	3621
ppw-50-5-2'-a	653	12	0	0	100.5	100.0	95.9	96.1	1.6	95.6	13.4	99.5	2.6	323	827	85	100.0	48
ppw-50-5-2'-b	351	6	0	0	100.3	100.0	91.1	92.9	5.3	99.0	10.6	99.4	6.9	973	2391	204	99.7	3619
ppw-50-5-2-a	587	12	1	2	100.0	100.0	96.7	98.3	1.5	93.6	11.1	98.9	2.1	828	880	949	100.0	126
ppw-50-5-2-b	357	6	0	0	100.0	100.3	92.3	93.4	11.3	96.7	13.3	96.7	6.5	514	2174	61	97.1	3625
ppw-50-5-3-a	586	12	1	3	100.2	100.0	94.8	95.4	20.5	92.2	12.4	95.9	2.0	152	728	20734	97.9	3630
ppw-50-5-3-b	381	6	0	0	100.0	100.0	91.2	94.4	28.3	95.9	8.4	96.8	6.4	493	3335	395	97.4	3644
ppw-100-5-0-a	1158	25	2	2	100.9	100.9	97.7	98.8	3.1	93.8	216.9	99.3	10.6	12	1417	1312	100.0	507
ppw-100-5-0-b	679	11	3	3	100.0	100.0	91.2	95.3	146.6	91.6	115.2	96.7	42.6	2019	5451	7	96.8	3819
ppw-100-5-2-a	1010	24	1	1	100.6	100.4	96.8	97.4	3.1	91.7	193.0	97.5	14.0	640	1672	5639	97.8	3735
ppw-100-5-2-b	569	12	1	1	100.0	100.2	94.8	95.6	99.5	90.0	110.1	96.0	30.8	1040	3019	302	96.2	3783
ppw-100-5-3-a	1068	23	2	3	100.7	100.7	97.1	98.2	29.1	93.6	146.8	98.6	8.8	561	1312	8235	98.9	3686
ppw-100-5-3-b	612	11	0	0	100.0	100.7	92.7	95.6	55.7	93.8	131.5	97.1	27.9	1105	5207	183	97.4	3733
ppw-100-10-0-a	1215	24	1	1	100.1	100.7	97.9	98.7	47.9	89.9	154.4	98.9	11.2	372	1040	2420	100.0	991
ppw-100-10-0-b	693	11	1	1	100.0	101.9	94.2	96.6	208.8	93.2	159.1	97.4	38.3	1006	4232	84	97.6	3869
ppw-100-10-2-a	1030	24	0	0	100.8	100.0	96.2	97.7	134.6	90.6	163.8	97.7	9.9	85	1182	6064	98.2	3853
ppw-100-10-2-b	582	11	0	0	100.0	100.9	93.4	95.7	127.1	91.2	133.3	96.4	54.6	829	4798	30	96.5	3809
ppw-100-10-3-a	1055	24	4	6	100.0	100.3	97.6	98.3	121.6	88.7	142.1	99.3	14.8	30	1327	245	100.0	424
ppw-100-10-3-b	608	11	1	2	100.0	101.6	91.2	92.9	98.7	89.7	123.0	94.6	46.8	919	6930	24	94.8	3773

Table EC.3 Results on Class A: different authors LRP based instances

Name	z^*	$\#r$	$\#f$	$\#c$	$\%UB_1$	$\%UB_2$	$\%LB_1$	$\%LB_2$	t_{DA}	$\%LB_C$	t_C	$\%LB$	t_{LB}	$\#cuts$	$\#cols$	#N	$\%Opt$	t_{TOT}
Christ-50x5	514	5	2	2	100.6	103.1	95.7	97.3	58.1	98.2	8.3	99.2	7.9	1086	3231	54	100.0	130
Christ-50x5_B	533	5	1	2	100.2	100.4	92.5	95.9	84.8	96.4	7.9	96.8	7.7	592	4284	175	97.2	3698
Christ-75x10	783	9	3	6	100.0	100.0	93.4	94.6	40.0	93.6	58.2	97.1	28.6	665	6206	22	97.3	3676
Christ-75x10_B	814	9	3	5	100.0	101.8	94.0	95.2	112.9	94.2	46.0	97.5	33.3	1096	6417	235	97.8	3750
Christ-100x10	831	8	0	0	100.0	101.4	92.9	93.8	321.8	94.8	63.6	96.8	126.1	1335	17779	5	96.9	3987
Gaskell-21x5	371	4	1	2	100.0	100.0	97.1	98.7	1.3	98.1	1.3	100.0	0.3	157	534	1	100.0	5
Gaskell-22x5	554	3	3	4	102.0	100.0	82.3	88.6	83.7	97.3	1.0	99.8	46.0	301	3410	5	100.0	145
Gaskell-29x5	503	4	1	1	102.2	100.0	88.3	94.0	119.1	93.7	1.6	98.0	67.1	223	1936	67	100.0	683
Gaskell-32x5-2	427	3	0	0	100.0	100.0	92.4	98.9	459.5	100.0	1.4	100.0	97.3	6	5890	1	100.0	567
Gaskell-32x5	479	4	1	1	100.0	100.0	91.7	95.7	224.3	98.7	2.4	100.0	49.8	330	2149	1	100.0	280
Gaskell-36x5	411	4	1	1	100.2	100.2	96.4	96.7	8.8	99.0	3.7	100.0	1.3	140	1875	1	100.0	17
Min-27x5	3083	4	1	1	100.0	100.0	89.4	95.2	17.1	99.2	1.7	100.0	1.5	364	1357	1	100.0	24
Perl83-12x2	100	2	0	0	100.0	100.0	92.1	99.3	0.8	100.0	0.1	100.0	0.1	25	417	1	100.0	2
Perl83-55x15	453	10	3	3	101.5	101.3	96.8	97.9	38.2	94.6	17.5	99.3	3.5	324	2422	467	100.0	278
Perl83-85x7	618	11	2	3	100.2	101.1	96.9	97.7	36.1	92.7	59.5	98.2	13.0	580	3516	1835	98.8	3736
P111112-100x10	1346	11	0	0	100.0	100.6	92.1	94.5	163.2	92.2	112.2	95.4	46.4	976	7651	196	95.6	3846
P111122-100x20	1252	11	1	2	100.0	102.6	93.8	96.1	453.1	93.3	148.3	98.7	58.2	70	5927	625	98.8	4138
P111212-100x10	1266	10	0	0	100.0	100.6	92.8	95.8	46.8	93.1	118.8	96.8	57.4	1144	4786	5	96.9	3718
P111222-100x20	1338	11	1	1	100.0	100.4	91.5	94.0	379.0	90.8	208.6	96.1	74.5	1542	5446	19	96.1	4053
P112112-100x10	1236	11	3	3	100.0	100.0	89.6	93.2	196.6	93.1	173.0	96.7	134.8	2344	9884	15	96.8	3889
P112122-100x20	1047	10	3	3	100.0	100.0	84.6	86.8	485.6	92.4	278.6	94.2	227.5	2348	16311	3	94.2	4177
P112212-100x10	892	11	2	2	100.4	100.0	89.0	90.6	220.3	88.6	107.2	92.1	83.5	1045	9920	15	92.1	3918
P112222-100x20	1006	10	1	1	100.0	103.0	93.4	94.4	93.4	94.7	170.8	95.7	176.1	1039	7578	8	95.7	3747
P113112-100x10	1158	11	0	0	100.0	102.9	89.5	91.8	319.1	93.8	176.5	94.0	58.5	1295	6782	8	94.2	4007
P113122-100x20	1190	11	4	6	100.0	102.3	87.8	90.3	227.5	92.9	221.0	96.1	163.0	2200	10465	13	96.1	3914
P113212-100x10	1154	10	1	1	100.0	104.9	92.9	93.6	48.9	93.1	125.8	95.2	63.8	927	4778	97	95.3	3717
P113222-100x20	1078	11	0	0	100.0	100.0	90.6	91.8	73.2	94.4	235.0	94.3	101.7	1164	5276	68	94.5	3748
P131112-150x10	1833	16	1	1	100.8	100.0	93.7	95.1	171.8	90.2	669.4	95.6	134.3	1112	7283	37	95.7	3946
P131122-150x20	1769	16	1	1	100.0	100.8	92.5	95.5	579.3	89.8	872.2	95.9	137.6	34	9383	56	95.9	4411
P131212-150x10	1802	16	1	2	100.0	101.2	93.5	96.2	318.2	91.1	500.7	97.1	171.2	2076	9637	18	97.2	4147
P131222-150x20	1802	15	2	2	100.0	100.5	93.2	95.2	600.7	89.8	892.7	95.6	119.4	870	7002	87	95.7	4371
P132112-150x10	1783	16	3	3	100.0	101.0	91.8	95.0	969.1	93.3	1147.1	96.5	286.1	3288	11779	60	96.7	4815
P132122-150x20	1541	15	1	1	100.0	100.1	88.8	90.4	731.2	91.0	1240.3	93.8	429.3	3329	23858	2	93.8	4508
P132212-150x10	1251	16	0	0	100.0	101.0	91.5	92.6	240.4	90.8	766.6	94.3	180.1	2126	11721	13	94.3	4073
P132222-150x20	1184	16	1	1	100.0	100.0	92.7	93.7	513.2	89.9	985.7	95.8	514.0	2487	11655	3	95.8	4368
P133112-150x10	1899	16	2	2	100.0	103.2	92.4	93.8	617.6	93.0	1182.5	94.7	409.5	2545	12407	6	94.8	4434
P133122-150x20	1498	16	2	2	100.0	100.8	92.5	93.4	1061.3	90.8	755.5	94.5	190.7	1966	9534	88	94.6	4908
P133212-150x10	1245	16	1	1	100.0	103.9	93.1	94.1	649.9	93.4	818.1	95.6	108.3	1602	7306	39	95.7	4510
P133222-150x20	1551	16	0	0	100.0	100.1	90.3	91.0	836.0	89.3	1069.9	91.5	321.5	1317	10764	222	91.6	4688

Table EC.4 Results on Class B: Akca et al. (2009) LRP based instances

Name	z^*	$\#r$	$\#f$	$\#c$	$\%UB_1$	$\%UB_2$	$\%LB_1$	$\%LB_2$	t_{DA}	$\%LB_C$	t_C	$\%LB$	t_{LB}	$\#cuts$	$\#cols$	#N	$\%Opt$	t_{TOT}
cr30x5a-1	4176	5	2	8	100.0	100.2	98.8	99.2	3.4	97.0	2.6	99.5	1.9	426	770	13	100.0	13
cr30x5a-2	4428	5	1	6	100.6	103.3	89.9	92.8	3.2	96.3	3.2	98.2	5.2	338	1431	50	98.3	3609
cr30x5a-3	3655	5	2	11	101.8	100.5	91.5	96.7	3.2	98.7	2.4	98.2	3.3	633	1610	34	100.0	24
cr30x5b-1	4844	5	1	5	102.4	100.0	94.7	97.1	3.0	97.3	3.0	98.4	5.9	893	1141	22	100.0	26
cr30x5b-2	4931	6	4	13	103.3	102.0	95.9	97.2	4.1	95.2	3.1	98.6	0.9	236	656	100	100.0	28
cr30x5b-3	4626	6	3	15	108.3	100.6	95.4	96.5	1.7	94.5	2.1	96.6	0.4	1	707	3330	98.9	3605
cr40x5a-1	4221	6	4	21	100.0	100.5	95.4	96.8	3.5	94.6	5.1	97.4	3.0	15	2112	90	100.0	50
cr40x5a-2	4804	6	4	17	102.0	101.4	91.6	96.9	4.3	94.6	7.1	97.3	5.4	2647	919	282	98.3	3616
cr40x5a-3	4577	6	4	25	106.9	103.9	94.9	97.4	7.4	96.7	3.7	98.8	10.4	1546	1605	213	100.0	2085
cr40x5b-1	6334	9	3	13	102.5	100.5	94.1	96.3	15.2	92.0	5.4	97.0	2.4	592	993	175	100.0	74
cr40x5b-2	5933	8	3	14	100.6	102.1	95.3	96.8	7.5	93.4	6.3	97.2	2.4	219	930	362	100.0	136
cr40x5b-3	5279	8	3	14	102.7	100.9	97.3	98.6	5.6	97.1	5.1	99.4	3.0	1610	955	45	100.0	33

Table EC.5 Results on Class B: Prins et al. (2004) LRP based instances

Name	z^*	$\#r$	$\#f$	$\#c$	$\%UB_1$	$\%UB_2$	$\%LB_1$	$\%LB_2$	t_{DA}	$\%LB_C$	t_C	$\%LB$	t_{LB}	$\#cuts$	$\#cols$	#N	$\%Opt$	t_{TOT}
ppw-20-5-0-a	1535	5	2	7	101.2	100.0	95.3	97.0	0.8	95.6	1.4	99.4	0.4	29	342	7	100.0	4
ppw-20-5-0-b	1315	3	1	8	103.3	100.0	93.0	94.4	1.3	99.2	0.6	99.6	7.6	632	3212	5	100.0	13
ppw-20-5-2-a	1488	5	3	11	102.1	100.0	94.8	95.6	2.1	92.4	1.4	97.3	0.4	300	324	133	100.0	11
ppw-20-5-2-b	1085	3	2	11	103.0	100.0	94.7	99.2	1.9	100.0	0.4	100.0	0.2	1	539	1	100.0	4
ppw-50-5-0-a	4159	12	3	12	102.3	100.7	97.5	99.3	1.7	93.5	15.7	99.3	0.9	7	650	645	100.0	96
ppw-50-5-0-b	2638	6	3	15	100.0	104.2	91.0	96.5	17.6	97.0	13.1	97.1	8.5	981	2751	283	97.3	3633
ppw-50-5-2'-a	4488	12	2	8	100.0	100.5	95.2	95.6	19.3	94.7	16.4	98.1	2.0	316	849	1057	100.0	214
ppw-50-5-2'-b	2439	6	1	7	100.9	100.3	90.8	92.5	7.7	97.7	12.1	98.9	9.8	630	2811	847	99.1	3633
ppw-50-5-2-a	3910	12	2	8	101.8	101.0	97.0	98.3	18.5	92.8	11.3	98.3	0.7	342	719	8849	100.0	946
ppw-50-5-2-b	2389	6	2	10	101.1	100.0	90.7	91.8	7.6	93.6	11.2	94.9	12.8	1414	2733	60	95.2	3623
ppw-50-5-3-a	3649	12	4	14	103.8	101.3	95.3	95.7	39.6	93.7	12.8	95.9	0.8	2	673	19638	98.3	3668
ppw-50-5-3-b	2421	6	3	25	100.0	100.6	90.0	93.8	4.1	92.9	5.7	95.7	9.6	1596	3006	199	96.2	3620
ppw-100-5-0-a	8009	25	3	10	101.5	101.1	97.2	98.5	3.1	93.3	210.1	98.7	7.2	11	1262	12954	99.6	3689
ppw-100-5-0-b	4629	11	4	13	100.0	103.7	93.0	95.7	219.0	92.4	116.9	97.3	50.9	3561	4394	38	97.4	3891
ppw-100-5-2-a	6838	24	3	13	104.1	100.0	96.6	97.2	105.8	91.6	169.9	97.3	6.5	645	1672	8072	97.7	3838
ppw-100-5-2-b	3925	11	3	20	100.4	100.0	93.6	94.8	110.1	89.8	133.6	94.9	17.5	657	2893	326	95.1	3793
ppw-100-5-3-a	7184	24	4	15	103.7	102.7	97.0	97.9	18.2	92.0	124.9	98.3	10.2	1291	1617	5523	98.7	3785
ppw-100-5-3-b	4141	11	4	21	100.0	101.5	93.3	95.3	27.3	92.0	133.4	96.1	44.6	3258	5104	50	96.3	3706
ppw-100-10-0-a	7960	24	8	28	101.2	100.0	95.5	96.8	166.0	88.3	109.0	96.8	5.9	2	1030	6269	97.4	3824
ppw-100-10-0-b	4698	26	6	35	100.0	103.6	93.4	95.3	61.8	91.7	164.9	95.6	24.0	274	3132	55	95.8	3721
ppw-100-10-2-a	6883	23	6	24	101.8	100.0	94.9	96.4	115.1	90.8	178.6	96.4	6.5	1	1988	2439	96.6	3831
ppw-100-10-2-b	3984	11	5	22	100.0	100.8	92.5	94.4	95.3	90.1	120.4	94.5	35.3	37	2982	71	94.6	3780
ppw-100-10-3-a	7060	24	9	32	102.3	100.0	95.6	96.4	147.3	87.4	124.5	97.1	10.5	5	1191	3973	97.5	3802
ppw-100-10-3-b	4081	11	6	22	100.0	102.0	92.2	94.1	75.2	90.5	138.9	95.2	68.3	1697	5956	92	95.3	3750

Table EC.6 Results on Class B: different authors LRP based instances

Name	z^*	$\#r$	$\#f$	$\#c$	$\%UB_1$	$\%UB_2$	$\%LB_1$	$\%LB_2$	t_{DA}	$\%LB_C$	t_C	$\%LB$	t_{LB}	$\#cuts$	$\#cols$	#N	$\%Opt$	t_{TOT}
Christ-50x5	3157	5	4	24	100.0	105.3	97.5	98.8	7.0	99.1	10.4	99.6	8.8	1410	2341	27	100.0	125
Christ-50x5_B	3238	5	5	21	103.9	103.0	96.5	98.5	6.7	99.0	8.8	99.3	11.1	1137	2467	63	99.7	3619
Christ-75x10	5241	9	5	17	100.0	101.4	92.1	92.9	60.4	90.8	45.9	94.4	42.1	1498	4130	83	94.6	3697
Christ-75x10_B	5394	9	5	18	100.0	102.5	93.5	94.6	50.8	92.2	37.8	96.0	55.8	1375	4226	3	96.0	3687
Christ-100x10	5377	8	5	34	100.0	103.9	92.3	93.8	70.2	93.3	63.8	94.9	91.9	813	9094	19	94.9	3735
Gaskell-21x5	2057	4	3	14	100.6	104.6	96.8	98.4	1.8	98.3	1.0	99.4	0.4	285	555	5	100.0	5
Gaskell-22x5	3097	3	3	14	105.5	100.0	78.4	75.9	86.7	96.4	0.8	99.6	147.4	560	3709	7	100.0	268
Gaskell-29x5	3035	3	2	5	103.9	105.0	91.4	96.2	103.1	93.7	1.6	99.4	177.0	69	3273	24	99.4	3708
Gaskell-32x5-2	2537	3	1	8	101.7	101.7	89.1	97.2	488.0	98.8	2.1	99.4	262.4	305	2208	39	100.0	1320
Gaskell-32x5	2691	4	2	9	105.1	100.0	97.1	99.2	206.7	99.3	1.7	99.5	55.2	24	2038	7	100.0	314
Gaskell-36x5	2355	4	3	24	109.8	100.0	98.7	98.8	1.3	99.8	3.8	100.0	6.8	1699	2661	5	100.0	19
Min-27x5	20229	4	2	10	103.9	102.2	82.6	93.8	4.0	95.1	1.4	96.5	4.9	368	1738	100	96.9	3848
Perl83-12x2	519	2	2	12	101.2	100.0	99.3	99.4	0.5	100.0	1.0	100.0	0.5	32	151	1	100.0	1
Perl83-55x15	2404	10	7	38	104.1	104.0	98.5	99.0	10.5	98.4	24.7	99.5	3.6	157	1867	137	100.0	221
Perl83-85x7	3994	11	6	33	101.3	101.1	97.1	97.9	44.4	95.3	62.8	98.0	17.1	1136	4049	247	98.3	3745
P111112-100x10	8778	11	6	36	100.0	102.3	93.4	95.2	58.8	92.2	90.8	95.8	43.9	325	5618	47	95.9	3737
P111122-100x20	8364	11	8	34	100.0	103.3	91.5	93.3	89.6	91.5	180.7	95.4	93.8	192	5649	26	95.4	3773
P111212-100x10	8371	10	8	47	100.9	100.0	92.4	94.8	33.2	93.4	163.3	95.2	57.4	2925	5442	21	95.3	3704
P111222-100x20	8732	11	8	44	100.0	105.3	91.2	92.8	43.0	92.2	227.5	95.5	121.6	442	8533	26	95.6	3718
P112112-100x10	8367	11	4	10	100.0	100.4	87.9	91.9	283.2	91.0	155.2	94.4	207.0	4431	8546	15	94.5	3976
P112122-100x20	6856	10	4	14	100.0	102.2	87.7	88.9	213.6	90.5	219.2	94.2	361.7	2199	14341	2	94.2	3905
P112212-100x10	6024	11	2	16	100.9	100.0	91.0	93.3	285.9	91.3	124.7	94.3	52.7	613	5383	274	94.4	3983
P112222-100x20	6869	10	3	15	100.0	103.0	94.4	95.3	91.1	95.0	221.4	96.5	301.9	5200	8629	28	96.6	3743
P113112-100x10	7987	10	3	11	100.6	100.0	86.8	89.0	276.6	87.0	116.8	90.3	150.0	4002	5866	18	90.5	3916
P113122-100x20	7573	11	4	18	100.0	105.4	92.3	93.9	90.0	93.5	176.8	96.7	624.9	8229	13890	13	96.8	3774
P113212-100x10	7888	10	2	6	100.0	106.0	94.9	95.6	258.7	94.6	153.7	97.1	62.6	725	5611	218	97.2	3925
P113222-100x20	7418	10	5	16	100.0	103.5	91.6	93.0	61.9	93.6	260.8	95.0	252.4	1191	8197	20	95.1	3733
P131112-150x10	12681	15	8	43	100.6	100.0	91.7	93.1	119.3	88.9	686.9	93.3	74.6	55	8146	61	93.4	3896
P131122-150x20	11881	16	14	68	100.0	102.6	91.2	93.2	163.5	90.0	917.0	94.2	175.2	225	9803	88	94.3	3978
P131212-150x10	12314	16	5	21	100.0	102.1	93.0	95.8	427.6	90.7	443.6	96.3	132.1	441	10244	24	96.4	4256
P131222-150x20	11858	15	10	74	100.0	102.1	93.6	94.7	159.9	89.3	748.6	95.1	118.1	459	7479	125	95.2	3906
P132112-150x10	11952	16	3	14	100.0	103.5	93.9	95.7	629.4	92.5	879.6	96.1	81.3	358	7255	22	96.2	4478
P132122-150x20	10198	15	7	37	100.0	101.3	90.5	91.3	569.3	90.8	1193.7	93.1	221.5	795	14115	42	93.1	4343
P132212-150x10	8683	16	1	9	100.0	101.1	91.9	93.0	577.9	90.8	893.1	93.7	243.7	2658	9313	25	93.7	4411
P132222-150x20	8218	16	3	20	100.0	102.2	92.8	94.0	543.6	91.1	955.8	94.3	201.0	2589	8535	37	94.3	4397
P133112-150x10	13043	16	5	11	100.0	105.3	91.7	93.3	152.4	92.0	1048.9	94.6	496.3	6615	11122	8	94.6	3970
P133122-150x20	10027	16	6	24	100.0	104.5	92.9	94.0	464.2	91.0	886.2	94.9	149.8	1643	8831	116	95.0	4300
P133212-150x10	8707	16	1	7	100.0	104.0	92.7	93.9	225.6	92.5	778.3	95.2	95.7	1548	7539	189	95.3	4088
P133222-150x20	10822	16	4	11	100.0	103.0	89.9	90.8	355.6	88.2	849.5	91.1	303.4	2350	9247	30	91.2	4209