Optimal sentencing with recurring crimes and adjudication errors

Alice Guerra[*] Tore Nilssen[†]

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Abstract

We analyze optimal sentence length for recurring crimes in the face of adjudication errors. We develop an infinite-horizon model where offenders are habitual—they repeat crimes whenever free. If apprehended, criminals may be wrongfully acquitted. Similarly, innocent people may be apprehended and wrongfully convicted. The key result shows how the risks of wrongful convictions and wrongful acquittals affect optimal sentencing. For reasonable ranges of parameter values, the two types of adjudication errors have the same qualitative effect on optimal sentencing: a greater risk of any of the two adjudication errors leads to a decrease in optimal sentencing.

Keywords: Recurring crime; Recidivism; Incapacitation; Adjudication errors; Sentencing
JEL Codes: K14; K42

1 Introduction

Some crimes have a high rate of recidivism, meaning that many offenders go back to repeat the crimes after possibly serving jail time. Crimes with such high rates of recidivism include intimate violence, child molesting, and embezzlement (see, e.g., Walsh and Beck,

[*]Department of Economics, University of Bologna, via Angherà 22, 47921 Rimini, Italy. ORCID: 0000-0003-1956-0270. E-mail: alice.guerra3@unibo.it.

[†]Department of Economics, University of Oslo, Norway. ORCID: 0000-0003-2632-3855. E-mail: tore.nilssen@econ.uio.no.

1990; Bureau of Justice Statistics, 2003; Kensey and Benaouda, 2011; Durose et al., 2014; Kang, 2017; Monnery et al., 2020). In such settings, a society may want to apprehend and convict these individuals to incarcerate them so that they are unable to commit additional offenses (the so-called "incapacitation effect;" e.g., Shavell, 1987, 2015; Dittmann, 2006; Owens, 2009; Abrams, 2012; Buonanno and Raphael, 2013; Di Tella and Schargrodsky, 2013; Barbarino and Mastrobuoni, 2014). A problem arises, though, in the presence of adjudication errors, which give rise to cases where criminals are acquitted and innocent people are sent to prison (Polinsky and Shavell, 2007). Wrongful acquittals imply that some criminals will escape convictions and be let out into society to repeat crime, while wrongful convictions mean that innocent people are put in prison.

The question is whether incarceration can be a good strategy in the face of recurring crimes when adjudication errors are possible. In this paper, we investigate this question: how do adjudication errors affect optimal sentencing in the context of recurring crimes? To this aim, we develop a model of a society where individuals live forever, some of them criminals and others non-criminals; to be specific, criminals do crime whenever they are at large, while non-criminals never do. If apprehended, an individual from any of the two groups—criminals or non-criminals—may get convicted, but a criminal gets convicted at a higher rate than a non-criminal. This model enables us to discuss the joint effect of crime recurrence and adjudication errors on optimal sentence length by exploring the properties of the society at the steady state.

Our results show how the risks of wrongful convictions and wrongful acquittals affect the optimal sentencing, revealing that this effect depends on the range of parameter values. The key finding is that, for a reasonable range of values, optimal sentencing is decreasing with the risk of any of the two adjudication errors—convicting non-criminals or acquitting criminals. Shortly speaking: the greater the error risks are, the shorter should be the sentencing.

Most previous work on sentencing in the presence of adjudication errors discusses crime as a one-time event and thus disregards the issue of recurring crimes (e.g., Lando and Mungan, 2018). Of course, exceptions exist: Rubinstein (1979), Chu et al. (2000), and Emons (2007) analyze recurring crimes and adjudication errors; however—while allowing for the possibility that some criminals are not convicted after committing a crime—they focus on wrongful convictions only.[1] The main finding of these studies is that repeat offenders should be punished harder than first-time offenders. Our model differs from the above in that we consider *both* wrongful convictions and wrongful acquittals. Criminals can, of course, go on with their crimes if they are not apprehended after committing a crime. In our analysis, allowing for wrongful acquittals in addition to wrongful convictions represents a crucial contribution to the discussion of crimes with high rates of recidivism, since such wrongful acquittals open up the possibility for criminals returning to crime not only because they may not be apprehended after committing a crime, but also because, even if apprehended, they may be wrongfully acquitted.

This paper is structured as follows. In Section 2, we present the model and derive the results. In Section 3, we summarize the key findings and conclude with suggestions for future research. The proofs of the propositions are reported in Appendix A.

2 Model and Analysis

Consider a society with a population of measure 1, where a fraction $\nu \in (0,1)$ are criminals, while the rest of the population are non-criminal. The society goes through an infinite number of periods. All individuals are present throughout, i.e., there are no births or deaths. A convicted individual serves $n \in \mathbb{N}$ periods in prison before being let back into society; so an individual convicted in period t stays in prison through period $t+n$ and is let out at the start of period $t+n+1$. As discussed in the introduction, our concern is mainly categories of crime where recidivism is high. In the model, we address this concern by assuming that crime always pays for criminals—so that, if a person is a criminal and not in prison in period t, then she does crime in that period.

An individual will be apprehended at the end of period t with probability $p \in (0,1)$; for simplicity and with little loss of generality, we assume that the apprehension rates of criminals and non-criminals are the same. If apprehended, a criminal is prosecuted and convicted with probability $(1-\beta)$, where $\beta \in [0,1)$ is the probability of a wrongful acquittal. A non-criminal will, if apprehended, be prosecuted and convicted with probability

[1] See, e.g., Chu et al. (2000), wherein those who commit crimes in period 1 but are not convicted are treated as first-time offenders.

$\alpha \in [0,1)$, which is the probability of a wrongful conviction. We make the following assumption:

Assumption 1. $\alpha + \beta < 1$.

This assumption means that the probability of being convicted is larger for an apprehended criminal than for an apprehended non-criminal: $1 - \beta > \alpha$.

Before we start deriving the optimal sentencing, we analyze the properties of the steady state. In the steady state, the fraction of the population in prison is constant. We have the following proposition.

Proposition 2.1. *In this society,*

(i) the number of crimes in each period equals

$$\frac{v}{1 + np(1 - \beta)}; \tag{2.1}$$

(ii) the fraction of the population innocently in prison at any time is

$$\frac{\alpha np(1 - v)}{1 + \alpha np}. \tag{2.2}$$

Part (i) of the Proposition shows that the number of crimes increases with the number of criminals but also with the extent of wrongful acquittals. In addition, the number of crimes decreases with the length of sentencing and the rate of apprehension. Part (ii) shows that the number of innocently jailed individuals decreases with the number of criminals but increases with the length of sentencing, the rate of apprehension, as well as the extent of wrongful convictions.

The government's concern in designing its judicial system is to minimize social costs. It is not concerned with the benefits of crime nor with criminals' disutility from being jailed. For simplicity, we disregard the government's costs of running the prison system. What the government does take into account are the victims' costs of crime and the social cost of having innocent people in prison. Considering the steady state as described in Proposition 2.1, we define H as victims per-period costs of crimes (or, equivalently, the harm caused by unincarcerated guilty people), and K as the per-period social cost of having innocent people in prison (or, equivalently, the harm caused by incarcerated innocent people). This means that the total per-period social cost, C, is

$$C = H + K \tag{2.3}$$

Assuming these costs are linear, with a per-period social cost of $k > 0$ of having an innocent individual in prison, and victims' costs per crime $h > 0$, we can use Proposition 2.1 to write

$$C = \frac{h\nu}{1 + np(1-\beta)} + \frac{k\alpha np(1-\nu)}{1+\alpha np}. \tag{2.4}$$

We make the following regularity assumption on $\frac{h}{k}$, which measures the social cost of a crime relative to the per-period social cost of keeping an innocent in prison:

Assumption 2. $\frac{h}{k} < \frac{1-\beta}{\alpha}\frac{1-\nu}{\nu}$.

Realistically, the expression to the right in this assumption is much larger than 1. The assumption is, therefore, a rather weak one. It means that the social cost per crime may be lower or greater than the per-period social cost of keeping an innocent in prison, but that there is a (weak) restriction on how many times greater it can be.[2]

To analyze how the social costs vary with sentencing, we disregard the fact that, in this model, the length of the prison term is an integer. This means that we can differentiate the social costs in (2.4) with respect to n to find the optimal sentencing. Clearly, the optimal n^* satisfies

$$\frac{\partial H}{\partial n} = -\frac{\partial K}{\partial n}. \tag{2.5}$$

We have the following proposition.

Proposition 2.2. *Disregarding the integer constraint on n, the optimal sentencing equals*

$$n^* = \begin{cases} 0, & \textit{if} \quad 0 < \frac{h}{k} \le \frac{\alpha}{1-\beta}\frac{1-\nu}{\nu}; \\ \frac{1-\sqrt{\frac{k}{h}\frac{\alpha}{1-\beta}\frac{1-\nu}{\nu}}}{\alpha p\left(\sqrt{\frac{k}{h}\frac{1-\beta}{\alpha}\frac{1-\nu}{\nu}}-1\right)}, & \textit{if} \quad \frac{\alpha}{1-\beta}\frac{1-\nu}{\nu} < \frac{h}{k} < \frac{1-\beta}{\alpha}\frac{1-\nu}{\nu}. \end{cases} \tag{2.6}$$

Note that the denominator in (2.6) is positive, by Assumption 2. Hence, whether optimal sentencing is positive or zero is determined by the numerator in (2.6).

[2]For example, if $\alpha = \beta = \nu = p = 0.05$, then the expression to the right in Assumption 2 is 361.

We can now find how adjudication errors affect optimal sentencing. We obtain the following result.

Proposition 2.3. *(i) If*

$$\frac{h}{k} \leq \frac{\alpha}{1-\beta}\frac{1-\nu}{\nu}, \tag{2.7}$$

then the optimal sentencing n^* is zero and independent of the probability of wrongful convictions, α, and the probability of wrongful acquittals, β.

(ii) If

$$\frac{h}{k} \in \left(\frac{\alpha}{1-\beta}\frac{1-\nu}{\nu}, \frac{4\alpha(1-\beta)}{(1+\alpha-\beta)^2}\frac{1-\nu}{\nu}\right), \tag{2.8}$$

then n^* is decreasing in both α and β.

(iii) If

$$\frac{h}{k} \in \left(\frac{4\alpha(1-\beta)}{(1+\alpha-\beta)^2}\frac{1-\nu}{\nu}, \frac{(1+\alpha-\beta)^2}{4\alpha(1-\beta)}\frac{1-\nu}{\nu}\right), \tag{2.9}$$

then n^* is decreasing in α and increasing in β.

(iv) If

$$\frac{h}{k} \in \left(\frac{(1+\alpha-\beta)^2}{4\alpha(1-\beta)}\frac{1-\nu}{\nu}, \frac{1-\beta}{\alpha}\frac{1-\nu}{\nu}\right), \tag{2.10}$$

then n^* is increasing in both α and β.

As shown in Proposition 2.2, a very low $\frac{h}{k}$ means that optimal sentencing is set at zero, to avoid the cost of jailing innocent people. When $\frac{h}{k}$ is higher, Proposition 2.3 reveals that optimal sentencing decreases with the probability of any of the two adjudication errors. For an even higher $\frac{h}{k}$, a greater risk of putting non-criminals in prison calls for decreasing the sentencing, while a greater risk of acquitting criminals calls for increasing the sentencing. Finally, when crime is very costly to society relative to the cost of having innocent people in prison, optimal sentencing increases with the probability of any of the two adjudication errors. This latter result occurs because it might delineate pathological cases, as we discuss below.

Our findings in Proposition 2.3 can be more easily interpreted if compared to those in

Proposition 2.2. Inspecting the expression for n^* in (2.6), we find that

$$n^* \leq \frac{1}{\alpha p} \text{ if and only if } \frac{h}{k} \leq \frac{(1+\alpha-\beta)^2}{4\alpha(1-\beta)}. \tag{2.11}$$

Note that αp is the unconditional probability that an innocent person is put in prison. Although it is a crucial aspect of our analysis that αp is positive (because α is positive), in most modern judicial systems this probability is regularly much lower than 1. This implies that the first statement in (2.11) encompasses a wide range of sentences, from zero to $1/\alpha p$.[3] The second statement in (2.11) narrows our attention to part (ii) of Proposition 2.3, where n^* is decreasing in both α and β. It might be argued that parts (iii) and (iv) of Proposition 2.3 delineate rather pathological cases where the optimal sentence is so long that an increased adjudication error actually reduces the harm from an even longer sentence. Therefore, the focus should be on part (ii) of Proposition 2.3 and, for completeness, the case of no sentencing in part (i): unless the optimal sentence is zero, it is decreasing in both types of adjudication errors.

3 Conclusion

We have developed an infinite-horizon crime model with adjudication errors. The key result shows how the risks of wrongful acquittals and wrongful convictions affect optimal sentencing. We show that, for reasonable ranges of parameter values, the two types of adjudication errors have the same qualitative effect on the optimal sentencing: a greater risk of any of the two adjudication errors leads to a decrease in optimal sentencing.

The purpose of our note is to show that *both* crime recurrence and adjudication errors should be considered in the discussion about the optimal design of criminal justice systems. Our analysis is intentionally simple and it opens up several avenues for future research. For example, it would be interesting to extend it by endogenizing adjudication errors as a function of the number of previous convictions, or including various degrees of recidivism. While prior literature generally models criminals—and non-criminals—as

[3] To follow up on the numerical example in footnote 2, if $\alpha = p = 0.05$, then $1/\alpha p = 400$.

agents, this paper has them behaving in a perfectly mechanical manner. This allows us to isolate a particular set of issues. That is, an increase in jail time has no impact on incentives, but has a benefit only through crime reduction. We leave for future research the exploration of models with crime recurrence and adjudication errors, where individuals are represented as agents capable of making decisions.

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Appendix A

Proof of Proposition 2.1. Let X be the population not in prison in period t, and let Y be the fraction of criminals among those outside prison. Out of the X individuals outside prison in period t, a further fraction $p\left[Y(1-\beta)+(1-Y)\alpha\right]$ will be put in prison for periods $t+1$ through $t+n$.

The population inside prison constitutes $1-X$ of the population. $\frac{1-X}{n}$ of them serve their first period in prison, while equally many serve their second period and third period, and so on. The people not in prison in period $t+1$ are those $\frac{1-X}{n}$ done with their prison term plus those at large in period t who are still out there in period $t+1$. In steady state, these fractions coincide:

$$X = \frac{1-X}{n} + X\{[1 - pY(1-\beta) + (1-Y)\alpha]\}, \text{ or:}$$

$$X = \frac{1}{1+np\left[Y(1-\beta)+\alpha(1-Y)\right]}.$$

Of those not in prison, a fraction Y do crime, so the number of criminals out of prison in each period is

$$\frac{Y}{1+np\left[Y(1-\beta)+\alpha(1-Y)\right]}; \tag{A.1}$$

this is also the number of crimes committed each period.
Those in prison in each period constitute a fraction

$$1-X=\frac{np\left[Y\left(1-\beta\right)+\alpha\left(1-Y\right)\right]}{1+np\left[Y\left(1-\beta\right)+\alpha\left(1-Y\right)\right]}.$$

It follows that a fraction

$$\frac{npY\left(1-\beta\right)}{1+np\left[Y\left(1-\beta\right)+\alpha\left(1-Y\right)\right]} \tag{A.2}$$

of the population are in prison and rightly convicted, *i.e.*, they are criminal, while a fraction

$$\frac{n\alpha p\left(1-Y\right)}{1+np\left[Y\left(1-\beta\right)+\alpha\left(1-Y\right)\right]} \tag{A.3}$$

are innocently jailed, *i.e.*, they are non-criminal.
Since the total number of criminals is ν, we have, from (A.1) and (A.2):

$$\frac{Y}{1+np\left[Y\left(1-\beta\right)+\alpha\left(1-Y\right)\right]}+\frac{npY\left(1-\beta\right)}{1+np\left[Y\left(1-\beta\right)+\alpha\left(1-Y\right)\right]}=\nu, \tag{A.4}$$

where the first term on the left-hand side is the number of criminals outside prison at any time, while the second term is the number of criminals in prison. Solving for Y in (A.4), we have

$$Y=\frac{\nu\left(1+\alpha np\right)}{1+np\left[\left(1-\nu\right)\left(1-\beta\right)+\alpha\nu\right]}. \tag{A.5}$$

It follows that the fraction outside prison is

$$X=\frac{1}{1+np\left[Y\left(1-\beta\right)+\alpha\left(1-Y\right)\right]}=\frac{1+np\left[\left(1-\nu\right)\left(1-\beta\right)+\nu\alpha\right]}{\left(1+\alpha np\right)\left[1+np\left(1-\beta\right)\right]},$$

where the second equality follows from insertions from (A.5).
The number of crimes per period is

$$XY=\frac{1+np\left[\left(1-\nu\right)\left(1-\beta\right)+\nu\alpha\right]}{\left(1+\alpha np\right)\left[1+np\left(1-\beta\right)\right]}\frac{\nu\left(1+\alpha np\right)}{1+np\left[\left(1-\nu\right)\left(1-\beta\right)+\alpha\nu\right]},$$

which can be written as (2.1); this is part (i) of the Proposition.

The fraction of the population that is innocently in prison is found by inserting from (A.5) into (A.3) to get (2.2); this is part (ii) of the Proposition. □

Proof of Proposition 2.2. To prove Proposition 2.2, we need to establish a condition ensuring that C is convex in n. The challenge is that C consists of two terms: the first one being convex in n; the second one being concave in n. By twice differentiation of (2.4), we get that C is locally convex in n if

$$\frac{h}{k} > \left(\frac{\alpha}{1-\beta}\right)^2 \frac{1-\nu}{\nu} \left(\frac{1+np(1-\beta)}{1+np\alpha}\right)^3. \tag{A.6}$$

The condition in (A.6) shows that the social costs C are convex in n if victims' costs are sufficiently high relative to the social costs of jailing innocent people, putting sufficient weight on the first term in (2.4) to make the total convex in n. Note that there always exist values of $\frac{h}{k}$ that satisfy both condition (A.6) and Assumption 2, as long as Assumption 1 holds.

Proposition 2.2 follows from noting that the interior solution in (2.6) satisfies (A.6) as long as Assumption 2 holds—yielding C to be locally convex at the only stationary point, hence n^* to be an optimum. □

Proof of Proposition 2.3. The optimal sentencing n^*, when positive, is:

(i) increasing in the probability of wrongful acquittals, β, if

$$\frac{4\alpha(1-\beta)}{(\alpha+(1-\beta))^2}\frac{1-\nu}{\nu} < \frac{h}{k} < \frac{1-\beta}{\alpha}\frac{1-\nu}{\nu},$$

and decreasing in β if

$$\frac{\alpha}{1-\beta}\frac{1-\nu}{\nu} < \frac{h}{k} < \frac{4\alpha(1-\beta)}{(\alpha+(1-\beta))^2}\frac{1-\nu}{\nu}.$$

(ii) increasing in the probability of wrongful convictions, α, if

$$\frac{(\alpha+(1-\beta))^2}{4\alpha(1-\beta)}\frac{1-\nu}{\nu} < \frac{h}{k} < \frac{1-\beta}{\alpha}\frac{1-\nu}{\nu},$$

and decreasing in α if

$$\frac{\alpha}{1-\beta}\frac{1-v}{v} < \frac{h}{k} < \frac{(\alpha+(1-\beta))^2}{4\alpha(1-\beta)}\frac{1-v}{v}.$$

Proposition 2.3 follows straightforwardly.

□

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