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Horizon wave-function and the quantum cosmic censorship

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ABSTRACT

We investigate the Cosmic Censorship Conjecture by means of the horizon wave-function (HWF) formalism. We consider a charged massive particle whose quantum mechanical state is represented by a spherically symmetric Gaussian wave-function, and restrict our attention to the superextremal case (with charge-to-mass ratio $\alpha > 1$), which is the prototype of a naked singularity in the classical theory. We find that one can still obtain a normalisable HWF for $\alpha^2 < 2$, and this configuration has a non-vanishing probability of being a black hole, thus extending the classically allowed region for a charged black hole. However, the HWF is not normalisable for $\alpha^2 > 2$, and the uncertainty in the location of the horizon blows up at $\alpha^2 = 2$, signalling that such an object is no more well-defined. This perhaps implies that a *quantum* Cosmic Censorship might be conjectured by stating that no black holes with charge-to-mass ratio greater than a critical value (of the order of $\sqrt{2}$) can exist.

1. Introduction

A complete understanding of the gravitational collapse of a compact object remains one of the most challenging issues in contemporary theoretical physics. The general relativistic (GR) description, resulting in the formation of a black hole (BH) or naked singularity (NS), was first investigated in the papers of Oppenheimer and co-workers [1]. Although the literature on the subject has grown immensely (see, e.g. Ref. [2]), many technical and conceptual issues remain. One of these is the famous Cosmic Censorship Conjecture (CCC), proposed by Penrose in 1969 [3], which states that no singularities will ever become visible to an outer observer in a generic gravitational collapse starting from reasonable non-singular initial states. To date, the conjecture remains unproved, and it is considered one of the most important open problems in gravitational physics. Another great open issue in GR is the problem of considering the quantum mechanical (QM) nature of the collapsing matter [4]. We will here address both issues for the Reissner–Nordström (RN) geometry, which describes charged BHs, a subject of many theoretical investigations in the past (see, e.g. Ref. [5]).

Most attempts at quantising BH metrics consider the gravitational degrees of freedom unrelated to the matter state that sources the geometry. More recently, the Horizon Wave Function (HWF) formalism was proposed [6], as a way of quantising the Einstein equation that determines the gravitational radius of a spherically symmetric matter source and its time evolution [7], which instead relates the quantum state of the horizon to the quantum state of matter. This formalism was then applied to a few different case studies [8–10], yielding apparently sensible results in agreement with (semi)classical expectations, and there is therefore hope that it will facilitate our understanding of the formation of BHs from QM particles. In particular, it seems natural to extend this formalism beyond Schwarzschild BHs and tackle the CCC from a quantum perspective by considering an electrically charged particle represented by a Gaussian wave-packet in the classical regime in which it would be an NS.

2. Electrically charged spherical sources

We start by recalling the classical RN metric can be written as

$$ds^2 = -f\, dt^2 + f^{-1}\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{1}$$

with

$$f = 1 - \frac{2\,\ell_p\, M}{m_p\, r} + \frac{Q^2}{r^2}, \tag{2}$$

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where M and Q respectively represent the ADM mass and charge of the source, ℓ_p is the Planck length and m_p is the Planck mass.[1] For $|Q| < \ell_p M/m_p$, the above metric contains two horizons, namely

$$R_\pm = \ell_p \frac{M}{m_p} \pm \sqrt{\left(\ell_p \frac{M}{m_p}\right)^2 - Q^2}, \tag{3}$$

and represents a BH. The two horizons overlap for $|Q| = \ell_p M/m_p$, the so-called extremal BH case, while for $|Q| > \ell_p M/m_p$ no horizon exists and the central singularity is therefore accessible to outer observers. This is the prototype of an NS, which we will refer to as the "superextremal geometry". It is in fact more convenient to express all relevant quantities in terms of the mass M and the (positive definite) specific charge

$$\alpha = \frac{|Q|\, m_p}{\ell_p M}. \tag{4}$$

Using this parameter, the above expression (3) becomes

$$R_\pm = \ell_p \frac{M}{m_p}\left(1 \pm \sqrt{1-\alpha^2}\right), \tag{5}$$

and the three regimes mentioned above are then explicitly parametrised as i) $0 < \alpha < 1$ for the BH with two horizons,[2] ii) $\alpha = 1$ for the extremal BH, and iii) $\alpha > 1$ for the superextremal geometry.

We shall now investigate the superextremal geometry from a quantum mechanical perspective by first determining the HWF for $\alpha < 1$ and then extending it continuously into the regime $\alpha > 1$.

2.1. HWF for Gaussian sources

The general procedure that leads to the HWF [6,7] is based on lifting the gravitational radius R_H of a spherically symmetric QM system to the rank of a quantum operator. This step can be physically motivated by first recalling that the coordinate r in a spherical metric like the one in Eq. (1) is invariantly related to the geometrical area $4\pi r^2$ of a sphere centred on the origin $r = 0$, and is therefore a natural candidate to become an observable in the quantum theory. Moreover, the specific property that qualifies $r = R_H$ is that it represents the location of trapping surfaces and thus determines the causal structure of the space–time, which one can assume will also ought to remain an observable property in the quantum theory. In detail, we recall that in a neutral spherically symmetric system, $R_H(r) = 2\,\ell_p M(r)/m_p$, where

$$M(r) = 4\pi \int_0^r \rho(\bar{r}, t)\, \bar{r}^2\, d\bar{r}, \tag{6}$$

is the Misner–Sharp mass. One should notice that M represents the *total* energy (thus, roughly speaking, including the negative gravitational energy) and is related to the energy density ρ of the source via the *flat* space volume. A specific value of r is then a trapping surface if $R_H(r) = r$, whereas, if $R_H(r) < r$, the gravitational radius is still well-defined but does not correspond to any peculiar causal surface. In the electrically charged case, we have two gravitational radii and corresponding operators, namely $\hat{R}_\pm$. The classical relation (5) will then be reinterpreted in this context as the operatorial equation relating $\hat{R}_\pm$ to the total energy $\hat{M}$ of the system, with $\hat{R}_\pm$ acting multiplicatively on the HWF and $\hat{M}$ acting multiplicatively on energy eigenstates. Finally, we will consider the ratio α as a simple parameter.

Let us consider as a source of the RN space–time an electrically charged massive particle at rest in the origin of the reference frame, represented by a spherically symmetric Gaussian wave-function

$$\psi_S(r) = \frac{e^{-\frac{r^2}{2\ell^2}}}{\ell^{3/2}\,\pi^{3/4}}. \tag{7}$$

We emphasise that the radial variable r in the above is interpreted to be the same as the coordinate r in the metric (1), and is therefore a measure of the particle's size as seen from an outer observer who, e.g. scatters particles against it. We shall also assume that the width of the Gaussian ℓ is the minimum compatible with the Heisenberg uncertainty principle, that is[3]

$$\ell = \lambda_m \simeq \ell_p \frac{m_p}{m}, \tag{8}$$

where λ_m is the Compton length of the particle of rest mass m. In momentum space, the wave-function of the particle described above is

$$\psi_S(p) = \frac{e^{-\frac{p^2}{2\Delta^2}}}{\Delta^{3/2}\,\pi^{3/4}}, \tag{9}$$

where $p^2 = \vec{p}\cdot\vec{p}$ is the square modulus of the spatial momentum, and the width $\Delta = m_p\,\ell_p/\ell \simeq m$. For the energy of the particle, we shall employ the relativistic mass–shell relation in flat space,

$$M^2 = p^2 + m^2, \tag{10}$$

in analogy with the expression of the Misner–Sharp mass (6). This choice does not therefore imply that the effects of curved space–time are *a priori* discarded, but rather that they should be included in the very definition of total energy of the system.[4]

For $\alpha < 1$, it is clear that one can now write an HWF for each of the two horizons. In fact, from the quantum version of Eq. (3), the total energy M can be expressed in terms of the two horizon radii as

$$\ell_p \frac{\hat{M}}{m_p} = \frac{\hat{R}_+ + \hat{R}_-}{2}, \tag{11}$$

and

$$\hat{R}_\pm = \hat{R}_\mp \frac{1 \pm \sqrt{1-\alpha^2}}{1 \mp \sqrt{1-\alpha^2}}. \tag{12}$$

Note that we promoted M, R_+, and R_- into operators $\hat{M}$, $\hat{R}_+$, and $\hat{R}_-$, which are related to the corresponding observables. Our specific choice is not unique, and it is associated with usual ambiguities when going from a classical to quantum formalism.

The unnormalised HWFs for R_+ and R_- are then obtained by expressing p from the mass–shell relation (10) in terms of the energy M in Eq. (11), and then replacing one of the relations in Eq. (12) into Eq. (9). The two HWFs are then given by

[1] We shall use units with $c = k_B = 1$, and always display the Newton constant $G = \ell_p/m_p$, so that $\hbar = \ell_p m_p$.

[2] We remark in passing that the inner horizon gives rise to an instability usually referred to as "mass inflation", but we have investigated under which conditions the inner horizon is actually realised in the quantum context in a separate work [11].

[3] A thorough analysis of the changes that occur by relaxing this condition can be found in Ref. [7].

[4] Let us also remark that, for a source of Planckian mass and size, quantum effects might make the very concept of "curved space–time" inadequate in its proximity, whereas determining (asymptotic) cross sections for particle collisions might still be meaningful.

$$\psi_H(R_\pm) = \mathcal{N}_\pm\, \Theta\left(R_\pm - R_{\min\pm}\right) \times \exp\left\{ -\frac{m_p^2\, R_\pm^2}{2\,\Delta^2\,\ell_p^2\,(1 \pm \sqrt{1-\alpha^2})^2} \right\}, \tag{13}$$

where the Heaviside function arises from the minimum energy in the spectral decomposition of the wave-function (7) being $M = m$, which corresponds to

$$R_{\min\pm} = \ell_p\, \frac{m}{m_p} \left(1 \pm \sqrt{1-\alpha^2}\right). \tag{14}$$

Finally, the normalisations $\mathcal{N}_\pm$ are fixed by assuming the scalar product[5]

$$\langle \psi_H \mid \phi_H \rangle = 4\pi \int_0^\infty \psi_H^*(R_\pm)\, \phi_H(R_\pm)\, R_\pm^2\, dR_\pm, \tag{15}$$

where, like in the previous equation, the upper signs are used for the normalisation of $\psi_H(R_+)$, while the lower signs are used when normalising $\psi_H(R_-)$.

The probability density that the particle lies inside its horizon of radius $r = R_\pm$ can now be calculated starting from the wave-functions (13) associated with (7) as

$$\mathcal{P}_{<\pm}(r < R_\pm) = P_S(r < R_\pm)\, \mathcal{P}_H(R_\pm), \tag{16}$$

where

$$P_S(r < R_\pm) = 4\pi \int_0^{R_\pm} |\psi_S(r)|^2\, r^2\, dr \tag{17}$$

is the probability that the particle is inside a sphere of radius $r = R_\pm$, and

$$\mathcal{P}_H(R_\pm) = 4\pi\, R_\pm^2 |\psi_H(R_\pm)|^2 \tag{18}$$

is the probability density that the sphere of radius $r = R_\pm$ is a horizon. Finally, one can integrate (16) over all possible values of the horizon radius R_+ to find the probability for the particle described by the wave-function (7) to be a BH, namely

$$P_{BH+} = \int_{R_{\min+}}^\infty \mathcal{P}_{<+}(r < R_+)\, dR_+. \tag{19}$$

The analogous quantity for R_-,

$$P_{BH-} = \int_{R_{\min-}}^\infty \mathcal{P}_{<-}(r < R_-)\, dR_-, \tag{20}$$

will instead be the probability that the particle lies further inside its inner horizon, and both R_- and R_+ are therefore realised (for more details about this case, see Ref. [11]).

Before moving to the specific topic of this work, let us note that for a particle with mass $M \simeq m$ smaller than the Planck scale, the classical values of $R_\pm$ from Eq. (5) are smaller than the Planck length. This however should not prevent one from considering the corresponding $\hat{R}_\pm$ as valid quantum operators representing the gravitational radii of the system. In fact, let us first point out that, for any value of the particle's mass, if $\langle \hat{R}_\pm \rangle \lesssim \ell$, these radii most likely do not correspond to horizons, exactly like in the classical theory there is no trapping surface of size r if the corresponding gravitational radius $R(r) = 2\,\ell_p\, M(r)/m_p < r$. Consequently, the probability that the particle is a BH decreases very fast below the Planck mass, as it was shown for the simpler case of an electrically neutral particle in Refs. [6,8,7], and is confirmed for the probability P_{BH+} of the RN BH in Ref. [11]. This means that, although one can have $\langle \hat{R}_\pm \rangle \lesssim \ell_p$, QM fluctuations due to the uncertainty in the particle's size dominate below the Planck energy and the horizon is simply not realised as an actual trapping surface (see also footnote 4). More realistically, it was shown in Ref. [9], that a BH has a significant probability to form from the collision of sub-Planckian particles only if the centre-of-mass energy reaches into the trans-Planckian regime. Finally, in Ref. [8], it was shown that the uncertainty in the horizon size obtained from the HWF, combined with the standard Heisenberg uncertainty for the particle's size, yields a minimum detectable length which is always larger than the Planck scale. In other words, a minimum detectable length of the order of the Planck scale naturally emerges within the HWF formalism, without the need to assume it, and without a modification of canonical commutation rules.

2.2. Superextremal geometry

We will now focus on studying overcharged sources, represented by the range of specific charge $\alpha > 1$. It is well known that in the classical theory of gravity, the CCC *a priori* forbids the existence of NSs. In the case of the classical charged BHs, this would precisely correspond to $\alpha > 1$, so it is interesting to investigate whether quantum physics leads to any modifications or *predictions* therein. Our guiding principle will be to assume that the quantum states in the regime $\alpha > 1$ can be obtained by extending continuously the HWF from the case $\alpha < 1$. Of course, this is by no means the only possible choice, but one can at least hope that it leads to consistent predictions for α not too much larger than the classical limiting value of 1.

The first problem that we encounter for $\alpha > 1$ is that the operators $\hat{R}_\pm$ are not Hermitian. We could stop right there and say that there are no observables which correspond to $\hat{R}_\pm$ in the regime $\alpha > 1$. However, we will make a simple (and perhaps non-unique) choice, which will allow us to proceed and probe the classically forbidden region. We will take the real parts of the multiplicative operators $\hat{R}_\pm$, which are certainly Hermitian, to correspond to quantum observables. With this choice, the modulus squared of the two HWFs are given from Eq. (13), for $R_\pm > R_{\min\pm}$, by

$$|\psi_H(R_\pm)|^2 = \mathcal{N}_\pm{}^2 \exp\left\{ -\frac{m_p^2\, R_\pm^2}{\Delta^2\,\ell_p^2\,(1 \pm \sqrt{1-\alpha^2})^2} \right\}, \tag{21}$$

which, for $\alpha > 1$, becomes one expression

$$|\psi_H(R)|^2 = \mathcal{N}^2 \exp\left\{ -\frac{2-\alpha^2}{\alpha^4}\, \frac{m_p^2 R^2}{\Delta^2 \ell_p^2} \right\}, \tag{22}$$

where R now replaces both R_+ and R_-. This HWF is still normalisable in the scalar product (15) if R belongs to the real axis and the specific charge lies in the range

$$1 < \alpha^2 < 2. \tag{23}$$

We could therefore infer that no normalisable quantum state with $\alpha^2 > 2$ is allowed, or that there is an obstruction that prevents the system from crossing $\alpha^2 = 2$. This point will be further clarified after we have fully determined the HWF.

The fact that the HWF (22) is the same for R_+ and R_- reflects the classical behaviour according to which the two real horizons

[5] The analytical expressions for the normalisation of the HWFs are very cumbersome and not particularly significant, thus we will omit them throughout the paper.

merge at the critical value of $\alpha = 1$, and then mathematically extend as one into the complex realm for $\alpha > 1$. We then need to address what happens to the Heaviside function in Eq. (13) when we extend it into the superextremal regime. First of all, we note that although Eq. (14) becomes complex for $\alpha > 1$, its real part is again the same for R_+ and R_-, namely

$$R_{\min} = \mathrm{Re}\left[\ell_p \frac{m}{m_p}\left(1 \pm \sqrt{1-\alpha^2}\right)\right] = \ell_p \frac{m}{m_p}. \tag{24}$$

We can then show that the same continuity principle, which led us to Eq. (22), requires that R be bounded from below by this $R_{\min}$. In fact, the expectation value for $\hat{R}$ is, in this case,

$$\langle \hat{R} \rangle = 4\pi \int_{R_{\min}}^{\infty} |\psi_H(R)|^2 R^3\, dR = \frac{2\,\ell_p^2\left(2-\alpha^2+\alpha^4\right)/\ell}{\sqrt{2-\alpha^2}\left[2\sqrt{2-\alpha^2}+\alpha^2 e^{\frac{2-\alpha^2}{\alpha^4}}\sqrt{\pi}\,\mathrm{erfc}\left(\frac{\sqrt{2-\alpha^2}}{\alpha^2}\right)\right]}, \tag{25}$$

and it matches exactly the corresponding expressions for $\alpha < 1$ [11],

$$\langle \hat{R}_\pm \rangle = 4\pi \int_{R_{\min\pm}}^{\infty} |\psi_H(R_\pm)|^2 R_\pm^3\, dR_\pm = \frac{4\,\ell_p^2\left(1 \pm \sqrt{1-\alpha^2}\right)/\ell}{2+e\sqrt{\pi}\,\mathrm{erfc}(1)}, \tag{26}$$

namely

$$\lim_{\alpha \searrow 1} \langle \hat{R} \rangle = \frac{4\,\ell_p^2/\ell}{2+e\sqrt{\pi}\,\mathrm{erfc}(1)} = \lim_{\alpha \nearrow 1} \langle \hat{R}_\pm \rangle. \tag{27}$$

One can likewise show that the uncertainty

$$\Delta R^2(\ell, \alpha > 1) = \langle \hat{R}^2 \rangle - \langle \hat{R} \rangle^2 \tag{28}$$

matches the corresponding uncertainties

$$\Delta R_\pm^2(\ell, \alpha < 1) = \langle \hat{R}_\pm^2 \rangle - \langle \hat{R}_\pm \rangle^2, \tag{29}$$

at the specific charge $\alpha = 1$, but we omit the explicit expressions since they are rather cumbersome. We just note that, for $\alpha = 1$, the width of the Gaussian $\ell > \langle \hat{R} \rangle$ for $m < \sqrt{2+e\sqrt{\pi}\,\mathrm{erfc}(1)}\, m_p/2 \simeq 0.8\, m_p$, so that quantum fluctuations in the source's size will dominate for masses significantly smaller than the Planck scale (in qualitative agreement with the neutral case [6,7,11]).

It is now interesting to analyse the limit $\alpha^2 \to 2$. One may have already noticed that

$$\langle \hat{R} \rangle \simeq \frac{2^{5/4}\,\ell_p^2/\ell}{\sqrt{\pi\left(\sqrt{2}-\alpha\right)}}, \tag{30}$$

so that the ratio $\langle \hat{R} \rangle/\ell$ blows up at $\alpha^2 = 2$ for any values of the mass $m = m_p\,\ell_p/\ell$. The same indeed occurs to the uncertainty, since

$$\Delta R \simeq \sqrt{3\pi/8 - 1}\,\langle \hat{R} \rangle \simeq 0.4\,\langle \hat{R} \rangle, \tag{31}$$

for $\alpha^2 \to 2$ (see also Fig. 1).

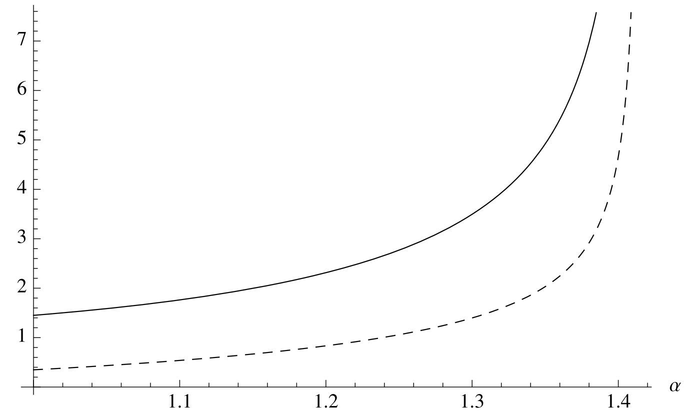

Fig. 1. The expectation value $\langle \hat{R} \rangle$ (solid line) and its uncertainty ΔR (dashed line) as functions of the specific charge for $1 < \alpha^2 < 2$ and $m = m_p$ ($\ell = \ell_p$).

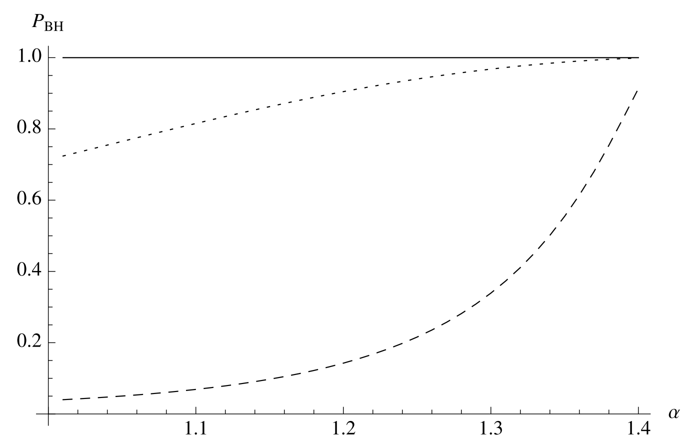

Fig. 2. P_{BH} as a function of α for $m = 2\,m_p$ (solid line), $m = m_p$ (dotted line) and $m = 0.5\,m_p$ (dashed line). Cases with $m \gg m_p$ are not plotted since they behave the same as $m = 2\,m_p$, i.e. an object with $1 < \alpha^2 < 2$ must be a BH.

Using Eq. (19), one can also calculate the probability P_{BH} that the particle is a BH for α in the allowed superextremal range (23). This probability is displayed in Fig. 2. One notices that, for a particle mass above the Planck scale, P_{BH} is practically one throughout the entire range of α (thus extending a similar result that holds for $\alpha < 1$ [11]). Moreover, even for m significantly less than m_p, P_{BH} approaches one in the limit $\alpha^2 \to 2$. We recall here that $P_{BH} \ll 1$ for small m is essentially related to $\ell \gg \langle \hat{R} \rangle$, and the system is thus dominated by quantum fluctuations in the source's position well below the Planck scale. On the other end, since both $\langle \hat{R} \rangle$ and ΔR blow up on approaching $\alpha^2 = 2$, the superextremal configurations with a significant probability of being BHs contain strong quantum fluctuations in the horizon's size.

Let us conclude this section by emphasising that, in order to achieve the above results, we had to choose a way to continue the HWF and operators that are straightforwardly defined for $\alpha < 1$ into the overcharged regime. Since this choice is not apparently unique, one might wonder whether different options would lead to significantly different outcomes. Although we have not performed a full survey, it is important to remark that we were able to ensure the continuity of expectation values across $\alpha = 1$, like in Eq. (27), by imposing simultaneously that $\hat{R}$ equals the real part of $\hat{R}_\pm$ and Eqs. (24) and (22), whereas we found no ways to make other smilingly natural options, like $R = |R_\pm|$, work.

3. Conclusions

From the above analysis we can learn two important things. First, quantum mechanical effects are perhaps able to continuously take us into the classically forbidden region of $\alpha > 1$. This means that even an overcharged object, with a charge-to-mass ratio greater than unity, can still make a quantum BH. The basic reason for this is that in our formalism the location of the horizon is not given by a sharp classical value, instead it is described by a quantum wave function with associated uncertainties. Second, the charge-to-mass ratio α cannot be arbitrarily large, even in the context of QM. We found that for $\alpha^2 > 2$ the HWF cannot be normalised, and thus it is not describing a well defined physical object. Moreover, at the same value of $\alpha^2 = 2$, the uncertainties in the location of the horizon become infinite, signalling again that such an object stops being well defined. We should warn the reader that the specific value of the upper limit $\alpha^2 = 2$ in Eq. (23) might simply be a consequence of describing the source as the Gaussian function (7), and should not be taken literally. However, it is likely that the overall qualitative picture remains in a more general context, and our results imply that perhaps a *quantum* version of the CCC might be formulated by stating that no BHs with the charge-to-mass ratio greater than a critical value (of order $\sqrt{2}$) can exist.

We should here recall that for the charged Reissner–Nordström metric with $\alpha \leq 1$ analysed in Ref. [11], as well for neutral sources [7,8,10], the single Gaussian constituent (7) leads to unacceptably large uncertainties in the horizon size of large astrophysical BHs. In fact, one has $\Delta R \sim \langle \hat{R} \rangle$, even for very large mass m, for which one expects a semiclassical behaviour for the horizon size R. The above quantum CCC will therefore have to be tested further, by considering models of BHs that allow for a semiclassical limit $\Delta R \ll \langle \hat{R} \rangle$. An example of such models is given by those in Refs. [12,10], which contain a very large number N of light constituents, whose wave-functions span the entire region inside R, and $\Delta R / \langle \hat{R} \rangle \sim N^{-1}$. The emerging picture is that BHs of any size should be treated as macroscopic quantum objects (just like superconductivity and superfluidity are macroscopic quantum phenomena at scales where one expects classical physics to be a good description).

Finally, let us point out that the analysis performed in this work, and the above quantum CCC, should hold for sources with mass m within a few orders of magnitude of the Planck mass. Primordial BHs formed in the early universe by large density fluctuations could have masses in this range. Moreover, it is also plausible that overcharged configurations with such small masses emerge from the gravitational collapse of astrophysical objects, acting as seeds for much larger BHs. Our results should then apply straightforwardly to these two cases.

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