Mathematics and Computers in Simulation 221 (2024) 210–221

Contents lists available at ScienceDirect

Mathematics and Computers in Simulation

journal homepage: www.elsevier.com/locate/matcom

Original articles

Linearising anhysteretic magnetisation curves: A novel algorithm for finding simulation parameters and magnetic moments

Daniele Carosi [a,*], Fabiana Zama [b], Alessandro Morri [a], Lorella Ceschini [a]

[a] *Department of Industrial Engineering, University of Bologna, Italy*
[b] *Department of Mathematics, University of Bologna, Italy*

ARTICLE INFO

Keywords:
Magnetic hysteresis model
Model parameters
Magnetic materials
Parameters identification

ABSTRACT

This paper proposes a new method for determining the simulation parameters of the Jiles–Atherton Model used to simulate the first magnetisation curve and hysteresis loop in ferromagnetic materials. The Jiles–Atherton Model is an important tool in engineering applications due to its relatively simple differential formulation. However, determining the simulation parameters for the anhysteretic curve is challenging. Several methods have been proposed, primarily based on mathematical aspects of the anhysteretic and first magnetisation curves and hysteresis loops. This paper focuses on finding the magnetic moments of the material, which are used to define the simulation parameters for its anhysteretic curve. The proposed method involves using the susceptibility of the material and a linear approximation of a paramagnet to find the magnetic moments. The simulation parameters can then be found based on the magnetic moments. The method is validated theoretically and experimentally and offers a more physical approach to finding simulation parameters for the anhysteretic curve and a simplified way of determining the magnetic moments of the material.

1. Introduction

Ferromagnetic materials have long presented a challenge in determining their magnetic constitutive laws. Numerous approaches and mathematical models have been developed to address this issue. The most accurate models, according to the literature, are the Brillouin and Langevin Functions for describing reversible magnetic transformations, which produce “anhysteretic curves”, and the Presiach and Jiles–Atherton Model for describing irreversible magnetic transformations, which produce the first magnetisation curve and hysteresis loop [1].

In daily applications, the magnetisation of a magnetic material due to an external generated magnetic field does not pass through equilibrium states but through non - equilibrium states, showing the phenomenon of hysteresis.

Many models to describe the hysteresis behaviour of a magnetic material have been developed in the years, like Preisach Model, Stoner–Wolfhart Model and so on. One of the most used, especially in engineering applications is the Jiles–Atherton Model (JA) [1].

This model describes the magnetisation of a ferromagnetic material in function of the external applied field with a first-order Ordinary Differential Equation (ODE) depending on several critical parameters related to the material and experiment conditions. To define the JA model of a given material, it is necessary to estimate the parameters from magnetisation measurements at different intensities of the applied magnetic field. Such a problem is a well-known, very difficult task, and many approaches can be found in the literature to simulate the first magnetisation curve and hysteresis loop. One such method is the genetic algorithm, which uses a penalty fitness function and boundary values [2]. Another method is the “Branch and bound method”, which uses boundary

* Corresponding author.
E-mail address: daniele.carosi2@unibo.it (D. Carosi).

https://doi.org/10.1016/j.matcom.2024.03.006
Received 13 June 2023; Received in revised form 26 February 2024; Accepted 4 March 2024
Available online 6 March 2024

Table 1
JA model parameters.

$a_J = k_B T/(\mu_0\ m)$	related to the shape of the anhysteretic curve
k_B	the Boltzmann constant
T	the temperature of the material in K
μ_0	the magnetic permeability of free space
m	the magnetic moment of a pseudo-domain
α	related to the interdomain coupling
M_s	the saturation magnetisation
k	related to the coercive field and the pinning sites
c	related to the reversible processes of magnetisation
$\delta = sign(dH/dt)$	related to the derivative of external applied magnetic field

conditions of the parameters and is mainly based on mathematical considerations [3]. A third method involves considering the anhysteretic function similar to the first magnetisation curve at the maximum applied field [4]. More recently, an improved genetic algorithm has been developed that uses a loss function to evaluate the distance between simulated and experimental hysteresis loops [5]. Additionally, neural networks have been used, with inputs such as frequency, maximum flux density and flux density, and a parameter indicating whether the magnetic field increases or decreases [6]. Alternatively, Silveyra and Conde Garrido [7,8] have proposed transforming the search space by expressing the model parameters in terms of a new set of parameters that can be easily optimised within known bounds.

The above methods determine the simulation parameters primarily based on mathematical aspects of the anhysteretic and first magnetisation curves and hysteresis loops, using differential or non-linear equations and differential susceptibilities. However, the parameters for simulating the anhysteretic curve are related to the magnetic moment m, which has yet to be determined.

The present work aims to investigate a robust way to define approximate parameter values using the material physical properties by introducing the linearisation of the anhysteretic magnetisation curves. This method offers several benefits, such as finding the magnetic moments of the material, finding the simulation parameters for the anhysteretic curve more physically, and finding the simulation parameters based solely on the value of initial anhysteretic susceptibility. The results show that it is possible to describe the anhysteretic magnetisation curve of a ferromagnetic material with a paramagnetic function linearly approximated for every value of the external applied field. This approach could also be used to define the starting guess of parameter estimation procedure, making it robust and efficient.

The paper is structured as follows. In Section 2, we present the JA model, delineating the involved parameters and physical quantities, and describe the algorithm for parameter estimation developed in the paper [9]. In Section 3, we propose a method to find the magnetic moment. This involves linearising the anhysteretic magnetisation curve and examining its susceptibilities. Additionally, we propose a robust and efficient algorithm to estimate the parameters of the JA model. Finally, in Section 4, we validate the proposed algorithm using both literature data and real measurements.

2. The problem

According to the JA model, the magnetisation M of ferromagnetic materials in function of an external applied field H_a is described by the following ODE:

$$\frac{dM}{dH_a} = \frac{1}{1+c}\frac{M_{an}(H_a) - M}{\delta k - \alpha(M_{an}(H_a) - M)} + \frac{c}{1+c}\frac{dM_{an}(H_a)}{dH_a} \tag{1}$$

where $M_{an}(H_a)$ is the anhysteretic magnetisation function, described as an implicit non-linear function - $f(H_a, M)$:

$$M_{an}(H_a) \equiv f(H_a, M) \equiv M_s\left(\coth\left(\frac{H_a + \alpha M}{a_J}\right) - \frac{a_J}{H_a + \alpha M}\right) \tag{2}$$

$M_s, a_J, c, \delta, k, \alpha$ are the model parameters, defined in Table 1, and αM is the molecular field [10].

If are considered only the anhysteretic values of a ferromagnetic material, then (2) becomes:

$$M_{an} = M_{an}(H_a) = M_s\left(\coth\left(\frac{H_a + \alpha M_{an}}{a_J}\right) - \frac{a_J}{H_a + \alpha M_{an}}\right) \tag{3}$$

because the magnetisation of the material is proportional to itself [11].

One of the most difficult tasks of such a modelling problem is the determination of the model parameters from measurements of the anhysteretic magnetisation M_{an} at different intensities of the external field H_a. In addition to the inherent difficulty of solving an ill-posed problem, JA model also presents extreme challenges in defining starting guesses and extreme sensitivity to their value.

To address this difficulty, we propose to improve the original approach in [9], based on the exploitation of physical relationships between different quantities that can be obtained from the measurements. The estimation procedure proposed in [9] exploits the quantities reported in Table 2 and reduces the dependence of all model parameters to a single parameter α which is heuristically set. Such procedure is outlined in algorithm 1. The `fit_condition` is usually represented by a test on the least squares distance or Mean Square Error between the simulated hysteresis loop and the experimental data. A known weakness of such an approach is the lack of guarantee that the exit condition can be fulfilled; therefore, a restart with different seeds might be required, depending on

Table 2
Physical quantities that can be extracted from measured data.

χ'_{in}	Initial differential susceptibility of the first magnetisation curve
χ'_{an}	Initial differential susceptibility of the anhysteretic magnetisation curve
χ'_{max}	Differential susceptibility at coercive field
χ'_{r}	Differential susceptibility at remanence point
χ'_{m}	Differential susceptibility at hysteresis loop tip
H_c	Value of coercive field
M_r	Value of magnetisation at remanence point
M_m	Value of magnetisation at loop tip
H_m	Value of external applied field corresponding to M_m

the measured data. One strength is the reduced computational cost consisting of the solution of two nonlinear equations for each iteration.

In the next section we introduce a method to find the magnetic moment m based on the linearisation of the anhysteretic magnetisation curve and its susceptibilities. This provides a simple way to find values of parameters a_J and α.

Algorithm 1 Estimation Algorithm Jiles [9]

1: Calculate the value of c with equation: $c = \frac{\chi'_{in}}{\chi'_{an}}$
2: **repeat**
3: Set a seed value of α
4: Calculate a first estimation value of a_J with equation: $a_J = \frac{M_s}{3}\left(\frac{1}{\chi'_{an}} + \alpha\right)$
5: Compute k as:

$$k = \frac{M_{an}(H_c)}{1-c}\left(\alpha + \frac{1}{\left(\frac{1}{1-c}\right)\chi'_{max} - \left(\frac{c}{1-c}\right)\frac{dM_{an}(H_c)}{dH}}\right)$$

6: Solve for α the following non-linear equation, using the current α estimate as initial guess:

$$M_r = M_{an}(M_r) + \frac{k}{\left(\frac{\alpha}{1-c}\right) + \frac{1}{\chi'_r - c\frac{dM_{an}(M_r)}{dH}}}$$

7: Update a_J solving numerically the following non-linear equation, using as initial guess the current value of a_J:

$$M_m = M_{an}(H_m) - \frac{(1-c)k\chi'_m}{\alpha\chi'_m + 1}$$

8: Solve (1) with the estimated parameters in the measured points
9: **until** `fit_condition`

3. Magnetic moments and simulation parameters of anhysteretic curve

Let us start considering an anhysteretic theoretic curve of a ferromagnetic material generated by (2) with given simulation parameters α and a_J.

Since for every value of external applied field H_a the curve has only one value of anhysteretic magnetisation M_{an}, the following injective function can describe its behaviour:

$$M_{an}(H_a) = M_s\left(\coth\left(\frac{H_a}{a}\right) - \frac{a}{H_a}\right) \tag{4}$$

with a simulation parameter $a \neq a_J$. Injective functions such as (4) usually describe the magnetic behaviour of a paramagnetic material [11]: since there is no interaction between the magnetic moments in paramagnetic materials, the parameter α is absent. Therefore, analogously to (2), the shape parameter a is:

$$a = \frac{k_B T}{\mu_0 m_1}, \tag{5}$$

where m_1 is the magnetic moment of the equivalent paramagnetic curve that can describe the ferromagnetic one by function (4).

A common experimental procedure to obtain the anhysteretic curve of a magnetic material consists of superimposing a steady external magnetic field on another magnetic field that varies between a minimum and maximum value. The varying magnetic field is responsible for creating the hysteresis loop. Gradually, the range of the varying magnetic field is reduced until it aligns with the value of the constant external magnetic field. Through this procedure, consistent magnetisation values of the material can be obtained that are free of hysteresis [12].

Considering then that the magnetisation curve of ferromagnetic materials is obtained with relatively small and constant values of the external applied field H_a, it is possible to obtain a simplified approximation of $M_{an}(H_a)$. Considering the series expansion of $\coth(x)$ for $x \neq 0$,

$$\coth(x) = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2x^5}{945} - \cdots, \ |x| < \pi$$

and taking the first two terms, we obtain the linearised approximation $M^a_{an}(H_a)$:

$$M^a_{an}(H_a) = M_s \left(\cancel{\frac{1}{\frac{H_a}{a}}} + \frac{\frac{H_a}{a}}{3} - \cancel{\frac{a}{H_a}} \right)$$

Substituting a from (5):

$$M^a_{an}(H_a) = \frac{M_s}{3} \frac{\mu_0 m_1}{k_B T} H_a \tag{6}$$

Using $M^a_{an}(H_a)$ we can define the anhysteretic susceptibility of the ferromagnetic material χ^a_{an} [11,12]:

$$\chi^a_{an} = \frac{M^a_{an}(H_a)}{H_a}. \tag{7}$$

Substituting into (6), we obtain the magnetic moment m_1 of the equivalent paramagnetic material for the external applied field related to the anhysteretic susceptibility of the ferromagnetic material:

$$m_1 = \frac{3k_B T \chi^a_{an}}{\mu_0 M_s}. \tag{8}$$

Going back to function (3) the linearisation of the anhysteretic behaviour of a ferromagnetic material becomes:

$$M_{an} = \frac{M_s}{3a_J}(H_a + \alpha M_{an}) = \frac{M_s}{3} \frac{\mu_0 m}{k_B T}(H_a + \alpha M_{an}) \tag{9}$$

To find the value m for the magnetic moment of the ferromagnetic material in (9), we substitute M_{an} with $M^a_{an} \equiv M^a_{an}(H_a)$:

$$M^a_{an} = \frac{M_s}{3} \frac{\mu_0 m}{k_B T}(H_a + \alpha M^a_{an}),$$

hence using (7)

$$\chi^a_{an} = \frac{M_s}{3} \frac{\mu_0 m}{k_B T}(1 + \alpha \chi^a_{an}).$$

we find the following expression for m:

$$m = \frac{3k_B T}{\mu_0 M_s} \frac{\chi^a_{an}}{(1 + \alpha \chi^a_{an})}. \tag{10}$$

However, since α is unknown this relation cannot be applied to compute m. Viewing the quantity $\frac{\chi^a_{an}}{(1+\alpha\chi^a_{an})}$ as the susceptibility χ_{param} of an equivalent paramagnetic curve of the ferromagnetic material, we obtain an alternative characterisation of m depending on the unknown susceptibility χ_{param} :

$$m = \frac{3k_B T}{\mu_0 M_s} \chi_{param}. \tag{11}$$

An estimate of χ_{param} can be obtained by substituting (11) in function (4) and solving the nonlinear equation:

$$M_{an} = M_{an}(H_a) = M_s \left(\coth\left(\frac{3\chi_{param} H_a}{M_s} \right) - \frac{M_s}{3\chi_{param} H_a} \right) \tag{12}$$

for properly chosen values H_a.

The idea is to choose very high values of the applied external field H^1_a - i.e. $H^1_a \approx 10^6[\frac{A}{m}]$, since the molecular field in the paramagnetic case does not act. Still, the saturation of the magnetisation is almost reached. With the instruments at our disposal, such as those from Borckhaus Messtechnik, it is impossible to measure the anhysteretic magnetisation curve for magnetic field values exceeding 10 kA/m due to the instrument's limitations. Instead of measuring the anhysteretic magnetisation curve at such high external magnetic field values, we utilise its equivalent paramagnetic curve with a magnetic moment m_1.

We can compute the corresponding magnetisation value M_{an_1} at (H^1_a) as:

$$M_{an_1} = M_s \left(\coth\left(\frac{\mu_0 m_1}{k_B T} H^1_a \right) - \frac{k_B T}{\mu_0 m_1 H^1_a} \right). \tag{13}$$

Since $M_{an_1} > M_{an}$ (see Appendix), we solve Eq. (12) with the magnetisation value M_{an} estimated as follows:

$$M_{an} \approx \eta^* M_{an_1}, \quad 0.9 \leq \eta^* < 1.$$

Table 3
Parameters of an electrical steel at room temperature and JA simulation parameters.

Ferromagnetic material parameters			JA parameters	
M_s	T	T_c	a_J	α
$1.6 \cdot 10^6 \left[\frac{A}{m}\right]$	$303.5\ [K]$	$1023.5\ [K]$	972	$1.4 \cdot 10^{-3}$
			972	$1.0 \cdot 10^{-3}$
			972	$1.8 \cdot 10^{-3}$
			800	$1.4 \cdot 10^{-3}$
			1000	$1.4 \cdot 10^{-3}$
			1200	$1.4 \cdot 10^{-3}$

Finally, we can simplify computation by considering the anhysteretic susceptibility of the equivalent paramagnetic curve

$$\chi_{an_1} = \frac{M_{an_1}}{H_a^1}$$

(see [11], eqn (3.35)). Substituting χ_{an_1} into (13) we obtain :

$$\eta^* \cdot \chi_{an_1} - \frac{M_s}{H_a^1}\left(\coth\left(\frac{3\chi_{param}H_a^1}{M_s}\right) - \frac{M_s}{3\chi_{param}H_a^1}\right) = 0. \tag{14}$$

After computing χ_{param} through the numerical solution of (14), we obtain the magnetic moment m through (11) and the parameter $a_J = \frac{k_B T}{\mu_0 m}$, then we can compute α by relations (10) and (11), i.e.:

$$\alpha = \frac{1}{\chi_{param}} - \frac{1}{\chi_{an}^a}. \tag{15}$$

considering χ_{an}^a as the initial anhysteretic susceptibility of the ferromagnetic material.

Once the parameters have been computed, we compute the approximate anhysteretic magnetisation value $\hat{M}_{an}$ correspondent to every value of the external applied field H_a, solving the following nonlinear equations for each value of external applied field H_a :

$$\hat{M}_{an} - M_s\left(\coth\left(\frac{H_a + \alpha\hat{M}_{an}}{a_J}\right) - \frac{a_J}{H_a + \alpha\hat{M}_{an}}\right) = 0. \tag{16}$$

Since η^* is not known, the idea is to evaluate (14) in a sequence $\{\eta_k\}_{k>0} \in [0.9, 1)$ and define

$$\eta^* = \arg\min_{\eta} \|\mathbf{r}^{(\eta)}\|_2, \quad \mathbf{r}^{(\eta)} = \mu_0\|\hat{M}_{an} - M_{an}\|_2$$

where $\mathbf{r}^{(\eta)}$ has components $r_i^{(\eta)}$ given by the difference between the magnetic anhysteretic data $(\mu_0 M_{an_i})$ and its approximation $(\mu_0 \hat{M}_{an_i})$ for each corresponding value of external applied field.

These steps are summarised in Algorithm JA_par.

From extended experimental tests with various materials, we verified that a value of $\eta \in [0.9, 1)$ is sufficient to obtain closely aligned ferromagnetic and equivalent paramagnetic curves, for very high values of the externally applied field.

4. Methodology validation and testing

This section validates the proposed method by investigating its robustness in presence of perturbations. Then the method is tested against data taken from literature and real measurements.

4.1. Methodology validation

Firstly, we demonstrate that the anhysteretic curve of a ferromagnetic material can be approximated using the linear approximation of a paramagnet for any given value of external applied field.

By utilising the anhysteretic susceptibilities of the ferromagnetic material's anhysteretic curve, which are described by Eq. (7) and substituted into Eq. (8), it becomes possible to approximate the curve for any external applied field value using Eq. (6) of an equivalent paramagnetic curve.

To accomplish this, we generate a theoretical anhysteretic curve by employing real parameters from a ferromagnetic material and synthetic simulation parameters. For example, we consider a carbon steel at room temperature, with the material parameters listed in Table 3. We then choose a set of simulation parameters for the JA model, as provided in Table 3, to generate an anhysteretic curve, which is depicted in blue in Figs. 3 and 4.

For each value of the external applied field and magnetisation along the anhysteretic curve of the ferromagnetic material, we compute the susceptibility (7) and the values of magnetic moment m_1 by applying Eq. (8). By substituting these values into Eq. (6),

Algorithm 2 Algorithm JA_par. INPUT: (H_a, M_{an}), k_B, T, M_s

Define M_{an}^{start} as the first experimental positive magnetisation value in the measurements set (H_a, M_{an})
Define H_a^{start} as the external applied field value corresponding to M_{an}^{start}
$\chi_{an}^a = \dfrac{M_{an}^{start}}{H_a^{start}}$;
$m_1 = \dfrac{3k_B T \chi_{an}^a}{\mu_0 M_s}$;
$a_1 = \dfrac{k_B T}{\mu_0 m_1}$;
$H_a{}^1 = 10^6$: $M_{an_1} = \left[M_s \left(\coth\left(\dfrac{H_a^1}{a_1}\right) - \dfrac{a_1}{H_a^1} \right) \right]$
$\chi_{an_1} = \dfrac{M_{an_1}}{H_a^1}$;
$\varepsilon = 10^{-5}$; ▷ step increment
$\eta_0 = 0.9$;
$k = 0$
Loop = true
while Loop **do**
    Compute χ_{param} by solving the nonlinear equation :

$$\eta_k \cdot \chi_{an_1} - \frac{M_s}{H_a^1}\left(\coth\left(\frac{3\chi_{param} H_a^1}{M_s}\right) - \frac{M_s}{3\chi_{param} H_a^1}\right) = 0;$$

    $m_2 = \frac{3k_B T \chi_{param}}{\mu_0 M_s}$;
    $a_J = \frac{k_B T}{\mu_0 m_2}$;
    $\alpha = \frac{1}{\chi_{param}} - \frac{1}{\chi_{an}^a}$;
    Solve the nonlinear system for $\hat{M}$:

$$\hat{M}_{an} - M_s\left(\coth\left(\frac{H_a + \alpha \hat{M}_{an}}{a_J}\right) - \frac{a_J}{H_a + \alpha \hat{M}_{an}}\right) = 0$$

    $r = \mu_0(\hat{M}_{an} - M_{an})$; ▷ Residual vector
    $k = k + 1$; $Nr(k) = norm(r)$; ▷ Norm of residual vector
    if $k > 1$ **then**
        Loop $= Nr(k) < Nr(k-1)$
    end if
    if Loop **then**
        $\eta_k = \eta_{k-1} + \varepsilon$
    end if
end while

we can evaluate the anhysteretic magnetisation for every value of the applied external field. In Fig. 1 we can appreciate the perfect agreement between the anhysteretic magnetisation curve of a ferromagnetic material and that of an equivalent paramagnetic curve for every value of external applied field. Such quality is preserved even for changes in the JA parameters a_J and α as reported in the examples represented in Fig. 2 where such parameters are modified according to Table 3. By defining the curve obtained using Eq. (4), with $a = \frac{\mu_0 m_1}{k_B T}$ where m_1 is calculated using the initial anhysteretic susceptibility (considering only the initial value of $H_a > 0$), as *Paramagnet equivalent 1*, we can observe in Fig. 3 that it tends to overestimate the anhysteretic magnetisation curve, particularly for small values of H_a.

On the other hand, if we consider the curve obtained by setting $\alpha = 0$ in Eq. (4), and using the value of a provided in the first row of table 3, we obtain an underestimating curve, referred to as *Paramagnet equivalent 2*. This curve demonstrates better agreement with the anhysteretic magnetisation curve, particularly for large values of the applied field H_a. Finally, we verify that by evaluating χ_{param} for extremely high values of external applied field and magnetisation, through the solution of Eq. (16), and calculating the initial anhysteretic susceptibility of the ferromagnetic material χ_{an}, we can determine the values of the parameters a_J and α that result in a reliable approximation of the experimental anhysteretic curve of the ferromagnetic material described by Eq. (2). We validate the evaluation of χ_{param} and χ_{an} by reporting in Figs. 4 the computed magnetisation curves varying a_J and α as in Table 3.

Again we can observe the perfect agreement between the theoretical and simulated anhysteretic curve with the simulation parameters' variation.

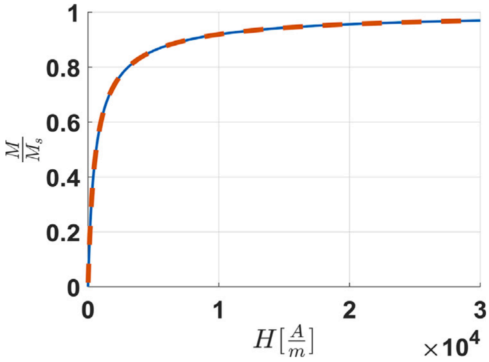

Fig. 1. Theoretical anhysteretic magnetisation curve and its linearisation with paramagnetic function. The field H is sampled in 10^5 uniformly distributed points with step $\delta_H = 10$ [A/m] in the range $[0, 10^6]$ [A/m]. The blue line represents the anhysteretic curve and the red dashed line is its linearisation with paramagnetic function. The maximum residual in $\mathcal{O}(10^{-9})$.

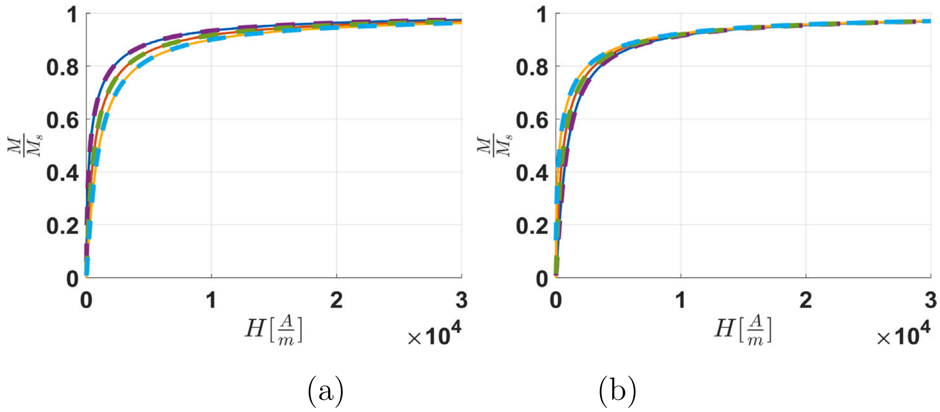

Fig. 2. Theoretical anhysteretic magnetisation curves and linearised approximations. (a) variation of a_J: 800, 1000, 1200 (b) variation of α: $1 \cdot 10^{-3}, 1.4 \cdot 10^{-3}, 1.8 \cdot 10^{-3}$. Continuous lines represent the anhysteretic magnetisation curves, dashed lines are their linearised approximations. The field H is sampled in 10^5 uniformly distributed points with step $\delta_H = 10$ [A/m] in the range $[0, 10^6]$ [A/m]. The solid lines represent the anhysteretic curves and the dashed lines are their linearisation with paramagnetic function. The maximum residual in $\mathcal{O}(10^{-9})$.

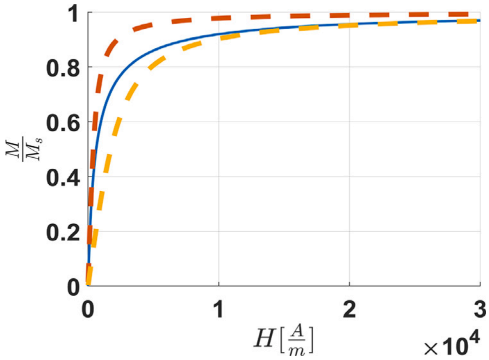

Fig. 3. Theoretical anhysteretic magnetisation curve and its paramagnetic curves. Blue line is the anhysteretic magnetisation curve. Red dashed line is the *paramagnet equivalent 1* obtained by Eq. (4), with $a = \frac{\mu_0 m_1}{k_B T}$. Yellow dashed line is the *paramagnet equivalent 2* obtained setting $\alpha = 0$ and a_J as in table 3. The field H is sampled in 10^5 uniformly distributed points with step $\delta_H = 10$ [A/m] in the range $[0, 10^6]$ [A/m]. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

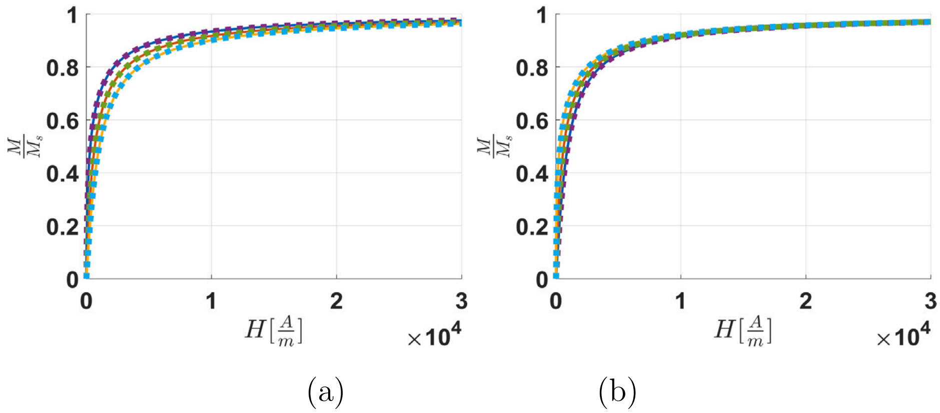

Fig. 4. Anhysteretic magnetisation curves obtained by χ_{param}, χ_{an} estimates and simulation curves varying parameters α and a_J. (a) variation of a_J: 800, 1000, 1200 (b) variation of α: $1 \cdot 10^{-3}, 1.4 \cdot 10^{-3}, 1.8 \cdot 10^{-3}$. Continuous lines represent the anhysteretic magnetisation curves, dotted lines are their linearised approximations. The field H is sampled in 10^5 uniformly distributed with step $\delta_H = 10$ [A/m] points in the range $[0, 10^6]$ [A/m].

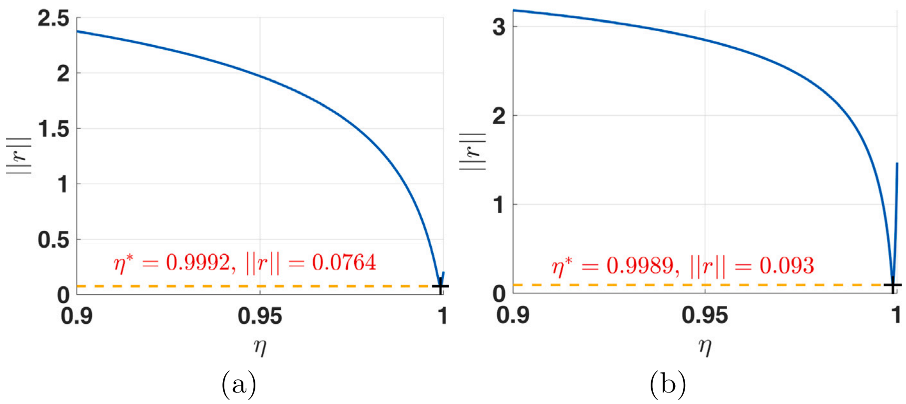

Fig. 5. Behaviour of the residual norm $\|\mathbf{r}\|$ for $\eta \in [0.9, 1)$. (a) Data in [13] taken from Fig. 2(a), "Experimental anhysteretic at zero stress" (b) Data in [10] taken from Fig. 8, "experimental".

Table 4
Parameters and residual obtained by algorithm 2.

Figure	Data	η^*	$\|\|\mathbf{r}\|\|$	χ_{param}	m_2	a_J	α
5(a)	Fig.2a - [13]	0.9992	0.0764	171.7717	$1.0779 \cdot 10^{-18}$	3088	0.0015
5(b)	Fig.8 - [10]	0.9989	0.093	406.5476	$2.5512 \cdot 10^{-18}$	1304.9	0.0021

4.2. Algorithm 2 testing

In this section, we evaluate the performance of the JA_par algorithm using data from papers [10,13], which were extracted using the web tool for data extraction called WebPlotDigitilizer [14].

Figs. 5 depict the residual behaviour within the interval $[0.9, 1)$, thereby confirming that the minimum value can be found in the given interval.

Additionally, these figures provide the optimal value η^* computed by the JA_par algorithm. The computed residual and parameters are presented in Table 4. The parameters a_J and α corresponding to η^* provide the best fit for the anhysteretic data (Figs. 6), making them the most representative of the magnetic material.

In Table 5 are shown the parameters found by proposed method - a_J, α and the Jiles' ones - $a_{J,J}, \alpha_J$: We observe that there is a good agreement between the parameters computed in the literature and those computed by our method. Moreover, we observe an improvement in the fitted curve compared to that reported in Fig.8 - [10].

From a computational efficiency perspective, we observe that the algorithm requires solving nonlinear equations in steps 11 and 15 of the JA_par algorithm. For this purpose, the function `fzero` is applied, using zero as the starting guess.

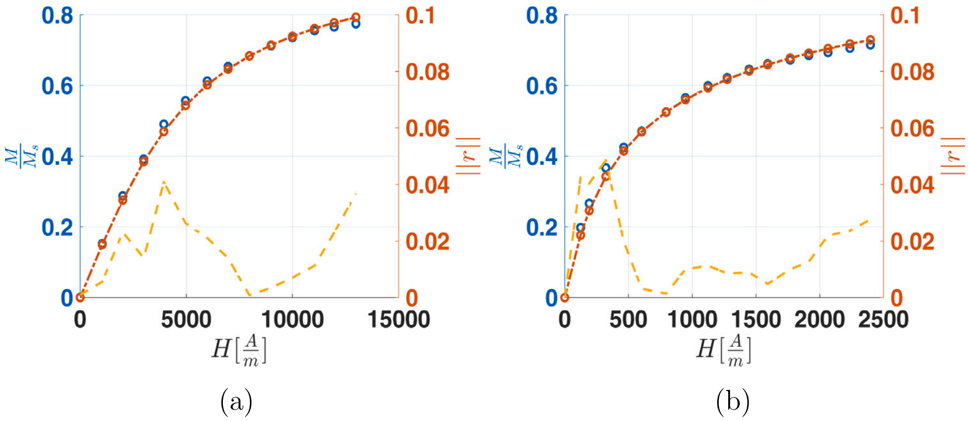

Fig. 6. Experimental anhysteretic magnetisation curves (blue circles) and JA_par simulations (orange line), residual curve (yellow line). (a) data [13] "Experimental anhysteretic at zero stress" (b) data [10] taken from Fig. 8, "experimental". (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 5
Values of simulation parameters with the proposed method and those found in Jiles' papers [13],[10].

Figure	Data	a_J	$a_{J,J}$	α	α_J
6(*a*)	Fig.2a - [13]	3088	3750	0.0015	0.0033
6(*b*)	Fig.8 - [10]	1304.9	1100	$2.1 \cdot 10^{-3}$	$1.6 \cdot 10^{-3}$

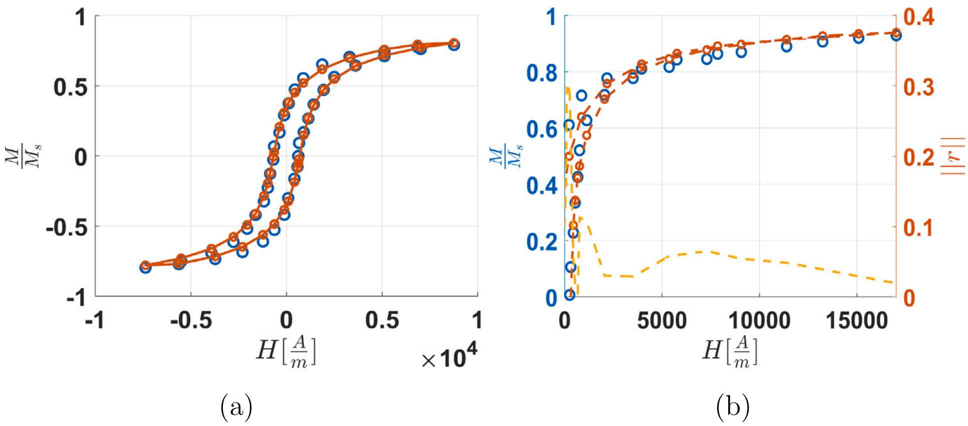

Fig. 7. Experimental hysteresis loop and its simulation from [9]. Data of figure (a) is taken from Fig. 6 and data of figure (b) from Fig. 9 of [9].

4.3. Experimental hysteresis validation

It is necessary to verify whether the parameters obtained through simulating the anhysteretic curve and solving Eq. (14) describe the hysteresis curve accurately. To this purpose, we set the simulation parameters c and k in (1) as follows:

- $c = \frac{\chi'_{in}}{\chi'_{an}}$;
- $k = H_c$;

where χ'_{in}, χ'_{an} are defined in Table 2.

The results are checked on the curves obtained from Jiles' paper [9] (Figs. 7) and real measurements (Fig. 8).

In the case of Fig. 7 the values of k and c are taken directly from Jiles' paper [9].

In Table 6 are shown material parameters taken from Jiles' paper [9]. The parameters k and c are calculated as in steps 1. and 5. of Jiles's algorithm 1, taking in account the approximation $\mu'_{an} \approx \chi'_{an}$, $\mu'_{in} \approx \chi'_{in}$ and $B_s \approx \mu_0 \cdot M_s$. In Fig. 7 is shown the comparison between data taken from Jiles' paper [9] and the hysteresis loop simulated with the proposed method to find parameters a and α. In Table 7 are shown simulation parameters between the ones found by Jiles - $a_{J,J}, \alpha_J$ and the ones found with the proposed method - a_J, α.

Table 6
Materials characteristics taken from [9].

Figure	Data	$B_s[T]$	μ'_{an}	μ'_{in}	$H_c[\frac{A}{m}]$
7(*a*)	Table IV - [9]	2	1343	142	693
7(*b*)	Table IV - [9]	2	5000	100	315

Table 7
Simulation parameters found by Jiles [9] – $a_{J,J}, \alpha_J$ and with the proposed method - a_J, α.

Figure	Data	a_J	$a_{J,J}$	α	α_J
7(*a*)	Fig.6 - [9]	2596	1000	$4.1 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$
7(*b*)	Fig.9 - [9]	1206	1085	$2.1 \cdot 10^{-3}$	$2 \cdot 10^{-3}$

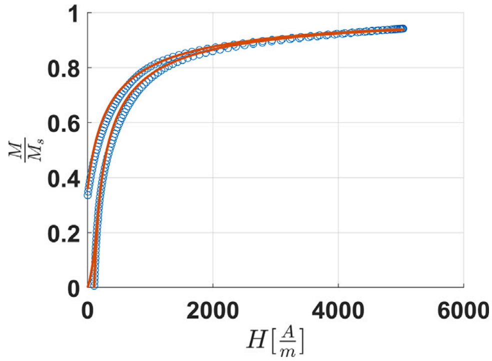

Fig. 8. Experimental hysteresis loop and its simulation. Blue circles represent the measurements, red line is the computed simulation.

In the case of real data, a hysteresis loop is obtained from a Non Oriented M 470-50 A produced by Marcegaglia Ravenna s.p.a with a Single Sheet Tester from Brockhaus Messtechnik. This machine has the following characteristics:

- Model: MPG100 D DC/AC
- frequency ranges: from 3 Hz to 10 kHz
- maximum polarisation: 2T
- measurement repeatability: ≤ 2 percent;
- 3631 sample points with external applied field range - $H_a = [-5000, 5000]\frac{A}{m}$.

The coercive field - H_c is directly given by Borckhaus instrument, while χ'_{in} is calculated as the initial slope of the differential susceptibility of the first magnetisation curve. Moreover χ'_{an} is approximated with the maximum value of differential susceptibility of hysteresis loop, that is the one corresponding to coercive field, since the anhysteretic magnetisation curve is not known.

From the result represented in Fig. 8, we can see is a good agreement between experimental data points and simulation.

5. Conclusion

This paper focused on the Jiles–Atherton Model, which is widely used in engineering applications, and presented a new approach for finding the simulation parameters for the anhysteretic curve of ferromagnetic materials. By using the material's susceptibility and linearising the anhysteretic magnetisation curve with a paramagnetic function, we could find the magnetic moments of the material and determine the simulation parameters in a more physical and simplified manner. Our results showed that it is possible to describe the anhysteretic magnetisation curve of a ferromagnetic material with a linear approximation of a paramagnet for every value of the external applied field. Validation of the proposed method with synthetic and experimental data has demonstrated its effectiveness and stability.

In conclusion, JA_par extends the approach of Algorithm 1 by improving the quality of parameter estimation without requiring the iterative solution of a system of ordinary differential equations (ODEs), which is computationally expensive and presents challenges in solving an inverse problem. This approach can be useful in many engineering applications requiring accurate ferromagnetic material characterisation.

CRediT authorship contribution statement

Daniele Carosi: Conceptualization, Software, Investigation, Writing – review & editing. **Fabiana Zama:** Methodology, Formal analysis, Writing – original draft. **Alessandro Morri:** Conceptualization, Methodology, Formal analysis. **Lorella Ceschini:** Resources, Data curation.

Acknowledgements

We would like to thank the reviewers for their insightful comments that have enabled us to enhance the quality of our work.

This work was financed by the European Union - NextGenerationEU (National Sustainable Mobility Center CN00000023, Italian Ministry of University and Research Decree n. 1033 - 17/06/2022, Spoke 11 - Innovative Materials & Lightweighting), and National Recovery and Resilience Plan (NRRP), Mission 04 Component 2 Investment 1.5 – NextGenerationEU, Call for tender n. 3277 dated 30/12/2021. Moreover this work was supported by Alessandro Ferraiuolo, R & D Manager of Marcegaglia Ravenna S.p.A., giving the material for experimental validations.

Appendix. Proof of $M_{an_1} > M_{an}$

For better readability and understanding, we first restate Eqs. (12) and (13):

$$M_{an}(H_a) = M_s\left(\coth\left(\frac{3\chi_{param}H_a}{M_s}\right) - \frac{M_s}{3\chi_{param}H_a}\right) \tag{17}$$

$$M_{an_1}(H_a) = M_s\left(\coth\left(\frac{\mu_0 m_1}{k_B T}H_a\right) - \frac{k_B T}{\mu_0 m_1 H_a}\right). \tag{18}$$

Our aim is to prove that $M_{an_1}(H_a) > M_{an}(H_a)$ for every value of $H_a > 0$. Considering small values of H_a, we express the magnetisations M_{an} and M_{an_1} through their linear approximations M^a_{an} and $M^a_{an_1}$:

$$\begin{cases} M^a_{an_1} = \frac{M_s}{3}\frac{\mu_0 m_1}{k_B T}H_a = \frac{M_s}{3a}H_a \\ M^a_{an} = \frac{M_s}{3}\frac{\mu_0 m}{k_B T}H_a = \frac{M_s}{3a_J}H_a \end{cases} \tag{19}$$

We establish the relationship between m and m_1 as:

$$\begin{cases} m_1 = \frac{3k_B T}{\mu_0 M_s}\chi^a_{an} \\ m = \frac{3k_B T}{\mu_0 M_s}\frac{\chi^a_{an}}{1+\alpha\chi^a_{an}} \end{cases} \rightarrow m = \frac{m_1}{1+\alpha\chi^a_{an}} \tag{20}$$

Therefore, we find that $m_1 > m$ for all $\alpha > 0$.

The expressions for the parameters a and a_J are given by:

$$a = \frac{k_B T}{\mu_0 m_1}, \qquad a_J = \frac{k_B T}{\mu_0 m}$$

Substituting from (20), we get:

$$a_J = \frac{k_B T}{\mu_0 m_1}(1+\alpha\chi^a_{an}) = a(1+\alpha\chi^a_{an}).$$

thus $a_J > a, \forall \alpha > 0$. Returning to (19), we derive:

$$M^a_{an} = \frac{M_s}{3a_J}H_a = \frac{M_s}{3a(1+\alpha\chi^a_{an})}H_a = \frac{M^a_{an_1}}{1+\alpha\chi^a_{an}}.$$

Hence $M^a_{an_1} > M^a_{an}$ when $H_a \gtrapprox 0$, and $a, \alpha > 0$. For sufficiently small values H_a the inequality holds also for M_{an_1} and M_{an}. Now we extend the inequality to $H_a \gg 0$. To simplify the analysis, we define the magnetisations M_{an_1} and M_{an} in terms of the function $f(x) \equiv \coth(x) - \frac{1}{x}$ for $x > 0$, setting:

- $M_{an}(H_a) \equiv M_s f(\gamma_1 H_a)$, where $\gamma_1 = \frac{3\chi_{param}}{M_s} \in \mathbb{R}^+$.
- $M_{an_1}(H_a) \equiv M_s f(\gamma_2 H_a)$, where $\gamma_2 = \frac{\mu_0 m_1}{k_B T} \in \mathbb{R}^+$.

We observe that the function $f(x)$ is strictly increasing for $x > 0$. This is established by computing the derivative:

$$f'(x) = \frac{1}{x^2} - \frac{1}{\sinh(x)^2}.$$

The derivative satisfies $0 < f'(x) < 1/3$ for all x in $\mathbb{R}^+$, with $\lim_{x\to 0} f'(x) = 1/3$ and $\lim_{x\to\infty} f'(x) = 0$.

For $H_a \gtrapprox 0$ the condition $M^a_{an_1} > M^a_{an}$ writes as $f(\gamma_2 H_a) > f(\gamma_1 H_a)$, now by the Lagrange mean value theorem:

$$0 < f(\gamma_2 H_a) - f(\gamma_1 H_a) = H_a(\gamma_2 - \gamma_1)f'(\xi_a), \quad \xi_a \in (H_a\gamma_1, H_a\gamma_2)$$

we can conclude that $\gamma_2 > \gamma_1$. Therefore, the inequality will hold for all $H_a \in \mathbb{R}^+$.

References

[1] Fausto Fiorillo, Carlo Appino, Massimo Pasquale, Hysteresis in magnetic materials, in: The Science of Hysteresis, Elsevier, 2006, pp. 1–190.
[2] Krzysztof Chwastek, Jan Szczyglowski, Identification of a hysteresis model parameters with genetic algorithms, Math. Comput. Simulation 71 (3) (2006) 206–211.
[3] Krzysztof Chwastek, Jan Szczygłowski, An alternative method to estimate the parameters of jiles–atherton model, J. Magn. Magn. Mater. 314 (1) (2007) 47–51.
[4] H. Hauser, Y. Melikhov, D.C. Jiles, Examination of the equivalence of ferromagnetic hysteresis models describing the dependence of magnetization on magnetic field and stress, IEEE Trans. Magn. 45 (4) (2009) 1940–1949.
[5] Qingsong Liu, Junjie Zhou, Jinwei Chu, Shunliang Wang, Qingming Xin, Chuang Fu, Identification of jiles–atherton model parameters using improved genetic algorithm, in: 2020 IEEE 1st China International Youth Conference on Electrical Engineering, CIYCEE, 2020, pp. 1–6.
[6] Mingxing Tian, Hongchen Li, Huiying Zhang, Neural network model for magnetization characteristics of ferromagnetic materials, IEEE Access 9 (2021) 71236–71243.
[7] Josefina María Silveyra, Juan Manuel Conde Garrido, On the modelling of the anhysteretic magnetization of homogeneous soft magnetic materials, J. Magn. Magn. Mater. 540 (2021) 168430.
[8] Josefina M. Silveyra, Juan M. Conde Garrido, A physically based model for soft magnets' anhysteretic curve, JOM (2023) 1–14.
[9] D.C. Jiles, J.B. Thoelke, M.K. Devine, Numerical determination of hysteresis parameters for the modeling of magnetic properties using the theory of ferromagnetic hysteresis, IEEE Trans. Magn. 28 (1) (1992) 27–35.
[10] D.C. Jiles, D.L. Atherton, Theory of ferromagnetic hysteresis, J. Magn. Magn. Mater. 61 (1–2) (1986) 48–60.
[11] G. Bertotti, Hysteresis in Magnetism, Elsevier, 1998.
[12] Slawomir Tumanski, Handbook of Magnetic Measurements, CRC Press, 2016.
[13] D.C. Jiles, D.L. Atherton, Theory of the magnetisation process in ferromagnets and its application to the magnetomechanical effect, J. Phys. D: Appl. Phys. 17 (6) (1984) 1265–1281.
[14] Ankit Rohatgi, 2022, Webplotdigitizer: Version 4.6, url https://automeris.io/WebPlotDigitizer.