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EDITORIAL

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7th Workshop on Matrix Equations and Tensor Techniques

The workshop series on *Matrix Equations* started in 2005 in Leipzig, Germany, and was continued biennially in 2007 in Chemnitz, Germany, and in 2009 in Braunschweig, Germany. To account for the many connections between matrix equations and tensor techniques, it was then decided to widen the scope of the series in this direction. The first such workshop on *Matrix Equations and Tensor Techniques (METT)* took place in Aachen, Germany, in 2011, followed by Lausanne, Switzerland, in 2013 and Bologna, Italy, in 2015.

The seventh METT workshop took place in Pisa, Italy, on February 13–14, 2017 (see Figure 1). With 18 talks, 13 posters, and 58 participants from 15 different countries, this workshop clearly showed the continued interest and high level of activity in the area. The local organizers were Dario Bini, Gianna Del Corso, Stefano Massei, Federico Poloni, and Sergio Steffé.

This special issue contains nine papers, reflecting the diversity of topics covered by the METT workshop in Pisa. In the following, we provide a brief summary of these papers, in the order in which they appear in this issue.

Damm et al.[1] investigate the numerical solution of medium-sized matrix equations related to Markov jump linear systems. While being similar to Lyapunov matrix equations, these matrix equations contain additional terms that significantly complicate the development of efficient algorithms. Different approaches are suggested and compared. In particular, fixed-point iterations and associated Krylov subspace formulations are treated, as well as a reformulation of the problem as an optimization problem and associated steepest descent, conjugate gradient, and trust-region methods.

Jarlebring et al.[2] study the efficient numerical solution of certain types of large-scale matrix equations. The left-hand side of such equations consists of the sum of a Sylvester and a positive operator, where the commutators of certain combinations of the coefficient matrices satisfy a low-rank property. Exploiting this fact, an effective subspace projection method is devised, which generalizes the extended Krylov subspace method for Lyapunov equations to the considered matrix equations. Numerical examples illustrate the effectiveness of the new method.

Gosea and Antoulas[3] develop a Loewner-based approach for the data-driven model order reduction of quadratic–bilinear systems. The reduction process involves singular value decompositions of Loewner matrices, which are shown to satisfy certain generalized Sylvester equations. The effectiveness of the proposed method is demonstrated by means of several numerical experiments, including applications to the Burgers' equation, a nonlinear transmission circuit, and the Chafee-Infante equation.

Boito et al.[4] consider shifted linear systems with a quasiseparable structure or, equivalently, a Sylvester matrix equation with a quasiseparable and a diagonal matrix coefficient. Such equations arise, for example, from the discretization of nonlocal boundary value problems and, more generally, the approximation of meromorphic functions. An efficient algorithm based on a structured QR decomposition is developed. The complexity of this algorithm grows linearly with the size of the quasiseparable matrix, which is confirmed by the numerical experiments.

Bini et al.[5] consider matrix equations of the kind $A_1X^2 + A_0X + A_{-1} = X$, where the matrix coefficients are semi-infinite quasi-Toeplitz matrices. Such equations arise in stochastic processes, in particular, certain quasi-birth–death processes. A numerical framework for approximating the minimal nonnegative solution of these equations that relies on semi-infinite quasi-Toeplitz matrix arithmetic is presented. Cyclic reduction can be effectively applied and can approximate the infinite-dimensional solutions with quadratic convergence at a cost that is comparable to that of the finite case.

Grasedyck and Löbbert[6] discuss the parallelization of operations with tensors in the so-called hierarchical Tucker format, a tree-based compressed format designed for tensors of arbitrarily high order. Assuming that the tensor data is distributed among several compute nodes and associating the tree structure with the data distribution, parallel tensor operations are introduced, which allow for the efficient implementation of iterative and multigrid solvers for the fast solution of linear systems of equations. Weak scaling studies indicate a growth of runtime logarithmic with respect to the order of the involved tensor.

Boussé et al.[7] consider the solution of tensor-structured linear systems. More specifically, it is assumed that the solution vector, reshaped as a tensor, is in canonical polyadic decomposition. For the case of rank one, an algebraic method is

FIGURE 1 Participants of the 7th Matrix Equations and Tensor Techniques (METT) Workshop in Pisa

derived. For the general case, the problem at hand is reformulated as an optimization problem and a trust-region method is applied. Several numerical experiments, including applications in face recognition and blind deconvolution, demonstrate the effectiveness of the proposed approach.

Heidel and Schulz[8] develop second-order Riemannian optimization methods for the tensor completion problem, which is concerned with filling in missing entries of a partially known tensor under a low-rank constraint. For this purpose, tensor completion is rephrased as an optimization problem on the manifold of tensors of fixed multilinear rank. An explicit formula for the Riemannian Hessian of this problem is derived and plugged into a trust-region method. Numerical experiments demonstrate that this approach results in a rapidly and robustly converging algorithm.

Meini and Poloni[9] study a second-order extension of the well-known PageRank problem for ranking websites. After imposing a rank-one structure, this extension leads to a vector equation that is bilinear in the entries of the vector and features additional constraints to ensure stochasticity. A fixed-point method and the Newton–Raphson method are applied to such a quadratic vector equation and shown to converge to the desired solution under certain conditions. As these conditions are not always met, a new fixed-point algorithm in combination with a continuation strategy is proposed. The resulting algorithm is demonstrated to be more reliable than existing alternatives.

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