



SIMULATION OF CATEGORICAL AND CONTINUOUS VARIABLES OF A SOUTH-AMERICAN COPPER DEPOSIT

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Abstract: Planning a mining operation requires modeling the metal grades contained in the ore of the deposit based on sampling data, which are inevitably limited relative to the volume of the deposit.

Unlike estimation, geostatistical simulation allows not only for the construction of a discrete three-dimensional model of the target grades, but also for the quantification of uncertainty through the generation of multiple equiprobable realizations.

This paper presents the construction of a three-dimensional block model of a porphyry copper deposit in South America. The study area covers approximately 1.4 km² to a depth of about 600 m, and the available data consist of approximately 40000 samples containing copper grade values and interpretive geological models provided by the data-owning company to define rock and mineral types. Since rock type is the primary control on the spatial distribution of metal grades, the work follows a hierarchical approach: simulating the geological domains first and then the grades within each of them allows this dependency to be accounted for and reduces the range of variability to be modeled within each domain. The geological domains are simulated using Truncated Plurigaussian Simulation, a method that allows for the reproduction of complex contacts between lithotypes; the total copper grade is then simulated within each domain separately using Turning Bands Simulation.

This paper has presented the methodology of how to obtain different block models of a South American copper deposit through the combined application of Truncated Plurigaussian Simulations for categorical variables and Turning Bands Simulations for continuous variables.

The geostatistical simulation approach offers clear advantages over traditional kriging: it reproduces the full variability of the deposit, avoids the smoothing effect, and provides multiple realizations that quantify uncertainty for mine planning purposes.

Keywords: Mineral resource estimation, geostatistical simulation, Turning Bands Simulations, Plurigaussian simulation, copper deposit, block model

1. Introduction

Human beings have always been dependent on natural materials to satisfy the needs of everyday life. All these materials must come from Earth, either grown as plants or animals or extracted as rocks, minerals, or fossil fuels. Some of the rocks and minerals that are important in everyday life are readily available, but many are not, and we must search for them: that search is what exploration is about. Some mineral resources are used directly, with little processing apart from shaping, polishing, crushing, or sizing; but many mineral resources are consumed in industrial processes that transform them into new products. The large group of rocks we call ores (e.g. iron, copper, and gold ores) are important because of the metals they contain. It is in this context that mining activities take place [1].

The workflow of a large scale mining operation involves many players with different specialties, but the aim is one: to exploit minerals that have to be processed, and then the refined mineral has to be sold on the market. In order to plan a mining activity around this business, economists and engineers require information on how much of this mineral it is possible to provide at each phase of the production system, in order

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to properly design them. Since every process has its own efficiency and requires certain characteristics for the input material, we have to start with quantifying the quantity of the material of interest that is naturally present in the portion of the earth's crust under investigation. This quantity will affect in a cascading manner all the quantities that will be used as inputs and outputs of every phase of the production process [2].

Geostatistics was developed for this purpose: to provide a more reliable estimation of the properties of a deposit, once the exploration phase has defined the borders of the study area. When the goal is to estimate the mineral grade, this activity is called mineral resource estimation.

Due to the potential consequences of uncertainty, mineral resources are estimated and the uncertainty of estimates is assessed using different methods [3]. The transition from exploration to mine planning is achieved by constructing a three-dimensional subsurface model. Historically, there are two approaches to create a three-dimensional block model of the subsurface: a traditional approach and a geostatistical approach.

According to systems theory, a deterministic model will always provide the same output given the same input. A stochastic process, in contrast, incorporates randomness in the model, so multiple runs of the same model may generate different values [4]. This random element forces the generation of many outcomes to explore the full range of possibilities.

Attribute values at unsampled locations may be assigned by estimation or by simulation. Estimation involves using the available data to identify a single value that is thought to lie close to the true but unknown value at that location. Simulation uses the available data to construct a probability distribution for each attribute at each location, and a full realization is then obtained by sampling from those distributions [5].

Geostatistics introduced the concept of regionalized variable (RV): a function that presents spatial continuity but also varies irregularly. The purpose of Geostatistics is the mathematical description of regionalized phenomena through the characterization of spatial variability as a function that depends not only on the distance between samples but also on their orientation in space [2].

A classic problem in geostatistics is illustrated in Fig. 1: when two points are at different distances from the estimation target, the spatially closer one does not necessarily provide the more similar value, if there exists a preferential direction of mineralization continuity.

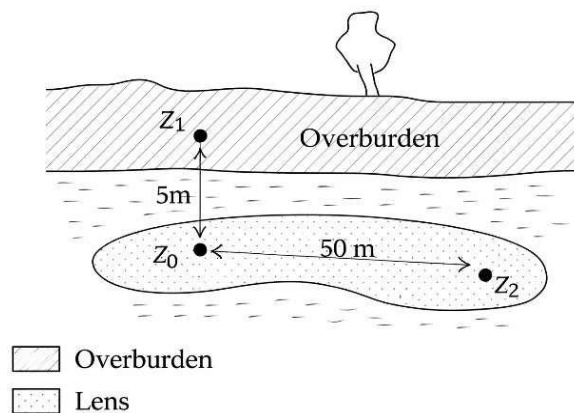


Fig. 1. Representation of the spatial problem: distance is not the only variable to consider when studying how a property varies in the subsurface

Notwithstanding the benefits of kriging over traditional estimation methods, Ordinary Kriging suffers from important weaknesses that limit its applicability in modern mineral resource estimation:

- Smoothing effect: kriging does not reproduce the extreme values of the variable, leading to a spatial model that is too regular and underestimates local variability.
- Single realization: kriging produces only one estimated value per location, preventing the quantification of uncertainty on the estimated metal grade.
- Lack of local information in variance: the Kriging Variance does not incorporate the actual sample data values, only their spatial configuration. Simulation methods overcome this limitation by including local information directly in the model [2].

Fig. 2 illustrates the smoothing effect intrinsic in kriging, which limits the reproduction of the natural variability of the phenomenon.

Geostatistical simulations address the limitations of kriging by generating multiple equally probable realizations of the deposit, each reproducing the spatial statistics of the conditioning data. This allows the propagation of geological uncertainty through subsequent mine planning stages, providing a probabilistic framework for decision-making rather than a single deterministic model [2].

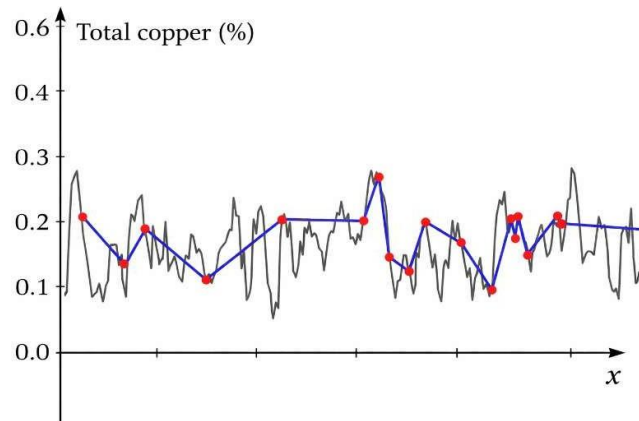


Fig. 2. Limitation in representing the natural variability of a phenomenon due to the smoothing effect intrinsic in kriging.
Adapted from Lantuéjoul (2002) [6].

2. Geostatistical Simulation Algorithms

Simulation models are descriptive, not prescriptive. They tell how a system works under given conditions, not how to arrange those conditions to make the system work best. A simulation does not optimize, it merely describes [4]. Stochastic simulations are characterized by the introduction of randomness in the model, guaranteeing that each realization of the reality will differ from the others.

2.1 Simulation of Continuous Variables: Turning Bands Simulation

The idea behind the Turning Bands Method (TBS) is to reduce the problem of simulating a multidimensional continuous variable to the problem of simulating a 1D continuous variable [6]. The method consists of two main parts: the generation of an unconditional simulation at all points in the space of interest, followed by the conditioning of the simulated variable at the sampled points [2, 6, 7, 8].

In the first part, a set of N lines passing through a point is generated. Each line is then simulated using functions that comply with the properties of a standard Gaussian variable $N(0,1)$. A generic point is projected onto each line, and the simulated value at that projection is identified. The average of these values across all lines gives one realization of the unconditional simulation at that point. This procedure is repeated for all points in the 3D model [2].

In the second part, the unconditional simulated values are conditioned at the exact locations of the data. The conditional simulated values $Y_{cs}(x)$ are expressed as [2]:

$$Y_{cs}(x) = Y_{us}(x) + [Y_{kc}(x) - Y_{ku}(x)] \quad (1)$$

where $Y_{us}(x)$ is the unconditional simulation, $Y_{kc}(x)$ is the kriging of the conditioning data, and $Y_{ku}(x)$ is the kriging of the unconditional simulated values at data locations. The conditioning term tends to zero as the distance from the conditioning data approaches the variogram range. The simulated values are finally back-transformed into the original scale of the variable.

2.2 Simulation of Categorical Variables

The first method for simulating geological facies and lithotypes was developed in the late 1980s to simulate oil reservoir lithotypes. The rationale is clear: simulating the geometry and internal architecture of the reservoir first, and then generating suitable petrophysical values conditioned on the lithotype, leads to a more accurate reserve evaluation [9].

The same logic applies in the mining industry: geometallurgical processing often depends on rock type rather than on grade alone. Therefore, for both mining deposits and oil reservoirs, the recommended workflow is: first simulate the geometry of the lithotypes or facies, and then simulate their properties as a function of that type [9].

2.3 Truncated Gaussian Simulations

Truncated Gaussian Simulation (TGS) is a method to simulate rock types that occur in sequential order. The idea is to simulate a Gaussian variable with a $N(0,1)$ distribution at every point in the domain, and then truncate this distribution using truncation thresholds. Each segment of the Gaussian curve represents a rock type, and each truncation threshold represents an admissible physical contact between adjacent rock types. After simulation, each Gaussian value is transformed back into the corresponding rock type [9].

As an example, Fig. 3 shows a rock type rule defining a stratigraphic sequence of rock types A, B, and C with proportions of 30%, 50% and 20% respectively, delimited by thresholds $t_1 = -0.52$ and $t_2 = 0.84$.

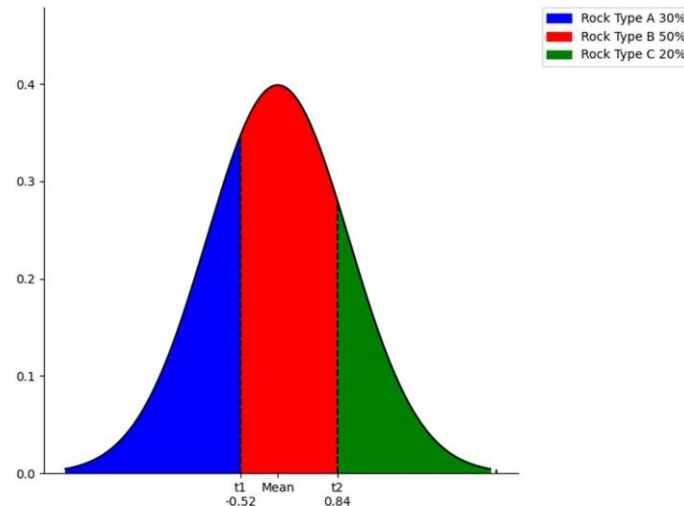


Fig. 3. An example of a rock type rule defining the sequential order of the lithotypes. Thresholds t_1 and t_2 indicate the admissible contacts and the values where the standard Gaussian distribution is truncated

In a Truncated Gaussian Simulation, four main steps are required [9]:

- Define the geological model as stratigraphic in nature (applicable to hydrocarbon or alluvial deposits).
- Estimate the truncation thresholds by applying Simple Kriging to the proportions of each rock type, and derive the variogram model of the Gaussian variable.
- Convert the categorical variable into indicators and exploit the mathematical relationship between indicator variograms and the variogram of the underlying Gaussian variable.
- Execute the Gibbs Sampler algorithm to simulate Gaussian values at the sample locations respecting the rock type rule, then apply one of the standard simulation algorithms (SGS or TBS) to obtain a full realization.

2.4 Truncated Plurigaussian Simulations

The natural extension of TGS is the Plurigaussian Simulation method. The main limitation of TGS is that it can only be applied when a clear sequential stratigraphy exists in the subsurface, since each lithotype can be in contact with at most two others. When the geological structure of the deposit is more complex, a more flexible model is required [9].

The Plurigaussian method overcomes this limitation by defining the rock type rule through two or more independent Gaussian random variables. As a consequence, the rock type rule is no longer a simple line of parallel thresholds, but a two-dimensional flag divided by threshold lines, as shown in Fig. 4. This allows for modeling a greater variety of possible contacts between rock types that would be impossible to reproduce with a single Gaussian variable [9].

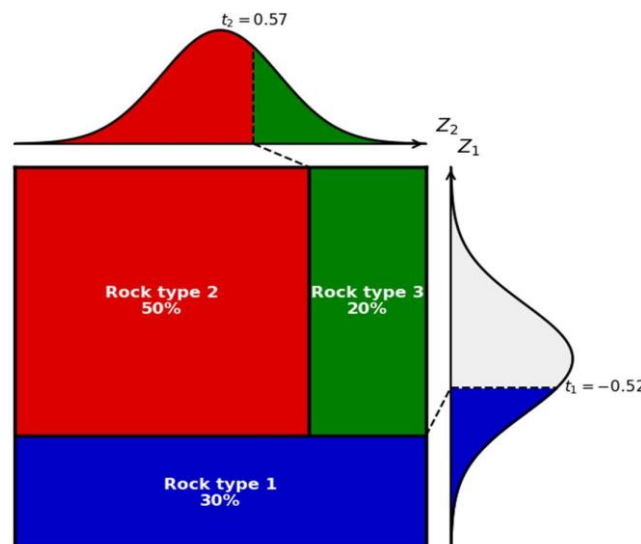


Fig. 4. An example of a rock type rule in the Truncated Plurigaussian model, with three rock types and proportions of 30%, 50% and 20% respectively. Adapted from Maleki (2016) [8] and Armstrong et al. (2011) [9]

The thresholds are computed sequentially. The first threshold t_1 assigns the bottom 30% of the domain to Rock type 1. The second threshold t_2 must be computed as the quantile associated with the cumulative probability $0.50/(1-0.30) \approx 0.71$, ensuring that the prescribed global proportions are correctly reproduced. When the two Gaussian random variables are independent, the truncation rule is applied in two steps: the realization of Z_1 determines whether each location belongs to Rock type 1 or to the remaining domain, and then Z_2 assigns the remaining locations to Rock type 2 or Rock type 3.

An important operational rule governs the order of simulation: the most recent mineralization or rock type must be simulated first using Z_1 , followed by the remaining types in reverse chronological order using Z_2 .

3. Case Study

After identifying the appropriate methodology for simulating both categorical and continuous variables, we present an application to a copper deposit in South America. For confidential reasons, the coordinates have been modified and the minerals are labeled with geological codes. The study area covers a surface of approximately $800 \text{ m} \times 1000 \text{ m}$, and the drill holes follow a mostly regular grid of $25 \text{ m} \times 40 \text{ m}$, reaching depths up to 600 m below the surface.

The dataset consists of a total of 44,920 samples, each analyzed for at least one of the following metals: total copper (CuT), molybdenum (ppm), and gold (ppm). In this article only the study of total copper is conducted. In contrast, the subsurface model obtained using geophysical exploration techniques shows a total of 22,458 samples where a categorical variable associated with different mineralization and/or rock types.

3.1 Identification of Estimation Domains

According to the chosen mineralization model, 22,030 out of 22,458 samples have been labeled with a categorical variable. Their original spatial distribution is shown in Fig. 5.

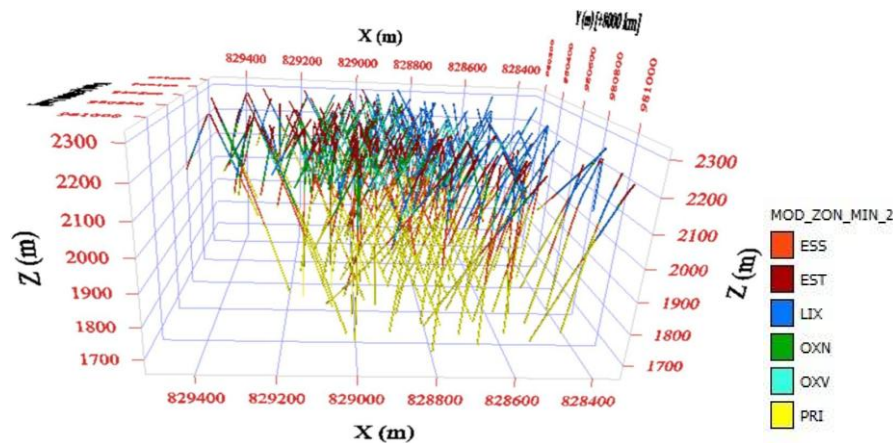


Fig. 5. Original mineralization distribution

In order to perform Plurigaussian Simulations, the number of different estimation domains is reduced by clustering geological domains with statistical similarities in total copper grade and spatial vicinity. According to Rossi & Deutsch (2014) [2], estimation domains must make spatial and geological sense: zones clustered into a single estimation domain must share geochemical (ore grade) and geometallurgical (ore recovery and grindability) properties [10].

The first statistical study regards the average–variance plot of the copper grade for each geological domain, shown in Fig. 6. The copper grade histograms for each geological code are presented in Fig. 7.

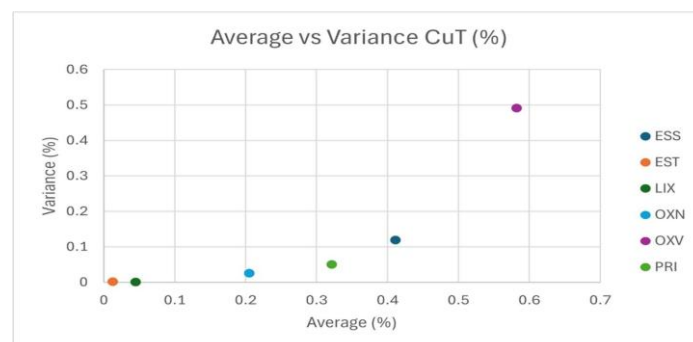


Fig. 6. Average vs. Variance total copper plot

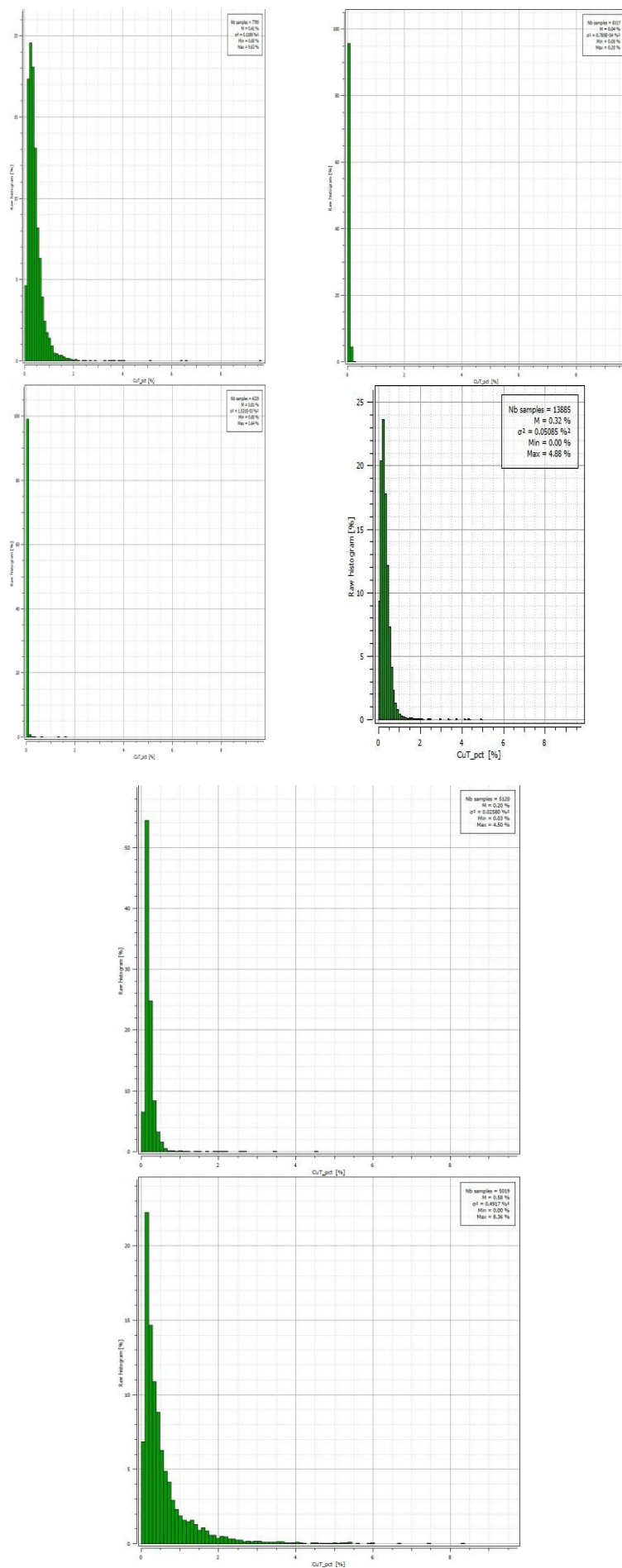


Fig. 7. Total copper histograms for each geological domain: (a) EST, (b) LIX, (c) OXN, (d) PRI, (e) ESS, (f) OXV

Q–Q plots of the clustered geological domains are shown in Fig. 8. The analysis reveals that EST, LIX and OXN are characterized by poor ore grades, while PRI and ESS show economically valuable copper grades for the current year (2026). OXV displays a high copper enrichment. EST and LIX are classified as waste; OXN, while having a slightly higher copper grade than waste, is not and will not be economically valuable for many years. ESS and PRI show clearly similar distributions. The first cluster will be labeled as RT1, OXV will be labeled as RT2 and the third cluster will be labeled as RT3.

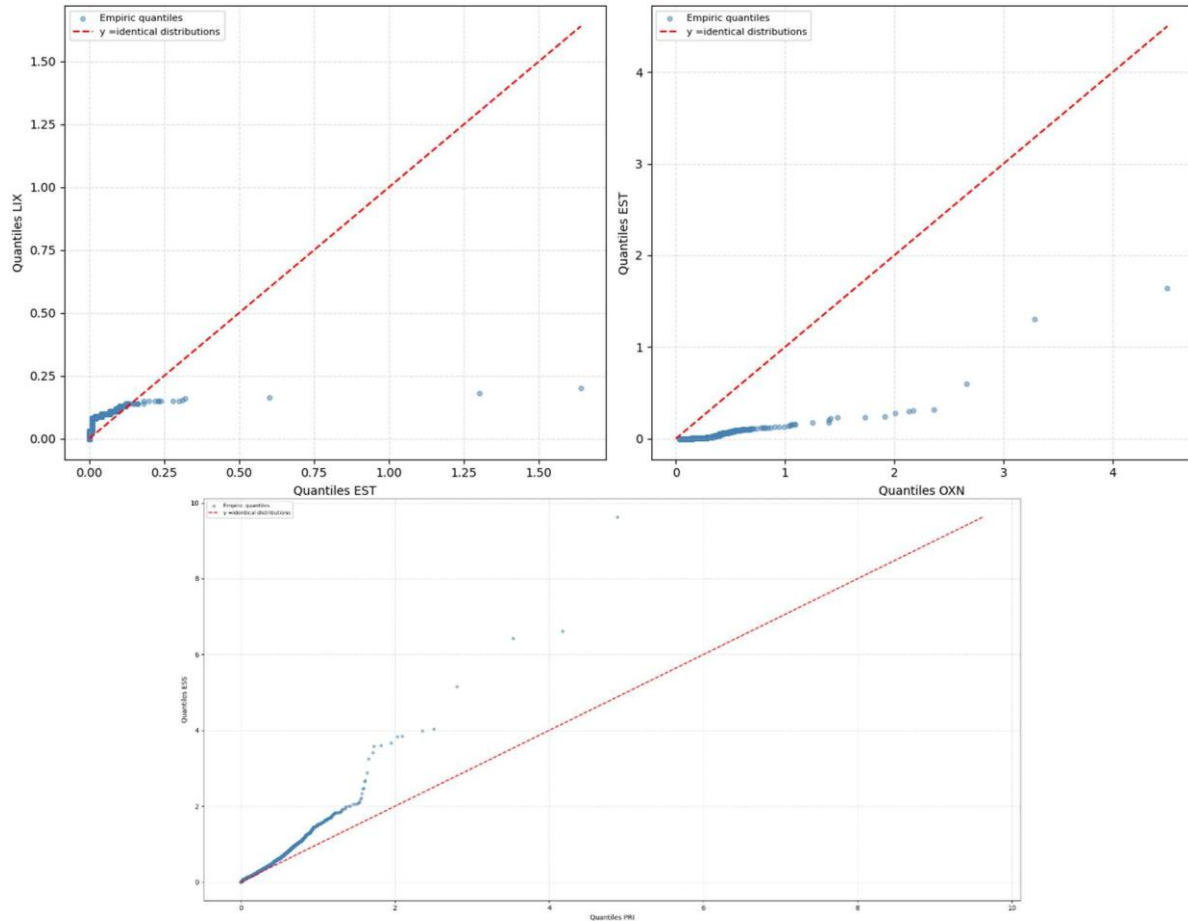


Fig. 8. Q–Q plots of the clustered mineralizations: (a) Second cluster RT1, (b) Third cluster RT3

Spatial vicinity was then assessed to justify the clustering of geological domains into estimation domains, as shown in Fig. 9. The mineralization labeled EST, LIX and OXN is mainly located at shallow depths, at elevations between 2320 m and 2040 m. OXV is located between 2250 m and 2070 m elevation, and ESS and PRI are located between 2100 m and 1700 m elevation.

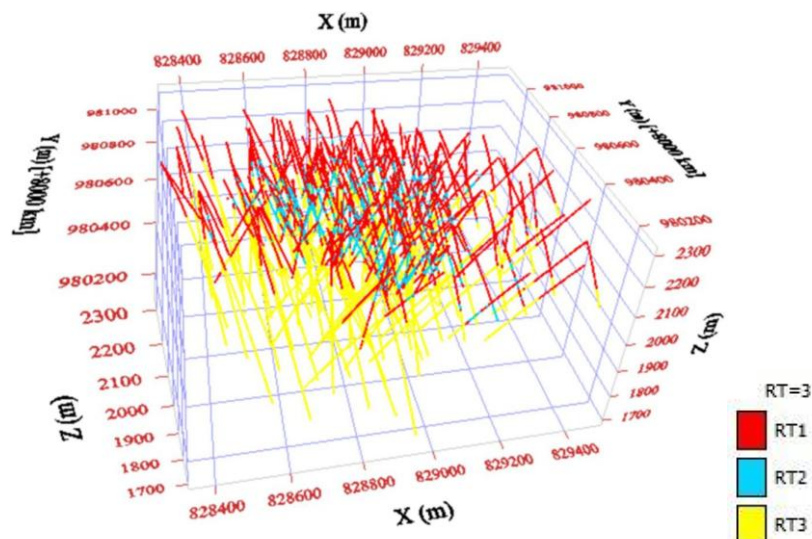


Fig. 9. Spatial distribution of the three clusters: (a) RT1 (EST, LIX, OXN), (b) RT2 (OXV), (c) RT3 (ESS, PRI)

3.2 Exploratory Data Analysis (EDA)

Compositing

As categorical attributes are assigned to supports of variable length, compositing to a standard length is required. Data compositing is performed creating a macro-variable characterized by samples with a length of 2 m, as shown in Fig. 10. The residuals are kept and the criterion to assign a geological code to each composite is the most frequent geological code within that sample interval.

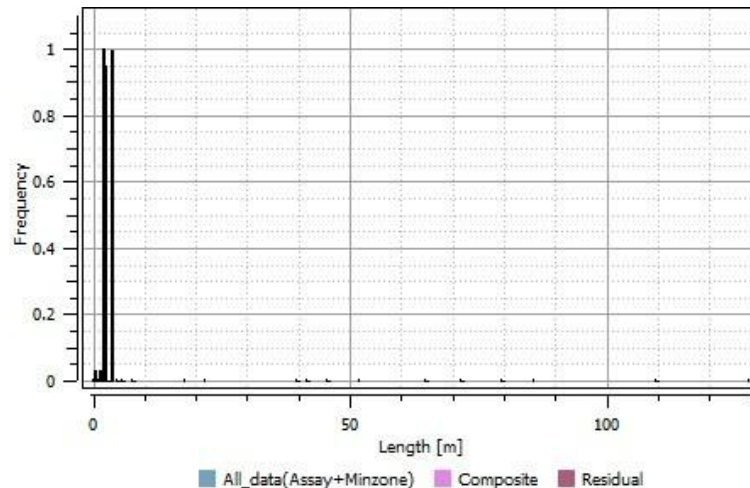


Fig. 10. Sample composited to a length of 2 m

Capping

Capping just above a threshold might lead to errors in the mineral resource estimation, since not all high ore grade values are outliers. Fig. 11 shows a box plot of the total copper grade (CuT).

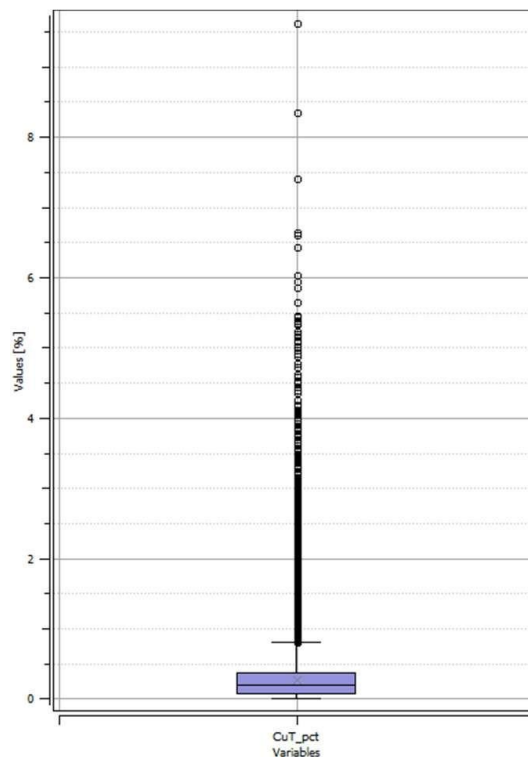


Fig. 11. Box plot of CuT.

Applying a global capping threshold of, for example, 3.0% would remove many samples from a specific region coded as OXV, as shown in Fig. 12. Since removing data from a dataset must be done carefully, capping by geological code is recommended in all cases where the data belong to a geological domain whose ore grade distribution justifies it, and where it is consistent with studies conducted in the same area and geological conditions, such as Singer et al. (1975) [11].

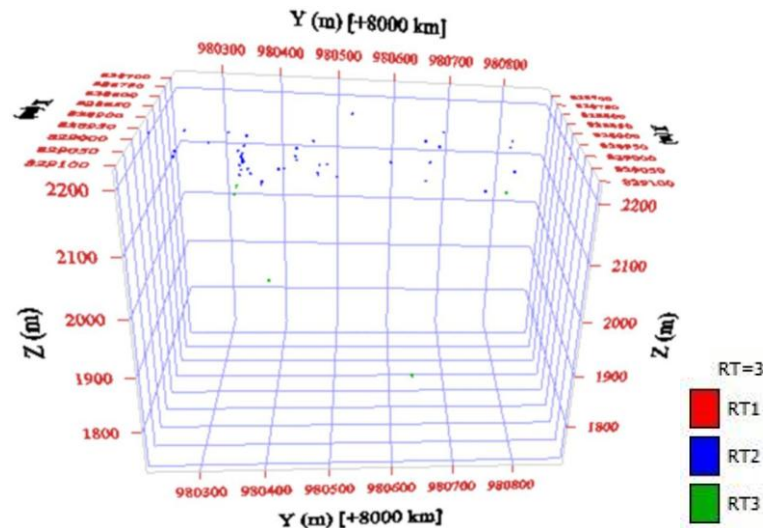


Fig. 12. Effect of applying a global capping threshold to all mineralization. The picture shows that a cluster of samples in a high-enrichment area and belonging to a specific mineralization (RT2) would be capped

Declustering

Many different declustering window sizes were tested, as shown in Fig. 13. However, none of the declustering configurations was ultimately applied, since the resulting weights would not have significantly affected the simulation outcomes.

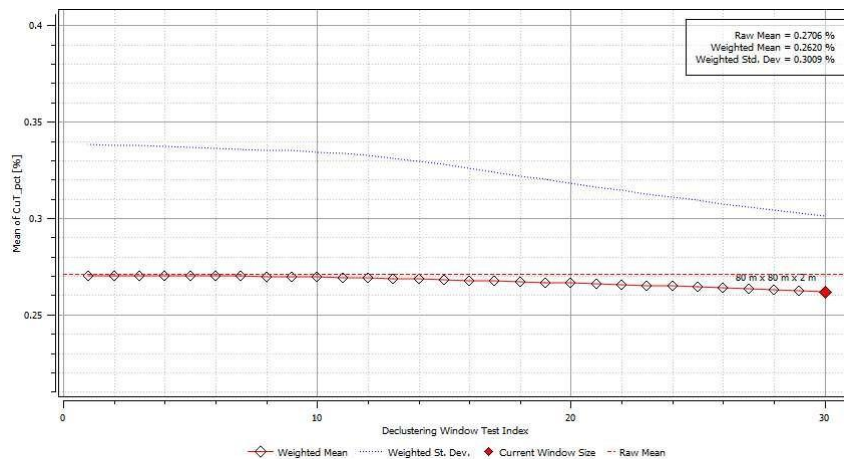


Fig. 13. Declustering performed for different window sizes

3.4 Simulation of categorical variables

Proportion modeling

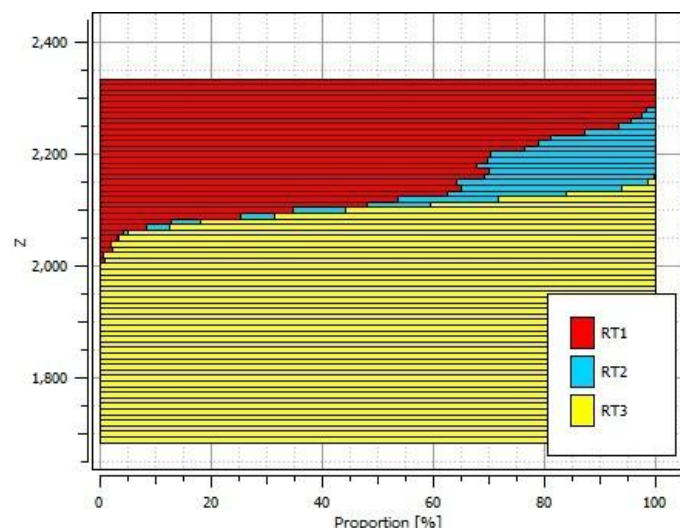


Fig. 14. A generic vertical section of clusters of mineralization proportion before estimating them

Proportion modeling is performed by estimating the local proportions of each cluster of mineralization recurring to Ordinary Kriging and using a unique neighborhood of 400 m x 400 m x 200 m and with a vertical discretization of 10 m. An example of a vertical section of the estimation block model is shown in Figure 14. In Figure 15 it's shown an instance of which blocks have a probability of at least 30 % to belong to the categorical value of OXV, also reported as RT2 as the label used for clusters.

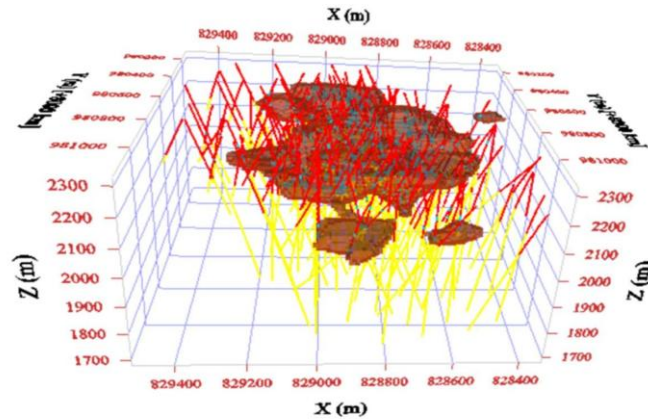


Fig. 15. In brown the blocks where a probability of RT2 to occur have been highlighted

Plurigaussian Simulation

Plurigaussian Simulation has been performed in a block model constituted of cubic blocks where each edge measures 10 m and using the proportion modeling discussed at the previous step. In Figure 16 the Rock Type Rule used is shown. In Figure 17 the variogram models for the two gaussian variables v1 and v2 are illustrated.

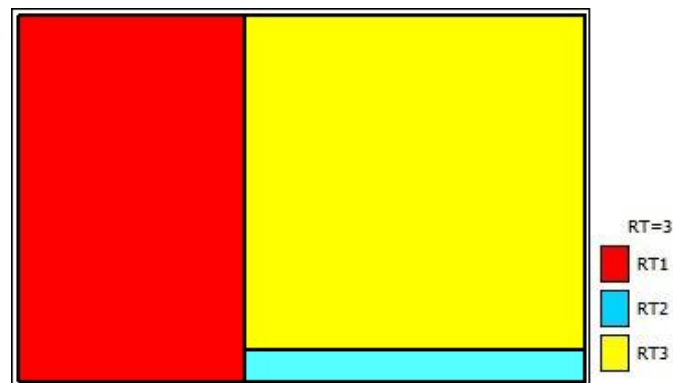


Fig. 16. Rock Type Rule (also known as "Flag") used for the Plurigaussian Simulation of categorical variables RT1, RT2 and RT3

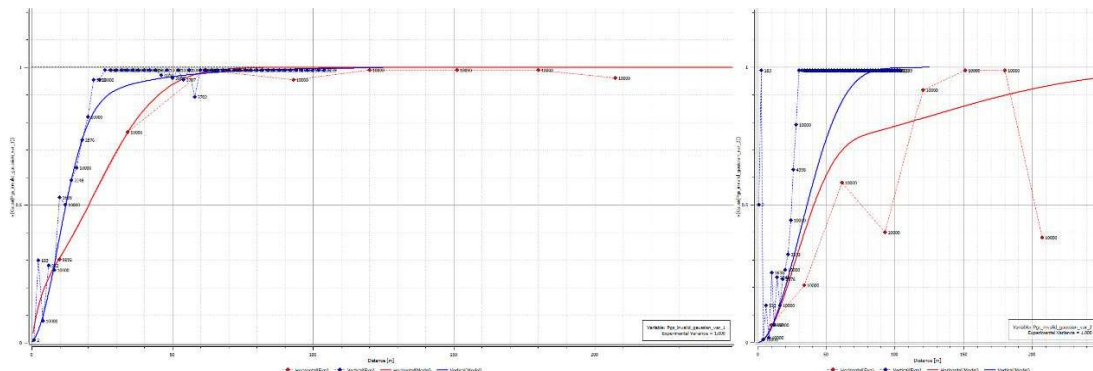


Fig. 17. On the left the plurigaussian variogram that describe the plurigaussian model where RT1 is simulated against the rest of the categorical domain. On the right the plurigaussian variogram that describe the plurigaussian model where RT2 is simulated against RT3

After performing 100 simulations, each one corresponding to a possible realization of the underlying plurigaussian model, in Figure 18 two central vertical sections of the most probable resulting block model are presented. Both show a slice with a width of 30 m, the first one in the Y-Z plane, the one on the right in the X-Z plane.

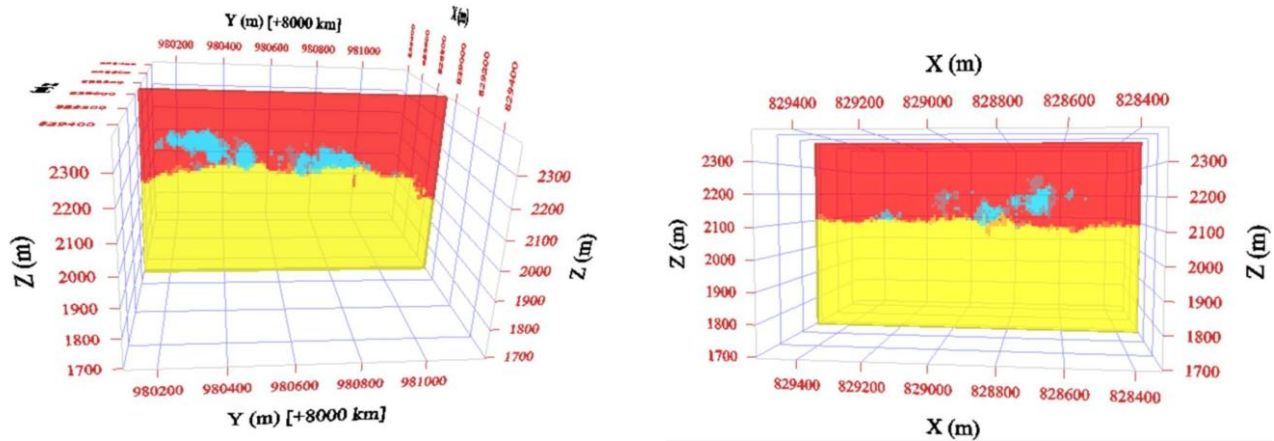


Fig. 18. On the left the plurigaussian variogram that describe the plurigaussian model where RT1 is simulated against the rest of the categorical domain. On the right the plurigaussian variogram that describe the plurigaussian model where RT2 is simulated against RT3

Plurigaussian Simulation

Once the Plurigaussian Simulation has been performed, what remains is just to simulate the copper grade using Turning Bands method.

The original values have to be normalized trough a NST (or anamorphosis). Then the variogram of the normalized variable has to be modeled, as shown in Figure 19.

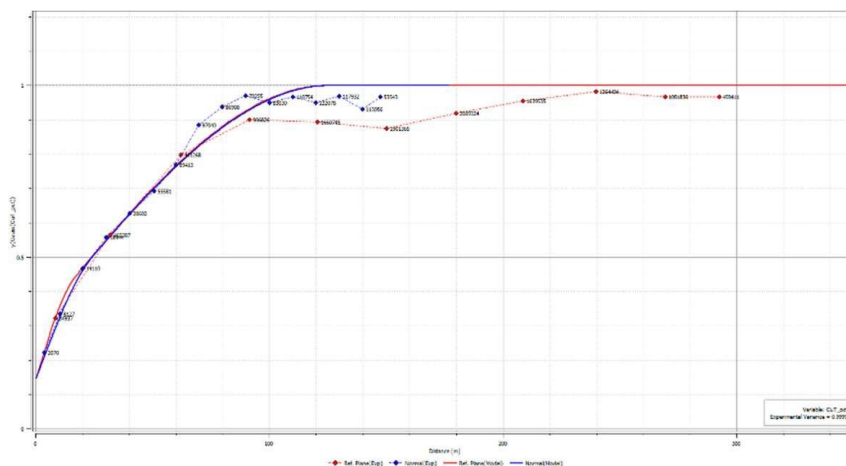


Fig. 19. Horizontal and vertical variogram modeled around experimental variograms in the two directions

After this transformation, 30 realizations have been performed with 1000 lines used to divide the space. In Figure 20, a section of the mean of 30 different TBS is reported.

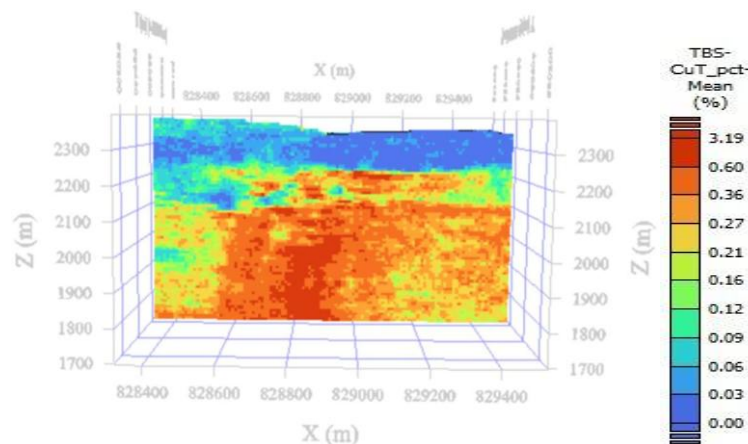


Fig. 20. A vertical section of a block model of the mean of 30 different simulations of total copper obtained by performing TBS

4. Conclusions

This paper has presented the methodology of how to obtain different block models of a South American copper deposit through the combined application of Truncated Plurigaussian Simulations for categorical variables and Turning Bands Simulations for continuous variables.

The exploratory data analysis demonstrated the importance of domain-specific data preparation, including compositing, capping by geological code rather than by global threshold, and the assessment of declustering weights. The statistical and spatial analysis of the six geological codes led to a rational reduction to three estimation domains (RT1, RT2, RT3), enabling the application of the Plurigaussian simulation framework.

The geostatistical simulation approach offers clear advantages over traditional kriging: it reproduces the full variability of the deposit, avoids the smoothing effect, and provides multiple realizations that quantify uncertainty for mine planning purposes.

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