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A Computational Model for Pragmatic Oddity

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Abstract. We introduce a computational model based on Deontic Defeasible Logic to handle the issue of Pragmatic Oddity. The key idea is that a conjunctive obligation is allowed only when each individual obligation is independent of the violation of the other obligations. The solution makes essential use of the constructive proof theory of the logic.

Keywords. Pragmatic Oddity, Defeasible Deontic Logic

1. Introduction

The problem of Pragmatic Oddity, one of the issues related to the formal treatment of the so called contrary-to-duty obligations, introduced by Prakken and Sergot [10], is illustrated by the scenario that when you make a promise, you have to keep. But if you do not, then you have to apologise. The oddity is that when you fail to keep your promise, you have the obligation to keep the promise and the obligation to apologise. In our view, what is odd, is not that the two obligations are in force at the same time, but that if one admits for form a conjunctive obligation from the two individual obligations then we get an obligation that is impossible to comply with. In the scenario, when the promise is broken, we have the conjunctive obligation of keeping the promise and to apologise from not having kept the promise. The Pragmatic Oddity problem arises when we have a conjunctive obligation, i.e., $\mathsf{O}(a \wedge b)$ derived from the two individual obligations ($\mathsf{O}a$ and $\mathsf{O}b$) where one of the conjuncts is contrary-to-duty obligations triggered by the violation of the other individual obligation, for example when $\neg a$ entails that $\mathsf{O}b$ is in force.

Most of the work on Pragmatic Oddity (e.g., [10, 3]) focuses on the issue of how to distinguish the mechanisms leading to the derivation of the two individual obligations, and create different classes of obligations. Consequently, the solution to the Pragmatic Oddity problem is to prevent the conjunction when the obligations are from different classes. Accordingly, if the problem is to prevent to have a conjunctive obligation in force when the individual obligations are in force themselves, the simplest solution is to have a deontic logic that does not support the aggregation axiom[2]:

$$(\mathsf{O}a \wedge \mathsf{O}b) \rightarrow \mathsf{O}(a \wedge b)$$

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[2] See, among others, [4].

However, a less drastic solution, advocated by Parent and van der Torre [8, 9], is to restrict the aggregation axiom to independent obligations (meaning that one obligation should not depend on the violation of the other obligation).

We are going to take Parent and van der Torre's suggestion and propose a simple mechanism in Defeasible Deontic Logic to guard the derivation of conjunctive obligations. The mechanism guarantees that the obligations in a conjunctive obligation are independent of the violations of the individual obligations. The mechanism is founded on the proof theory of the logic.

2. Defeasible Deontic Logic

Defeasible Deontic Logic [5] (DDL) is a sceptical computationally oriented rule-based formalism designed for the representation of norms. The logic extends Defeasible Logic [1] with deontic operators to model obligations and (different types of) permissions and provides an integration with the logic of violation proposed in [7]. The logic is based on a constructive proof theory that allows for full traceability of the conclusions In the rest of this section we are going to show how the proof theory can be use to propose a simple and (arguably) elegant treatment of the issue of Pragmatic Oddity. To this aim, here, we restrict ourselves to the fragment of DDL that excludes permission and permissive rules, since they do not affect the way we handle Pragmatic Oddity: Definition 10 describing the mechanisms for Pragmatic Oddity, is independent from any issue related to permission, and can be used directly in the full version of the logic. Accordingly, We consider a logic whose language is defined as follows.

Definition 1. Let PROP be a set of propositional atoms, O the modal operator for obligation. The set Lit = PROP $\cup \{\neg p \mid p \in \text{PROP}\}$ denotes the set of *literals*. The *complement* of a literal q is denoted by $\sim q$; if q is a positive literal p, then $\sim q$ is $\neg p$, and if q is a negative literal $\neg p$, then $\sim q$ is p. The set of *deontic literals* is DLit $= \{\mathsf{O}l, \neg\mathsf{O}l \mid l \in \text{Lit}\}$. If $c_1, \dots, c_n \in$ Lit, then $\mathsf{O}(c_1 \wedge \dots \wedge c_n)$ is a *conjunctive obligation*.

We introduce the compensation operator $\otimes$. This operator builds chains of compensation called $\otimes$-expressions, where an $\otimes$-expression is a sequence of one or more literals concatenated by the $\otimes$ operator. In addition we stipulate that $\otimes$ obeys the following property (duplication and contraction on the right):

$$\bigotimes_{i=1}^{n} a_i = \Big(\bigotimes_{i=1}^{k-1} a_i\Big) \otimes \Big(\bigotimes_{i=k+1}^{n} a_i\Big)$$

where there exists j such that $a_j = a_k$ and $j < k$.

Given an $\otimes$-expression A, the *length* of A is the number of literals in it. Given an $\otimes$-expression $A \otimes b \otimes C$ (where A and C can be empty), the *index* of b is the length of $A \otimes b$. We also say that b appears at index n in $A \otimes b$ if the length of $A \otimes b$ is n.

The meaning of a compensation chain $c_1 \otimes c_2 \otimes \dots \otimes c_n$ is that $\mathsf{O}c_1$ is the primary obligation, and when violated (i.e., $\neg c_1$ holds), then $\mathsf{O}c_2$ is in force and it compensates for the violation of the obligation of c_1. Moreover, when $\mathsf{O}c_2$ is violated, then $\mathsf{O}c_3$ is in force, and so on until we reach the end of the chain when a violation of the last element is a non-compensable violation where the norm corresponding to the rule in which the chain appears is not complied with.

We adopt the standard DL definitions of *strict rules*, *defeasible rules*, and *defeaters* [1]. However, for the sake of simplicity, and to better focus on the non-monotonic aspects that DDL offers, in the remainder, we use only defeasible rules and defeaters.

Definition 2. Let Lab be a set of arbitrary labels. Every rule is of the type $r: A(r) \hookrightarrow C(r)$ where $r \in$ Lab is the name of the rule; $A(r) = \{a_1, \ldots, a_n\}$, the *antecedent* (or *body*) of the rule, is the set of the premises of the rule (alternatively, it can be understood as the conjunction of all the elements in it). Each a_i is either a literal, a deontic literal or a conjunctive obligation; $\hookrightarrow \in \{\Rightarrow, \Rightarrow_{\mathsf{O}}, \leadsto, \leadsto_{\mathsf{O}}\}$ denotes the type of the rule. If $\hookrightarrow$ is $\Rightarrow$, the rule is a *defeasible rule*, while if $\hookrightarrow$ is $\leadsto$, the rule is a *defeater*. Rules without the subscript O are constitutive rules, while rules with such a subscript are prescriptive rules, and in case the rule is defeasible, the conclusion derived from the rule is an obligation. $C(r)$ is the *consequent* (or *head*) of the rule is a single literal for defeaters and constitutive rules, and an $\otimes$-expressions for prescriptive defeasible rules.

Given a set of rules R, we use the following abbreviations for specific subsets of rules: R_{def} denotes the set of all defeaters in the set R; $R[q,n]$ is the set of rules where q appears at index n in the consequent. The set of (defeasible) rules where q appears at any index n is denoted by $R[q]$; R^{O} denotes the set of all rules in R with O as their subscript. $R^{\mathsf{O}}[q,n]$ is the set of (defeasible) prescriptive rules where q appears at index n. The set of (defeasible) prescriptive rules where q appears at any index n is denoted by $R^{\mathsf{O}}[q]$;

Definition 3. A *Defeasible Theory* is a structure $D = (F, R, >)$, where F, the set of facts, is a set of literals and modal literals, R is a set of rules and $>$, the superiority relation, is a binary relation over R.

A theory corresponds to a normative system, i.e., a set of norms, where every norm is modelled by some rules. The superiority relation is used for conflicting rules, i.e., rules whose conclusions are complementary literals, in case both rules fire. Namely, the superiority just determines the relative strength between two rules.

Definition 4. A *proof* P in a defeasible theory D is a linear sequence $P(1) \ldots P(n)$ of *tagged literals* in the form of $+\partial$, $-\partial$, $+\partial_{\mathsf{O}} q$ and $-\partial_{\mathsf{O}} q$, where $P(1) \ldots P(n)$ satisfy the proof conditions given in Definitions 8–10.

The tagged literal $+\partial q$ means that q is *defeasibly provable* as an institutional statement, or in other terms, that q holds in the normative system encoded by the theory. The tagged literal $-\partial q$ means that q is *defeasibly refuted* by the normative system. Similarly, the tagged literal $+\partial_{\mathsf{O}} q$ means that q is *defeasibly provable* in D as an obligation, while $-\partial_{\mathsf{O}} q$ means that q is *defeasibly refuted* as an obligation. The initial part of length i of a proof P is denoted by $P(1..i)$.

A rule is *applicable* for a literal q if q occurs in the head of the rule have already been proved with the appropriate mode. On the other hand, a rule is *discarded* if at least one of the literals in the antecedent has not been proved. However, as literal q might not appear as the first element in an $\otimes$-expression in the head of the rule, some additional conditions on the consequent of rules must be satisfied. Defining when a rule is applicable or discarded is essential to characterise the notion of provability for constitutive rules and then for obligations ($\pm\partial_{\mathsf{O}}$).

Definition 5. A rule $r \in R[q,j]$ is *body-applicable* iff for all $a_i \in A(r)$:
1. if $a_i = \mathsf{O}l$ then $+\partial_{\mathsf{O}} l \in P(1..n)$;
2. if $a_i = \neg\mathsf{O}l$ then $-\partial_{\mathsf{O}} l \in P(1..n)$;
3. if $a_i = \mathsf{O}(c_1 \wedge \cdots \wedge c_m)$ then $+\partial_{\mathsf{O}} c_1 \wedge \cdots \wedge c_m \in P(1..n)$;

4. if $a_i = l \in \text{Lit}$ then $+\partial l \in P(1..n)$.

A rule $r \in R[q, j]$ is *body-discarded* iff $\exists a_i \in A(r)$ such that

1. if $a_i = \mathsf{O}l$ then $-\partial_{\mathsf{O}} l \in P(1..n)$;
2. if $a_i = \neg\mathsf{O}l$ then $+\partial_{\mathsf{O}} l \in P(1..n)$;
3. if $a_i = \mathsf{O}(c_1 \wedge \cdots \wedge c_m)$ then $-\partial_{\mathsf{O}} c_1 \wedge \cdots \wedge c_m \in P(1..n)$;
4. if $a_i = l \in \text{Lit}$ then $-\partial l \in P(1..n)$.

Definition 6. A rule $r \in R^{\mathsf{O}}[q, j]$ such that $C(r) = c_1 \otimes \cdots \otimes c_n$ is *applicable* for literal q at index j, with $1 \leq j < n$, in the condition for $\pm\partial_{\mathsf{O}}$ iff

1. r is body-applicable; and
2. for all $c_k \in C(r)$, $1 \leq k < j$, $+\partial_{\mathsf{O}} c_k \in P(1..n)$ and $+\partial{\sim}c_k \in P(1..n)$.

Conditions (1) represents the requirements on the antecedent stated in Definition 5; condition (2) on the head of the rule states that each element c_k prior to q must be derived as an obligation, and a violation of such obligation has occurred.

Definition 7. A rule $r \in R[q, j]$ such that $C(r) = c_1 \otimes \cdots \otimes c_n$ is *discarded* for literal q at index j, with $1 \leq j \leq n$ in the condition for $\pm\partial_{\mathsf{O}}$

1. r is body–discarded; or
2. there exists $c_k \in C(r)$, $1 \leq k < l$, such that either $-\partial_{\mathsf{O}} c_k \in P(1..n)$ or $+\partial c_k \in P(1..n)$.

In this case, condition (2) ensures that an obligation prior to q in the chain is not in force or has already been fulfilled (thus, no reparation is required).

For space reasons we only provide the proof conditions for the positive tags. The definitions of the negative tags can be obtained from the definition of the corresponding positive tag by apply the principle of strong negation (that transform the Boolean operators and quantifiers in their dual, and swapping "applicable" and "discarded" [2, 6]. We now introduce the proof conditions for ∂ and ∂_{O}.

Definition 8 (Defeasible provability for an institutional statement)**.**
$+\partial$: If $P(n+1) = +\partial q$ then
(1) $q \in F$ or
  (2.1) ${\sim}q \notin F$ and
  (2.2) $\exists r \in R[q]$ such that r is applicable, and
  (2.3) $\forall s \in R[{\sim}q]$, either
    (2.3.1) s is discarded, or either
    (2.3.2) $\exists t \in R[q]$ such that t is applicable and $t > s$.

The proof conditions for $\pm\partial$ are the standard conditions in defeasible logic, see [1] for the full explanations.

Definition 9 (Defeasible provability for an obligation)**.**
$+\partial_{\mathsf{O}}$: If $P(n+1) = +\partial_{\mathsf{O}} q$ then
(1) $\mathsf{O}q \in F$ or
  (2.1) $\mathsf{O}{\sim}q \notin F$ and $\neg\mathsf{O}q \notin F$ and
  (2.2) $\exists r \in R^{\mathsf{O}}[q, i]$ such that r is applicable for q, and
  (2.3) $\forall s \in R^{\mathsf{O}}[{\sim}q, j]$, either
    (2.3.1) s is discarded, or either
    (2.3.2) $s \in R^{\mathsf{O}}$ and $\exists t \in R^{\mathsf{O}}[q, k]$ such that t is applicable for q and $t > s$.

To show that q is defeasibly provable as an obligation, there are two ways: (1) the obligation of q is a fact, or (2) q must be derived by the rules of the theory. In the second case, three conditions must hold: (2.1) q does not appear as not obligatory as a fact, and $\sim q$ is not provable as an obligation using the set of deontic facts at hand; (2.2) there must be a rule introducing the obligation for q which can apply; (2.3) every rule s for $\sim q$ is either discarded or defeated by a stronger rule for q.

We are now ready to provide the proof condition under which a conjunctive obligation can be derived. The condition essentially combines two aspects: the first that a conjunction holds when all the conjuncts hold (individually). The second aspect is to ensure that the derivation of one of the individual obligations does not depend on the violation of the other conjunct. To achieve this, we determine the line of the proof when the obligation appears, and then we check that the negation of the other elements of the conjunction does not occur in the previous derivation steps.

Definition 10 (Defeasible provability for a conjunctive obligation)**.**
If $P(n+1) = +\partial_{\mathsf{O}} c_1 \wedge \cdots \wedge c_m$, then
$\forall c_i$, $1 \leq i \leq m$,
(1) $+\partial_{\mathsf{O}} c_i \in P(1..n)$ and
(2) if $P(k) = +\partial_{\mathsf{O}} c_1 \wedge \cdots \wedge c_m$, $k \leq n$, then $\forall c_j$, $1 \leq j \leq m$, $c_j \neq c_i$, $+\partial{\sim}c_j \notin P(1..k)$.

Again, the proof condition to refute a conjunctive obligation is obtained by strong negation from the condition to defeasibly derive a conjunctive obligation.

In what follows we use $\cdots \Rightarrow c$ to refer to an applicable rule for c where we assume that the elements are not related (directly or indirectly) to the other literals used in the examples.

Compensatory Obligations The first case we want to discuss is when the conjunctive obligation corresponding to the Pragmatic Oddity has as conjuncts an obligation and its compensation. This scenario is illustrated by the rule:

$$\cdots \Rightarrow_{\mathsf{O}} a \otimes b$$

In this case, it is clear that we cannot derive the conjunctive obligation of a and b, since the proof condition that allows us to derive $+\partial_{\mathsf{O}} b$ explicitly requires that $+\partial{\sim}a$ has been already derived (condition 2 of Definition 6). In this case, it is impossible to have the obligation of b without the violation of the obligation of a.

Contrary-to-duty The second case is when we have a CTD. The classical representation of a CTD is given by the following two rules:

$$\cdots \Rightarrow_{\mathsf{O}} a \qquad\qquad \neg a \Rightarrow_{\mathsf{O}} b$$

In this case, it is possible to have situations when the obligation of b is in force without having a violation of the obligation of a, namely, when a is not obligatory. However, as soon as we have $\mathsf{O}a$, we need to derive $\neg a$ to trigger the derivation of $\mathsf{O}b$ (Definition 5).

Pragmatic Oddity via Intermediate Concepts The situations in the previous two cases can be easily detected by a simple inspection of the rules involved; there could be more complicated cases. Specifically, when the second conjunct does not immediately depends on the first conjunct, but it depends through a reasoning chain. The simplest structure for this case is illustrated by the following three rules:

$$\cdots \Rightarrow_{\mathsf{O}} a \qquad \neg a \Rightarrow b \qquad b \Rightarrow_{\mathsf{O}} c$$

To derive $\mathsf{O}c$, we need to prove b. To prove b we require that $\neg a$ has already been proved.

Pragmatic Un-pragmatic Oddity What about when there are multiple norms both prescribing the contrary-to-duty obligation, and at least one of the norms is not related to the violation of the primary norm?

$$r_1\colon \cdots \Rightarrow_{\mathsf{O}} a \otimes b \qquad r_2\colon \cdots \Rightarrow_{\mathsf{O}} b \qquad \neg a$$

In this situation you can have the following two proofs:

(1)	$+\partial\neg a$	fact
(2)	$+\partial_{\mathsf{O}} a$	from r_1
(2)	$+\partial_{\mathsf{O}} b$	from r_1 and (1) and (2)

(1)	$+\partial_{\mathsf{O}} a$	from r_1
(2)	$+\partial_{\mathsf{O}} b$	from r_2
(3)	$+\partial\neg a$	fact
(4)	$+\partial_{\mathsf{O}} a \wedge b$	from (1) and (2)

In the proof on the left $\mathsf{O}b$ ($+\partial_{\mathsf{O}} b$) depends on the violation of the primary obligation of r_1. In this case, we cannot derive the conjunctive obligation of a and b. However, in the other proof, that demonstrates the independence of $\mathsf{O}b$ from $\neg a$, given that the derivation of $\neg a$ occurs in a line after the line where $+\partial_{\mathsf{O}} b$ is derived.

3. Summary

We have proposed an extension of Defeasible Deontic Logic able to handle the so called Pragmatic Oddity paradox. The mechanism we used to achieve this result was to provide a schema that allows us to give a guard to the derivation of conjunctive obligations ensuring that each individual obligation does not depend on the violation of the other obligation. The mechanism is given by the proof theory of defeasible logic. The next steps are (1) to study the complexity of the approach and to verify that the logic obtained is still computationally feasible (a prima facie analysis, based on the structure of the proof conditions for conjunctive obligations, seems to suggest the complexity to be quadratic and then still feasible, and mostly practical for real life applications, where it is unlikely to have many conjunction obligations, and they have a small number of conjuncts); (2) to devise efficient algorithms to implement the novel proof conditions.

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