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This is the final peer-reviewed author's accepted manuscript (postprint) of the following publication:

Published Version:
Borovykh, A., Pascucci, A., Oosterlee, C.W. (2017). Bermudan option valuation under state-dependent models. Springer New York LLC [10.1007/978-3-319-66536-8_6].

Availability:
This version is available at: https://hdl.handle.net/11585/613260 since: 2018-10-24

Published:

DOI: http://doi.org/10.1007/978-3-319-66536-8_6

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13 September 2026

Chapter 1
Bermudan option valuation under state-dependent models

Anastasia Borovykh, Andrea Pascucci and Cornelis W. Oosterlee

Abstract

We consider a defaultable asset whose risk-neutral pricing dynamics are described by a state-dependent SDE with jumps and default. This class of models allows for a local volatility, local default intensity and a locally dependent Lévy measure. We present a pricing method for Bermudan options based on an analytical approximation of the characteristic function combined with the COS method. Due to a special form of the obtained characteristic function the price can be computed using a fast Fourier transform-based algorithm resulting in a fast and accurate calculation.

1.1 Introduction

In order to price derivatives in finance one requires the specification of the underlying asset dynamics. This is usually done by means of a stochastic differential equation. In this work we consider the flexible dynamics of a state- and time-dependent model, in which we account for a local volatility function, a local jump measure such that the jumps in the underlying arrive with a state-dependent intensity and a local default intensity, so that the default time depends on the underlying state. One of the problems when considering such a state-dependent model is the fact that there is no explicit density function or characteristic function available. In order to still be able to price derivatives, we derive the characteristic function by means on an advanced Taylor expansion of the state-dependent coefficients, as first presented in [7] for a simplified model and similar to the derivations in [1] for the local Lévy model. This Taylor expansion allows one to rewrite the fundamental solution of the related Cauchy problem in terms of solutions of simplified Cauchy problems, which we then solve in the Fourier space to obtain the approximated characteristic

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function. Once we have an explicit approximation for the characteristic function we use a Fourier method known as the COS method, first presented in [2], for computing the continuation value of a Bermudan option. Due to a specific form of the approximated characteristic function the continuation value can be computed using a Fast Fourier Transform (FFT), resulting in a fast and accurate option valuation.

1.2 General framework

We consider a defaultable asset S whose risk-neutral dynamics are given by:

$$\begin{aligned} S_t &= \mathbb{1}_{\{t<\zeta\}} e^{X_t}, \\ dX_t &= \mu(t,X_t)dt + \sigma(t,X_t)dW_t + \int_{\mathbb{R}} d\tilde{N}_t(t,X_{t-},dz)z, \\ d\tilde{N}_t(t,X_{t-},dz) &= dN_t(t,X_{t-},dz) - \nu(t,X_{t-},dz)dt, \\ \zeta &= \inf\{t \geq 0 : \int_0^t \gamma(s,X_s)ds \geq \varepsilon\}, \end{aligned}$$

where $\tilde{N}_t(t,x,dz)$ is a compensated random measure with state-dependent Lévy measure $\nu(t,x,dz)$. The default time ζ of S is defined in a canonical way as the first arrival time of a doubly stochastic Poisson process with local intensity function $\gamma(t,x) \geq 0$, and $\varepsilon \sim \text{Exp}(1)$ and is independent of X. Thus the model features:

- a local volatility function $\sigma(t,x)$;
- a local Lévy measure: jumps in X arrive with a state-dependent intensity described by the local Lévy measure $\nu(t,x,dz)$. The jump intensity and jump distribution can thus change depending on the value of x. A state-dependent Lévy measure is an important feature because it allows to incorporate stochastic jump-intensity into the modeling framework;
- a local default intensity $\gamma(t,x)$: the asset S can default with a state-dependent default intensity.

We define the filtration of the market observer to be $\mathscr{G} = \mathscr{F}^X \vee \mathscr{F}^D$, where $\mathscr{F}^X$ is the filtration generated by X and $\mathscr{F}_t^D := \sigma(\{\zeta \leq u\}, u \leq t)$, for $t \geq 0$, is the filtration of the default. We assume

$$\int_{\mathbb{R}} e^{|z|}\nu(t,x,dz) < \infty,$$

and by imposing that the discounted asset price $\tilde{S}_t := e^{-rt}S_t$ is a $\mathscr{G}$-martingale, we get the following restriction on the drift coefficient:

$$\mu(t,x) = \gamma(t,x) + r - \frac{\sigma^2(t,x)}{2} - \int_{\mathbb{R}} \nu(t,x,dz)(e^z - 1 - z).$$

1.3 The characteristic function

Is it well-known (see, for instance, [4, Section 2.2]) that the price V of a European option with maturity T and payoff $\Phi(S_T)$ is given by

$$V_t = \mathbb{1}_{\{\zeta>t\}} e^{-r(T-t)} E\left[e^{-\int_t^T \gamma(s,X_s)ds}\varphi(X_T)|X_t\right], \qquad t \le T,$$

where $\varphi(x) = \Phi(e^x)$. Thus, in order to compute the price of an option, we must evaluate functions of the form

$$u(t,x) := E\left[e^{-\int_t^T \gamma(s,X_s)ds}\varphi(X_T)|X_t = x\right]. \tag{1.2}$$

Under standard assumptions, u can be expressed as the classical solution of the following Cauchy problem

$$\begin{cases} Lu(t,x) = 0, & t \in [0,T[,\ x \in \mathbb{R}, \\ u(T,x) = \varphi(x), & x \in \mathbb{R}, \end{cases} \tag{1.3}$$

where L is the integro-differential operator

$$\begin{aligned} Lu(t,x) = {} & \partial_t u(t,x) + r\partial_x u(t,x) + \gamma(t,x)(\partial_x u(t,x) - u(t,x)) + \frac{\sigma^2(t,x)}{2}(\partial_{xx} - \partial_x)u(t,x) \\ & - \int_{\mathbb{R}} \nu(t,x,dz)(e^z - 1 - z)\partial_x u(t,x) + \int_{\mathbb{R}} \nu(t,x,dz)(u(t,x+z) - u(t,x) - z\partial_x u(t,x)). \end{aligned} \tag{1.4}$$

Define $\Gamma(t,x;T,y)$ to be the fundamental solution of the Cauchy problem (1.3). The function u in (1.2) can be represented as an integral with respect to $\Gamma(t,x;T,dy)$:

$$u(t,x) = \int_{\mathbb{R}} \varphi(y)\Gamma(t,x;T,dy). \tag{1.5}$$

Here we notice explicitly that $\Gamma(t,x;T,dy)$ is not necessarily a standard probability measure because its integral over $\mathbb{R}$ can be strictly less than one; nevertheless, with a slight abuse of notation, we refer to its Fourier transform

$$\hat{\Gamma}(t,x;T,\xi) := \mathscr{F}(\Gamma(t,x;T,\cdot))(\xi) := \int_{\mathbb{R}} e^{i\xi y}\Gamma(t,x;T,dy), \qquad \xi \in \mathbb{R},$$

as the characteristic function of $\log S$. Following the method developed in [1] we use an adjoint expansion of the state-dependent coefficients

$$a(t,x) := \frac{\sigma^2(t,x)}{2}, \qquad \gamma(t,x), \qquad \nu(t,x,dz),$$

around some point $\bar{x}$. The coefficients $a(t,x)$, $\gamma(t,x)$ and $\nu(t,x,dz)$ are assumed to be continuously differentiable with respect to x up to order $N \in \mathbb{N}$. Introducing the n-th order Taylor approximation of the operator L to be (1.4):

$$\begin{aligned} L_n = {} & L_0 + \sum_{k=1}^{n} \Big((x-\bar{x})^k a_k(\partial_{xx} - \partial_x) + (x-\bar{x})^k \gamma_k \partial_x - (x-\bar{x})^k \gamma_k \\ & - \int_{\mathbb{R}} (x-\bar{x})^k \nu_k(dz)(e^z - 1 - z)\partial_x + \int_{\mathbb{R}} (x-\bar{x})^k \nu_k(dz)(e^{z\partial_x} - 1 - z\partial_x)\Big), \end{aligned}$$

where

$$L_0 = \partial_t + r\partial_x + a_0(t)(\partial_{xx} - \partial_x) + \gamma_0(t)\partial_x - \gamma_0(t) - \int_{\mathbb{R}} \nu_0(t,dz)(e^z - 1 - z)\partial_x + \int_{\mathbb{R}} \nu_0(t,dz)(e^{z\partial_x} - 1 - z\partial_x),$$

and

$$a_k = \frac{\partial_x^k a(\bar{x})}{k!}, \qquad \gamma_k = \frac{\partial_x^k \gamma(\bar{x})}{k!}, \qquad \nu_k(dz) = \frac{\partial_x^k \nu(\bar{x},dz)}{k!}, \qquad k \ge 0.$$

Let us assume for a moment that L_0 has a fundamental solution $G^0(t,x;T,y)$ that is defined as the solution of the Cauchy problem

$$\begin{cases} L_0 G^0(t,x;T,y) = 0 & t \in [0,T[,\ x \in \mathbb{R}, \\ G^0(T,\cdot;T,y) = \delta_y. \end{cases}$$

In this case we define the nth-order approximation of Γ as

$$\Gamma^{(n)}(t,x;T,y) = \sum_{k=0}^{n} G^k(t,x;T,y),$$

where, for any $k \geq 1$ and (T,y), $G^k(\cdot,\cdot;T,y)$ is defined recursively through the following Cauchy problem

$$\begin{cases} L_0 G^k(t,x;T,y) = -\sum\limits_{h=1}^{k} (L_h - L_{h-1}) G^{k-h}(t,x;T,y) & t \in [0,T[,\ x \in \mathbb{R}, \\ G^k(T,x;T,y) = 0, & x \in \mathbb{R}. \end{cases}$$

Correspondingly, the nth-order approximation of $\hat{\Gamma}$ is defined to be

$$\hat{\Gamma}^{(n)}(t,x;T,\xi) = \sum_{k=0}^{n} \mathscr{F}\left(G^k(t,x;T,\cdot)\right)(\xi) := \sum_{k=0}^{n} \hat{G}^k(t,x;T,\xi), \qquad \xi \in \mathbb{R}.$$

Now, by transforming the simplified Cauchy problems into adjoint problems and solving these in the Fourier space we find

$$\begin{aligned} \hat{G}^0(t,x;T,\xi) &= e^{i\xi x} e^{\int_t^T \psi(s,\xi) ds}, \\ \hat{G}^k(t,x;T,\xi) &= -\int_t^T e^{\int_s^T \psi(\tau,\xi) d\tau} \mathscr{F}\left(\sum_{h=1}^{k} \left(\tilde{L}_h^{(s,\cdot)}(s) - \tilde{L}_{h-1}^{(s,\cdot)}(s) \right) G^{k-h}(t,x;s,\cdot) \right)(\xi) ds, \end{aligned}$$

with

$$\psi(s,\xi) = i\xi(r + \gamma_0(s)) + a_0(s)(-\xi^2 - i\xi) - \int_{\mathbb{R}} \nu_0(s,dz)(e^z - 1 - z) i\xi + \int_{\mathbb{R}} \nu_0(s,dz)(e^{iz\xi} - 1 - iz\xi),$$

the characteristic exponent of the Lévy process with coefficients $\gamma_0(s)$, $a_0(s)$ and $\nu_0(s,dz)$, and

$$\begin{aligned} \tilde{L}_h^{(s,y)}(s) - \tilde{L}_{h-1}^{(s,y)}(s) = {} & a_h(s) h(h-1)(y-\bar{x})^{h-2} + a_h(s)(y-\bar{x})^{h-1}\left(2h\partial_y + (y-\bar{x})(\partial_{yy} + \partial_y) + h\right) \\ & - \gamma_h(s) h (y-\bar{x})^{h-1} - \gamma_h(s)(y-\bar{x})^h(\partial_y + 1) \\ & + \int_{\mathbb{R}} \nu_h(s,dz)(e^z - 1 - z)\left(h(y-\bar{x})^{h-1} + (y-\bar{x})^h \partial_y\right) \\ & + \int_{\mathbb{R}} \bar{\nu}_h(s,dz)\left((y+z-\bar{x})^h e^{z\partial_y} - (y-\bar{x})^h - z\left(h(y-\bar{x})^{h-1} - (y-\bar{x})^h \partial_y\right)\right). \end{aligned}$$

From these results one can already see that the dependency on x comes in through $e^{i\xi x}$ and after taking derivatives the dependency on x will take the form $(x-\bar{x})^m e^{i\xi x}$: this fact will be crucial in our analysis. After some algebraic manipulations, see for details [1], we find that the approximation of order n is a function of the form

$$\hat{\Gamma}^{(n)}(t,x;T,\xi) := e^{i\xi x} \sum_{k=0}^{n} (x-\bar{x})^k g_{n,k}(t,T,\xi), \tag{1.6}$$

where the coefficients $g_{n,k}$, with $0 \le k \le n$, depend only on t, T and ξ, but not on x. The approximation formula can thus always be split into a sum of products of functions depending only on ξ and functions that are linear combinations of $(x-\bar{x})^m e^{i\xi x}$, $m \in \mathbb{N}_0$.

1.4 Bermudan option valuation

A Bermudan option is a financial contract in which the holder can exercise at a predetermined finite set of exercise moments prior to maturity, and the holder of the option receives a payoff when exercising. Consider a Bermudan option with a set of M exercise moments $\{t_1, ..., t_M\}$, with $0 \le t_1 < t_2 < \cdots < t_M = T$. When the option is exercised at time t_m the holder receives the payoff $\Phi(t_m, S_{t_m})$. For a Bermudan put option with strike price K, we simply have $\varphi(t,x) = (K - e^x)^+$. By the dynamic programming approach, the option value can be expressed by a backward recursion as

$$v(t_M, x) = \mathbb{1}_{\{\zeta > t_M\}} \varphi(t_M, x)$$

and

$$\begin{cases} c(t,x) = E\left[e^{\int_t^{t_m} -(r+\gamma(s,X_s))ds} v(t_m, X_{t_m}) | X_t = x \right], & t \in [t_{m-1}, t_m[\\ v(t_{m-1}, x) = \mathbb{1}_{\{\zeta > t_{m-1}\}} \max\{\varphi(t_{m-1}, x), c(t_{m-1}, x)\}, & m \in \{2, \ldots, M\}. \end{cases} \tag{1.7}$$

In the above notation $v(t,x)$ is the option value and $c(t,x)$ is the so-called continuation value. The option value is set to be $v(t,x) = c(t,x)$ for $t \in]t_{m-1}, t_m[$, and, if $t_1 > 0$, also for $t \in [0, t_1[$.

Remark 4.0. Since the payoff of a call option grows exponentially with the log-stock price, this may introduce significant cancellation errors for large domain sizes. For this reason we price put options only using our approach and we employ the well-known put-call parity to price calls via puts. This is a rather standard argument (see, for instance, [8]).

1.4.1 An algorithm for pricing Bermudan put options

The COS method as proposed in [2] is based on the insight that the Fourier-cosine series coefficients of $\Gamma(t,x;T,dy)$ (and therefore also of option prices) are closely related to the characteristic function of the underlying process. Remembering that the expected value $c(t,x)$ in (1.7) can be rewritten in integral form as in (1.5),

$$c(t,x) = e^{-r(t_m - t)} \int_{\mathbb{R}} v(t_m, y) \Gamma(t,x;t_m,dy), \qquad t \in [t_{m-1}, t_m[,$$

we apply the COS formulas to find the approximation:

$$\begin{aligned} \hat{c}(t,x) &= e^{-r(t_m-t)} {\sum_{k=0}}' \mathrm{Re}\left(e^{-ik\pi \frac{a}{b-a}} \hat{\Gamma}\left(t,x;t_m, \frac{k\pi}{b-a}\right)\right) V_k(t_m), \qquad t \in [t_{m-1}, t_m[\\ V_k(t_m) &= \frac{2}{b-a} \int_a^b \cos\left(k\pi \frac{y-a}{b-a}\right) \max\{\varphi(t_m, y), c(t_m, y)\} dy, \end{aligned} \tag{1.8}$$

with $\varphi(t,x) = (K - e^x)^+$.

Next we recover the coefficients $(V_k(t_m))_{k=0,1,\ldots,N-1}$ from $(V_k(t_{m+1}))_{k=0,1,\ldots,N-1}$. To this end, we split the integral in the definition of $V_k(t_m)$ into two parts using the early-exercise point x_m^*, which is the point where the continuation value is equal to the payoff, i.e. $c(t_m,x_m^*)=\varphi(t_m,x_m^*)$; thus we have

$$V_k(t_m)=F_k(t_m,x_m^*)+C_k(t_m,x_m^*), \qquad m=M-1,M-2,...,1,$$

where

$$F_k(t_m,x_m^*):=\frac{2}{b-a}\int_a^{x_m^*}\varphi(t_m,y)\cos\left(k\pi\frac{y-a}{b-a}\right)dy,$$
$$C_k(t_m,x_m^*):=\frac{2}{b-a}\int_{x_m^*}^{b}c(t_m,y)\cos\left(k\pi\frac{y-a}{b-a}\right)dy,$$

and $V_k(t_M)=F_k(t_M,\log K)$.

Remark 4.0. Since we have a semi-analytic formula for $\hat{c}(t_m,x)$, we can easily find the derivatives with respect to x and use Newton's method to find the point x_m^* such that $c(t_m,x_m^*)=\varphi(t_m,x_m^*)$. A good starting point for the Newton method is $\log K$, since $x_m^*\leq\log K$.

The coefficients $F_k(t_m,x_m^*)$ can be computed analytically using $x_m^*\leq\log K$. On the other hand, by inserting the approximation (1.8) for the continuation value into the formula for $C_k(t_m,x_m^*)$ have the following coefficients $\hat{C}_k$ for $m=M-1,M-2,...,1$:

$$\hat{C}_k(t_m,x_m^*)=\frac{2e^{-r(t_{m+1}-t_m)}}{b-a}{\sum_{j=0}^{N-1}}'V_j(t_{m+1})\int_{x_m^*}^{b}\mathrm{Re}\left(e^{-ij\pi\frac{a}{b-a}}\hat{\Gamma}\left(t_m,x;t_{m+1},\frac{j\pi}{b-a}\right)\right)\cos\left(k\pi\frac{x-a}{b-a}\right)dx.$$

Similar to the FFT-based algorithm in [2] for an exponential Lévy process with constant coefficients, the continuation value in case of the state-dependent coefficients can also be calculated using the FFT. Using the structure of the characteristic function (1.6) we write the continuation value in vector form as:

$$\hat{\mathbf{C}}(t_m,x_m^*)=\sum_{h=0}^{n}e^{-r(t_{m+1}-t_m)}\mathrm{Re}\left(\mathbf{V}(t_{m+1})\mathscr{M}^h(x_m^*,b)\Lambda^h\right),$$

where $\mathbf{V}(t_{m+1})$ is the vector $[V_0(t_{m+1}),...,V_{N-1}(t_{m+1})]^T$ and $\mathscr{M}^h(x_m^*,b)\Lambda^h$ is a matrix-matrix product with $\mathscr{M}^h$ being a matrix with elements

$$M_{k,j}^h(x_m^*,b)=\frac{2}{b-a}\int_{x_m^*}^{b}e^{ij\pi\frac{x-a}{b-a}}(x-\bar{x})^h\cos\left(k\pi\frac{x-a}{b-a}\right)dx, \qquad k,j=0,...,N-1$$

and Λ^h is a diagonal matrix with elements

$$g_{n,h}\left(t_m,t_{m+1},\frac{j\pi}{b-a}\right), \qquad j=0,\ldots,N-1.$$

It can be shown using standard trigonometric that the matrix $\mathscr{M}$ can be rewritten as a sum of a Hankel and Toeplitz matrix such that $\mathscr{M}=\mathscr{M}_H+\mathscr{M}_T$ with elements

$$M_j^h(x_m^*,b)=\frac{1}{b-a}\int_{x_m^*}^{b}\cos\left(ij\pi\frac{x-a}{b-a}\right)(x-\bar{x})^h dx+\frac{1}{b-a}\int_{x_m^*}^{b}\sin\left(ij\pi\frac{x-a}{b-a}\right)(x-\bar{x})^h dx.$$

Using the split into sums of Hankel and Toeplitz matrices we can write the continuation value in matrix form as:

$$\hat{C}(t_m,x_m^*) = \sum_{h=0}^{n} e^{-r(t_{m+1}-t_m)} \mathrm{Re}\left((\mathscr{M}_H^h + \mathscr{M}_T^h)u^h\right),$$

where $\mathscr{M}_H^h = \{M_{k,j}^{H,h}(x_m^*,b)\}_{k,j=0}^{N-1}$ is a Hankel matrix and $\mathscr{M}_T^l = \{M_{k,j}^{T,h}(x_m^*,b)\}_{k,j=0}^{N-1}$ is a Toeplitz matrix and $u^h = \{u_j^h\}_{j=0}^{N-1}$, with $u_j^h = g_{n,h}\left(t_m,t_{m+1},\frac{j\pi}{b-a}\right)V_j(t_{m+1})$ and $u_0^h = \frac{1}{2}g_{n,h}(t_m,t_{m+1},0)V_0(t_{m+1})$. It is well known that a product of a Hankel or Toeplitz matrix with a vector can be calculated using FFTs, see [1] for the full details. Using the fact that an FFT can be computed with computational complexity $O(N\log_2 N)$, we find that for a Bermudan option with M exercise dates the overall computational complexity is $O((M-1)N\log_2 N)$.

1.5 Numerical experiments

In this section we apply the method developed in Section 1.4 to compute the European and Bermudan option values with various underlying stock dynamics. The computer used in the experiments has an Intel Core i7 CPU with a 2.2 GHz processor. We use the second-order approximation of the characteristic function.

For the COS method, unless otherwise mentioned, we use $N = 200$ and $L = 10$, where L is the parameter used to define the truncation range $[a,b]$ as follows:

$$[a,b] := \left[c_1 - L\sqrt{c_2+\sqrt{c_4}}, c_1 + L\sqrt{c_2+\sqrt{c_4}}\right],$$

where c_n is the nth cumulant of log-price process $\log S$ calculated using the 0th-order approximation of the characteristic function. We compare the approximated values to a 95% confidence interval computed with a Longstaff-Schwartz method with 10^5 simulations and 250 time steps per year. Furthermore, in the expansion we always use $\bar{x} = X_0$.

1.5.1 Tests under CEV-Merton dynamics with default

Consider a process under the CEV-Merton dynamics:

$$dX_t = \left(r - a(X_t) - \lambda\left(e^{m+\delta^2/2}-1\right)\right)dt + \sqrt{2a(X_t)}dW_t + \int_{\mathbb{R}} d\tilde{N}_t(t,dz)z,$$

with

$$\begin{aligned}
a(x) &= \frac{\sigma_0^2 e^{2(\beta-1)x}}{2},\\
\nu(dz) &= \lambda\frac{1}{\sqrt{2\pi\delta^2}}\exp\left(\frac{-(z-m)^2}{2\delta^2}\right)dz,\\
\psi(\xi) &= -a_0(\xi^2+i\xi) + ir\xi - i\lambda\left(e^{m+\delta^2/2}-1\right)\xi + \lambda\left(e^{mi\xi-\delta^2\xi^2/2}-1\right),
\end{aligned}$$

We use the following parameters $S_0 = 1$, $r = 5\%$, $\sigma_0 = 20\%$, $\beta = 0.5$, $\lambda = 30\%$, $m = -10\%$, $\delta = 40\%$. The results are compared to a widely used method for valuing Bermudan options, the Least-Squares Monte Carlo method (LSM), see [6].

1.5.1.1 Time-independent CEV-Merton model

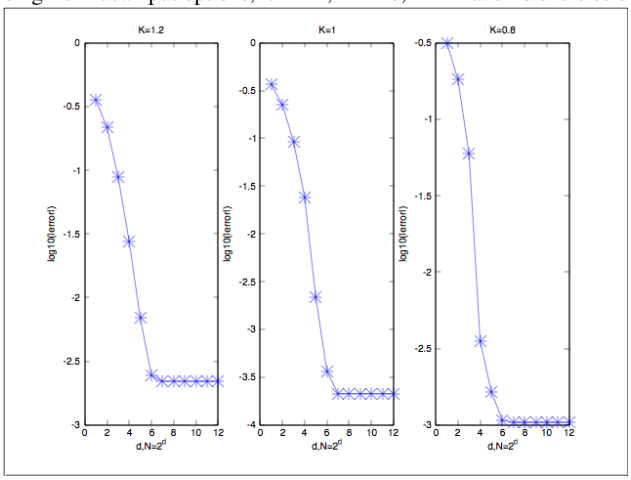
We compute the European and Bermudan option values in Table 1.1. The error in our approximation consists of the error of the COS method and the error in the adjoint expansion of the characteristic function. In particular for low strikes the method seems to be more sensitive to the approximation, as the approximated value does not always fall into the LSM confidence interval.

Table 1.1 Prices for a European and a Bermudan put option (expiry $T = 1$ with 10 exercise dates) in the CEV-Merton model for the 2nd-order approximation of the characteristic function, and a Monte Carlo method

	European		Bermudan	
K	MC 95% c.i.	Value	MC 95% c.i.	Value
0.6	0.006136-0.006573	0.006579	0.006307-0.006729	0.006096
0.8	0.02526-0.02622	0.02581	0.02595-0.2689	0.02520
1	0.08225-0.08395	0.08250	0.08480-0.08640	0.08593
1.2	0.1965-0.1989	0.1977	0.2097-0.2115	0.2132
1.4	0.3560-0.3589	0.3574	0.3946-0.3957	0.3954
1.6	0.5341-0.5385	0.5364	0.5930-0.5941	0.5932

In Figure 1.1 the convergence results of the COS method using the 2nd-order approximation of the characteristic function for $T = 1$ and 10 exercise dates are presented. We choose $L = 10$ and $N = 2^d$ and see that a very quick convergence is obtained.

Fig. 1.1 Error convergence for pricing Bermudan put options, $N = 2^d$, $L = 10$, $T = 1$ and 10 exercise dates and strikes $K = 0.8, 1, 1.2$



1.5.1.2 Time-dependent CEV-Merton model

Here we consider the CEV-Merton model but now with a time-dependent local volatility, similar to the structure in [5]. We define the local volatility as

$$a(t,x) = \frac{\sigma_0(t)^2 e^{2(\beta-1)x}}{2},$$
$$\sigma_0(t)^2 = \sigma_0^2 \left(1 + b_0 \exp\left(-\frac{(t-t_0)^2}{b_1}\right).\right)$$

In this case the volatility term structure can be interpreted as pulses of a surge or a drop in the market volatility centered at t_0. The width of the pulses is determined by b_1. We choose the following parameters for the time-dependency to be $\sigma_0 = 20\%$, $b_0 = 1$, $b_1 = 0.01$, $t_0 = 0.5$. The results are given in Table 1.2. We observe that the effect of the term structure for a maturity of $T = 1$ is indeed small, as also noted in [5], but the European option price is observed to be slightly larger compared to the case without the term-structure. For the Bermudan option the effect is hardly visible.

Table 1.2 Put prices for a European and a Bermudan option (10 exercise dates, expiry $T = 1$) in the CEV-Merton model with a volatility term-structure for the 2nd order approximation of the characteristic function, and a Monte Carlo method

	European		Bermudan	
K	MC c.i.	Value	MC c.i.	Value
0.8	0.02824-0.02926	0.02641	0.02859-0.02956	0.02500
1	0.08690-0.08865	0.08777	0.08949-0.09113	0.08580
1.2	0.2011-0.2036	0.2052	0.2130-0.2150	0.2134
1.4	0.3575-0.3604	0.3626	0.3950-0.3961	0.3955
1.6	0.5373-0.5406	0.5388	0.5925-0.5936	0.5932

1.5.2 Tests under a CEV-like Lévy process with a state-dependent measure

In this section we consider a model similar to the one used in [3]. The model is defined with local volatility and a state-dependent Lévy measure as follows:

$$\begin{aligned} a(x) &= \frac{1}{2}(b_0^2 + \varepsilon_1 b_1^2 \eta(x)), \\ \nu(x,dz) &= \varepsilon_3 \nu_N(dz) + \varepsilon_4 \eta(x) \nu_N(dz), \\ \eta(x) &= e^{\beta x}. \end{aligned} \tag{1.9}$$

We will consider Gaussian jumps, meaning that

$$\nu_N(dz) = \lambda \frac{1}{\sqrt{2\pi\delta^2}} \exp\left(\frac{-(z-m)^2}{2\delta^2}\right) dz.$$

In Table 1.3 the results are presented for a model as defined in (1.9) with a state-dependent jump measure, so $\nu(x,dz) = \eta(x)\nu_N(dz)$. In this case we have

$$\psi(\xi) = ir\xi - a_0(\xi^2 - i\xi) - \lambda\nu_0(e^{m+\delta^2/2} - 1)i\xi + \lambda\nu_0(e^{mi\xi - \delta^2\xi^2/2} - 1),$$

where $a_0 = \frac{1}{2}b_1^2 e^{\beta\bar{x}}$ and $\nu_0(dz) = e^{\beta\bar{x}}\nu_N(dz)$. The other parameters are chosen as: $b_1 = 0.15$, $b_0 = 0$, $\beta = -2$, $\lambda = 20\%$, $\delta = 20\%$, $m = -0.2$, $S_0 = 1$, $r = 5\%$, $\varepsilon_1 = 1$, $\varepsilon_3 = 0$, $\varepsilon_4 = 1$, the number of exercise dates is 10 and $T = 1$. Again the method performs accurately, but for out-of- and at-the money strikes the approximation tends to under- and over-estimate the LSM value.

Table 1.3 Prices for a European and a Bermudan put option (10 exercise dates, expiry $T = 1$) in the CEV-like model with state-dependent measure for the 2nd-order approximation characteristic function, and a Monte Carlo method.

	European		Bermudan	
K	MC 95% c.i.	Value	MC 95% c.i.	Value
0.8	0.01025-0.01086	0.009385	0.01068-0.01125	0.01024
1	0.04625-0.04745	0.04817	0.05141-0.05253	0.05488
1.2	0.1563-0.1582	0.1564	0.1942-0.1952	0.1952
1.4	0.3313-0.3334	0.3314	0.3927-0.3934	0.3930
1.6	0.5207-0.5229	0.5218	0.5919-0.5926	0.5920
1.8	0.7103-0.7124	0.7122	0.7906-0.7913	0.7910

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