Productivity Crowding-out in Labor Markets with Motivated Workers

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Francesca Barigozzi†, Nadia Burani‡and Davide Raggi§

University of Bologna

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Abstract

When workers' intrinsic motivation matters, a wage increase has mixed consequences on applicants' productivity and motivation, as shown in public service, healthcare, education and politics. In a simple theoretical framework where ability and motivation are workers' private information, we rationalize these differentiated responses and identify intuitive conditions for higher wages inducing self-selection of more (or less) productive and motivated workers. The selection patterns depend both on the statistical association between workers' characteristics and on the difference between the incentivised returns to ability across sectors. We emphasize a crowding-out effect of wage on workers' productivity that has not been analyzed in the theoretical literature before.

JEL Classification: D82, J3, J2.

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†Department of Economics, University of Bologna, P.zza Scaravilli 2, 40126 Bologna (Italy). E-mail address: francesca.barigozzi@unibo.it

‡Department of Economics, University of Bologna, Strada Maggiore 45, 40125 Bologna (Italy). E-mail address: nadia.burani@unibo.it

§Department of Economics, University of Bologna, P.zza Scaravilli 2, 40126 Bologna (Italy). E-mail address: davide.raggi@unibo.it

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1 Introduction

When deciding whether to apply for and to accept a job, prospective civil servants, teachers, health professionals, and politicians do not only respond to standard extrinsic motivations (i.e. wages) but also, possibly, to non-pecuniary ones. Teachers often care about the achievements of their students, many health professionals are intrinsically interested in the well-being of their patients, dedicated bureaucrats value their public service motivation, and politicians may be publicly-spirited regarding their political duties.

In these environments, the empirical evidence on the impact of a wage increase on the characteristics of the (incoming) workforce is controversial. Several papers focus on workers' skills, and find that higher wages increase applicants' ability, measured by different proxies.[1] Important exceptions are Merlo *et al.* (2009) and Fisman *et al.* (2015) who document the opposite result in the case of politicians: in their studies, higher wages decrease members of the Parliament's quality. Workers' motivation has also attracted some attention. On the one hand, the field experiment analyzed in Dal Bó *et al.* (2013) shows that higher salaries attract workers who are more skilled and have a higher public service motivation. On the other hand, in the lab experiment by Banuri and Keefer (2016), higher salaries lead to a less socially motivated pool of public servants, but do not affect skills as measured by education and income. Our theoretical model provides a general and unifying explanation about such contrasting evidence and offers intuitive conditions for higher wages to attract more (or less) skilled and motivated workers.[2]

We study workers' self-selection in labor markets where intrinsic motivation may matter (i.e. in vocational labor markets), workers are heterogeneous with respect to both their ability and intrinsic motivation and the latter characteristics are workers' private information. We show that the consequences of changes in the wage offered in the vocational market on the characteristics of the pool of applicants fundamentally depend on how ability and motivation are statistically associated in the population of potential workers and on the relative steepness of incentive schemes across the vocational and the non-vocational sectors.[3] We provide an encompassing theory of how the degree of association between workers'

[1] See Gagliarducci and Nannicini (2013), Ferraz and Finan (2011) and Dolton and Marcenaro-Gutierrez (2011), among others.

[2] More details about the empirical and the scant theoretical literature are provided in Section 4.

[3] We call 'steepness of the incentive scheme' the degree to which the workers' information rent is increasing in their ability.

ability and motivation determines the impact of a wage increase on the composition of the active workforce in vocational sectors. Such a unified theoretical framework accounting for the functioning of different labor markets where intrinsic motivation matters is new and reconciles the contrasting empirical evidence (discussed in detail in Section 4.2). We also document some striking and unexpected results. We show that when the statistical association between workers' attributes is positive and large and the incentive scheme is less effective in the vocational than in the non-vocational sector, then higher wages in the vocational sector attract workers with lower motivation *and* lower ability on average. This reduces productivity in the vocational sector.

Although our results show that the statistical association between ability and motivation plays a key role, it has been overlooked in the literature on screening and sorting of workers into different sectors of the labor market. We can expect that this statistical association varies in interesting and systematic ways across sectors with different vocational jobs and/or various cultural and historical backgrounds (see also the empirical works described in Sections 4.2). These elements and the broad picture we provide are useful for the ongoing debate on how to increase the supply and the quality of the workforce in vocational markets, as we elaborate in Section 5.

Our model is also remarkably simple. We begin with the classical lemons problem *à la* Akerlof (1970), where firms cannot condition their wage payments upon workers' productivity while workers' opportunity cost of accepting the job is increasing in their productivity, i.e. some returns to ability exist in the opt-out sector. Given that only relatively less capable workers are willing to accept the job at any given wage, the relevant sector will be characterized by an inefficiently low employment rate and by a low expected productivity of active workers. Moreover, the expected productivity of active workers will always be monotonically increasing in the wage rate: higher salaries will attract more productive workers in the relevant sector.

Consider now a vocational market in which workers also privately know how much they are motivated for the job offered by firms in the vocational sector. How does this second source of workers' private information affect the lemons problem? How does the pool of active workers change with the wage rate? In particular, do higher salaries still attract more productive workers?

In our setting, intrinsic motivation is interpreted as a benefit accruing to workers when they are employed in the vocational sector. Hence, highly motivated workers have lower reservation wages than poorly motivated colleagues because the former receive a larger premium for they motivation. *Ceteris*

Specifically, a high-powered incentive scheme translates into a wage schedule characterized by high returns to ability.

paribus, this implies a *propitious selection* into the vocational sector because only the most motivated workers are willing to apply for the job.[4]

For a close comparison with the standard lemons model, we analyze first the case in which in the vocational market the wage is uniform (no incentives). In the opt-out sector, instead, a screening mechanism is in place so that some 'incentivised' returns to ability exist. We show that, in a vocational market where both adverse selection about productivity and propitious selection about motivation coexist, the lemons problem is less severe with respect to the situation in which workers only differ in their productivity and motivation plays no role. Indeed, when intrinsic motivation matters, the expected productivity of active workers increases and thus production inefficiencies due to adverse selection are mitigated. Although workers' motivation has no direct impact on the firms' output in our setting, it indirectly affects the sector's overall production by means of the self-selection mechanism of workers into the vocational market.

As a consequence of both adverse and propitious selection effects, we expect more skilled but less motivated workers to enter the market as the wage increases.[5] Assuming that workers' attributes are normally distributed, the previous intuition proves to be correct when: (1) productivity and motivation are independently distributed or negatively correlated, or (2) productivity and motivation are positively correlated *and* the incentivised return to ability in the alternative sector is neither too high nor too low. Importantly, we also show that, when productivity and motivation are positively correlated, two other less intuitive sorting patterns emerge: (2.*i*) *Productivity crowding-out:* both average productivity and average motivation of active workers monotonically decrease as the wage rate increases; this pattern occurs when the incentivised return to ability in the opt-out sector is sufficiently low. (2.*ii*) *Motivation crowding-in:* both average productivity and average motivation of active workers monotonically increase in the wage rate; this pattern occurs when the incentivised return to ability in the alternative sector is sufficiently high.[6]

We thus provide very simple necessary and sufficient conditions for crowding effects to realize, that depend on the incentivised return to ability in the opt-out sector and on the degree of correlation between ability and motivation in the population of potential applicants. These conditions can be tested

[4] *Propitious* or *advantageous selection* are the terms used in opposition to *adverse selection* to indicate the possible propitious self-selection of low-risk consumers into the insurance market when bidimensional private information on consumer's characteristics exists (see the excellent survey by Einav and Finkelstein 2011).

[5] On this point see, among others, Heyes (2005), Delfgaauw and Dur (2007) and Dal Bò *et al.* (2013).

[6] This selection pattern has been analyzed in Dal Bò *et al.* (2013), but only in the particular case in which ability and motivation are perfectly and positively correlated (see Subsection 4.1).

empirically, as we discuss in Section 5.

The intuition for the crowding effects is the following. Consider motivation crowding-in first. When the incentivised return to ability in the alternative sector is high, the outside option is relatively more attractive for high productivity workers, who prefer the non-vocational to the vocational sector. Thus, at a given level of the fixed wage offered in the vocational sector only the least skilled applicants are attracted, who are the first to enter the vocational sector. But, because of the positive correlation between workers' characteristics, applicants with low productivity also have low motivation on average. Then, as the wage offered in the vocational sector increases, applicants with higher and higher productivity and motivation are induced to enter the vocational sector and the average levels of both ability and motivation among workers in the vocational sector both increase. Second, consider productivity crowding-out. If the incentivised return to ability in the opt-out sector is low, the outside option is not very attractive for high-ability workers and motivation becomes the main driver of workers' decision to enter the vocational sector. At a given level of the fixed wage offered in the vocational sector only the most motivated applicants are attracted, who are the first to enter the vocational sector. Given the positive correlation between the workers' characteristics, applicants with high motivation also have high productivity on average. Once the workers with the highest motivation and productivity entered the vocational sector, a wage increase induces other applicants to enter, characterized by lower and lower motivation and productivity. Hence, as the wage rate increases, the quality of the pool of active workers in the vocational sector necessarily deteriorates.

We then derive sufficient conditions for the crowding effects to occur, that hold for fully general joint distributions of workers' characteristics and, as an application, we consider the case of a bivariate exponential distribution highlighting the role of the positive association between ability and motivation (Subsection 3.2).

Finally, we further generalize the analysis allowing for incentivised returns to ability in the vocational sector too. We show that all our results are robust to this extension, provided that the power of incentives is higher in the opt-out sector than in the vocational sector, as commonly documented (Subsection 3.3).

The rest of this paper is organized as follows. Section 2 sets up the model and shows how motivation changes the well-known adverse selection problem. In Section 3, we study how monetary compensations affect the composition of the pool of applicants in vocational labor markets. Subsection 3.1 considers a simple environment with jointly normally workers' characteristics; in Subsection 3.2 we allow for fully general distributions of productivity and motivation; Subsection 3.3 generalizes the results considering

incentives in the vocational sector. Section 4 relates our model and results to the empirical and theoretical literature. Specifically, Subsection 4.3 provides a validation of our main results using data from previous studies. Section 5 concludes with relevant policy implications. All proofs are relegated in the Appendix.

2 Model

Consider a labor market consisting of two sectors. In the *vocational sector* (for instance, the market for health professionals, teachers, civil servants or politicians) motivated workers obtain a vocational premium on top of their monetary compensation. Alternatively, workers may decide to enter the *opt-out sector* which is non-vocational.

In both sectors, firms produce with the same constant returns technology, using labor as the only input. Since we are mainly interested in the supply side of the vocational sector, we do not extensively model firms' behavior.[7]

Workers are heterogeneous with respect to both productivity and intrinsic motivation, which are not observable to firms. The productivity parameter $\theta \in \mathbb{R}_{++}$ measures the number of units of output workers produce if hired in either of the two sectors. Following Heyes (2005) and Delfgaauw and Dur (2010), workers obtain the monetary equivalent of a vocational premium $\gamma \in \mathbb{R}_{+}$ when employed in the vocational sector.[8,9] Let $F(\theta, \gamma)$ and $H(\theta)$ and $G(\gamma)$ be the cumulative joint and marginal distribution functions, respectively, of workers' attributes. Average productivity and average motivation are given by $E[\theta] = \int \theta dH(\theta) \equiv \mu_\theta$ and $E[\gamma] = \int \gamma dG(\gamma) \equiv \mu_\gamma$, respectively.

A very simple and neat way to model Akerlof (1970)'s adverse selection problem in the labor market is with a uniform wage and an outside option that depends on productivity.[10] In the opt-out (non-vocational) sector, workers obtain a payoff $\beta(\theta)$ which is assumed to be strictly increasing in productivity

[7] Barigozzi and Burani (2016a) analyze screening for workers' ability and motivation in a monopsonistic labor market. Barigozzi and Burani (2016b) study a setting with screening and competition between two firms/sectors. In these models firms are explicitly modeled and act strategically. See Delfgaauw and Dur (2008) for a screening model in which the relevant sector is represented by a public agency.

[8] Motivation is assumed not to affect workers' productivity. This allows us to compare our results with those in the standard market for lemons where motivation is absent. In addition this allows us to focus on the 'indirect' effect of motivation on the sector's production, i.e. the self-selection mechanism of workers into the vocational market through the degree of association in workers' characteristics.

[9] Our setting is similar to the one in Delfgaauw and Dur (2010), who consider a public (vocational) and a private sector, both perfectly competitive, with observable workers' ability and motivation.

[10] See also Jovanovic (1982) and Mas-Colell *et al.* (1995), chapter 13.

θ. The opportunity cost of accepting employment in the vocational sector increases in productivity θ because, for example, some (imperfect) screening is in place in the opt-out sector. We will refer to $\beta'(\theta)$ as the *incentivised return to ability in the opt-out sector*. If employed in the vocational sector, workers receive instead a flat wage w. As we show in Subsection 3.3, our results also hold when some screening mechanism is in place in the vocational sector too, provided that the incentivised return to ability in the opt-out sector is higher than the one in the vocational sector. Low-powered incentives in vocational sectors (which imply lower returns to ability) are consistent with the fact that vocational markets are frequently characterized by institutional constraints and less flexibility in the design of wage schemes. Moreover, the tasks that workers are required to perform in vocational sectors generally involve some non-contractible components so that workers' productivity is often more difficult to measure with respect to alternative sectors.[11] A lower return to ability in vocational sectors is consistent with the empirical literature on the wage differential between non-profit and for-profit firms and between public and private organizations showing that wage differentials are almost absent at the lowest job levels but increase at the higher job levels (which translates into a lower wage dispersion in the public and in the non-profit sector with respect to the private/for-profit one).[12]

A prospective worker with characteristics (θ, γ) accepts a job in the vocational sector if the total monetary benefit is higher than the outside option (for convenience the worker accepts when indifferent),[13] that is if

$$w + \gamma \geq \beta(\theta) \tag{1}$$

holds. Since the vocational premium γ is only obtained when a worker is employed in the vocational sector, *ceteris paribus,* the higher the worker's intrinsic motivation, the higher the total benefit from the job in the vocational sector, and the higher the likelihood of accepting employment in that sector. Moreover, *ceteris paribus,* the higher the worker's productivity, the higher the reward from the job in

[11] The quality of services is particularly relevant in vocational jobs and is typically non-contractible (although it is observable by the recipients of services) and thus it is not rewarded directly. At school, for example, the quality of teaching is higher if a teacher promotes curiosity and creative thinking and refines students' oral and written communication skills, but this goes beyond his/her explicit duties.

[12] See, among others, De Varo *et al.* (2017) for the wage differential between for-profit and non-profit firms and Melly (2005) for the wage differential between public and private organizations. The well known theoretical prediction that workers' intrinsic motivation stemming from the employer's pro-social mission allows for economizing on monetary incentives (see, among others, Francois 2000) has also been recently confirmed in the lab (see Cassar 2017).

[13] Although w will be treated as an exogenous parameter in our main analysis, we show in Appendix A.3 that it can be rationalized as an equilibrium in the vocational market.

the alternative sector, and the lower the worker's likelihood of accepting employment in the vocational sector. In other words, potential applicants who are more likely to accept the job in the vocational sector are those characterized by high motivation and/or low productivity. Thus, in the vocational sector, we obtain *propitious selection* as for motivation, together with *adverse selection* as for productivity and we can investigate the interaction between these two phenomena.

Let us define *marginal workers* as those workers who are just indifferent between accepting the job in the vocational sector or opting out.

Definition 1 ***Marginal workers.*** *Given a fixed compensation w, marginal workers are those applicants with characteristics (θ, γ) such that*

$$\gamma = \gamma(\theta) \equiv \beta(\theta) - w.$$

Figure 1 represents the set of potential workers in the plane (θ, γ) when the function $\beta(\theta)$ is strictly concave. The curve of marginal workers $\gamma(\theta)$ divides the set of applicants in two regions. The area to the left of $\gamma(\theta)$ represents all workers accepting the job in the vocational sector given the flat wage w, whereas the complementary region contains all types opting out. In Figure 1, the horizontal intercept of the curve of marginal workers is positive and given by $\theta = \beta^{-1}(w)$. At that point, the wage w is high enough to allow some low-productivity workers without motivation (i.e. such that $\gamma = 0$) to enter the vocational sector.

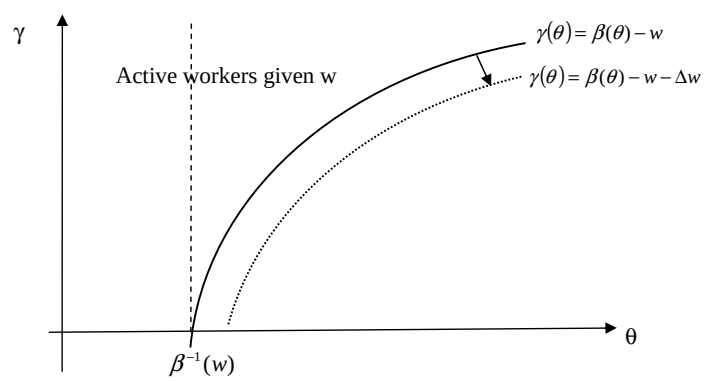

Figure 1: The curve of marginal workers and the set of workers active in the vocational market given wage w.

Similarly, we define the reservation wage as the wage which, given the characteristics (θ, γ) of a worker, makes such worker indifferent between accepting employment in the vocational sector or opting out.

Definition 2 ***Reservation wage***. *Given a worker's characteristics* (θ, γ), *the reservation wage* w_R *corresponds to the lowest compensation that the worker is ready to accept to enter the vocational sector, i.e.*

$$w_R = w(\theta, \gamma) \equiv \beta(\theta) - \gamma.$$

In order to highlight how the adverse selection problem changes with the introduction of workers' heterogeneity in (and private information about) intrinsic motivation, let us start with considering the following benchmark case describing the standard lemons problem.

3 Productivity crowding-out and motivation crowding-in

To set the scene, momentarily assume that motivation is irrelevant, i.e. applicants enjoy no vocational premium from the job in the relevant sector. The workers' self-selection condition is simply given by

$$w \geq \beta(\theta). \tag{2}$$

Since the population of potential workers can still be described by the distribution $F(\theta, \gamma)$, the set of active workers is represented by the area to the left of the vertical line passing through $\theta = \beta^{-1}(w)$ in Figure 1. Being the production technology the same in the two sectors, the adverse selection problem materializes, as usual, with an inefficiently low employment and an inefficiently low level of output in the relevant sector. It is also simple to show the following intuitive result.

Lemma 1 *When motivation plays no role, then:* (i) *the impact of a wage increase on the marginal workers' productivity,* $\theta = \beta^{-1}(w)$, *is always positive;* (ii) *the average productivity of active workers is monotonically increasing in the wage.*

Back to intrinsic motivation, we show that the lemons problem is less severe when intrinsic motivation is relevant, that is when potential workers enjoy a non-monetary premium γ if employed in the relevant sector.

Lemma 2 *The presence of intrinsic motivation* γ *in applicants' self-selection condition (1) is such that:* (i) *it increases the employment in the relevant (i.e. vocational) sector and* (ii) *it increases the average productivity of active workers* $E[\theta \mid w_R \leq w]$.

Point (i) follows from the comparison of the relative sizes of the areas depicting active workers at the benchmark and with motivation in Figure 1. The incentivised return to ability in the opt-out sector,

i.e. $\beta'(\theta)$, plays a crucial role in determining the efficiency gain due to intrinsic motivation. Consider a subset of potential workers characterized by high ability and high motivation and their decision whether or not to enter the vocational sector at wage w. When $\beta'(\theta)$ is low, i.e. when the incentivised return to ability outside the vocational sector is small, the curve of marginal workers $\gamma(\theta)$ is flat and working in the vocational sector becomes relatively more attractive. When, on the contrary, the locus of marginal workers is steep, the incentivised return to ability in the opt-out sector is high so that working in the vocational sector becomes less attractive for high-productivity applicants. Point (ii) follows from the fact that the flatter the curve of marginal workers, the larger the average productivity of active workers in the vocational sector.[14]

We now turn to the analysis of main interest, namely how a wage increase affects the characteristics of the pool of active workers in the vocational sector.

3.1 Normal distribution and linear incentives

Consider a simple environment in which workers' productivity and motivation are normally distributed, and the incentive scheme in the opt-out sector is linear. In particular, ability and motivation follow a bivariate normal distribution with parameters μ_θ, μ_γ (as for marginal means), σ_θ^2, σ_γ^2 (as for marginal variances), and covariance $\sigma_{\theta\gamma}$. Let us also assume that the ratios between marginal averages and standard deviations ($\frac{\mu_\theta}{\sigma_\theta}$ and $\frac{\mu_\gamma}{\sigma_\gamma}$) are large, which implies that the probability of observing negative values for each one of the workers' characteristic is close to zero.[15]

Definition 3 ***Conditional expectations***. *Assume that θ and γ follow a bivariate normal distribution. The conditional expectations of θ and γ are*

$$\begin{aligned} E[\gamma|\theta] &= a + b\theta, \ \textit{with } b = \frac{\sigma_{\theta\gamma}}{\sigma_\theta^2} \\ E[\theta|\gamma] &= a' + b'\gamma, \ \textit{with } b' = \frac{\sigma_{\theta\gamma}}{\sigma_\gamma^2} = b\frac{\sigma_\theta^2}{\sigma_\gamma^2}. \end{aligned}$$

where a and a' are constants.

We also consider a simple linear wage scheme in the opt-out sector. In particular, let w_V be the uniform wage offered in the vocational sector and let $\beta(\theta) = w_O + \beta\theta$ be the wage scheme offered in

[14] In Appendix A.3, we compare the equilibium wage that would emerge in the relevant market when motivation matters and when it does not.

[15] For example, when the ratios $\frac{\mu_\theta}{\sigma_\theta}$ and $\frac{\mu_\gamma}{\sigma_\gamma}$ are larger than 3.5, the probability that the random variables θ and γ take negative values is smaller than 0.02 percent.

the opt-out sector, where w_O is the flat component and $\beta\theta$ represents the linear approximation of the workers' information rent. Hence, β represents the incentivised return to ability in the outside-option sector.

These assumptions will allow us to analyze the comparative statics of the wage in a simple way and will be relaxed in the general analysis of the next sections.

Denoting the net flat compensation by $w = w_V - w_O > 0$, the workers' self-selection condition slightly changes so that inequality 1 now becomes

$$w + \gamma \geq \beta\theta. \tag{3}$$

Since an increase in the wage w_V translates, other things equal, into an equivalent change in the net flat wage w, we will still focus our comparative statics on w. Also notice that Definition 1 slightly changes in that the locus of marginal workers, given the net wage w, is now described by the line $\gamma(\theta) = \beta\theta - w$.

Workers entering the vocational sector at the net flat compensation w are those characterized by reservation wage $w_R \leq w$. Hence, we can compute the average productivity of these workers, given the wage w, by considering applicants' average productivity conditional on w_R, i.e. $E[\theta|w_R]$, and cumulating it up to w

$$E[\theta|w_R \leq w] = \int_{w_R \leq w} E[\theta|w_R]\, k(w_R, w)\, dw_R, \tag{4}$$

where $k(w_R, w) > 0$ is the distribution of w_R in the subset $w_R \leq w$. In the same way, to compute the average motivation of workers entering the vocational sector at the wage w, we consider applicants' average motivation conditional on w_R, i.e. $E[\gamma|w_R]$, and cumulate it up to w

$$E[\gamma|w_R \leq w] = \int_{w_R \leq w} E[\gamma|w_R]\, k(w_R, w)\, dw_R \tag{5}$$

Given Definitions 2 and 3, the expectations of the reservation wage w_R conditional on θ and γ, respectively, are the following linear functions

$$E[w_R|\theta] = \beta\theta - E[\gamma|\theta] \tag{6}$$

$$E[w_R|\gamma] = \beta E[\theta|\gamma] - \gamma. \tag{7}$$

Expressions (6) and (7) are crucial for the results that follow.

Lemma 3 ***Impact of*** w ***on the composition of the pool of active workers.*** *When θ and γ follow a bivariate normal distribution and $\beta(\theta)$ is linear, then:*

(i) the average productivity in the vocational sector at wage w, i.e. $E[\theta|w_R \leq w]$, is monotonically increasing (decreasing) in w if the expected reservation wage conditional on θ, i.e. $E[w_R|\theta]$, is increasing (decreasing) in θ;

(ii) the average motivation in the vocational sector at wage w, i.e. $E[\gamma|w_R \leq w]$, is monotonically increasing (decreasing) in w if the expected reservation wage conditional on γ, i.e. $E[w_R|\gamma]$, is increasing (decreasing) in γ.

The proof of Lemma 3 relies on the observation that functions $E[\theta|w_R \leq w]$ and $E[\gamma|w_R \leq w]$ are increasing (decreasing) in w if an only if functions $E[\theta|w_R]$ and $E[\gamma|w_R]$ are increasing (decreasing) in w_R, respectively. In turn, the slopes of functions $E[\theta|w_R]$ and $E[\gamma|w_R]$ have the same sign as the slopes of their inverse functions $E[w_R|\theta]$ and $E[w_R|\gamma]$, respectively.

As a next step, we now study the signs of $\frac{\partial}{\partial\theta}E[w_R|\theta]$ and $\frac{\partial}{\partial\gamma}E[w_R|\gamma]$ and relate them to the correlation between θ and γ. This will allow us to ascertain under what conditions crowding effects occur.

Proposition 1 ***Independence or negative correlation between θ and γ.*** *When θ and γ are jointly normally distributed, either independently or with negative correlation, and $\beta(\theta)$ is linear, then:*

(i) the average productivity of workers in the vocational sector, i.e. $E[\theta|w_R \leq w]$, is monotonically increasing in w;

(ii) the average motivation of workers in the vocational sector, i.e. $E[\gamma|w_R \leq w]$, is monotonically decreasing in w.

Under independence or negative correlation between θ and γ, higher wages in the vocational market always attract more productive but less motivated workers. This result is the more natural one because it corresponds to the combination of the two selection effects that one would observe under unidimensional asymmetric information: adverse selection with respect to ability on the one hand (see Lemma 1 on this point), and propitious selection with respect to motivation on the other hand (see Heyes 2005 and Delfgaauw and Dur 2007).

We now consider the most interesting case in which the correlation between workers' characteristics is positive.

Proposition 2 ***Positive correlation between θ and γ and crowding effects.*** *Assume that θ and γ are jointly normally distributed with positive correlation, and that $\beta(\theta)$ is linear, then:*

(i) if $\beta < \frac{\sigma_{\theta\gamma}}{\sigma_\theta^2}(=b)$, both average productivity and average motivation in the vocational sector are monotonically decreasing in w ***(productivity crowding-out)****.*

(ii) if $\beta > \frac{\sigma_\gamma^2}{\sigma_{\theta\gamma}} (= 1/b')$, *both average productivity and average motivation in the vocational sector are monotonically increasing in* w ***(motivation crowding-in)***.

(iii) if $\frac{\sigma_{\theta\gamma}}{\sigma_\theta^2} \leq \beta \leq \frac{\sigma_\gamma^2}{\sigma_{\theta\gamma}}$, *average productivity in the vocational sector is non-decreasing in* w, *whereas average motivation is non-increasing in* w.

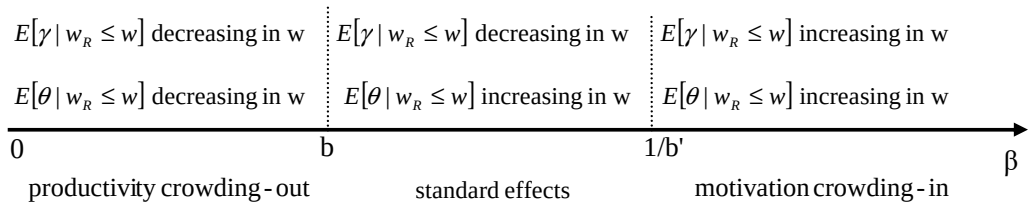

Figure 2: Bivariate normal distribution with positive correlation between θ and γ. (The length of the segments is arbitrarily taken).

Figure 2 provides a concise representation of the three cases outlined in Proposition 2; Figure 4 in Appendix A.7 plots the results from a Monte Carlo experiment reproducing the possible selection patterns described in Propositions 1 and 2.

When the correlation between productivity and motivation is positive, the effect of an increase in the wage w offered in the vocational sector affects applicants' selection patterns depending on the sign of the derivatives of the conditional expectations of reservation wages, i.e. on the sign of the slopes of functions $E[w_R|\theta] = \beta\theta - E[\gamma|\theta]$ and $E[w_R|\gamma] = \beta E[\theta|\gamma] - \gamma$. When $\beta < b$ (see Point (i) in Proposition 2), the function $E[w_R|\theta]$ is decreasing in θ meaning that, on average, workers with high productivity are also characterized by low reservation wages. Thus, at a given w, only the most productive applicants are ready to enter the vocational sector. As the wage offered in the vocational sector progressively increases, workers with lower and lower productivity enter the vocational sector determining a decrease in the average productivity of workers in the vocational sector. Moreover, given that $\beta < b < 1/b'$, also the function $E[w_R|\gamma]$ is decreasing in γ. Thus, on average, workers with high motivation are also characterized by low reservation wages. This means that, at a given w, only the most motivated applicants are willing to enter the vocational sector. As the wage offered in the vocational sector increases, workers with lower and lower motivation accept employment, so that the average motivation of the pool of active workers monotonically decreases in w. To sum up, when the slope of the regression line is higher than the incentivised return to ability in the opt-out sector, higher wages attract less productive and also less motivated workers on average, and the productivity crowding-out effect realizes. To the best of our

knowledge ours is the first theoretical model able to explain productivity crowding-out as a selection mechanism into vocational markets.[16]

The opposite mechanisms drive motivation crowding-in (see Point (*ii*) of Proposition 2). When $\beta > 1/b'$, the function $E[w_R|\gamma]$ is increasing in γ implying that, on average, workers characterized by high motivation also have high reservation wages. As the uniform wage offered in the vocational sector progressively increases, workers with higher and higher motivation enter the vocational sector and the expected motivation of the pool of active workers is monotonically increasing in w. Moreover, the function $E[w_R|\theta]$ is increasing in θ. Then also the average productivity of the pool of workers active in the vocational sector increases with w.

In the intermediate case in which $b < \beta < 1/b'$, $E[w_R|\theta]$ is increasing in θ whereas $E[w_R|\gamma]$ is decreasing in γ so that an increase in the uniform wage offered in the vocational sector always attracts more productive but less motivated workers (see Point (*iii*) of Proposition 2). Now the same pattern as with independence or negative correlation between ability and motivation realizes (see Proposition 1). In the particular case in which $\beta = b$, average productivity is constant whereas average motivation decreases monotonically with the wage w; when instead $\beta = 1/b'$ average motivation is constant whereas average productivity is monotonically increasing in the wage w.

Notice that, under perfect positive correlation between productivity and motivation, it is always the case that $b = 1/b'$, implying that Point (*iii*) in Proposition 2 is no longer relevant.[17] In words, when the correlation between the workers' characteristics equals one, only two selection patterns are possible: average productivity and average motivation of active workers in the vocational sector are either both increasing or both decreasing in the wage rate.

As an intuition for the results outlined in Propositions 1 and 2, notice that a worker's reservation wage depends on the worker's pecuniary payoff in the opt-out sector and on the non-pecuniary payoff in the vocational sector. Specifically, a worker's reservation wage rises in the degree to which firms in the non-vocational sector can tie workers' pecuniary compensation to their productivity, and it falls in the degree to which workers' motivation is correlated with ability, given that workers receive no vocational premium when they accept employment in the opt-out sector. Hence, when the correlation between ability and motivation is zero or negative, expression $\frac{\partial}{\partial\theta}E[w_R|\theta] = \beta - \frac{\partial}{\partial\theta}E[\gamma|\theta] = \beta - b$ is necessarily

[16] Barigozzi and Turati (2012) consider a discrete model with four worker types and $\beta = 1$. In that case, irrespective of the joint distribution of workers' attributes, one crowding effect *always* occurs for a subset of wage rates; which one realizes depends on the relative impact of productivity and motivation on workers' reservation wages.

[17] The equality $b = 1/b'$ comes from the Cauchy-Schwarz inequality that, under perfect correlation, writes as $\sigma^2_{\theta\gamma} = \sigma^2_\gamma \sigma^2_\theta$.

positive (being it either a positive term or the sum of two positive terms). Thus, higher wages offered in the vocational sector attract workers with higher reservation wages who are characterized by higher productivity. Moreover, because of negative correlation, the workers entering the vocational sector are also characterized by low motivation. Overall this defines the selection pattern described in Proposition 1.

When instead the correlation between ability and motivation is positive, workers' selection pattern crucially depends on the magnitude of the incentivised return to ability in the alternative sector β, because $\frac{\partial}{\partial\theta}E[\gamma|\theta] = b > 0$ holds and the sign of $\frac{\partial}{\partial\theta}E[w_R|\theta]$ depends on whether β exceeds or falls short of b (see Proposition 2). At any given wage w, high-ability and high motivated workers prefer the vocational sector when the payoff to ability in the opt-out sector β is low. In this case, as the wage w offered in the vocational sector increases, the vocational sector becomes attractive for other workers (who previously preferred the opt-out sector), whose ability is on average lower than one of the colleagues who are already in. Thus, average ability in the vocational sector falls (productivity crowding-out). And because of the positive correlation between ability and motivation, motivation in the vocational sector also falls as w increases. The opposite pattern realizes when the return to ability in the non-vocational sector is high. In this case, for any wage w, the proportion of high-ability applicants willing to enter the vocational sector is low. Hence, when w increases, the new workers being attracted to the vocational sector have higher average ability than those already in, thus raising average productivity. Because of the positive correlation between ability and motivation, high-ability workers preferring the vocational sector are also characterized by high motivation, so that average motivation in the vocational sector also increases as the wage w increases (motivation crowding-in).

The following corollary is a direct implication of Proposition 2.

Corollary 1 *Crowding effects never occur together.*

In different words, it is never possible that, for the same values of the flat wage w, the average productivity of workers entering the vocational market is decreasing while the average motivation is increasing in w.

We just showed that crowding effects may occur only when workers' characteristics are positively correlated. Is positive correlation between workers' productivity and motivation empirically relevant? The field experiment by Dal Bó *et al.* (2013) suggests that ability and intrinsic motivation may be positively correlated in the case of civil servants. The literature on public administration documents that public service motivation is positively correlated with job performance in the public sector (see, among

others, Naff and Crum 1999, Park and Rainey 2008 and Ritz 2009), which again is consistent with a positive correlation between ability and motivation.

3.2 General distributions and non-linear incentives

Consider now a general joint probability distribution for ability and motivation, and a nonlinear function $\beta(\theta)$ as for worker's remuneration in the alternative sector. When the joint distribution of θ and γ is arbitrary, Lemma 3 does not hold because it relies on the linearity of conditional expectations, which is not guaranteed anymore; in addition, the distribution of the random variable w_R is not known. Nevertheless, we provide sufficient conditions for crowding effects to occur for a non-empty subset of wage levels.

Proposition 3 ***Sufficient conditions for crowding effects.*** (i) *Productivity crowding-out: if a wage rate* w_0 *exists such that* $E[\theta|w_R \leq w_0] > \mu_\theta$, *then* $E[\theta|w_R \leq w]$ *must be decreasing in the wage rate for at least some wage levels higher than* w_0. (ii) *Motivation crowding-in: if a wage rate* w_{00} *exists such that* $E[\gamma|w_R \leq w_{00}] < \mu_\gamma$, *then* $E[\gamma|w_R \leq w]$ *must be increasing in the wage rate for some wage levels higher than* w_{00}.

The intuition for the above proposition is that the average characteristics of active workers in the vocational sector must ultimately converge to the population averages, once the uniform wage offered in the vocational sector has increased so much that all potential applicants prefer to enter the vocational rather than the outside-option sector. If $E[\theta|w_R \leq w_0]$ is above the population average μ_θ, then $E[\theta|w_R \leq w]$ must be decreasing in w for some $w > w_0$ in order to converge to μ_θ; likewise, if $E[\gamma|w_R \leq w_{00}]$ is below the population average μ_γ, then $E[\gamma|w_R \leq w]$ must be increasing in w for some $w > w_{00}$ in order to converge to μ_γ. Condition $E[\theta|w_R \leq w_0] > \mu_\theta$ implicitly requires that a big mass of workers with high productivity enter the vocational sector for $w = w_0$, while condition $E[\gamma|w_R \leq w_{00}] < \mu_\gamma$ requires that a big mass of workers with low motivation enter the vocational sector for $w = w_{00}$. Hence, not only do these sufficient conditions rely on the degree of association between the workers' characteristics, but they also incorporate information about the workers' reservation wage, because they refer to the averages of workers' characteristics conditional on the wage rate. In Appendix A.8, we derive the technical counterpart of conditions (i) and (ii) in Proposition 3 and we emphasize that those conditions are consistent with the hypothesis of a positive association between θ and γ in the population of potential candidates.

Interestingly, Proposition 2 shows that, when workers' attributes follow a bivariate normal distribution, crowding effects are verified not only for some subset of wage rates, but for all possible values of w, so that monotonicity is obtained (see also Figure 4).

As an example of a joint distribution characterized by a non-linear dependence in mean between θ and γ, let us now turn to analyze a bivariate exponential distribution and check that productivity crowding-out can easily be obtained.

Bivariate exponential distribution. Due to its mathematical tractability, the bivariate exponential distribution allows to derive analytical results and evaluate condition (i) of Proposition 3 in practice. Bivariate exponential distributions with different parameterization have been analyzed in Gumbel (1960). In particular, let us consider the following joint distribution of (θ, γ):

$$p(\theta, \gamma) = e^{-\theta-\gamma}\left[1 + b(2e^{-\theta} - 1)(2e^{-\gamma} - 1)\right],$$

with $(\theta, \gamma) \in \mathbb{R}_+ \times \mathbb{R}_+$, and where the parameter $b \in (-1, 1)$ measures the association between the two characteristics. It can be easily shown that the coefficient of correlation between ability and motivation is $\frac{b}{4}$. With this parameterization, the marginal means are equal to one for both variables, i.e. $\mu_\gamma = \mu_\theta = 1$. It is worth noting that the conditional expectations $E[\theta|\gamma]$ and $E[\gamma|\theta]$ are non-linear functions and that their curvature depends on the coefficient b. We derive analytical results by assuming that $\beta(\theta) = \theta^{1/2}$ is the non-linear wage scheme offered in the outside-option sector. The curve of marginal workers, thus, is given by $\gamma = \theta^{1/2} - w$. We first check with mathematical programming that condition (i) in Proposition 3 (or condition 18 in Appendix A.8) is satisfied when the correlation between θ and γ is positive, provided that the wage rate is sufficiently low. We then compute the average productivity of active workers in the vocational sector and verify that, with a positive correlation, it is first decreasing and then increasing in the wage (see Figure 3 below). This confirms that productivity crowding-out occurs for a sufficiently low wage rate. Instead, with a negative correlation, the average productivity of active workers is always increasing in w.

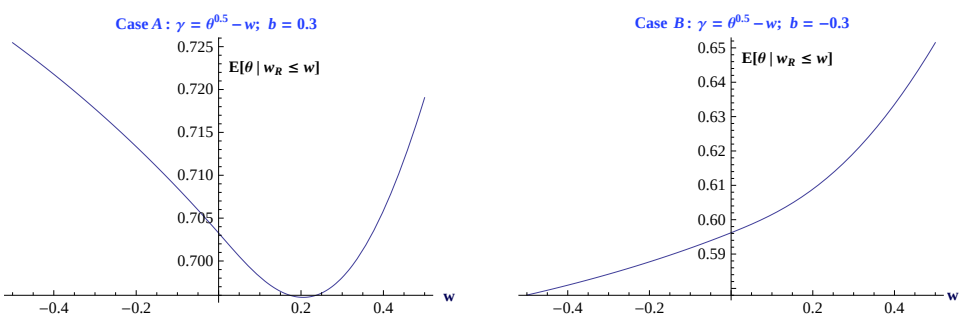

Figure 3: Workers' characteristics follow a bivariate exponential distribution and $\beta(\theta) = \theta^{1/2}$. In graph A correlation is positive; in graph B correlation is negative.

3.3 Incentives in the opt-out and in the vocational sector

So far we have assumed that there is no screening in the vocational sector. Here we show that our results are robust to the introduction of some incentives in the vocational sector too.

Suppose that, in both sectors, workers' remuneration is characterized by a wage scheme of the form:

$$W_j(\theta) = w_j + \beta_j(\theta)$$

where $j \in \{V, O\}$, with V referring to the vocational and O to the opt-out sector. We call w_j the flat component of the compensation, whereas $\beta_j(\theta)$ is the incentive scheme that rewards the workers' productivity, with $\beta_j'(\theta) > 0$.

The workers' self-selection constraint now becomes

$$w_V + \beta_V(\theta) + \gamma \geq w_O + \beta_O(\theta), \tag{8}$$

where the left-hand side of the inequality indicates workers' overall remuneration when accepting the job in the vocational sector and the right-hand side the workers' remuneration when working in the opt-out sector.

Setting $w = w_V - w_O > 0$ and $\beta(\theta) = \beta_O(\theta) - \beta_V(\theta)$, we are back to inequality (1). All our results still hold as long as the vocational sector is characterized by a lower incentivised return to ability, i.e. $\beta_O'(\theta) > \beta_V'(\theta)$ for all θ, so that $\beta'(\theta) > 0$. From the previous discussion, our results can be extended as follows.

Proposition 4 ***Incentives in the vocational sector.*** *Assume that a screening mechanism is in place in both sectors of the labor market but that it is relatively more effective in the opt-out sector. Propositions 1, 2 and 3 still hold. Crowding effects now depend on the association between workers' characteristics being positive and on the difference in the incentivised returns to ability across sectors.*

Specifically, Proposition 1 and 2 are relevant when $\beta_O(\theta)$ and $\beta_V(\theta)$ are both linear and the workers' characteristics follow a bivariate normal distribution. Proposition 3 instead applies for non-linear wage schemes and a general joint distribution.

In the case of positions of leadership, incentives in the vocational sector may be particularly likely. What lessons can be drawn from our model when we deal with managers? As mentioned in the model setup, the empirical literature on wage differentials shows that substantial wage penalties exist at the top of the wage distribution in both the public and the non-profit sectors as opposed to the private and the for-profit sectors, respectively. Conversely wage differentials are almost absent at the lowest wage levels. This

evidence suggests that pecuniary compensations are likely to be tied to managers' productivity also in the vocational sector, but less tightly than in alternative sectors. Moreover, the relevance of wage penalties at the top of the wage ladder translates into the difference in the incentivised returns to ability across sectors β being relatively high for managers. In addition, managers are likely to have high ability on average and, typically, they have high responsibilities, high autonomy in decision-making and, they perform interesting and challenging tasks that contribute to make their motivation particularly salient.[18] This hints at ability and motivation being positively correlated within the population of managers. To sum up, in the case of managers, Proposition 2 is likely to be relevant and, provided that β be sufficiently high, we could be in a situation in which a wage increase attracts workers who are more skilled and more motivated on average. Overall, our model offers a justification for having low incentives in vocational sectors: with a flat wage in the vocational sector, the difference in the incentivised returns to ability across sectors is the largest possible and this fact, coupled with a positive correlation between potential applicants' attributes, increases the likelihood of the motivation crowding-in effect (requiring that $\beta > 1/b' > 0$).

4 Results in perspective

The problem of how to attract high-quality applicants in specific vocational markets has been investigated, both theoretically and empirically, for example in the market for civil servants and bureaucrats, the market for politicians, teachers and nurses.[19] We first discuss the related theoretical literature in the light of our results. Then, we illustrate the controversial empirical evidence on the effects of a wage increase on the characteristics of the pool of active workers in those sectors. Finally, we confirm our theory using data from two experiments on civil servants: Banuri and Keefer (2016) and Dal Bó *et al.* (2013).

4.1 Related theories

Our paper relates to the "motivation crowding theory" according to which monetary rewards may undermine intrinsic motivation and may reduce individuals' prosocial and/or productive activities (Titmuss 1970). Different mechanisms have been proposed to explain the unintended consequences of monetary

[18] See Deci and Ryan (1985)'s self-determination theory, according to which conditions supporting the individual's experience of autonomy, competence, and relatedness strongly foster intrinsic motivation.

[19] As for civil servants see, for example, Francois (2000), Delfgaauw and Dur (2007) and (2010). About politicians, among many others, see Caselli and Morelli (2004), Messner and Polborn (2004), Gagliarducci *et al.* (2010). As for teachers, Figlio and Lawrence (2007) and Dolton and Marcenaro-Gutierrez (2011) are important examples. Health professionals have been considered by Heyes (2005), Barigozzi and Turati (2012) and Barigozzi and Burani (2016b).

incentives: extrinsic motivations might interact with intrinsic motivations (see Deci 1975 and Frey 1997); extrinsic incentives might destroy trust in a principal-agent relationship (see Falk and Kosfeld 2006, Fehr and List 2004, Fehr and Falk 2002); the introduction of extrinsic motives might shift an individual's decision frame from a social to a monetary frame (see, Heyman and Ariely 2004); extrinsic incentives might interact with image motivation by diluting the signaling value of prosocial behavior (Bénabou and Tirole 2006); performance incentives may signal some information that negatively impacts on workers' willingness to exert effort (Bénabou and Tirole 2003, Sliwka 2007). In this paper, we instead study how the correlation between workers' characteristics in the population of applicants interacts with monetary compensations and ultimately affects the self-selection decisions of potential applicants.

Starting from Roy (1951)'s seminal contribution, many authors have analyzed the sorting of workers into alternative sectors of the labor market. Typically, workers' self-selection decisions depend on the distribution of workers' attributes (generally workers' productivity) in the population of applicants and on the differences in compensation across sectors (see, in particular, Jovanovic 1982). Along these lines, Dal Bó *et al.* (2013) develop a theoretical model to guide their empirical inquiry and investigate the effect of wage changes on the average levels of productivity and motivation. In their model, workers decide whether to apply for a position of social promoter which is associated with a given salary. Their self-selection condition is very similar to ours but, as for association between ability and motivation in the population of potential employees, they only consider two extreme cases: either ability and motivation are independent and uniformly distributed, or they are perfectly positively correlated. The authors find conditions such that higher wages increase average motivation in the applicant pool. However, their model predicts that higher wages *always* increase the average skills of candidates.[20] Thus, their results correspond to a special case of point (*ii*) of our Proposition 2 and exclude productivity crowding-out effects.

In the market for politicians, some papers examine the theoretical relationship between politicians' wages and politicians' quality, performance, and willingness to run for office. Messner and Polborn (2004) find a non-monotonic relationship between the salary of elected officials and their average quality. In Mattozzi and Merlo (2008), an increase in the salary a politician receives while in office decreases the average quality of individuals who become politicians. Conversely, Gagliarducci *et al.* (2010) find advantageous sorting into politics: given the outside opportunities for "moonlighting politicians", high-

[20] In their Proposition 1, they state that "an increase in wages increases the average ability of the applicant pool" and that it also increases the average motivation of the pool, provided that the correlation between ability and motivation is perfect and positive and that an increase in motivation increases the ability of a candidate at a rate faster than one for one.

ability individuals are more likely to run for office because they can use the political office to boost their private returns. With respect to all these papers, our analysis rationalizes a broad spectrum of responses, also providing intuitive conditions on workers' characteristics and incentives across sectors.

4.2 Related empirical evidence: reconciling contrasting results

Civil servants In a field experiment on a large-scale recruitment drive by the Mexican government, Dal Bó *et al.* (2013) examine applicants to the position of community development agents, i.e. social promoters expected to enhance state presence in Mexico's most marginalized municipalities. Two different salaries were announced randomly across recruitment sites, and job offers were subsequently randomized in order to measure the role of financial incentives in attracting a larger and more qualified pool of applicants. In a screening session, three broad categories of applicants' personal characteristics were measured: personality traits (through the Big 5), values and motives (e.g., goals, interests and especially inclination toward public sector employment measured using Perry's 1996 scale of Public Service Motivation) and abilities (verbal, quantitative, and spatial intelligence measuring IQ through the Raven's test). During the screening exam, candidates were also asked to provide information about their last employments (position and length of employment, previous wages). The authors find that higher wages attract more able applicants as measured by their Raven score and personality traits, by their past earnings and also by their occupational profiles. In addition, higher wages also attract workers with higher proclivity toward public sector work. Overall, their results provide evidence of a positive effect of higher wages on both ability and motivation. Moreover, the authors find a positive correlation between ability and motivation in the sample of self-selected candidates and they state that this feature is consistent with the existence of a positive correlation between the two characteristics in the whole population of potential workers (see page 1199). In the next subsection we show that the documented selection pattern corresponds to the motivation crowding-in effect illustrated in our Proposition 2(*ii*) which requires a positive association between ability and motivation in the population of potential employees.

Banuri and Keefer (2016) design a lab experiment in order to study the interaction of motivation and wages in pro-social organizations and provide different results. Their subjects are university students in Indonesia, including students who are committed to work in the public sector. As a measure of pro-social motivation, they use the subjects' donations to a charity, the Indonesian Red Cross, in a dictator game. The amount gained by experimental subjects in a pay-for-effort initial task serves instead as a measure of ability. Subjects then undergo a choice task designed to mimic the choice between working for a

private for-profit organization or for a public pro-social one. They have to select between two different contracts: one in which they receive a piece rate payment scheme such that their reward is proportional to productivity, and the other in which subjects receive a flat salary, but their productivity is rewarded in terms of donations to the charity. Importantly, two treatments are considered, with two different flat salaries offered in the public sector contract. Results show that raising public sector wages reduces the likelihood that intrinsically motivated individuals enter the public sector and has no significant impact on the likelihood that high-skilled individuals choose the public sector. In addition, within the whole pool of experimental subjects, the correlation between ability and motivation is weakly positive but not significant. As we show in the next subsection, the selection pattern documented by Banuri and Keefer (2016) confirms our Proposition 1, which is indeed relevant when workers' attributes are not correlated.

Politicians Fisman *et al.* (2015) focus their analysis on the members of the European Parliament (MEPs) and employ a differences-in-differences approach exploiting the introduction of the 2009 law that equalized MEPs' salaries. The authors show that higher salaries induce a negative selection effect on MEPs' quality as measured not only by their education level but also by the ranking of their educational institutions. Indeed, doubling the salary decreases the probability that an elected MEP attended a top college by 15%. Hence, higher salaries lower the human capital of MEPs by way of lowering the quality of their undergraduate institution. At the country level, Merlo *et al.* (2009) study the labor supply of Italian politicians using data about the national Parliament. They account that, between 1985 and 2004, the average total annual real income of Italian legislators grew at an average annual growth rate of 3.8%. Despite that, Italian representatives are much less educated and have lower outside opportunities in recent years than in the past. Here again, higher salaries lower the human capital of members of the Italian Parliament. Neither Merlo *et al.* (2009) nor Fisman *et al.* (2015) have data about candidates' non-pecuniary motivation. Nonetheless, their evidence is consistent with the productivity crowding-out effect (see Proposition 2(i)) and their finding could be explained by the selection mechanism in place when ability and motivation are strongly positively correlated (see also below).[21]

At the local level, the evidence about the selection mechanisms is different. In a quasi-experimental setup, Gagliarducci and Nannicini (2013) use data on Italian municipal governments from 1993 to 2001,

[21] Both studies share some important features. On the one hand, the salaries of elected members of the European or Italian Parliament are very high, possibly much higher than the returns in alternative sectors. Thus one would expect the self-selection condition into the public sector to be satisfied very often. On the other hand, political parties play a critical role in determining which candidates run for election and this might interfere with the selection mechanisms we outline.

exploiting the fact that the wage of mayors varies discontinuously according to the size of the resident population. The authors apply a regression discontinuity design to the threshold of 5,000 resident inhabitants, which determines a 33% wage increase for the elected mayor. They show that this wage increase attracts more educated candidates (0.905 years of schooling more), which means an increase in education of about 6.4% (with respect to an average of 14 years of schooling in municipalities between 3,000 and 5,000). In addition, the wage increase attracts more candidates employed in white-collar occupations, such as lawyers and managers. This also translates into more-educated (0.879 years of schooling more) and higher-skilled elected mayors.[22] Again, no data on potential candidates' motivation are available in this study, but, in terms of our theory, its evidence is consistent with both Propositions 1 and 2 (the latter as for points (*ii*) and (*iii*)) which predict that higher wages attract more skilled workers. In other words, the selection pattern documented by Gagliarducci and Nannicini (2013) excludes productivity crowding-out and thus the existence of a positive and strong correlation between politicians' measured skills and unmeasured motivation.

The different selection patterns documented for Italian politicians at the national and at the local level is particularly striking. Using insights from our model, we can propose an explanation based on the different returns to ability in the opt-out sectors for the two groups of politicians. Merlo *et al.* (2009) report that members of the Italian Parliament are largely represented by candidates coming from flat wage professions: former teachers and professors (from public schools and public universities), employees of public firms, party and trade unions officials (see Table 1, page 56). Instead, Gagliarducci and Nannicini (2013) report that elected mayors are mainly lawyers, managers and entrepreneurs (see Table 2, pages 381-2) characterized by high opportunity costs into politics. Hence, on average, the return to ability in the opt-out sectors, i.e. β, can be conceived of as being higher at the local than at the national level. As already mentioned, information on politicians' motivation is not available. However, admitting that ability and motivation may be positively correlated in the population of potential politicians, the empirical study by Merlo *et al.* (2009) corresponds to a case with low return to ability in the opt-out sector and productivity crowding-out. Gagliarducci and Nannicini (2013) represents instead an instance in which there are high returns to ability and motivation crowding-in (see Figure 2).[23]

[22] Along the same line, Ferraz and Finan (2011), examining Brazil's municipalities, show that higher salaries attract candidates that are more educated and have more experience.

[23] As an anonymous referee pointed out, the probability to be re-elected can be interpreted as an additional reward entering the payoff of elected politicians and depending on ability: $p(\theta)$. From (1), the self-selection condition of politicians can thus be rewritten as:

$$w + p(\theta) + \gamma \geq \beta(\theta).$$

Teachers Dolton and Marcenaro-Gutierrez (2011) study the variation in teachers' pay across OECD countries and its impact on educational outcomes: they find a significant correlation between higher relative teachers' salaries and higher standardized pupil scores across countries. The authors conclude that higher wages attract teachers with higher degrees and improve students' performance. Although no information on teachers' motivation is available for a more precise identification of the two alternative explanations, this evidence fits again Propositions 1 and 2 (points (*ii*) and (*iii*)).

4.3 Confirming our theory with data

Two of the previous papers allow us to go a step further and check empirically whether the mechanism at play is fully consistent with our theory. In particular, we use the data on ability and motivation of these papers and check that the observed consequences of changing wages are indeed coherent with the cases identified in our Propositions 1 and 2.[24]

We first use the publicly available data of Banuri and Keefer (2016). A big "plus" with these experimental data is the possibility to measure ability and motivation of the entire relevant population with no issues of selection. We first confirm, as the authors do, that no significant correlation between ability and motivation exists in their data.[25] With no significant correlation our Proposition 1 becomes relevant. We then divide the population into four groups depending on salaries and observed choices between the private and the public organization, respectively when subjects opt for a piece-rate contract or a flat salary, following the authors' interpretation. In group (1) subjects prefer the private organization to a public organization that offers a low salary, in (2) subjects prefer the private organization to a public one with high salary, in (3) subjects opt for the public organization with low salary, and in (4) subjects opt for the public organization with high salary.

We observe a higher average ability in groups (1) and (2) opting for the private sector. In particular, we find the highest level of average ability in group (2) opting for the private sector when the wage in the public organization is high. We observe low average skills in the subjects opting for the public organization

The net incentivised return to ability in the alternative sector then becomes $\beta'(\theta) - p'(\theta) = \beta$. In Italy, a two-term limit for mayors exists, suggesting that $p(\theta)$ is likely to be steeper for members of Parliament, who don't face ay constraint, than for mayors. Hence β would be lower at the national than at the local level reinforcing the result that productivity crowding-out is observed for members of the Italian Parliament but not for Italian mayors.

[24] We thank a referee for suggesting us this possibility.

[25] Motivation is measured by their variable *dict_send* whereas ability is measured by *t1_effort*. We checked that the assumption of a normal distribution for the two variables is reasonable.

in general (groups 3 and 4) and, in particular, the lowest average skills in group (4) of subjects entering the public sector for a high salary (although this difference is not statistically significant). To sum up, subjects with higher skills are attracted by the higher pay-for-effort they can obtain in the private sector, and particularly so when the wage in the public sector is high. This implies that subjects with the highest average ability (i.e. the ones opting for the private sector when the wage in the public sector is high, i.e. group 2) would require an even larger wage to move to the public sector. Overall, average productivity is monotonically increasing in w as contemplated in part (i) of Proposition 1.

Concerning motivation, we find that the group deciding to enter the public organization for a low salary exhibits the highest average motivation. A slightly lower average motivation is observed in the group entering the public organization for an high salary. The average motivation in the other two groups is strictly lower. Expected motivation in the vocational sector is thus monotonically decreasing in the flat wage w, as contemplated in part (ii) of Proposition 1.[26]

The authors of Dal Bó *et al.* (2013) kindly provided us with part of their dataset, consisting in applicants' Perry's index of public sector motivation, the Raven IQ scores they use as a measure for ability and current or past earnings. A word of caution is in order. These data refer to applicants that already self-selected into the public sector, i.e. that already showed up for the job interview and took the screening exam. As the authors point out in their paper, the positive correlation between ability and motivation that they identify may not be representative of the entire population of potential applicants and might be spurious.[27] With this caveat in mind, we compute the correlation between ability and motivation and find it is significant, positive and equal to 0.112, close to the authors' estimate (Table A2 in the on-line appendix of Dal Bó *et al.* 2013). This places us in the environment of Proposition 2. As mentioned above, the authors find evidence that higher financial incentives attract more motivated and more productive applicants which corresponds to Proposition 2 point (ii). The next three steps show estimates that indeed identify that case. First, we measure how ability, as captured by the Raven test scores, increases the applicants' earnings potential outside the public sector, which corresponds to our coefficient β. A unit variation on the Raven test increases the candidates' current or past earnings by 3.03%, with a standard error of .0072, which is statistically significant. Second, we estimate that a unit variation on the Raven score induces an increase of 0.529% of motivation, with a standard error of 0.001,

[26] See Table 1 in Appendix A.9 summarizing our analysis using Banuri and Keefer (2016)'s dataset.

[27] For example, if correlation is nil for the entire population, self selection with a positively sloped function $\beta(\theta)$ might generate positive correlation in the subsample of candidates that sorted into the public sector.

which is statistically significant. This figure can be interpreted as the coefficient b in Definition 3. Third, the estimated elasticity of ability with respect to motivation is 0.37%, statistically significant at the 1% level. This estimate can be interpreted as the parameter b' in Definition 3, with its inverse being equal to 2.703. Taken together, these three estimates confirm the conditions $\beta > \frac{1}{b'} > b$ which identify motivation crowding-in as in Proposition 2 (*ii*). Clearly, if sorting into the public sector induces an overestimation bias of the correlation between ability and motivation, the actual b and $\frac{1}{b'}$ might be respectively smaller and higher, in which case the first inequality might be violated. In case of underestimation, instead, it may be strengthened.

Although we cannot directly test our theory because the dataset of Banuri and Keefer (2016) and Dal Bó *et al.* (2013) were not collected with this aim, the set of results we obtain with these data confirm our theory, the first identifying Proposition 1 and the second Proposition 2.

5 Conclusion and policy implications

Do higher salaries always attract more productive workers? We have shown that this is not necessarily the case when workers are heterogeneous with respect to two characteristics, productivity and motivation, that are not observable to their employers. Our analysis identifies conditions implying that a negative relationship between the wage rate and applicants' ability exists (at least for a subset of possible wage levels), based on the degree of association between workers' characteristics in the population and on the difference between incentivised returns to ability across sectors. What are the policy implications of our results?

Suppose that, given the wage scheme that is in place in the vocational sector, policy-makers observe a shortage of workers, as is actually the case in the market for nurses in many countries.[28] Our paper shows that a wage increase as a policy to overcome the shortage may have a serious drawback. Indeed, higher salaries may trigger a decrease in the average ability of the workforce together with a decrease in average motivation. Thus, an increase in the wage may deteriorate the quality of workers with respect to both dimensions (recall that motivation does affect the firms' output indirectly, i.e. through the workers' selection pattern into the sector).

In order to ascertain whether this negative outcome is just a theoretical possibility or a substantial risk, one should measure the association between the workers' attributes in the population of potential candidates. The statistical association between motivation and productivity for prospective nurses is a

[28] See, among others, Antonazzo *et al.* (2003) and Shields (2004).

reasonable proxy and could be measured using data collected from students attending nursing schools. Amabile *et al.* (1994) prepared a Work Preference Inventory (WPI) consisting of 30 questions designed to assess college students' overall intrinsic and extrinsic motivation toward their (future) work. Matching results from WPI with information about students' academic performance would then deliver insights about the statistical association between prospective nurses' motivation and skills. In case the distribution of students' characteristics approximate a bivariate normal distribution, our analysis indicates that a null or a negative correlation between motivation and skills would imply that higher salaries will always attract nurses with higher skills but lower motivation on average. Given a positive correlation, instead, we are able to predict which one of the two crowding effects realizes, if any, by comparing rewards in different sectors. One has to estimate the rate of return to the profession which represents the best alternative to becoming a nurse for prospective nursing students. Morris and McGuire (2002) consider "obtaining a university degree" as the best alternative to entering the nursing profession. Then, it suffices to compare the rate of return to "obtaining a university degree", with the degree of positive correlation between the workers' characteristics previously obtained for nursing students. In particular, if the rate of return to obtaining a university degree is lower than the (positive) slope of the regression line of motivation conditional on productivity, our model suggests that increasing the wage rate reduces the productivity of active workers.[29]

The market for politicians represents a second example. The existing evidence relative to the European Union or to Italy, discussed in Subsection 4.2, shows that higher remunerations tend to be associated with less educated and less productive members of the Parliament. As mentioned above, our model offers a possible explanation of this phenomenon based on the selection mechanism of citizens into politics at the national level that may cause the crowding-out of politicians' ability in those countries.[30] Moreover, in

[29] We reckon, however, that estimating the incentivised return to ability in the alternative sector is difficult and the obtained measure is necessarily noisy. Hence, in the case of positive correlation between ability and motivation, it may be difficult to reach the precision required to yield a prediction on crowding effects.

[30] In Italy, the current compensation of politicians is generally perceived as too generous given that MPs' salaries are 5 times larger than the average country wage. Before being appointed as Italy's Prime Minister, Matteo Renzi promised that he would cut the exorbitant cost of Italian politics by halving both the number of Members of Parliament (MPs) and the salaries of the representatives. As an engagement to reduce MPs' compensation, Five Star Movement's representatives are forced by their party to devote a fraction of their salary to prosocial projects (see "In Italy, members of parliament make five times more than the average worker", Quartz.com, March 4th, 2014, available at http://qz.com/183305, and "Rewarding work" The Economist, July 15th 2013). According to our model, the policy measure of reducing the parliamentary compensation may not only reduce inequality and public expenditure, as the advocates of a reduction assert, but might also improve the quality of politicians.

some countries, it is debated whether the remuneration of elected representatives should be increased.[31] To this respect, our model suggests that increasing the remuneration of elected representatives may have unintended consequences on the Members of Parliament's quality and that the effects of wage policies ultimately rest on the correlation between ability and motivation in the population of potential candidates.

Overall we advocate that in vocational labor markets, wage policies should be supported by prior sector-specific knowledge of the statistical association between workers' attributes. We expect the latter to vary in a systematic way with the type of vocational job and/or cultural and historical backgrounds.

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A Appendix

A.1 Proof of Lemma 1

(i) The marginal workers in the benchmark case without motivation have productivity θ such that $\beta(\theta) - w = 0$. Totally differentiating the previous equation with respect to θ and w one gets $\beta'(\theta)\, d\theta - dw = 0$. Since $\beta'(\theta) > 0$, the first claim is obtained. Obviously, the higher $\beta'(\theta)$, the lower the impact of a wage increase on the productivity of marginal types.

(ii) Given w, the marginal worker's productivity at the benchmark in which motivation is irrelevant is $\theta = \beta^{-1}(w)$. Let's consider the case in which $\beta^{-1}(w) > 0$ as in Figure 1. The same reasoning can be applied when $\beta^{-1}(w) < 0$. Letting $f(\theta, \gamma)$ be the joint probability density function of θ and γ, the probability that workers enter the relevant sector at a given salary w in the benchmark case in which motivation is irrelevant is

$$A_1 = \int_0^{+\infty} \int_0^{\beta^{-1}(w)} f(\theta, \gamma)\, d\theta d\gamma, \tag{9}$$

corresponding to the area at the left of the vertical dotted line passing through $\theta = \beta^{-1}(w)$ in Figure 1. Hence, the expected value of θ given the salary w in the benchmark without vocational premia is

$$E_B[\theta | \beta(\theta) \leq w] = \frac{\int_0^{+\infty} \left[\int_0^{\beta^{-1}(w)} \theta f(\theta, \gamma)\, d\theta \right] d\gamma}{A_1}$$

where the subscript B stands for benchmark.

Let $h(\theta) = \int_0^{+\infty} f(\theta, \gamma)\, d\gamma$; hence we have

$$E_B[\theta | \beta(\theta) \leq w] = \frac{\int_0^{\beta^{-1}(w)} \theta h(\theta)\, d\theta}{\int_0^{\beta^{-1}(w)} h(\theta)\, d\theta}.$$

We now compute the derivative of $E_B[\theta | \beta(\theta) \leq w]$ with respect to the wage w and show that it is always positive

$$\frac{\partial}{\partial w} E_B[\theta | \beta(\theta) \leq w] = \frac{h(\beta^{-1}(w))\beta^{-1}(w) \frac{\partial \beta^{-1}(w)}{\partial w} \int_0^{\beta^{-1}(w)} h(\theta) d\theta - h(\beta^{-1}(w)) \frac{\partial \beta^{-1}(w)}{\partial w} \int_0^{\beta^{-1}(w)} \theta h(\theta) d\theta}{\left[\int_0^{\beta^{-1}(w)} h(\theta) d\theta \right]^2}$$

The sign of $\frac{\partial}{\partial w} E_B[\theta | \beta(\theta) \leq w]$ is the same as the sign of its numerator, which can be rewritten as

$$N = h(\beta^{-1}(w)) \frac{\partial \beta^{-1}(w)}{\partial w} \left[\int_0^{\beta^{-1}(w)} \left[\beta^{-1}(w) h(\theta) - \theta h(\theta) \right] d\theta \right].$$

Since $\beta'(\theta) > 0$ and $\theta \in \left[0, \beta^{-1}(w)\right]$, N is always non-negative.

A.2 Proof of Lemma 2

(i) This result trivially follows from the comparison of the relative sizes of the areas depicting active workers at the benchmark and with motivation in Figure 1.

(ii) As already mentioned in the proof of Lemma 1, the marginal worker's productivity at the benchmark in which motivation is irrelevant is $\theta = \beta^{-1}(w)$. For the same wage rate, the productivity of marginal workers when motivation matters is $\theta = \beta^{-1}(w+\gamma)$. We can compare the productivity of marginal workers in the two scenarios. Since $\gamma \geq 0$, it is true that $\beta^{-1}(w+\gamma) \geq \beta^{-1}(w)$. In other words, for every strictly positive motivation level and for every uniform wage w, marginal workers have higher productivity when motivation matters than at the benchmark.

Given w, we now compare average productivity of active workers in the two scenarios. The probability that workers enter the vocational sector conditional on w when motivation matters is $A_1 + A_2$, where A_1 has been derived in (9) whereas A_2 is the area in between the dotted vertical line and the curve of marginal workers in Figure 1 and writes as

$$A_2 = \int_0^{+\infty} \int_{\beta^{-1}(w)}^{\beta^{-1}(w+\gamma)} f(\theta,\gamma)\, d\theta d\gamma.$$

The expected value of θ given w when motivation matters is

$$E[\theta | w_R \leq w] = \frac{\int_0^{+\infty}\left[\int_0^{\beta^{-1}(w)} \theta f(\theta,\gamma) d\theta\right] d\gamma + \int_0^{+\infty}\left[\int_{\beta^{-1}(w)}^{\beta^{-1}(w+\gamma)} \theta f(\theta,\gamma) d\theta\right] d\gamma}{A_1+A_2} = \frac{A_1'+A_2'}{A_1+A_2}$$

where $A_1' = \int_0^{+\infty}\left[\int_0^{\beta^{-1}(w)} \theta f(\theta,\gamma)\, d\theta\right] d\gamma$ and $A_2' = \int_0^{+\infty}\left[\int_{\beta^{-1}(w)}^{\beta^{-1}(w+\gamma)} \theta f(\theta,\gamma)\, d\theta\right] d\gamma$. We now prove that $E[\theta | w_R \leq w] \geq E_B[\theta | \beta(\theta) \leq w]\ \forall w$, or that

$$\frac{A_1' + A_2'}{A_1 + A_2} \geq \frac{A_1'}{A_1},$$

which simplifies as follows

$$\frac{A_2'}{A_2} \geq \frac{A_1'}{A_1}. \tag{10}$$

The ratio $\frac{A_1'}{A_1}$ is the expected value of θ in the interval $\left(0, \beta^{-1}(w)\right]$, while $\frac{A_2'}{A_2}$ is the expected value of θ in the interval $\left(\beta^{-1}(w), \beta^{-1}(w+\gamma)\right]$. The two expected values lie, respectively, in the two intervals that are not overlapping and then $\frac{A_1'}{A_1} \in \left(0, \beta^{-1}(w)\right]$ and $\frac{A_2'}{A_2} \in \left(\beta^{-1}(w), \beta^{-1}(w+\gamma)\right]$. Inequality (10) is thus always valid, for any given w, provided that the probabilities A_1 and A_2 are different from zero.

A.3 Equilibrium

Point (ii) of Lemma 2 has implications for market equilibrium. Because intrinsic motivation has no direct impact on the firms' output, the equilibrium wage only depends on the expected productivity of active

workers. As Lemma 2 illustrates, the average productivity of active workers is higher when workers are intrinsically motivated for the job than when they are not. This thus translates in higher salaries in sectors where workers are motivated.[32]

To see this formally, let us indicate with $\Theta(w)$ the set of active workers given the wage w offered by firms in the vocational sector and consider rational expectations on the part of firms: in equilibrium, firms correctly anticipate the average productivity of those workers who accept employment in the vocational sector.[33]

Definition 4 ***Competitive equilibrium.*** *A* competitive equilibrium *is a wage rate w^* and a set Θ^* of active workers such that:*

$$\Theta^* = \{(\theta, \gamma) : w^* + \gamma \geq \beta(\theta)\} \text{ and } w^* = E[\theta | w_R \leq w^*].$$

With Definition 4 and Lemma 2 the following result holds.

Corollary 2 ***Equilibrium wage.*** *If an equilibrium wage exists, then it is weakly higher when prospective workers enjoy a non-monetary premium reflecting their intrinsic motivation for the job.*

Hence, the model predicts that the equilibrium wage emerging in the relevant market when motivation matters (i.e. in the lemons model with intrinsic motivation) is higher than the one emerging in the relevant market when motivation does not matter (i.e. in the standard lemons model).

A.4 Proof of Lemma 3

(i) Under the assumption that (θ, γ) are jointly normal, also $w_R = \beta\theta - \gamma$ is normal. As a consequence, the conditional expectation of θ given the reservation wage w_R can be written as

$$E[\theta | w_R] = c_\theta + d_\theta w_R \tag{11}$$

where c_θ and d_θ are constants, and d_θ measures the correlation between θ and w_R.

Our objective is to understand how $E[\theta | w_R \leq w]$, as expressed in (4), changes with w. Hence we are interested in

$$\text{sign}\left(\frac{\partial}{\partial w} E[\theta | w_R \leq w]\right) = \text{sign}\left(\frac{\partial}{\partial w} \int_{w_R \leq w} E[\theta | w_R] k(w_R, w)\, dw_R\right)$$

[32] Clearly, salaries in the labor market for civil servants, politicians, teachers and health professionals (at least when the last two categories of workers are employed in the public sector) reflect market forces but are also the result of collective bargaining and lobbing activities.

[33] See Jovanovic (1982) and Mas-Colell *et al.*(1995) chapter 13, for the competitive equilibrium concept. The competitive equilibrium need not be unique.

Note that $E[\theta|w_R \leq w]$ depends on the function $E[\theta|w_R]$ given by (11). To start with, suppose that d_θ in (11) is positive, then higher levels of reservation wages imply larger productivities on average. Since higher flat wages w attract workers with higher reservation wages w_R, and since $E[\theta|w_R]$ is increasing in w_R, then the cumulative average $E[\theta|w_R \leq w]$ is increasing in w. Conversely, if $d_\theta < 0$, then $E[\theta|w_R \leq w]$ is decreasing in w. In short, we can write that

$$\text{sign}\left(\frac{\partial}{\partial w}E[\theta|w_R \leq w]\right) = \text{sign}\left(\frac{\partial}{\partial w_R}E[\theta|w_R]\right). \tag{12}$$

Now, because of the joint normality of (θ, w_R), $E[w_R|\theta]$ defined in (6) can be also expressed as follows

$$E[w_R|\theta] = c'_\theta + \kappa_\theta d_\theta \theta, \quad \text{with } \kappa_\theta > 0. \tag{13}$$

From (11) and (13) it follows that

$$\text{sign}\left(\frac{\partial}{\partial w_R}E[\theta|w_R]\right) = \text{sign}\left(\frac{\partial}{\partial \theta}E[w_R|\theta]\right). \tag{14}$$

Comparing (12) and (14), yields

$$\text{sign}\left(\frac{\partial}{\partial w}E[\theta|w_R \leq w]\right) = \text{sign}\left(\frac{\partial}{\partial \theta}E[w_R|\theta]\right)$$

which explains the statement in part (i) of the proposition.

(ii) The proof relative to the derivative of the conditional average motivation with respect to the flat compensation w, i.e. $\frac{\partial}{\partial w}E[\gamma \mid w_R \leq w]$, follows the same lines and is therefore omitted.

A.5 Proof of Proposition 1

Consider functions $E[w_R|\,\theta]$ and $E[w_R|\,\gamma]$ as expressed in (6) and (7), respectively. Using Definition 3 to compute the derivatives of $E[\gamma|\,\theta]$ and $E[\theta|\,\gamma]$, we obtain

$$\frac{d}{d\theta}E[w_R|\theta] = \beta - \frac{\partial}{\partial \theta}E[\gamma|\,\theta] = \beta - b \tag{15}$$

$$\frac{\partial}{\partial \gamma}E[w_R|\gamma] = \beta\frac{\partial}{\partial \gamma}E[\theta|\gamma] - 1 = \beta b' - 1 = \beta b\frac{\sigma_\theta^2}{\sigma_\gamma^2} - 1 \tag{16}$$

Because of independence or negative correlation between θ and γ, $b \leq 0$. (i) From (15), $\text{sign}\left(\frac{d}{d\theta}E[w_R|\theta]\right)$ $= \text{sign}(\beta - b) > 0$, $\forall\theta$ because $\beta > 0$ and $b \leq 0$. Hence, from Lemma 3, $E[\theta|\,w_R \leq w]$ is monotonically increasing in w. (ii) From (16), $\text{sign}\left(\frac{\partial}{\partial \gamma}E[w_R|\gamma]\right) = \text{sign}\left(\beta b\frac{\sigma_\theta^2}{\sigma_\gamma^2} - 1\right) < 0$, $\forall\gamma$ given that $\beta > 0$ and $b' = b\frac{\sigma_\theta^2}{\sigma_\gamma^2} \leq 0$. As a consequence of Lemma 3, $E[\theta|\,w_R \leq w]$ is monotonically decreasing in w.

A.6 Proof of Proposition 2

Notice that, under positive correlation, $b = \frac{\sigma_{\theta\gamma}}{\sigma_\theta^2} > 0$. From Lemma 3 and expression (15) we have that: (a) if $\beta < \frac{\sigma_{\theta\gamma}}{\sigma_\theta^2}$, then $\frac{d}{d\theta}E[w_R|\theta] < 0$, $\forall\theta$, which implies that $E\left[\theta|\, w_R \leq w\right]$ is monotonically decreasing in w (productivity crowding-out); (b) if, instead, $\beta > \frac{\sigma_{\theta\gamma}}{\sigma_\theta^2}$, then $\frac{d}{d\theta}E[w_R|\theta] > 0$, $\forall\theta$, which implies that $E\left[\theta|\, w_R \leq w\right]$ is monotonically increasing in w. Similarly, from Lemma 3 and expression (16) the following holds: (a) if $\beta > \frac{1}{b'} = \frac{1}{b}\frac{\sigma_\gamma^2}{\sigma_\theta^2}$, then $\frac{\partial}{\partial\gamma}E[w_R|\gamma] > 0$, $\forall\gamma$, which implies that $E\left[\gamma|\, w_R \leq w\right]$ is monotonically increasing in w (motivation crowding-in); (b) if, instead, $\beta < \frac{1}{b'} = \frac{1}{b}\frac{\sigma_\gamma^2}{\sigma_\theta^2}$, then $\frac{\partial}{\partial\gamma}E[w_R|\gamma] < 0$, $\forall\gamma$, which implies that $E\left[\gamma|\, w_R \leq w\right]$ is monotonically decreasing in w. Finally notice that $b < \frac{1}{b'} = \frac{1}{b}\frac{\sigma_\gamma^2}{\sigma_\theta^2}$ always holds. If this were not the case, i.e. if $b > \frac{1}{b}\frac{\sigma_\gamma^2}{\sigma_\theta^2}$, then we would have $\sigma_{\theta\gamma}^2 > \sigma_\theta^2\sigma_\gamma^2$, which contradicts the Cauchy-Schwarz inequality. Putting all these results together, three different selection patterns into the vocational market exist, and the statement of the Proposition holds.

A.7 Monte Carlo simulations

We consider a sample of 200,000 subjects with characteristic (θ_i, γ_i) from a bivariate Gaussian random variable such that $\theta \in [2,4]$, $\gamma \in [0,1]$, $\mu_\theta = 3$, $\mu_\gamma = 0.5$, $\sigma_\theta^2 = 0.25$ and $\sigma_\gamma^2 = 0.125$. We can reproduce all the four cases described in Propositions 1 and 2 by varying the values of β and b. Expected values of γ and θ given the wage level w are computed through Monte Carlo integration. Marginal standard errors have been chosen to keep the truncation rate of our Monte Carlo experiment lower than 0.01%.

As an example, we report the outcome of the following parametrization displayed in Figure 4. In panel A, we set $b = -0.4$ and $\beta = 0.7$. Correlation is negative and, as expected, $E[\theta|w \geq w_R]$ is monotonically increasing while $E[\gamma|w \geq w_R]$ is monotonically decreasing in w. In all other cases, correlation is positive, in particular we set $b = 0.4$. Panel B depicts the case in which $\beta = 0.3 < b$, so that productivity crowding-out realizes and both $E[\theta|w \geq w_R]$ and $E[\gamma|w \geq w_R]$ are monotonically decreasing in w. In panel C, $\beta = 0.6$ and $b < \beta < b' = \frac{1}{b}\frac{\sigma_\gamma^2}{\sigma_\theta^2}$ because $b' = \frac{1}{b}\frac{\sigma_\gamma^2}{\sigma_\theta^2} = 0.625$. Here, as in panel A, $E[\theta|w \geq w_R]$ is monotonically increasing while $E[\gamma|w \geq w_R]$ is monotonically decreasing in w. Finally, in panel D, $\beta = 0.7 > b' = \frac{1}{b}\frac{\sigma_\gamma^2}{\sigma_\theta^2}$, motivation crowding-in verifies, with $E[\theta|w \geq w_R]$ and $E[\gamma|w \geq w_R]$ being both monotonically increasing in w.

A.8 Proposition 3 and the statistical association between θ and γ

To explain how the sufficient conditions in Proposition 3 translate in terms of the degree of association between workers' characteristics, we first need the following definition.

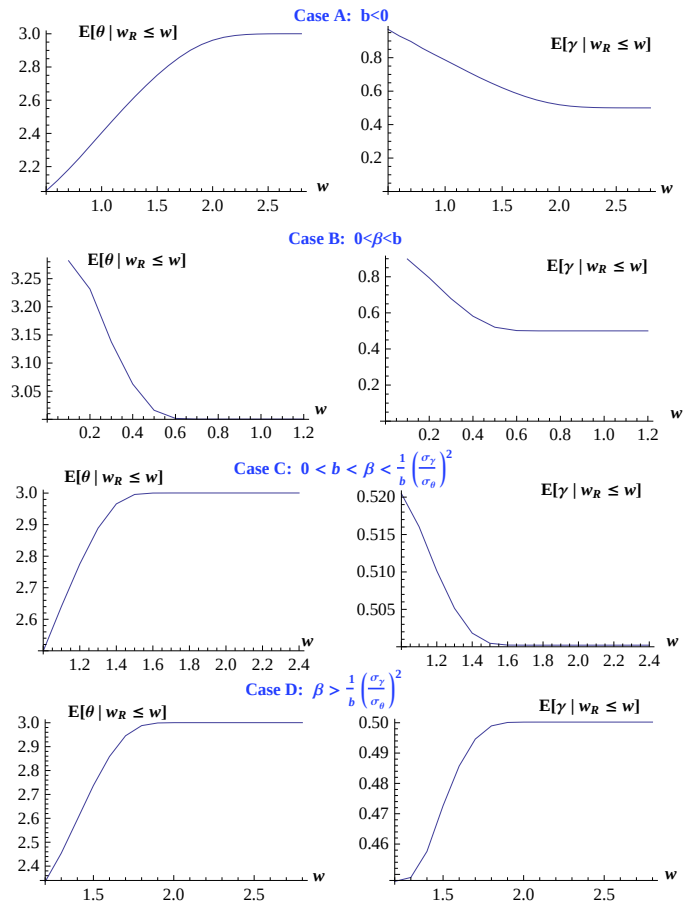

Figure 4: Monte Carlo experiment with a bivariate normal distribution: we display all the possible selection patters by changing the incentivised return to ability β and the slope of the regression line b.

Definition 5 (i) *Let $S_{B,w_0} \cup S_{A,w_0}$ denote the set of all workers entering the vocational sector given the wage rate w_0. Let S_{B,w_0} be the subset of workers whose productivity is below the population average, i.e. such that $\theta \leq \mu_\theta$; instead, let S_{A,w_0} be the subset of workers whose productivity is above the population average, i.e. $\theta \geq \mu_\theta$.*

(ii) *In the same way, let $M_{B,w_{00}} \cup M_{A,w_{00}}$ denote the set of all workers entering the vocational sector given the wage rate w_{00}. Let $M_{B,w_{00}}$ be the subset of workers whose motivation is below the population average, i.e. such that $\gamma \leq \mu_\gamma$; instead, let $M_{A,w_{00}}$ be the subset of workers whose motivation is above the population average, i.e. $\gamma \geq \mu_\gamma$.*

It immediately follows that: $E[\theta|S_{A,w_0}] > \mu_\theta$, $E[\theta|S_{B,w_0}] < \mu_\theta$ and $P(S_{B,w_0}) = 1 - P(S_{A,w_0})$, where P denotes the probability associated with each subset. Moreover, $E[\gamma|M_{A,w_{00}}] > \mu_\gamma$, $E[\gamma|M_{B,w_{00}}] < \mu_\gamma$ and

$P(M_{A,w_{00}}) = 1 - P(M_{B,w_{00}})$.

Let us consider part (i) of Proposition 3. The second part is equivalent and therefore omitted.

The two non-overlapping subsets S_{B,w_0} and S_{A,w_0} described in Definition 5 are such that $S_{B,w_0} \cup S_{A,w_0}$ is the set of workers entering the vocational sector at a given salary w_0. The law of iterated expectations allows us to write

$$E[\theta|w_R \leq w_0] = P(S_{B,w_0})E[\theta|S_{B,w_0}] + P(S_{A,w_0})E[\theta|S_{A,w_0}]$$

From the condition in Proposition 3 (a), given w_0, productivity crowding-out occurs if

$$E[\theta|w_R \leq w_0] = (1 - P(S_{A,w_0}))\, E[\theta|S_{B,w_0}] + P(S_{A,w_0})E[\theta|S_{A,w_0}] \geq \mu_\theta. \tag{17}$$

Define $\epsilon_1 = E[\theta|S_{B,w_0}] - \mu_\theta < 0$ and $\epsilon_2 = E[\theta|S_{A,w_0}] - \mu_\theta > 0$. If (17) holds, then necessarily

$$(1 - P(S_{A,w_0}))\, \epsilon_1 + P(S_{A,w_0})\epsilon_2 > 0.$$

Substituting ϵ_1 and ϵ_2 and rearranging, the previous inequality can be rewritten as

$$\frac{P(S_{A,w_0})}{1 - P(S_{A,w_0})} \geq -\frac{E[\theta|S_{B,w_0}] - \mu_\theta}{E[\theta|S_{A,w_0}] - \mu_\theta}. \tag{18}$$

Hence, Proposition 3 can be restated as follows.

Proposition 5 ***Sufficient conditions for crowding effects.*** *(i) If a wage w_0 exists such that condition (18) holds, then $E[\theta \mid w_R \leq w_0] > \mu_\theta$, and $E[\theta \mid w_R \leq w]$ is decreasing in the wage rate for a non-empty subset of wage levels. (ii) If a wage w_{00} exists such that*

$$\frac{P(M_{B,w_{00}})}{1 - P(M_{B,w_{00}})} \geq -\frac{E[\gamma|M_{A,w_{00}}] - \mu_\gamma}{E[\gamma|M_{B,w_{00}}] - \mu_\gamma}, \tag{19}$$

then $E[\gamma|w_R \leq w_{00}] < \mu_\gamma$, and $E[\gamma|w_R \leq w]$ is increasing in the wage rate for a non-empty subset of wage levels.

The left-hand side of inequality (18) is always positive and unbounded, whereas the quantity on the right-hand side is positive and finite as long as $\mu_\theta < \infty$. So, a salary w_0 that satisfies the condition may indeed exist. Similarly, the left-hand side of inequality (19) is always positive and unbounded, whereas the quantity on the right-hand side is positive and finite as long as $\mu_\gamma < \infty$. So, again, a wage w_{00} that satisfies the condition may well exist.

Inequality (18) states that, for a given wage rate w_0, productivity crowding-out occurs if: (a) the probability of observing highly productive workers $P(S_{A,w_0})$ is greater than the probability of observing workers with below-average productivity, $1 - P(S_{A,w_0})$; (b) the ratio of these two probabilities is greater

than the ratio of the differences between the two conditional means, $E[\theta|S_{B,w_0}]$ and $E[\theta|S_{A,w_0}]$ respectively, and the marginal mean. Thus, not only should the probability $P(S_{A,w_0})$ be large enough to cause $E[\theta|w_R \leq w]$ to be decreasing in wage for some $w > w_0$, but the average productivity in S_{A,w_0} should be sufficiently greater than μ_θ, while the average productivity in S_{B,w_0} has to be close enough to μ_θ. Condition (19) follows the same logic and requires that a sufficiently large mass of workers who enter the vocational sector be characterized by an average motivation sufficiently lower than the population average μ_γ.

In addition, note that Condition (18) requires a positive association between θ and γ conditional on w_0. Similarly, Condition (19) asks for a positive association between θ and γ conditional on w_{00}. In turn, both conditions are consistent with the hypothesis of a positive association between θ and γ in the population of potential candidates. In fact, Condition (18) requires a large mass in the upper-right part of the set of potential workers, while Condition (19) holds if a large mass is associated with the bottom-left part of the set of potential workers.

A.9 Appendix on Subsection 4.3: Confirming our theory with data

Table 1: Expected ability (θ) and motivation (γ) for different wages. Estimates have been obtained through OLS with robust standard errors by regressing ability or motivation with respect to the dummy variables that label each of the four groups.

	E[θ|w]			E[γ|w]		
	Coeff.	Std.Err	p-value	Coeff.	Std.Err	p-value
Accept/high salary	75.9459	2.3839	0.0000	620.297	62.3971	0.0000
Accept/low salary	78.7083	3.1629	0.0000	729.208	107.863	0.0000
Refuse/high salary	83.3621	1.4193	0.0000	582.414	69.3404	0.0000
Refuse/low salary	81.3529	1.2736	0.0000	421.706	53.1331	0.0000