Legislative Dialogues with Incomplete Information

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Abstract. This paper extends previous work by presenting a framework for modelling legislative deliberation in the form of dialogues with incomplete information. Roughly, in such legislative dialogues coalitions are initially equipped with different theories which constitute their private knowledge. Under this assumption they can dynamically change and propose new legislation associated with different utility functions.

Keywords. Legislation, Theory revision, Argumentation games, Dialogues

1. Introduction

This paper shows how to formally model legislative deliberation involving coalitions which express public interests. In this sense, it offers a conceptual and technical machinery suitable for designing new decision-support tools for e-Democracy. The contribution follows the general methodology of Governatori *et al.* [2] and extends [5]'s analysis to cover the case of deliberation with incomplete information.

As done with [5], we assume that the legislative procedure can be analysed into two different components: deliberation—the preparatory process of legislation, which runs in the form of a dialogue involving coalitions of agents—and voting (for a critique of this distinction, see [9]). Informally, the idea of legislative dialogue in [5] was the following:

- given an initial theory $\mathscr{T}_0$—intuitively corresponding to the current legislative corpus or a part of it—coalitions propose in a dialogue the legislative theory that amends $\mathscr{T}_0$ and that they would prefer;
- each theory is associated with an utility that measures the impact of the proposed changes given the utility of $\mathscr{T}_0$; the intended reading could be, for example, in terms of the consequence for the society if all agents would conform to such norms (as suggested by rule utilitarianism [7]);
- coalitions deliberate in a different way depending on which of the above theories are employed to compute the utility;
- we may have more rounds in which coalitions amend theories proposed earlier;
- the process does not require that coalitions are fixed during the debate.

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Several rationality criteria can be introduced to guide the legislative dialogue and the amendments proposed by coalitions [2,5]. For the sake of simplicity, in this paper we only consider one type of utility maximisation among those proposed in [5]. The contribution of this paper goes far beyond [5]'s framework and shows how legislative dialogues work when we abandon the simplistic idea that coalitions in legislative dialogues have complete information, i.e., that the structure of the dialogue (typically, the set of all possible arguments) is common knowledge among the coalitions. This is clearly an oversimplification, as in many real-life contexts players in legislation do not know the entire structure of the argumentation game: in fact, each of them does not know what arguments its opponent will employ and thus takes part in the dialogue in a strategic way. While this thesis was previously defended for legal disputes (see [3]), the point has not yet been extensively analysed for modelling legislative dialogues.

Another contribution is an investigation on how the assumption of incomplete information interplays with the fact that coalitions search to express a majority within the set Ag of agents forming them. This is a very complex research issue, which we address here with some basic and preliminary remarks.

This paper is methodologically aligned with some general approaches developed in law and economics. In particular, we were inspired by the so-called Political Economy School [8], which is based on the following principles:

- private individuals respond to legal rules in an economic fashion;
- private individuals have predominantly self-interested preferences;
- the influence of legal rules is mediated the rational calculus of agents to maximise their preferences;
- public officials are also self-interested[2];
- legislation can be viewed as the product of interest group politics; the problem is then to form coalitions among interests.

While there is a large literature using argumentation for modelling joint deliberation among agents (see [1]), to the best of our knowledge no systematic investigation has been developed combining means-ends rationality principles, theory revision in the law and formal dialogues. The proposal of Shapiro and Talmon [9] is a recent exception, which shares with us the idea that the legislative process proceeds in rounds of deliberation focused on editing a legal text, but the authors do not consider utility criteria guiding the procedure; on the contrary, they analyse voting outcomes—which we do not discuss here—upon a range of conditions, including reaching consensus, a Condorcet-winner, a time limit, or a stalemate. More specifically, we are not aware of any work that combined approaches like the above with the assumption of dialogues with incomplete information.

The layout of the paper is the following. Section 2 recalls basic concepts introduced in [5]. Section 3 shows how legislative dialogues with incomplete information work. Section 4 offers some remarks on how majority dynamics of coalitions interplay with information asymmetries in legislation. Some conclusions end the paper.

[2]We should notice that this assumption does not necessarily mean that public officials work for their direct real benefits. Rather, we have to assume that they are faithful representatives of different social interests coming from groups of private individuals.

2. Background

In this section we recall basic concepts introduced in [5].

2.1. Building Blocks

Let us first give a basic language setting. A literal is a propositional atom or the negation of a propositional atom. Given a literal ϕ, its complementary literal is a literal, denoted as $\sim\phi$, such that if ϕ is an atom p then $\sim\phi$ is its negation $\neg p$, and if ϕ is $\neg q$ then $\sim\phi$ is q. If $Prop$ is a set of propositional atoms then $Lit = Prop \cup \{\neg p \mid p \in Prop\}$ is a set of literals. Rules have the form $\psi_1, \ldots, \psi_n \Rightarrow \phi$ $(0 \leq n)$, $\psi_1, \ldots, \psi_n, \phi \in Lit$. The set of all rules from this language is denoted by Rul.

A corpus of legislative provisions in a given legal system can be defined as a set of legislative rules equipped with priority criteria to rank such rules and solve possible conflicts between them:

Definition 1 (Legislative theory)**.** *A **legislative theory** is a tuple $\mathscr{T} = \langle \mathscr{R}, \succ \rangle$ where $\mathscr{R}$ is a set of rules, and $\succ \subseteq \mathscr{R} \times \mathscr{R}$ is a superiority relation over the rules.*

The legislative deliberation process involves a legislative body of lawmakers (such as the members of a parliament), which we generically call legislative agents, in short *agents*. During the deliberation process, agents can dynamically form coalitions. Typically, at the beginning of the deliberation, coalitions correspond to political-party groups in the legislative body.

Definition 2 (Legislative coalition)**.** *Let Ag be a finite set of agents. A **legislative coalition** in Ag is a subset of agents in Ag. The set 2^{Ag} of all coalitions is denoted by $\mathscr{C}$.*

For brevity we will often speak of coalitions instead of legislative coalitions.

When legislative agents, i.e., the members of the legislative body, argue about theories to govern their own society, they form coalitions proposing theories that represent social interests corresponding to the utility resulting from such theories.

Definition 3 (Coalition social theory utility distribution)**.** *Let $\mathfrak{T}$ be a set of theories, $\mathbb{V}$ an ordered set of values (on which the social utility functions are computed), and $\mathscr{C}$ the set of all legislative coalitions. A **coalition social theory utility distribution** is a function*

$$U : \mathfrak{T} \to \prod_{0}^{|\mathscr{C}|} \mathbb{V}.$$

Given a theory $\mathscr{T}$ and n agents, the function returns a vector of $2^n + 1$ values, which define the value of the theory for each possible coalition in Ag and where the first value, conventionally, indicates the aggregated welfare for all coalitions. Thus, the overall coalitions' utility corresponds in the vector to projection $\pi_0(U(\mathscr{T}))$, while the value of the theory for any specific coalition i corresponds to the projection on the i-th element of the vector, $U_i(\mathscr{T}) = \pi_i(U(\mathscr{T}))$.[3]

[3] In the remainder, $U_i(\mathscr{T})$ denotes the utility of any coalition $i \in \mathscr{C}$. Also, we abuse notation and write $U_{\mathscr{C}}(\mathscr{T})$ to denote the overall coalitions' utility, i.e., $U_j(\mathscr{T})$ where $j = \bigcup_{k \in \mathscr{C}} k$. Accordingly, the overall coalitions' utility corresponds in the vector to projection $\pi_0(U(\mathscr{T}))$.

In line with ideas developed, e.g., by rule utilitarianism, we can determine what is the value of a theory (for each coalition, in our case, and based on the context in which the theory is used) with respect to some inference mechanism [6]. In particular, an approach to articulate the way in which utility springs from any theory $\mathscr{T}$ can be based on the utility of conclusions that follow from arguing on $\mathscr{T}$.

For each literal l in a set *Lit* of literals and given a (possibly different) set of literals $\{l_1,\ldots,l_n\}$, we can define a function λ that assigns for each coalition i in $\mathscr{C}$ an utility value, i.e., the utility that the state of affairs denoted by l brings to i in a context described by $l_1,\ldots,l_n$.

Definition 4 (Coalition literal valuation)**.** *Let $\mathscr{C}$ and $\mathbb{V}$ be, respectively, a set of coalitions and an ordered set of values. A* ***coalition literal valuation*** *is a function*

$$\lambda : \mathscr{C} \times Lit \times \mathrm{pow}(Lit) \rightarrow \mathbb{V}.$$

If $E(\mathscr{T}) = \{c_1,\ldots,c_m\}$ is the set of conclusions of a theory $\mathscr{T}$, then a coalition utility can be given by agglomerating the values of all conclusions. Following an intuition from rule utilitarianism, the agglomeration can simply correspond to the sum of individual valuations with respect to any coalition i [7]:

$$U_i(\mathscr{T}) = \sum_{l \in E(\mathscr{T})} \lambda(i,l,E(\mathscr{T})). \tag{1}$$

2.2. Objectives of the Legislative Procedure

In [5] some objectives for the legislative procedure have been also proposed. Among them, one seems of paramount importance: *legislation must produce as an output an optimal theory from the utility point of view*. This requirement can amount to different rational criteria, which include in [5] those producing overall agents' utility optimal theories, i.e., theories maximising the coalitions' utility, or (strong) 'Pareto optimal theories', i.e., theories for which no coalition can be made better off by making some coalitions worse off, or 'maximin optimal theories', i.e., theories maximising the utility of the worst off coalitions, or, finally, theories satisfying Kaldor-Hicks efficiency, i.e., theories in which any coalitions which are made better off could in theory compensate those which are made worse off and so produce a Pareto efficient outcome.

Here, we just recall one of them, i.e., coalitions' utility optimality:

Definition 5 (Coalitions' utility optimal theory)**.** *Let $\mathscr{C}$ be a set of coalitions. A theory $\mathscr{T}^*$ is a* ***coalitions' utility optimal theory*** *amongst a set of theories $\mathfrak{T}$ iff there is no theory $\mathscr{T} \in \mathfrak{T}$ such that $U_{\mathscr{C}}(\mathscr{T}) > U_{\mathscr{C}}(\mathscr{T}^*)$.*

2.3. Legislative Amendments in Dialogues

As argued in [5], a legislative dialogue is the process through which coalitions propose their normative theories with the aim to improve on the current legislative corpus of provisions. The normative system resulting from the dialogue is taken to be justified and so it is suitable for the voting stage.

Several operations can be applied to the the current legislative corpus of provisions in order to revise it. Consider the following very basic operations [4]:

Definition 6 (Theory Revision [4]). *Let $\mathscr{T} = \langle \mathscr{R}, \succ \rangle$ be a legislative theory.*

*The **contraction** of $\mathscr{T}$ with respect to some set R of rules is defined as follows:*

$$(\mathscr{T})^{-R} = \langle \mathscr{R} - R, \succ' \rangle$$

where $R \subseteq \mathscr{R}$ and $\succ' = \succ - \{(r,s) \mid r \in R \text{ or } s \in R\}$.

*The **expansion** of $\mathscr{T}$ with respect to some set R of rules is defined as follows:*

$$(\mathscr{T})^{+R} = \langle \mathscr{R} \cup R, \succ' \rangle$$

where $\succ' = \succ \cup \{(r,s) \mid r \in R, s \in \mathscr{R} \text{ and } C(s) = \neg C(r)\}$.

Definition 6 identifies the legal ways through which legislative theories can be amended: coalitions propose possible amendments in dialogues.

3. Legislative Dialogues with Incomplete Information

Let us now define the notion of legislative dialogue with incomplete information. When legislative dialogues have incomplete information, this means that, while all coalitions share some common knowledge—in addition to the current legislative corpus of provisions (which is assumed to be known by all agents)—they know different ways in which such a corpus can be revised, i.e., they are initially equipped with different additional set of rules which constitute their private knowledge, being unknown by the other parties: each coalition does not know what rules are taken to be valid by the other parties in the game for revising the corpus. *Intuitively, that different coalitions have private knowledge means that they can operate strategically in the dialogue by having ways for promoting their view in the deliberation* and achieving the best results from their viewpoint. In the following definition we assume that all coalitions share knowledge of legislative rules in the legal corpus, plus, possibly, some more legal rules that can be used to revise such corpus. For clarity reasons, we will speak only of this additional set of rules as *the common knowledge of all coalitions* in the dialogue.

Definition 7 (Legislative Dialogue with Incomplete Information). *A **legislative dialogue with incomplete information** (henceforth, dialogue) d is a sequence of triples*

$$\langle \mathscr{T}_k, \mathsf{Pr}_k, \mathsf{Com}_k \rangle_{k=0,\ldots K}$$

where each

$$\mathscr{T}_k = \langle \mathscr{R}_k, \succ_k \rangle$$

is a legislative theory, and

$$\mathsf{Pr}_k = \{R_k^{i_j} | \forall i_j \in \mathfrak{C}, R_k^{i_j} \subseteq Rul, R_k^{i_j} \cap \mathscr{R}_k = \emptyset\}$$
$$\mathsf{Com}_k = \{r | r \in Rul, r \notin \mathscr{R}_k \cup (\bigcup_{\forall i_j \in \mathfrak{C}} R_k^{i_j})\}$$

are, respectively, the private knowledge of each coalition with respect to k, and the common knowledge of all coalitions with respect to k that in not contained in $\mathscr{T}_k$.

The dialogue d is such that

- *theory* $\mathscr{T}_0 = \langle \mathscr{R}_0, \succ_0 \rangle$ *is the initial theory;*
- *for each coalition* $i_j \in \mathscr{C}$, $R_0^{i_j}$ *is the initial private knowledge of* i_j*;*
- Com_0 *is the initial common knowledge of all coalitions which is not in* $\mathscr{T}_0$*;*
- *for every triple* $\langle \mathscr{T}_k, \mathsf{Pr}_k, \mathsf{Com}_k \rangle$, $k > 0$, *there is a set of theories* $\mathfrak{T}^k = \{\mathscr{T}_{i_1}^k, \ldots, \mathscr{T}_{i_n}^k\}$ *where* $\{i_1, \ldots, i_n\} \subseteq \mathscr{C}$ *(i.e., theories individually proposed by coalitions* $i_1, \ldots, i_n$*) such that each* $\mathscr{T}_{i_j}^k$ *is either*
  - $(\mathscr{T}_{k-1})^{-R}$ $(1 \leq j \leq n)$ *for some set* $R \subseteq \mathscr{R}_{k-1}$ *of rules, or*
  - $(\mathscr{T}_{k-1})^{+R}$ $(1 \leq j \leq n)$ *for some set* $R \subseteq R_{k-1}^{i_j} \cup \mathsf{Com}_{k-1}$ *of rules, so that the private knowledge of* i_j *with respect to k is* $R_{k-1}^{i_j} \setminus \mathscr{R}_k$*;*
- *triple* $\langle \mathscr{T}_{k+1}, \mathsf{Pr}_{k+1}, \mathsf{Com}_{k+1} \rangle$ *is such that the theory* $\mathscr{T}_{k+1} = Choice(\mathfrak{T}^k)$*, where*
  - *Choice is a function that selects theory* $\mathscr{T}_{k+1}$ *out of a non-empty set* $\mathfrak{T}^k$*;*
  - $\mathsf{Com}_{k+1} = \mathsf{Com}_k \cup (\bigcup_{i_j \in \{i_1, \ldots, i_n\}} \mathscr{R}_{i_j}^k \setminus \mathscr{R}_{k+1})$*;*
- *triple* $\langle \mathscr{T}_K, \mathsf{Pr}_K, \mathsf{Com}_K \rangle$ *is terminal iff* $\mathfrak{T}^K = \emptyset$*.*

Some (but not necessarily all) coalitions start the dialogue by proposing some revisions of the initial legislative theory. At each round of the dialogue the choice function obeys certain rational criteria (such as coalitions' utility maximisation) and aims at ensuring a utility improvement with respect to previous rounds. Legislative revisions proposed by coalitions, if they implement theory expansions, may resort to coalitions' private information, thus adding new rules to coalitions' common knowledge. Notice that all new rules that are proposed by coalitions but are not used for revising the current theory become anyway common knowledge.

Definition 8 (Theories proposed in a dialogue)**.** *The set of theories* $\mathfrak{T}^d$ *proposed in a dialogue* $d = \langle \mathscr{T}_k, \mathsf{Pr}_k, \mathsf{Com}_k \rangle_{k=0,\ldots K}$ *is* $\bigcup_{k \in \{0,\ldots K\}} \mathfrak{T}^k$*.*

We can note that theory $\mathscr{T}_k$ may be included in $\mathfrak{T}^k$, possibly leading to some sort of equilibrium. However, we are not interested in computing *equilibria* as we deal with principles and not with *moves* as in standard game theoretic approaches. For this reason, we rely on dialogues and not on games, though our dialogues may be seen as *mirroring* such games.

A dialogue is sound if, and only if, the choice function is sound. We concentrate on one sound *Choice* function:

Definition 9 (Coalitions' utility maximising choice)**.** *The choice function of a dialogue* $\langle \mathscr{T}_k, \mathsf{Pr}_k, \mathsf{Com}_k \rangle_{k=0,\ldots K}$ *is a* ***coalitions' utility maximising choice function*** *iff any theory* $\mathscr{T}_k$ $(2 \leq k)$ *is a coalitions' utility optimal theory amongst the set of theories* $\mathfrak{T}^{k-1}$*.*

Example 1 (Running example)**.** *Let us consider three fixed coalitions: coalition* i_1 *representing people with high incomes because of their high salary, coalition* i_2 *representing those with high incomes because of tax evasion, and coalition* i_3 *representing those with low incomes.*

Suppose the initial theory $\mathscr{T}_0$ *comprises the following:*

$$\begin{aligned}\mathscr{R} = \{&r_1 : \mathit{UpperClass} \Rightarrow \mathit{RaiseTax},\\ &r_2 : \mathit{TaxEvader} \Rightarrow \mathit{SeverePunishment},\\ &r_3 : \mathit{LowerClass} \Rightarrow \mathit{Subsidies},\\ &r_4 : \mathit{LowerClass}, \mathit{TaxEvader} \Rightarrow \neg \mathit{Subsidies},\\ &r_5 : \mathit{TaxEvader} \Rightarrow \mathit{PoorCountry},\\ &r_6 : \quad \Rightarrow \mathit{LowerClass},\\ &r_7 : \quad \Rightarrow \mathit{TaxEvader},\\ &r_8 : \quad \Rightarrow \mathit{InItaly}\}\\ \succ = \{&\langle r_4, r_3\rangle\}\end{aligned}$$

The conclusions of $\mathscr{T}_0$ are the following:

$$\begin{aligned}E(\mathscr{T}) = \{&\mathit{SeverePunishment}, \neg \mathit{Subsidies}, \mathit{PoorCountry},\\ &\mathit{LowerClass}, \mathit{TaxEvader}, \mathit{InItaly}\}.\end{aligned}$$

We also have the following: $\mathsf{Pr}_0 = \{R_0^{i_1}, R_0^{i_2}, R_0^{i_3}\}$ *where*

$$\begin{aligned}R_0^{i_1} &= \{r_9 : \mathit{UpperClass} \Rightarrow \neg \mathit{RaiseTax}\}\\ R_0^{i_2} &= \{r_{10} : \mathit{InItaly} \Rightarrow \mathit{Subsidies}\}\\ R_0^{i_3} &= \emptyset\end{aligned}$$

Finally, $\mathsf{Com}_0 = \emptyset$.

Consider, for example, coalition i_2 and assume that the λ function is defined as follows (we omit the literals that are not logically derived):

$$\begin{aligned}&\lambda(i_2, \mathit{SeverePunishment}, E(\mathscr{T})) = -10\\ &\lambda(i_2, \neg \mathit{Subsidies}, E(\mathscr{T})) = -5\\ &\lambda(i_2, \mathit{PoorCountry}, E(\mathscr{T})) = -2\\ &\lambda(i_2, \mathit{LowerClass}, E(\mathscr{T})) = 0\\ &\lambda(i_2, \mathit{TaxEvader}, E(\mathscr{T})) = 18\\ &\lambda(i_2, \mathit{InItaly}, E(\mathscr{T})) = 0.\end{aligned}$$

Hence, the overall utility of $\mathscr{T}_0$ for i_2 is 1. *Similarly, we could assume that λ works for coalitions i_1 and i_3 such that the overall utility for the former is* 3 *and* 1 *for the latter. If the global utility is the sum of individual coalitions utility, the utility distribution for $\mathscr{T}_0$ is* $[5,3,1,1]$.

What should coalition i_2 do? Although it represents tax evaders (leading for them to a significant positive utility: 15*) and their being free-riders, which makes poor the country, only slightly impacts on them personally (*-2*), the overall utility is positive but small. Hence, coalition i_2 knows that $\mathscr{T}_0$ can be improved. This can be done, for example, by directly working on rules leading to negative utilities, i.e., rules r_2, r_4, r_5 and r_6. For*

instance, i_2 could propose to amend theory $\mathscr{T}_0$ by expanding the theory and add the following rule from i_2's private knowledge:

$$r_{10} : \mathit{InItaly} \Rightarrow \mathit{Subsidies}$$

If the underlying semantics of reasoning is defeasible reasoning under grounded semantics [5], by expansion $(\mathscr{T}_0)^{+\{r_{10}\}}$ we would block $\neg$Subsidies and the overall utility of the new theory would be 6 for i_2.

Of course, this is i_2's view but the other coalitions play in the debate and work differently. Assume that the new theory $\mathscr{T}_1$ resulting from the debate involving all coalitions goes against the interests of coalition i_2, since the final utility distribution is $U(\mathscr{T}_1) = [8,2,0,6]$ (i.e., taxes are slightly raised for upper classes, tax evasion is more severely punished, and public subsidies are raised for lower classes). If the coalitions' utility maximising choice is adopted then $\mathscr{T}_1$ is elicited.

Assume that i_1 unsuccessfully proposed to add r_9, which was discarded through the deliberation choice. Clearly, the triple

$$\langle \mathscr{T}_1, \mathsf{Pr}_1, \mathsf{Com}_1 \rangle$$

is as follows:

- $\mathscr{R}_1 = \mathscr{R}_0 \cup \{r_{10}\}$;
- $R_1^{i_2}$ *in* Pr_1 *is now* $\emptyset$;
- $\mathsf{Com}_1 = \{r_9\}$.

Notice that all results proved in [2,5] hold, too, in the case of incomplete information.

Proposition 1. *The terminal theory of a dialogue d with a coalitions' utility maximising choice function is coalitions' utility optimal amongst the set of theories $\mathfrak{T}^d$ proposed in the dialogue if for any $\mathscr{T}_k$, it holds that $\mathscr{T}_k \in \mathfrak{T}^k$.*

Definition 10 (Coalitions' utility improving theory)**.** *Let $\mathscr{C}$ a set of coalitions. A theory $\mathscr{T}^*$ is a **coalitions' utility improvement** of a theory $\mathscr{T}$ iff $U_{\mathscr{C}}(\mathscr{T}^*) > U_{\mathscr{C}}(\mathscr{T})$.*

Proposition 2. *A theory is a coalitions' utility optimal theory amongst a set of theories $\mathfrak{T}$ iff there exist no coalitions' utility improvements in $\mathfrak{T}$ of the theory.*

Proposition 3. *The terminal theory of a dialogue d with a coalitions' utility maximising choice function is coalitions' utility optimal amongst the set of theories $\mathfrak{T}^d$ proposed in the dialogue and it is a coalitions' utility improvement of the initial theory, if for any $\mathscr{T}_k$, it holds that $\mathscr{T}_k \in \mathfrak{T}^k$, and there exists a theory $\mathscr{T}_k$ which is a coalitions' utility improvement of $\mathscr{T}_{k-1}$.*

4. Majority Dynamics and Incomplete Information

So far coalitions adopt only some type of means-ends rationality. However, deliberative procedures usually assume that some other basic constraints apply to them. In particular, *coalitions naturally search to express a majority* within the set Ag of agents.

As suggested in [5], we should notice that Definition 7 does not require that coalitions are fixed in the dialogue, but simply that at each turn in the dialogue some coalitions individually propose some revised theories. Hence, if the legislative body works on the basis of the *majority principle* as applied to the agents forming the coalitions, it is obvious that such coalitions could change during the dialogue.

This means that an additional criterion for dialogues can be added.

Definition 11 (Coalitions' majority optimal choice)**.** *The choice function of a dialogue* $\langle \mathscr{T}_k, \mathsf{Pr}_k, \mathsf{Com}_k \rangle_{k=0,\ldots K}$ *is a* ***coalitions' majority optimal choice function*** *iff any theory* $\mathscr{T}_k = \mathscr{T}_k^{i_j}$ *(*$2 \leq k$*) amongst the set of theories* $\mathfrak{T}^{k-1}$ *is such that* $|i_j| > |Ag|/2$*.*

In other words, a coalitions' majority optimal choice ensures that each theory selected at each turn is proposed by a majoritarian coalition in Ag (since the size of the coalition i_j must exceed the half of the size of the set of agents). Definition 11 works with simple majority, but other requirements such as supermajority or unanimity can be easily implemented. It is easy to prove the following result also with incomplete information:

Proposition 4. *The terminal theory of a dialogue d with a coalitions' majority optimal choice is majority optimal amongst the set of theories* $\mathfrak{T}^d$ *proposed in the dialogue if for any* $\mathscr{T}_k$*, it holds that* $\mathscr{T}_k \in \mathfrak{T}^k$*.*

Of course, as done with utility maximisation (see Definition 10), we can imagine that dialogues aim at maximising majorities by reconfiguring coalitions during the debate.

Definition 12 (Majority improving theory)**.** *Let* $\mathscr{C}$ *a set of coalitions and* $i_j, i_k \in \mathscr{C}$*. A theory* $\mathscr{T}_*^{i_j}$ *is a* ***coalitions' majority improvement*** *of a theory* $\mathscr{T}^{i_k}$ *iff* $|i_j| > |i_k|$*.*

If Definition 5 applies to dialogues, coalitions improvement may impact of their private knowledge.

Let Ag be a set of agents:

$$Ag = \{ag_1, ag_2, ag_3\}.$$

All possible coalitions are trivially the following:

$$\begin{aligned} i_1 &= \{ag_1\} \\ i_2 &= \{ag_2\} \\ i_3 &= \{ag_3\} \\ i_4 &= \{ag_1, ag_2\} \\ i_5 &= \{ag_2, ag_3\} \\ i_6 &= \{ag_1, ag_3\} \\ i_7 &= \{ag_1, ag_2, ag_3\} \end{aligned}$$

Assume that in the dialogue only coalitions i_3 and i_4 propose at round 1 revisions of $\mathscr{T}_0$ and suppose that i_4 successfully revises theory $\mathscr{T}_0$ by adding rules from its private knowledge $R_0^{i_4}$, thus resulting into $\mathscr{T}_1$. Nothing prevents that some private information from $R_1^{i_3} \cup \mathsf{Com}_1$, when combined with $R_1^{i_4}$, may support a utility improvement, this time being based on the largest coalition i_7. If this happens, private information no longer exists, as all rules are common knowledge among agents and possible coalitions.

Proposition 5. *Let $\mathscr{C}$ a set of coalitions, $i_j, i_k \in \mathscr{C}$ and R any set of rules. If theory $\mathscr{T}_*^{i_j}$ is a coalitions' majority improvement of a theory $\mathscr{T}_\sharp^{i_k}$ in dialogue $\langle \mathscr{T}_k, \mathsf{Pr}_k, \mathsf{Com}_k \rangle_{k=0,\ldots K}$, where $\mathscr{T}_*^{i_j} = (\mathscr{T}_\sharp^{i_k})^{+R}$, then $\mathsf{Com}_*^{i_j} \subseteq \mathsf{Com}_\sharp^{i_k}$.*

5. Summary

In this paper we extended Governatori *et al.* [2,5]'s framework to the legal domain for modelling legislative deliberation with incomplete information. As done in [5], we assumed that the legislative procedure can be analysed into two different components: deliberation—the preparatory process of legislation, which runs in the form of a dialogue involving coalitions of agents—and voting—which was not discussed here.

The idea of legislative deliberation consists in revising the current legislative corpus or a part of it, where agents's coalitions propose in a dialogue legislative theories that amends such corpus. Each revision is associated with an utility that measures the impact of the proposed changes. Several rationality criteria can be described according to which coalitions deliberate.

In this sense, we argued that this work is methodologically aligned with some general approaches developed in law and economics. In particular, we were inspired by the so-called Political Economy School, where legislation can be viewed as the product of interest group politics and the problem is then to form coalitions among interests.

The current contribution extended this analysis by making [5]'s original framework more realistic: indeed, coalitions in the dialogue can be strategic and exploit in a convenient ways their private knowledge. Once the search of majorities is added to the framework, this integration exhibits some expected but interesting interactions with the dynamics of information asymmetries.

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