# Appendices

## A Existence of the oligopoly equilibrium

The utility of consumers $x_{12}(p_1^*, p_2^*)$ and $x_{23}(p_2^*, p_3^*)$, under Assumption A1 is $V - c - \frac{1}{144}(3-2a)(15-26a)t$, which is positive if, and only if $V > c + \frac{1}{144}(3-2a)(15-26a)t$. The right-hand side of this inequality is the blue function in Figure 1. The utility of the consumers in 0 and 1, at the equilibrium oligopoly prices, is $V - c - \frac{1}{12}\left(8a^2 - 4a + 3\right)t$, which is positive if $V > c + \frac{1}{12}\left(8a^2 - 4a + 3\right)t$. The right-hand side of the foregoing inequality is the red line in Figure 1.

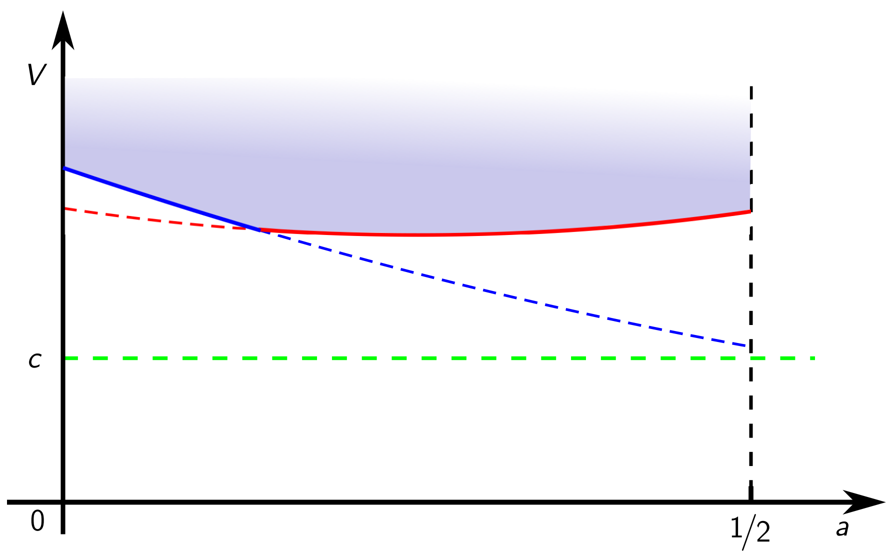


Figure 1: Region of existence of the oligopoly equilibrium, $c = .6, t = .8$.

Because both these conditions are necessary to have full market coverage at the Nash equilibrium prices, the desired condition is

$$V > \max\{c + \frac{1}{144}(3-2a)(15-26a)t, c + \frac{1}{12}\left(8a^2 - 4a + 3\right)t\} \tag{A.1}$$

This corresponds to the shaded region in the Figure. Notice that any marginal cost reduction following process R&D will lower the prices. Therefore, at the innovation stage the above condition is sufficient for market coverage.

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# B Asymmetric production costs.

A natural question is whether the symmetric-cost results are confirmed in the case where firms have different costs, in particular if the cost structure is such that the central firm has higher costs than the peripheral ones. In the following we are going to assume that the peripheral firms are characterized by the same marginal production cost $c_1 = c_2 = c$, and that the central one has a marginal production cost equal to $c_2 > c > 0$.[13]

**Assumption 2** (A2)**.** *$c_2 > c > 0$.*

Under A2 the advantage of the central location is countered by the higher marginal cost of the central firm. The prices set by the firms are

$$p_k(c, c_2, a) = \frac{1}{12}\left\{[3 - 4a(1+a)]t + 4c_2 + 8c\right\}, \text{ for } k \in \{1, 3\} \tag{A.2}$$

$$p_2(c, c_2, a) = \frac{1}{12}\{3 - [4a(2-a)]t + 4c + 8c_2\}. \tag{A.3}$$

Let $\Delta c = c_2 - c > 0$, then at the foregoing prices, the quantities sold by the firms are

$$q_k(\Delta c, a) = \frac{1}{12}\left[3 + 2a + \frac{4\Delta c}{(1-2a)t}\right], \text{ for } k \in \{1, 3\}, \quad q_2(\Delta c, a) = \frac{1}{6}\left[3 - 2a - \frac{4\Delta c}{(1-2a)t}\right] \tag{A.4}$$

It is apparent that, while $q_1(\Delta c, a)$ and $q_3(\Delta c, a)$ are always positive,

$$q_2(\Delta c, a) > 0 \Leftrightarrow \Delta c < \left[\frac{3}{4} - (2-a)a\right] t. \tag{A.5}$$

The equilibrium profits are

$$\pi_1(\Delta c, a) = \frac{\{[3 - 4a(a+1)]t + 4\Delta c\}^2}{144(1-2a)t}, \text{ for } k \in \{1, 3\} \quad \pi_2(\Delta c, a) = \frac{\{[3 - 4(2-a)a]t - 4\Delta c\}^2}{72(1-2a)t}. \tag{A.6}$$

[13]Like in the equal ex-ante costs case, the gross utility $V$ needs to be large enough to guarantee that, at the optimal prices the consumers at the extremes of the firms' demands enjoy a non-negative utility, so that the market is covered. The relative condition are qualitatively equivalent to those in the equal-cost case, though more involved, and are available upon request.

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By comparing $\pi_1(\Delta c, a)$ and $\pi_2(\Delta c, a)$ it is easily obtained that

$$\pi_1(\Delta c, a) > \pi_2(\Delta c, a) \Leftrightarrow \frac{1-2a}{4}\left(9-2a-6\sqrt{2}\right)t < \Delta c < \frac{1-2a}{4}\left(9-2a+6\sqrt{2}\right)t. \tag{A.7}$$

Direct inspection reveals that the upper bound in (A.7) is larger than the cutoff for the positivity of $q_2(\Delta c, a)$ as defined in (A.5). This means that as long as the central firm is active on the market, it reaps lower profits than the peripheral ones if its cost disadvantage $\Delta c$ is large enough for any value of $a$.

Consider the marginal incentives of firms to undertake process innovation. Under Assumption A2 they are

$$B_1(\Delta c, a) = \left|\frac{\partial \pi_1^*}{\partial c_1}\right|_{c_1=c_3=c} = B_3(\Delta c, a) = \left|\frac{\partial \pi_3^*}{\partial c_3}\right|_{c_1=c_3=c} = \frac{5}{72}\left[3+\frac{4\Delta c}{(1-2a)t}+2a\right], \tag{A.8}$$

$$B_2(\Delta c, a) = \left|\frac{\partial \pi_2^*}{\partial c_2}\right|_{c_1=c_3=c} = \frac{1}{9}\left[3-\frac{4\Delta c}{(1-2a)t}-2a\right], \tag{A.9}$$

which are all positive as long as (A.5) is satisfied. The difference in the marginal benefit from process innovation between the central and firm 1 is

$$\Delta B(\Delta c, a) = B_2(\Delta c, a) - B_1(\Delta c, a) = \frac{1}{72}\left[9-26a+\frac{52\Delta c}{(1-2a)t}\right]. \tag{A.10}$$

Easy computations reveal that (A.10) is positive for

$$\Delta c < \frac{1}{52}\left(52a^2-44a+9\right)t. \tag{A.11}$$

By combining the conditions in (A.5), (A.7) and (A.11) we obtain

**Proposition 4.** *Under A2 the central firm has a larger marginal benefit from profit innovation than the peripheral ones although it reaps a lower pre-innovation profit for*

$$\frac{1-2a}{4}\left(9-2a-6\sqrt{2}\right)t < \Delta c < \frac{1}{52}\left(52a^2-44a+9\right)t. \tag{A.12}$$

As for the case of equal costs for all firms, we consider the business stealing and size

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effects, which are, under A2:

$$B_1^{BS}(\Delta c, a) = B_1^{SE}(\Delta c, a) = \frac{5}{144}\left[3 + \frac{4\Delta c}{(1-2a)t} + 2a\right], \tag{A.13}$$

$$B_2^{BS}(\Delta c, a) = B_2^{SE}(\Delta c, a) = \frac{1}{18}\left[3 - \frac{4\Delta c}{(1-2a)t} - 2a\right]. \tag{A.14}$$

It is immediate to show that the business stealing effect and the size effect are larger for firm 2 than for firm 1 if, and only if

$$\Delta c < \frac{1}{52}\left(52a^2 - 44a + 9\right)t, \tag{A.15}$$

namely the same condition making the total benefit from a marginal cost reduction larger for firm 2. As long as $a < 9/26$, so that the central position of firm 2 is undisputed, this firm may still be more inclined to invest in innovation than the peripheral ones though it reaps a lower profit. However, this happens only if the cost disadvantage of that firm is low enough. Conversely, if $\frac{9}{26} < a < \frac{1}{2}$ the central firm has always lower incentives (although it *may* earn a larger profit) than the peripheral ones. Proposition 4 strengthens our claims that location in the product space is a key determinant of the innovation incentives. In fact, the cost disadvantage of the central firm drags down its profit relative to that of the peripheral ones. Nonetheless, so far as the cost disadvantage is not overly burdensome the central producer is still more inclined to invest in process innovation, provided that its centrality is not challenged.

Like in the case of identical *ex-ante* costs, a deeper analysis of the BS and SE effects reveals that the central firm exploits the reduction in marginal costs to increase its sales, rather than its mark-up. Interestingly, this holds true even if the cost disadvantage of the central firm makes its equilibrium demand lower than that of the peripheral firms. Niche firms channel product innovation into a larger margin, with a lower increase in sales.[14]

[14]The marginal effects $\left|\frac{\partial p_1^*}{\partial c_i} - 1\right|$ and $\left|\frac{\partial q_i^*}{\partial c_i}\right|$ are identical to these with equal *ex-ante* costs, because of the linearity of prices and quantities in own marginal costs. With asymmetric pre-innovation costs the margin of the niche firms are still larger than that of the central one: $p_i^* - c_i > p_2^* - c_2, i = 1, 3$. By contrast, and rather intuitively, the quantity of the niche firms *may be* larger than that of the central one, if the cost asymmetry is large: $q_i^* \gtreqless q_2^* \Leftrightarrow \Delta c \gtreqless \frac{(1-2a)^2}{4}$.

**Global innovation incentives.** It is a matter of algebra to observe that the global innovation incentives with asymmetric costs are

$$B(\Delta c, a) \equiv \sum_{i=1}^{3} B_i(\Delta c, a) = \frac{1}{36}\left(27 + 2a + \frac{4\Delta c}{(1-2a)t}\right). \tag{A.16}$$

It is immediate to conclude that, like in the equal-cost case, agglomeration increases the global innovation incentives. Here, in addition, we observe that the global innovation incentives are also increasing in the before-innovation cost differential $\Delta c$.[15]

A more efficient technology adds up to the positive effect of gaining market shares *via* an increase in $a$, whence the result.

As a last remark, it is worth pointing out that the message conveyed by the foregoing Proposition is valid so long as all the firms are active on the market. As we have already remarked (see inequality A.5), if $\Delta c$ is larger than a function of $a$ and $t$ the output of the central firm may vanish, and with it its profit and innovation incentives.

# C Linear transportation costs

In this case the utility of a consumer who purchases the good from firm $i$ is $V - p_i - td_i$. The indifferent consumers are now

$$x_{12} = \frac{1}{4} + \frac{a}{2} + \frac{p_1 - p_2}{2t}, \quad x_{23} = \frac{3}{4} - \frac{a}{2} + \frac{p_2 - p_3}{2t} \tag{A.17}$$

The demands and profits are obtained like in the quadratic cost case. The optimal prices, in this case are,

$$p_1^* = p_k(\boldsymbol{c}, a) = \quad [7c_1 + 4c_2 + c_3 + 2t(3+2a)]/12, \tag{A.18}$$
$$p_2^* = p_2(\boldsymbol{c}, a) = \quad [c_1 + 4c_2 + c_3 + t(3-2a)]/6, \tag{A.19}$$
$$p_3^* = p_3(\boldsymbol{c}, a) = \quad [c_1 + 4c_2 + 7c_3 + 2t(3+2a)]/12. \tag{A.20}$$

At these prices, the quantities are

[15] For the sake of precision, $\pi_1(\Delta c, a)$ and $\pi_3(\Delta c, a)$ are increasing in $a$ as long as $t < \max\{\frac{4\Delta c}{1+4a-12a^2}, \frac{4\Delta c}{3-4(2-a)a}\}$, beyond this threshold, transportation costs become so high that the reduction in these firms' prices due to increased agglomeration offsets the increase in demand. This notwithstanding, the marginal benefit from R&D of the niche firms remains positive (due to the increased sales) and more than compensates the central firm's incentive reduction.

$$
\begin{aligned}
q_1^* &= q_1(\boldsymbol{c}, a) = [-5c_1 + 4c_2 + c_3 + 2t(3+2a)]/(24t), && \text{(A.21)}\\
q_2^* &= q_2(\boldsymbol{c}, a) = [c_1 - 2c_2 + c_3 + t(3-2a)]/(6t), && \text{(A.22)}\\
q_3^* &= q_3(\boldsymbol{c}, a) = [c_1 + 4c_2 - 5c_3 + 2t(3+2a)]/(24t). && \text{(A.23)}
\end{aligned}
$$

Finally, the equilibrium profits are

$$
\begin{aligned}
\pi_1^* &= \pi_1(\boldsymbol{c}, a) = \frac{(1/8)[-5c_1 + 4c_2 + c_3 + 2t(3+2a)]^2}{36t}, && \text{(A.24)}\\
\pi_2^* &= \pi_2(\boldsymbol{c}, a) = \frac{[c_1 - 2c_2 + c_3 + t(3-2a)]^2}{36t}, && \text{(A.25)}\\
\pi_3^* &= \pi_3(\boldsymbol{c}, a) = \frac{(1/8)[c_1 + 4c_2 - 5c_3 + 2t(3+2a)]^2}{36t}. && \text{(A.26)}
\end{aligned}
$$

Like in the case of quadratic transportation costs, we assume that the process innovation costs are equal across firms, so we evaluate the impact of a marginal change in $c_i$ (obviously it suffices to compare firms 1 and 2):

$$
\begin{aligned}
B_1(\boldsymbol{c}, a) = \left|\frac{\partial \pi_1^*}{\partial c_1}\right| &= \frac{5[-5c_1 + 4c_2 + c_3 + 2t(3+2a)]}{144t}, && \text{(A.27)}\\
B_2(\boldsymbol{c}, a) = \left|\frac{\partial \pi_2^*}{\partial c_2}\right| &= \frac{c_1 - 2c_2 + c_3 + t(3-2a)}{9t}. && \text{(A.28)}
\end{aligned}
$$

It is a matter of simple algebra to ascertain that the difference in the investment incentives between the central firm and –say– firm 1 is

$$
\Delta B(\boldsymbol{c}, a) = B_2(\boldsymbol{c}, a) - B_1(\boldsymbol{c}, a) = \frac{1}{72}\left(D(\boldsymbol{c}) + 9 - 26a\right), \tag{A.29}
$$

where $D(\boldsymbol{c}) \equiv \frac{41c_1 - 52c_2 + 11c_3}{t(1-2a)}$, so it clearly coincides with (14).

Furthermore, simple computations allow to ascertain that, under Assumption A1 we have $\pi_2(c, a) \gtreqless \pi_1(c, a) \Leftrightarrow a \lesseqgtr \frac{1}{2}\left(9 - 6\sqrt{2}\right)$. These observations immediately entail:

**Corollary 3.** *Under Assumption A1, Lemma 1, Proposition 1 and Corollary 1 hold with linear transportation costs too.*

The underlying intuitions are the same as well.[16]

[16] The algebra behind the results corresponding to Lemma 1 and Proposition 1 is unchanged relative to the quadratic case, that of Lemma 1 is slightly different, but completely equivalent.

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If we consider the case of asymmetric production costs (Assumption A2) we obtain the same results of the quadratic cost case, with a simpler algebra. We summarize them in the following

**Corollary 4.** *Under A2 the central firm has a larger marginal benefit from profit innovation than the peripheral ones although it reaps a lower pre-innovation profit for*

$$\frac{9-2a-6\sqrt{2}-2a}{2} < \Delta c < \frac{9t-26(at+\Delta c)}{72t}. \tag{A.30}$$

The proof follows the same steps of the quadratic-cost case.

To conclude, let us consider the aggregate private incentives toward process innovation. As in the quadratic case, they are the sum of the individual ones.

With linear transportation costs, the total marginal incentive is

$$B(\Delta c, a) = \frac{(27+2a)t+22\Delta c}{36t}. \tag{A.31}$$

Like in the quadratic case, the benefit is increasing in the level of agglomeration $a$ and in the pre-innovation technological asymmetry of the firms $\Delta c$.

# D Non-address product differentiation

## D.1 Asymmetric competition

Consider the following demand system with $0<\gamma<\beta<1<\alpha$, and $\alpha\gamma>\beta$:

$$\begin{pmatrix} q_1(\boldsymbol{p}) \\ q_2(\boldsymbol{p}) \\ q_3(\boldsymbol{p}) \end{pmatrix} = \begin{pmatrix} -\alpha\gamma & \beta & \gamma \\ \beta & -\alpha\beta & \beta \\ \gamma & \beta & -\alpha\gamma \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} + \begin{pmatrix} v \\ v \\ v \end{pmatrix}. \tag{A.32}$$

Products are differentiated and substitutable, however, competition is not symmetrical among the three firms. Indeed, we posit that competition between products 1 and 2, and 3 and 2, is more intense than competition between products 1 and 3. In particular, the matrix of the cross-price derivatives implies that products 2 is a "closer" substitute of products 1 and 3, the substitutability between 1 and 3 being instead lower.

The parameter $\gamma$ captures the extent of this "lower" substitutability. Indeed, a marginal decrease in –say– $p_1$, $dp_1$, corresponds to an marginal decrease $\beta dp_1$ in $q_2$, the "close" substitute's quantity, but only of $\gamma dp_1$ in the "distant" substitute's quantity $q_3$. On the other hand, a marginal increase in own price has a larger impact $(-\alpha\beta)$ for good 2 than for goods 1 and 3 $(-\alpha\gamma)$. This is consistent with the idea that an increase in $p_2$ displaces the demand of firm 2 away from its good more than does an increase of $p_1$ on the demand of firm 1 (and $p_2$ on that of good 2). This assumption "mimics" the idea that, loosely speaking, good 2 is directly competing against goods 1 and 3, while the latter only compete with good 2. It should be noted that, although $\gamma$ directly determines the extent of competitive pressure between firms 1 and 3, it also indirectly influences the pressure on firm 2 through strategic interaction.

Firms bear constant marginal production costs $v > c_i > 0, \forall i \in \{1, 2, 3\}$ The firms' second-period profits are thus

$$\pi_i(\boldsymbol{p}) = q_1(\boldsymbol{p})(p_i - c_i) \tag{A.33}$$

By means of the first-order conditions we obtain the best replies for firm $i$, with $i, j \in \{1, 3\}, i \neq j$, and firm 2:

$$p_i(\boldsymbol{p_{-i}}) = \frac{c_i}{2} + \frac{v + \beta p_2 + \gamma p_j}{2\alpha\gamma}, \quad p_2(\boldsymbol{p_{-2}}) = \frac{c_2}{2} + \frac{v + \beta(p_1 + p_3)}{2\alpha\beta}. \tag{A.34}$$

Let $\boldsymbol{\delta} = (\alpha, \beta, \gamma)$, standard computations then yield the equilibrium prices:

$$\begin{aligned} p_i^*(\boldsymbol{c}, \boldsymbol{\delta}) =& \frac{\alpha\{\{\beta[-c_i + (1 + 2\alpha)c_2 + c_j]\} + 2\alpha\gamma(2\alpha c_1 + c_j)\} + v(2\alpha + 1)^2}{2\left(4\alpha^3\gamma - 2\alpha\beta - \alpha\gamma - \beta\right)}, \\ p_2^*(\boldsymbol{c}, \boldsymbol{\delta}) =& \frac{v(2\alpha\gamma + 2\beta - \gamma) + \alpha\beta\gamma(c_1 + 2\alpha c_2 - c_2 + c_3)}{2\beta[\alpha\gamma(2\alpha - 1) - \beta]}. \end{aligned} \tag{A.35}$$

At these prices, the quantities are:

$$\begin{aligned} q_i^*(\boldsymbol{c}, \boldsymbol{\delta}) =& \frac{\alpha\gamma\left\{v(1 + 2\alpha)^2 + \alpha\beta[c_j + (1 + 2\alpha)] + 2\alpha\gamma\left[\alpha c_j + (1 - 2\alpha^2)c_i\right] + (3\alpha + 2)\beta c_i\right\}}{2\left(4\alpha^3\gamma - 2\alpha\beta - \alpha\gamma - \beta\right)}, \\ q_2^*(\boldsymbol{c}, \boldsymbol{\delta}) =& \frac{v\alpha(2\alpha\gamma + 2\beta - \gamma) + \alpha\beta[\alpha\gamma(c_1 - 2\alpha c_2 + c_2 + c_3) + 2\beta c_2]}{2[(\alpha(2\alpha - 1)\gamma\beta]}. \end{aligned} \tag{A.36}$$

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The equilibrium profits write:

$$\pi_i^*(\boldsymbol{c},\boldsymbol{\delta}) = \frac{\alpha\gamma\left\{v(1+2\alpha)^2+\alpha\beta[c_j+(1+2\alpha)]+2\alpha\gamma\left[\alpha c_j+(1-2\alpha^2)c_i\right]+(3\alpha+2)\beta c_i\right\}^2}{4\left(4\alpha^3\gamma-2\alpha\beta-\alpha\gamma-\beta\right)^2} = \frac{(q_i^*(\cdot))^2}{\alpha\gamma},$$

$$\pi_2^*(\boldsymbol{c},\boldsymbol{\delta}) = \frac{\{v\alpha(2\alpha\gamma+2\beta-\gamma)+\alpha\beta[\alpha\gamma(c_1-2\alpha c_2+c_2+c_3)+2\beta c_{]}\}^2}{4\alpha\beta[(\alpha(2\alpha-1)\gamma\beta]^2} = \frac{(q_2^*(\cdot))^2}{\alpha\beta}. \quad \text{(A.37)}$$

We now define the marginal benefit from process innovation for firm $i=1,3$ as

$$\begin{aligned} B_i(\boldsymbol{c},\boldsymbol{\delta}) &= \left|\frac{\partial\pi_i(\cdot)}{\partial c_i}\right| = \alpha\gamma\left[4\alpha^3\gamma-(3\alpha+2)\beta-2\alpha\gamma\right]\times \\ &\times\frac{[v(1+2\alpha)^2+2\beta c_i+\alpha\beta(3c_i+2\alpha c_2+c_2+c_j)-2\alpha\gamma\left(2\alpha^2c_i-c_i-\alpha c_j\right)]}{2\left(4\alpha^3\gamma-2\alpha\beta-\alpha\gamma-\beta\right)^2} = \\ &= \frac{4\alpha^3\gamma-3\alpha\beta-2\alpha\gamma-2\beta}{4\alpha^3\gamma-2\alpha\beta-\alpha\gamma-\beta}q_i^*(\cdot), \end{aligned} \quad \text{(A.38)}$$

and, for firm 2:

$$\begin{aligned} B_2(\boldsymbol{c},\boldsymbol{\delta}) &= \left|\frac{\partial\pi_2(\cdot)}{\partial c_2}\right| = \\ &= \frac{\{\alpha[\alpha(1-2\alpha)\gamma+2\beta][v(1-2\alpha)\gamma-2\beta(v+\beta c_2)+\alpha\beta\gamma(c_1-(2\alpha-1)c_2)+c_3]\}}{2[\alpha(2\alpha-1)\gamma-\beta]^2} = \\ &= \frac{\alpha(2\alpha-1)\gamma-2\beta}{\alpha(2\alpha-1)\gamma-\beta}q_2^*(\cdot). \end{aligned} \quad \text{(A.39)}$$

In order keep the algebra manageable, in the following, we make

**Assumption 3** (A3)**.** $c_i = 0\forall i = 1,2,3$.

We state two preliminary results.

**Lemma 2.** *Under Assumption A3*

*(i) The equilibrium prices, quantities and profits of all the firms are positive.*

*(ii) The profit of firm 2 is larger than that of firm $i=1,3$ if, and only if, $\gamma < \frac{4\beta}{(2\alpha-1)^2} \equiv \gamma^{\pi}$.*

*Proof.* (i) The equilibrium prices, quantities and profits under Assumption A3 are,

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respectively:

$$p_1^*(0,\boldsymbol{\delta}) = p_3^*(0,\boldsymbol{\delta}) = \frac{v(2\alpha+1)}{2[\alpha(2\alpha-1)\gamma-\beta]}, \quad p_2^*(0,\boldsymbol{\delta}) = \frac{v(2\alpha\gamma+2\beta-\gamma)}{2\beta[(\alpha(2\alpha-1)\gamma-\beta]}, \tag{A.40}$$

$$q_1^*(0,\boldsymbol{\delta}) = q_3^*(0,\boldsymbol{\delta}) = \frac{v\alpha\gamma(2\alpha+1)}{2[\alpha\gamma(2\alpha-1)-\beta]}, \quad q_2^*(0,\boldsymbol{\delta}) = \frac{v\alpha(2\alpha\gamma+2\beta-\gamma)}{2[\alpha(2\alpha-1)\gamma-\beta]}, \tag{A.41}$$

and

$$\pi_1^*(0,\boldsymbol{\delta}) = \pi_2^*(0,\boldsymbol{\delta}) = \frac{v^2\alpha\gamma(2\alpha+1)^2}{4[\alpha(2\alpha-1)\gamma-\beta]^2}, \quad \pi_2^*(0,\boldsymbol{\delta}) = \frac{\alpha[v(2\alpha-1)\gamma+2v\beta]^2}{4\beta[\alpha(2\alpha-1)\gamma-\beta]^2}. \tag{A.42}$$

Direct inspection reveals that they are all positive.

(ii) The difference between the profit of firm 2 and –say– 1 is

$$\pi_2^*(0,\boldsymbol{\delta}) - \pi_1^*(0,\boldsymbol{\delta}) = \frac{v^2\alpha(\beta-\gamma)\left[4\beta-(2\alpha-1)^2\gamma\right]}{4\beta[\alpha(2\alpha-1)\gamma-\beta]^2}, \tag{A.43}$$

and its sign matches that of the term in the square brackets in the numerator, which is positive if, and only if, $\gamma < \frac{4\beta}{(2\alpha-1)^2} \equiv \gamma^\pi$.

□

**Lemma 3.** *Under A3,*

*(i) The innovation benefit of firm $i = 1, 3$ is positive if $\gamma > \frac{(3\alpha+2)\beta}{2\alpha(2\alpha^2-2)} \equiv \gamma_1^+$, the innovation benefit of firm 2 is positive if and only if, $\gamma > \frac{2\beta}{\alpha(2\alpha-1)} \equiv \gamma_2^+$.*

*(ii) The innovation benefit of firm 2 is larger than that of firm $i = 1, 3$ if, and only if, $\gamma > \frac{4\beta}{\alpha(4\alpha-3)} \equiv \gamma^B$.*

*(iii) $\gamma^B > \gamma_i^+$ and $\gamma^B > \gamma_2^+$.*

*Proof.* (i) We start by firms 1 and 3. The marginal benefit of firm $i = 1, 3$, under A3, is

$$B_i(0,\boldsymbol{\delta}) = \frac{v\alpha\gamma\left[2\alpha(2\alpha^2-2)\gamma-(3\alpha+2)\beta\right]}{2[\alpha(2\alpha-1)\gamma-\beta]^2} \tag{A.44}$$

The sign of the above expression coincides with that of the term within square brackets in the numerator, which is positive so long as $\gamma > \frac{(3\alpha+2)\beta}{2\alpha(2\alpha^2-2)}$.

Move now to the marginal benefit of firm 2 under A3, which is:

$$B_2(0, \boldsymbol{\delta}) = \frac{v\alpha[(2\alpha - 1)\gamma + 2\beta][\alpha(2\alpha - 1)\gamma - 2\beta]}{2[\alpha(2\alpha - 1)\gamma - \beta]^2} \tag{A.45}$$

It is evident that the sign of the benefit matches that of the second term in square brackets in the numerator, which is positive so long as $\gamma > \frac{2\beta}{\alpha(2\alpha-1)}$.

(ii) The difference in the innovation benefit of firm 2 and 1 is

$$B_2(0, \boldsymbol{\delta}) - B_1(0, \boldsymbol{\delta}) = \frac{v\alpha(\beta - \gamma)[\alpha(4\alpha - 3)\gamma - 4\beta]}{2[\alpha(2\alpha - 1)\gamma - \beta]^2}. \tag{A.46}$$

Like above, the sign of (A.46) is determined by term in the square brackets in the numerator, which is positive if, and only if, $\gamma > \frac{4\beta}{\alpha(4\alpha-3)}$.

(iii) Follows from direct comparison in the admissible parameter constellation.

□

The combination of the foregoing Lemmata yields

**Corollary 5.** *Under Assumption A3,*

*(i)* $\gamma_2^+ < \gamma^B < \gamma^\pi$;

*(ii)* $\gamma_i^+ < \gamma^B$.

This ultimately entails

**Proposition 5.** *(i) For* $\max\{\gamma^+, \gamma_2^+\} < \gamma < \gamma_B$ *firms 1 and 3 have larger innovation incentives, although reaping lower profits, than firm 2.*

*(ii) For* $\alpha^B < \gamma < \gamma^\pi$ *firm 2 has larger innovation incentives, and reaps a larger profit, than firms 1 and 3.*

*(iii) For* $\gamma > \gamma^\pi$ *firm 2 has larger innovation incentives, although reaping a lower profit, than firms 1 and 3.*

The foregoing Proposition confirms that there is no direct relationship between pre-innovation profit and incentives to innovate. Inspection of (A.38) and (A.39) also suggests that there is no direct relationship either between the sales volume and the incentives to innovate.

We now delve into the business stealing and size effects. Simple, but tedious, calculations show that they coincide for each firm.

$$\begin{aligned}
B_i^{BS}(\boldsymbol{c},\boldsymbol{\delta}) &\equiv \left|(p_i^*(\cdot)-c_i)\frac{\partial q_i^*(\cdot)}{\partial c_i}\right| = B_i^{SE}(\boldsymbol{c},\boldsymbol{\delta}) = \left|q_i^*(\cdot)(\frac{\partial p_i(\cdot)}{\partial c_i}-1)\right| = \\
=&\frac{\pi_i^*(\boldsymbol{c},\boldsymbol{\delta})[4\alpha^3\gamma-(3\alpha+2)\beta-2\alpha\gamma]}{v(2\alpha+1)^2+2\beta c_i+\alpha\beta[3c_i+(2\alpha+1)c_2+c_j]+2\alpha\gamma\left[(2\alpha^2+1)c_i-\alpha c_j\right]}.
\end{aligned} \tag{A.47}$$

$$\begin{aligned}
B_2^{BS}(\boldsymbol{c},\boldsymbol{\delta}) &\equiv \left|(p_2^*(\cdot)-c_2)\frac{\partial q_2^*(\cdot)}{\partial c_2}\right| = B_2^{SE}(v,\boldsymbol{c},\gamma) = \left|q_2^*(\cdot)(\frac{\partial p_2(\cdot)}{\partial c_2}-1)\right| = \\
=&\frac{\pi_2^*(\boldsymbol{c},\boldsymbol{\delta})\beta[\alpha(2\alpha-1)\gamma-2\beta]}{v(2\alpha-1)\gamma+2\beta(v+\beta c_2)+\alpha\beta\gamma(c_1-(2\alpha-1)c_2+c_3)}.
\end{aligned} \tag{A.48}$$

We state

**Proposition 6.** *Under A3 the business stealing and size effect are larger for the firm enjoying the larger marginal benefit from innovation.*

*Proof.* The difference between, say, the business stealing effect of firms 2 and 1, under zero ex-ante marginal costs is

$$B_2^{BS}(0,\boldsymbol{\delta})-B_1^{BS}(0,\boldsymbol{\delta}) = \frac{v\alpha(\beta-\gamma)[\alpha(4\alpha-3)\gamma-4\beta]}{4[\alpha(2\alpha-1)\gamma-\beta]^2}. \tag{A.49}$$

which is half of the difference between the marginal benefits. □

This result confirms the findings of the Hotelling framework, in that it shows that the BS and SE effects are both larger for the firm with the largest marginal benefit from innovation.

Last, we turn to the global innovation incentives. Like in the Hotelling case, we define the aggregate innovation incentives as:

$$B(\boldsymbol{c},\boldsymbol{\delta}) = \sum_{i=1}^{3} B_i(\boldsymbol{c},\boldsymbol{\delta}) = \frac{\alpha}{2(2\alpha+1)[\alpha(1-2\alpha)\gamma+\beta]^2}(\Phi+\Psi+\Xi), \tag{A.50}$$

with $\Phi \equiv v(2\alpha+1)\left\{\alpha[4\alpha(3\alpha-1)-3]\gamma^2-2[2(3-\alpha)\alpha-1]\beta\gamma-4\beta^2\right\}$, $\Psi \equiv \gamma(c_1+c_3)\left\{4(\alpha-1)\alpha^2\left(2\alpha^2-1\right)\gamma^2+4(\alpha+1)^2\beta^2+\alpha\{\alpha[3-2\alpha(2\alpha+7)]+8\}\beta\gamma\right\}$ and

$\Xi \equiv \beta c_2 \left\{\alpha^2 \left[2\alpha \left(4\alpha^2 - 6\alpha - 1\right) + 5\right] \gamma^2 + (8\alpha + 4)\beta^2 + 2\alpha[\alpha(3 - 8\alpha) + 4]\beta\gamma\right\}$.

To study the effects of increased competition (equivalent to the shift toward the center of the segment of the niche firms), we shall consider the impact of an increase in $\gamma$ on the global innovation incentives. Unfortunately the first-order partial derivative of $B(\cdot)$ w.r.t. $\gamma$ is uninformative, due to its cumbersomeness. However, some insights may be drawn by restricting the attention to the case of zero pre-innovation marginal costs. We have:

$$\frac{\partial B(0, \boldsymbol{\delta})}{\partial \gamma} = \frac{a\alpha\beta\left[\left(6\alpha^2 + 2\alpha + 1\right)\beta - 2\alpha\left(2\alpha^3 - \alpha^2 - 1\right)\gamma\right]}{[\alpha(2\alpha - 1)\gamma + \beta]^3}, \tag{A.51}$$

We state

**Proposition 7.** *Under Assumption A3, the aggregate innovation incentives are increasing in the competitiveness of the market as long as*

$$\frac{\gamma}{\beta} < \frac{6\alpha^2 + 2\alpha + 1}{(1 - \alpha)\alpha\left(2\alpha^2 + \alpha + 1\right)} \equiv b(\alpha). \tag{A.52}$$

Simple computations show that $b(\alpha)$ tends to infinity if $\alpha \to 1$, that it is always positive and decreasing for $\alpha > 1$ and that it tends to zero for $\alpha \to \infty$. Because $\gamma < \beta$ by assumption, a necessary condition for the non-emptiness of the foregoing space is that $b(\alpha) < 1$, which in turn requires that $\alpha > 2.21$. Incidentally, notice that $\alpha > 2.21$ is sufficient to insure that all the innovation benefits are positive under A3.

The economic intuition behind the foregoing Proposition is that increased competition, here represented by goods 1 and 3 becoming more substitutable with each other is beneficial for innovation only if (i) the own-price effects are large enough, and (ii) products remain differentiated enough. Because $b(\alpha)$ is decreasing, the stronger are the cross price effects, the larger the differentiation must remain in order to stimulate aggregate innovation incentives.

## D.2 Symmetric competition

Consider the following demand system with $\gamma \in ]0, 1[$:

$$\begin{pmatrix} q_1(\boldsymbol{p}) \\ q_2(\boldsymbol{p}) \\ q_3(\boldsymbol{p}) \end{pmatrix} = \begin{pmatrix} -1 & \gamma & \gamma \\ \gamma & -1 & \gamma \\ \gamma & \gamma & -1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}. \tag{A.53}$$

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Products are differentiated and substitutable. Firms bear constant marginal production costs $c_i > 0$ such that $v_i > c_i > 0 \forall i \in \{1, 2, 3\}$ The firms' second-period profits are thus

$$\pi_i(\boldsymbol{p}) = q_1(\boldsymbol{p})(p_i - c_i) \tag{A.54}$$

By means of the first-order conditions we obtain the best replies for firm $i$, with $i, j, k \in \{1, 2, 3\}, i \neq j \neq k$:

$$p_i(\boldsymbol{p_{-i}}) = \frac{v_i + c_i + \gamma(p_j + p_k)}{2}. \tag{A.55}$$

Standard computations then yield the equilibrium prices:

$$p_i^*(\boldsymbol{c}, \boldsymbol{v}, \gamma) = \frac{(2-\gamma)(v_i + c_i) + \gamma(v_j + a_k + c_j + c_k)}{2\left(2 - \gamma - \gamma^2\right)}. \tag{A.56}$$

At these prices, the quantities are:

$$q_i^*(\boldsymbol{c}, \boldsymbol{v}, \gamma) = \frac{(2-\gamma)v_i + \gamma(v_j + a_k + c_j + c_k) - (2 - \gamma - 2\gamma^2)c_i}{2(2 - \gamma - \gamma^2)}. \tag{A.57}$$

In the ensuing analysis, we are going to make the following

**Assumption 4** (A4)**.** $\gamma < \frac{\sqrt{17}-1}{4} \approx .78$.

This assumption insures that the equilibrium quantities are decreasing in their own marginal cost; intuitively, it imposes an upper bound to the substitutability among products, which, in turn, moderates the competitive pressure in the market.[17]

The equilibrium profit of firm $i$ writes:

$$\pi_i^*(\boldsymbol{c}, \boldsymbol{v}, \gamma) = \frac{\left[(2-\gamma)v_i + \gamma(v_j + a_k + c_j + c_k) - (2 - \gamma - 2\gamma^2)c_i\right]^2}{4\left(2 - \gamma - \gamma^2\right)^2} = \left(q_i^*\right)^2. \tag{A.58}$$

We now define the marginal benefit from process innovation as

$$\begin{aligned} B_i(\boldsymbol{c}, \boldsymbol{v}, \gamma) = \left|\frac{\partial \pi^*}{\partial c_i}\right| =& \frac{(2 - \gamma - 2\gamma^2)\left[(2-\gamma)v_i + \gamma(v_j + a_k + c_j + c_k) - (2 - \gamma - 2\gamma^2)c_i\right]}{2\left(2 - \gamma - \gamma^2\right)^2} = \\ =& \Gamma\sqrt{\pi_i^*(\boldsymbol{c}, \boldsymbol{v}, \gamma)}, \end{aligned} \tag{A.59}$$

[17] If this condition is violated, then an increase in $c_1$ leads to a direct increase in $p_i$, and to an indirect increase in $\boldsymbol{p_{-i}}$ due to the strategic interaction. The first effect reduces the demand for good $i$, while the second boosts it. The high product subsititutability however entails that this strategic increase of the rivals' prices is large, which more than offsets the negative demand effect of the increase in own price.

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with $\Gamma \equiv \frac{2-\gamma-2\gamma^2}{2-\gamma-\gamma^2} > 0$ under A4. We state

**Proposition 8.** *Under Assumption A4, firm i's marginal benefit from process innovation is larger than firm j's one if, and only if, its pre-innovation profit is also larger.*

*Proof.* Directly follows from the fact that

$$B_i(\cdot) - B_j(\cdot) = \Gamma\left(\sqrt{\pi_i^*(\cdot)} - \sqrt{\pi_j^*(\cdot)}\right). \tag{A.60}$$

□

The message conveyed by this Proposition significantly diverges from the one of Propositions 1 and 2, which maintain that, under spatial differentiation the central firm may have larger incentives than the niche ones, even if reaping lower profits. This cannot happen under the "unlocalized" product differentiation of this Section. In fact, here firms are by construction affected symmetrically by changes in the rivals' prices, so that e.g., a drop in $p_1$ directly (and symmetrically) reduces $q_2$ and $q_3$, and the identity of the firm is immaterial as far as a price's impact on own demand. In the Hotelling setup, by contrast, a reduction in $p_1$ affects indirectly $q_3$ only through the (direct) effect it has on $p_2$. Furthermore, an increase in $p_2$, the "central" price, reduces $q_2$ more than the same increase of $p_1$ (the "niche" price) does on $q_1$, or $p_3$ on $q_3$.

Because $\sqrt{\pi_i^*(\cdot)} = q_i^*(\cdot)$ the foregoing Proposition implies

**Corollary 6.** *Under A4 sales volumes are positively correlated with innovation incentives.*

This also differs from the Hotelling results. There, with symmetric production costs, the central firm always has a larger output than the niche ones, irrespective of the relative innovation incentives. Asymmetric marginal costs lead to the same outcome, provided that the cost disadvantage of the central firm is not overly large.

We now delve into the business stealing and size effects. Simple, but tedious, calculations show that they coincide:

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$$
\begin{aligned}
B_i^{BS}(\boldsymbol{c},\boldsymbol{v},\gamma) \equiv \left|(p_i^*(\cdot)-c_i)\frac{\partial q_i^*(\cdot)}{\partial c_i}\right| &= B_i^{SE}(\boldsymbol{c},\boldsymbol{v},\gamma) = \left|q_i^*(\cdot)(\frac{\partial p_i(\cdot)}{\partial c_i}-1)\right| = \\
&= \frac{(2-\gamma-2\gamma^2)\left[(2-\gamma)v_i+\gamma(v_j+a_k+c_j+c_k)-(2-\gamma-2\gamma^2)c_i\right]}{4\left(\gamma^2+\gamma-2\right)^2} = \\
&= \frac{\Gamma}{2}\sqrt{\pi_i^*(\cdot)}.
\end{aligned} \tag{A.61}
$$

We can thus conclude that

**Proposition 9.** *Under A4 the business stealing and size effect are larger for the firm reaping the larger profit, thus enjoying the larger marginal benefit from innovation.*

This result partly confirms the findings of the Hotelling framework, in that it shows that the BS and SE effects are both larger for the firm with the largest marginal benefit from innovation. With non-address differentiation, however, a larger marginal benefit corresponds to a lager profit, which ultimately implies that, again, the firm with the larger pre-innovation profits also benefits more from the business stealing and size effects. This again confirms the pivotal role of volumes and profits in assessing the incentives to innovate of firms engaged in non-localized competition.

Last, we turn to the global innovation incentives. Like in the Hotelling case, we define the aggregate innovation incentives as:

$$
B(\boldsymbol{c},\boldsymbol{v},\gamma) = \sum_{i=1}^{3} B_i(\boldsymbol{c},\boldsymbol{v},\gamma) = \frac{(2-\gamma-2\gamma^2)\left[a_1+a_2+a_3-(1-2\gamma)(c_1+c_2+c_3)\right]}{2(1-\gamma)^2(\gamma+2)}, \tag{A.62}
$$

which are clearly non-negative under A4. To study the effects of increased competition (equivalent to the shift toward the center of the segment of the niche firms), we shall consider the effects of an increase in $\gamma$ on the global innovation incentives. Unfortunately the first-order partial derivative of $B(\cdot)$ w.r.t. $\gamma$ is uninformative, due to its cumbersomeness. However, some insights may be drawn by restricting the attention to the symmetric case, where $v_i = a$ and $c_i = c \forall i \in \{1,2,3\}$. In this case, we have:

$$
\frac{\partial B(a,c,\gamma)}{\partial \gamma} = \frac{3a\left\{2-\gamma[2+\gamma(\gamma+2)]\right\}-3c[\gamma(7\gamma-2)-2]}{(1-\gamma)^3(2+\gamma)^2}. \tag{A.63}
$$

Unlike the Hotelling case, an increase in the competitiveness of the market has not a univocal effect on the aggregate innovation incentives. In fact, tedious algebra yields

**Proposition 10.** *Under A4, the aggregate innovation incentives are increasing in $\gamma$ if, and only if $\gamma < .644$.*

This result should be contrasted with the one obtained with spatial competition. There, so long as all the firms are active on the market, increased agglomeration, which results in fiercer competition, raises the aggregate innovation incentives. Here, instead this holds true insofar the substitutability parameter $\gamma$ (whence the competitive pressure) is low enough.

# E Mergers

## E.1 Merger between niche firm 1 and central firm 2

Following a merger between the central firm 2 and the niche firm 1, the merged entity's profit is

$$\pi_{12}(\boldsymbol{p}, c_1, c_2) = \pi_1(p_1, p_2, c_1) + \pi_2(\boldsymbol{p}, c_2), \tag{A.64}$$

while the profit of firm 3 is as in (10). By taking the FOCs and solving them for the appropriate price we get, with a slight abuse of terminology, the best replies:

$$\begin{aligned} p_1(p_2; c_1, c_2) &= \frac{1}{8}\left[4c_1 - 4c_2 + 8p_2 + t(1-4a^2)\right], & \text{(A.65)} \\ p_2(p_1, p_3; c_1, c_2) &= \frac{1}{8}\left[(1-2a)^2 t - 2c_1 + 4c_2 + 4p_1 + 2p_3\right], & \text{(A.66)} \\ p_3(p_2; c_3) &= \frac{1}{8}\left(4c_3 + t - 4a^2 t + 4p_2\right). & \text{(A.67)} \end{aligned}$$

Clearly, (A.67) coincides with the best reply of firm 3 in the non-merger scenario, (A.65) is the optimal price of the niche good, which is not exposed to "external" competition and (A.66) is the best reply of the merged firm against good 3, which also accounts for the price of good 1, part of the product array of the merged entity. The solution to the system of first-order conditions yields the following equilibrium prices:[18]

$$
\begin{aligned}
p_1^{1+2}(\boldsymbol{c}, a) &= \frac{1}{24}\{[17-4a(a+8)]t+12c_1+4c_2+8c_3\}, &\text{(A.68)}\\
p_2^{1+2}(c_2, c_3, a) &= \frac{1}{12}\{([4(a-4)a+7]t+8c_2+4c_3\}, &\text{(A.69)}\\
p_3^{1+2}(c_2, c_3, a) &= \frac{1}{12}\{[5-4a(a+2)]t+4c_2+8c_3\}, &\text{(A.70)}
\end{aligned}
$$

which are all positive under the model's assumptions. Contrary to the non-merger case, the optimal prices of the central good 2 and of good 3 only depend on $c_2$ and $c_3$, but *not* on $c_1$. The intuition can be grasped by inspecting the fist-order conditions for profit maximization of the merged entity. The prices (A.65) and (A.66) positively depend on the other good's price and on own marginal cost, but also *negatively* on the other product's marginal cost. With this in mind, let us spell out the two forces at work here. The first one is related to strategic complementarity: an increase in –say– $c_1$ makes the optimal $p_1$ increase, which, in turn causes an increase in $p_2$. However, the best reply function $p_2$ is shifted downward by an increase in $c_1$. This is so because the merged entity wants to serve the consumers located between products 1 and 2 at the lowest cost, so if $c_1$ increases, it wants to divert consumers from good 1 to good 2, and it does so by optimally lowering $p_2$. This second force counters the one driven by strategic complementarity and, in our model, they precisely offset each other, such that any increase in $c_1$, which would otherwise lead to a strategic increase in $p_2$, is exactly counterbalanced by a decrease in $p_2$ aimed at serving consumers at the lowest cost. Consequently, changes in $c_1$ do not translate into changes in $p_2$. Because $p_2$ is the channel conveying the effects of $c_1$ on $p_3$,the equilibrium value of $p_3$ is not a function of $c_1$ either –contrary to what happens in our Spatial Model in Section 2.

By comparison with the pre-merger equilibrium (as in Section 2 for the Hotelling case) it can be checked that at symmetric costs the niche firm 3 sets a lower price than the product of the central firm, while $p_2 < p_3$ in the pre-merger equilibrium, see (8) and (9).

It is a matter of simple algebra to ascertain that –as expected- all the post-merger

[18]The profit function of the merged firms is jointly concave in $p_1$ and $p_2$.

prices are higher than their non-merger counterparts. The corresponding quantities are

$$q_1^{1+2}(\boldsymbol{c},a) = \frac{1}{8}\left[1+2a+\frac{4(c_2-c_1)}{t(1-2a)}\right], \tag{A.71}$$

$$q_2^{1+2}(\boldsymbol{c},a) = \frac{1}{24}\left[11-10a+\frac{4(3c_1-5c_2+2c_3)}{t(1-2a)}\right], \tag{A.72}$$

$$q_3^{1+2}(\boldsymbol{c},a) = \frac{1}{12}\left[5+2a+\frac{4(c_2-c_3)}{t(1-2a)}\right]. \tag{A.73}$$

Backward substitution returns the equilibrium, pre-innovation, profits of the merged firm

$$\begin{aligned}\pi_{12}^{1+2}(\boldsymbol{c},a) = {} & \frac{1}{576(1-2a)t}\{(1-2a)^2\left(52a^2-76a+205\right)t^2+8(2a-1)t[9(2a+1)c_1 \\ & +(19-26a)c_2+4(2a-7)c_3] \\ & +16\left(9c_1^2-18c_1c_2+13c_2^2-8c_2c_3+4c_3^2\right)\}\end{aligned} \tag{A.74}$$

and those of firm 3

$$\pi_3^{1+2}(\boldsymbol{c},a) = \frac{\{[5-4a(2+a)]t+4c_2-4c_3\}^2}{144(1-2a)t}. \tag{A.75}$$

The benefits from innovation of the merged firm, for each production process are:

$$\left|\frac{\partial\pi_{12}^{1+2}(\cdot)}{\partial c_1}\right| = B_1^{1+2}(c_1,c_2,a) = \frac{1}{8}\left[1+2a-\frac{4(c_1-c_2)}{t(1-2a)}\right], \tag{A.76}$$

$$\left|\frac{\partial\pi_{12}^{1+2}(\cdot)}{\partial c_2}\right| = B_2^{1+2}(\boldsymbol{c},a) = \frac{1}{72}\left[19-26a+\frac{4(9c_1-13c_2+4c_3)}{t(1-2at)}\right]. \tag{A.77}$$

Two things are worth noticing here. First, $B_1^{1+2}(\cdot)$ is larger the larger is the pre-innovation cost advantage in producing good 1 relative to good 2 $(c_1-c_2)$. Likewise, $B_2^{1+2}(\cdot)$ positively depends on (a function of) the cost advantage of producing good 2 relative to good 1 $(13c_2-9c_1)$, but also increases with the cost of the rival good 3. Second, at equal pre-innovation costs, the benefit from process 1 increases, and that from process 2 decreases, if products cluster toward the center.

The sum of the two preceding expressions is the total innovation incentive of the merged firm:

$$B_{12}^{1+2}(c_2, c_3, a) = B_1^{1+2}(\cdot) + B_2^{1+2}(\cdot) = \frac{1}{18}\left[7 - 2a - \frac{4(c_2 - c_3)}{t(1-2a)}\right]. \tag{A.78}$$

It instructive that the total incentives of the merged entity only depend on the marginal cost of the central good $c_2$ and on the rival's cost $c_3$. The marginal cost of niche good 1, which is not exposed to external competition, does not determine the total innovation benefit. Because the merged firm completely internalizes the demand displacement between goods 1 and 2, as clarified above, it sets the innovative effort so that the positive effect of a reduction in $c_1$ on the stream of profits from good 1 and the negative one on that from good 2 cancel each other out. Therefore, the total innovative benefit of the merged firm only depends on its cost (dis)advantage in the production of the good exposed to external competition: $c_2 - c_3$. Last, the innovation incentives of firm 3 are:

$$B_3^{1+2}(c_2, c_3, a) = \frac{1}{18}\left[5 + 2a + \frac{4(c_2 - c_3)}{t(1-2a)}\right]. \tag{A.79}$$

The incentives are decreasing in own marginal cost, and increasing in the rival's one; moving toward the center increases the benefit from innovation.

We are going to restrict our attention to the case of *ex-ante* identical marginal costs ($c_i = c$). Simple calculations show that, in this case, the merger is profitable:

$$\pi_{12}^{1+2}(a) - [\pi_1^*(a) + \pi_2^*(a)] = \frac{1}{576}(1-2a)[4(a-7)a + 97]t > 0. \tag{A.80}$$

We shall now consider the effect of a merger on the distribution of incentives in the industry. Simple algebra yields:

**Lemma 4.** *Under A1, a merger between firms 1 and 2:*

*(i) Makes the innovation incentives of firm 3 larger than the incentives of the merged firm on each of its production processes.*

*(ii) Makes the total innovation incentives of the merged firm larger than those of firm 3.*

*Proof.* (i) The difference between the incentives of firm 3 and those of the merger firm on processes 1 and 2 are, respectively: $B_3^{1+2}(a) - B_1^{1+2}(a) = \frac{1}{72}(11 - 10a) > 0$ and $B_3^{1+2}(a) - B_2^{1+2}(a) = \frac{1}{72}(34a + 1) > 0$.

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(ii) The difference between the incentives of the merged firm and firms 3 is $B_{12}^{1+2}(a) - B_3^{1+2}(a) = \frac{1}{9}(1-2a) > 0$.

$\square$

Differently from the non-merger case, the niche firm 3 has always larger incentives to reduce its marginal cost than does the merged entity on each of its production processes. This notwithstanding, the total innovation incentives of the merged entity exceed those of firm 3.

Let us compare the innovation incentives before and after the merger:

**Lemma 5.** *Under A1, a merger between firms 1 and 2*

*(i) Reduces innovation incentives of the merged firms on processes 1 and 2.*

*(ii) Increases the innovation incentives of the non-merging firm 3.*

*(iii) Reduces the global innovation incentives, which now no longer depend on the location of firms.*

*Proof.* (i) The difference between the pre- and post-merger incentives on processes 1 and 2 are, respectively $B_1(a) - B_1^{1+2}(a) = \frac{1}{36}(3-4a) > 0$ and $B_2(a) - B_2^{1+2}(a) = \frac{5}{72}(2a+1) > 0$.

(ii) The difference between the pre- and post-merger innovation incentives of firm 3 is $B_3(a) - B_3^{1+2}(a) = \frac{1}{72}(5-2a) > 0$.

(iii) The global innovation incentives after the merger are

$$B^{1+2} = \sum_{i=1}^{3} B_i^{1+2}(a) = \frac{2}{3}, \tag{A.81}$$

It is immediate to observe that $B^{1+2}$ does not depend on $a$, and to compute that $B(a) - B^{1+2} = \frac{1}{36}(3+2a) > 0$.

$\square$

The merger between firms 1 and 2 affects the incentives to innovate through two channels. On the one hand, the merger results in an increase in the generalized price

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level, which has a positive effect on the willingness to invest on each innovative endeavor.[19] On the other hand, the merged entity now internalizes the fact that a lower –say– $c_2$ reduces the marginal benefit from innovating on $c_1$: $\frac{\partial B_1^{1+2}(\cdot)}{\partial c_2} > 0$, and likewise for $B_2^{1+2}(\cdot)$ and $c_1$. This reduces the willingness to invest. The second effect offsets the first, so that the merger reduces the propensity to innovate.

The foregoing Lemma conveys an array of interesting consequences. First, it shares the concerns regarding the possible R&D-hindering effects of mergers, see, e.g. Denicolò and Polo (2018); Motta and Tarantino (2021); Bourreau *et al.* (2025) and the references therein. It also confirms that the reduction does not regard all the firms in the industry. Indeed, firm 3 experiences an increase in its innovation incentives compared to the non-merger scenario. This is in line with the findings of Motta and Tarantino (2021), who show that, while the merged entity reduces the total investment relative to the pre-merger levels, the outsiders are more inclined to invest. The intuition is that, although all prices increase, that of the outsider firm 3 increases less, which boosts its sales, whence the marginal benefit from investment.

It is finally worth pointing out that, in our setup, any change in locations affects the individual innovation incentives, but not the global ones which are fixed at $2/3$. This follows from the fact that the merger restricts market competition to the merged firm selling product 2 against the niche product 3. The increase in sales by firm 3 following an increase in $a$ amounts exactly to the total sales reduction of the merged entity.[20] Consequently, any loss in incentives by the merged entity is exactly compensated by an equal increase by firm 3.[21]

We now move to the case a merger involving the niche firms 1 and 3.

## E.2 Merger between niche firms 1 and 3

A merger between firms 1 and 3 is qualitatively different from a merger between 1 and 2 because now the goods of the merging firms are not directly competing with each

[19] The innovation benefit from a reduction of $c_1$ on the stream of profits coming from good 1 only, evaluated at the post-merger prices (and equal marginal costs), is larger than the pre-merger benefit: $\left|\frac{\partial \pi_1(p_1^{1+2}(\cdot),p_2^{1+2}(\cdot),c_1,c_2)}{\partial c_1}\right|\Big|_{c_1=c_2=c} > B_1(a)$. The same observation applies to the stream of profits from good 2.

[20] The decrease in the sales of good 2 is larger than the increase of good 3, due to the increase in volumes of good 1.

[21] Remember that the merged entity completely internalizes the effect of a reduction in $c_1$ on the sales of good 2, see (A.78).

other. Similarly to the previous case, the profit of the merged firm is

$$\pi_{13}(\boldsymbol{p}, c_1, c_3) = \pi_1(p_1, p_2, c_1) + \pi_3(p_2, p_3, c_3), \tag{A.82}$$

that of firm 2 being like in (11). It is immediate to observe that, contrary to the previous case, the stream of profits from product 1 does not depend on $p_3$, and that from product 3 does not depend on $p_1$, which entails that, at given costs, the merger does not generate price effects. Stated differently, although the merged entity is, in principle, able to internalize the cross price effects (and demand displacements) between the goods it produces, the fact that the demand of good 1 does not directly depend on $p_3$ (and the converse) results in the merged entity setting the same optimal prices as the independent firms 1 and 3. The immediate consequence is that the outsider firm 2 also sets the same price as in the non-merger case.[22] Because prices coincide, also demands and profits coincide, pre-innovation.

It is easy to compute the innovation benefit of the merged entity on process $i = 1, 3$, with $j = 1, 3, i \neq j$:

$$B_i^{1+3}(\boldsymbol{c}, a) = \frac{6t - 8a(a+1)t + 5c_i - 13c_j + 8c_2}{36t(1-2a)} \tag{A.83}$$

Contrary to the preceding case, the benefit from reducing $c_i$ is increasing with a function of the cost dis-advantage of producing good $i$ relative to $j$ ($5c_i - 13c_j$), and also positively depends on the rival's cost $c_2$. It is finally easy to ascertain that, at equal marginal costs, these incentives are increasing in $a$. Firm 2's benefit coincides with $B_2(\boldsymbol{c}, a)$, as in (13).

The merger still influences the merged entity's innovation incentives, but, differently from the case of the merger involving firms 1 and 2, here the cross-effect of the marginal costs $c_1$ and $c_3$ in determining the benefits from innovation passes through their influences on the goods prices. Needless to say, the positive effect brought about by the reduced competitive pressure is absent.

We report our findings, for the usual case of equal ex-ante costs, in the following.

**Lemma 6.** *Under A1, a merger between firms 1 and 3:*

[22] This is a "P-neutral merger", in the terminology of Bourreau *et al.* (2025): a merger that, in itself, *does not affect the products prices.* In the present paper, however, the neutrality *does not* follow from merger-generated production synergies that compensate the price effects, like in the aforementioned paper.

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*(i) Makes the innovation incentives of firm 2 larger than the incentives of the merged firms on each of its production processes.*

*(ii) Makes the total innovation incentives of the merged firm larger than those of firm 2.*

*Proof.* (i) The difference between the incentives of firm 2 and those of the merger firm on processes $i = 1, 3$ are: $B_2^{1+3}(a) - B_i^{1+3}(a) = \frac{1}{6}(1 - 2a) > 0$.

(ii) The difference between the incentives of the merged firm and firm 2 is $B_{13}^{1+3}(a) - B_2^{1+3}(a) = \frac{4a}{9} > 0$.

□

Before briefly commenting on this Lemma, we compare the pre- and post-merger incentives:

**Lemma 7.** *Under A1, a merger between firms 1 and 3*

*(i) Reduces innovation incentives of the merged firms on processes 1 and 3.*

*(ii) Does not affect the innovation incentives of the non-merging firm 2.*

*(iii) Reduces the global innovation incentives, which now no longer depend on firm location.*

*Proof.* (i) The difference between the pre- and post-merger incentives on processes $i = 1, 3$ are, respectively $B_i(a) - B_i^{1+3}(a) = \frac{1}{72}(2a + 3) > 0$.

(ii) The difference between the pre- and post-merger innovation incentives of firm 3 is $B_2(a) - B_2^{1+3}(a) = 0$.

(iii) The global innovation incentives after the merger are

$$B^{1+3} = \sum_{i=1}^{3} B_i^{1+3}(a) = \frac{2}{3}, \tag{A.84}$$

It is immediate to observe that $B^{1+2}$ does not depend on $a$, and to compute that $B(a) - B^{1+2} = \frac{1}{36}(3 + 2a) > 0$.

□

The two preceding Lemmata confirm, from different perspective, the already mentioned results. Here, however, it is worth observing that, because of the absence of price effects, the innovation incentives of the central firm are unaffected relative to the non-merger scenario. Also, they are larger than each of the incentives of the merged entity, as this firm, as already pointed out, experiences an unambiguous innovation incentives reduction. Our last step is to compare, across merger types, the incentives on each production process. They are summarized in the following.

**Lemma 8.** *Under A1:*

*(i) The relative location of products determines whether the innovation incentives on process 1 are larger after a merger between firms 1 and 2 or 1 and 3.*

*(ii) The innovation incentives on process 2 are larger if firms 1 and 3 merge than if 1 and 2 merge.*

*(iii) The innovation incentives on process 3 are larger if firms 1 and 2 merge than if 1 and 3 merge.*

*Proof.* (i) $B_1^{1+2}(a) - B_1^{1+3}(a) = \frac{1}{72}(10a - 3) \gtreqless 0 \Leftrightarrow a \gtreqless \frac{3}{10}$.

(ii) $B_2^{1+2}(a) - B_2^{1+3}(a) = -\frac{5}{72}(2a + 1) < 0$.

(iii) $B_3^{1+2}(a) - B_3^{1+3}(a) = \frac{1}{9} > 0$.

□

We lastly state:

**Lemma 9.** *1. The global innovation incentives are independent of the identity of the merging firms.*

*Proof.* Follows from the proofs of point (iii) of Lemmata 5 and 7. □

We shall comment on this set of results by starting from the last one. Like for a merger between firms 1 and 2, the location in the product space of the merging firms determines the individual innovation incentives, yet, the global incentives are independent of $a$. Although the result coincides with that of the merger between 1 and 2, the intuition is slightly different. Here, a rise in $a$ increases the competitive pressure on the outsider's product 2. This results in a decrease in all prices, with a more pronounced drop in $p_2$, as this good faces greater competition on both the ends of its demand. This lowers the central firm's innovation incentives, and increases the merged entity's ones on both its production processes, like in the independent firms case. Yet, as highlighted earlier, the merged entity internalizes the negative cross-effect of process innovation, which counters the increase in the innovation incentives, relative to the no merger case. In our setup the (dampened) increase in the incentives of the merged entity exactly offsets the reduced one of the central firm, resulting in global incentives independent of location. Points (ii) and (iii) confirm the result that the merger outsider firm has larger incentives to undertake process innovation than the insiders. Point (i) instead shows that the incentives to reduce the cost associated to process 1 (which is always controlled by the merged entity) are larger in the case the merger involves the producer of the central good if, and only if, firms are relatively clustered toward the center ($a > \frac{3}{10}$). To have an intuition for this, remember that clustering towards the center increases the competitive pressure particularly on good 2, which is threatened by substitute products on both sides. The larger $a$, the larger the loss of profitability of good 2. In case of a merger involving firms 1 and 2, by lowering $c_1$ the merged entity increases the margin on the good that is shielded from competition, so as to reap a larger margin on it. The closer to the center are the niche products, the larger this incentive. Eventually, for $a$ large enough, this makes the innovation incentives larger in a merger between firms 1 and 2 than with 1 and 3.