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Original Research Paper

Comparing practice-ready forecast models for weekly and monthly fluctuations of average daily traffic and enhancing accuracy by weighting methods

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HIGHLIGHTS

- Three practice-ready models by the literature have been used to predict weekly/monthly fluctuations for average daily traffic.
- They demonstrate a good combination of accuracy and ease of calibration, requiring conventional skills and analysis tools.
- The research shows possibilities for improvements by resorting to an appropriate linear combination of individual forecasts.

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ABSTRACT

Knowing daily traffic for the current year is recognized as being essential in many fields of transport analysis and practice, and short-term forecasting models offer a set of tools to meet these needs. This paper examines and compares the accuracy of three representative parametric and non-parametric prediction models, selected by the analysis of the numerous methods proposed in the literature for their good combination of forecast accuracy and ease of calibration, using real-life data on Italian motorway stretches. Non-parametric *K*-NN regression model, Gaussian maximum likelihood model and double seasonality Holt–Winters exponential smoothing model confirm their goodness to predict the weekly and monthly fluctuations of average daily traffic with varying degrees of performance, while maintaining an easy use in professional practice, i.e. requiring ordinary professional skills and conventional analysis tools. Since combining several prediction models can give, on average, more accuracy than that of the individual models, the paper compares two weighting methods of easy implementation and susceptible to a direct use, namely the widely used information entropy method and the less widespread Shapley value method. Despite being less common than the information entropy method, the Shapley value method proves to be more capable in better combining single forecasts and produces improvements in the predictions for test data. With these remarks, the paper might be of interest to traffic technicians or analysts, in various and not uncommon tasks they might find in their work.

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1. Introduction

Traffic flow data represent basic information for many aspects in traffic assessment issues, which may be related, for example, to the calibration and validation of simulation models, the design and operation of road facilities, the development and management of any information transport system (ITS).

The need for current-year traffic flow data is a problem for transport planners since such data aren't available for ongoing transport studies. In this sense short-term forecasting models offer, therefore, a set of tools to meet these needs. If in general it is important to know the current level of data and to predict its evolution, a noticeable issue when it has to do with short-term daily traffic forecasts concerns the data fluctuations that occur with time periods of varying length (Yu et al., 2016). Vehicular flow presents fluctuations during 24 h of the day, and fluctuations can also be recorded with respect to the volumes of traffic on different days of the week, or over the months that compose the solar year. The experience highlights important and very significant seasonal components, especially for motorway infrastructures, and short-term forecasts should properly consider them.

In recent years new technologies and the gradual reduction of construction and maintenance costs for traffic monitoring systems have gradually expanded data availability relating on transport infrastructures operation. Nowadays, especially in motorway systems, monitoring equipment are widely diffused along the infrastructure and even on vehicles: dual loop detectors, radar devices, systems with radio-frequency identification, video image processing systems, detectors for portable phones (with or without GPS or Wi-Fi devices) allow for constantly storing large databases of traffic time series. This provides a sufficient amount of information for traffic data mining to investigate traffic trends and seasonality and to predict their short-term evolution. At the same time, in the field of traffic analysis and forecasting, this large availability has powered a huge increase in research. Many studies proposed theoretical and practical applications, both on-line and off-line, but scholars are always looking for more efficient algorithms to analyse large amounts of data in less time.

The paper is organized as follows. Following this first section that gives an overview of the topic, a brief review of approaches in the past-to-present literature and the general criteria for models selection are reported in Section 2. Section 3 explores the main features of three selected models, taking into account either individually or in combined form as a synthetic linear weighting model. Two weighting methods are considered, namely the IE method and SV method, pointing to the most important aspects and the computational steps. The section shows, therefore, some key metrics or indexes to test the goodness of the predictions, and then describes the nature of the traffic data for the use in the real cases. Section 4 is devoted to the application of the three single forecasting models and of the two combining methods with real-life data from Italian motorway stretches. In this section we discuss the results obtained from the application of the three models, evaluating the goodness of each to predict the sample data. The analysis of the indexes shows the valence of the different methods, even with respect to the set used for the validation excluded from the original sample, clarifying also the opportunity of introducing the weighing methods in improving prediction performances. Finally, Section 5 explains the main outcomes and their usefulness in the practical applications.

2. State of the art

2.1. *Brief literature review*

The research about short-term traffic forecasting covers a period of almost 40 years. In the first part of this period, most of the research employed "classical" statistical approaches to predicting traffic; going forward over the years, data driven approaches have become the most discussed field of analysis in the literature, with a rich variety of algorithmic specifications, as effectively exposed by Vlahogianni et al. (2014). The authors reviewed the last decade of literature, starting from 2004, citing for the previous period three papers: by Vlahogianni et al. (2004), for short-term traffic forecasting literature and related conceptual and methodological issues up to 2003; by Adeli (2001) and by Van Lint and Van Hisbergen (2012), for neural network and artificial intelligence applications to short-term traffic forecasting.

Thus, a large number of studies about the short-term traffic flow forecasts is well known, extensively dealt using analytical and data driven modelling approaches. Trying to summarize the approaches used in relation to the specific topic of this paper, this section provides some essential references to the literature, in relation to the main parametric and non-parametric approaches.

Most parametric approaches relate to the analysis of time series models, especially based on autoregressive integrated moving average model (ARIMA) (Ahmed and Cook, 1979; Davis et al., 1991; Hamed et al., 1995; Lee and Fambro, 1999), even with introducing of seasonal components (SARIMA) (Cools et al., 2009; Shekhar and Williams, 2008; Szeto et al., 2009; Williams and Hoel, 2003), multivariate vector auto regressive (VAR) time series models (Chandra and Al-Deek, 2008, 2009), structural time-series model (STM) (Ghosh et al., 2009), or exponential smoothing (ES) (Li et al., 2008; Smith and Demetsky, 1997; Williams et al., 1998) and in particular by recourse to the Holt–Winters method (Castro-Neto et al., 2009; Daraghmi and Daadoo, 2015; Ghosh et al., 2005; Yu et al., 2016).

The most popular non-parametric approaches include non-parametric regression (NPR), and in particular *K*-nearest neighbour (*K*-NN) approach (Davis and Nihan, 1991; Lam et al., 2006a; Smith and Demetsky, 1997; Smith et al., 2002; Tang et al., 2003), support vector machines (SVM) (Su et al., 2007; Yao et al., 2008; Zhang and Liu, 2009; Zhang and Xie, 2008) and neural networks (NN) (Chen et al., 2001; Dia, 2001; Dougherty and Cobbett, 1997; Nagare and Bhatia, 2012; Smith and Demetsky, 1994; Vlahogianni et al., 2005; Yin et al., 2002).

These different approaches have been widely discussed in the literature, trying to find the best combination of forecast accuracy (often preferring parametric models) and ease of calibration and use in real cases (preferring, instead, non-parametric models) (Lam et al., 2006a, b). In general, a model that is always the best does not exist; on the contrary there is at each point in time a model that can be identified as the best (Timmermann, 2006). Jointly with the comparison, surely we find the theme of the combination of the models; "compare models or combine forecasts" is actually one of the 10 challenges proposed by Vlahogianni et al. (2014) about directions for further research in traffic forecasts. The combination of different models has often been taken as an attractive strategy in various fields, to have better forecasts on average than individual forecasting models and enhanced by diversification. Since various researches have demonstrated that combining several prediction models can give on average more accuracy than that of the individual models (Timmermann, 2006), sometimes the single-model approach has been exceeded and mixed or hybrid models have gradually become popular. Regarding the use of mixed or hybrid models in traffic forecasts, Tan et al. (2009) propose an aggregation based on the moving average (MA), ES, ARIMA and NN models, Zhang et al. (2011) propose a hybrid method that combines SARIMA and SVM models to take advantage of the two models, and Wang et al. (2014) combine ARIMA, Kalman filter (KF) and NN models by a Bayesian method.

It should also be considered that some recent studies have focused their attention on the impact of the periodic characteristic of traffic data, highlighting that the explanation of these periodic cycles may be an advantage for models. Zou et al. (2015) propose a hybrid prediction approach to consider the cyclical characteristics of freeway speed data collected from detectors located on a freeway segment in Minnesota. This approach decomposes speed into two different components: a periodic component for cyclical pattern over different weekdays, which is modelled by a trigonometric regression function and a residual part after removing the periodic component modelled by the space-time (ST), vector auto regressive model (VAR), and ARIMA model. The authors demonstrate that modelling the periodicity and the residual part separately can better interpret the underlining structure of the data, highlighting the advantages of the ST model. Tang et al. (2017) propose an evolving fuzzy neural network method (EFNN) to forecast travel speed based on speed data collected from remote traffic sensors located on a segment of a ring road in Beijing City and compare that with other forecasting methods, such as NN, SVM, ARIMA and VAR. EFNN model considers also a trigonometric regression function to capture the daily similarity of peaks and trough hours between days in the raw speed data. The results suggest that the prediction performances of EFNN are better than those of traditional models, with the advantages of consider the periodic pattern of data.

2.2. *Criteria and choices for models selection*

In this research we present, the literature cited above has been examined with respect to a specific need: predicting the average traffic during the seven typical days of the week in the various months of the year on the basis of past data recorded for a number of earlier years.

This is a problem that often occurs in preparing traffic studies, or in designing and developing decision support system or data mining tools in the motorway sector, when one needs to estimate the average expected traffic patterns in a typical week on the basis of an archive of past data and to predict the typical fluctuation of the average daily traffic throughout the year.

It is important to note that while the traffic fluctuation and peak-hour traffic within a day are information required by transportation agencies for the management of highway traffic operations, with particular reference to their real-time forecast, the same companies need different data for other aspects that fall within their management tasks. Among these aspects we can find, for example, the various issues related to the economic and financial planning of the activities by the concessionaire company, and the regulation of its relationship with the grantor. The reference horizons for these activities go beyond the single hour or the specific day, turning to periods that consider larger portions of the year (weeks, months, quarters and semesters), for which reliable estimates are required and, however, these forecasts can be made by estimating the average daily traffic during the chosen period.

Being aware of the high number of approaches proposed in the literature, with extremely refined theoretical and practical implications, the aim of this study is to select some methods with a simple analytical definition and ready to use, for solving the mentioned prediction problem. The purpose is to find some models easily and directly applicable by technicians and professionals (i.e., practice-ready), to solve a usual problem they meet in their normal work. The ease of implementing is a very significant issue in traffic flow forecasting models, taking into account that the more the model is simple the more its application is generalisable and usable frequently with a not too high research specialization by practitioners.

Model selection, testing and comparison are among the key themes of the research and application of forecasting models. The criteria can be multiple, always calibrated to the specific needs. However some guiding elements need to be considered as basic, such as accuracy, time and effort for model development, technical skills and expertise required, transferability and adaptability and so on (Vlahogianni et al., 2014).

In line with that and with the purpose of the work, the attention has been addressed to those methods with few requirements in the specification and calibration process, highly implementable without having an in-depth statistical training, as well as in-depth modelling/programming skills. A good compromise, such as combination of accuracy of forecasting and development readiness using ordinary professional skills and conventional analysis tools, appears in three models of the past and present literature panorama: two non-parametric methods, namely non-parametric *K*-NN regression model (NPR) and Gaussian maximum likelihood model (GML); a parametric method, namely exponential smoothing approach for time series based on double seasonality Holt–Winters model (DSHW). Finally, considering what mentioned above with regard to the potential profitability by combining multiple models, this paper calls into question a linear combination approach with two weighting methods. Even in this case, the choice fell on two weighing methods of more easy understanding and implementation, susceptible to a direct use in professional practice, namely information entropy (IE) method and Shapley value (SV) method.

In this way, this paper proposes the study and application of three prediction models, both in single and combined manner, easy to use in professional practice (even using only spreadsheets), but which still keep up high levels of accuracy in forecasting, as demonstrated through the analysis of four cases with real-life data on Italian motorway stretches characterized by different seasonal patterns. On the whole, and briefly, the three individual models and the two combining methods have been identified testing the compliance with the following selection criteria.

- proven uses in literature, with availability of tried and tested applications.
- reliable input data availability for their calibration and validation.
- few requirements in the specification and calibration process.
- highly implementability and usability without having an in-depth statistical training.
- highly implementability and usability without having an in-depth modelling/programming skills.
- good compromise between forecasting accuracy and practical usability.

3. Materials and methods

3.1. *Prediction models*

In recent years the studies about short-term traffic forecast models have increased enormously, due to the growth of available data and the needs of implementing ITSs to traffic management. Vlahogianni et al. (2004, 2014) review decades of research in short-term prediction models taking into account several issues as scope, methodologies, type of output and input data and their quality, intending to provide a logical flow for developing short-term traffic forecasting algorithms. In this highly diversified context, characterized by enormous literature production, this paper aims to offer a comparison of a number of high-potential alternative models that prove their usefulness for predicting the average traffic during the seven days of the week in the various months of the year on the basis of past data recorded for a number of earlier years with easily generalisable models, which can have a direct use in professional practice.

As highlighted in the introduction, in this section the paper explores the main features of the three models already present in the literature review and assumed as references for the paper purpose: the non-parametric regression model (NPR) and Gaussian maximum likelihood (GML), which are non-parametric type; the double seasonality Holt–Winters model (DSHW), which is a parametric exponential smoothing model for time series.

For these three models, that will be further processed jointly, in what follows the paper also describes two weighing models and the main steps for their application to the case under discussion.

3.1.1. *Non-parametric regression model*

In general, non-parametric prevision methods are based on data-driven models, allowing observations to highlight the underlying structures without requiring the explanation of the relations between inputs and outputs. The primary purpose of these methods is to identify data clusters with characteristics similar to the current state for a certain interval of prediction, and then to define the same prediction from these. In this way, it is not necessary to assume a forecasting equation expressed mathematically by a set of parameters, as it happens to the parametric approach.

The most popular non-parametric approaches include NPR and NN techniques. As NN are data-driven models based on pattern classification and recognition capabilities of the artificial-intelligence approach, NPR models make their prediction searching for the most similar case regarding the current prediction state among all observations. After decades of research, the NPR technical applications have been numerous in different fields of analysis, including transport and traffic engineering in particular using the *K*-NN approach, which predicts the forecasting value based on a similarity measure by distance functions. Davis and Nihan (1989) suggest *K*-NN approach as a candidate alternative method to Parametric approaches in short-term motorway traffic forecasting. They compare *K*-NN to simple univariate linear time series forecasts discussing the profitability of the non-parametric approach. Smith and Demetsky (1997) show the superiority of *K*-NN approach in terms of robustness and regarding different data, analysing the differences with NN and ARIMA models.

Smith et al. (2002) compare parametric ARIMA and non-parametric NPR models, pointing out that the former is more powerful than the latter, while being the ARIMA model more complicated in specifying, calculating and updating. Tang et al. (2003) introduce NPR and Gaussian maximum likelihood (GML), compared with ARIMA and NN, to estimate the daily flow and calculate the annual average daily traffic (AADT) for the current year in Hong Kong. The results of the four models are compared with the real data for validation, showing greater ease of implementation of the NPR and GML models with respect to ARIMA and NN models, which require extensive data calibration. Lam et al. (2006a) chose again an NPR model (specifically *K*-NN), for short-term traffic forecasting in Hong Kong, showing the comparison with the already mentioned GML that we will discuss below in this paper. Lam et al. (2006b) extend once again the analysis comparing the two non-parametric models, *K*-NN NPR and GML, with ARIMA and NN approaches, revealing that *K*-NN NPR and GML models can offer better estimations of traffic data. Resuming the studies above, the essential element of these non-parametric methods is to find in the previous data a range with similarity characteristics compared to what is being forecast. From this point of view Huang et al. (2011) generalize a method for the search and characterization of the similarities in the previous data, suggesting and combining pattern matching and forecasting algorithms.

For the purposes of this paper, we resume the NPR model proposed by Tang et al. (2003) and used by Lam et al. (2006a,b), to which we refer for a complete description of the method. In general, we can summarize that the NPR model produces its prediction as output, on the basis of a group of past states identified as similar with respect to the current (forecasting) state, which is the input for the same prediction. The similar early states, identified in the number *K*, are defined as those closest (nearest neighbor), on the basis of a certain measure of distance, with respect to the forecasting state, i.e. *K*-NN.

Going back to our case, each state that composes the database is identified by the day d of the week (Sunday = 1 to Saturday = 7), the month m (January = 1 to December = 12) and the year y (first year in database = 1 to last year in database = l), and it is characterized by a traffic value $Y_{d,m,y}$. For each state (d; m; y) in the database, the variables to be considered are:

- $Y_{d,m-1}$: traffic of the day d of month preceding m, which is m-1;
- $Y_{d,m-2}$: traffic of the day d of the month preceding m-1, which is m-2;
- $\overline{Y_{d,m}}$: traffic of the day d averaged over all the months m in the database (for various years 1 to l);
- $\overline{Y_{d,m-1}}$: traffic of the day d averaged over all the relative months m-1 in the database (for various years 1 to l).

As the aim is to forecast the average (typical) traffic in the chosen day $\bar{d}$ of the week (Sunday to Saturday) for the selected month $\bar{m}$ (January to December) for a certain year $\bar{y}$ (e.g. $\bar{y} = l + 1$), the distance in the *K*-NN model is between variables in forecasting state ($\bar{d}$; $\bar{m}$; $\bar{y}$) and variables for each past state ($\bar{d}$; m = 1, …, 12; y = 1, …, l) in the database. Choosing the expression of the Euclidean distance, it holds Eq. (1)

$$\Delta\left\{\left(\bar{d},\bar{m},\bar{y}\right),\left(\bar{d},m,y\right)\right\} = \sqrt{\left(Y_{\bar{d},m-1}-Y_{\bar{d},\bar{m}-1}\right)^2+\left(Y_{\bar{d},m-2}-Y_{\bar{d},\bar{m}-2}\right)^2+\left(\overline{Y_{\bar{d},m}}-\overline{Y_{\bar{d},\bar{m}}}\right)^2+\left(\overline{Y_{\bar{d},m-1}}-\overline{Y_{\bar{d},\bar{m}-1}}\right)^2} \quad (1)$$

Once identified a suitable value for *K*, which in this case may be equal to 3 as suggested by Tang et al. (2003) and Lam et al. (2006a, b), the $K = 3$ smallest values, Δ', Δ'' and Δ''' of the distance between the prediction state and all states of the database can be identified. So, the traffic expected for the state ($\bar{d}$; $\bar{m}$; $\bar{y}$) may be estimated by averaging.

$$\hat{Y}_{\bar{d},\bar{m},\bar{y}} = \frac{\Delta' + \Delta'' + \Delta'''}{3} \quad (2)$$

3.1.2. *Gaussian maximum likelihood model*

As stated in the literature framework for NPR method, Tang et al. (2003) introduced another non-parametric method based on the GML model for the short-term traffic forecast, also proposed by Lam et al. (2006a, b). For this paper, we resume GML model summarizing the general aspects and the computational steps of the method, but making reference to the original articles cited above for a complete description of the methodology.

Bearing in mind the terms of reference for the analysis, we can compose a traffic time series as a sequence of states identified by the average day of the week d (Sunday = 1 to Saturday = 7), the month m (January = 1 to December = 12) and the year y (first year in database = 1 to last year in database = l), each of which characterized by the traffic $Y_{d,m,y}$.

We denote that by $Y_{\bar{d},m,y}$ the consecutive observations of the daily traffic obtained for a selected day $\bar{d}$ varying the month m with $m = 1, 2, \ldots, 12$ and the year with $y = 1, 2, \ldots, l$; by $FY_{\bar{d},m,y}$ the consecutive observations of the increment of daily traffic for the selected day $\bar{d}$ of month m obtained with respect to daily traffic at day $\bar{d}$ of month m-1, i.e., $FY_{\bar{d},m,y} = Y_{\bar{d},m,y} - Y_{\bar{d},m-1,y}$, with m that varies between 1 and 12 for years y 1 to l (noted that if $m = 1$, m-1 = 12 of the previous year y-1).

By considering the data, we can define $\mu_{Y,\bar{d},\bar{y}}$ and $\sigma^2_{Y,\bar{d},\bar{y}}$ as the mean and the variance of the consecutive observations of the daily traffic $Y_{\bar{d},m,\bar{y}}$ (traffic in a selected day $\bar{d}$ and year $\bar{y}$, varying the month m), and $\mu_{FY,\bar{d},\bar{m}}$ and $\sigma^2_{FY,\bar{m},\bar{y}}$ as the mean and the variance of the consecutive observations of daily traffic increment $FY_{\bar{d},\bar{m},y}$ (traffic increment in a selected day $\bar{d}$ and month $\bar{m}$, varying the year y 1 to l).

Supposing that $Y_{d,m}$ and $FY_{d,m}$ are normally distributed, Lam et al. (2006a, b) prove the existence of a closed-form solution for the traffic estimate in a selected day $\bar{d}$ of a month $\bar{m}$ for the year $\bar{y}$ derived from the maximization of the likelihood function and taking account of both the flows and flow increments simultaneously. Referring to the articles cited above for the detailed demonstration, the traffic forecast for the day $\bar{d}$ of a month $\bar{m}$ for the year $\bar{y}$ can be calculated using the following Eq. (3).

$$\hat{Y}_{\bar{d},\bar{m},\bar{y}} = \frac{\sigma^2_{Y,\bar{d},\bar{y}-1}\left(\mu_{FY,\bar{d},\bar{m}} - Y_{\bar{d},\bar{m},\bar{y}-1}\right) + \sigma^2_{FY,\bar{d},\bar{m}}\mu_{Y,\bar{d},\bar{y}-1}}{\sigma^2_{Y,\bar{d},\bar{y}-1}\sigma^2_{FY,\bar{d},\bar{m}}} \quad (3)$$

3.1.3. *Double seasonality Holt–Winters model*

The double seasonal triple exponential smoothing, also called double seasonal Holt–Winters method (DSHW), is a version proposed by Taylor (2003) to take into account the double seasonality problem within the standard Holt–Winters (HW) seasonal ES model. Winters (1960) introduces HW model as a time series forecasting method in the presence of seasonality by extending the Holt model (Holt, 1957), which considers a double exponential smoothing for level and trend. This triple smoothing model is commonly used for modelling auto-correlated time series with seasonality using additive or multiplicative seasonal patterns in many fields of application. ES methods are widespread used in many fields, especially when an automated procedure is preferable to work with many time series (Taylor, 2008). Williams et al. (1998) introduce their usage in the analysis of traffic data, highlighting that ES models are useful to deal with seasonality in traffic time series, and propose a comparison between seasonal ARIMA and HW in traffic flow prediction.

In the last few years the already mentioned research by Taylor (2003) has resolved one of the main problems of the single-seasonality model, namely the impossibility of treating multiple seasonal phenomena, with its formulation of DSHW model. Although the author applies the extended model to the case of electricity demand, the proposed solution is also applicable in any case of double seasonal, and then in traffic time series in which this effect is relevant.

Recently Daraghmi and Daadoo (2015) use a DSHW method for a short-term traffic flow prediction to determine the status of all roads on routes for implementing dynamic route guidance as a part of advance traveller information systems in Taipei City. The authors prove the accuracy of the proposed method, which outperforms other methods achieving a lower root mean square error. Yu et al. (2016) investigate the possibility of using the DSHW method to account the double seasonality in traffic time series. They prove how DSHW model outperforms the standard Holt–Winters method and the ARIMA method in short-term traffic forecasting, allowing to take into consideration both within-day and within-week seasonal cycles.

From this considerations, the suitability of DSHW model in our case study is clear, having to deal with a double seasonality relates to an intra-week component (average days per week) with period s_1 and an intra-annual component (weeks in the months of the year) with period s_2. In particular, as in Taylor (2003) and many other applications including Daraghmi and Daadoo (2015), and Yu et al. (2016), in this case we refer to the multiplicative model, as the size of the seasonal fluctuations are not fixed but vary depending on the overall level of the traffic series.

As for the other models in Sections 3.1.1 and 3.1.2, each traffic time series consists in a sorted sequence of states t (t = 1, …, N) identified by the average day of the week d (Sunday = 1 to Saturday = 7, and then $s_1 = 7$), the month m (January = 1 to December = 12, and then $s_2 = 12 \times 7 = 84$) and the year y (first year in database = 1 to last year in database = l) with traffic $Y_{d,m,y}$. In line with this concept a time origin o for the forecasting is set, which may be the state corresponding to the last typical day of the last month of the last year (i.e. $t = N$), which is part of the database.

Once selected a reference time o for the forecasts, we can write the selected time $\bar{t}$ for the out-of-sample prediction (i.e., a selected day $\bar{d}$ of a month $\bar{m}$ for the year $\bar{y}$) as $\bar{h}$ step-ahead from origin o, and traffic forecast $\hat{Y}_{\bar{t}}$ as $\bar{h}$ step-ahead forecast from the same origin.

$$\hat{Y}_{\bar{d},\bar{m},\bar{y}} = \hat{Y}_{\bar{t}} = \hat{Y}_o\left(\bar{h}\right) = \left(S_o + \bar{h}T_o\right)W_{o-s_1+\bar{h}}M_{o-s_2+\bar{h}} \tag{4}$$

where S_o is the level (an estimate of the local mean), T_o is the trend (an estimation of the local trend, i.e., the variation between successive time points), W_o and M_o are the two seasonal components (the estimation of the deviation from the local mean for seasonality) for the weekly and monthly cycles with their related periods s_1 and s_2, whose update are made in consideration of four smoothing equations.

$$S_o = \alpha\left[Y_o/\left(W_{o-s_1}M_{o-s_2}\right)\right] + (1-\alpha)(S_{o-1} + T_{o-1}) \tag{5}$$

$$T_o = \gamma[Y_o/(S_o + S_{o-1})] + (1-\gamma)T_{o-1} \tag{6}$$

$$W_o = \delta\left[Y_o/\left(S_oM_{o-S_2}\right)\right] + (1-\delta)W_{o-s_1} \tag{7}$$

$$M_o = \omega[Y_o/(S_oW_{o-s_1})] + (1-\omega)M_{o-s_2} \tag{8}$$

where α, γ, δ and ω are the smoothing parameters, which can be estimated on the basis of the sample data (e.g. by minimizing the squared errors between predicted and measured data). Smoothing parameters close to 1 emphasize the most recent observations, while values close to 0 assign more influence on the past ones.

It is worth specifying that this model is able to provide the explanation of the periodic characteristic of annual traffic, i.e. the daily and monthly cycles, which is recognized as an advantage in modelling and forecasting traffic (Tang et al., 2017; Zou et al., 2015). This model, in fact, allows expressing in a direct way the double frequency in predicting daily traffic values through the two seasonal components W_o and M_o as the estimation of the deviation from the local mean (i.e., the level S_o) for the weekly and monthly cycles with their related periods s_1 and s_2.

It should be emphasized that the model implementation needs starting values for the level, the trend and the two seasonal components. Going backwards with respect to the origin o, the starting values must be calculated for the first s_2 observations of the time series, which form the orderly succession of the daily traffic on days d = 1, …, 7 and for months m = 1, …, 12 of the first year y = 1. This allows the definition of the updating algorithm, also making possible on-sample data forecasts even for the first s_2 observations of the series. In this case, in fact, once selected the origin o so that $d = 1$, $m = 1$ and y = 1 (i.e. $t = 1$), S_o, T_o, W_o and M_o would not be defined by the general equations.

We address first the case of short seasonality W, with period s_1, whose starting value W_{*j} for the first j = 1, …, s_2 observations of the time series are obtained by repeating s_2/s_1 times the following values W_{*z} for the first s_1 observations (z = 1, …, s_1) of the same time series.

$$W_{*z} = \frac{\overline{RW}_d}{\Sigma_d \overline{RW}_d / s_1} \quad (9)$$

$$\overline{RW}_d = \frac{\Sigma_{m,y}(RW_d)_{m,y}}{l(s_2/s_1)}$$

$$(RW_d)_{m,y} = \frac{(Y_d)_{m,y}}{\Sigma_d (Y_d)_{m,y} \big/ s_1}$$

where $d = 1, \dots, 7$; $m = 1, \dots, 12$; $y = 1, \dots, l$.

Then we address the long seasonality M with period s_2, whose starting value M_{*j} for the first s_2 ($j = 1, \dots, s_2$) observations of the time series are obtained by the following.

$$M_{*j} = \frac{\overline{RM}_{d,m}}{\sum_{d,m} \overline{RM}_{d,m} \Big/ s_2} \quad (10)$$

$$\overline{RM}_{d,m} = \frac{\sum_y (RM_{d,m})_y}{l}$$

$$(RM_{d,m})_y = \frac{(Y_{d,m})_y}{\sum_{d,m} (Y_{d,m})_y \Big/ s_2}$$

where $d = 1, \dots, 7$; $m = 1, \dots, 12$; $y = 1, \dots, l$.

For trend and level, we can estimate T_* and S_* as in Hyndman et al. (2017):

$$T_* = \frac{\sum_{j=1}^{s_2} \left(\frac{Y_j - Y_{j+s_2}}{s_2} + X_{*j} \right)}{2s_2} \quad (11)$$

$$X_{*j} = Y_j - Y_{j-1}$$

$$X_{*1} = 0$$

$$S_* = \frac{\sum_{j=1}^{2s_2} Y_j}{2s_2} - \left(s_2 + \frac{1}{2} \right) T_* \quad (12)$$

Definitely, the traffic estimate in a selected day $\overline{d}$ of a month $\overline{m}$ for the year $\overline{y}$, i.e., $\widehat{Y}_{\overline{d},\overline{m},\overline{y}}$, can be obtained using the prediction and smoothing equations declared above, considering an adequate number of $\overline{h}$ step-ahead from o. From this point of view, if the origin o of the current forecasts (and then the last state in past database) is the average Saturday ($d = 7$) in December ($m = 12$) of the last year l of the database, the forecast for the average Friday ($\overline{d} = 6$) in February ($\overline{m} = 2$) of year $\overline{y} = l + 1$ is obtained considering $\overline{h} = 13$ step-ahead from o.

3.2. *Combined prediction model*

The pioneers in the theoretical study of the combination of forecasts are Bates and Granger (1969). These authors propose techniques to obtain a combined forecast from linear combinations of two individual forecasts, whose weights are obtained from their forecast error variances. Subsequently Newbold and Granger (1974) extend the linear combination of forecast models from two to many, and then various forms of combination have been developed using more sophisticated methods, both objective (i.e., using a mathematical function) and subjective (i.e., human judgements, such as experts' opinions) (Aline and Werner, 2013).

Considering the simple linear combination model (Bates and Granger, 1969; Newbold and Granger, 1974), in presence of P prediction models the form in which a combined model can be expressed is

$$\widehat{Y}_{\overline{d},\overline{m},\overline{y}} = \sum_{i=1}^{P} \omega_i \left(\widehat{Y}_{\overline{d},\overline{m},\overline{y}} \right)_i \quad (13)$$

where ω_i is a weighting coefficient for each single model, considering that $\omega_i \geq 0$ and $\sum_i \omega_i = 1$.

Since the combined prediction model in this paper is the combination of $P = 3$ models, we can set $(\widehat{Y}_{\overline{d},\overline{m},\overline{y}})_1$ for the NPR model, $(\widehat{Y}_{\overline{d},\overline{m},\overline{y}})_2$ for the GML model and $(\widehat{Y}_{\overline{d},\overline{m},\overline{y}})_3$ for the DSHW model, getting

$$\widehat{Y}_{\overline{d},\overline{m},\overline{y}} = \omega_1 \left(\widehat{Y}_{\overline{d},\overline{m},\overline{y}} \right)_1 + \omega_2 \left(\widehat{Y}_{\overline{d},\overline{m},\overline{y}} \right)_2 + \omega_3 \left(\widehat{Y}_{\overline{d},\overline{m},\overline{y}} \right)_3 \quad (14)$$

In the formulation above, the combination of weighting coefficients would affect the accuracy and the success of the combined forecast model (Timmermann, 2006) and for this reason their right choice is an important and interesting problem.

In what follows, this paper examines two weighing methods for combining models: the widely used information entropy method (IE) and the Shapley value (SV) method, which is less widespread.

3.2.1. *Information entropy method*

Entropy is a thermodynamics concept introduced into the information theory by Shannon (1948) to measure the order and quality of information expressed by a system: a more orderly system has the lower entropy and has the greater information, and vice versa.

The concept of entropy is very used to combine forecasting methods. In this case, after choosing a certain prediction model, its weight in the combined model can be calculated measuring the coefficient of variation of an appropriate index according to entropy theory, agreeing with the fact that to a greater index variation degree corresponds a lower weight (Huang et al., 2011; Liu and Li, 2015). The major differences in applying the method derive from different choices of indexes and, from this point of view, various options are possible in real application. In this paper, we use relative forecasting error as the evaluation index, and the entropy weighing algorithm is described as follows.

Suppose the actual average daily traffic volume in a time t (to indicate a day d in a given month m during the year y) is Y_t and the forecasting value based on the three models is $\widehat{Y}_{it}$, with $i = 1, 2, 3$.

Let e_{it} denotes relative forecasting error for i-th forecasting method in t time and P_{it} is the normalized relative prediction error, regarding the entire prediction interval ($t = 1, \dots, N$), described as

$$e_{it} = \begin{cases} 1 & \text{if } \left| \left(Y_t - \widehat{Y}_{it} \right) \big/ Y_t \right| \geq 1 \\ \left| \left(Y_t - \widehat{Y}_{it} \right) \big/ Y_t \right| & \text{if } 0 \leq \left| \left(Y_t - \widehat{Y}_{it} \right) \big/ Y_t \right| < 1 \end{cases} \quad (15)$$

Table 1 – Brief description and calculation formula for five forecasting performance metrics.

Index	Calculation	Description
MAPE	$\frac{1}{N}\sum_{t=1}^{N}\left\|\frac{Y_t-\hat{Y}_t}{Y_t}\right\|$	Relative and dimensionless index, expressing the average absolute percentage deviation between pairs of measured and modelled values.
RMSE	$\sqrt{\frac{1}{N}\sum_{t=1}^{N}(Y_t-\hat{Y}_t)^2}$	Absolute index dimensionally the same as that of the variable, which expresses the standard deviation of the errors.
RMSPE	$\sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(\frac{Y_t-\hat{Y}_t}{Y_t}\right)^2}$	Relative and dimensionless index, calculated from the RMSE and normalizing errors with respect to the actual value of the variable.
UII	$\frac{\sqrt{\sum_{t=1}^{N}(\hat{Y}_t-Y_t)^2}}{\sqrt{\sum_{t=1}^{N}Y_t^2}}$	Relative and dimensionless index, which can be interpreted as the RMSE of the proposed forecasting model divided by the RMSE of a no-change model.
SSLAR	$\sum_{t=1}^{N}\left(\ln\frac{\hat{Y}_t}{Y_t}\right)^2$	Relative and dimensionless index, calculated from the square logarithm of predicted value divided by actual value.

$$P_{it} = e_{it}\Big/\sum_{t=1}^{N}e_{it} \tag{16}$$

For the i-th model the entropy value of relative forecasting error is

$$h_i = -\frac{1}{\ln(N)}\sum_{t=1}^{N}P_{it}\ln(P_{it}) \tag{17}$$

Then, for each single forecasting model we can introduce the relative weight in the combined model by IE method as

$$\omega_i = D_i\Big/\sum_{t=1}^{3}D_t \tag{18}$$

$$D_i = 1 - h_i \tag{19}$$

3.2.2. *Shapley value method*

Shapley Value calculation is a mathematical method proposed by Shapley (1953) and used to solve cooperation games with multiple players. This method allows achieving a fair and efficient allocation of team total revenue among the various members that are cooperative players.

From this point of view, the aim of limiting the total prediction error of the combined model can be considered as a cooperative game, where each of the single models is a cooperative player in the process of combined forecasting (Feng et al., 2014).

We denote by $I = \{1, 2,\ldots, P\}$ the set of P models taking part in the combined model, which in our specific case is $I = \{1, 2, 3\}$, by E the total error of the combined prediction and E_i the absolute value of error of the i-th forecasting model. If s is any subset of I, we can identify with $E(s)$ the combined error of this subset s. In general, the following relationships apply

$$E_i = \frac{1}{N}\sum_{t=1}^{N}\left|Y_t - \hat{Y}_{it}\right| \quad i = 1, 2, \ldots, P \tag{20}$$

$$E = \frac{1}{P}\sum_{i=1}^{P}E_i \tag{21}$$

where N is the number of samples and $\left|Y_t - \hat{Y}_{it}\right|$ is the prediction error of getting $\hat{Y}_{it}$ prediction by i-th forecasting method, regarding to t-th time in the prediction interval.

Shapley value model considers that

$$E_i = \sum_{s_i\in S_i}\omega(|s_i|)[E(s_i) - E(s_i - \{i\})] \tag{22}$$

$$\omega(|s_i|) = \frac{(n-|s_i|)!(|s_i|-1)!}{n!} \tag{23}$$

where s_i is a model combination set containing prediction model i, among all sets S_i containing the same original model, $|s_i|$ is the number of prediction models in the combination and s_i-{i} is a new set obtained from s_i by removing i-th model.

Each forecasting model i appear in the combined prediction model with a weight that is

$$\omega_i = \frac{1}{n-1}\frac{E-E_i}{E} \tag{24}$$

In this case, as we are dealing with three initial models (NPR, GML and DSHW) respectively numbered as 1, 2 and 3, then $I = \{1, 2, 3\}$ and all the possible subsets are $E\{1\}$, $E\{1,2\}$, $E\{1,3\}$ and $E\{1,2,3\}$, containing model 1; $E\{2\}$, $E\{1,2\}$, $E\{2,3\}$ and $E\{1,2,3\}$, containing model 2; $E\{3\}$, $E\{1,3\}$, $E\{2,3\}$ and $E\{1,2,3\}$, containing model 3. Taking as an example the model 1, it holds

$$\begin{aligned}E_1 &= \omega(|\{1\}|)[E(\{1\}) - E(\{1\}-\{1\})] + \omega(|\{1,2\}|)[E(\{1,2\}) \\ &-E(\{1,2\}-\{1\})] + \omega(|\{1,3\}|)[E(\{1,3\}) - E(\{1,3\}-\{1\})] \\ &+ \omega(|\{1,2,3\}|)[E(\{1,2,3\}) - E(\{1,2,3\}-\{1\})] \\ &= \frac{(3-1)!(1-1)!}{3!}[E(\{1\})] + \frac{(3-2)!(2-1)!}{3!}[E(\{1,2\}) - E(\{2\})] \\ &+ \frac{(3-2)!(2-1)!}{3!}[E(\{1,3\}) - E(\{3\})] \\ &+ \frac{(3-3)!(3-1)!}{3!}[E(\{1,2,3\}) - E(\{2,3\})]\end{aligned} \tag{25}$$

In a similar way E_2 and E_3 may be obtained. From them, the weights $\omega_1, \omega_2, \omega_3$ of NPR, GML and DSHW (respectively numbered as model 1, 2 and 3) are obtained for the combined forecasting model.

3.3. *Forecasting performance metrics*

To check the accuracy of the proposed algorithms in forecasting results, some evaluation metrics have been introduced using a number of error testing indexes.

Table 2 – AADT variability for different stretches and years (veh/d).

Stretch	AADT	Year				
		2012	2013	2014	2015	2016
(a)	Mean	30,458	29,932	30,367	31,141	31,464
	Stand. Dev.	4956	4971	4789	4856	4752
	Max variation	17,845	18,100	16,415	16,135	17,586
	Coef. Var.	0.163	0.166	0.158	0.156	0.151
(b)	Mean	31,832	31,640	32,165	33,567	34,398
	Stand. Dev.	6442	6882	6714	7099	6874
	Max variation	22,318	23,310	23,898	23,962	22,681
	Coef. Var.	0.202	0.218	0.209	0.211	0.200
(c)	Mean	11,940	11,730	11,801	12,106	12,357
	Stand. Dev.	5008	4845	4887	4968	5136
	Max variation	9724	9491	9607	9825	9989
	Coef. Var.	0.419	0.413	0.414	0.410	0.416
(d)	Mean	44,091	43,714	44,425	46,334	47,581
	Stand. Dev.	6464	6540	6187	6596	6304
	Max variation	15,847	15,702	15,926	15,688	15,631
	Coef. Var.	0.147	0.150	0.139	0.142	0.132

The error testing indexes that have been considered are some most used in the literature, namely: mean absolute percentage error (MAPE); root mean square error (RMSE); root mean square percentage error (RMSPE); Theil inequality II coefficient (UII) (Bliemel, 1973; Theil, 1966); sum squared log accuracy ratio (SSLAR) (Toffalis, 2015).

Taking up the notation already used, let Y_t denotes the actual value and $\hat{Y}_t$ the predicted value, with reference to the entire interval (t = 1, …, N), Table 1 shows the calculation formula and a brief description for each index. As it is known, the best model has the lowest indexes.

3.4. *Daily traffic volume collection and treatment*

Real-life data for describing average daily traffic fluctuations during weeks and months concern some stretches (basic freeway segments) on the Italian motorway network. The complete dataset contains daily traffic volumes for several motorway stretches for five years between 2012 and 2016, as resulting from accounting tickets/payments, distinguishing travel direction and vehicle type (light and heavy vehicles).

For this paper, four stretches have been extracted from the overall database, named later in the paper as (a), (b), (c), and

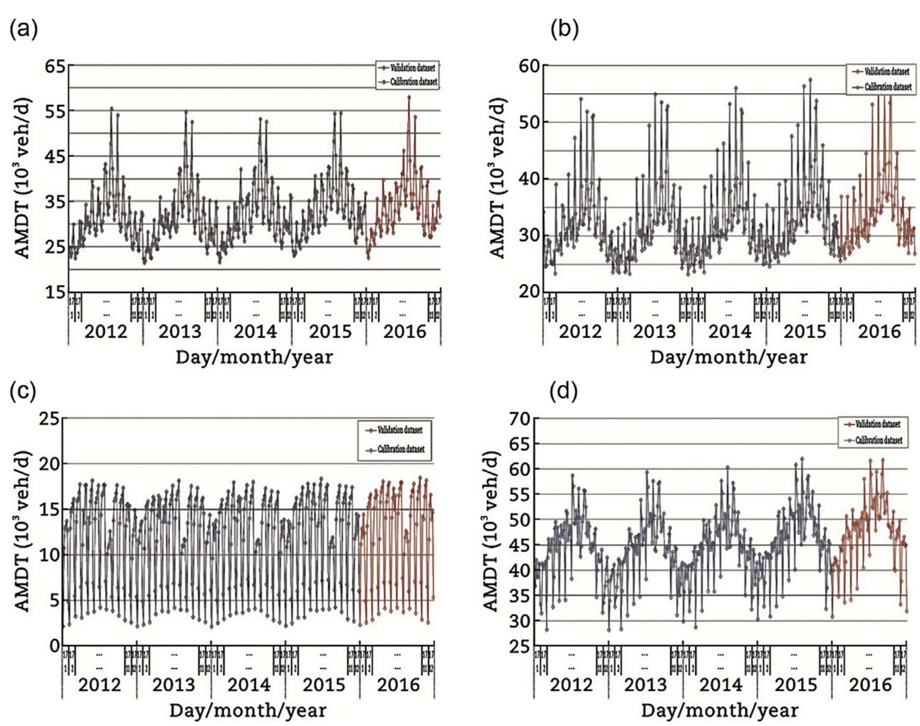

Fig. 1 – Time sequences for average daily traffic, with weekly and monthly fluctuation – year 2012–2015 (calibration set) and 2016 (validation set). (a) Stretch (a) - light vehicles. (b) Stretch (b) - light vehicles. (c) Stretch (c) - heavy vehicles. (d) Stretch (d) - light + heavy vehicles.

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Table 3 – DSHW optimized parameters for on-sample data (2012–2015).

Stretch	Model	Optimized parameters			
		α	γ	δ	ω
(a)	DSHW	0.889	0.001	1.000	1.000
(c)	DSHW	0.907	0.011	1.000	0.010
(d)	DSHW	0.569	0.001	0.575	0.950
(e)	DSHW	0.802	0.007	0.373	0.010

(d). The four selected stretches are located on Autostrada A4, an important Italian motorway that goes through the Po Valley and connects several major cities between Turin and Trieste in Northern Italy. The A4 is one of the busiest in the Italian motorway system and in the last years it has been the subject of different researches about the fundamental diagram and the traffic flow quality (Pompigna and Rupi, 2015), and the lane distributions of the macroscopic variables of the traffic (Pompigna and Rupi, 2017). Table 2 shows the annual average daily traffic (AADT) values and some statistics (standard deviation, max variation and coefficient of variation) that express the data variability within each year and for each stretch.

For these stretches, the average traffic volumes Y for each typical day of the week, i.e., from Sunday to Saturday, have been calculated over the 12 months of each year from 2012 to 2016. In particular, for stretch (a) and (b) the average traffic data relate to light vehicles, for stretch (c) data refer to heavy vehicles, while for stretch (d) the same data concern total vehicles (light and heavy). This choice was made to analyse the behaviour of the forecasting models for different patterns of seasonality, mainly related to the type of vehicles or the place of the stretch.

The sequences $Y_{d,m,y}$ constructed ordering the data by days d 1 (Sunday) to 7 (Saturday) of the average week in the months m 1 (January) to 12 (December) of the years y 2012 to 2016, allow describing the trend and the seasonal fluctuations of average monthly daily traffic (AMDT) for each motorway stretch.

It is necessary to specify that the overall database of the values obtained for the seven days of the average week for each month in 5 years, have been divided into two portions: data for years 2012–2015, which have been used for models training and calibration (on-sample data); data for the year 2016, which have been reserved for models validation and accuracy assessment (out-of-sample data).

Fig. 1 shows the traffic $Y_{d,m,y}$ time series, distinguishing the model calibration dataset (2012–2015) from the validation dataset (2016). The double seasonality of the data is clear by figures: a first seasonality regarding the daily fluctuation within a week; a second seasonality regarding the monthly fluctuation within a year.

4. Results and discussion

The single models have been implemented using standard functions provided by any commercially available software package for spreadsheets, to test and make sure their complete definition in a substantially simple way and by using commonly available tools. It should be specified that the two Non-Parametric models have no calibration parameter, and their complete definition about on-sample data only depends on their adequate treatment.

The same thing does not happen for the Parametric model DSHW, the smoothing parameters of which (i.e. α, γ, δ, and ω) have been optimized by minimizing the squared errors between predicted $\hat{Y}_t$ and actual Y_t average daily traffic for the years 2012–2015.

Table 3 shows the values obtained for the calibration parameters of the model DSHW. Also in this case the optimization has been performed using the normal functionality of spreadsheets (Namely Microsoft Excel Solver). It is evident that the values obtained for model parameters are extremely variable, depending on the motorway stretch under consideration.

Table 4 shows the calculated values for the five error indexes in Table 1, for the three proposed models and with reference to the on-sample data. It should be specified that the metrics have been applied to the differences between the predicted values and the sample values only for the years 2013–2015. Due to the nature of the GML model, in fact, a prediction for the first year in time series cannot be provided and, for this reason, the year 2012 has been excluded. Despite being the same possible for the other two models, however, for a homogeneous and concordant discussion the first year has been excluded from the calculation of the error testing indexes. To assess the relative order of the three models proposed in terms of average accuracy with regard to all the indexes, an average

Table 4 – Performances metrics for the three models considering on-sample data (2013–2015).

Stretch	Model	MAPE (%)	RMSE	RMSPE (%)	UII	SSLAR	AV RANK
(a)	NPR	2.16	871.28	2.98	0.03	0.220	1.0
	GML	7.50	3227.24	10.03	0.10	2.865	3.0
	DSHW	2.90	1131.12	3.89	0.04	0.377	2.0
(b)	NPR	3.11	1385.86	4.47	0.04	0.489	1.2
	GML	8.81	4000.60	11.43	0.12	3.821	3.0
	DSHW	3.60	1219.13	4.79	0.04	0.577	1.6
(c)	NPR	3.00	552.51	4.40	0.04	0.467	1.0
	GML	10.95	1972.67	15.96	0.15	8.419	3.0
	DSHW	5.85	1101.87	8.69	0.09	1.720	2.0
(d)	NPR	2.64	1566.91	3.59	0.03	0.327	1.2
	GML	7.45	4265.76	9.71	0.10	2.696	3.0
	DSHW	2.67	1327.89	3.77	0.03	0.348	1.6

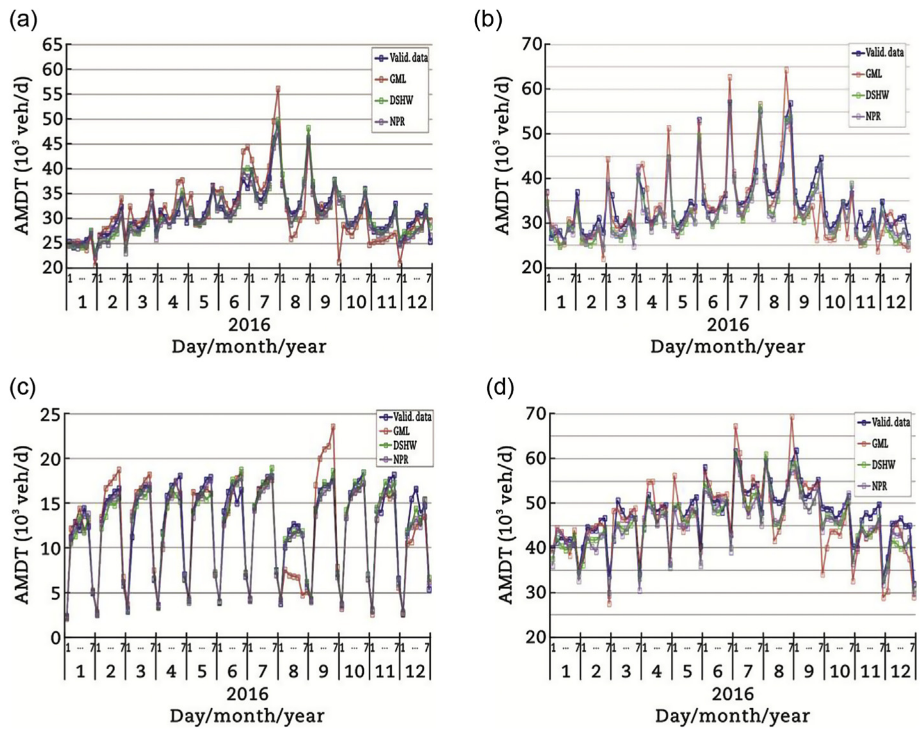

Fig. 2 – Single models forecast performances for out-of-sample data (Valid. data for 2016 validation set). (a) Stretch (a) - light vehicles. (b) Stretch (b) - light vehicles. (c) Stretch (c) - heavy vehicles. (d) Stretch (d) - light + heavy vehicles.

rank (AV RANK) has been introduced. The AV RANK is calculated as the average value of the accuracy orders of the models with respect to each index, evaluated for each stretch by attributing: the value 1 for the model with the smallest index (the best performer); the value 3 for the model with the largest index (the poorest performer); the value 2 for the remaining model.

It should first be noted that the response of the three models is generally good as shown by the values of the indexes. Taking into consideration the mean absolute percentage error (MAPE) we have values that, at most, slightly exceed 10%. The highest values for the MAPE are shown by the GML model; for the other two models, these values do not exceed 6% and, on average, reach about 3%. Essentially, the same finding emerges considering also the other indexes.

In comparing the performance of the three models for on-sample data prediction, it is still clear that the poorest performing model is steadily, for all the four stretches, the GML model. This model presents, in fact, an average rank order that it is always equal to 3. This means that the GML model presents, for the four stretches, the less prediction accuracy with respect to all the five errors test indexes. The NPR and DSHW models share out, in substance, the role of the best performer with a slight predominance of the non-parametric regression model.

Switching to the prediction for 2016, i.e., out-of-sample data, Fig. 2 shows the evolution of the forecast for each single model, and the comparison with the actual data. Table 5 presents a comparison among the indexes for the validation year.

For the validation set, forecasting accuracy ensured by the three models is satisfactory. Taking into consideration once again the MAPE, values are all below the 12%. The highest values are shown also in this case by the GML model; for the

Table 5 – Single models performances metrics for out-of-sample data (2016).

Stretch	Model		MAPE (%)	RMSE	RMSPE (%)	UII	SSLAR	AV RANK
(a)	Single	NPR	3.86	2877.79	4.69	0.093	0.195	2.0
	Single	GML	7.24	3014.58	9.55	0.095	0.872	3.0
	Single	DSHW	3.60	1408.03	4.48	0.045	0.172	1.0
(b)	Single	NPR	6.06	3473.17	7.51	0.104	0.539	2.2
	Single	GML	8.22	3397.72	10.76	0.098	1.125	2.6
	Single	DSHW	6.13	2490.46	7.46	0.075	0.528	1.2
(c)	Single	NPR	5.71	1884.12	7.20	0.146	0.459	1.6
	Single	GML	11.71	1934.31	16.18	0.145	3.105	2.8
	Single	DSHW	6.26	1079.22	7.97	0.083	0.546	1.6
(d)	Single	NPR	5.57	3630.00	6.42	0.079	0.377	2.4
	Single	GML	6.54	3402.00	8.75	0.072	0.731	2.6
	Single	DSHW	4.89	2693.00	5.76	0.059	0.303	1.0

Table 6 – IE and SV weights for single models calculated for on-sample data (2012–2015).

Stretch	Model	Weights ω_i	
		IE	SV
(a)	NPR	0.351	0.521
	GML	0.312	0.025
	DSHW	0.337	0.454
(b)	NPR	0.385	0.487
	GML	0.295	0.059
	DSHW	0.320	0.453
(c)	NPR	0.325	0.538
	GML	0.351	0.090
	DSHW	0.324	0.372
(d)	NPR	0.343	0.475
	GML	0.285	0.053
	DSHW	0.372	0.473

other two models, these values do not exceed 6.3%, reaching on average about 5. The other indices confirm the substance of what has been found above. The reciprocal comparison between the predictive capabilities of the three models, made clear through the AV RANK, shows that the best performances are those of DSHW model, which detaches the second best model, i.e., NPR, and the third in order of accuracy, i.e. GML.

Once assessed the goodness of prediction of the individual models, we move to test the performances of the combined models. As already mentioned, these models are obtained from the weighing of the individual models and these weights are calculated with the IE and SV methods explained in section 3.3.

Table 6 shows the EI and SV values of the weights calculated on the training data, and which are later used for weighing the forecast for the year 2016. The weights shown in Table 5 have been calculated on 2012 to 2015 data because it is assumed that the measured values are not known (and the forecasting errors cannot be calculated) for 2016. These weights seem to be very different for the two methods: the IE method, in fact, produces weights that are more and more balanced between them and values are not too different from one-third each for NPR, GML and DSHW models.

The SV method shows more spaced weights between the three models, and in all cases it is clear the reduced contribution (and therefore low weight) for the GML model and the predominance (with the weight values close to each other) of the other two models.

Looking at the weighted combinations of each model, Fig. 3 shows the evolution of the joint forecast using IE and SV weights, compared with the actual data, while Table 7 contains the relative indexes for errors and the average accuracy.

The results presented in Table 7 highlights that the best combination of models is obtained considering the SV method, using the relative weights. The cooperative method allows, in fact, finding the best values of the indexes (e.g. MAPE that does not exceed 6%), with an AV RANK that it is always close to one. This confirms the best forecasting performance of the Shapley value method for all the motorway stretches considered.

Comparing the indices in Tables 7 and 5, and looking at the synthesis in Table 8 especially for normalized AV RANK, the SV weights allows getting slightly better forecast in linear combinations than those obtained with each model individually considered. One exception appears for the stretch (d), for which the best performances are obtained

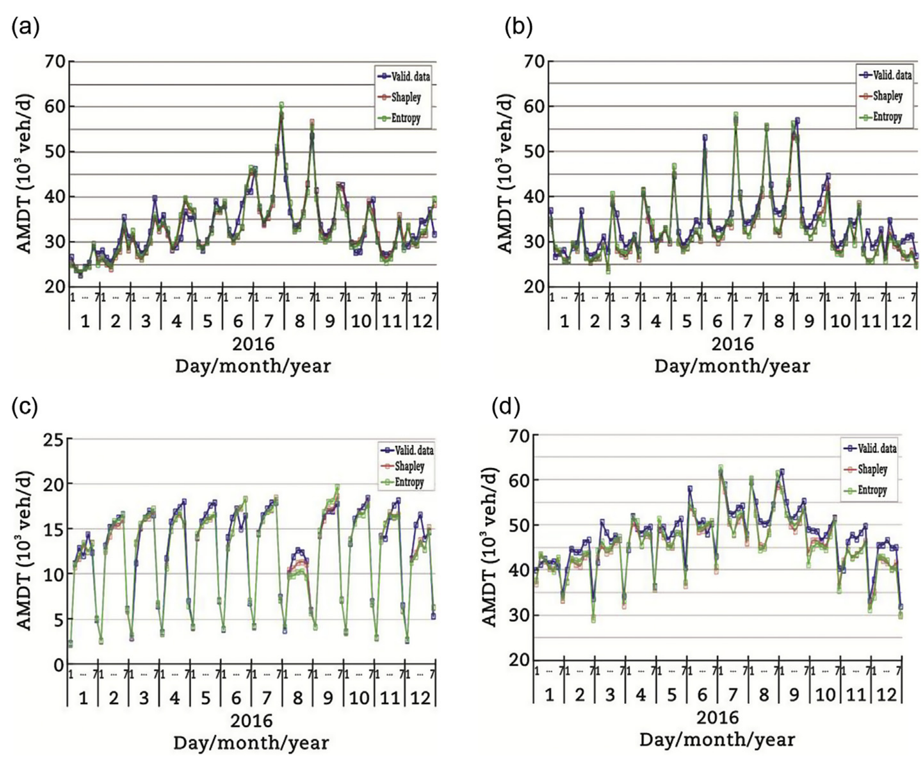

Fig. 3 – IE (Entropy) and SV (Shapley) combined models forecast performances for out-of-sample data (Valid. data for 2016 validation set). (a) Stretch (a) - light vehicles. (b) Stretch (b) - light vehicles. (c) Stretch (c) - heavy vehicles. (d) Stretch (d) - light + heavy vehicles.

Table 7 – Combined models performances metrics for out-of-sample data (2016).

Stretch	Model		MAPE (%)	RMSE	RMSPE (%)	UII	SSLAR	AV RANK
(a)	Combined	IE	3.99	1605.086	5.03	0.051	0.219	2.0
	Combined	SV	3.59	1359.745	4.35	0.044	0.166	1.0
(b)	Combined	IE	6.01	2567.133	7.43	0.076	0.524	2.0
	Combined	SV	5.87	2462.019	7.20	0.074	0.493	1.0
(c)	Combined	IE	6.62	1177.686	8.78	0.090	0.703	2.0
	Combined	SV	5.63	980.085	7.18	0.075	0.450	1.0
(d)	Combined	IE	5.10	2795.599	6.02	0.061	0.332	1.8
	Combined	SV	5.10	2743.672	5.88	0.060	0.314	1.2

Table 8 – AV Rank and Norm. AV Rank for single and combined IE and SV models for out-of-sample data (2016).

Model	AV RANK				NORM. AV RANK			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
NPR	3.4	4.2	3.0	4.4	3	4	2	4
GML	5.0	4.6	4.8	4.6	5	5	5	5
DSHW	2.0	2.8	2.6	1.0	2	2	2	1
IE	3.6	2.4	3.6	2.8	4	3	4	3
SV	1.0	1.0	1.0	2.2	1	1	1	2

with the DSHW model followed by the models combination in accordance with the SV weights. In the latter case, however, it should be noted the extreme closeness of the indexes when comparing singular DSHW forecasts and weighted SV predictions.

It should be noted, moreover, as the linear combination with IE weights doesn't improve the results of the best single model in all the stretches. The indices relating IE joint forecasts are worse than those of the combined SV, worse than those of the single DSHW and, for two stretches, even than those of the single NPR model.

5. Conclusions

This paper deals with a very common issue for professionals in analysis of transport systems, especially of motorway infrastructures, which is predicting the weekly fluctuations of traffic on various months of the year, on the basis of past data recorded for a number of earlier years. This problem occurs when practitioners need to estimate the average expected traffic patterns in a typical week on the basis of an archive of past data and, therefore, to predict the typical fluctuation of the average daily traffic throughout the current year. This often happens to those who are professionally involved in motorway traffic, such as in activities for traffic studies and for forecast report arrangements to support the concessionary company or the grantor.

Following a literature review, three short-time prediction models for average daily traffic have been identified: the non-parametric regression model (NPR) and Gaussian maximum likelihood (GML), which are non-parametric type: the double seasonality Holt–Winters model (DSHW), which is a parametric exponential smoothing model for time series.

On account of the fact that many approaches in the literature are very hardly applicable in professional tasks, due to the complexity of underlying theory and the mathematical formulation of solutions algorithms, our attention has been addressed to those three because of their few requirements in the specification and calibration process. These characteristics make them highly generalisable and implementable without necessarily having an in-depth statistical training, as well as in-depth modelling and programming skills.

The three models have been applied to four stretches of the Italian motorway network, characterized by different shapes of weekly and monthly seasonality. The overall data for each stretch have been divided into two datasets, with the values for the years 2012–2015 as the training data and the values for 2016 as the validation data. To check the predictive accuracy of these models for both data sets, five common performance metrics for forecast error have been introduced (namely MAPE, RMSE RMSPE, UII, SSLAR), with an average rank indicator (AV. RANK) that allows to rate the goodness of the indexes with respect to each model.

For both training and validation data, the response of the three models discussed is generally good. The Parametric DSHW model and non-parametric *K*-NN NPR model prove to be the best ones; if the latter proves slightly better on the dataset of calibration, the former is more accurate on the validation dataset, but the distance between them is always minimal. More detached is the GML model, with slightly higher forecast errors. Beyond the mutual comparison, all three models confirm prediction performances that are widely acceptable to the requirements of the problem we face here.

Given the fact that the possibility of a combined use of the single models as enhancing factor for accuracy is a well-known finding in the forecast literature, two linearly combined predictions have been tested and compared by computing weight coefficients through Information Entropy and Shapley Value methods. Regarding the validation year 2016, the relative performances for these two weighted predictions have been discussed comparatively and respect to the three single models, to understand their strengths in improving the forecast for the four motorway stretches. Despite being less popular than EI, the SV method proves to be more capable in better combining forecasts and produces improvements in the predictions for test data than the individual models.

In conclusion, being known that estimating the current traffic level and predicting the typical fluctuation of the average daily traffic throughout the year are essential in many fields of transport analysis and practice, this paper draws reader's attention to three models reviewed by the literature as valuable and viable ready-to-practice tools for prediction purposes. Referring to these models and using real-life data, this research has highlighted a good balance of forecasting accuracy and ease development using ordinary professional skills and conventional analysis tools, also showing the possibility for improvement by resorting to an appropriate linear combination of individual forecasts. In our view these remarks highlighted by the paper could be a useful reference and aid for traffic technicians or analysts, in various and not uncommon tasks they might find in their work.

Conflicts of interest

The authors do not have any conflict of interest with other entities or researchers.

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