

The Logic of Proportionality: Reasoning with Non-Numerical Magnitudes

By *Giovanni Sartor**

A. Introduction

This paper aims at explaining the basic logical structure of proportionality assessments, under the assumption that such assessments are based on quantitative reasoning, even when no numbers are given.

First, an analysis of practical rationality is proposed, including the endorsement of values, the adoption of goals to be pursued, and the selection of plans of actions to be executed. In particular it is argued that possible plans for action are to be assessed with regard to their impacts on all relevant values.

Then the way in which norms affect practical reasoning is considered, distinguishing two kinds of norms: value-norms and action-norms. Value-norms determine what values an agent should consider, in what scale of importance, to assess the merits of goal-directed choices. Action-norms determine what actions an agent should execute.

Compliance with value-norms and with action-norms needs to be conceptualized differently. Compliance with value-norms requires that the agent implement or respect the concerned values. In particular, respect of a value-norm requires the agent not to make choices that diminish the realisation of a certain value, unless that loss is outweighed by the increased realisation of other values. Compliance with an action-norm requires the performance of the prescribed action under the indicated conditions. This analysis is then brought to bear on rights and their constitutional guarantees. It is argued that rights may be protected not only through action-norms, but also through value-norms.

The assessment of the merits of an action with regard to a set of values is then considered. It is argued that this assessment is usually performed by processing mental magnitudes. These magnitudes concern the impact of the action on the realisation of values, the proportional utilities so delivered, and the weights of the values. Even though this processing does not use symbolically expressed numbers, it still deals with quantities, and

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has to comply with the common laws of arithmetic. Relying on some work on cognitive and evolutionary psychology, it is observed that processing non-symbolic approximate magnitudes is a fundamental human cognitive capacity. We share it with animals and it supports our understanding of numerical mathematics and our ability to learn it.

Moving more specifically into the analysis of proportionality assessment in constitutional adjudication, it is argued that this assessment consists in determining whether value-norms directed at the legislator have been duly taken into account. The traditional standards of suitability, necessity and proportionality are examined and reformulated under the perspective of reasoning with non-numerical magnitudes.

Finally some specific issues are considered, such as how the adoption of action-norms can be justified by referring to values, and how proportionality assessments are constrained by the requirement of consistency with precedents.

B. Teleological Reasoning in Practical Rationality

Practical cognition, namely, the cognitive process through which a rational agent determines what to do, may be viewed as a process involving at least four steps:¹

- (1) Value-adoption: The determination to positively evaluate certain aspects of future states of affairs, viewing such aspects—the values—as criteria to assess the merits of one's choices;
- (2) Goal-adoption: The determination to pursue certain aims in order to advance one's values, viewing such aims—the goals—as future objectives to be reached through appropriate plans of action;
- (3) Plan-adoption: The determination to perform certain future actions or combinations of actions, given certain circumstances, as means to achieve certain goals, viewing such actions or combinations of them—the plans—as commitments to be executed in the future;
- (4) Action-adoption: The determination to presently perform a chosen action, given that appropriate conditions are met.

This multistep reasoning process enhances our rational capacities, by enabling us to use in the present the outcomes of our past deliberations. As a simple example, assume that a person values her fitness (has adopted this value). She may consequently adopt the goal of getting in shape during the holidays. To achieve that goal she may then adopt the plan to

¹ See JOHN L. POLLOCK, *COGNITIVE CARPENTRY: A BLUEPRINT FOR HOW TO BUILD A PERSON* (1995) [hereinafter *COGNITIVE CARPENTRY*]; JOHN L. POLLOCK, *THINKING ABOUT ACTING: LOGICAL FOUNDATIONS FOR RATIONAL DECISION MAKING* (2006) [hereinafter *THINKING ABOUT ACTING*]. See also Cristiano Castelfranchi & Fabio Paglieri, *The Role of Beliefs in Goal Dynamics: Prolegomena to a Constructive Theory of Intention*, 15 *SYNTHESE* 237–63 (2007) (arguing that intentions and values are not primitive notions, given that they may be analyzed into combinations of goals and beliefs).

exercise thirty minutes every morning. Finally when a particular morning comes, to implement this plan, she may adopt the present-time intention to exercise for thirty minutes.

Organizations too can instantiate the model of practical rationality just described. An organization may adopt values, namely, abstract criteria for assessing the organisation's performance—such as, for instance, shareholder's value, workers' well-being, social responsibilities, etc. On the basis of such values, the organization may adopt goals, and then plans to achieve those goals. Obviously, in an organization determinations pertaining to different steps of a single decisional process can be allocated to different individuals, according to the particular competence and role of those individuals.

Theoretical analyses of practical rationality are often limited to the third step we have described, i.e., planning for a goal through means-end reasoning. The other steps are, however, equally necessary for enabling successful actions by a bounded agent. In particular value-adoption represents a key aspect of rationality. Without the ability to examine our values critically in light of their implications, to change or modify them, and to reassess their relative importance—is it really so important for me to get rich, how important is environmental protection as compared with economic performance?—we would only be *rational fools*², blindly pursuing objectives that may obnoxious to us or to what we care for—other people, communities, culture, environment, etc.

Nevertheless, teleological reasoning in a strict sense, i.e., planning, (step 3, above), undoubtedly constitutes the core of practical rationality. In teleological reasoning an agent constructs and tests possible plans of action to achieve a goal. Once the agent is satisfied that one of these plans appropriately implements the goal, the agent adopts that plan, i.e., it adopts the intention to implement it. Ideally, the chosen plan should maximize the benefit to the agent, i.e. be superior to any other possible plan. Unfortunately, teleological optimality usually cannot be achieved in practical matters, and it would be usually impossible even to know whether it has been achieved. Thus the agent needs to settle on a lower standard, which we may call teleological acceptableness. A teleologically acceptable plan does not need to be optimal. It is sufficient that it is (1) better than inactivity, and (2) not worse than any other plan the agent has been able to conceive of so far through an adequate inquiry.³

² See Amartya Sen, *Rational Fools: A Critique of Behavioral Foundation of Economic Theory*, 6 PHIL. & PUB. AFF. 317 (1977).

³ HERBERT A. SIMON, REASON IN HUMAN AFFAIRS 63 (1983). For a discussion of the idea of satisficing, as developed by theorists of bounded rationality, see HERBERT A. SIMON, THE SCIENCES OF THE ARTIFICIAL 139 (2d. 1981).

We cannot within practicable computational limits generate all the admissible alternatives and compare their respective merits. Nor can we recognize the best alternative, even if we are fortunate enough to generate it early, until we have seen all of them. We satisfice by

To conclude this summary analysis of practical rationality, some considerations may be useful.

First of all, the steps of this multi-layered model of rationality are naturally ordered in a sequence: on the basis of our values we adopt our goals, on the basis of our goals (and values) we adopt our plans, and on the basis of our plans we adopt our actions. However, the corresponding activities usually proceed in parallel: according to the circumstances and the problems we encounter we switch from one activity to the other. In particular, we usually only partially specify our values and have only very vague ideas concerning their relative importance. Only when facing concrete decisional issues, at the planning stage, do aspects of our values become more precisely defined.⁴

Secondly, the progress of our practical reasoning depends on epistemic information, namely, on our beliefs concerning the implication of our values and their interdependencies, the causal connections according to which we may hope to implement our goals, and the circumstances in which we should act to implement our plans. As new factual information comes in, plans and values may need to be changed.

Thirdly, changes in our higher-level determinations lead us to revise our lower-level determinations. In particular a change in the values of an agent may lead it to change its goals, and revise its plans. For example, an individual may determine that enjoying her work is more important than getting rich and consequently change his job; similarly, a

looking for alternatives in such a way that we can generally find an acceptable one after only moderate search.

Id.

⁴ HERBERT A. SIMON, ADMINISTRATIVE BEHAVIOR: A STUDY OF DECISION-MAKING PROCESSES IN ADMINISTRATIVE ORGANIZATIONS 63 (2d ed. 1957).

The fact that goals may be dependent for their force on other more distant ends leads to the arrangement of these goals in a hierarchy each level to be considered as an end relative to the levels below it and as a means relative to the levels above it. Through the hierarchical structure of ends, behavior attains integration and consistency, for each member of a set of behavior alternatives is then weighed in terms of a comprehensive scale of values, the ultimate ends. In actual behavior, a high degree of conscious integration is seldom attained. Instead of a single branching hierarchy, the structure of conscious motives is usually a tangled web, or more precisely, a disconnected collection of elements only weakly and incompletely tied together; and the integration of these elements becomes progressively weaker as the higher levels of the hierarchy the more final ends are reached.

Id.

political body may consider that environmental protection is more important than increasing GDP, and consequently change its energy policy.

Fourthly, there is no direct clash between determinations at different levels, since they play different roles in decision-making. In particular, we should not address an apparent conflict between a value and a plan—*e.g.*, between the value of environmental protection and the choice to construct a coal power station—as a clash between two reasons for action, to be decided by evaluating their respective strength. This is because the lower-level determination, such as the choice of building the power station, has no independent merit; its merit depends on the fact that it appropriately balances certain values.⁵ We should not weigh the plan against a particular value it interferes with, but rather we should weigh, as we shall see, the impacts of the plan on all the values it promotes or demotes and decide accordingly whether to keep it or to abandon it. Weighing a plan of action only against the values on which it negatively interferes, *i.e.*, whose satisfaction it diminishes, would lead us to mistaken conclusions. This is because, as we shall see the merit of a plan is the difference between the utility gains and losses that it produces relatively to all relevant values, so that by matching the plan's merit against the losses it causes, losses would be counted twice.⁶

C. Value-Norms and Action-Norms

On the basis of the above analysis of practical reasoning, we can distinguish two kinds of norms: value-norms and action-norms. These norms differ in three regards: the aspects of decision making they affect, the ways in which compliance with them can be assessed, and

⁵ This can also be expressed by saying that a plan operates as an exclusionary reason with regard to the values that were considered in its adoption, though this terminology may be misleading. *See, e.g.*, Joseph RAZ, PRACTICAL REASON AND NORMS (1975).

⁶ Let me clarify this issue with an economic example. Assume that the choice to switch to a new product in a company would cause a loss of 100 for the abandonment of the old product, which still has some market, and a gain of 150 from the new product. Then we can say that the plan would provide a net gain of 50. Now if we were to consider whether to adopt the plan by comparing the net gain it provides (50), with the loss it causes (100) it seems that we should not implement the change, since the loss is higher than the benefit provided by the plan, so the result of the plan seems to be negative: $50 - 100 = -50$. But this calculation is wrong, because we have double-counted the loss: first, we subtracted it to compute the net outcome of the plan, and then we subtracted it again when comparing the net outcome of the plan against the loss it causes. The same kind of reasoning applies when losses and gains concern different values. Assume for instance that a decision has to be taken concerning whether to endow an existing power plant with new expensive anti-pollution measures. Assume that environmental benefit outweighs the additional costs, providing a utility 1.5 times bigger than such costs. By comparing the net benefit of the plan (0.5 times the costs) against its costs (1 times themselves), it seems again that the power plant should not be endowed with the anti-pollution measures, since the wrong computation gives a negative result: $0.5 - 1 = -0.5$.

the cognitive attitudes they postulate in their addressees.⁷ Let us first consider the way in which these norms affect decision-making and then move to the other aspects just mentioned.

- *Action-norms*: An action-norm is meant to govern the plans of action that one or more agents deliberate to pursue. An action-norm may require, given certain conditions, (1) the performance of an action, or (2) the abstention from an action.
- *Value-norms*: A value-norm is meant to govern the values that one or more agents adopt for assessing plans of action; it identifies values or establishes their importance. A value-norm may require (1) the *promotion* of a value, i.e., its increased satisfaction (2) the *respect* of a value, its non-diminished satisfaction or (3) the *irrelevance* of a value, i.e., its non-consideration.

Let us now consider *compliance* with the two kinds of norms.

- *Action-norms*: (1) An action norm requiring the performance of an action under certain conditions, is complied with in case the agent accomplishes the action under those conditions; (2) an action-norm requiring the omission of an action under certain conditions is complied with in case the agent does not accomplish the action under those conditions.
- *Value-norms*: (1) A value-norm requiring the promotion of a value is complied with when the agent chooses, among the available courses of action, the one providing a higher realisation of that value, unless superior satisfaction could only be obtained through a more important sacrifice of different values; (2) a value-norm requiring the respect of a value is complied with where the agent does not choose a course of action that decreases the realisation of that value unless the sacrifice is needed for obtaining a more significant increase in the realisation of other values; (3) a value-norm requiring the irrelevance of a value is complied with where the agent does not choose a course of action which provides a higher realisation of the irrelevant value, while providing a lower result with regard to the other values at issue.

The *intentional attitudes* that are required for the intentional compliance with such norms are also to be distinguished.

- *Action-norms*. The agent must have the intention to perform/omit the required action, under the indicated conditions.
- *Value-norms*: The agent must take into account the value, and give it an adequate importance in the assessment of the teleological merit of courses of action having an impact on the value.

⁷ See Manuel Atienza & Juan Ruiz Manero, *Permissions, Principles and Rights. A Paper on Statements Expressing Constitutional Liberties*, 9 *RATIO JURIS* 236 (1996), which discusses the distinction between action-norms and norms having a purposive content, which they call policies.

The distinction between value-norms and action-norms can be mapped into the dichotomy of rules and principle provided by Robert Alexy,⁸ who defines principles as *optimisation commands*: Requirements to realize a value as much as possible. However, the notion of value-norm here proposed is broader than Alexy's idea of a principle, since besides norms requiring the promotion of a value—which may be understood as optimisation's commands, in Alexy's terms, though not literally, since as we have observed optimisation is usually unachievable—it also includes norms requiring the respect of a value or its irrelevance.

The distinction between action-norms and value-norms can also be mapped into Luhmann's opposition between conditional-programs, *Konditionalprogramme*, and goal-programs, *Zweckprogramme*.⁹ According to conditional programs the determination of whether an action is legal or illegal depends on ascertainable pre-existing conditions, while according to goal programs this determination depends on the future consequences of the action (its impact on the relevant goals and values). Luhmann argues that legal decisions must be based on conditional-programs, since otherwise they would fail to provide certainty. I believe that Luhmann underestimates the possibility of assessing an action on the basis of its impacts on the relevant values, and consequently he underestimates the possibility of viewing teleologically mistaken choices as legally mistaken and allowing for remedies again such choices. There are undoubtedly uncertainties related with making legal decisions challengeable for failing to adequately realize the intended goals, since the assessment of the future effects of a choice are uncertain on both factual and normative grounds, and can change as time goes by and the situation evolves. However, these uncertainties can be reduced in various ways: maintaining the legal validity of the mistaken choice up to the moment when it is reviewed, setting thresholds of teleological mistakeness for revision—for instance, requiring that the choice be not just suboptimal, but unreasonably defective—requiring teleological mistakes to exist with regard to the epistemic situation of the moment when the decision was taken, regardless of future changes, etc.¹⁰

D. Implications of the Distinction between Value-Norms and Action-Norms

Value-norms and action-norms intervene at different stages in the decision-making process of their addressees and are meant to play different roles in it. Value-norms govern teleological reasoning, they are meant to shape the teleological reasoning by their addressees, and in particular their assessment of whether an action is teleologically

⁸ ROBERT ALEXY, A THEORY OF CONSTITUTIONAL RIGHTS 44–110 (2002).

⁹ NIKLAS LUHMANN, DAS RECHT DER GESELLSCHAFT 195 (1996).

¹⁰ See *id.* at 198.

appropriate, since it adequately takes into account the values at stake. On the contrary, action-norms side-step teleological reasoning, since they are meant to directly provide their addressees with the intention to execute the required action, once the agent believes that the relevant circumstances have been met. Their addressees do not need to engage in teleological reasoning, unless this reasoning is required for finding a suitable way to accomplish the required action. When an agent is committed to comply with an action norm, the agent should perform the required action even when it believes that a different action would be teleologically preferable—would better implement/respect the values at stake –, up to the point where the agent considers that it should withdraw or hedge its commitment.

Value-norms are most important in the regulation of the activity of public bodies. They govern the discretionary activity of such bodies, assigning them values to be taken into account—e.g., economic prosperity, environmental quality, cultural development, education, cost-effectiveness and also, as we shall see, respect and promotion of fundamental rights. They also determine which specific purposes are to be achieved by each public body, according to its particular focus.

In particular, legislative action is typically governed by value-norms, within the constraints provided by constitutional action-norms that rule out particular legislative choices—e.g., laws allowing for torture, detention without judicial control, racial discrimination, death penalty, etc. However, value-norms cannot completely regulate the value-standards that inspire the legislative activity, since there must be a space for political choice, namely, for the choice of what values to take into account with what urgency. The same applies when value-norms are directed to other authorities having a degree of autonomy in establishing the goals and values of their activities such as regional or local governments, and more generally deliberative forums involving stakeholder or their representatives.

Value-norms, though being mainly directed to public bodies, may also find limited application to private actors, though this seems to conflict with the Kantian idea that legal norms should only limit the sphere of permissible actions, leaving private agents free in their choice of what values to consider and what goals to pursue. Consider for instance the requirement that children's interests are taken into account by their parents or also the requirement that human dignity and solidarity are respected in private interactions. The latter value-norm may be linked to another Kantian idea: an agent, even when primarily focused on its own interests, should still pay some attention to the interests of others, should still view its fellows as valuable ends in themselves, and thus avoid inflicting upon them disproportioned hardships.

Action-norms are the primary form of regulation of private activities, but they also address public activities. Consider for instance how public bodies have to behave according to the procedural rules that apply to them, or how public organizations have to comply with general rules such as, for instance, the law of contracts, torts or speed limits.

The action-norm/value-norm dichotomy does not coincide with other classifications of legal norms. First of all, it must be distinguished from the opposition between defeasibility and indefeasibility, which concerns the extent to which a norm is susceptible to being overridden by reasons against its application in particular cases. Action-norms too can be defeasible. For instance, a contractual default may not be applicable if the parties express a different intention. Similarly, a speed limit may be inapplicable when a car race takes place in an urban circuit or may be defeated when one is rushing to the hospital in an emergency. More generally, excuses provide exceptions to criminal and civil liability.

Secondly, the action-norm/value-norm dichotomy must be distinguished from the opposition between determinacy and indeterminacy, which concerns how precisely a norm characterizes the requested behavior and its circumstances. Even action-norms may be very indeterminate; consider for instance the obligation to act in good faith, or not to damage others recklessly. Action-norms and value-norms remain distinct, even when the content of an indeterminate norm can be filled on the basis of teleological consideration, as when the notion of recklessness is determined with reference to the objective of minimizing social costs; by fixing that content, on teleological consideration, we obtain an action rule telling what we should do under certain conditions.

The distinction between action-norms and value-norms overlaps with another significant distinction, the distinction between a yes/no state of affairs and a scalable state of affairs. Action-norms concern the realisation of yes/no states of affairs, while value-norms concern the realisation of scalable states of affairs, which can be realized up to different extents. For instance, while being a citizen is a yes/no state, being free or unfree is a scalable state of affair, since this is a function of the number and quality of the options within one's reach. When two conflicting duties concern the realisation of a yes/no state of affairs, preference should be given to one duty to the exclusion of the other. By contrast, when two scalable values are in conflict, the best compromise usually requires that neither of them be completely neglected to the advantage of the other, given that the satisfaction of values provides a decreasing marginal benefit, as we shall see in the following. A scalable value—e.g., the value of not distressing people, when they are questioned or detained—can become an action-duty below a certain threshold—the duty not to torture people.

E. Value-Norms and Legal Rights

According to the so-called benefit-theory of rights, advanced by authors such as Jeremy Bentham and Rudolf Jhering, a right protects an individual interest or opportunity. Thus, the statement that “*j* has a right to ω toward *k*”—where *j* is the beneficiary of the right, *k*

is the counterpart, and ω is the opportunity that the right is meant to provide—expresses two components:¹¹

- An axiological component. The law positively values the situation where j enjoys opportunity ω , it characterizes j 's enjoyment of ω as a valuable individual interest, namely, as a valuable benefit pertaining to particular individuals separately considered—my freedom to speak, your freedom to speak, etc.
- A guarantee component. There exist certain guarantees aimed at facilitating j 's enjoyment of ω , which bear upon counterpart k , guarantees that may be specified by other norms or may have to be argued from general principles.

The guarantees of a right may include both value-norms and action-norms. This is relevant in particular when rights are directed toward public authorities.

A value-based guarantee of j 's right to opportunity ω , toward the authority k , pertains to k 's duty to view j 's enjoyment of ω as a value, and to give this value appropriate consideration—e.g., the duty to adequately consider freedom of speech when introducing a regulation aimed at protecting privacy.

The action-duty guarantees of j 's right to opportunity ω , toward the authority k , may consist in various specific action-duties of k . For instance, with regard to freedom of speech toward the government, the duty not to prevent the exercise of the right can be viewed, in liberal regimes, as a perfect action-duty, i.e., the prohibition that government takes any action restricting with individual freedom of speech, unless specific conditions exist for a legitimate limitation. Under such conditions freedom of speech would only be the object of a value-duty. The prevention of private interferences hindering the expression of unwanted opinions, and the provisions of resources enabling every individual or group to speak to the public, can only be viewed as values, at this level of abstraction, i.e., as scalable goals to be taken into account. These value-duties may be accompanied by specific action-duties, which emerge when a legal norm establishes the obligation to achieve the goal in a certain way or up to a certain threshold—e.g., a norm requiring that a government shall give to all candidates in an election certain minimal financial resources to be used in their campaign.¹²

In order for a right to exist, it is not necessary that full protection be provided, through enforceable action-duties whose implementation would insure the satisfaction of the individual interests at issue. The protection of certain rights—e.g., some social rights, such as the right to work or to housing—may only consist in value-duties, often not judicially

¹¹ See Giovanni Sartor, *Fundamental Legal Concepts: A Formal and Teleological Characterisation*, 21 *ARTIFICIAL INTELLIGENCE & L.* 101 (2006).

¹² For a discussion on the different norms related to a constitutional right, see Atienza & Manero, *supra* note 7.

enforceable through the right-holder's individual action. This would provide a lesser, but not irrelevant, protection of the corresponding individual interests.

Some rights may operate in different ways with regard to different counterparts. For instance, the right to privacy may be protected by a negative action-duty with regard to administrative authorities, which are prohibited from using personal data unless specifically allowed by the law, but only as a value-duty with regard to the legislature, who can limit the protection of privacy through legislation, to advance other values such as security or freedom of information. Usually in the case of a constitutional right, a constitutional value-duty directed at public authorities requires the right to be taken into account in public decision-making. This right is complemented by various value-duties and action-duties upon administrative authorities, resulting from the interpretation of the constitution or from ordinary legislation, prescribing actions advancing the right or prohibiting actions that would impair it. Certain rights, such as social rights, may be protected only by a value-duty at a constitutional level, being complemented by value- and action-duties at the legislative level, according to statutory norms granting certain social benefits to citizens.

Thus, the idea that constitutional rights identify valuable interests protected through value-duties is consistent with the view that the same rights may also be protected through a range of defeasible or even indefeasible action-norms. However, outside the domain where an action-norm is to be applied, e.g., the prohibition against torture, value-norms—e.g., those requiring the respect of individual self-determination and integrity—would still operate. The view that constitutional norms prescribe value-duties is also consistent with the assumption that certain individualized values—the enjoyment of civil and political liberties—carry more weight than other values, and in particular more than certain collective interests, as we shall see in the following. Thus, it is true that the adoption a value-based understanding of right-norms necessarily involves what has been disparagingly called a “utilitarianism of rights”,¹³ namely, the view that trade-offs may be justified between different rights and even between individual rights and public interests. However, a legal system may constrain such trade-offs by stating action norms and specifying priorities over values.

F. Quantitative Reasoning without Symbolically Expressed Numbers

When we are to assess whether a decision α has failed with regard to certain values, we need to compare the extent to which the relevant values are realized by α and the extent they would be realized if a different choice β were made instead of α —where β may consist in not interfering with the status quo, or in changing it in a different way. Moreover, as we shall see, we need to take into account the differential benefit, or utility,

¹³ ROBERT NOZICK, ANARCHY, STATE AND UTOPIA 30 (1974), 28.

that is provided by the implementation of different choices. This raises the issue of how we are going to determine the impact of a choice on the relevant values and aggregate such impacts into a determination of the overall benefit or loss that is provided by that choice, as compared with other possible choices.

If we could obtain appropriate numbers,¹⁴ it seems that some mathematics should easily provide the answer on the merit of a choice. For this purpose we should need numbers expressing the different impacts of our choices on the realisation of the values at stake. Moreover we need functions connecting such impacts to further numbers expressing the corresponding benefits or losses in single currency. However, in most legal cases, we do not seem to have sensible ways of assigning such numbers and constructing the corresponding utility functions. Nor have we an exhaustive set of preferences between all possible combinations of the different realisations of values, which may be represented as a utility function, in accordance with the representation theorems used in economics.¹⁵ This makes quantitative methods used in decision theory and cost-benefit analysis not directly applicable to many legal contexts, and in particular, to constitutional decisions involving impacts on different values.¹⁶

It seems that to explain how we are able to make reasonable choices on the basis of their impacts on different values, even though we cannot sensibly express these impacts through numbers, we have to assume that people, in particular, legislators and judges, possess some, more or less inborn, capacity to engage in non-numerical quantitative reasoning.

There are two possible alternative approaches to account for this capacity. The first approach consists in assuming that this non-numerical capacity for quantitative reasoning is limited to ordinal comparison between quantities: Without numbers we are able to assess that a certain object possesses more or less of a certain quality—such as length, volume, weight, speed, etc.— than another object, but we cannot say how much each object possesses of that quality. The second approach, which I find more plausible, consists in assuming that this capacity also covers cardinal measures: Even without numbers we are able to assess, though in a very approximate way, the extent—i.e., a cardinal quantity—up

¹⁴ I use the term *number* to refer only to the cases where a quantity is expressed with the symbols, the numerals, or a particular number system. When a quantity is represented, e.g. graphically, or mentally, without the use of such symbols, I use the term *magnitude*.

¹⁵ According to the so-called Morgenstern-Von Neumann representation theorem, if we have a set of preferences among alternatives, and these preferences are complete, transitive, independent and continuous, then we can build an utility function assigning a numerical utility to each alternative, in such a way that any alternative being strictly preferred to another would have a higher utility than the latter.

¹⁶ This does not exclude that the methods of decision theory and cost-benefit analysis can be usefully deployed in many cases. For a technical account of multi-criteria decision-making, see RALPH L. KEENEY & HOWARD RAIFFA, DECISIONS WITH MULTIPLE OBJECTIVES: PREFERENCES AND VALUE TRADE OFFS (1993).

to which an object possesses a certain quality, or the extent of its difference from another object. To express such non-numerical cardinal evaluations, we often refine our ordinal assessment with adverbs: We say, for instance that an object is a little, fairly, a lot larger, smaller, or quicker, than another object. We can sometimes map such evaluations into numbers, even without referring to a general unit of measure and without engaging in explicit numerical computations: We may just say that an object is about a half, two times, three times larger, or smaller, or quicker than another. For instance not only can we compare two lines and establish which one is longer, but we can say that one line is twice the length of the other, or that a line is the sum of the two lines of different sizes, without the need of making numerical calculations.

This capacity is not peculiar to humans. Cognitive scientists affirm that we share it with various animals, such as monkeys, rats and pigeons:

There is considerable experimental literature demonstrating that laboratory animals reason arithmetically with real numbers. They add, subtract, divide, and order subjective durations and subjective numerosities; they divide subjective numerosities by subjective durations to obtain subjective rates of reward; and they multiply subjective rates of reward by the subjective magnitudes of the rewards to obtain subjective incomes.¹⁷

In fact, it seems that animals are not only able to order objects according to size, but they can also perform tasks that involve processing magnitudes. Animals can compute distances by summing up the extent of successive displacements, they make visits to caches according to the difference between the time when the food was stored and its expected rotting time, they remain in different locations according to ratios between time spent and rewards obtained, etc. From this evidence the hypothesis has been made that a developed mathematical competence is quite widespread in the animal kingdom:

Research with vertebrates, some of which have not shared a common ancestor with man since before the rise of the dinosaurs, implies that they represent both countable and uncountable quantity by means of mental magnitudes. The system of arithmetical reasoning with these mental magnitudes is closed under the basic operations of arithmetic, that is, mental magnitudes may be mentally added, subtracted, multiplied, and divided without restriction.¹⁸

¹⁷ C.R. Gallistel, Rochel Gelman & Sara Cordes, *The Cultural and Evolutionary History of the Real Numbers*, in *EVOLUTION AND CULTURE: A TYSSON FOUNDATION SYMPOSIUM 247*, 255 (Stephen C. Levinson & Pierre Jaisson eds., 2006).

¹⁸ *Id.* at 259.

Similarly, experiments with humans show that we can do computations with quantities without associating numerical symbols to such quantities.¹⁹ Thus, it seems that there exists an inborn ability to represent and mathematically process mental magnitudes, which is deployed without translating these magnitudes into the linguistic symbols of a number system. Contrary to a famous statement by the mathematician Leopold Kronecker that “God made the integers; all else is the work of man”, it seems that God, or natural evolution, endowed us with the primitive ability to store and process continuous, though approximated or noisy, mental magnitudes.²⁰ These magnitudes are mappable only into real numbers—since they include also negative numbers, fractions, and even irrational numbers. In addition to this ability, humans have also the option of using symbols for expressing such quantities and making them more precise. Our mind, however, continues to map numerical values into analogical magnitudes, and we resort to the latter when making quick, unreflected judgments. According to John Pollock this capacity for intuitive cardinal assessment of quantities, which he calls “analogical quantitative cognition”, applies not only to lengths, weights or volumes, but also to our likes and dislikes, and to the realisation of our value.²¹

I shall accept the assumption that humans have a basic and largely inborn—though improvable by training and experience—intuitive capacity for non-symbolic quantitative reasoning. This capacity includes not only assessing and comparing magnitudes, but also performing on such magnitudes approximate mathematical operations: sums, subtractions, proportions, multiplications and divisions and even approximate differentiation and integration. I shall also accept that this capacity is involved in assessing impacts on values. We can deploy it in making choices concerning our private life. We do this, for instance, when we choose a car or a computer by balancing design, performance, and cost or when we choose a restaurant by considering quality of food, service and price, or when we decide on a course of studies balancing interests and work-opportunities. The same process is undertaken when public choices have to be made. For engaging in this kind of intuitive or *analogical* quantitative reasoning, we do not need to translate quantities into numbers through measurement, which is an ability that only humans possess, and in many domains only after adequate schooling. We just rely on our intuitive appreciation of the quantities involved and of their relations. When more precision is needed and numerical quantification makes sense, we may turn to numbers and use

¹⁹ C. R. Gallistel & Rochel Gelman, *Mathematical Cognition*, in THE CAMBRIDGE HANDBOOK OF THINKING AND REASONING 559 (Keith J. Holyoak & Robert G. Morrison eds., 2005).

²⁰ Thus, apparently, these findings of contemporary cognitive science seem to validate Leibniz’s principle of continuity, often expressed by the saying *natura non facit saltus*, at least with regard to the mental processing of quantitative information. See NICHOLAS RESCHER, G.W. LEIBNIZ’S MONADOLGY: AN EDITION FOR STUDENTS § 10 (1991): “I also take it for granted that every created being is subject to change . . . and even that this change is continuous in each.”

²¹ POLLOCK, THINKING ABOUT ACTING, *supra* note 1, at 37–54.

numerical methods to test and refine our intuitions. Thus mathematical relationships do not hold only between symbolically expressed numbers; they also constrain the process of our intuitive-analogical quantitative reasoning. Such relationships can be used as a standard of rationality for that reasoning, and for facilitating the transition into numerical quantification, when possible and convenient.

The assumption that we can reason with approximate quantities does not entail that we can precisely determine such quantities, nor that we can always establish with certainty whether one object's magnitude is bigger than another's. For instance, we may sometimes, though not in most cases, remain uncertain when comparing the lengths of two twisted lines, or the volumes of two solid objects. Similarly we may sometimes, though not in most cases, remain in doubt concerning the impacts of our choices on our values, and the comparative merits of such choices.

G. Quantitative Evaluations without Numbers: Basic Concepts

In the following sections, I shall examine teleological reasoning in law as an instance of non-numerical quantitative reasoning and I shall derive some implications of this idea. First of all, I shall specify certain notions that are needed in order to proceed in the analysis. For this purpose I shall introduce some definitions, assumptions and corollaries, identifying each definition, assumption or corollary with a progressive number.

I assume that we can approximately quantify the quantity of the realisation of a value in a particular situation—where a situation is an actual or possible set of circumstances, including social and institutional arrangements.

*Definition 1 (Realisation-Quantity of a Value): The realisation-quantity of a value v in a particular situation is the extent up to which v is realized in case that situation occurs.*²²

We can express our assessment of the realisation-quantity of a value in non-numerical term—e.g., we may say that privacy is granted to a sufficient extent in Country x , and to an insufficient extent in Country y , or that a large freedom of speech is enjoyed by the citizen of Country x , and even a larger one by the citizens of y . When appropriate numerical indicators are available, we may also express in numerical terms such quantities, for example, the GDP per head or the employment rate of a country. For some values—such as transparency, democracy, economic freedom, equality, and non-discrimination—proxies are available according to various measurements, such as those that are used for ranking countries according to their levels of welfare or of protection of human rights. However,

²² Using a logical notation to make our concept more precise, we may write $\text{Real}_v(s)$ to denote the realisation-quantity of value v in situation s . We correspondingly denote as $\text{Real}_v(s_c)$, or simply Real_v , the level of realisation of v in the current situation s_c (the present state of affairs). Thus $\text{Real}_v = q$ means that in the current situation the value v is realized in quantity q .

even when no such proxies are available, or when they are not known to us, we still engage in quantitative assessments, being aware that such assessments are inevitable noisy, approximate and revisable.

Such assessments may be different according to different conceptions of the values at issue but, I would argue, different people would show some consistency in making them. For instance, I think that very few people would disagree that a 50% increase in the revenue per head, with the same distribution, would result in a much greater welfare, that storing personal data for a longer time or making that data accessible to a larger set of people would determine a restriction of privacy, that extending considerably the time for detention without judicial authorization would greatly restrain individual liberties.

We may wonder, however, if it is really possible to compare situations where values are realized in different ways, and different aspects of the same value are in competition. Assume for instance that we have to choose between a situation where privacy is well protected against governmental interference but much less protected against commercial interference, and a situation where privacy is well-protected against commercial interference and much less protected against governmental interference. Similarly, assume that given a fixed amount of resources available for the welfare of dependent people, we can increase the welfare of old people only by decreasing welfare for children. For the purpose of the application of the model here presented, these cases, involving a conflict between different aspects of the same value, can be addressed by viewing these aspects as distinct values, both of which need to be taken into account to assess the merit of choices affecting them.

Besides assuming that we can quantify the extent of the realisation of a value, we assume that there is also a quantity consisting in the benefit or utility that is delivered by the fact that a value is realized up to a certain extent in a certain situation.

*Definition 2 (Utility-Quantity Concerning Value): The utility-quantity concerning a value v , in a certain situation, is the amount of utility provided by the realisation of v in that situation.*²³

Note that here I use *utility* as a “neutral” term denoting the amount of goodness (or badness, when the utility is negative) that is provided by a choice, without making any assumption on the nature or distribution of such goodness. Thus the “utility” of a choice

²³ We write $Ut_{v,s}$ to denote the utility that is obtained with regard to value v , in situation s , i.e., the utility that is delivered by the fact that v is realised up to extent denoted by $Real_v(s)$. Thus, $Ut_{v,s} = q$ means that such utility is quantity q .

includes the assessment all of its aspects and consequences that interfere with relevant values, increasing or decreasing their realisation.²⁴

In the following I shall consider how to move from the realisation-quantity of a value to the utility that is provided by the fact that the value is realized in that quantity. Obviously, people's assessment of the utility of the realisation of a value may be quite variable, and in particular, more variable than their assessment of the realisation-quantity of a value. However, some relations between such assessments may be considered to be invariant.

First of all, since values are by definition good things, we can assume that the utility provided by the realisation of a value increases as the realisation-quantity of that value increases.

*Assumption 1 (Increasing Utility from Values): A higher realisation of a value provides a higher utility. In other words, when the realisation-quantity of value v increases, also the utility-quantity resulting from this realisation increases.*²⁵

If the realisation-quantity of v in a situation s_2 is higher than the realisation-quantity of v in s_1 , correspondingly the utility-quantity of v in situation s_2 is higher than its utility-quantity in s_1 .²⁶ Moreover, the utility-quantity with regard to a value can increase, given the current situation, only if the realisation-quantity of that value increases.

Thus the utility resulting from the realisation of a value v will increase progressively, when v 's realisation-quantity increases. For instance, a higher level of a value such as health, or environmental quality, or privacy, or freedom of speech, gives more utility than a lower level of the same value. As we shall see in the following, we assume that usually the extent of this increase progressively diminishes, as the realisation-quantity of the value gets higher, i.e., we assume that there is a diminishing marginal utility.

On the basis of the notions just introduced, we can address impacts of choices of action on the realisation of values. We use Greek letters α, β , etc., as variables ranging over actions. Such actions may change the status quo, and this change may affect the values at issue.

²⁴ In particular, I do not assume a utilitarian approach, according to which utility is to be viewed as happiness or preference satisfaction. On the contrary, here utility refers to the sum of all impacts on all legally relevant, communal and individual, values, and may be specified in such a way that the distribution of individual opportunities is subject to some fairness requirements.

²⁵ In mathematical terms, we can say the relation between the realisation of a value and the corresponding utility is a monotonic function, and indeed a strictly increasing one. However, we take this only as a defeasible assumption, which expresses what is usually the case, and does not exclude that in certain cases over realisation of a value can be counterproductive.

²⁶ In other words, if $\text{Real}_v(s_1) < \text{Real}_v(s_2)$, then $\text{Ut}_v(s_1) < \text{Ut}_v(s_2)$.

For simplicity's sake, we assume that each action results in only one outcome. This means that we assume a deterministic framework, where the effects of each action are precisely determined. The unique outcome of an action α is the situation that would result from performing α , in the current situation. We use the symbol \emptyset to denote the null action, which just consists in leaving things as they are, or better put, in letting things evolve without the intervention by the agent we are considering.

We are now able to specify the impact of an action α on a value v , namely, the change the action α can make to the realisation of v . This is the difference between the extent up to which v would be realized by α , and the extent up to which it would be realized by not doing anything, i.e., by the null action \emptyset .

Definition 3 (Realisation Impact): The realisation-impact of an action α on a value v is the difference between the realisation-quantities of v resulting from α and \emptyset .²⁷

For instance if α is a law prohibiting the use of a polluting substance which is currently in use in industrial processes, the realisation-impact of α on health is the increased level of health that results from α , while the realisation-impact of α on productivity is the decreased level of productivity which results from α .

The notion of a realisation-impact allows us to define what it means to promote or demote a value. Promoting means increasing the value's level of realisation and demoting means decreasing the same level, as compared to \emptyset .

Definition 4 (Promotion and Demotion of a Value): An action α promotes a value v if its realisation-impact on v is positive;²⁸ it demotes v if its realisation-impact on v is negative.²⁹

Thus a legislative choice that prohibits the use of a polluting substance may promote health and demote productivity. A legislative measure that makes Internet providers liable for violations of data-protection committed by their subscribers may promote data protection while demoting freedom of speech.

We can similarly characterize the utility-impact of an action with regard to a value as the differential utility provided by that action with regard to that value. This is a measure of the difference in utility that is provided by the fact that the value is realized to a higher or lower extent.

²⁷ Let us denote the outcome of action α , namely, the situation resulting from its performance, in the current situation, as $out(\alpha)$ and the realisation impact, the differential realisation, of an action α on a value v , as $\Delta Real_v(\alpha)$. Then $\Delta Real_v(\alpha) = Real_v(out(\alpha)) - Real_v(out(\emptyset))$.

²⁸ $\Delta Real_v(\alpha) > 0$.

²⁹ $\Delta Real_v(\alpha) < 0$.

Definition 5 (Utility-impact of an Action on a Value): The utility-impact of an action α on a value v is the difference between the utility-quantities provided by α and by \emptyset , relatively to realisation of v .³⁰ The utility-impact of an action α on a value v is also called the utility of α concerning v .

Thus in the above case of the prohibition α of the use of a polluting substance, we can say that α increases the utility concerning health, and decreases the utility concerning productivity, since, as compared with the status quo, α promotes the value of health and demotes the value of productivity,

Corollary 1 (Realisation and utility-impact of \emptyset): The realisation-impact of \emptyset on any value is 0, and so is \emptyset 's utility impact, on the basis of Definitions 3 and 5.

H. Quantitative Evaluations without Numbers: Impacts on Single Values

The notions we have described enable us to compare the impact of different choices on different values. First of all we need to introduce a way to express that a choice α is superior to a choice β with regard to its aggregate impact on a set of values.

Definition 6 (Superiority with Regard to a Set of Values): We say that choice α is superior to choice β with regard to a set of values $\{v_1, \dots, v_n\}$, and write $\alpha \succ_{\{v_1, \dots, v_n\}} \beta$, if α 's utility-impact on this set is higher than β 's utility-impact on the same set.³¹

Note that since the utility-impact of \emptyset is null (0), then a choice α is superior to \emptyset with regard to a set of value, wherever α has a positive utility-impact on that set. Thus, for instance, an environmental measure α that prohibits the use of polluting substance, promoting health and demoting productivity, is superior to the null action \emptyset , in case α 's utility-impact with regard to the combination of health and productivity is positive. In such a case we would write: $\alpha \succ_{\{health, productivity\}} \emptyset$.

We will come back later on how to establish superior utility with regard to a set of values. Let us first address impacts on a single value. When we are considering just one value, we can say that whenever the realisation-impact on that value is positive, then the utility-impact on it is also positive. In fact, according to Assumption 1, the higher realisation

³⁰ Let us denote the utility impact of an action α on a value v , as $\Delta Ut_v(\alpha)$. Then $\Delta Ut_v(\alpha) = Ut_{v,out}(\alpha) - Ut_{v,out}(\emptyset)$.

³¹ In other words, $\alpha \succ_{\{v_1, \dots, v_n\}} \beta$ if and only if $\Delta Ut_{\{v_1, \dots, v_n\}} \alpha > \Delta Ut_{\{v_1, \dots, v_n\}} \beta$.

of a value provides a higher utility concerning that value. But a higher utility concerning a value entails superiority with regard to that value. This leads us to the following corollary.³²

Corollary 2 (Superiority, with Regard to a Value by Superior Realisation): Whenever α 's realisation-impact on value v is higher than β 's, then α is superior to β with regard to v ($\alpha \succ_v \beta$).³³

Note that this corollary also applies to the comparison of a choice α with the null action \emptyset . Since \emptyset provides a null differential contribution to the realisation of any value—the differential contribution of a choice being its difference with regard to the contribution of \emptyset —any choice having a positive differential contribution would be better than \emptyset , and any choice having a negative differential contribution would be worse than \emptyset .

For instance, assume that α denotes the action of issuing a new law allowing wiretapping only on the basis of a judicial warrant while in the existing status quo, denoted by null action \emptyset , police authorities are allowed to wiretap any communication in their criminal investigations. Since α , in comparison to \emptyset , has a positive impact on privacy and a negative impact on crime prevention we can say that $\alpha \succ_{privacy} \emptyset$, while $\emptyset \succ_{crime_prevention} \alpha$.

I. Quantitative Evaluations without Numbers: Pareto Superiority

We can extend the comparison to choices having an impact on multiple values. For this purpose, I shall make a simplifying assumption, namely, the assumption that the utilities resulting from the realisation of different values are independent, so that the utility-impact of a choice with regard to a set of values is just the sum of the utility impacts it has on the separate realisation of each of such values. This assumption may be questionable. For instance it may be argued that we enjoy more a given amount of freedom of speech, or access to culture, when we are not starving, or when we have political liberties. However, for the sake of simplicity, it is convenient to make the following independence assumption.³⁴

Assumption 2 (Independence of the Utilities from Different Values): Given a choice α having an impact on values v_1, \dots, v_n , the utility-impact of α with regard to the set of those values

³² In other words, since (1) $\Delta\text{Real}_v\alpha > \Delta\text{Real}_v\beta$ entails $\Delta\text{Ut}_v\alpha > \Delta\text{Ut}_v\beta$ and (2) the latter entails $\alpha \succ_v \beta$, we can conclude (3) that $\Delta\text{Real}_v\alpha > \Delta\text{Real}_v\beta$ entails $\alpha \succ_v \beta$. This is an application of the propositional inference according to which premises $A \rightarrow B$ and $B \rightarrow C$ entail conclusion $A \rightarrow C$.

³³ $\Delta\text{Real}_v\alpha > \Delta\text{Real}_v\beta$ entails $\alpha \succ_v \beta$.

³⁴ On how to handle cases when this assumption does not hold, see POLLOCK, THINKING ABOUT ACTING, *supra* note 1, at 13.

is the sum $i_1 + \dots + i_n$ of the utility impacts i_1, \dots, i_n of α with regard to each such values.³⁵

For instance, consider a law exempting host providers from liability for the privacy violations committed by their users, as compared to a situation where providers are considered to be liable for such a violation. The total utility provided by such a law results from the sum of the utility impacts it has on the different values involved: its positive-utility impacts on freedom of expression, freedom of information and economic efficiency, and its negative utility-impact on privacy.

The easy case is when α as compared to β provides a higher realisation of some values, and does not provide a lower realisation of any other value. In this case we say that α is Pareto-superior to β .

Definition 7 (Pareto Superiority): We say that choice α is Pareto superior to a choice β if there exists a value v_i such that the realisation-impact of α on v_i is higher than β 's and there exists no value v_j , such that the realisation-impact of β on v_j , is higher than α 's. In this case we also say that β is Pareto-inferior to α .³⁶

The following corollary follows from the definition of Pareto superiority, the assumption that a higher realisation of a value provides a higher utility, and the assumption of the independence of utilities from values.

Corollary 3 (Pareto Superiority Entails Overall Superiority): If α is Pareto superior to β with regard to a set of values then α is superior tout court to β with regard to the same set.³⁷

Consider for instance the legislative choice to raise the length of copyright from seventy to ninety years after the death of the author, and assume that the two lengths are equivalent with regard to the incentive to produce new works, but the shorter term contributes more to the value of knowledge. In such a case we can say that the choice of keeping the shorter term, i.e., leaving the status quo unchanged, is Pareto superior and thus superior tout court to the choice of adopting the longer term.

A legislator's choice that, like this one, is Pareto inferior to the status quo is particularly condemnable, since it makes things worse in some regards, while providing no other

³⁵ In other words, $\Delta Ut_{\{v_1, \dots, v_n\}} \alpha = \Delta Ut_{v_1} \alpha + \dots + \Delta Ut_{v_n} \alpha$.

³⁶ In other words, α is Pareto-superior to β with regard to a value-set $\{v_1, \dots, v_n\}$ if (a) there exists a $v_i \in \{v_1, \dots, v_n\}$ such that $\Delta Real_{v_i} \alpha > \Delta Real_{v_i} \beta$ and (b) for all $v_j \in \{v_1, \dots, v_n\}$, $\Delta Real_{v_j} \alpha \geq \Delta Real_{v_j} \beta$.

³⁷ Whenever some x_i is such that $x_i > y_i$ and for all x_j is such that $x_j \geq y_j$, then $(x_1 + \dots + x_n) > (y_1 + \dots + y_n)$. Thus if α is Pareto-superior to β with regard to $\{v_1, \dots, v_n\}$ then $\alpha >_{\{v_1, \dots, v_n\}} \beta$.

advantage. Such choices may however take place, as a consequence of mistakes in appreciating the social impacts of a new regulation, or as a consequence of the fact that legislators pursue their merely private interests, disregarding public values.

Note that the cognitive capacities that are needed for assessing Pareto superiority are limited. For this purpose, we just need to be able to assess whether the differential impacts on the realisation of the concerned values are positive or negative. We do not need to establish the magnitude of such impacts, nor the magnitude of the utility they deliver.

J. Quantitative Evaluations without Numbers: Comparative Analysis without Pareto Superiority

In many cases legislative choices are not Pareto inferior to the status quo. They promote some values and demote some other values. For instance, a regulation increasing privacy protection may decrease freedom of speech, a regulation increasing environmental protection may decrease productivity, or a regulation beneficial to public health may diminish economic freedom.

To evaluate choices having such impacts, we need to find a way of adding up gains and losses, providing a single outcome, on which to base an evaluation of the whole. This means that the utilities provided by impacts on distinct values must somehow be comparable, and subject to elementary arithmetical operations, such as sum, subtraction, and multiplication.

Let us assume, as above, that we have an approximate way of assessing the current realisation-quantity of a value v , which may or not be expressed numerically, and a way of assessing the impact of a particular action α on the realisation of v , i.e., an assessment of the extent to which α increases or decreases the realisation of v . Given this information, we want to determine the utility-impact of α on v , i.e., the differential utility resulting from α 's impact on the realisation of v . And we want to express this impact as a cardinal quantity that is homogenous to the quantities representing α 's utility impacts on other values, so that we can compute the overall utility of α by adding up all these quantities.

The assessment of the utility-impact of a choice α on a single value may be divided in two-steps. First of all, we try to determine the importance of α 's impact on the realisation of the value, as an abstract quantity, which does not depend on using a particular unit of measurement in assessing impacts; then we try to determine the quantity of utility provided by that realisation.³⁸

³⁸ In the concrete reasoning of a decision-maker the two steps do not need to be clearly distinguished. They may be merged in the intuitive assessment of the importance of the negative or positive impact on the value at issue, an assessment that also reflects emotional responses.

To express a value impact without using a unit of measure, we may specify the gain or loss in proportional terms. For instance, we may say that a certain increase in the GDP per head in a poor county while being small in absolute terms—e.g., measured in a particular currency—still is a big increase relative to the previous level of the GDP per head of that country. Similarly, we may say that a liberalization measure in an authoritarian regime, while represents a little increase in freedom of the press in absolute terms, still is a huge increase relatively to the previous level. Different frames of reference may be used for such proportional judgments. In particular, we may quantify increases or decreases either as proportions of what the full realisation of that value would be, or as proportions of the current realisation level of that value.

Here I follow the first approach, by quantifying the level of realisation of a value as a proportion of the maximum realisation of that value that is concretely available under the existing conditions, i.e., as the maximum realisation resulting from the actions that we view as feasible. As a common-sense example, consider a person who is considering what career to undertake, and is considering what kind of revenue and work satisfaction he may obtain from different professions. The range of revenue-quantities and satisfaction-quantities the person is considering would probably end at the top of the levels of revenue and satisfaction that person considers to be reasonably achievable to him, given his qualifications and the opportunities offered to him by the work market. The same analysis takes place with regard to public choices, whose impacts on the relevant values are to be considered within a feasibility horizon. For instance, changes in the GDP would be assessed with reference to the maximal achievable GDP, and similarly changes in privacy or freedom of speech. Thus, the agent can define the proportional impact of action α on the realisation level of value v as the proportion between the increase or a decrease in the realisation of v brought about by α and the maximum amount of such realisation that is achievable.

Definition 8 (Proportional Impact on the Realisation of a Value): The proportional impact of an action α to the realisation of a value v is the proportion between α 's realisation-impact on v and the reasonably achievable maximum level of v .³⁹

Similarly, we can define the proportional utility impact of an action on the utility deriving from the realisation of a value, as a proportion of the utility that can be obtained by the maximal feasible realisation of that value. Thus, for instance, the proportional utility impact of a legislative action increasing the protection of privacy—for instance, by prohibiting unauthorised commercial spamming—would involve determining how much this contributes to the benefit resulting from a full protection of privacy.

³⁹ $\Delta\text{PropReal}_v\alpha = \frac{\Delta\text{Real}_v\alpha}{\text{MaxReal}_v}$, where MaxReal_v is the maximum, reasonably achievable, realisation of v .

Definition 9 (Proportional Impact on the Utility concerning a Value): The proportional impact of an action α on the utility provided by the realisation of value v is the proportion between α 's utility-impact on v and the utility provided by the maximal, reasonably achievable, realisation of v .⁴⁰ The proportional impact of action α on the utility concerning value v is also called the "proportional utility of α concerning v ".

The next step is to determine the change in the proportional utility that corresponds to a change in proportional realisation. The relation between the two changes is not constant, since the realisation of a value has decreasing marginal utility, i.e., the same quantitative change in the realisation of a value provides less (more) utility the higher (the lower), the position of the realisation interval at issue.

Assumption 3 (Decreasing Marginal Utility of the Realisation of a Value): A change in the realisation-quantity of value v , from quantity q_1 to quantity q_2 provides a smaller utility-difference the higher is the position of interval $[q_1, q_2]$.⁴¹

Thus, for instance, a proportional loss in the realisation of revenue, or of privacy, of 1/10 determines a higher loss, if it concerns the passage from 5/10 to 4/10 than if it concerns the passage from 9/10 to 8/10.

Corollary 4 (From Decreasing Marginal Utility): Assumption 3 has the following implications:

- *The utility loss resulting from a diminution in the realisation of a value is higher than the utility gain which is provided by an equal increase in the realisation of the same value.*
- *A greater decrease in the realisation of a value causes a proportionally greater decrease in the utility concerning value. A greater increase in the realisation of a value causes a proportionally smaller increase in the utility concerning that value.*

After establishing the proportional contribution of a choice to the utility concerning a certain value we need to find a way of having homogeneous quantities for the utilities provided by the realisation of different values. For this purpose we need to assign weights to the values.

Definition 10 (Weight of a Value): The weight of a value v is a quantity expressing the importance of v relatively to the other values.⁴²

⁴⁰ $\Delta \text{PropUt}_v \alpha = \frac{\Delta \text{Ut}_v \alpha}{\text{MaxUt}_v}$.

⁴¹ In mathematical terms, we would say that the function connecting a value to its utility is such that its second derivative is negative. This too, however, has to be taken as what happens in most of the cases, namely, as a defeasible assumption.

Obviously, more important values, such as personal freedom, or freedom of speech will have a higher weight, while less important values, such as privacy or transparency will have a lower weight. The idea of assigning weights to values may seem to introduce some arbitrariness, due to the difficulty of comparing different values. However, our experience tells us that we often engage in such comparisons, and we can come to determinations (approximate quantities) that are sufficient to support our choices, and even to come to shared conclusions.⁴³ We are now in a condition to provide a quantitative characterization of the absolute utility of an action with regard to a value.

Definition 11 (Utility-impact on a Value): The utility-impact of action α on value v , also called the utility of action α concerning value v , is the proportional utility of α concerning v , multiplied by the weight of v .⁴⁴

The notion of a utility-impact enables us to give precise content to the idea that the utility of a choice is the sum of its contributions to all values at stake. The elements to be summed up consist of the utility impacts concerning each value, which are obtained by multiplying the proportional utility-impact on a value, for the weight of that value.

Definition 12 (Utility of an Action): The utility of action α with regard to a set of values $\{v_1, \dots, v_n\}$, is the sum $i_1 + \dots + i_n$ of the utility impacts of α on each of such values. In this sum each element i_i is the proportional utility of α concerning v_i multiplied by the weight of v_i .⁴⁵

By separating, in the set of the utility impacts of a choice α , positive and negative elements we get the notion of outweighing: the positive value impact of α on a set of values outweighs its negative value impact on these value, if the sum of α 's positive impacts is higher than the sum of α 's negative impacts.

Definition 13 (Positive Value Impact, Negative Value Impact, and Outweighing): The positive utility-impact of action α on value-set $\{v_1, \dots, v_n\}$ is the sum of its utility-impacts on

⁴² Let us denote the weight of a value v with w_v .

⁴³ Amartya Sen observes that there are facts that there are "reasonable variations, or inescapable ambiguities, in the choice of relative weights" does not exclude a shared assessment, with sufficient precision, from being made under many circumstances. In particular, agreement on the fact that the weights at issue fall within certain ranges is often sufficient. For some references to more technical contributions, see AMARTYA SEN, *The Idea of Justice* 297 (2009).

⁴⁴ $\Delta Ut_v \alpha = \Delta Prop Ut_v \alpha * w_v$.

⁴⁵ $\Delta Ut_{\{v_1 \dots v_n\}} \alpha = \Delta Ut_{v_1} \alpha + \dots + \Delta Ut_{v_n} \alpha$.

the values whose realisation it increases; α 's negative utility-impact on $\{v_1, \dots, v_n\}$ is the sum of its utility-impacts on the values whose realisation it decreases.⁴⁶

Corollary 5 (From the Notion of Outweighing): The following statements are equivalent:

- α 's utility concerning values $\{v_1, \dots, v_n\}$ is larger than 0;⁴⁷
- α 's positive utility-impact on values in $\{v_1, \dots, v_n\}$, is larger than α 's negative utility-impact on values in $\{v_1, \dots, v_n\}$;⁴⁸
- α 's positive utility-impact on values in $\{v_1, \dots, v_n\}$, outweighs α 's negative utility-impact on values in $\{v_1, \dots, v_n\}$;
- the proportion between α 's positive utility-impact on values in $\{v_1, \dots, v_n\}$, and α 's negative utility-impact on values in $\{v_1, \dots, v_n\}$, is bigger than 1.

Consequently also the following are equivalent:

- α 's utility concerning values $\{v_1, \dots, v_n\}$ is smaller than 0;⁴⁹
- α 's negative utility-impact on values in $\{v_1, \dots, v_n\}$, is larger than α 's positive utility-impact on values in $\{v_1, \dots, v_n\}$;⁵⁰
- α 's negative utility-impact on values in $\{v_1, \dots, v_n\}$, outweighs α 's positive utility-impact on values in $\{v_1, \dots, v_n\}$;
- the proportion between α 's negative utility-impact on values in $\{v_1, \dots, v_n\}$, and α 's positive utility-impact on values in $\{v_1, \dots, v_n\}$, is bigger than 1.⁵¹

The last item of Corollary 5⁵² provides a generalization of the so-called weight formula proposed by Robert Alexy.⁵³ Finally, we can define the utility of an action α relatively to an alternative action β .

⁴⁶ The positive impact can be expressed as follows: $\Delta\text{PosUt}_{\{v_1 \dots v_n\}}\alpha = \sum_{\Delta\text{Ut}_{v_i} > 0} \Delta\text{Ut}_{v_i}$. The negative impact is correspondingly: $\Delta\text{NegUt}_{\{v_1 \dots v_n\}}\alpha = \sum_{\Delta\text{Ut}_{v_i} < 0} |\Delta\text{Ut}_{v_i}|$. We use positive quantities for negative impacts—given that the absolute value $|-x|$ a negative number $-x$, is the positive number (x) —since we want to express the negative impact through a positive quantity, which can be compared with the quantity of the positive impact.

⁴⁷ $\Delta\text{PosUt}_{\{v_1 \dots v_n\}}\alpha \geq 0$

⁴⁸ $\Delta\text{PosUt}_{\{v_1 \dots v_n\}}\alpha > \Delta\text{NegUt}_{\{v_1 \dots v_n\}}\alpha$

⁴⁹ $\Delta\text{PosUt}_{\{v_1 \dots v_n\}}\alpha \geq 0$

⁵⁰ $\Delta\text{PosUt}_{\{v_1 \dots v_n\}}\alpha > \Delta\text{NegUt}_{\{v_1 \dots v_n\}}\alpha$.

⁵¹ $\frac{\Delta\text{PosUt}_{\{v_1 \dots v_n\}}\alpha}{\Delta\text{NegUt}_{\{v_1 \dots v_n\}}\alpha} > 1$.

⁵² $\frac{\Delta\text{NegUt}_{\{v_1 \dots v_n\}}\alpha}{\Delta\text{PosUt}_{\{v_1 \dots v_n\}}\alpha} > 1$.

⁵³ See ALEXY, *supra* note 8, at 388. Alexy provides the proportion between negative and positive impacts on two values, in a particular case, through the formula $W_{v_i, v_j} = \frac{I_{v_i} * w_{v_i}}{I_{v_j} * w_{v_j}}$, where v_i is the promoted value and v_j is the demoted value, I_{v_i} and I_{v_j} are the intensities of the interference on the two values, and w_{v_i} and w_{v_j} are the abstract weights of such values, and W_{v_i, v_j} expresses, in Alexy's terminology, the "concrete weight" of the demoted value v_i relatively to the promoted value v_j . In the framework here presented, the intensity of the

Definition 14 (Utility of an Action Relatively to Another Action): The utility of action α relatively to action β , with regard to a set of values $\{v_1, \dots, v_n\}$, is the difference between the utilities of α and β concerning those values.⁵⁴

Finally it may be useful to specify the notion of utility tout court and of relative utility with regard to the legal values, namely, with regard to all values that are established or recognised by the law.

Definition 15 (Legal Utility and Relative Legal Utility): The legal utility of action α is its utility with regard to the set of all values that are established or recognised by the law. The legal utility of action α relatively to action β is the difference between the legal utilities of α and of β .

In the following when speaking of utilities, I will always refer to legal utilities.

Definition 14 entails that superiority can also be specified on the basis of utilities.

Corollary 6 (Superiority as positive relative utility): Action α is superior to action β when the utility of α relatively to β is positive.⁵⁵

Another interesting corollary is that it may happen that given a set of actions, the action that is superior to all actions in the set is not superior to all of them with regard to any single value.

Corollary 7 (Superiority and maximality): Superiority does not necessarily require maximality with regard to a single value, when at least three choices are compared with regard to at least two values.⁵⁶

interference of an action α on a value v corresponds to $\Delta\text{PropUt}_v\alpha$, i.e., the proportional utility of α concerning v . By multiplying $\Delta\text{PropUt}_v\alpha$ for v 's weight w_v , we obtain the utility of α concerning v , i.e., $\Delta\text{Ut}_v\alpha$. Thus, in our framework Alexy's "concrete weight" of a demoted value v_i relatively to a promoted value v_j becomes the proportion $W_{v_i, v_j}\alpha = \frac{|\Delta\text{Ut}_{v_i}\alpha|}{\Delta\text{Ut}_{v_j}\alpha}$, which amounts to $\frac{|\Delta\text{PropUt}_{v_i}\alpha| * w_{v_i}}{\Delta\text{PropUt}_{v_j}\alpha * w_{v_j}}$. This proportion gives a number larger than 1 when α 's negative utility-impact concerning v_i is larger than α 's positive utility-impact concerning v_j .

⁵⁴ $\text{Ut}_{\{v_1, \dots, v_n\}}(\alpha, \beta) = \Delta\text{Ut}_{\{v_1, \dots, v_n\}}\alpha - \Delta\text{Ut}_{\{v_1, \dots, v_n\}}\beta$.

⁵⁵ $\alpha \succ_{\{v_1, \dots, v_n\}} \beta$ if and only if $\Delta\text{Ut}_{\{v_1, \dots, v_n\}}(\alpha, \beta) > 0$.

⁵⁶ More precisely, given an option set $\{o_1, o_2, \dots, o_m\}$ and a value-set $\{v_1, \dots, v_n\}$ it is possible that there is an option $o^* \in \{o_1, o_2, \dots, o_m\}$ such that $o^* \succ_{\{v_1, \dots, v_n\}} o_i$ for every $o_i \neq o_k$ but there is no $v_j \in \{v_1, \dots, v_n\}$ such that for every $o_i, o^* \succ_{\{v_j\}} o_i$.

For instance, given three possible choices α , β and γ , it may be the case that γ is superior to both α and β with regard to value set $\{v_1, v_2\}$, while being inferior to α with regard to v_1 and to β with regard to v_2 .⁵⁷ This is the case when γ represents a superior compromise between two values one of which is best promoted by α while the other is best promoted by β . For instance, the choice of keeping DNA data from suspects only for a short time, with appropriate warranties, may be preferable, all things considered, to both the most privacy-favourable option (not storing the data at all) and the most security-favourable option (keeping the data indefinitely).

K. Assessing Compliance with Value-norms

We now deploy the concepts just defined in order to assess compliance with value-norms. Let us focus on norms requiring the respect of a value, though the analysis can easily be extended to norms requiring the promotion or the irrelevance of a value. We have said above that a norm requiring the respect of a value is satisfied where the agent does not choose a course of action that sacrifices the value, unless the sacrifice is needed for obtaining a more significant increase in the satisfaction of other values. This means that such a norm is violated where the agent makes a choice that demotes the value, and the overall utility sum—considering all impacts on all relevant values—is negative. In this sum we have to include all values whose consideration is prescribed by the legal system, plus those values that have been chosen by the decision maker, to the exclusion of the values whose consideration is prohibited. The weight to be attributed to such values is the weight that is prescribed by the legal system, and for the permissible values chosen by the decision maker, the weight that is given to them by the decision maker itself, within the boundaries established by the legal system.

This idea can be expressed by the following two conditions. A value-norm prescribing the respect of value v is violated by legislative measure α in case that:

- α demotes value v , and
- either (1) α provides a negative utility relatively to the null action \emptyset or (2) α provides a negative utility relatively to an alternative choice β , when such an alternative choice should have been taken into account.

Let us see how this idea can be matched with the traditional proportionality texts. According to the reconstruction proposed by Alexy,⁵⁸ a legislative norm interfering with rights protected through a constitutional value-norm—a principle, in Alexy's terminology—is only legitimate when it meets the following tests:

⁵⁷ This happens when $0 < \Delta Ut_{\{v_1\}}(\alpha, \gamma) < \Delta Ut_{\{v_2\}}(\gamma, \alpha)$ and $0 < \Delta Ut_{\{v_2\}}(\beta, \gamma) < \Delta Ut_{\{v_1\}}(\gamma, \beta)$.

⁵⁸ Robert Alexy, *Constitutional Rights, Balancing, and Rationality*, 16 *RATIO JURIS* 131, 135 (2003) (role of proportionality in judicial practice). See also Alec Stone Sweet & Jud Mathews, *Proportionality Balancing and Global Constitutionalism*, 47 *COLUM. J. TRANSNAT'L L.* 68 (2008).

1. Suitability, which excludes “the adoption of means obstructing the realisation of at least one principle without promoting any principle or goal for which they were adopted”;⁵⁹
2. Necessity, which requires, with regard to principles P_1 and P_2 , “that of two means promoting P_1 that are, broadly speaking, equally suitable, the one that interferes less intensively in P_2 ought to be chosen”;⁶⁰
3. Balancing in strict sense, which requires that “the greater the degree of non-satisfaction of, or detriment to, one principle, the greater the importance of satisfying the other.”⁶¹

The three tests provide independently necessary and jointly sufficient conditions for teleological correctness. For instance, as Alexy observes, a legislative norm requiring tobacco producers to place health warnings in their products could pass the proportionality test, since the German Constitutional Court considered that (1) this norm served a suitable end, i.e., health, (2) there were no alternative measures achieving that end that would be less interfering upon the economic freedom of tobacco producers; (3) the advantage this measure provided with regard to health outweighed the minor interference it caused on economic freedom.

In the following I shall address the three tests using the model presented above, starting with choices affecting only two values: The promoted goal-value v_g , pursued by the agent, e.g., the legislator, and the demoted normative value v_n to be respected according to a value-norm.

1. Suitable choice: A choice α demoting a normative value v_n is suitable if it also promotes a permissible goal-value v_g .⁶²
2. Necessary choice: A choice α demoting a normative value v_n is necessary if α also promotes a permissible goal-value v_g and there exists no alternative choice β , being both a non inferior to α with regard to goal-value v_g and superior to α with regard to the normative value v_n .⁶³
3. Balanced choice. Two notions of a balanced choice seem to be present in the discussion on proportionality.

⁵⁹ *Id.* at 135.

⁶⁰ *Id.*

⁶¹ *Id.*

⁶² $\alpha \succ_{v_g} \emptyset$, which entails $\Delta\text{PropReal}_{v_g}\alpha > 0$.

⁶³ $\alpha \succ_{v_g} \emptyset$, and there exist no β such that $\beta \succcurlyeq_{v_g} \alpha$ and $\beta \succ_{v_n} \alpha$.

3.1. A choice α demoting a normative value v_n and promoting a goal-value v_g is balanced if α is not inferior to \emptyset relatively to the value-set $\{v_g, v_n\}$.⁶⁴

3.2. A choice α demoting a normative value v_n and promoting a goal-value v_g is balanced if there exists no alternative β such that β would be superior to α relatively to v_n and superior to α also relatively to the set $\{v_g, v_n\}$.⁶⁵

Note that the Definition 3.2 provides a much stricter standard than Definition 3.1: while Definition 3.1. only requires non-inferiority with regard to the status quo, Definition 3.2. requires non-inferiority with regard to any possible decision interfering to a lesser extent with the normative value. If brought to the extreme, the latter characterisation of a balanced choice would severely restrict the possibility for a decision-maker to adopt a decision having a negative impact on a normative value and escape censorship. The reviewer would be free to imagine possible alternative decisions that the decision maker did not consider, speculate on their possible effects and merits, and condemn the decision-maker as soon as the latter's decision could be shown to be suboptimal. Thus, I believe, this kind of review needs to be strongly constrained, for instance by requiring that the adoption of α appears to have been an unreasonable mistake, given the evidence available when α was adopted.

By denying the conditions above, we get three conditions under which a choice infringes a value-norm.

1. Unsuitable choice: A choice α demoting a normative value v_n in order to pursue a goal value v_g is unsuitable if it fails to promote v_g . An unsuitable choice is Pareto-inferior to the status quo, i.e., to the null action \emptyset . This may depend on the fact that the chosen action is incapable of promoting the pursued goal—its adoption is based on mistaken factual assumptions—or on the fact that the pursued goal is impermissible, and thus irrelevant according to a value-norm.

2. Unnecessary choice. A choice α demoting a normative value v_n in order to pursue a goal value v_g is unnecessary if there exists an alternative choice β , which is superior to α relatively to v_n and non-inferior relatively to v_g . Thus the unnecessary α is Pareto-inferior to the β relatively to $\{v_g, v_n\}$.

3. Unbalanced choice:

3.1. Unbalanced choice-1. A choice α demoting a normative value v_n and promoting a goal value v_g is unbalanced if α is inferior to \emptyset relatively to $\{v_g, v_n\}$. This means that α 's positive utility concerning v_g is outweighed by its disutility concerning v_n .

⁶⁴ $\Delta Ut_{v_g} \alpha + \Delta Ut_{v_n} \alpha \geq 0$.

⁶⁵ There exists no β such that $\beta \succ_{v_n} \alpha$ and $\beta \succcurlyeq_{v_g} \alpha$.

3.2. Unbalanced choice-2. A choice α demoting a normative value v_n and promoting a goal-value v_g is unbalanced if there exists an alternative choice β such that β is superior to α relatively to v_n , and is superior to α also relatively to $\{v_g, v_n\}$.

We can generalize these notions to cover decisions affecting multiple values.

1. Suitable choice: A choice α demoting a normative value is suitable if it promotes another normative value or a permissible goal-value.
2. Necessary choice: A choice α demoting a normative value is necessary if it promotes a normative value or a permissible goal-value and there exists no alternative choice β which is better than α with regard to the demoted normative value and non-inferior to α with regard to any other normative value or permissible goal-value.
3. Balanced choice:
 - 3.1. Balanced choice-1: A choice α demoting a normative value is balanced if the utility provided by α 's impact on all other relevant values outweighs the disutility provided by its negative impact the normative value.
 - 3.2. Balanced choice-2: A choice α demoting a normative value is balanced if there exists no alternative β such that β be superior to α relatively to the normative value and would be superior to α also relatively to the set of all legally relevant values.

Further refinements and specifications are possible of the notions just introduced. For instance, a different notion of necessity is needed for covering cases where a choice α is qualified as “unnecessary” when α is slightly superior to the alternative β relatively to the goal-value, but largely inferiority to β relatively to the normative value. This will be left to further research.⁶⁶

As these definitions should have made clear, what is at issue in a proportionality assessment concerning a decision α affecting two values v_1 and v_2 is not a comparison of the weights of v_1 and v_2 , but rather a comparison of α 's impacts on such values. Consequently, the fact that value v_1 is more important, i.e., has a higher weight than value v_2 , does not necessarily entail that α 's utility-impact on v_1 is larger than its utility-impact on v_2 : The utility-impact on a value depends on both (1) the proportional utility-impact on that value—the extent to which the benefit deriving from the realisation of the value is increased or decreased—and (2) the weight the value.⁶⁷ This is affirmed with particular clarity by the Israeli judge Aharon Barak:

⁶⁶ Some aspects have been addressed in Giovanni Sartor, *Doing Justice to Rights and Values: Teleological Reasoning and Proportionality*, 18 ARTIFICIAL INTELLIGENCE & L. 175 (2010).

⁶⁷ More exactly, in our framework, the utility impact of a decision α on two values is the sum of its utility impacts on each of them, where each utility impact is the result of the proportional impact on a value for the weight of that value: $\Delta Ut_{\{v_1, v_2\}}\alpha = \Delta PropUt_{v_1}\alpha * w_{v_1} + \Delta PropUt_{v_2}\alpha * w_{v_2}$. Thus the condition for the first term—in hypothesis, the negative impact—to be bigger than the second term is that $|\Delta PropUt_{v_1}\alpha * w_{v_1}| > \Delta PropUt_{v_2}\alpha *$

[T]he comparison is not between the advantages gained by realizing the goal in contrast to the effect brought by limiting the right. Nor is it between security and liberty. The comparison is between the marginal benefit to security and the marginal harm to the right caused by the restricting law and as such, the comparison is concerned with the marginal and the incremental.⁶⁸

Thus, it may happen that in a certain case the impact of a measure α on value v_1 outweighs α 's impact on v_2 , while in another case the impact of a different measure β on v_2 outweighs β 's impact on v_1 . To explain this it is not necessary to assume that the weights of v_1 and v_2 have changed, being "context dependent". A more plausible explanation may be provided by the fact that the proportional impacts on v_1 and on v_2 were different in the two cases, the weights remaining the same, namely, that v_1 was affected by α in the first case more than it was affected by β in the second case, or that v_2 was affected by β in the second case more than it was affected by α in the first case.

L. Teleological Reasoning and the Choice of Rules

The evaluation of value-impacts pertains not only to the adoption of individual decisions but also to the adoption of general rules.

A value-based choice of rules takes place in the teleological interpretation of legislative texts, which requires choosing the interpretation that most realizes the legislator's goals and the values the legislators considered, or should have considered according to the law. Within constitutional review, a similar reasoning pattern is used in the so called "definitional balancing", i.e., when a court does not only affirm that a certain law is disproportionate, but explains this statement considering that any law having a certain kind of content would be disproportionate and would therefore violate the constitution. It seems that two teleological arguments are involved in this reasoning.

First, a specific legislative choice is assessed, having regard to its impacts on the values at issue in the individual case. Secondly, a rule generalising the outcome of the case is stated, having regard to the value-impacts of the application of that rule in future cases.

w_{v_2} which is equivalent to $\frac{w_{v_1}}{w_{v_2}} > \frac{\Delta\text{PropUt}_{v_2}\alpha}{|\Delta\text{PropUt}_{v_1}\alpha|}$. This inequality can be falsified even when the weight w_{v_1} of value v_1 is much larger than the weight w_{v_2} of v_2 . This happens when the proportion between $\Delta\text{PropUt}_{v_2}\alpha$, and $\Delta\text{PropUt}_{v_1}\alpha$ is larger than the proportion between w_{v_1} and w_{v_2} .

⁶⁸ Aharon Barak, *Proportionality and Principled Balancing*, 4 L. & ETHICS OF HUM. RTS. 1, 8 (2010).

Let me give an example to explain how a court may engage in definitional balancing, or may rather refrain from it. The European Court of Human Rights has recently addressed a case concerning a man and a woman who were both unaffected carriers of mucoviscidosis, a very serious genetic disease, and thus had a high risk (1/4) of having children affected by this illness (*Costa and Pavan v. Italy*, application no. 54270/10). The claimants, who had already generated an affected child, attacked an Italian law, Law 40/2007, which prohibited pre-implantation tests. They argued that this law impeded them from using a medical procedure that would have eliminated the risk of having an affected child. This procedure involved the in-vitro production of embryos and the implantation of a non-affected one. The Court affirmed that “the interference with the applicants’ right to respect for their private and family life was disproportionate, in breach of Article 8” of the Charter of Human Rights, since it negatively affected the right to a private life of the claimants to an extent that was not compensated by its alleged benefits concerning other interests at stake, such as protecting the life of embryos and preventing eugenic practices.

In this case the judges did not state any general rule to explain or justify their decision, though, it may be argued, they had many rule-choices available to them, including the following: (1) Any prohibition of pre-implantation testing for mucoviscidosis is disproportionate, with regard to couples who have already generated an affected child, (2) any prohibition of pre-implantation testing for mucoviscidosis is disproportionate, even for carriers of the disease not having already generated an affected child, (3) any prohibition of pre-implantation testing for any genetic disease is disproportionate, even when concerning genetic problems different from mucoviscidosis, (4) any prohibition of pre-implantation testing is disproportionate, even when the test serves non-therapeutic purposes, such as sex selection, (5) any prohibition of pre-implantation interventions is disproportionate, even when the intervention goes beyond mere testing, as for cloning or genetic engineering.

The adoption of any one of these “definitional” rules by the competent court would enable subsequent judges to decide certain cases through rule-based reasoning, rather than through balancing. The subsequent judge could simply check whether a prohibition of pre-implantation testing has the properties that make it disproportionate according to the rule and decide the case accordingly.⁶⁹

Since disproportionateness of a legislative measure entails that it should not be taken according to the Constitution, such definitional rules could be re-expressed as prohibitions of adopting laws having the indicated content. For instance, assume that the judges in this case, rather than being silent, stated rule (3), as the *ratio decidendi* of the case, i.e., they explained their conclusion on the disproportion of the Italian law by arguing that any

⁶⁹ In Alexy’s terms this would be the passage from the application of principles to the application of rules, in Luhmann’s terms, the passage from goal-programs to conditional-programs

prohibition of pre-implantation testing for genetic diseases is disproportionate, and that the prohibition to test for mucoviscidosis, as entailed by the Italian law, was indeed such a prohibition. But given premises (a) “laws prohibiting pre-implantation testing for genetic diseases are disproportionate”, and (b) “the legislator should not adopt disproportionate laws”, we can infer the conclusion (c) “the legislator should not adopt laws prohibiting pre-implantation testing for genetic diseases.” Thus a prohibitive action-norm addressed to the legislator would be implied by the judicial definition.

It seems that the judges having established that a law produces a disproportionate outcome in a particular case face a choice concerning how to frame their decision. Their options include choosing one of many available rule-based explanations, or choosing not to provide any such explanation. On what grounds should they make such a choice? The answer, I think, requires another appeal to teleological reasoning. They should adopt the generalization that could best realize the legal values at issue, through the subsequent application of the generalization by judges, legislators, and citizens, in the given institutional framework—as characterized by applicable action-norms, judicial powers, legislative competence, interpretive practices, existing precedents and social norms. On the basis of the same teleological reasoning the judges could also conclude that perhaps stating no rule is the best solution. They could reach the latter conclusion, for instance, having regard to the high degree of uncertainty in the matter at stake, which prevents them from stating with sufficient confidence even a highly defeasible general rule.

Thus, a judgment on the proportionality of may involve two teleological assessments. The first assessment concerns establishing that the legislative decision α in the current case has a negative overall impact on the values at stake and that therefore α is disproportionate. In our example this is the assessment that the prohibition of pre-implantation testing in the case of a man and a woman both carrier of mucoviscidosis, who already had an affected child, has a disproportionately negative impact on their private lives. The second assessment involves two steps: (1) developing, through abductive reasoning, a set of possible explanations as to why α 's utility-impact is negative in that case, each explanation appealing to a (different) rule saying that legislative actions having certain features are disproportionate (2) selecting, for the rule-based justification of the decision, the rule whose future adoption by courts, legislators, and citizens, in the given institutional and socio-economic context, is likely to provide the highest utility impact, or choosing to provide no rule-based justification in case no advantageous rule can be found.

Thus this second teleological assessment takes place at a meta-level, concerning the choice between alternative patterns for decision in future cases. It compares the utility impacts provided by the future application of different action rules, and also the utility-impact of not having any such rule, and thus entrusting future decisions to case-by-case assessments.

I think that the formulation of action rules based on a proportionality assessment is also at the basis of the attempts to specify the essential content, or the core, of a right. The

essential core of a right is indeed identified by infeasible prohibitions and obligations of performing certain actions, for the sake of the right at issue. Whether an action rule protecting a right could be viewed as being defeasible or rather infeasible seems again to be supported by teleological reasoning. It should be preferable that the rule is viewed as infeasible—i.e., as concerning the essential core of the right at issue—when the following conditions hold: (1) The action it prohibits, e.g., torture, has a negative impact on a value, this impact being so significant that it is very unlikely that it will be compensated by positive impacts on other values, and (2) the costs of mistaken exceptions to the rule, i.e., of failing to apply it when its application would provide a higher benefit—are presumably higher than the costs of its overreaching applications, i.e., of applying it when its non-application would be more beneficial. Under such conditions we should indeed accept the view that the rule is infeasible and we should be ready to pay the costs of the rare disutility of its counterproductive application in extreme cases for the benefit of preventing the disutility of its counterproductive disapplication in a larger or more important set of cases.⁷⁰ Moreover, it may be argued that in the cases when the application of a beneficial rule may lead to most tragic consequences—the so called ticking time bomb-scenarios—it is likely that the rule will not be applied anyway, since the concerned agent will do what is needed to prevent an irreparable loss, relying also on the possibility to avoid criminal sanctions (appealing to the state of necessity) even when breaking the rule.

M. Consistency in Balancing

The idea that quantitative reasoning with non-numerical magnitudes has a valuable function in the application of the law can be challenged by pointing to the arbitrariness of the inputs of such reasoning. Even though the balancing is constrained by arithmetic, it operates on magnitudes that are idiosyncratic contents of the minds of individual decision-makers or reviewers. How can there be convergence in the outcomes of the reasoning of different individuals, and how can any social control over such outcomes be effective, if any outcome would be obtainable by changing subjective input quantities?

I am unable to develop here even a preliminary account of how social learning can take place with regard to input magnitudes for teleological reasoning and with regard to methods for processing such magnitudes. I think, however, that the following hypothesis can be made. We learn magnitudes for our values—i.e., the proportional utilities delivered by the realisation of the values and the weights of the values—by processing inputs we get from multiple sources, such as our inborn attitudes, education, and personal experience.

⁷⁰ On the need to complement proportionality with deontological considerations, see Mattias Kumm, *Political Liberalism and the Structure of Rights: On the Place and Limits of the Proportionality Requirement*, in *LAW, RIGHTS AND DISCOURSE: THE LEGAL PHILOSOPHY OF ROBERT ALEXI* 131 (George Pavlakos, ed., 2007). Such deontological worries, however, can be addressed, I would argue, by applying proportionality analysis at a higher level, namely, at the level of the justification of adopting an action-rule.

This processing involves possibly inductive and abductive patterns of reasoning which deliver adjustments and explanations of our intuitive assessments. When we adopt the legal point of view, our value assessments are additionally constrained by the requirement that such assessments fit with the legal materials, such as value-norms contained in constitutions and legislative documents, decisions of individual cases involving impacts on such values, legislative rules addressing value conflicts, soft legal sources including explicit statements on the absolute and comparative importance of impacts and values, and shared social attitudes.

I cannot here provide a general analysis of how we can determine the measure of fit of a new assessment with a certain past history of teleological reasoning in the legal domain.⁷¹ I shall only consider two basic cases, where reasoning *a fortiori* on the basis of previous assessments gives clear indications. Assume that a choice α , involving a demotion of value v_d and the promotion of value v_p , was assessed as being proportionate, v_d and v_p being the only values at stake. Now consider a decision β involving a smaller, in absolute number, demotion of v_d and an equal or greater promotion of v_p . Clearly, β must be considered proportionate as well.⁷²

Assume, on the contrary that a choice α , involving a demotion of a value v_d and the promotion of legislative value v_p , was assessed as non-proportionate. Consider a decision β involving a greater demotion of v_d and a smaller or equal promotion of v_p : Clearly, β must be considered disproportionate as well.⁷³

These ideas can be further generalized, as the following simple example will show. Assume that we have to assess whether an acceptable balance is provided by the choice β to store DNA samples of all citizens for twenty years, a choice which demotes citizen's privacy and promotes their security.⁷⁴ Assume also that in the past the choice α of storing DNA

⁷¹ On the idea of fit, see RONALD DWORKIN, *LAW'S EMPIRE* 225–75 (1986). On the connection between value-based reasoning and the interpretation of rules or the determination of their priorities, see Trevor Bench-Capon & Giovanni Sartor, *A Model of Legal Reasoning with Cases Incorporating Theories and Values*, 150 *ARTIFICIAL INTELLIGENCE* 97 (2003); Henry Prakken, *An Exercise in Formalising Teleological Case-Based Reasoning*, 10 *ARTIFICIAL INTELLIGENCE & L.* 113 (2000).

⁷² This follows from inequalities $|\Delta\text{PropUt}_{v_d}\alpha| > |\Delta\text{PropUt}_{v_d}\beta|$ and $\Delta\text{PropUt}_{v_p}\alpha \leq \Delta\text{PropUt}_{v_p}\beta$. In fact, such inequalities entail that $\Delta\text{Ut}_{v_p}\alpha - |\Delta\text{Ut}_{v_d}\alpha| < \Delta\text{Ut}_{v_d}\beta - |\Delta\text{Ut}_{v_p}\beta|$, i.e., that $\Delta\text{Ut}_{\{v_d, v_p\}}\alpha < \Delta\text{Ut}_{\{v_d, v_p\}}\beta$. Since we know that the α 's utility ($\Delta\text{Ut}_{\{v_d, v_p\}}\alpha$) is superior to 0 (α is proportionate), and that β 's utility ($\Delta\text{Ut}_{\{v_d, v_p\}}\beta$) is superior to α 's, it follows that also β 's utility is superior to 0, namely, that also β is proportionate.

⁷³ This follows from inequalities $|\Delta\text{PropUt}_{v_d}\alpha| < |\Delta\text{PropUt}_{v_d}\beta|$ and $\Delta\text{PropUt}_{v_p}\alpha \geq \Delta\text{PropUt}_{v_p}\beta$. Such inequalities entail that $\Delta\text{Ut}_{v_p}\alpha - |\Delta\text{Ut}_{v_d}\alpha| > \Delta\text{Ut}_{v_d}\beta - |\Delta\text{Ut}_{v_p}\beta|$, i.e., that $\Delta\text{Ut}_{\{v_d, v_p\}}\alpha > \Delta\text{Ut}_{\{v_d, v_p\}}\beta$. Since we know that the α 's utility is inferior to 0, α being disproportionate, and that the β 's utility is inferior to α 's, it follows that also β 's utility is inferior to 0, namely, that also β is disproportionate.

⁷⁴ Whether $\Delta\text{PropUt}_{\{v_d, v_p\}}\alpha \geq 0$, where v_d is the demoted value, i.e., privacy, while v_p is the promoted value, i.e., security.

samples of all citizens accused of a crime for ten years was considered to be unacceptable, since its negative impact on privacy outweighed the benefit on security. Assume also that it is agreed that by doubling the conservation time, the damage to privacy is proportionally increased to a large extent, while the benefit for security is increased to a much smaller proportion.⁷⁵ Given such premises, any assessment according to which the new law β would provide a positive balance, by subtracting losses and adding benefits, would be inconsistent with the previous assessment concerning α .⁷⁶ More generally, any assignments of weights to the values of privacy and security that would satisfactorily explain the disproportionality of the ten years term, would also determine the disproportionality of the twenty year term. Thus, the need to explain the disproportionality of the ten year storage term would impose assignment of weights entailing the disproportionality the twenty year term.

N. Conclusions

I have developed here an analysis of proportionality based on a view of teleological reasoning as a rational activity, which can be governed through value-norms. I have argued that teleological reasoning includes the assessment of impacts upon relevant values, which is to be viewed as a kind of quantitative reasoning, even when we are unable to assign symbolic numerals to the concerned magnitudes. We engage in this reasoning both when making common-sense private choices and when participating in public decision-making. From an evolutionary perspective, the capacity for non-symbolic quantitative reasoning can be viewed as the antecedent of our numerical competence, an antecedent that is still operating in the background whenever we engage in explicit quantitative reasoning. Non-numerical quantitative reasoning involves certain rationality conditions, and primarily the respect of the usual arithmetical relationships, which indicate general constraints over the

$${}^{75} \frac{\Delta \text{PropUt}_{v_d} \beta}{\Delta \text{PropUt}_{v_d} \alpha} > \frac{\Delta \text{PropUt}_{v_p} \beta}{\Delta \text{PropUt}_{v_p} \alpha}.$$

⁷⁶ On the basis of the previous definitions we have the following equalities: $\Delta \text{Ut}_{\{v_d v_p\}} \alpha = \Delta \text{Ut}_{v_d} \alpha + \Delta \text{Ut}_{v_p} \alpha = \Delta \text{PropUt}_{v_d} \alpha * w_{v_d} + \Delta \text{PropUt}_{v_p} \alpha * w_{v_p}$. We know from the previous assessment that α was unbalanced, i.e., that $\Delta \text{Ut}_{\{v_d v_p\}} \alpha < 0$. This entails that the negative quantity $\Delta \text{PropUt}_{v_d} \alpha * w_{v_d}$ is greater, in absolute value, than the positive quantity $\Delta \text{PropUt}_{v_p} \alpha * w_{v_p}$. Thus, whatever quantities w_{v_d} and w_{v_p} we attribute to the weight of v_d and v_p consistently with the precedent, it must be the case that $|\Delta \text{PropUt}_{v_d} \alpha * w_{v_d}| > \Delta \text{PropUt}_{v_p} \alpha * w_{v_p}$. We also know that the new decision β decreases privacy to a larger proportion than it increases security, in comparison to α : $\frac{\Delta \text{PropUt}_{v_d} \beta}{\Delta \text{PropUt}_{v_d} \alpha} > \frac{\Delta \text{PropUt}_{v_p} \beta}{\Delta \text{PropUt}_{v_p} \alpha}$. Then by multiplying $|\Delta \text{PropUt}_{v_d} \alpha * w_{v_d}|$ for $\frac{\Delta \text{PropUt}_{v_p} \beta}{\Delta \text{PropUt}_{v_d} \alpha}$ and simplifying we get $\Delta \text{PropUt}_{v_d} \beta * w_{v_d}$, and similarly by multiplying $\Delta \text{PropUt}_{v_p} \alpha * w_{v_p}$ for $\frac{\Delta \text{PropUt}_{v_p} \beta}{\Delta \text{PropUt}_{v_p} \alpha}$ and simplifying we get $\Delta \text{PropUt}_{v_p} \beta * w_{v_p}$. Thus we must conclude that also these results must be, in absolute value, such as the first one is bigger than the second: $|\Delta \text{PropUt}_{v_d} \beta * w_{v_d}| > \Delta \text{PropUt}_{v_p} \beta * w_{v_p}$. This follows indeed from the fact that for all numbers a_1, a_2, b_1, b_2 if $a_1 > b_1$ and $a_2 > b_2$ then also $a_1 * a_2 > b_1 * b_2$. Therefore also the second sum must give a negative result: $\Delta \text{PropUt}_{v_d} \beta * w_{v_d} + \Delta \text{PropUt}_{v_p} \beta * w_{v_p} < 0$. This means that $\Delta \text{Ut}_{\{v_d v_p\}} \beta < 0$, i.e., that β is unbalanced too.

processing of quantitative information. Thus arithmetical relationships can be viewed as standards of rationality to be applied by both decisions makers and reviewers of their decisions. In particular they apply to both legislators and courts charged with constitutional review, on the basis of constitutional value-norms.

I hope that this approach can help in addressing to some extent the critiques against the possibility of protecting rights also through value-norms, and of enforcing such norms through judicial review.⁷⁷

There are many further issues that need be addressed to account for value-based assessments in the legal domain, such as the following:

- the discretion of decision-makers in pursuing permissible values as well as their margins of appreciation concerning the weights of normative values;
- the deference of reviewers to the empirical and normative judgments of decision-makers,
- the need to disaggregate value-impacts, considering how particular individuals or categories are affected relatively to a particular value, to take into account the so-called separateness of persons and, more generally, issues pertaining to distributive justice and equality,
- rationales and methods for giving priority to fundamental liberties, even without attributing them a lexical priority over other values,
- methods for linking quantitative value-assessments to arguments and counterarguments, concerning the foundations of such assessments, such as the identification of values, the determination of their priorities, and the quantification of impacts.

I am aware that here I have just provided a very preliminary sketch of the view of value-assessments as quantitative reasoning with non-numerical magnitudes, without developing such implications into a coherent and comprehensive theory, nor providing sufficient empirical data to validate my hypotheses. I think however, that this perspective could provide us with a fresh understanding of the prospects and the limitations of rationality in legislative reasoning and in proportionality assessments, which may lead to interesting developments, especially when merged with other insights on the evolutionary and cognitive bases of legal reasoning.⁷⁸

⁷⁷ For some critical observations on teleological reasoning in the law and value-based constitutional review in the law, see JURGEN HABERMAS, *BETWEEN FACTS AND NORMS: CONTRIBUTIONS TO A DISCOURSE THEORY OF LAW AND DEMOCRACY* 259 (1999); NIKLAS LUHMANN, *ZWECKBEGRIFF UND SYSTEMRATIONALITÄT: ÜBER DIE FUNKTION VON ZWECKEN IN SOZIALEN SYSTEMEN* (1973); Bernhard Schlink, *Der Grundsatz der Verhältnismäßigkeit*, in *FESTSCHRIFT 50 JAHRE BUNDESVERFASSUNGSGERICHT 445* (Peter Badura & Horst Dreier eds., 2001).

⁷⁸ See WOJCIECH ZALUSKI, *EVOLUTIONARY THEORY AND LEGAL PHILOSOPHY* (2009); *5 STUDIES IN PHILOSOPHY OF LAW: LAW AND BIOLOGY* (Jerzy Stelmach, Bartosz Brożek & Marta Soniewicka eds., 2011).